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by

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ABSTRACT

A device is repaired at failure. With probability p , it is returned to the "good as new" state (perfect repair), with probability $1 - p$ it is returned to the functioning state, but it is only as good as a device of age equal to its age at failure (imperfect repair). Repair takes negligible time. We obtain the distribution F_p of the interval between successive good as new states in terms of the underlying life distribution F . We show that if F is in any of the life distribution classes: IFR, DFR, IFRA, DFRA, NEU, NWU, DMRL, or EMRL, then F_p is in the same class. Finally, we obtain a number of monotonicity properties for various parameters and random variables of the stochastic process. The results obtained are of interest in the context of stochastic processes in general, as well as being useful in the particular imperfect repair model studied.

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Imperfect Repair.

1. Introduction and Summary.

It is well known in practice that repair of a failed item may not yield a functioning item which is "as good as new". In this paper, we treat a model for imperfect repair somewhat related to the model of "minimal repair at failure". See Barlow and Proschan (1965), pp. 96-98. Our results may be of interest in renewal theory as well as in reliability theory.

1.1. Model. An item is repaired at failure. With probability p , it is returned to the "good as new" state (perfect repair), with probability $q = 1 - p$, it is returned to the functioning state, but it is only as good as an item of age equal to its age at failure (imperfect repair). Repair takes negligible time. The process of alternating failure and repair continues indefinitely over time; we call it a failure process.

Suppose the item has underlying life distribution F . To avoid inessential technicalities, we assume the failure rate function $r(t)$ exists.

In Section 2, we obtain the distribution F_p of the interval between successive good-as-new states (regeneration points of the stochastic process). We show that the failure rate function of F_p is simply $p r(t)$. Although the proof is trivial, the result is interesting in two ways:

- (1) It furnishes useful information in the imperfect repair model of 1.1.
- (2) It shows how devices with proportional failure rates or proportional hazard functions may arise in a naturally occurring physical situation.

We show in Section 2 that if F is in any of the classes: IFR, DFR, IFRA, DFRA, NEU, NWU, DMRL, or IMRL, then F_p is in the same class. We are unable to obtain a corresponding preservation result for F NBUE or F NWUE, although we conjecture such a result holds.

In Section 3, we derive a variety of monotonicity properties and inequalities for various parameters and random variables of interest. For example, for F NBUE, the mean time $\mu(p)$ between perfect repairs is bounded above by $\frac{1}{p} \mu(1)$. (Note that $\mu(1)$ is the mean of F.) Under slightly stronger assumptions, $p \mu(p)$ is shown to be increasing in $p \in (0, 1]$. We also show that for F NBU(NWU), the number $N_p(t)$ of failures in $[0, t)$ is stochastically decreasing (increasing) in p . We also obtain monotonicity results for the waiting time until the next failure starting in the steady state of the failure process assuming first F IFR(DFR) and alternatively F DMRL (IMRL). Finally, for F DFR we obtain a variety of monotonicity results for the waiting time until the next failure in the transient state.

The methods used in obtaining the results described above are of interest in their own right. They should be applicable in other stochastic processes.

Also, the imperfect repair model treated in this paper appears to be useful in studying properties of proportional hazard families of distributions.

One final word: The problem of imperfect and even destructive repair is an important practical one in reliability, although few results are available in the statistical literature. We are preparing additional papers studying other aspects of this problem.

2. Preservation of Aging Properties.

To obtain our first result, note that the choice of $p = 0$ results in a nonhomogeneous Poisson process with intensity function $r(t)$. This process has the same distribution as the process of upper record values derived from i.i.d. observations with distribution F . Specifically, let $\{X_i, i \geq 1\}$ be i.i.d. observations from F . Define $Y_1 = X_1$, $N_2 = \{\min i: X_i > X_1\}$, $Y_2 = X_{N_2}$, $N_3 = \{\min i: X_i > Y_2\}$, $Y_3 = X_{N_3}$, etc. Then $\{Y_i, i \geq 1\}$ generates a nonhomogeneous Poisson process with intensity function $r(t)$.

The choice $0 < p < 1$ leads to a process in which the time epochs of perfect repairs are regeneration points. The interval between successive regeneration points is the waiting time for a perfect repair starting with a new component.

Let F_p denote the waiting time distribution for a perfect repair starting with a new component. Let r_p denote its failure rate function. Then we show:

2.1. Lemma. (i) $r_p(t) = pr(t)$. (ii) $F_p(t) = F^p(t)$.

Proof. (i) Given that no perfect repairs occur in $[0, t)$, the item present at time t behaves as an item of age t . Its failure rate is thus $r(t)$. After a failure occurs, repair is perfect with probability p . Thus the conditional intensity of a perfect repair at time t given no perfect repairs in $[0, t)$ is $r_p(t) = pr(t)$.

(ii) Since $F_p(t) = \exp\{-\int_0^t r_p(s)ds\}$, it follows that $F_p(t) = F^p(t)$. ||

Since the original failure rate function is simply multiplied by p , it follows that many of the important classes of distributions characterized by aging properties are preserved in the following sense: If F has a given aging property, then F_p also has this property for $0 < p < 1$. We state this formally in:

2.2. Theorem. Let F be in any of the classes: IFR, DFR, IFRA, DFRA, NBU, NWU, DMRL, or IMRL. Then F_p is in the same class.

Proof. For the first six classes, the conclusion follows directly from $r_p(t) = pr(t)$.

Suppose now F is DMRL. Let Y have distribution F_p . Define $\tau_k(t)$ to be the time of the k^{th} failure after time t . Since for $s < t$, $\tau_k(s) \leq \tau_k(t)$ deterministically, it follows that $\tau_k(t)$ is stochastically increasing in t .

Define $g(t) = E(X - t | X > t)$. Then:

$$(2.1) \quad E(Y - t | Y > t) = g(t) + \sum_{k=1}^{\infty} q^k Eg(\tau_k(t)).$$

Since F is DMRL, g is decreasing. Thus $Eg(\tau_k(t))$ is decreasing. From (2.1) it follows that Y is DMRL.

A similar argument holds for F IMRL. ||

Remark. We do not know whether Theorem 2.2 can be extended to the NBUE and NWUE cases.

3. Monotonicity Properties.

Define $\mu(p) = \int x dF_p(x) = \int x F_p(x) dx$. Theorem 3.1 presents monotonicity properties of $\mu(p)$.

3.1. Theorem. (i) Let F be NBUE. Then $\mu(p) \leq \frac{1}{p} \mu(1)$ for $0 < p \leq 1$.

(ii) Let F_p be NBUE for all p in $(0, 1]$. Then $p \mu(p)$ is increasing for $p \in (0, 1]$. In particular, this monotonicity holds for F NBU or DMRL.

(iii) Dual results hold for the NWUE, NWU, and DMRL classes.

Proof. (i) Let Y have distribution F_p and N denote the number of the failure which results in the first perfect repair. Since F is NBUE, then $E(Y|N = n) \leq n \mu(1)$. Thus $\mu(p) = EY = E(E(Y|N)) \leq \mu(1)EN = \frac{1}{p} \mu(1)$.

(ii) Since $F_{p_1}(t) = [F_{p_2}(t)]^{p_1/p_2}$ and F_{p_2} is NBUE, it follows from (i) that for $p_1 < p_2$, $\mu(p_1) \leq \frac{p_1}{p_2} \mu(p_2)$. Thus $p \mu(p)$ is increasing. If F is NBU (DMRL), then by Lemma 2.1, F_p is also NBU (DMRL), and is thus NBUE.

(iii) The results follow by the arguments in (i) and (ii). ||

The inequality: $p_1 \mu(p_1) \leq p_2 \mu(p_2)$ for $p_1 < p_2$ when F_p is NBUE for all $p \in [0, 1]$ can be interpreted as: F_{p_1} is smaller in expectation than a geometric sum (with parameter p_1/p_2) of i.i.d. random variables having distribution F_{p_2} . When F is NBU, "smaller in expectation" can be strengthened to "stochastically smaller".

3.2. Theorem. Assume that F is NBU and $p_1 < p_2$. Suppose that $Y \sim F_{p_1}$, $\{Z_i, i \geq 1\}$ is an i.i.d. sequence with distribution F_{p_2} , and that N is a geometric random variable with parameter p_1/p_2 , independent of $\{Z_i, i \geq 1\}$. Then Y is stochastically smaller than $\sum_{i=1}^N Z_i$. A dual result holds for F NWU.

Proof. We prove the result for $p_1 = p$, $p_2 = 1$. By the argument in the proof of Th. 3.1(ii) and by the fact that F_p is NBU (Th. 2.2), the desired conclusion will follow in the general case.

First note that $Y = \sum_{i=1}^N X_i$, where X_i denotes the interval between the $(i-1)^{\text{st}}$ and the i^{th} failures, and N is a geometric random variable with parameter p . Since F is NBU, X_i is stochastically smaller than Z_i given $X_1, \dots, X_{i-1}$, $\{N \geq i\}$, for each i . We can therefore construct a version of $(N, X_1, \dots, X_N, Z_1, \dots, Z_N)$ with desired marginal distributions for $(N, X_1, \dots, X_N)$ and $(N, Z_1, \dots, Z_N)$ with $X_i \leq Z_i$, $i = 1, \dots, N$, a.s. Thus $\sum_{i=1}^N X_i \leq \sum_{i=1}^N Z_i$ a.s., from which it follows that $Y = \sum_{i=1}^N X_i$ is stochastically smaller than $\sum_{i=1}^N Z_i$. ||

Let $N_p(t)$ denote the number of failures in $[0, t)$ for the failure process in which perfect repair has probability p . In Corollary 3.4 we prove that for F NBU, $N_p(t)$ is stochastically decreasing in p . First we need a preliminary lemma.

3.3. Lemma. Let F be NBU and $p = 0$. Let $\tau_k(t)$ denote the time of the k^{th} failure after time t . Then $\tau_k(t) - t$ is stochastically smallest when $t = 0$.

Proof. $N_0(t+x) - N_0(t)$ is governed by a Poisson distribution with parameter $\int_t^{t+x} r(s)ds$. Since F is NBU, then $N_0(t+x) - N_0(t)$ is stochastically smallest for $t = 0$. But $P[\tau_k(t) - t > x] = P[N_0(t+x) - N_0(t) \leq k-1]$, which is maximized for $t = 0$. ||

3.4. Corollary. Let F be NBU(NWU). Then $N_p(t)$ is stochastically decreasing (increasing) in p .

Proof. Let $\{N_i, i \geq 1\}$ be i.i.d. geometric random variables with parameter p_2 , $S_j = \sum_{i=1}^j N_i$, $\{N'_i, i \geq 1\}$ be i.i.d. geometric random variables with parameter p_1/p_2 , $S'_j = \sum_{i=1}^j N'_i$, $S_j^* = S_{S'_j}$, and $D_j^* = S_j^* - S_{j-1}^*$. Then $\{D_j^*, j \geq 1\}$ is a sequence of i.i.d. geometric random variables with parameter p_1 .

Next we construct two failure processes as follows. Process 1, with parameter p_1 , has perfect repair of the S_1^{*th} failure, of the S_2^{*th} failure, ...; Process 2, with parameter p_2 , has perfect repair of the S_1^{th} failure, of the S_2^{th} failure, Let $T_i(k)$ denote the time of the k^{th} failure in Process i , $i = 1, 2$. We wish to prove that $T_1(k)$ is stochastically smaller than $T_2(k)$ for all k , from which the desired conclusion immediately follows.

By Theorem 3.2, $T_1(S_j^*)$ is stochastically smaller than $T_2(S_j^*)$ for all $j \geq 1$. Now condition on $\{S_j^*, j \geq 1\}$. Given k , find j such that $S_{j-1}^* \leq k < S_j^*$. Now $T_1(k) = T_1(S_{j-1}^*) + A_1$, where A_1 is distributed as the time until the $(k - S_{j-1}^*)^{th}$ failure in a failure process with $p = 0$.

Also $T_2(k) = T_2(S_{j-1}^*) + A_2$, where A_2 is distributed as the conditional distribution of the time until the $(k - S_{j-1}^*)^{th}$ failure in a failure process with parameter p_2 , given the information as to which, if any, of the $k - S_{j-1}^*$ repairs are perfect. It follows from Lemma 2.1 that A_2 is stochastically larger than A_1 . Since $T_2(S_{j-1}^*)$ is stochastically greater than $T_1(S_{j-1}^*)$, A_1 and $T_1(S_{j-1}^*)$ are independent, and A_2 and $T_2(S_{j-1}^*)$ are independent, the desired conclusion follows. ||

3.5. Theorem. Let F be IFR. Let Z_p denote the waiting time until the next failure, starting in steady state. Let h_p denote the failure rate function of Z_p . Then, (i) for each $t \geq 0$, $h_p(t)$ is decreasing in p ; (ii) as a consequence, Z_p is stochastically increasing in p . (iii) Dual results hold for F DFR.

Proof. (i) The effective age of a component in steady state is the time since the last perfect repair. Since the successive time points of perfect repair form a renewal process with interarrival time distribution F_p , the

steady state age density is $\bar{F}^p(x)/\mu(p)$. Since for $p_1 < p_2$, $\bar{F}^{p_1}(x)/\bar{F}^{p_2}(x)$ is increasing in x , it follows that the family of densities:

$$(3.1) \quad \gamma_p(x) = \frac{\gamma(x)\bar{F}^p(x)}{\int_0^\infty \gamma(x)\bar{F}^p(x)dx}, \quad x \geq 0,$$

has decreasing monotone likelihood ratio in p for each nonnegative function γ satisfying $\int_0^\infty \gamma(x)\bar{F}^p(x)dx < \infty$. Thus the distribution with density γ_p is stochastically decreasing in p , $0 < p \leq 1$.

Next note that:

$$\begin{aligned} h_p(t) &= \frac{\int_0^\infty \frac{\bar{F}^p(x)}{\mu(p)} \frac{\bar{F}(t+x)}{\bar{F}(x)} r(t+x)dx}{\int_0^\infty \frac{\bar{F}^p(x)}{\mu(p)} \frac{\bar{F}(t+x)}{\bar{F}(x)} dx} \\ &= \int_0^\infty r(t+x) \gamma_p(x)dx, \end{aligned}$$

with γ_p defined by (3.1) with $\gamma(x) = \bar{F}(t+x)/\bar{F}(x)$. Since r is increasing and γ_p is stochastically decreasing in p , then $h_p(t)$ is decreasing in p .

(ii) Since $\bar{F}_{Z_p}(s) = e^{-\int_0^s h_p(t)dt}$, it follows that Z_p is stochastically increasing in p .

(iii). The dual results are obtained in a similar fashion. ||

A weaker conclusion is obtained under the weaker assumption of F DMRL(IMRL):

3.6. Corollary. Let F be DMRL(IMRL). Then EZ_p is increasing (decreasing) in p .

Proof. Let F be IMRL. Then $g(t) = E(X - t|X > t)$ is decreasing. We may express:

$$EZ_p = \int \frac{F_p(t)}{u(p)} g(t) dt.$$

Since $\frac{F_p(t)}{u(p)}$ is the density of a distribution stochastically decreasing in p , it follows that EZ_p is increasing in p .

A similar argument yields the result for F IMRL. ||

In all the results above, conclusions were obtained for dual families of distributions corresponding to deterioration with age and improvement with age. The following result applies for DFR distributions, but the dual result is known not to necessarily hold for IFR distributions.

3.7. Theorem. Let F be DFR. Let $Z_p(t)$ denote the waiting time at t for the next failure to occur; let $Z_p^*(t)$ denote the waiting time at t for the next perfect repair. Let $m_p(t)$ denote the failure intensity at t and let $m_p^*(t)$ denote the renewal density at t for the renewal process with interarrival time distribution F_p . Finally, let $A_p(t)$ denote the effective age at time t , i.e., the time elapsed since the last perfect repair. Then:

(i) $A_p(t)$, $Z_p(t)$, and $Z_p^*(t)$ are stochastically increasing in t for fixed p ; $m_p(t)$ and $m_p^*(t)$ are decreasing in t for fixed p .

(ii) $A_p(t)$ and $Z_p(t)$ are stochastically decreasing in p for fixed t , $m_p(t)$ is increasing in p for fixed t .

Proof. (i) Since F_p is DFR, it follows from Brown (1980), Theorem 3, that $A_p(t)$ and $Z_p^*(t)$ are stochastically increasing in t and that $m_p^*(t)$ is decreasing in t . Since $P[Z_p(t) > x] = \int \frac{F(x+y)}{F(y)} dF_{A_p}(y) \frac{\bar{F}(x+y)}{\bar{F}(y)}$ is increasing in y , and $F_{A_p}(t)$ is stochastically increasing in t , it follows that $P[Z_p(t) > x]$ is increasing in t . Finally, $m_p(t) = \frac{1}{p} m_p^*(t)$, so that $m_p(t)$ is decreasing in t .

(ii) For $p_1 < p_2$, $r_{p_2}(t) = p_2 r(t) \geq p_1 r(t) = \sup_{s \geq t} r_{p_1}(s)$. It follows from Theorem 1 of Brown (1980) that we can construct a random variable C with distribution F_{p_1} , a sequence $\{B_i, i \geq 1\}$ of i.i.d. random variables with distribution F_{p_2} , and a stopping time N such that $C = \sum_{i=1}^N B_i$ a.s. By employing this construction, we generate a bivariate failure process with marginals which are failure processes with parameters p_1, p_2 , and for which every p_1 perfect repair time epoch is a p_2 perfect repair time epoch. Thus under this construction, $A_{p_1}(t) \geq A_{p_2}(t)$ deterministically. It follows that $A_p(t)$ is stochastically decreasing in p for fixed t . Since $P[Z_p(t) > x] = \int \frac{\bar{F}(x+y)}{\bar{F}(y)} dF_{A_p(t)}(y)$, it follows that $Z_p(t)$ is stochastically decreasing in p for fixed t . Since $m_p(t) = \text{Er}(A_p(t))$, r is decreasing and $A_p(t)$ is stochastically decreasing in p , it follows that $m_p(t)$ is increasing in p . ||

REFERENCES

Richard E. Barlow and Frank Proschan (1965). Mathematical Theory of Reliability. Wiley, New York.

Mark Brown (1980). Bounds, inequalities, and monotonicity properties for specialized renewal processes. Ann. Probability 8.

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