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Optical Turbulence Forecasting: A Tutorial

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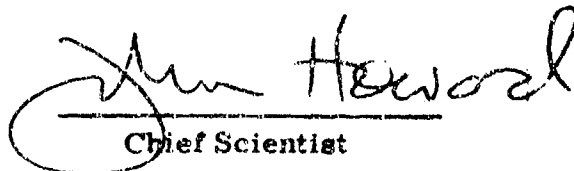
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>This report is a tutorial on the effects of atmospheric turbulence upon systems which rely upon the propagation of LASER beams. In addition to providing a simplified presentation of turbulence theory and optical effects, it describes the state of the art of the new technique of radiosonde estimation of index of refraction fluctuations. Suggestions are given for future research which will help to answer current Air Force needs. The feasibility of some laser systems will depend upon the value of $r_0$ the coherence length (which |                                                                                      |                                             |

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20. Abstract (Continued)

is related to  $C_n^2$  which in turn is related to the degree of turbulence). At present, the statistics of  $C_n^2$  are inadequate.

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## Summary

The Air Force has a need for better estimates of a parameter known as " $C_n^2$ ". It also needs to know how it ( $C_n^2$ ) varies with geographic location, season, time of day and weather conditions. This need arises in connection with optical (LASER) communication, navigation and weapon systems applications.  $C_n^2$  is related to turbulence-induced, index of refraction variations in the atmosphere.

The main purposes of this report are to explain the physical nature of  $C_n^2$ , to describe the current state of the art in this area, and to make suggestions for future research.

Section 2 shows how the parameter  $C_n^2$  arises in atmospheric turbulence. Quantitatively,  $C_n^2 = (\theta')^2 / l_o^{2/3}$  where  $(\theta')^2$  is the mean square variation of the index of refraction (these fluctuations being caused by atmospheric turbulence mixing effects) and  $l_o$  is the approximate size of the largest eddies. It will be explained that the  $C_n^2$  parameter can only be employed in a certain domain of turbulence scales (eddy sizes), namely the "inertial" range in which there is a "cascade" of energy flow from the large eddy scale sizes to the small ones where the viscous dissipation occurs. The so-called "structure function" of the index of refraction,  $\mathcal{D}_n(r)$ , (defined as the mean square difference of index of refraction measured at two places a distance  $r$  apart) is equal to  $C_n^2 r^{2/3}$  in the inertial range. Section 2 displays both an heuristic and a mathematical treatment of the turbulence concepts which make up the context of  $C_n^2$  (the mathematical treatment being given in an appendix).

Section 3 is devoted to the display of examples of how  $C_n^2$  affects optical systems. The turbulent eddies can be described as a random array of lenses or blobs

of air with index of refraction differing slightly from their surroundings. It is shown that the  $C_n^2$  effects on electromagnetic propagation can be fairly accurately explained on this simple basis. The two main effects are phase modulations ( and their associated wavefront distortions), and "scintillation" which is the random fading in and out of the signal. The "twinkling" of stars is precisely this effect (also known to astronomers as "bad seeing" conditions). Scintillation can be explained as being due to the focusing and defocusing effects of the "blob lenses" already mentioned.

Section 4 describes the current state of the art of estimating  $C_n^2$  by means of standard "weather balloon" (that is, rawinsonde-radiosonde) observations. The objective of such research is to convert the entire data-bank of such observations into a comprehensive set of estimates of  $C_n^2$  over a wide range of spatial locations and times. This would make possible, presumably, climatological and forecasting capabilities for  $C_n^2$ .

Section 5 considers the problems to be solved by future research, and possible techniques to consider. In essence, the weakest part of the current analysis for radiosonde measurements concerns the lowest altitudes (6 km to ground). More experimental investigation is urgently needed here. Another question, the most crucial in fact, is whether or not there exists a significant difference in  $C_n^2$  as a function of location and season. "Anecdotally," the placement of a telescope site for optimum "seeing" conditions would seem to indicate that  $C_n^2$  does depend on location; but, at this writing the question does not seem to be answered to everyone's satisfaction. In contrast, it has already been established that there is a significant diurnal effect on  $C_n^2$ . For the purpose of making low altitude  $C_n^2$  measurements by radar, it will be necessary to make use of a powerful radar system equipped with a steerable dish (Lincoln Labs, for example, has such a piece of equipment). In addition, the "microshears" which cause the  $C_n^2$ -producing turbulence can be measured by in-house generated technology involving smoke trails. The latter type of measurements would be useful for verifying current  $C_n^2$  models for all altitudes. Balloon-borne instrumentation techniques (also generated in-house) should also be very useful.

The report concludes with an annotated bibliography for those who may wish to read further on this subject.

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## Optical Turbulence Forecasting: A Tutorial

### 1. INTRODUCTION

Astronomers have long known that atmospheric turbulence can cause optical propagation effects. The famous "twinkling" of stars is due to such turbulence. When turbulence is at a minimum, astronomers can photograph planets etc., with a minimum of blur, because with less turbulence there is less so-called "image dancing." Brightness fluctuations, or scintillations, also diminish when there is low turbulence. In general, such conditions are called "good seeing conditions." The basic cause of the optical effects is the presence of fluctuations in the index of refraction along the optical path. These fluctuations are caused by temperature fluctuations and, in the case of microwave frequencies, fluctuations of humidity. The inhomogeneous nature of such quantities is due to the (incomplete) mixing effects of turbulence. In a homogeneous or "already totally mixed" region of atmosphere, turbulence will have no optical effect. So, variations of index of refraction can be caused by turbulence; but there can be (in principle) turbulence without index of refraction variations. The time rate of optical modulations due to the fluctuations depends both on the sizes of the eddies (or inhomogeneities) and on the mean motion of air which carries these fluctuations across the optical path.

The Air Force is currently involved with communication and navigation systems which use laser beams to carry signals. Such systems can involve

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ground-to-space and space-to-ground propagation as well as air-to-space, ground-to-ground, etc., links as well. The capabilities of such systems could be significantly affected by the turbulence-induced fluctuations mentioned above. In addition, photography from high altitudes can also be influenced by these effects as well as any weapon system which depends upon the propagation of a laser beam. Refraction fluctuations are not bad for all Air Force propagation systems, however. For example, "over the horizon" or "troposcatter" communication systems cannot function at all unless the fluctuations are present. For these reasons it would be desirable to have some way to forecast those conditions which give rise to the fluctuations. Two important and interesting questions are whether there are geographic, diurnal, seasonal, or weather conditions which are especially conducive to fluctuations (or to their absence) and whether the current, large data base of radiosonde measurements can be tied to the value of  $C_n^2$  (a measure of index of refraction fluctuations) in such a way that it can be used to investigate the dependence of  $C_n^2$  upon other measurable — perhaps predictable — parameters. One purpose of this report is to examine the current state of the art along these lines. In particular, we shall examine the best methods available to relate radiosonde estimates of  $C_n^2$  as a function of altitude and time to simultaneous colocated radar measurements.

It is hoped that a non-expert in this field will be able to see from this report how  $C_n^2$  is related to turbulence-induced index of refraction fluctuations and how the latter can affect system performance. Also he may see, hopefully, how one may someday succeed in connecting rawinsonde measurements to the relative performance of such systems.

## 2. $C_n^2$ AND TURBULENCE-INDUCED INDEX OF REFRACTION FLUCTUATIONS

In Section 3 we shall consider the connection between index of refraction fluctuations, as parameterized by  $C_n^2$ , and optical system performance. It will be shown that, in many ways, the turbulence-induced inhomogeneities can act like a random assembly of lenses. The signal amplitude changes, for example, can be explained in terms of focusing and defocusing effects due to these lenses as they are convected across the optical path. In the present section, however, we shall concentrate on the physics behind the parameter  $C_n^2$  which will appear in all equations describing the effects of turbulence on electromagnetic propagation. Our starting point is the concept of the "structure function."

## 2.1 The Structure Function

The index of refraction depends exclusively upon temperature for some electromagnetic frequencies (that is, for optical frequencies) whereas for radar frequencies it depends on both temperature and specific humidity. For simplicity we shall initially ignore this double dependence and imagine the case where only the temperature\* is important (the dependence upon water vapor will be treated at the end of Section 2). For generality we can start by considering an arbitrary passive scalar and call it " $\theta$ ." The structure function is defined, not in terms of  $\theta$ , but in terms of  $\theta'$ , that is, the "fluctuation" of  $\theta$ , which is defined by

$$\theta = \bar{\theta} + \theta' \quad (1)$$

Thus  $\theta'$  is the deviation from the mean of  $\theta$  (denoted by  $\bar{\theta}$ ). The structure function,  $\mathcal{D}_\theta(\vec{r})$  for  $\theta$  is thus

$$\mathcal{D}_\theta(\vec{r}) = \langle [\theta'(\vec{r}_1 + \vec{r}) - \theta'(\vec{r}_1)]^2 \rangle \quad (2)$$

The angular brackets  $\langle \rangle$  signify an ensemble average but in practical terms it may in fact be a time average. The vector  $\vec{r}$  is the distance between the two locations of the measurements:  $\vec{r}_1 + \vec{r}$  and  $\vec{r}_1$ . When the turbulence is isotropic and homogeneous,  $\mathcal{D}_\theta(r)$  will depend neither upon location  $r_1$  or the direction of  $\vec{r}$ . We shall consider only such simplified turbulence from now on. Fortunately this is a good approximation for our purposes. For this reason,  $\vec{r}$  will be represented as a scalar.

During the development of turbulence theory by such innovators as G.I. Taylor, work was done with the so-called autocorrelation function,  $B_\theta(r)$ , which is defined by

$$B_\theta(r) = \langle \theta'(\vec{r}_1 + r) \theta'(\vec{r}_1) \rangle \quad (3)$$

The quantities  $B_\theta(r)$ , and  $\mathcal{D}_\theta(r)$  are closely related. In particular (Tatarski, <sup>1</sup> p. 10)

$$\frac{1}{2} \mathcal{D}_\theta(r) = [B_\theta(0) - B_\theta(r)] \quad (4)$$

1. Tatarski, V.I. (1961) Wave Propagation in a Turbulent Medium, McGraw-Hill, N.Y.

\*Temperature in the following will always be considered as a passive-scalar additive to the fluid even though in actual fact it is not. The approximation, however, in the cases of present interest, is a good one.

The structure function, a Russian development, has now acquired universal usage because of certain practical advantages it affords. Whereas  $B_\theta(r)$  is advantageously used in connection with turbulence theory (due to its very simple connection with the spatial frequency power spectra which are intrinsic to much of that theory) the structure function has the following experimental advantages (Panchev,<sup>2</sup> p. 69). The fluctuation measurements are almost always associated with "slow variations" or "trends." The structure constant, being a first difference, automatically filters out these "annoyances." It is also less sensitive to error due to random deviations. It also provides advantages for instrumental design in turbulence measurements. Finally,  $\mathcal{D}_\theta(r)$  has the advantage that, theoretically, it has a beautifully simple form when one is dealing with "inertial range" or "cascade" turbulence. It is precisely this simplicity which gave rise to the concept of  $C_n^2$ . We shall, in the present context call it  $C_\theta^2$ , and this more general term is called the structure constant (or rather  $C_\theta$ , not squared, is the structure constant). As will be shown below for inertial range turbulence,

$$\mathcal{D}_\theta(r) = C_\theta^2 r^{2/3} \quad , \quad (5)$$

Equation (5) was the original form given by Kolmogorov; however, the one-dimensional spectrum which is associated with Eq. (5) depends on the spatial frequency,  $k$  (or  $(2\pi)/\text{wavelength}$ ) as  $k^{-5/3}$ . One usually hears of "Kolmogorov's minus five-thirds law"; in truth he discovered the "two-thirds law" of Eq. (5).

The relationship between  $\mathcal{D}_\theta(r)$  and  $B_\theta(r)$  as well as the variance,  $\sigma_\theta^2$ , is graphically illustrated in Figure 1 (cf. Panchev,<sup>2</sup> p. 69).

Now that  $\mathcal{D}_\theta(r)$  has been defined we can now travel on a path through turbulence theory which leads to a rather physical derivation of Eq. (5) (see Appendix A for more detail). Only "cascade" turbulence will be considered.

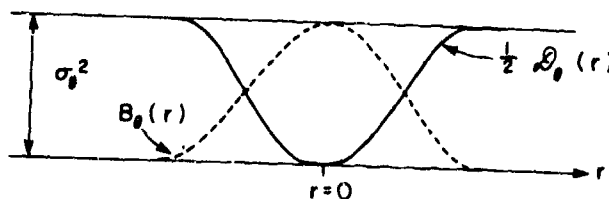


Figure 1. Relation Between Structure Function,  $\mathcal{D}_\theta(r)$ , Autocorrelation,  $B_\theta(r)$ , and Variance,  $\sigma_\theta^2$

2. Panchev, S. (1971) Random Functions and Turbulence, Pergamon Press, N.Y.

## 2.2 The Inertial Range and $D_\theta(r) \propto C_\theta^2 r^{2/3}$

In the following we shall consider a simplified form of turbulence theory. The main objectives are to derive the structure function from cascade theory and to relate the structure constant to the energy dissipation rate of the turbulence\*,  $\epsilon$ , and the "dissipation rate,"  $\epsilon_N$ , related to the molecular smoothing out of inhomogeneities so that, at the end, we can arrive at the relation for  $C_n^2$  used by VanZandt et al.<sup>3</sup> in their work on radiosonde estimation of  $C_n^2$ . Our treatment mathematically follows Tatarski<sup>1</sup> but the physical discussions are added.

For the sake of definiteness we shall replace the meaning of  $\theta$ , which has been "passive scalar" up to this point, with potential temperature.<sup>†</sup> In this way  $\theta$  will, in the case where humidity is not important, be closely related to index of refraction. The exact connection is given at the end of Section 2.

Next, we make certain simplifying assumptions. In addition to homogeneity and isotropy of turbulence mentioned above, we introduce "steady state," that is, we assume that as much energy flows into the turbulence as is dissipated in the form of heat (due to viscous dissipation). Furthermore we shall resort to the famous "cascade" picture, first introduced by Richardson in the form of a poem and then put effectively into mathematical form by Kolmogorov (and subsequently but independently by Onsager and von Weizsäcker). In the so-called "inertial range" the large eddies break down into smaller eddies, due to fluid dynamic instabilities, and these smaller eddies likewise break down into still smaller ones. This "cascade of energy" from large sizes or "scales" to smaller scales continues until the scale becomes so small that viscosity effects become important, at which point the energy flows out of the "cascade" and into the "sink" of molecular heat. As Richardson said:

"Big whirls have little whirls,  
Which feed on their velocity;  
And little whirls have lesser whirls,  
And so on to viscosity (in the molecular sense)."

The simplest derivation of the structure function in this inertial range can now be given provided one knows that, in addition to  $r$ , only two other parameters are of any physical relevance, namely  $\epsilon$  and  $\epsilon_N$  mentioned above. This is true

\*  $\epsilon$  represents heating due to turbulence and  $\epsilon_N$  represents molecular smoothing (diffusion) of turbulent inhomogeneities.

† Potential temperature is used instead of temperature in a compressible gas in order to remove the complication that a drop in pressure decreases the temperature; the potential temperature in this case would remain constant.

3. VanZandt, T.E., Green, J.L., Gage, K.S., and Clark, W.L. (1978) Vertical profiles of refractivity turbulence structure constant: Comparison of observations by the Sunset Radar with a new theoretical model, Radio Science 13:818-829.

because we have a steady state and the cascade must depend only on the rates of dissipation (heat =  $\epsilon$ , smoothing =  $\epsilon_N$ ) at the small end. Once this is granted, one can write

$$\mathcal{D}_\theta(r) = \frac{\alpha \epsilon_N}{\epsilon^{1/3}} r^{2/3} \quad (6)$$

This follows from dimensional analysis. The symbol  $\alpha$  is for a constant of order unity. We shall start again, however, from the "equations of motion" and rederive Eq. (6) with a little more detail in order to gain more insight into its meaning and to obtain information which will enable us to put Eq. (6) into a form more amenable to experimental requirements.

The starting point is the molecular diffusion equation. This is, in fact, the "equation of motion" for the passive scalar (potential temperature in our case),  $\theta$ .

$$\frac{d\theta}{dt} + \text{div}(-\nu_\theta \text{grad } \theta) = 0, \quad (7)$$

where  $\nu_\theta$  is the molecular coefficient of diffusion for  $\theta$ , and where

$$\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + \vec{v} \cdot \text{grad } \theta, \quad (8)$$

that is, the "mobile derivative" and  $\vec{v}$  the velocity,  $\text{div}$  is the divergence, and  $\text{grad}$  the gradient. Since it turns out that the dynamics will not be significantly affected by compressibility, we make the usual assumption that the air can be considered incompressible in this context. Mathematically, then, we put

$$\text{div}(\vec{v}) = 0 \quad (9)$$

and this makes it possible to write

$$\vec{v} \cdot \text{grad}(\theta) = \text{div}(\vec{v}\theta) \quad (10)$$

Thus Eq. (7) becomes

$$\frac{\partial \theta}{\partial t} + \text{div}(\vec{v}\theta - \nu_\theta \text{grad } \theta) = 0 \quad (11)$$

and this is the basic equation. At this point we again consider fluctuations. Reynolds was the first to introduce the "decomposition" of a variable into its mean and fluctuating part. We do this for both  $\theta$  and  $v_i$  where  $v_i$  is the  $i^{\text{th}}$  component of the velocity vector,  $\vec{v}$ .

$$\theta = \bar{\theta} + \theta' \quad (12)$$

$$v_i = \bar{v}_i + v'_i$$

where, as before, the overbar is an average. In Appendix A it is shown how the insertion of Eq. (12) into Eq. (11) and subsequent multiplication by  $\theta'$  and averaging etc. lead to a relation between  $\langle \theta'^2 \rangle$  (that is, the mean square fluctuations of  $\theta'$ ) and  $\epsilon_N$ . The relation

$$K_\theta (\text{grad } \bar{\theta})^2 = \nu_\theta \langle (\text{grad } \theta')^2 \rangle \quad (13)$$

is the result from such manipulations. The angular brackets are an average, and  $K_\theta$  is the so-called eddy diffusivity to be discussed and defined below. The right side of Eq. (13) represents the rate that the average fluctuation or inhomogeneity,  $\overline{\theta'^2}$ , is wiped out per unit of time by molecular diffusivity,  $\nu_\theta$ . The right side defines  $\epsilon_N$ . The left side, which we shall call P, represents the "production term," that is, the rate of production of the fluctuations per unit of time.

$$P \equiv K_\theta (\text{grad } \bar{\theta})^2 \quad (14)$$

The eddy diffusivity,  $K_\theta$ , can be approached in several ways. Its dimensions are  $[L^2][T^{-1}]$ . It could be written as

$$K_\theta = v' l' \quad (15)$$

that is, as a product of a turbulent velocity scale and a turbulent length scale. This is its basic physical meaning.<sup>4</sup> However, this leaves the value of  $l'$  unknown and for present purposes we can solve this problem by defining  $K_\theta$  by

$$K_\theta = - \langle \theta' \bar{v}' \rangle / (\text{grad } \bar{\theta}) \quad (16)$$

While it is clear that the dimensions of  $K_\theta$  are correct,  $(v' \cdot l')$ , and that  $l'$  is appropriately defined in terms of a length relating to  $\bar{\theta}$  variation, it is not obvious yet how one arrives at Eq. (16) directly. The best way to derive Eq. (16) is to make use of an analogy with Fourier's law for heat flow. Calling the flux of temperature flow  $q_\theta$ , we have

4. Pasquill, F. (1974) Atmospheric Diffusion (2nd Ed.), John Wiley & Sons, New York.

$$\bar{q}_\theta = -K_\theta \overline{\text{grad } \theta} \quad (17)$$

But the turbulent flux  $\bar{q}_\theta$  is just the flow of the fluctuations of  $\theta'$  as carried by the velocity fluctuations  $\bar{v}'$ . This in turn is given by

$$\bar{q}_\theta = \langle \bar{v}' \theta' \rangle \quad (18)$$

which physically means that the flux is due to the correlation between velocity and potential temperature fluctuations. Substitution of Eq. (18) into Eq. (17) (that is, eliminating  $\bar{q}_\theta$ ) one arrives at Eq. (16) directly.

At this point we wish to note a certain analogy. Eq. (14) defines the productivity of turbulent fluctuations while, as we have already stated,

$$\epsilon_N = \nu_\theta \langle (\text{grad } \theta')^2 \rangle \quad (19)$$

Notice the similarity in form for  $P$  and  $\epsilon_N$ . We shall make use of this shortly.

Next we shall consider the physical meaning of  $P$ . After all, we did not derive Eq. (13) here explicitly (see Appendix A), therefore we resort to "understanding" it from its physical interpretation. To do this we start by making a simplification which turns out to be the usual and justifiable one for the atmosphere, namely that the mean value of  $\theta$ , that is,  $\bar{\theta}$ , depends only upon the altitude,  $z$ . In this way we can write

$$P = \langle \theta' v'_z \rangle \frac{d\bar{\theta}}{dz} \quad (20)$$

Now we shall examine why  $P$  is physically the rate of production of  $\overline{\theta'^2}$  per unit time. Figure 2 makes possible a very simple explanation of this. In this figure, we imagine a parcel of air originally located at altitude  $z_1$  and let it be transported (by an eddy, say) to a higher altitude  $z_2$  which is at a height  $\Delta z$  above  $z_1$ . We shall assume that this parcel originally had the average temperature  $\bar{\theta}$  at  $z_1$ . At its destination  $z_2$ , it still has this potential temperature (because potential temperature does not change as the parcel expands to a larger volume at the higher altitude and because we assume that the parcel keeps its identity, that is, it does not mix). When the parcel is at  $z_2$ , however, its temperature now differs from the mean,  $\bar{\theta}$ , at this new altitude. This difference is given in Figure 2 and we call it  $\Delta\theta$ .  $\Delta\theta$  is, in effect, the value of  $\theta'$  for the parcel. Notice the important fact that the gradient,  $d\bar{\theta}/dz$ , is necessary for the development of  $\Delta\theta$ . Mathematically

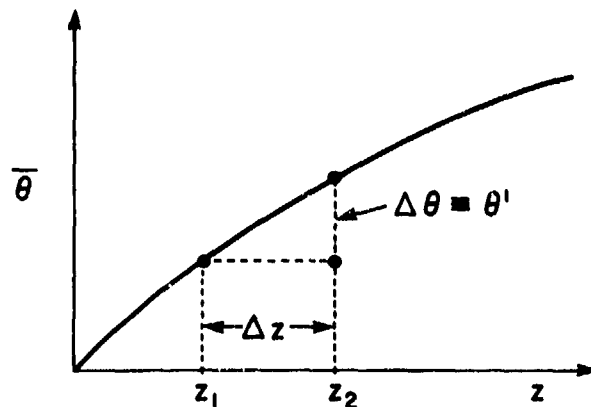


Figure 2. Generation of Fluctuation,  $\theta'$ , By Vertical Transport

$$\theta' = \frac{d\bar{\theta}}{dz} \cdot \Delta z \quad (21)$$

Now multiply Eq. (21) by  $\theta'$  and perform an average,  $\langle \rangle$ , to obtain

$$\langle \theta'^2 \rangle = \langle \theta' \Delta z \rangle \frac{d\bar{\theta}}{dz} \quad (22)$$

In order to estimate the rate of change of  $\langle \theta'^2 \rangle$  with respect to time, we divide Eq. (22) by  $\Delta t$  and identify  $\Delta z / \Delta t$  with the  $z$  component of the eddy velocity fluctuations,  $v'_z$ , to obtain

$$P \equiv \frac{\langle \theta'^2 \rangle}{\Delta t} = \langle \theta' v'_z \rangle \frac{d\bar{\theta}}{dz} \quad (23)$$

and this is precisely Eq. (20). Using the definition for  $K_\theta$  given by Eq. (16), we arrive at Eq. (14).

It is worthwhile to restate the physical meaning of  $P$ . It represents a conversion of the gradient of  $\bar{\theta}$  (a large scale variation) into the fluctuations  $\theta'$  (a small scale variation) of the turbulence. It is brought about in part by the turbulent fluctuations in velocity  $\vec{v}'$ . At the smallest end of the cascade, the fluctuations  $\langle \theta'^2 \rangle$  are smoothed out by molecular diffusion. The latter occurs at the rate  $\epsilon_N$ . The assumption of "steady state" is nothing other than the requirement given by

$$P = \epsilon_N \quad (24)$$

Now consider Eq. (19) and let us compare it to P. Compare, in other words,

$$P = K_{\theta} \left( \frac{d\bar{\theta}}{dz} \right)^2 \quad (25)$$

with

$$\epsilon_N = \nu_{\theta} \langle (\text{grad } \theta')^2 \rangle \quad (26)$$

These are of exactly the same form. In a manner similar to P,  $\epsilon_N$  can be viewed as a transition between a relatively large scale of variation of  $\theta$  to a small scale. The big difference is that  $\nu_{\theta}$  relates to the conversion to molecular variation. Thus  $K_{\theta}$  parameterizes the conversion from mean variation to eddy variation and  $\nu_{\theta}$  does the same for conversion from eddy fluctuations to molecular fluctuations.\* At this point the physical meaning of Eq. (13) should be in evidence.

We now turn to the next step in deriving the structure function. This consists on making an estimate of  $\langle \theta'^2 \rangle$  in terms of  $\epsilon_N$ ,  $\epsilon$ , and the scale of turbulence or size of the eddies which we shall here call  $l$ . To start this we make a simple estimate of the quantity  $\epsilon_N$ . To do this, consider Eq. (26) and approximate  $\langle (\text{grad } \theta')^2 \rangle$  by  $\{\langle \theta'^2 \rangle / (l)^2\}$ . Thus

$$\epsilon_N = \frac{\langle \theta'^2 \rangle \nu_{\theta}}{l^2} \quad (27)$$

Physically we imagine  $\theta'$  as the fluctuation of  $\theta$  across the eddy of size  $l$ . Quantity  $l$  here is defined as the "representative scale" of the turbulence. In order to have a cascade of the type discussed above, however, the molecular dissipation rate of the largest eddies must be very much smaller than the rate that they give up their fluctuations to the next smaller size of eddies ("big blobs become smaller blobs"). Consider an eddy of scale  $l$ . It converts itself into a group of smaller blobs in a time  $t_l$ . It is known in turbulence theory that this time  $t_l$  is very short. In fact, it is one "turn over time" of the eddy. Let  $v_l$  be the internal velocity associated with the eddy ( $v_l$  is the same thing as the velocity fluctuation,  $v'$ , across  $l$ ). Then

$$t_l \equiv \frac{l}{v_l} \quad (28)$$

---

\* Of course,  $\theta$  must be interpreted as a "passive scalar" in this context since temperature is meaningless at the molecular level.

Now in order that a cascade exist, it is required that

$$\epsilon_N \ll \frac{\langle \theta'^2 \rangle}{t_1} = \frac{\langle \theta'^2 \rangle v_l}{l} \quad (29)$$

but from Eq. (27) and Eq. (15) we see that Eq. (29), written as

$$\epsilon_N = \frac{\langle \theta'^2 \rangle}{l^2} \nu_\theta \ll \frac{\langle \theta'^2 \rangle}{l^2} v_l l = \frac{\langle \theta'^2 \rangle K_\theta}{l^2} \quad (30)$$

which implies that for a cascade the condition is

$$K_\theta \gg \nu_\theta \quad (31)$$

In other words, the big eddies must break up at a much faster rate than the action of molecular effects upon them. Alternatively, diffusion due to turbulence is much greater than that due to molecular transport. At the small scale end of the cascade, where the scales are so small that molecular effects are relatively large, we have the condition

$$v_1 l_1 \sim \nu_\theta \quad (32)$$

where  $v_1$  and  $l_1$  are the velocity and spatial scales of the dissipating small scale eddies. This defines the "dissipation scale,"  $l_1$  of the cascade.

We need another relationship for  $\epsilon_N$  because we wish to relate it to  $\epsilon$ . To do this we use the steady state condition, Eq. (24), or

$$\epsilon_N = P = \langle \theta' v' \rangle \frac{d\bar{\theta}}{dz} \quad (33)$$

and make the approximation that  $(\theta' \sim l(d\bar{\theta}/dz))$ . In this way

$$\epsilon_N \sim \frac{\langle \theta'^2 \rangle v_l}{l} \quad (34)$$

Now, recall the definition of  $\epsilon$ . It is the rate that the kinetic energy per unit mass of the velocity fluctuations  $\langle v'^2 \rangle$  becomes converted into heat through molecular dissipation effects. We need an estimate for  $\epsilon$  in terms of  $v_l$  in order to eliminate the latter from Eq. (34). The physical-dimensional argument goes as follows. The time scale for the conversion of an eddy into a number of smaller ones, that is, the time scale for an eddy to give up its energy, is  $(l/v_l)$  as has already been

explained. Suppose now that all of the kinetic energy of the large eddies of size  $l$  is transferred down to the next smaller size of eddies in this time; then, this would represent (in steady state) the rate of energy flow (per unit mass) into the top end of the cascade of  $(v_l^2)$  per interval of time  $(l/v_l)$ . Ultimately, in analogy with  $P = \epsilon_N$ , this must equal  $\epsilon$ . Thus,

$$\epsilon = \frac{v_l^2}{l/v_l} = \frac{v_l^3}{l} = \frac{v_1^3}{l_1} \quad * \quad (35)$$

This equation for  $\epsilon$ , in terms of the scales  $v_l$  and  $l$ , as simple as it may appear and as "dimensional" and non-rigorous as it may seem at first sight, is one of the "corner stone" assumptions of turbulence theory.<sup>5</sup> It is due to the validity of Eq. (35) that one can say turbulence is a "strongly damped, non-linear stochastic system." The dimensional arguments which are characteristic of our treatment up to this point are also typical of much of turbulence theory (due to the difficulties introduced by non-linearity). If they appear repugnant to the reader who may prefer exact analytic approaches, it should be pointed out that it is precisely these "similarity" techniques which can claim a large part of the useful practical aspects of turbulence theory (taken, of course, in conjunction with experimental measurements).<sup>†</sup>

Now using Eq. (35) to eliminate  $v_l$  from Eq. (34)

$$\epsilon_N = \frac{\langle \theta_l'^2 \rangle (\epsilon l)^{1/3}}{l} \quad (36)$$

and solving for  $\langle \theta_l'^2 \rangle$  we have

$$\langle \theta_l'^2 \rangle = \frac{\epsilon_N l^{2/3}}{\epsilon^{1/3}} \quad (37)$$

Eq. (37) is, in effect, an estimation of the structure function for the special case where  $r = l$ . For smaller  $r$ , the structure function must be

\*  $v_1$  and  $l_1$  refer to dissipation scales to be explained.

† The analytic approach has been worked out for the inertial range. See Appendix B.

5. Tennekes, H., and Lumley, J. L. (1972) A First Course in Turbulence, MIT Press, Cambridge, MA.

$$\mathcal{D}_\theta(r) = a \left( \frac{\epsilon_N}{\epsilon^{1/3}} \right) r^{2/3} \quad (38)$$

where  $a$  is a constant of order unity. The range of validity of Eq. (38) is confined to the cascade range. Denoting  $l_0$  as the size of the largest eddies or "outer length," or the size of the "energy-containing eddies," or the so-called "integral scale",<sup>6</sup> and denoting the small-scale end of the cascade by  $l_1$  (cf. Eq. (32)), we can denote the range of validity of Eq. (38) by

$$(l_1 \ll r \ll l_0)$$

and using  $v_l$  from Eq. (35) in Eq. (32) we can solve for  $l_1$  to obtain

$$l_1 = \left[ \frac{v_\theta^3}{\epsilon} \right]^{1/4} \quad (39)$$

This length,  $l_1$ , is called the "inner length" or "microscale."

Eq. (38) is also written as

$$\mathcal{D}_\theta(r) = C_\theta^2 r^{2/3} \quad (40)$$

where  $C_\theta$  is called the structure constant as mentioned in Section 2, thus we have the relation (see Eq. (38))

$$C_\theta^2 = \frac{a \epsilon_N}{\epsilon^{1/3}} \quad (41)$$

The quantity  $C_\theta^2$  is an object which very closely resembles the quantity  $C_n^2$  as we shall see.

What is the physical interpretation of  $C_\theta^2$ ? From Eqs. (37) and (41) we find

$$C_\theta^2 = \frac{\langle \theta'^2 \rangle}{l^{2/3}} \quad (42)$$

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6. Hinze, J.O. (1975) Turbulence (2nd Ed.), McGraw Hill.

Thus,  $C_\theta^2$  is the ratio of the mean square fluctuations of  $\theta$  to the  $2/3$  power of the dimension  $l$  of the eddies. In Eq. (42) the term  $l$  means the representative length scale of the eddies. Thus we can also write

$$\mathcal{D}_\theta(r) \sim \langle \theta'^2 \rangle \left( \frac{r}{l} \right)^{2/3} \quad (43)$$

where  $\langle \theta'^2 \rangle$  corresponds to the variations over a distance  $l$  of the scale of turbulence. When  $r = l$ ,  $\mathcal{D}_\theta = \langle \theta'^2 \rangle$ .

For completeness we add the form for  $\mathcal{D}_\theta(r)$  which is valid for the case where  $r \ll l_1$ . It is (Tatarski, <sup>1</sup> p. 46)

$$\mathcal{D}_\theta(r) = C_\theta^2 l_1^{2/3} \left( \frac{r}{l_1} \right)^2. \quad (44)$$

In order to illustrate the experimental significance of the structure function and the structure constant we consider a specific example. Consider a device which measures temperature at two points separated by a fixed horizontal distance  $r'$ . (We shall let  $\theta$  represent ordinary temperature.) Assume that  $r'$  is chosen so that it satisfies  $l_1 \ll r' \ll l_0$  for the case of interest. Suppose further that this device (a pair of temperature-measuring instruments in effect) is attached to a balloon and sent aloft to measure the  $\mathcal{D}_\theta(r)$  of various altitude regions of the atmosphere. The first question we wish to answer is: "How is the average in Eq. (2) actually performed, the latter equation giving the definition of  $\mathcal{D}(r)$ ?" More specifically we wish to know whether it is a spatial average or an average over time. The answer is that it is a time average; but, since in general such measurements must be made from a location relatively far below the balloon (to avoid balloon-induced artifacts), there will be a relative wind velocity between the measuring device and the air which immediately surrounds it. This would be due to the fact that in general there are wind shears present and while the balloon might be nearly at rest with respect to its surroundings, the gondola below would be expected to be moving with respect to its environment. Very often in such situations the so-called "frozen turbulence" hypothesis is used. This means that the measuring device traverses the turbulent field with enough velocity to render the turbulent velocity fluctuations very small by comparison. Thus one can imagine that the turbulent eddies remain fixed or "frozen in place" as the measuring device traverses them. Under these conditions the time average in the definition of  $\mathcal{D}_\theta(r)$  is actually a spatial average so far as its physical significance is concerned.

The quantity  $\langle \theta'^2 \rangle$  is, ideally, a one-point ensemble average. It is given by  $B_\theta(0)$ . From Eq. (4), however, we see that this implies that

$$\mathcal{D}_\theta(r) = \langle \theta'^2 \rangle - B_\theta(r) \quad (45)$$

In view of this, why physically is  $\mathcal{D}_\theta(l) = \langle \theta'^2 \rangle$ ? The answer is that when  $r$  reaches beyond the value of the representative scale  $l$ , the correlation  $B_\theta(r)$  falls off rapidly (this in fact is related to the empirical definition of  $l$ ). Thus, to a good approximation we can imagine that when  $r$  is somewhat larger than  $l$ , Eq. (43) no longer applies (we're out of the inertial range) and  $B_\theta(r)$  goes to zero because there is no correlation when  $r$  is larger than the existing energy-containing eddies. This completes our discussion of the physical significance of  $C_n^2$ . We now turn to the problem of converting  $C_\theta^2$  into a form more amenable to experimental measurement.

### 2.3 More Useful Forms for $C_n^2$ Calculations

From Eqs. (24) and (25) we have

$$\epsilon_N = K_\theta \left( \frac{d\bar{\theta}}{dz} \right)^2 \quad (46)$$

One can derive, in a very similar manner, the following relation for  $\epsilon$

$$\epsilon = K_M \left( \frac{d\bar{U}}{dz} \right)^2 \quad (47)$$

where  $\bar{U}$  is the mean horizontal velocity and  $K_M$  is the eddy diffusivity for momentum transfer.<sup>5, 7, 8, 9</sup> By inserting these forms for  $\epsilon$  and  $\epsilon_N$  into the equation for  $C_\theta^2$  Eq. (41), we obtain

$$C_\theta^2 = \frac{a K_\theta \left( \frac{d\bar{\theta}}{dz} \right)^2}{\left[ K_M \left( \frac{d\bar{U}}{dz} \right)^2 \right]^{1/3}} \quad (48)$$

7. Lumley, J.L., and Panofsky, H.A. (1964) The Structure of Atmospheric Turbulence, Interscience Pub., N.Y.
8. Lin, J.T., Panchev, S., and Cermak, J. (1969) Turbulence spectra in the buoyancy subrange of thermally stratified shear flows, Project Themis, Tech. Report No. 1, College of Engineering, Colorado State Univ.
9. Dewan, E.M. (1976) Theoretical Explanation of Spectral Slopes in Stratospheric Turbulence Data and Implications for Vertical Transport, AFGI-TR-76-0247, AD A036 307.

As we have already seen, eddy diffusivity is essentially the product of the velocity and spatial scale, that is,  $v_l l$ . Thus we can use

$$K_M = (v_{l_o}) \cdot (l_o) \quad (49)$$

(where subscript "o" refers to outer scale)\* for the momentum diffusion coefficient. The velocity scale for momentum transfer is given by

$$v_{l_o} = l_o \left( \frac{d\bar{U}}{dz} \right) \quad (50)$$

Eq. (50) is, in effect, the definition of the velocity scale (or the "velocity of the energy containing eddies"). Thus

$$K_M = l_o^2 \left( \frac{d\bar{U}}{dz} \right) \quad (51)$$

Solving this for  $(d\bar{U}/dz)$  and inserting into Eq. (48)

$$C_\theta^2 = a^\dagger \left[ \frac{K_\theta}{K_M} \right] l_o^{4/3} \left( \frac{d\bar{\theta}}{dz} \right)^2 \quad (52)$$

This equation for  $C_\theta^2$  is the form used in estimating  $C_\theta^2$  by means of radiosondes to be described below.

We now turn to the actual value of  $C_n^2$ . The latter is given by Eq. (52) when  $\theta$  is the index of refraction. The latter (in the case of radar) depends on both potential temperature and specific humidity. The latter dependence is especially strong at the lowest altitudes. In Eq. (52)  $(d\bar{\theta}/dz)$  must be replaced by

$$\frac{dN}{dz} \propto \left( \frac{\partial N}{\partial \bar{\theta}} \frac{d\bar{\theta}}{dz} + \frac{\partial N}{\partial q} \frac{dq}{dz} \right) \quad (53)$$

where  $N$  is defined as  $(n-1) 10^6$  and where  $n$  is the index of refraction. Here  $\bar{\theta}$  is the average potential temperature and  $q$  is the ratio of the mass of water vapor per unit volume to the mass of the moist air per unit volume. From such considerations, Tatarski<sup>1</sup> (p. 57) has shown that

\* In shear layer turbulence,  $l \neq l_o$  but  $l > l_o$ . There seems to be some vagueness in the definition of "outer length." Here  $l_o$  would be layer thickness and  $l \equiv (v_{rms})/(d\bar{U}/dz)$ .

†  $a = 2.8$  (Ref. 3).

$$C_n^2 = a \left( \frac{K\theta}{K_M} \right) (l_o^{4/3} M^2) \quad (54)$$

where

$$M = \frac{-79 \times 10^{-6} P}{T^2} \left( 1 + \frac{1.55 \times 10^4 q}{T} \right) \left( \frac{dT}{dz} + \gamma_a - \frac{7800 dq/dz}{1 + \left( \frac{1.55 \times 10^4 q}{T} \right)} \right) \quad (55)$$

(see Ref. 1, p. 57) where  $\gamma_a$  is the adiabatic lapse rate of 9.8°C per 1000 m, T is the absolute temperature, and P is the atmospheric pressure given in millibars. VanZandt et al<sup>3</sup> have modified this slightly to make it more convenient and have obtained

$$M = -77.6 \times 10^{-6} \left( \frac{P}{T} \right) \left( \frac{\partial \ln \bar{\theta}}{\partial z} \right) \left[ 1 + \frac{15500 q}{T} \left( 1 - \frac{1}{2} \frac{\left( \frac{\partial \ln q}{\partial z} \right)}{\left( \frac{\partial \ln \bar{\theta}}{\partial z} \right)} \right) \right] \quad (56)$$

where  $\bar{\theta}$  is the average potential temperature. The natural logarithm is designated by  $\ln$ .

### 3. $C_n^2$ AND ATMOSPHERIC TURBULENCE EFFECTS UPON OPTICAL SYSTEMS

#### 3.1 A List of Optical Effects

Since the laser systems under consideration all involve propagation through a portion of the atmosphere, these would all be subject to a number of degrading influences. In Pratt<sup>10</sup> many designs of receivers are described. In some of these receivers, such as the heterodyne and homodyne types, a "photomixing process" is used to mix the received laser beam with a locally generated laser beam in order to create a low-beat frequency in the process of signal detection. Such systems would be very sensitive to any corruption of the signal phase. On the other hand, all receivers would be subjected to the "fading" or scintillation effects. The following is a somewhat comprehensive list of turbulent optical (or electro-magnetic in general) effects.

1. Beam steering: The laser beam can be deviated from the line of sight so that part of it (or, in the worst case, all of it) will miss the receiving aperture.

10. Pratt, W.K. (1969) Laser Communication Systems, John Wiley & Sons, Inc., N. Y.

This is most likely for the case where the receiver is in space and the transmitter is on the ground (partly because a deviated beam will drift farther from the target in proportion to its subsequent distance of travel). Beam steering effects arise, when the size,  $l$ , of the atmospheric inhomogeneity (what we have called the eddy) is larger than the width of the beam. This occurs almost exclusively on the uplink of an earth - space channel.

2. Image dancing: The atmosphere can cause a modulation of the angle of arrival of the beam's wavefront. This will cause the image of the beam in the receiver (or on the photographic plate in the case of photography) to be focused at different places in the focal plane. In a photographic image this would cause blurring, for example. In a communication receiver this effect would necessitate a larger aperture and hence introduce more noise.

3. Beam spreading: In this case, numerous inhomogeneities distributed across the beam cross-section cause many small angle scattering events to occur. This has the effect of spreading the beam energy over a wider cross-section. This, in turn, dilutes the signal at the receiver and, in consequence, introduces a decrease in the signal-to-noise ratio. In this case the beam width must be significantly larger than the eddies, therefore beam spreading is a problem for the downlink of an earth-space laser system (receiver on the ground and transmitter in space or high altitude aircraft).

4. Spatial coherence degradation: Losses of phase coherence, across the wavefront of the beam occurs due to the inhomogeneities in the beam's path. This rapid change of phase with respect to the radial position within the beam cross-section interferes with the photomixing process mentioned above in connection with the mixing of received signal and local laser beams for detection purposes. In the case of photographic systems, this effect can cause blurring.

5. Scintillation: Within the beam cross-section, interference effects can cause destructive and constructive reinforcement. This causes the power to vary widely from point to point within the cross-section, and it is a spatial form of scintillation. A temporal form of scintillation also exists which consists of a fading in and out of the signal in time. These effects are due to the focusing and defocusing effects of the turbulent eddies. As will be explained below, scintillation or amplitude variation effects can be explained by regarding the eddies, or index of refraction variations, as a random distribution of weak lenses distributed within a spatial volume. This volume is to be envisioned as being convected across the beam and thus causing effects that depend on time.

6. Temporal phase fluctuations: These can introduce a spurious modulation which can destroy the amplitude modulation (AM) characteristics of a laser or radio signal.

The above list illustrates the corrupting effects of turbulence on electromagnetic propagation. Of course, in scatter communication systems, the lack of turbulence effects would have the result of destroying the effectiveness of the system. In any case,  $C_n^2$  plays an important role. We now turn to the quantitative role played by  $C_n^2$  in the above mentioned effects.

### 3.2 Amplitude Fluctuations (Scintillation)

Derivations of the equations given below will be found in the references cited. In the present section we are concerned with amplitude variations of the carrier signal. In the case of these fluctuations, a "log-normal distribution" is involved. Tatarski<sup>1</sup> (p. 169) has shown that the mean square fluctuations of the logarithmic level of the signal amplitude,  $A$ , are related to  $C_n^2$

$$\left\langle \left[ \ln \left( \frac{A}{A_0} \right) \right]^2 \right\rangle = 0.56 k^{7/6} \int_0^L C_n^2(\vec{r}) x^{5/6} dx \quad (57)$$

where  $A_0$  is the mean amplitude,  $k$  is the wave number of the radiation and  $x$  is the distance along the path of the beam.  $L$  is the total length of the beam path and  $\vec{r}$  is the position vector. It is assumed that  $l_1 \ll \sqrt{\lambda L} \ll l_0$ , that is, that the quantity  $\sqrt{\lambda L}$ , which is known as the size of the first "Fresnel zone," falls into the size range of inertial range eddies ( $l_1$  and  $l_0$ , the reader will recall, are the "inner" and "outer" lengths respectively).

It is important to notice that these scintillation effects depend on where they occur along the optical path. This is indicated by the term  $x^{5/6}$  in Eq. (57) under the integral sign, and  $x$  is to be interpreted as the distance from the receiver.

In the case where  $\sqrt{\lambda L} \ll l_1$  (that is, the Fresnel zone is much smaller than the inner scale), then, as Tatarski<sup>1</sup> (p. 169) has shown

$$\left\langle \left( \ln \frac{A}{A_0} \right)^2 \right\rangle = 7.37 l_1^{-7/3} \int_0^L C_n^2(\vec{r}) x^2 dx \quad (58)$$

In the simple case where  $C_n^2$  is constant along the path, as it might be for the case of surface-to-surface communication for example, then the latter may be taken out from the integral and,

$$\left\langle \left( \ln \left( \frac{A}{A_0} \right) \right)^2 \right\rangle = 0.31 C_n^2 k^{7/6} L^{11/6} \quad (59)$$

for  $l_1 \ll \sqrt{\lambda L} \ll l_0$  and

$$\left\langle \left[ \ln \left( \frac{A}{A_0} \right) \right]^2 \right\rangle = 7.37 l_0^{-7/3} C_n^2 \frac{L^3}{3} \quad (60)$$

for  $l_1 \gg \sqrt{\lambda L}$

(Ref. 1, p. 169).

The interested reader may wish to read the next section which gives a physical explanation of  $C_n^2$  induced scintillations in terms of the random lens picture of turbulence. The above formulas can be derived directly from this simple picture.

### 3.3 Eddies as Lenses (An Addendum)

The equations in subsection 3.1 can be obtained in a very heuristic and visualizable manner by considering the turbulent medium as an array of blobs which can act as weak lenses. First we shall follow Clifford<sup>11</sup> (pp. 37-39) and consider the following two questions: "What is the optimum size of an eddy, in relation to the wavelength of radiation, for causing fading effects?" and "How does the weighting effect,  $x^{5/6}$ , arise in Eq. (57)?" After treating these two questions we turn to the details of deriving the scintillation formulae from lens-focusing effects.

#### 3.3.1 OPTIMUM EDDY SIZE AND WEIGHTING EFFECTS

Figure 3 depicts an eddy of dimension  $l$  perpendicular to the path of propagation of an electromagnetic signal. This path is from left to right in the figure and ends at the distance  $L$ . The distance between the eddy (at  $z$ ) and the receiver (at  $L$ ) is marked  $y$  and is equal to  $(L - z)$ . For maximum destructive interference between waves propagating from point  $P$  to  $L$  and waves from  $z$  to  $L$ , the difference in the path lengths  $P - L$  and  $y$  should be one-half the wavelength.

Geometrically

$$(P L)^2 = l^2 + y^2 \quad (61)$$

and setting

$$(P L - y) = \frac{\lambda}{2} \quad (62)$$

11. Clifford, S. F. (1978) The Classical Theory of Wave Propagation in a Turbulent Medium, in J. W. Strohbehn (editor), Laser Beam Propagation in the Atmosphere, Springer Verlag, N. Y.

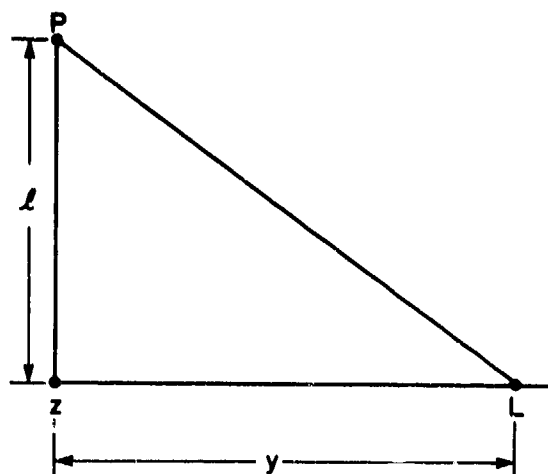


Figure 3. Path Differences From Two Edges of an Eddy of Size  $l$

and eliminating  $PL$  in Eq. (61) by Eq. (62) and ignoring  $(\lambda/2)^2$  we obtain

$$l = \sqrt{\lambda y} = \sqrt{\lambda(L - z)} \quad (63)$$

for the first interference condition. Note that this says  $l$  is equal to "a Fresnel zone for the remaining length of optical path."

This minimum size,  $l$ , for the irregularity is, it turns out, the most effective one for producing interference scintillation. This is true because smaller eddies are less effective since the turbulent cascade is characterized by ever smaller amounts of turbulent intensity and fluctuation as the wave number increases. As Kolmogorov has shown this dependence is  $k^{-5/3}$  for the one-dimensional spatial frequency fluctuation spectrum. In the case of three-dimensional spectra, the index of refraction fluctuation spectrum,  $\phi_n(k)$  depends on  $k$  and  $C_n^2$  in the following way (Ref. 11, p. 19).

$$\phi_n(k) = 0.033 C_n^2 k^{-11/3} \quad (64)$$

( $l_1 \ll k^{-1} \ll l_0$ ) which is a result directly derivable from

$$\mathcal{D}_n(r) = C_n^2 r^{2/3} \quad (65)$$

---

\*This corresponds to  $k^{-5/3}$  in the one-dimensional spectrum.

From Eq. (64), then, it is clear that the changes of index of refraction are a decreasing function of eddy sizes and that, therefore, eddies smaller than  $l$  given by Eq. (63) are less effective in causing effects on the electromagnetic waves. Now consider eddies larger than  $l$ . As eddies increase in size another factor enters which decreases their ability to cause interference. As  $l$  becomes larger, the blob becomes less effective in its ability to cause interference because an eddy of size  $l$  can diffract a wave through an angle of size  $\lambda/l$  (in radians). However, in order that diffracted light from P reach L, it must be diffracted through an angle given by  $l/(L - z)$ . But since we are considering an eddy larger than the  $l$  given by Eq. (63), this condition cannot be met; that is, the larger eddy cannot diffract radiation through a large enough angle and hence there is no interference. From these considerations we see that  $l$  given by Eq. (63) causes the maximum destructive interferences in the waves.

Next, we consider  $W$ , the weighting function  $x^{5/6}$  in Eq. (57). The spatial frequency distribution for the index of refraction inhomogeneities along the optical path is described by the one-dimensional Kolmogorov distribution which is, as was already mentioned, dependent on the wave number as  $k^{-5/3}$ . Inserting the value  $l = \sqrt{\lambda(L - z)}$  into this form, that is,  $l^{+5/3}$ , we arrive at the weighting function  $W$  given by

$$W \sim [L - z]^{5/6} \quad (66)$$

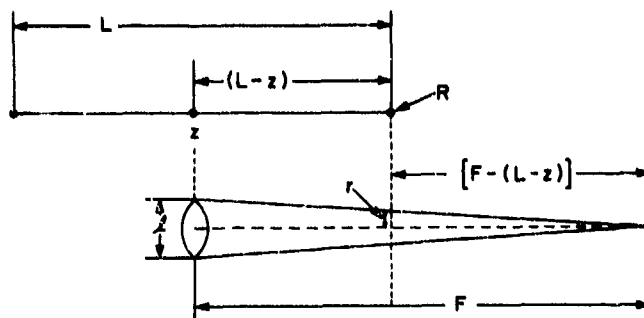
This is the physical reason for  $x^{5/6}$  under the integral sign in Eq. (57).

### 3.3.2 THE "WEAK LENS" APPROACH TO $C_n^2$ EFFECTS

In the following we base our treatment on that of Strohbehn<sup>12</sup> (p. 72) but we sacrifice some generality in the cause of simplicity and brevity. We now set out to derive Eqs. (59) and (60) by simply regarding the blobs as simple lenses which can be described by geometric optics. Figure 4 shows the path length  $L$ , the "lens" diameter,  $l$ , and lens focal length  $F$ . Before going any further one might ask, "What values of  $F$  would these lenses have for typical atmospheric turbulence?" According to Strohbehn<sup>12</sup>  $F$  is in the range of 1000 m to 100,000 km. In other words, these turbulent blobs are very weak lenses with very long focal lengths. To continue, let the refractive index of the lens be given by  $\Delta n = (n - 1)$ . For such a lens, with diameter  $l$ , the focal length can be taken as

12. Strohbehn, J. W. (1978) Modern theories in the propagation of optical waves in J. W. Strohbehn (editor), Laser Beam Propagation in the Atmosphere, Springer Verlag, N. Y.

13. Jenkins and White (1957) Fundamentals of Optics (3rd Ed.), McGraw-Hill, N. Y.



$$F = \frac{l}{\Delta n} \quad (67)$$

$$A_0^2 \left(\frac{\ell}{2}\right)^2 = A^2 r^2 \quad (68)$$

$$\delta A = A_0 \left( \frac{l}{2r} - 1 \right) \quad (69)$$

$$\frac{(l/2)}{F} = \frac{r}{F - (L - z)} \quad (70)$$

Solving this for  $l/2r$  and inserting the result in Eq. (69) we arrive at

$$\frac{\delta A}{A_0} = \frac{L - z}{F - (L - z)} \sim \frac{L - z}{F} \quad (71)$$

where we used  $(L - z) \ll F$  as an approximation.

Next, we consider an array of such lenses with indices of refraction which depend upon the sizes in the manner of the inertial range structure function (Eq. (42))

$$\langle \Delta n^2(l) \rangle \sim C_n^2 l^{2/3} \quad (72)$$

where we have identified  $\theta^2$  with  $\langle \Delta n^2 \rangle$  and  $C_\theta^2$  with  $C_n^2$ . Of course, Eq. (72) is correct only when  $l_1 \ll l \ll l_0$ . In the case where  $l \ll l_1$ , it can be shown that one must replace  $l$  by  $l^3/l_1^2$  in Eq. (72). Using Eq. (72) in Eq. (67)

$$F = \frac{l}{C_n \cdot (l')^{1/3}} \quad (73)$$

where we define

$$\left. \begin{aligned} l' &= l, & (l_1 \ll l \ll l_0) \\ l' &= \frac{l^3}{l_1^2}, & (l \ll l_1) \end{aligned} \right\} \quad (74)$$

From Eq. (71) then,

$$\frac{\delta A}{A_0} \sim \frac{C_n \cdot (l')^{1/3}}{l} (L - z) \quad (75)$$

Thus

$$\begin{aligned} \frac{\delta A}{A_0} &\sim C_n l_1^{-2/3} (L - z), & (l < l_1) \\ \frac{\delta A}{A_0} &\sim C_n l^{-2/3} (L - z), & (l_1 \ll l \ll l_0) \end{aligned} \quad (76)$$

If we assume that  $l_1$  is larger than the value  $l_c = \sqrt{\lambda L}$  (the most effective scattering length), then in general we can make the approximation that eddies of size  $l_1$ , do all of the scattering or focusing etc. and

$$\left(\frac{\delta A}{A_0}\right)^2 \sim C_n^2 l_1^{-4/3} (L - z)^2 \quad (77)$$

Next we must calculate the average value of  $(\delta A/A_0)^2$ . The number of inhomogeneities along the optical path we shall assume is  $N = L/l_1$ . In effect we are using a model where all the eddies are of size  $l_1$  and touching each other. Then

$$\left\langle \left(\frac{\delta A}{A_0}\right)^2 \right\rangle = C_n^2 l_1^{-4/3} \overline{(L - z)^2} \frac{L}{l_1} \quad (78)$$

where  $\overline{(L - z)^2}$  is the average over the path. Taking this as  $L^2$  we arrive at

$$\left\langle \left(\frac{\delta A}{A_0}\right)^2 \right\rangle \sim C_n^2 L^3 l_1^{-7/3} \quad (79)$$

But since

$$\frac{\delta A}{A_0} \doteq \delta(\ln A) = \ln A - \ln A_0 = \ln \left( \frac{A}{A_0} \right) \quad (80)$$

we can write Eq. (79) as

$$\left\langle \left[ \ln \left( \frac{A}{A_0} \right) \right]^2 \right\rangle \sim C_n^2 L^3 l_1^{-7/3} \quad (81)$$

where

$$(l_1 \ll l \ll l_0) \text{ and } (l_c < l_1) \quad .$$

This is Eq. (60) which has been derived (originally by means of rigorous analytic methods). The above simple model arrives at the same result with all but the constant of order unity.

In the alternative case of  $l_c > l_1$  one must replace  $l_1$  by  $\sqrt{\lambda L}$  since in this case  $l_c$  is the most efficient size of eddy, and

$$\left\langle \left[ \ln \left( \frac{A}{A_0} \right) \right]^2 \right\rangle \sim C_n^2 k^{7/6} L^{11/6} \quad (82)$$

where

$$l_c > l_1 \text{ (or } l_1 < \sqrt{\lambda L}) \text{ .}$$

This is the same as Eq. (59) and again we have everything but the numerical factor from the simple model.

### 3.3.3 SATURATION EFFECTS

As more eddies enter the beam, multiple scattering can occur whereby a scattered ray can be rescattered. As this process becomes more dominant, the phase of the radiation across the beam becomes more and more incoherent. When the point is reached where there is no longer any coherence across a distance of the size of the first Fresnel zone ( $\sqrt{\lambda L}$ ) then the above treatment breaks down, and, as a result, the scintillation effects become no longer dependent on an increase of path length. This phenomenon is known as "saturation" and it has received a lot of attention in the literature. A detailed explanation of this in terms of random lenses will be found in Strohbehn<sup>12</sup> p. 79.

### 3.3.4 THE DIFFRACTION GRATING PICTURE

For completeness it should be mentioned that, in addition to the "blob lens" approach, there is another simple picture available for describing  $C_n^2$  effects due to Tatarski,<sup>14</sup> p. 114. This is a model based on the idea that a random array of diffraction gratings of random spatial frequencies can explain the formula for the "scattering cross-section" of turbulent volumes of atmosphere. The eddies have not only random periods but random orientations. The "grating" picture arises from the fact that this particular theory makes intrinsic use of the spatial frequency spectra.

### 3.4 Phase and Ray Angle Fluctuations

As first shown by Tatarski,<sup>11</sup> p. 170, and mentioned in Pratt,<sup>10</sup> p. 135, the variance in phase as a function of separation,  $\rho$ , across a beam cross-section, that is to say the structure function of the phase ( $\phi$ ) fluctuations between two points of separation  $\rho$ , is given by

14. Tatarski, V.I. (1971) The Effects of the Turbulent Atmosphere on Wave Propagation, National Science Foundation, TT-68-50464.

$$\mathcal{D}_{\phi}(\rho) = \begin{cases} 1.46 k^2 \rho^{5/3} \int_0^L C_n^2(r) dx & , \quad (l_1 \ll \rho \ll \sqrt{\lambda L}) \\ 2.91 k^2 \rho^{5/3} \int_0^L C_n^2(r) dx & , \quad (l_0 \gg \rho \gg \sqrt{\lambda L}) \end{cases} \quad (83)$$

The integration here is carried out along the ray starting from the receiver or observing point to the source. In the case where  $C_n^2$  is constant along the path, the integrals can be replaced by  $C_n^2 L$ .

Phase fluctuations are caused by two physically different processes. In one case there are the effects of wavefront distortions over the beam cross-section. In the other case there is the effect of beam steering or image dancing where the wavefront is tilted through the angle  $\beta$ , where

$$\sigma_{\beta} = \frac{[\mathcal{D}_{\phi}(\rho)]^{1/2}}{k \rho} \quad (84)$$

where  $\phi$  is the phase of the radiation (Ref. 10, p. 135).

### 3.5 Spatial Coherence Degradation

One of the important parameters in the design of optical receivers is the parameter  $r_0$  (Pratt,<sup>10</sup> p. 140-141 and Fried<sup>15</sup>). This quantity,  $r_0$ , is a sort of coherence length. Physically  $r_0$  is the distance such that, if an antenna diameter is increased beyond  $r_0$ , there is significantly less improvement in performance. In other words,  $r_0$  represents the coherence across the beam cross-section. As the turbulence degrades the phase of the radiation,  $r_0$  is reduced in size. As  $C_n^2$  becomes larger,  $r_0$  becomes smaller. As Fried<sup>15</sup> has shown (by means of the structure functions of both amplitude and phase fluctuations)

$$r_0 = [1.2 \times 10^{-8}] (\lambda)^{6/5} L^{-3/5} C_n^{-6/5} \quad (85)$$

where  $L$  is the path length, constant  $C_n$  is assumed along the path, and  $\lambda$ , as usual, is the wavelength of the radiation.

In addition to the determination of the optimum aperture size, the value of  $r_0$  is used to calculate the resolution of focused images.<sup>16</sup> It is also an important

15. Fried, D.L. (1967) Optical heterodyne detection of an atmospheric distorted signal wave front, Proc. IEEE 55:57.

16. Beran, M.J., and Parrent, G.B. (1964) Theory of Partial Coherence, Prentice-Hall, N.J.

parameter in the calculation of so-called "saturation effects" (that is, where an increase of the path length of the ray no longer increases the value of  $\langle(\delta A/A_0)^2\rangle$ ).

From a practical point of view in the context of systems design (where adaptive optics is employed) the most important application of  $r_0$  is to the determination of the number of activator elements needed to achieve a given performance. Thus  $r_0$  determines the number of segments, for example, in a compensating mirror. If  $r_0$  is too small, it might imply that the present state of the art of electronic optical compensation is insufficient for certain applications; thus, this parameter is indeed of paramount importance.

It is of interest to understand the physical significance of  $r_0$ . Figure 5 shows this diagrammatically. Consider two rays, 1 and 2, which travel from left to right in the figure. In Figure 5A, these rays are separated by a distance  $r < r_0$  in the plane of the beam cross-section. The value of  $r_0$  corresponds to the size of the "eddy lens" affecting the phases of the rays. As can be seen, since both rays go through the same "lens," one can assume (in this model) that they both suffer the same change in phase and hence maintain their phase relation after the interaction. The alternative case is shown in Figure 5B. Here  $r > r_0$  and the two rays go through different lenses of different index of refraction. This would destroy their original phase relation. After a number of such encounters the two rays would be "incoherent" with respect to each other. This, in essence, is the physical meaning of  $r_0$ . The following description, which is based on a number of papers by Yura,<sup>17</sup> will render the above qualitative description more quantitative and at the same time explain Eq. (85).

Let  $\Delta S_1$  be the change in phase between rays 1 and 2 (initially with the same phase) caused by going through an inhomogeneity where  $r$ , again, is the distance between the rays. Let  $\Delta n$  be the difference of index of refraction between the two eddies traversed by the rays. Then the difference in phase,  $\Delta S_1$  caused by this interaction is

$$\Delta S_1 = k\Delta n \cdot r \quad (86)$$

In Eq. (86) it is assumed that  $r$  is also the diameter of each of two round eddies as in Figure 5B. Note that  $r\Delta n$  is the optical path difference and that the subscript, 1, denotes a single interaction.

If one recalls the definition of the structure function (Eq. (2)) one obtains

$$\langle\Delta n^2\rangle = \mathcal{D}_n(r) \quad (87)$$

17. Yura, H. T. (1974) J. Opt. Soc. Am. 64:59, 1526, 357, 1211.

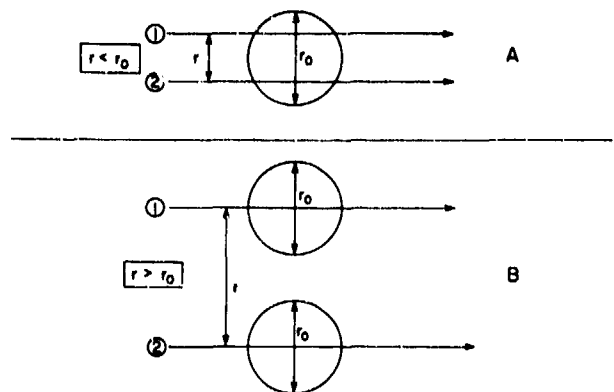


Figure 5. Coherence Distance  $r_0$  As Eddy Diameter. A: Two Rays Through One Eddy. B: Two Rays Through Two Eddies

( $r$  is the diameter of the "lens" or  $\sim$  outer length) thus

$$\langle \Delta S_1^2 \rangle \sim k^2 r^2 \mathcal{D}_n(r) \quad . \quad (88)$$

But in the inertial range we have  $\mathcal{D}_n(r) \sim C_n^2 r^{2/3}$ , thus

$$\langle \Delta S_1^2 \rangle \sim k^2 r^{8/3} C_n^2 \quad . \quad (89)$$

We must now consider multiple scattering. The number of encounters,  $N$ , with "lenses," along a path of length  $L$  (between source and receiver) can be estimated by

$$N = \frac{L}{r} \quad (90)$$

where we have assumed homogeneity for simplicity. In order to obtain  $\langle \Delta S^2(r) \rangle$  due to  $N$  events, we must remember that the phase changes would occur in a random manner. For this reason  $\langle \Delta S^2(r) \rangle$  would increase as  $N$  rather than  $N^2$  (Jenkins and White,<sup>13</sup> p. 218). Thus

$$\langle \Delta S^2(r) \rangle = N \langle \Delta S_1^2(r) \rangle \sim k^2 C_n^2 L r^{5/3} \quad . \quad (91)$$

When  $r = r_0$  we assume that there is incoherence. For the latter we take

$$\langle \Delta S^2(r_0) \rangle = \pi^2 \quad (92)$$

and insert this into Eq. (91) to obtain

$$r_0 \sim \left[ \frac{\pi^2 a}{k^2 C_n^2 L} \right]^{3/5} \quad (93)$$

which is within a constant of Eq. (85).

Finally we conclude with a very simple description of why, physically,  $r_0$  determines the maximum aperture mentioned previously in connection with image resolution. This explanation is due to George Parrent (private communication). First it must be recalled that the resolution of an image is determined by the diffraction pattern of the effective aperture. The latter is controlled by the destructive interference between the two most distant parts of the incident wavefront. But interference (or the concept of wavefront for that matter) demands the presence of coherence. As we have seen  $r_0$  is the coherent limit; therefore  $r_0$  is the effective size of the aperture so far as interference is concerned. If, for example, an aperture had a radius of  $4 r_0$ , it would gather much more light than one of radius  $r_0$ ; regarding interference effects it is as if there were several different lenses operating with different wavefronts without any mutual interference effects so that the diffraction pattern would be that of one lens of radius  $r_0$ . It is now obvious why an aperture larger than  $r_0$  would not help much with resolution problems.

### 3.6 Beam Spreading

Since a light beam, or any electromagnetic beam, does not have sharp edges (due to diffraction) its width is often defined by its Gaussian (1/e) radius. Designating this width by  $a_{\text{eff}}$  it has been shown (Ishimaru, <sup>18</sup> p. 137) that

$$\left. \begin{aligned} a_{\text{eff}}^2 &= a_0^2 + 1.6 C_n^2 k^{1/3} a_0^{1/3} L^{8/3}, \quad (\Omega \gg 1) \\ a_{\text{eff}}^2 &= \left( \frac{z}{ka_0} \right)^2 + 1.6 C_n^2 a_0^{-1/3} L^3, \quad (\Omega \ll 1) \end{aligned} \right\} \quad (94)$$

18. Ishimaru, A. (1978) The beam wave case and remote sensing in J. W. Strohbehn (editor), Laser Beam Propagation in the Atmosphere, Springer Verlag, N. Y.

where  $L$  is path length,  $a_0$  is initial Gaussian width and  $\Omega = (\alpha L)^{-1}$  where  $\alpha$  is defined by

$$\alpha = \frac{\lambda}{\pi a_0^2} \quad (95)$$

Notice that as  $C_n^2$  increases,  $a_{\text{eff}}^2$  increases linearly.

### 3.7 Typical $C_n^2$ Values – Optical Compensation Techniques

In the above subsections we have seen quantitatively how  $C_n^2$  affects design parameters of optical systems. We now complete Section 3 by considering two additional related topics, namely: (a) what are typical values of  $C_n^2$  in the atmosphere, and (b) what are some of the techniques available for the purpose of minimizing  $C_n^2$  effects on systems. According to Davis<sup>19</sup>

$$\text{Weak turbulence} \quad C_n = 8 \times 10^{-9} \text{ m}^{-1/3*} \quad (96)$$

$$\text{Intermediate turbulence} \quad C_n = 4 \times 10^{-8} \text{ m}^{-1/3} \quad (97)$$

$$\text{Strong turbulence} \quad C_n = 5 \times 10^{-7} \text{ m}^{-1/3} \quad (98)$$

These are meant to characterize average daytime conditions a few meters above the ground.  $C_n$  decreases roughly as the  $-1/3$  power of altitude within the first 100 m of the surface.<sup>19</sup> Hufnagel and Stanley are quoted by Davis as having integrated  $C_n^2$  along a vertical path through the atmosphere under daytime turbulence conditions. They found

$$C_n = 1.3 \times 10^{-11} \text{ m}^{-1/3} \quad (99)$$

Clifford<sup>11</sup> has quoted the following values from Obukov. In the daytime, 84 percent of observations fell within

$$2.3 \times 10^{-7} \text{ m}^{-1/3} < C_n < 7.4 \times 10^{-7} \text{ m}^{-1/3} \quad (100)$$

At night, 59 percent (note, a larger spread due to the smaller percent) fell within

$$7.4 \times 10^{-8} \text{ m}^{-1/3} < C_n < 2.3 \times 10^{-7} \text{ m}^{-1/3} \quad (101)$$

\*m for meters.

19. Davis, J. I. (1967) Consideration of atmospheric turbulence in laser systems design, Applied Optics 5:139-146.

Clifford<sup>11</sup> also quotes Neff's published values

$$\langle C_n^2 \rangle^{1/2} \sim 2.6 \times 10^{-8} \text{ m}^{-1/3} \pm 3.9 \times 10^{-8} \text{ m}^{-1/3} \quad (102)$$

for early morning, and

$$\langle C_n^2 \rangle^{1/2} \sim 5.5 \times 10^{-8} \text{ m}^{-1/3} \pm 1.4 \times 10^{-7} \text{ m}^{-1/3} \quad (103)$$

for afternoon measurements.

Shapiro<sup>20</sup> has described some of the compensation techniques used to minimize  $C_n^2$  problems. The following are some brief descriptions of these.

Interferometry: Consider the classical problem of measuring the distance between two very distant stars. Such a measurement would be impossible on the earth's surface, due to  $C_n^2$  effects, if the latter were not somehow eliminated. It turns out, however, that the image of the interference pattern caused by two slits is not displaced by turbulence-induced phase distortions. An interference pattern is the autocorrelation of the optical signal. From this fact one can derive the result that it is phase independent. The general problem of measuring the distance between two self-luminous objects can be solved in this manner up to limits which do not depend on turbulence.

Phase compensated imaging: As surprising as it may seem to the uninitiated, it is now possible to actively measure and compensate for turbulence-induced phase perturbations in real time (to some useful degree). This is made possible by the fact that atmospheric phase modulations occur at a rate less than 1 kHz,<sup>21</sup> and that recent advances in optical and electronic technology can enable the compensation to be made automatically. The device which does this is made up of a wave sensor (or phase-estimator array), a wavefront corrector (deformable mirror) and a feedback system which adjusts the corrector in a manner which maintains a constant image. The term "rubber mirror" has been used to describe this device and the reader will find a number of references in Shapiro.<sup>20</sup>

Diversity reception:<sup>20</sup> Fading can be minimized by using a number of receivers which are sufficiently separated (so that they occupy different parts of the reception pattern) and by combining their signals in an optimal manner. This is used in downlink space to earth channels and is a well known technique in communication engineering. For obvious reasons it cannot be used to decrease scintillation effects in the uplink channel.

20. Shapiro, J. H. (1978) Imaging and optical communication through atmospheric turbulence, in J. W. Strohbehn (editor), Laser Beam Propagation in the Atmosphere, Springer Verlag, N. Y.

21. Greenwood, D. P. (1977) Bandwidth specification for adaptive optics systems, Optical Soc. of Am. 67:390-392.

Adaptive variable rate communication: Kennedy et al<sup>22</sup> have suggested an idea that may have use. As has been already pointed out, the amplitude fluctuations are log-normally distributed.\* This implies that the distribution is highly skewed and it has been observed that amplitude fluctuations that far exceed the mean are not improbable. For example, astronomers report that with small apertures sudden increases of intensity of 400 percent above mean values over a time of 1/100 sec. are not unusual. By adjusting the bit rate of transmission it may be possible to take advantage of such periods of high gain. The practicality of such a scheme seems not to have been investigated, however.

#### 4. STATE OF THE ART FOR ESTIMATION OF $C_n^2$ FROM RAWINSONDE DATA

##### 4.1 State of the Art

VanZandt et al<sup>3</sup> have devised a method to relate standard rawinsonde measurements to direct radar measurements of  $C_n^2$ . If this leads to a comprehensive way to deduce  $C_n^2$  (for most altitudes of interest) from already existing rawinsonde data, it will be possible to use the large data bank of the latter in order to ascertain geographical, seasonal, diurnal and synoptic weather pattern effects on  $C_n^2$ . As we shall see, however, some problems remain to be solved before such is possible.

The work of VanZandt et al<sup>3</sup> is the second version of the model that they have proposed. The theory behind this model is based in part on a model of clear air turbulence which assumes that the shear instability is the sole cause of the turbulence. Some of the work by Rosenberg and Dewan<sup>23</sup> was incorporated into their model. While this latter paper was originally intended primarily for describing turbulence in the stratosphere it is also appropriate for the modeling of clear air turbulence in the troposphere (known popularly as CAT). The main difference between CAT in the troposphere and stratospheric turbulence is that, in the troposphere, the stability is significantly lower (that is, the buoyancy frequency is down by about a factor of two). In addition, there are other causes of turbulence in the troposphere having to do with convection and weather fronts, etc. At the lowest altitudes (of order 1 km) there is the so-called boundary layer where turbulence effects depend on factors relating to surface influences.

In order to explain the model of VanZandt et al<sup>3</sup> it is necessary to review the nature of stratified turbulence. (A much more comprehensive treatment will be

22. Kennedy, R.S., and Karp, S. (1969) Optical Space Communication, NASA SP-217.

23. Rosenberg, N.W., and Dewan, E.M. (1975) Stratospheric Turbulence and Vertical Effective Diffusion Coefficients, AFCRL-TR-75-0519, AD A019 708, Air Force Geophysics Lab., Bedford, MA.

\*This may be disputed somewhat, but it does not affect the argument.

found in Rosenberg and Dewan.<sup>23)</sup> The mechanism for turbulent breakdown in stratified fluids is the Kelvin-Helmholtz instability (KH). The crucial parameter in the theory of this instability is the Richardson number defined by

$$R_i = \frac{g}{\bar{\theta}_0} \frac{d\bar{\theta}/dz}{(d\bar{U}/dz)^2} = \frac{g}{T_0} \frac{(dT/dz + \gamma)}{(d\bar{U}/dz)^2} \quad (104)$$

where  $\theta$  is the potential temperature,  $T$  is the absolute temperature,  $\gamma$  is the adiabatic lapse rate,  $\bar{U}$  is the average horizontal velocity and the subscript naught indicates a larger scale average over the region of interest. When  $R_i < 1/4$  it is possible (but not mathematically guaranteed) that turbulence breakdown will occur; in other words, it has been shown<sup>24</sup> that  $R_i < 1/4$  is a necessary but not sufficient condition for turbulence. In nature it seems safe to assume that it is generally also a sufficient condition.<sup>25, 26</sup> For a simple energetic argument for why this is true, the reader may consult Businger<sup>27</sup> who demonstrated that  $R_i = 1/4$  is the condition where the kinetic energy (available from the shear across a layer of given thickness) is equal to the work necessary to overcome the buoyancy force (due to atmospheric stability) in the process of overturning the layer. Figure 6 illustrates the sequence of configurations (from the generation of a wave to the turbulent breakdown) in the development of the K-H instability. Notice that before the breakdown occurs, the wave "rolls up." The actual final breakdown seems to be due to the fact that when roll up occurs there are layers of dense fluid over less dense fluid and numerous local convective "turnover" events simultaneously occur within these superposed layers.

Figure 7 shows a diagram based on the one given by Browning and Watkins.<sup>28</sup> Those investigators worked with radar observations of CAT in the lower troposphere. It may be of interest that such radar sightings were historically known

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- 24. Miles, J. W. (1961) On the stability of heterogeneous shear flows, part I, J. Fluid Mech. 10:496-512.
  - 25. Woods, J. D. (1968) An investigation of some physical processes associated with the vertical flow of heat through the upper ocean, The Meteorological Mag. 97:65-72.
  - 26. Thorpe, S. A. (1973) Turbulence in stably stratified fluids: A review of laboratory experiments, Boundary Layer Met. 5:94-119.
  - 27. Businger, J. A. (1969) On the energy supply of clear air turbulence, in Y. H. Pao and A. Boldburg (editors), Clear Air Turbulence and its Detection, Plenum Press, 100-108.
  - 28. Browning, K. A., and Watkins, C. D. (1970) Observations of clear air turbulence by high power radar, Nature 227:260-263.

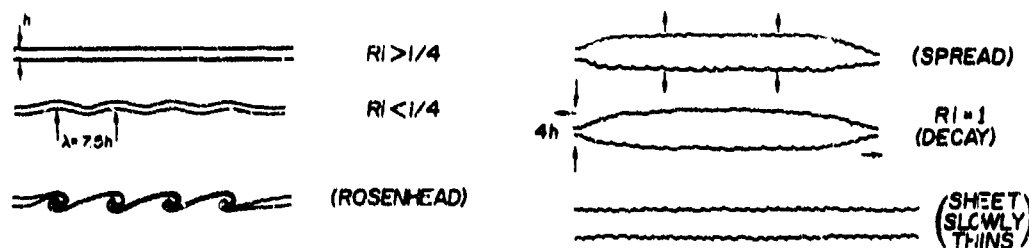


Figure 6. Development of Billow and Subsequent Turbulence (After Woods)

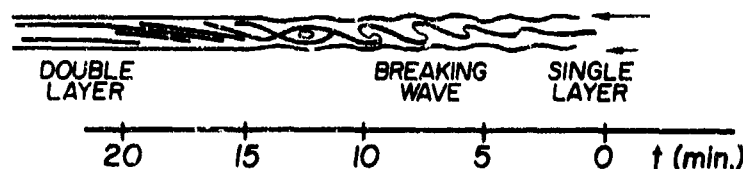


Figure 7. Browning and Watkins' Description of Radar History of CAT

as "braid angels" before their nature was understood. Atlas et al<sup>29</sup> seem to be the first to have identified these as CAT.

Before the turbulence commences there is a difference of potential temperature across the layer. This is caused by the fact that the atmosphere (in the absence of convection caused by heating from below) is usually stable. When turbulence takes place, the mixing causes parcels of air that were initially at different altitudes (having different potential temperatures) to be moved together. This causes the index of refraction to vary wildly (in comparison to the original smooth configuration) and hence to scatter the radar signal back to the receiver. In this manner radar can measure the  $C_n^2$  in its beam.

To relate  $C_n^2$  to rawinsonde data one, ideally, would plot  $R_i$  as a function of altitude and infer from such stability profiles an estimate of the turbulent layer configurations (that is, by designating as turbulent, or about to be turbulent, those regions where  $R_i < 1/4$ ).

Of course, the rawinsonde measurements have low resolution — too low in fact to directly measure the turbulence configuration directly.

29. Atlas, D., Metcalf, J. I., and Gossard, E. E. (1970) The birth of "CAT" and microscale turbulence, *J. Atm. Sci.* 27:903-913.

To actually obtain an estimate of  $C_n^2$ , one makes use of Eq. (54) repeated here for convenience.

$$C_n^2 = (\text{CONST.}) l_0^{4/3} M^2 \quad (54)$$

where  $M^2$  can be estimated from the rawinsonde data (which gives temperature and specific humidity as a function of altitude). Notice in Eq. (54) there is an unknown,  $l_0$ . The constant is known provided that the ratio of eddy diffusivities is taken equal to unity.

The radar estimate for  $C_n^2$  is obtained from<sup>3</sup>

$$C_n^2(\text{radar}) = 19.7 \times 10^{-30} [T_c + 1875] r^2 \left( \frac{S}{N} \right) [m^{-2/3}] \quad (105)$$

where  $r$  is range,  $(S/N)$  is the signal-to-noise ratio, and  $T_c$  is the cosmic noise temperature. By comparing results from Eq. (105) and Eq. (54) they were able to refine Eq. (54) in a manner which made it useful; but, first they had to solve certain problems.

The radar observations had a resolution of about 1 km. In contrast one would expect the turbulent layers to have an individual thickness of one-tenth this size. Another problem is introduced by the fact that the wind and temperature are telemetered at intervals of about 100 m. Furthermore it was found necessary to average this data over several such altitudes in order to obtain reliable data. By means of such running averages, profiles of average shear,  $\langle S^* \rangle$ , were obtained. As already mentioned such shears would not reveal any regions where  $R_i < 1/4$ . To overcome this problem, VanZandt et al<sup>3</sup> inferred the existence of "micro-shears" or "fine structure shears" which would cause turbulent layers to form. They devised a formalism to relate the population of micros shears to  $\langle S \rangle$ . In order to do this they used the following simple and ingenious technique. First they assumed that the values of the micros shears were normally distributed about  $\langle S \rangle$  and that the variance of this distribution was a fixed quantity. Thus, starting with a measured  $\langle S \rangle$  profile it was possible for them to infer the statistical probability of the occurrence of turbulence layers. In particular, they could infer the fraction of altitude, occupied by a given slab, which is expected to be in the state of turbulence on the basis of  $\langle S \rangle$  and the value of the standard deviation,  $\sigma$ . Like  $l_0$  above,  $\sigma$  was initially an unknown quantity. Since the values of  $\langle S \rangle$  were usually far too small to cause turbulence, it was only the shears in the "tails" of the Gaussian distributions which would cause  $R_i < 1/4$  and hence turbulence. As will be discussed below,

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\* Caution: this symbol was previously used for phase.

$\sigma$  and  $l_0$  were determined by means of an optimal fit with the data when rawinsonde were compared to radar measurements of  $C_n^2$ .

In order to make possible the comparison with 1-km resolution radar measurements, they calculated the mean values of  $C_n^2$  within the slabs, labeled "i", by means of

$$\overline{C_{n,i}^2} = C_{n,i}^2 F_i \quad (106)$$

where  $C_{n,i}^2$  is the mean value of  $C_n^2$  in turbulent layers in slab i and  $\overline{C_{n,i}^2}$  is the mean value including both the turbulent layers and non-turbulent regions.  $C_{n,i}^2$  is determined from rawinsonde data by Eq. (54) and by Eq. (105), and  $F_i$  is the mean fraction of the slab that is turbulent. Calling  $\langle C_n^2(\text{model}) \rangle$  the mean square of  $C_n^2$  over a kilometer (in order to make a comparison with 1-km resolution radar), we have

$$\langle C_n^2(\text{model}) \rangle = \sum_i \overline{C_{n,i}^2} \left[ \frac{\Delta z_i}{(1 \text{ km})} \right] \quad (107)$$

where  $\Delta z_i$  is the thickness of the  $i^{\text{th}}$  slab and where the sum is taken so that the  $\Delta z_i$  adds to 1 kilometer. This leads to

$$\langle C_n^2(\text{model}) \rangle = \sum_i a^2 \alpha' l_0^{4/3} M_i F_i \left[ \frac{\Delta z_i}{(1 \text{ km})} \right]^* \quad (108)$$

where  $a^2$  is about 2.8 and  $\alpha' \sim 1$ . Both the value of  $l_0$  and  $\sigma$  are determined (once and for all) by an optimum fitting to the data which involves a careful comparison between  $\langle C_n^2(\text{model}) \rangle$  with  $\langle C_n^2(\text{radar}) \rangle$ , the latter being obtained via Eq. (105) and measurements of (S/N) from radar returns. The radiosonde and radar measurements are made as close together in space and time as is practical in order to make the appropriate estimates of  $l_0$  and  $\sigma$ , and VanZandt et al.<sup>3</sup> found  $l_0 = 10 \text{ m}$ , and  $\sigma = 0.010 \text{ s}^{-1}$  for the troposphere and  $l_0 = 10 \text{ m}$ ,  $\sigma = 0.015 \text{ s}^{-1}$  for the stratosphere. These values of  $\sigma$  are almost exactly the same as those of Rosenberg and Dewan,<sup>23</sup> for example,  $\sigma = 0.014 \text{ s}^{-1}$  was given there for the stratospheric case. As for their value of  $l_0 = 10 \text{ m}$ , this "outer length" was close to that used by Crane on the basis of isotropy (that is,  $l_0$  for Crane, was its value when the turbulence became anisotropic: private communication). The value of  $l_0 = 10 \text{ m}$

\* M is defined by Eq. (56) and subscript "i" refers to  $i^{\text{th}}$  layer.

is of order 1/10th of the thickness of an average turbulent layer in the stratosphere (as seen by balloon measurements, Barat,<sup>30</sup> and by  $R_1$  number profiles, Rosenberg and Dewan,<sup>23</sup>) which is about 200 m. Indeed one would expect 10 m to be near the scale where isotropy breaks down<sup>31</sup>; but, in the case of VanZandt et al,<sup>3</sup> the value of  $l_0$  may actually have more to do with the scale at which high gradients of  $C_n^2$  would be expected to occur.

The main question that now arises is: "How well does the model of VanZandt et al<sup>3</sup> actually work?" In their own words, "The resulting profiles of  $\langle C_n^2 \text{ (model)} \rangle$  agree well with the measured profiles of  $\langle C_n^2 \text{ (radar)} \rangle$  in general shape, in changes from day to day, and in many details from km to km. This agreement implies that: (1) The vertical profile of  $C_n^2$  can be measured by Doppler Radar, (2) The vertical profile of  $C_n^2$  can also be estimated by calculation from routine rawinsonde profiles, using our theoretical model." They also pointed out that the agreement in absolute values, on the other hand (while very good) only implies that, by adjusting  $l_0$  and  $\sigma$  properly, one can obtain a good fit to the data.

What factor was the most important for the success of the model of VanZandt et al?<sup>3</sup> Their answer to this was that it is the variation of the quantity  $F_1$  (the fraction of turbulence) which was most responsible for the agreement between  $\langle C_n^2 \text{ (model)} \rangle$  and  $\langle C_n^2 \text{ (radar)} \rangle$ . This accounted for both the day-to-day variation and the variation with altitude. Both types of variations were found to be very large, that is, over an order of magnitude in some cases.

The largest values of  $C_n^2$  are found at the lowest altitudes. To illustrate this we give Figure 8 which is from Crane.<sup>32</sup> As can be seen, there is a very steep increase of  $C_n^2$  as one approaches the ground. In this respect it is unfortunate\* that the model of VanZandt et al<sup>3</sup> has not really been tested at the lower altitudes (below 7 km there are problems obtaining data and below 4 km none are available).<sup>33</sup> The reasons for data failure where  $C_n^2$  is measured at low altitudes are presented in the following list by VanZandt et al:<sup>3</sup>

1. The radar receiver may be overloaded by the much stronger echoes at the lower heights.

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\* Some applications would not be harmed by this however.

30. Barat, J. (1975) Etude experimentale de la structure du champ de turbulence dans la moyenne stratosphere, C.R. Acad. Sc. Paris, 280, Ser. B, 691-693.
31. Pond, S., Stewart, R.W., and Burling, R.W. (1963) Turbulence spectra in the wind over waves, J. Atm. Sci. 20:319-321.
32. Crane, R.K. (1977) Stratospheric Turbulence Analysis, AFGL-TR-77-0297, AD A047 740.
33. Shannon, R.R., Smith, W. Scott, et al (1978) Atmospheric Phase Distortions, AFGL-TR-78-0178, AD A059 402.

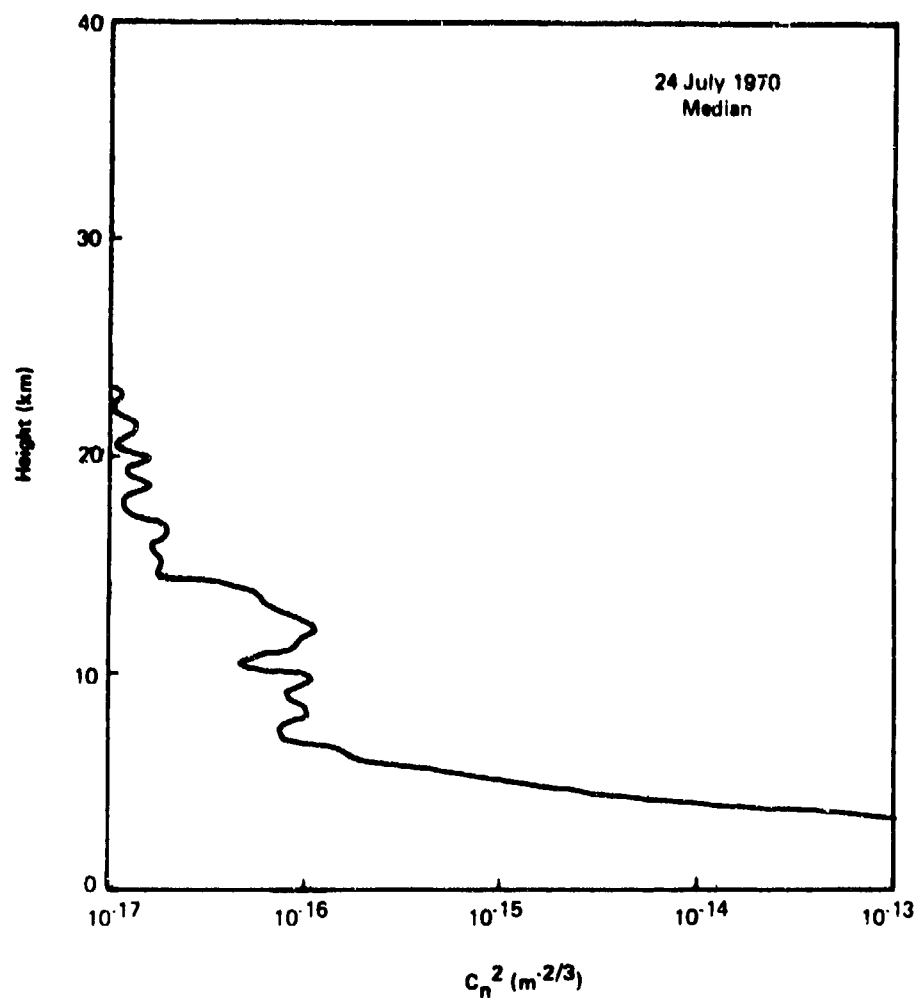


Figure 8. Crane's Data on  $C_n^2$  via Radar

2. The effect of the separation between radar site and location of radiosonde should be larger at lower heights.
3. The radiosonde measurement of humidity was quite poor in comparison to that of temperature. At lowest altitudes, the role of humidity is dominant. Therefore, the data from the radiosondes would be expected to become degraded at lowest altitudes.
4. In addition there is some evidence that the value of  $I_0$  may be, on occasion, radically different near the ground than at higher elevations, and this is not taken into account by the model.

To the above list we may also add the possibility that at lower altitudes, the boundary layer effect may become important (despite the fact that the latter extends upwards for only about 1 km on average). If this is correct, one would have to incorporate other parameters in addition to  $R_1$ , parameters which are used to characterize boundary layer turbulence. The model also leaves out effects of convective storms.

In a private communication, Dr. VanZandt mentioned that the following improvements will be included in the next  $C_n^2$  model his group will employ:

"The two major conceptual defects in the model of VanZandt et al.<sup>3</sup> are, first, the neglect of fluctuations of stability, and second, the use of a constant value of the outer scale  $l_0$ . The method of correcting the first defect is described in VanZandt et al.<sup>34</sup> The second defect must be corrected by introducing a probability distribution of layer thicknesses, but determination of this distribution has proved to be difficult."

The above will greatly reduce the need to "fit the data" as one must in the present model (for determining  $l_0$  and  $\sigma$ ). Since this work is at the forefront, we await the results of the new model with interest.

#### 4.2 Previous Work

Crane<sup>32</sup> has made observations of  $C_n^2$  with the Millstone Radar facility. In addition, he too had the problem of estimating the fraction of turbulence which occurs below the scale of the radiosonde radar resolution (again of order 1 km). In addition, he has estimated  $C_T^2$  profiles,  $\epsilon$  (dissipation rate) and  $K_H$  (local eddy diffusivity) on the basis of  $C_n^2$  profiles determined via radar. It is useful to compare some of the things that he did which might have value as parts of future  $C_n^2$  models.

Crane, who is a pioneer in this field, made his first contribution in this area with his Ph.D thesis Microwave Propagation Through a Turbulent Atmosphere.<sup>35</sup> In this he used a formulation where  $\epsilon$ , in contrast to Eq. (35) or to Eq. (47), is not given by

$$\epsilon = K_M (S)^2 \quad (109)$$

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- 34. VanZandt, T.E., Gage, K.S., Warnock, J.M. (1979) A statistical model for the probability of turbulence and the calculation of vertical profiles of turbulence parameters, Proc. Conf. Fourth Symp. on Turbulence, Diffusion, and Air Pollution, Jan. 15-18, 1979, Reno, Nev., pp. 52-55.
  - 35. Crane, R.K. (1969) Microwave Propagation Through a Turbulent Atmosphere, Ph.D thesis, Dept. of Electrical Engineering, Worcester Polytechnic Institute.

(where  $S$  is mean shear). The more accurate relation given by Crane is

$$\epsilon = K_M S^2 - B \quad (110)$$

where  $B$  is the rate of buoyancy dissipation given by

$$B = - \langle \theta' w' \rangle \frac{g}{\bar{\theta}_0} \quad (111)$$

( $w'$  is the vertical velocity fluctuation)

This represents the rate at which energy is given to the potential energy cascade. Using the definition of  $K_\theta$  given by Eq. (16) in a form appropriate for the present situation, that is

$$K_\theta = \frac{-\langle \theta' w' \rangle}{(d\bar{\theta}/dz)} \quad (112)$$

we obtain

$$B = \frac{g}{\bar{\theta}_0} K_\theta \left( \frac{d\bar{\theta}}{dz} \right) = K_\theta N_B^2 \quad (113)$$

where  $N_B$  is the buoyancy frequency defined by

$$N_B^2 = \frac{g}{\bar{\theta}_0} \frac{d\bar{\theta}}{dz} \quad (114)$$

then from Eq. (110)

$$\epsilon = K_M S^2 - K_\theta N_B^2 = K_M (S^2) \left[ 1 - \frac{K_\theta}{K_M} R_i \right] \quad (115)$$

Defining an "eddy Prandtl number" by  $P_\epsilon$  given by

$$P_\epsilon = K_M / K_\theta \quad (116)$$

we have

$$\epsilon = K_M S^2 (1 - P_\epsilon^{-1} R_i) \quad (117)$$

At this point, Crane made a model for  $P_g^{-1}$  based on the work of Ellison.<sup>36</sup> It was

$$P_g^{-1} = (1 + 0.4 R_i)^{-1} \left[ 1 - 0.946 \left( \frac{\tan^{-1}(R_i)}{\pi/2} \right) \right]^{1/2} . \quad (118)$$

In a report to be published,<sup>37</sup> he has simplified this to

$$P_g = 4 R_i . \quad (119)$$

I do not know if these refinements, that is, that of taking buoyancy dissipation into account or  $P_g$  into account, are of much significance; but, this can only be decided later when more refined measurements and models have been made.

The other interesting point made in the work by Crane<sup>32</sup> is that one can estimate the fraction of altitude which is turbulent in the following way.

Let  $\Delta V$  be the difference in velocity across a given height region as measured by means of a radiosonde. The resolution of such a measurement is expected to be sufficiently bad as to be far too smeared out to allow one to find layers where  $R_i < 1/4$  (as was mentioned in the above descriptions of the work of VanZandt et al<sup>3</sup>). To circumvent this difficulty (that is, to estimate what VanZandt et al<sup>3</sup> called  $F$ ), Crane<sup>32</sup> proceeded as follows: Let  $\Delta v$  be the velocity differential across a layer of thickness below the radiosonde height resolution, such that the layer will be unstable and become turbulent. Let

$$\Delta v = \Delta V . \quad (120)$$

This is simply a bald assumption. Next, define  $R_i$  as the gradient Richardson's number based on average vertical potential temperature gradient and mean vertical shear of the horizontal winds, thus

$$R_i = N_B^2 / [\Delta V/h]^2 \quad (121)$$

where  $h$  is the height resolution of the radiosonde. Now, the Richardson number across the actual turbulent layer thickness,  $d$  (where  $d$  cannot be measured directly) is given by

36. Ellison, T. H. (1957) Turbulent transmission of heat and momentum from an infinite rough plane, J. Fluid Mech. 2:456-466.

37. Crane, R.K. (1980) Radar observations of turbulence in the lower stratosphere, Radio Science, special issue (in press).

$$R_l = \frac{N_B^2}{(\Delta v/d)^2} = \frac{1}{4} \quad (122)$$

In other words we assume: (a)  $N_B^2$  across  $d$  and  $h$  are the same; and (b) when  $R_l = \frac{1}{4}$ , the layer will become turbulent. By eliminating  $N_B^2$  from Eqs. (121) and (122) we obtain

$$\left(\frac{d}{h}\right)^2 = \frac{1}{4} (R_l)^{-1} \quad (123)$$

and this is Crane's estimate of  $F$ , that is,  $(d/h)$  is the fraction of  $h$  that he estimates to be turbulent.

How does this compare to the method of VanZandt et al?<sup>3</sup> On one hand it is simple. On the other hand, it appears to be rather arbitrary. With regard to "success," it has not been tested in the same manner as has the other model (that is, via  $\langle C_n^2(\text{model}) \rangle$  vs  $\langle C_n^2(\text{radar}) \rangle$  comparisons). Yet, according to Crane<sup>37</sup> the average value of  $F$  (call it  $\bar{F}$ ) obtained in the above manner from his Millstone data is (for the stratosphere)

$$\bar{F}_{\text{CRANE}} = 0.026 \quad (124)$$

which is very close to that found by VanZandt et al,<sup>3</sup> that is,

$$\bar{F}_{\text{V. Z.}} = 0.03 \quad (125)$$

(Incidentally,  $\bar{F} = 0.1$  for the troposphere.)  
V. Z.

Thus, the recent work of Crane is in average agreement with the very different approach of VanZandt et al<sup>3</sup> in regard to  $F$  calculations. The two approaches thus lend to each other some support.

## 5. SUGGESTIONS FOR FUTURE RESEARCH

### 5.1 Are There Geographic and/or Seasonal Dependencies of $C_n^2$ ?

This is a very interesting and, as yet, unanswered question. Shannon et al<sup>33</sup> have said "the probability of geographic variation can be considered. The four

sites used "(Florida, Colorado, California and Arizona)" were chosen for various specific reasons, however, the statistics of the phase power spectrum values were generally independent of geographic location. Differences noted were due to the diurnal cycle, but not (sic.) consistent differentiation due to season or locality was noted." In other words they did not see a geographic or seasonal dependence of  $C_n^2$ . It would be premature (on the basis of one effort) to declare that in fact these results are true in general. We know that under certain circumstances, geographic location does indeed affect  $C_n^2$ . For example, a telescope sited on a remote mountain has far better "seeing" than one at low altitude and "looking" through the atmosphere over a large city. But, such effects may be due to altitude (and the man-made heat in cities, etc.) and geographic location per se may have no effect. What does affect  $C_n^2$ ? That, of course, is the main unanswered question. Time-of-day effects are accepted at present (as has been already mentioned more than once in this report). The effect of synoptic weather conditions is not yet known.

#### 5.2 What are the Highest Priority Gaps in the $C_n^2$ Estimation Capability via Radiosonde Measurements?

From the above it is clear that the greatest deficiency of the  $C_n^2$  model is to be found at the lower height regions (below 6 to 8 km) and extending all the way to the ground. Improvement of tests of the existing model of VanZandt et al<sup>3</sup> to include the lower altitudes is of high priority due to the fact that the highest values of  $C_n^2$  are also located in this lowest height region. Of course, due to weighting effects, higher altitude  $C_n^2$  can exert a disproportionate influence. As already has been mentioned, some applications do not involve lower altitudes. Nevertheless, a comprehensive model would be desirable. As has also been mentioned, however, work is already in progress to improve the model itself. (At present, however, there is no work being done which takes into account the boundary layer effects upon  $C_n^2$ .)

#### 5.3 Experimental Suggestions

1. The report by Shannon et al<sup>33</sup> used laser sources mounted on a U-2 aircraft and measuring devices on the ground. In this way they ascertained optical information about the  $C_n^2$  properties of the atmosphere between the source and receiver. The integrated values of  $C_n^2$  could then be compared with radar measurements. Since this report represented the only comparison between radar and optical measurements that I could find, it would appear to be an important source of information regarding future experiments. One of the most important facts to emerge from this report was that, "It is in fact a conclusion of this study that no further attempts should be made to obtain atmospheric data from thermosondes."

Thus, unless there were an attempt to advance the thermosonde technology (addressed to the problems encountered by Shannon et al<sup>33</sup>) one should avoid making such measurements. On the other hand, it may be possible to overcome the problems mentioned provided the difficulties can be pinpointed and appropriately handled without undue cost.

2. Simultaneous radar, radiosonde and smoke trail measurements would be desirable. Since the work of VanZandt et al<sup>3</sup> has relied completely upon "micro-shears" or high wind shears below the resolution of the radiosondes, it would be important to measure high resolution shears with the aid of smoke trails. Our laboratory has developed a technique for measuring vertical wind shears of horizontal winds in the stratosphere by means of time lapse photography and computerized triangulation methods in conjunction with smoke trails. The same technology could be used to obtain wind shears in the troposphere as well. While rockets were previously needed in order to lay the trails in the stratosphere, techniques involving balloon-borne smoke devices (subsequently dropped) could be used to measure the actual microshears. Above the boundary layer (1-3 km) and in the absence of weather fronts, storms, etc., the troposphere behaves in a way quite analogous to the stratosphere. The radiosonde and radar observations could be used to calculate  $C_n^2$  in the manner described in Section 4 (or by a more advanced method if available at the time) and one could compare measured microshears to the theoretical microshears. One anticipated problem is that in the troposphere, eddy-diffusion rates would be larger than in the stratosphere. This may affect the time history of the smoke trails in an adverse way. For example, if the eddy diffusivity were very large, the trail might spread too fast to allow an accurate measurement. This would have to be calculated in the design of the experiment. Values of  $C_n^2$  would also be available from measured microshears and radiosonde temperature profiles.

3. Low-altitude radar measurements (below 5 km) which cannot be made with dipole-array type radar instruments would be available from steerable dish radars, such as the one located at Millstone (Lincoln Laboratory). It would be of importance to actually make such measurements in conjunction with radiosonde measurements to see how the  $\langle C_n^2 \text{ (radar)} \rangle$  and  $\langle C_n^2 \text{ (balloon)} \rangle$  actually compare. Since specific humidity is of paramount importance at the lowest altitudes, it would be necessary to pay special attention to the precision of the humidity-measuring instruments.

4. It would be desirable to have in situ measurements of  $C_n^2$  as measured from a balloon-borne device. Since, it is not yet obvious that thermosondes are good for this purpose, it would be very valuable if the precise causes (of the in-situ measurement problem) were ascertained. It would be an important breakthrough if a technique were devised so that such measurements could be rendered possible.

A suggested cause of thermosonde artifact is that the balloon rises with such high velocity that the thermosonde measures only wake effects. If this is correct, then it might imply that the thermosonde device itself might be adequate but that a modified balloon vehicle would be needed. In other words, it may be necessary to construct a small balloon of very slow ascent (or descent) in order to make in situ  $C_n^2$  measurements. Electronic problems also seem to exist as well and these too would have to receive attention.

5. Research on the nature of clear air turbulence at all altitudes is needed. This would represent the only hope for a method to predict  $C_n^2$  variations. Along these lines, a report on the nature of CAT was put out by this organization<sup>38</sup> which indicated that aircraft in situ measurements of supposed turbulence may often measure gravity waves instead of turbulence. This may imply that gravity waves, perhaps trapped gravity waves, are the cause of the turbulence (and associated  $C_n^2$ ). If this were the case, then predictive capability may be made possible from better information on the source of gravity waves. Radar observations of CAT in the troposphere and stratosphere over long periods of time and with high resolution (preferably of order 100 m or better) would be able to answer many of the presently unanswered questions along the lines of: (a) Are there preferred altitude regions for turbulence to occur? (b) How long do these last? (c) Does the  $C_n^2$  pulsate as if the turbulence were due to shears generated by a traveling wave (or waves)? (d) From the specular reflection of stable layers can one ascertain if there are regions of gravity waves without turbulence? (e) From radar measurements of velocity, can the wind profiles "trap" observed waves?

Many other experiments along these lines are suggested by the observations of Woods and Wiley<sup>39</sup> in the upper ocean such as "are there ensembles of turbulent layers?" etc. Perhaps high-resolution radar measurements are the only feasible ones at present to investigate such questions; however, balloon-borne anemometers must also be given due consideration.

## 6. CONCLUSIONS

Optical turbulence is a crucial factor in the design of some of the laser defense systems now under development. An improved data base is therefore needed in this connection as well as a better physical understanding of the nature of the phenomenon itself.

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38. Dewan, E. M. (1979b) Stratospheric wave spectra resembling turbulence, Science 204:832.

39. Woods, J. D., and Wiley, R. L. (1972) Billow turbulence and ocean micro-structure, Deep Sea Res. 19:87-121.

This subject is somewhat urgent due to the fact that turbulence compensation systems, if underdesigned, will be unreliable, whereas if overdesigned will incur very great but unnecessary financial cost as well as delay in acquisition.

It is hoped that the preceding pages will acquaint the reader with some of the issues on this subject and introduce him to some of the literature.

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## Appendix A

### Mathematical Approach of Tatarski

In the text, a physical approach was given in order to derive  $C_\theta^2$  in terms of  $\varepsilon_N$  and  $\varepsilon$ . A more basic and exact approach is available, and in this appendix the latter is described. We shall be following, in a simplified way, the treatment given by Tatarski.<sup>1</sup> Our objective will be to arrive at the following relation

$$K_\theta (\text{grad } \bar{\theta})^2 = \nu_\theta \langle (\text{grad } \theta')^2 \rangle \quad (\text{A1})$$

where  $\theta$ , as in the text, corresponds to an additive scalar,  $\theta'$  is the fluctuation value,  $\bar{\theta}$  the average value, and  $\langle \rangle$  is also a symbol for averaging. Thus

$$\theta \equiv \bar{\theta} + \theta' \quad (\text{A2})$$

The starting point for this treatment is the molecular diffusion equation

$$\frac{d\theta}{dt} + \text{div} (-\nu_\theta \text{grad } \theta) = 0 \quad (\text{A3})$$

where  $\nu_\theta$  is the molecular diffusion coefficient for  $\theta$  and

$$\frac{d\theta}{dt} \equiv \frac{\partial \theta}{\partial t} + \vec{v} \cdot \text{grad } \theta \quad (\text{A4})$$

is the mobile derivative. Assuming incompressibility, which in the present case is permitted, we take

$$\text{div } \vec{v} = 0 \quad (\text{A5})$$

therefore

$$\vec{v} \cdot \text{grad } \theta = \text{div } (\vec{v} \theta) \quad (\text{A6})$$

and

$$\frac{\partial \theta}{\partial t} + \text{div } (\vec{v} \theta - \nu_{\theta} \text{grad } \theta) = 0 \quad (\text{A7})$$

The velocity can be decomposed in the manner of Eq. (A2)

$$v_i = \bar{v}_i + v_i' \quad (\text{A8})$$

where we now resort to cartesian vector notation. By definition

$$\langle v_i' \rangle = 0 ; \langle \theta' \rangle = 0 ; v_i = \langle v_i \rangle ; \bar{\theta} = \langle \theta \rangle . \quad (\text{A9})$$

Inserting Eq. (A2) and Eq. (A8) into Eq. (A7) and averaging (making use of Eq. (A9)) one arrives at

$$\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial}{\partial x_i} \left( \bar{v}_i \bar{\theta} + \langle v_i' \theta' \rangle - \nu_{\theta} \frac{\partial \bar{\theta}}{\partial x_i} \right) = 0 \quad (\text{A10})$$

The quality within the parentheses in Eq. (A10) is the flux of  $\theta$ . The quantity

$$\vec{q}_m \equiv -\nu_{\theta} \text{grad } \bar{\theta} \quad (\text{A11})$$

is that part of the flux of  $\theta$  which is due to the molecular diffusion,

$$\vec{q}_a \equiv \bar{\vec{v}} \bar{\theta} \quad (\text{A12})$$

is the flux of  $\theta$  due to the mean flow and

$$\vec{q}_T \equiv \langle \vec{v}' \theta' \rangle \quad (\text{A13})$$

is the turbulent flux of  $\theta$ . Notice that we switch from vector notation to component notation as convenience dictates in this treatment. As in the text we define the eddy diffusion coefficient by  $\theta$  by

$$\bar{q}_T = -K_\theta \text{grad } \bar{\theta} \quad (\text{A14})$$

Next, a measure of fluctuation inhomogeneity,  $G$ , is defined by

$$G = \frac{1}{2} \int_V \langle \theta'^2 \rangle dV \quad (\text{A14})$$

If  $\langle \rangle$  were zero everywhere inside of volume  $V$ , then  $G = 0$ . Subtracting Eq. (A10) from Eq. (A7) one can obtain the equation for the time rate of change of  $\theta'$ . First we use Eq. (A2) and Eq. (A8) in Eq. (A7) before carrying out the subtraction in order to decompose it into

$$\frac{\partial(\bar{\theta} + \theta')}{\partial t} - \frac{\partial \bar{\theta}}{\partial t} + \frac{\partial}{\partial x_i} \left[ (\bar{v}_i + v'_i)(\bar{\theta} + \theta') - \nu_\theta \frac{\partial(\bar{\theta} + \theta')}{\partial x_i} - \bar{v}_i \bar{\theta} - \langle v'_i \theta' \rangle + \nu_\theta \frac{\partial \bar{\theta}}{\partial x_i} \right] = 0 \quad (\text{A16})$$

and, using Eq. (A10), that is, subtracting the average equation, we obtain

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_i} \left[ v'_i \bar{\theta} + \bar{v}_i \theta' + v'_i \theta' - \langle v'_i \theta' \rangle - \nu_\theta \frac{\partial \theta'}{\partial x_i} \right] = 0 \quad (\text{A17})$$

At this point, we use  $v_i = \bar{v}_i + v'_i$  in the coefficient of  $\theta'$  to simplify Eq. (A17)

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_i} \left[ v'_i \bar{\theta} + \theta' v_i - \langle v'_i \theta' \rangle - \nu_\theta \frac{\partial \theta'}{\partial x_i} \right] = 0 \quad (\text{A18})$$

We may now employ Eq. (A18) to obtain the equation of motion for the quantity  $\langle \theta'^2 \rangle$ . The average mean square of the fluctuation is the crucial quantity in turbulence studies. Multiply Eq. (A18) by  $\theta'$

$$\theta' \frac{\partial \theta'}{\partial t} + \theta' \frac{\partial}{\partial x_i} \left[ v'_i \bar{\theta} + \theta' v_i - \langle v'_i \theta' \rangle - \nu_\theta \frac{\partial \theta'}{\partial x_i} \right] = 0 \quad (\text{A19})$$

and employ the following relations from elementary calculus

$$\theta' \frac{\partial \theta'}{\partial t} = \frac{\partial}{\partial t} \left( \frac{1}{2} \theta'^2 \right) \quad (\text{A20})$$

$$\theta' \frac{\partial}{\partial x_i} (v_i' \theta') = \frac{\partial}{\partial x_i} (v_i' \frac{1}{2} \theta'^2) \quad (A21)$$

$$\theta' \frac{\partial}{\partial x_i} (v_i' \bar{\theta}) = \theta' v_i' \frac{\partial \bar{\theta}}{\partial x_i} \quad (A22)$$

where  $v_i'$  is assumed homogeneous (that is, independent of  $x_i$ ) in the derivation of the above. For reasons which will be more apparent later, we need also an expression for  $\theta' \frac{\partial}{\partial x_i} \left( \nu_\theta \frac{\partial \theta'}{\partial x_i} \right)$  which is in the form related to a divergence. This is

$$\theta' \frac{\partial}{\partial x_i} \left[ \nu_\theta \frac{\partial \theta'}{\partial x_i} \right] = \frac{\partial}{\partial x_i} \left[ \nu_\theta \theta' \frac{\partial \theta'}{\partial x_i} \right] - \left( \frac{\partial \theta'}{\partial x_i} \right)^2 \nu_\theta \quad (A23)$$

To show this to be correct one must carry out the divergence on the right hand side and regard  $\nu_\theta$  as independent of  $x_i$ . When Eq. (A20) to Eq. (A23) are all substituted into Eq. (A19), we obtain, resorting again to vectors,

$$\begin{aligned} \frac{\partial}{\partial t} \left( \frac{1}{2} \theta'^2 \right) + \text{div} \left[ \frac{\bar{v}}{2} \theta'^2 - \nu_\theta \theta' \text{grad } \theta' \right] + \nu_\theta (\text{grad } \theta')^2 + \theta' v_i' \text{grad } \bar{\theta} \\ - \theta' \text{grad } \langle \bar{v}' \theta' \rangle = 0 \end{aligned} \quad (A24)$$

Eq. (A24) must now be averaged, and, in doing so we make use of a relation

$$\left\langle \theta' \left[ \frac{\partial}{\partial x_i} \langle v_i' \theta' \rangle \right] \right\rangle = 0 \quad (A25)$$

This follows from

$$\left\langle \theta' \left[ \frac{\partial}{\partial x_i} \langle v_i' \theta' \rangle \right] \right\rangle = \langle \theta' \rangle \frac{\partial}{\partial x_i} \langle v_i' \theta' \rangle \quad (A26)$$

in conjunction with  $\langle \theta' \rangle = 0$ .

Averaging Eq. (A24) with the use of Eq. (A25) we obtain the relation for  $\langle \theta'^2 \rangle$  that we sought, namely

$$\begin{aligned} \left( \frac{1}{2} \right) \frac{\partial \langle \theta'^2 \rangle}{\partial t} + \text{div} \left[ \frac{1}{2} \langle \bar{v} \theta'^2 \rangle - \nu_\theta \langle \theta' \text{grad } \theta' \rangle \right] + \langle \theta' \bar{v}' \rangle \cdot \text{grad } \bar{\theta} \\ + \nu_\theta \langle (\text{grad } \theta')^2 \rangle = 0 \end{aligned} \quad (A27)$$

One integrates Eq. (A27) over a volume  $V$  and makes use of the well-known fact that the volume integral of a divergence is equal to a surface integral of the normal flux. Assuming the turbulence to be homogeneous,\* the net flux integrates out to zero over the surface, and therefore

$$\frac{\partial G}{\partial t} + \int_V \left\{ \langle \theta' \vec{v}' \rangle \frac{\partial \bar{\theta}}{\partial x_i} + \nu_\theta \left\langle \left( \frac{\partial \theta'}{\partial x_i} \right)^2 \right\rangle \right\} dV = 0 \quad (A28)$$

where we have used

$$\langle \theta' \vec{v}' \rangle = 0 \quad (A29)$$

in the second term (of (A27)) (that is, the total vector  $\vec{v}$  is replaced by  $\vec{v}'$ ).

Recalling Eq. (A14), the definition of  $K_\theta$ , and the definition of  $q_T$  given by Eq. (A13) we obtain

$$\frac{\partial G}{\partial t} = \int_V [K_\theta (\text{grad } \bar{\theta})^2 - \nu_\theta \langle (\text{grad } \theta')^2 \rangle] dV = 0 \quad (A30)$$

Next we assume stationarity or

$$\frac{\partial G}{\partial t} = 0 \quad (A31)$$

and continue to assume that the turbulence in the volume we are considering is homogeneous throughout that volume. This takes us from Eq. (A30) to

$$K_\theta (\text{grad } \bar{\theta})^2 = \nu_\theta \langle (\text{grad } \theta')^2 \rangle \quad (A32)$$

which is the objective we were seeking.

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\*Alternatively, the surface can be located outside of the turbulent region.

## Appendix B

### Discussion of the Analytic Approach to Inertial Range Turbulence

In the text, the commonly used dimensional argument was employed to derive the  $r^{2/3}$  law for the structure function. This leads, via an approach which uses the Fourier transform, to the  $k^{-5/3}$  law for the inertial range power spectrum. The reader, however, may have mistakenly received the impression that no direct, analytic approach exists which arrives at this result starting from the Navier Stokes equations of fluid motion. He may also not be aware of the experimental evidence which has conclusively established this law. The purpose of this appendix is to elaborate these two facts.

With regard to the experimental evidence, the classic example is to be found in Grant, H. L., Stewart, R. W. and Moillet, A. (1962) Turbulence spectra from a tidal channel, J. Fluid Mech. 12:241. For recent references with exhaustive references to early work see Gibson, Stegun and Williams (1970), J. Fluid Mech. 41:153; Boston and Burling (1972) J. Fluid Mech. 55:473; and Wyngaard and Pao (1972) Statistical Models of Turbulence, Springer Verlag, Ed. by Rosenblatt and Van Atta, pp 384-401. The refined version of Kolmogorov's  $k^{-5/3}$  theory seems to be experimentally well confirmed at this time.

Regarding the analytic approach, the most successful is that of Kraichnan; and, a book by Leslie (1973) Developments in the Theory of Turbulence, Clarendon Press, reviews this approach in detail. The most recent paper by Kraichnan along these lines seems to be Kraichnan (1974) On Kolmogorov's inertial-range theories, J. Fluid Mech. 62:305-330. Reviews will be found in (a) Orszag (1977) Lectures

on the statistical theory of turbulence in Fluid Dynamics edited by Ballan and Peube, Gordon and Breach; and (b) Leslie (1973) Review of developments in turbulence theory in Rep. Prog. Phys. 36:1365-1424.

The analytic approach leads to a constant of proportionality for the  $k^{-5/3}$  law of 1.77 which compares favorably with the experimental value of 1.5 in view of the scatter in the data.

For a one page summary of Kraichnan's remarkable achievement the reader may consult Hinze (1975) Turbulence, McGraw Hill, Inc., pp 249-250.

## Appendix C

### A Guide to Further Reading

The following bibliography overlaps some of the references; but, the present list is singled out for annotation because it contains key reviews with many references and pieces of important information. In other words, these would be of high priority to those readers of this tutorial who wish to read further on this subject. The list is biased very strongly in the direction of atmospheric physics rather than optical physics, but some of the reviews will nevertheless help to lead the optical reader to the literature he seeks. All of the articles are widely available.

#### A. Books and Reviews

1. "Vertical profiles of refractivity turbulence structure constant: Comparison of observations by the Sunset Radar with a new theoretical model" by T. E. VanZandt, J. L. Green, K. S. Gage and W. L. Clark in Radio Science 13:819-829 (1978).

This is the article referenced in Section 4 as "VanZandt et al" repeatedly. In my opinion it is the most advanced model available in the literature for relating radiosonde data to radar data. The beauty of the model is the combination of success and simplicity.

2. Wave Propagation in a Turbulent Medium by Tatarski (McGraw-Hill, 1961). This is "the classic" on atmospheric turbulence effects on radio wave, radar and light propagation. It even includes effects on acoustic wave propagation (which is useful for acoustic  $C_n^2$  measurements if one were to consider them). This reference is required for anyone working in this field.

3. Laser Beam Propagation in the Atmosphere edited by Strohbehn (Springer-Verlag, 1978). This reference includes review articles by several experts. For lists of references on  $C_n^2$  for both atmospheric turbulence and optical effects, it is probably the best and the most up to date. The present report references this book very frequently (for example, the "blob lens approach").

4. Laser Communications Systems, Pratt, W.K. (John Wiley & Sons, 1969). This book is written at a simple and concise level, and it reviews mainly the optical and electronic sides of the problem. Someone who wanted to know the "hardware side" of  $C_n^2$  dependent systems in a short time would do well to look over this book.

5. Wave Propagation in a Random Medium by Chernov (1960). This is a classic from Russia which is often quoted in conjunction with Tatarski. It seems to go into more detail in regard to the electromagnetic side of the  $C_n^2$  picture.

6. The Effects of the Turbulent Atmosphere on Wave Propagation by Tatarski, Israel Program for Scientific Translations, Jerusalem, 1971 (available from NTIS TT68-50464). This is a more comprehensive and updated version of No. 2 above.

7. "Optical propagation through the turbulent atmosphere" in Progress in Optics, Vol. 9, 1971 edited by Wolf, Ch. III. Also, Optical and Millimeter Line-of-sight Propagation Effects in the Turbulent Atmosphere in Boundary-Layer Meteorology 4, 1973, 397-422.

Both of these review articles are by Strohbehn who edited reference no. 3 above. He seems to be the foremost reviewer in this field.

8. "Consideration of atmospheric turbulence in laser systems design" by J. I. Davis in Applied Optics, Vol. 5, pp 139-146. This is the main reference used by Pratt (reference 4 above) in his one chapter on the  $C_n^2$  effects of the atmosphere. The present report relied on it heavily.

9. Two issues of Radio Science have appeared which contain a very large number of articles on atmospheric turbulence and its effects on radio wave propagation ( $C_n^2$ ). These are Radio Science 4, 1969, Vol. 12, and Radio Science 10, 1975, #1.

These are both special issues which are exclusively devoted to the present subject.

10. Refraction, Attenuation and Backscattering of Electromagnetic Waves in the Troposphere: A Revision of Chapter 9, Handbook of Geophysics and Space Environments, by V.J. Falcone, Jr. and R. Dyer.

This is an in-house report; however, the "handbook" is in the open literature and the number for this report is AFCRL 70-0007 Jan. 1970 for those who have access to DDC or who would wish to write to those authors. This report is a short and authoritative review.

## B. Important Articles

11. "The birth of 'CAT' and microscale turbulence" by D. Atlas, J. Metcalf, J. Richter and E. Gossard in Journal of the Atmospheric Sciences 27, 1970, pp 903-913)

This is a classic because it seems to be the first published observation of CAT (seen as CAT) by means of radar.

12. "Structure of the atmosphere in the vicinity of large amplitude Kelvin-Helmholtz billows," by K. Browning in J. Roy. Met. Soc. Quart 97:283-299 (1971).

This is a classic article because it is the first to use a combination of radar and balloon measurements to observe Kelvin-Helmholtz shear-induced turbulent layers. Their radiosonde work was of such high resolution that they could "watch" the  $R_i$  number go below  $1/4$  and then see the turbulence occur subsequently on the radar (about one half hour after  $R_i$  went below critical). On occasion  $R_i$  went back up above  $1/4$  before the turbulence had a chance to develop. When the mixing occurred, the subsequent balloon observations of the temperature clearly showed the effect of this mixing. In all, 17 mixing events were observed.

13. "Billow turbulence and ocean microstructure" by J. P. Woods and R. L. Wiley in Deep Sea Research, 1972, pp 87-121.

I regard this paper as the description of an analogue computer model of stratospheric turbulence and CAT in the troposphere. A careful reading of this paper will suggest many things to a person interested in the nature of atmospheric turbulence.

14. "The Dynamics of the Upper Ocean" by O. M. Phillips, Cambridge Univ. Press, 1977, 2nd edition. Chapter 5 on internal waves is crucial to an understanding of certain key factors in atmospheric buoyancy (gravity) waves and turbulence.

15. "Turbulence in stably stratified fluids: A review of laboratory experiments" by S. A. Thorpe in Boundary Layer Met. 5 (1973) (p 94-119). This is an excellent review of the subject.

16. "Instability and turbulence in a stratified fluid with shear" by Koop and Browand in J. Fluid Mech. 93 (1979) pp 135-153. This is the most recent work, I think, on laboratory studies of the shear instability.

17. The following reports that I have published are of interest in connection with the physical nature of clear air turbulence.

Rosenberg, N. and Dewan, E., Stratospheric Turbulence and Vertical Effective Diffusion Coefficients, AFCRL-TR-75-0519. This paper has been quoted by several authors whose work is in the radar detection of turbulence such as VanZandt et al, Balsley et al, Crane, and finally Evans. It provides a review of clear air turbulence structure and physics, and it contains useful references.

Dewan, E. "Stratospheric wave spectra resembling turbulence," Science 204:832-835 (1979). This paper gives evidence and theory for the possibility that waves and turbulence may be parts of a single cascade phenomenon.

Of some possible use for background information are "Mixing in Billow Turbulence and Stratospheric Eddy Diffusion," AFGL-TR-79-0091, "Estimates of Vertical Eddy Diffusion Due to Turbulent Layers in the Stratosphere," AFGL-TR-79-0042, and "Theoretical Explanation of Spectral Slopes in Stratospheric Turbulence Data and Implications for Vertical Transport," AFGL-TR-76-0247.