



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

ADA087005

11-11-88

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER NRL Memorandum Report 14264	2. GOVT ACCESSION NO. 10-4087005	3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) A NON-RECURSIVE INCOMPLETE CHOLESKY DECOMPOSITION METHOD FOR THE SOLUTION OF LINEAR EQUATIONS WITH A SPARSE MATRIX	5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing NRL problem.	6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s) K. Hain	8. CONTRACT OR GRANT NUMBER(s) Memorandum rpt.		
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 627044; DNA-S99QAXH; 066/12; W.U.12; 67-0892-0-0		
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Nuclear Agency Washington, D.C. 20305	12. REPORT DATE June 10, 1980	13. NUMBER OF PAGES 25	14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12-76
	15. SECURITY CLASSIFICATION (of this report) UNCLASSIFIED	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. 18 SBIH			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) 19 AD-E000 4821			
18. SUPPLEMENTARY NOTES This research was sponsored by the Defense Nuclear Agency under Subtask S99QAXHC066, work unit 12, and work unit title "Late Time Electrostatic Investigations."			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Solution of linear equations Sparse matrix Non-recursive operations Electrostatic potential solutions			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The incomplete Cholesky decomposition and the subsequent iterative solution by the conjugate gradient method has been described recently by D. Kershaw [1]. The drawback of a triangular decomposition on a vector machine is the need for recursive computations. This paper proposes a method which eliminates the need for recursive computations. They are replaced by a number of non-recursive operations. This method can be utilized in the solution of potential equations in late time electrostatic codes.			

20-1754

CONTENTS

I. INTRODUCTION 1

II. ALGORITHM 1

 1) General considerations 1

 2) Expansion in connections 3

 3) Example 7

 4) The conjugate gradient method 9

 5) Non-symmetric matrices 10

III. NUMERICAL RESULTS 11

IV. CONCLUSION 15

V. ACKNOWLEDGEMENT 15

VI. REFERENCES 16

VII. DISTRIBUTION LIST 17

ACCESSION for		
NTIS	White Section	<input checked="" type="checkbox"/>
DDC	Buff Section	<input type="checkbox"/>
UNANNOUNCED		<input type="checkbox"/>
JUSTIFICATION _____		
BY _____		
DISTRIBUTION/AVAILABILITY CODES		
Dist.	AVAIL. and/or	SPECIAL
A		

A NON-RECURSIVE INCOMPLETE CHOLESKY DECOMPOSITION METHOD FOR THE SOLUTION OF LINEAR EQUATIONS WITH A SPARSE MATRIX

I. Introduction

The algorithm described in this paper is applicable to diagonally dominant matrices. It makes use of the fact that an incomplete Cholesky or LU decomposition which imposes a certain sparsity is actually an expansion in powers of off diagonal elements. It is therefore pointless to compute the elements of the tridiagonal matrices and the solution to infinite order by recursive procedures. It is self-consistent and sufficient to compute these quantities to the same order as the one introduced by the imposed sparsity of the tridiagonal decomposition.

The first section describes the method for symmetric matrices. The extension to non-symmetric matrices is achieved by iterating the solutions for the symmetric component of the matrix where the source terms contain the non-symmetric contributions from the previous iteration.

The second section gives the times and number of iterations used for a test case taken from a NRL electrostatic code. It involves the solution of an elliptic partial differential equation with variable coefficients in the two dimensions.

The third section gives some conclusions.

II. Algorithm

1) General considerations

The linear system of equations to be solved for the vector x can be written in the form

$$Mx = y \quad (1)$$

x, y are vectors of length N and $\text{Det}(M) \neq 0$.

In order to explain the following concepts one must introduce the definition of weakly and strongly diagonally dominant matrices. A matrix A is defined to be weakly diagonally dominant if

$$W_{ij}(A) \equiv 1 - \frac{|A_{ij}|}{\min(|A_{ii}|, |A_{jj}|)} \geq 0 \quad (2)$$

for all $i, j; i \neq j$

A matrix A is defined to be strongly diagonally dominant if

$$S_j(A) \equiv 1 - \sum_{k \neq j} (|A_{jk}| |A_{kj}| / |A_{kk}|) / |A_{jj}| \geq 0 \quad (3)$$

for all j .

Let L and U be two lower and upper triangular matrices respectively, subject to the following two conditions

$$a) \quad W(L^{-1}MU^{-1}) \geq W(M) \geq 0 \quad (4)$$

b) The inversions

$$z = L^{-1}y$$

$$x = U^{-1}z \quad (5)$$

with z a vector of length N can be performed exactly.

Equation (1) can be transformed into

$$L^{-1}MU^{-1}(UX) = L^{-1}y \quad (6)$$

It is obvious that Eq. (6) is easier to solve than Eq. (1). Any iteration scheme will use fewer iterations. One should also remark that neither L , U , or L^{-1} , U^{-1} have to be known, but only the results of L^{-1} , U^{-1} applied to a vector.

The real question is how one finds good matrices L , U with a minimum of operations, such that Eq. (6) can be solved with a few

iterations. This is especially important in time-dependent problems where a good approximation to the solution is known from the previous timestep. It is clear that with more operations one could find a better L and U, thereby reducing the number of iterations needed to solve Eq. (6). The problem is to find an algorithm which minimizes the total number of operations.

For most physical problems (elliptic equations in two or three dimensions) the matrix M is sparse with an (average) bandwidth $B \ll N$. The number of operations Op should be

$$Op \approx B \cdot N f(N) \quad (7)$$

where $f(N)$ is a weakly increasing function of N . Also, the algorithm should not contain recursive formula, which invoke scalar operations on a vector computer (ASC, CRAY, . . .) or parallel computer (ILLIAC).

2) Expansion in connections

In order that the following approximation for L and U be valid the matrix M must have two properties

- a) the average bandwidth $B \ll N$
- b) M must be expandable.

Condition b) will be explained now.

A connection of v^{th} order is to be defined as

$$\epsilon_{ik}^{v+1} = \sum_{\substack{j \neq i \\ j \neq k}} \epsilon_{ij}^v M_{jk} / M_{jj} ; v \geq 1 \quad (8)$$

with

$$\epsilon_{ik}^1 = M_{ik} \quad (9)$$

Then M is expandable if for all ν

$$|\epsilon_{ik}^{\nu+1}| \leq \max_j (|\epsilon_{ij}^\nu|, |M_{jk}|) \quad (10)$$

It is obvious that the condition of strong diagonal dominance

$$S(M) > 0$$

is sufficient. Furthermore, if M is expandable then

$$W(M) > 0$$

M is weakly diagonally dominant. Therefore, the necessary and sufficient conditions for expandability lie somewhere between weak and strong diagonal dominance. The computation of L, U will be an expansion in the number of connections, thus avoiding recursion procedures. In general, given a recursion formula

$$\mu_i = \sigma_i + \epsilon \mu_{i-1} \quad i = 1, n \quad (11)$$

with $0 < \epsilon < 1$.

The iterative formula

$$\begin{aligned} \eta_i^\nu &= \sigma_i + \epsilon \eta_{i-1}^{\nu-1} & i &= 1, n \\ \eta_i^0 &= \sigma_i \end{aligned} \quad (12)$$

will give the same results after n iterations. Terminating after the ν^{th} iteration gives the results to order $\epsilon^{\nu+1}$. All recursion formula which do occur will be replaced by an iteration with a very small ν . The matrices L, U will be computed to a certain order in the expansion in the number of connections. In order to explain this procedure the triangular decomposition is briefly described here without proof.

Let

$$1/D_j = L_{jj} = U_{jj} = M_{jj} - \sum_{k < j} L_{jk} D_k U_{kj}$$

$$L_{ji} = 0 \quad j < i$$

$$L_{ji} = M_{ji} - \sum_{k < i} L_{jk} D_k U_{ki} \quad j \geq i$$

$$U_{ji} = 0 \quad j > i$$

$$U_{ji} = M_{ji} - \sum_{k > i} L_{jk} D_k U_{ki} \quad j \leq i. \quad (13)$$

Then

$$M_{ij} = \sum_k L_{ik} D_k U_{kj} \quad (14)$$

and one can solve

$$z_i = D_i (y_i - \sum_{k < i} L_{ik} z_k)$$

$$x_i = z_i - \sum_{k > i} U_{ik} D_k x_k \quad (15)$$

directly.

The incomplete Cholesky or incomplete LU decomposition consists of imposing a specified sparsity on L, U. The order of approximation is

$$\eta = v_{(s)} + 1 \quad (16)$$

where $v_{(s)}$ is the highest possible connectivity given by the sparsity imposed on L and U (or η is first one neglected).

The recursion formulae in computing L, U (Eq. (13)) are reset by iteration to the order μ , which leaves an error of $O(\mu + 1)$. These iterations can be written as follows

$$\begin{aligned}
 1/D_i^\nu &= M_{ji} - \sum_{k=1}^{i-1} L_{ik}^{\nu-1} D_k^{\nu-1} U_{ki}^{\nu-1} \\
 L_{ji}^\nu &= M_{ji} - \sum_{k=1}^{j-1} L_{jk}^{\nu-1} D_k^{\nu-1} U_{ki}^{\nu-1} \quad \nu = 1, \mu \\
 U_{ji}^\nu &= M_{ji} - \sum_{k=1}^{j-1} L_{jk}^{\nu-1} D_k^{\nu-1} U_{ki}^{\nu-1}
 \end{aligned} \tag{17}$$

with

$$\begin{aligned}
 1/D_i^0 &= M_{ii} \\
 L_{ji}^0 &= M_{ji} \quad j \geq i \\
 U_{ji}^0 &= M_{ji} \quad j \leq i \\
 z_i^\nu &= D_i^\mu (y_i - \sum_{k=1}^{i-1} L_{ik}^\mu z_k^{\nu-1}) \\
 x_i^\nu &= z_i^\mu - \sum_{k=1}^{i-1} U_{ik}^\mu D_k^\mu x_k^{\nu-1} \quad \nu = 1, \mu
 \end{aligned} \tag{18}$$

with

$$\begin{aligned}
 z_i^0 &= D_i^\mu y_i \\
 x_i^0 &= z_i^\mu
 \end{aligned} \tag{19}$$

This procedure gives a matrix

$$L^{-1} M U^{-1}$$

whose off diagonal elements consists of errors of order $\nu_{(S)} + 1$ left by imposing a specified sparsity S , and errors of order $\mu + 1$ caused by resetting the recursion formula by iteration.

3) Example

As an example the five point difference formula for an elliptic equation in two dimensions is taken. M at one point, j, k is then

$$M = \begin{array}{ccc|c} & -\mu_y & & k-1 \\ -\mu_x & 1 & -\mu_x & k \\ & -\mu_y & & k+1 \\ \hline j-1 & j & j+1 & \end{array} \quad (20)$$

Then imposing the same sparsity on L and U and for $\nu = 1, 2$ gives:

$$L^{(\nu)} = \begin{array}{ccc|c} & 0 & & k-1 \\ 0 & 1/D^{(\nu)} & -\mu_x & k \\ & -\mu_y & & k+1 \\ \hline j-1 & j & j+1 & \end{array} \quad (21)$$

$$U^{(\nu)} = \begin{array}{ccc|c} & -\mu_y & & k-1 \\ -\mu_x & 1/D^{(\nu)} & 0 & k \\ & 0 & & k+1 \\ \hline k-1 & k & k+1 & \end{array} \quad (22)$$

with $D^{(1)} = 1$ $D^{(2)} = \frac{1}{1-\mu_x^2-\mu_y^2}$ (23)

This yields:

$$(L^{-1}MU^{-1})^{(1)} = \begin{array}{cccc|c}
 0 & 0 & -\mu_y^2 & 0 & 0 & k-2 \\
 0 & -2\mu_x\mu_y & 0 & -\mu_x\mu_y & 0 & k-1 \\
 -\mu_x^2 & 0 & 1-\mu_x^2\mu_y^2 & 0 & -\mu_x^2 & k \\
 0 & -\mu_x\mu_y & 0 & -2\mu_x\mu_y & 0 & k+1 \\
 0 & 0 & -\mu_y^2 & 0 & 0 & k+2 \\
 \hline
 j-2 & j-1 & j & j+1 & j+2 &
 \end{array} \quad (24)$$

This is not a good approximation to a unity matrix. Therefore, one more iteration step has to be taken;

$$(L^{-1}MU^{-1})^{(2)} = \begin{array}{cccccc|c}
 0 & 0 & 0 & -\mu_y^3 & 0 & 0 & 0 & k-3 \\
 0 & 0 & -3\mu_y^2\mu_x & 0 & -\mu_y^2\mu_x & 0 & 0 & k-2 \\
 0 & -3\mu_y\mu_x^2 & 0 & -\mu_y\mu_x^2 & -\mu_y\mu_x & -\mu_y\mu_x^2 & 0 & k-1 \\
 -\mu_x^3 & 0 & -\mu_y^2\mu_x & 1 & -\mu_y^2\mu_x & 0 & -\mu_x^3 & k \\
 0 & -\mu_y\mu_x^2 & -\mu_y\mu_x^2 & -\mu_y\mu_x^2 & 0 & -3\mu_y\mu_x^2 & 0 & k+1 \\
 0 & 0 & -\mu_y^2\mu_x & 0 & -3\mu_y^2\mu_x & 0 & 0 & k+2 \\
 0 & 0 & 0 & -\mu_y^3 & 0 & 0 & 0 & k+3 \\
 \hline
 j-3 & j-2 & j-1 & j & j+1 & j+2 & j+3 &
 \end{array}$$

This is a much better approximation to the unity matrix correct to terms of third order, except for the off diagonal terms $\mu_x\mu_y$ caused by the sparsity imposed on L and U. A complete

recursion would give only the expansion of those off diagonal terms. A third order $(L^{-1}MU^{-1})^{(3)}$ with the inclusion of these off diagonal elements will give 4th order accuracy.

4) The conjugate gradient method

A short description of the conjugate gradient method which is used to solve the equation

$$L^{-1}MU^{-1}(Ux) = L^{-1}y \quad (26)$$

is given for completeness. The procedure is taken from Kershaw's [1] paper where further references can be found. It minimizes $(x, Mx-y)$. For other norms see the paper by Petravic [2]. The conjugate gradient method can only be applied to symmetric matrices. The handling of asymmetric matrices will be discussed in the next paragraph.

Let the zero order approximation vector r and p be defined by

$$\begin{aligned} r^{(0)} &= y - Mx^{(0)} \\ p^{(0)} &= (LL^T)^{-1} r^{(0)} \end{aligned} \quad (27)$$

where $x^{(0)}$ is an approximation to the solution x . Compute two auxiliary vectors as

$$\begin{aligned} q^{(\lambda)} &= Mp^{(\lambda)} \\ s^{(\lambda)} &= (LL^T)^{-1} r^{(\lambda)} \end{aligned} \quad (28)$$

and two scalar products

$$\begin{aligned} \alpha^{(\lambda)} &= (r^{(\lambda)}, s^{(\lambda)}) \\ \beta^{(\lambda)} &= (p^{(\lambda)}, q^{(\lambda)}) \end{aligned} \quad (29)$$

The next iteration vectors are then given by

$$\begin{aligned}
x^{(\lambda+1)} &= x^{(\lambda)} + \frac{\alpha^{(\lambda)}}{\beta^{(\lambda)}} p^{(\lambda)} \\
r^{(\lambda+1)} &= r^{(\lambda)} - \frac{\alpha^{(\lambda)}}{\beta^{(\lambda)}} p^{(\lambda)} \\
p^{(\lambda+1)} &= s^{(\lambda+1)} + \frac{\alpha^{(\lambda+1)}}{\alpha^{(\lambda)}} p^{(\lambda)}
\end{aligned}
\tag{30}$$

5) Non-symmetric matrices

Non-symmetric matrices M arise in physical problems from different sources: gradients that can also be present in elliptic equations, non-separable coordinate systems and most important from Neumann boundary conditions. The method proposed by Kershaw [1] did not work very well in test problems. It consists essentially of multiplying

$$L^{-1}MU^{-1}(Ux) = L^{-1}y \tag{31}$$

by the transposed matrix $(L^{-1}MU^{-1})^T$ and then solving the resulting linear system. One can easily see that off diagonal elements are multiplied essentially by two. Therefore, the convergence of the conjugate gradient method is slowed. Even worse is to take $M^T M$. Then the condition of expandability is not fulfilled for elliptic equations. The method used here is to solve for the asymmetry by iteration. Let

$$\begin{aligned}
M_s &= \frac{1}{2} (M + M^T) \\
\delta M &= \frac{1}{2} (M - M^T)
\end{aligned}
\tag{32}$$

Let σ denote the iteration for the asymmetry, then

$$M_s x^{(\sigma)} = y - \delta M x^{(\sigma-1)} \tag{33}$$

One can go one step further and correct $x^{(\sigma-1)}$ as used in the above equation. Let

$$r^{(\sigma)} = S - Mx^{(\sigma)} - \delta M \tilde{x}^{(\sigma-1)} \quad (34)$$

Then

$$M_S (r^\sigma + \delta x) = 0$$

or

$$M_S \delta x = - r^{(\sigma)} \quad (35)$$

Now use LL^T in above equation and define

$$\tilde{x}^{(\sigma)} = x^{(\sigma)} - (LL^T)^{-1} r^{(\sigma)} \quad (36)$$

for the right hand side of Eq. (33).

III. Numerical results

Test runs have been made with an elliptic equation which arises in numerical simulations of electrostatic plasma flow:

$$\left(\frac{\partial}{\partial x} \sigma \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \sigma \frac{\partial}{\partial y} \right) f = \frac{\partial \sigma}{\partial x} \quad 0 < x, y < 1 \quad (37)$$

$$\sigma = 1 + \beta \cdot \exp \left[- \left(\frac{x-x_0}{l_x} \right)^2 - \left(\frac{y}{l_y} \right) \right]$$

The partial differential equation was translated into a five point formula in two different ways; a) by leaving σ inside of the second derivative and b) by taking the differentiation of σ out of the second derivatives and treating the first derivatives of σ separately.

$$\begin{aligned}
& \frac{1}{dx^2} \left[(\sigma_{j+1,k} + \sigma_{j,k}) (f_{j+1,k} - f_{j,k}) - (\sigma_{j,k} + \sigma_{j-1,k}) (f_{j,k} - f_{j-1,k}) \right] \\
& + \frac{1}{dy^2} \left[(\sigma_{j,k+1} + \sigma_{j,k}) (f_{j,k+1} - f_{j,k}) - (\sigma_{j,k} + \sigma_{j,k-1}) (f_{j,k} - f_{j,k-1}) \right] \\
& = \frac{1}{2dx} (\sigma_{j+1,k} - \sigma_{j-1,k}) \tag{38}
\end{aligned}$$

while b) gives

$$\begin{aligned}
& \frac{1}{dx^2} \left[(f_{j+1,k} - 2f_{j,k} + f_{j-1,k}) + \frac{1}{4\sigma_{j,k}} (\sigma_{j+1,k} - \sigma_{j-1,k}) (f_{j+1,k} - f_{j-1,k}) \right] \\
& + \frac{1}{dy^2} \left[(f_{j,k+1} - 2f_{j,k} + f_{j,k-1}) + \frac{1}{4\sigma_{j,k}} (\sigma_{j,k+1} - \sigma_{j,k-1}) (f_{j,k+1} - f_{j,k-1}) \right] \\
& = \frac{1}{2\sigma_{j,k} dx} (\sigma_{j+1,k} - \sigma_{j-1,k}). \tag{39}
\end{aligned}$$

Version a) is symmetric; b) is not.

Test runs have been made with β from 10^4 to .1, l_x , l_y from .1 to .5. The number of points in x and y has been varied between 25 and 100. The rms error:

$$\text{rms} \cong \sqrt{\frac{(Mx-s)^2}{s^2}} \text{ varied between } 10^{-3} \text{ and } 10^{-5}.$$

The general experience gained by the test runs can be summarized as follows.

- 1) The need for double precision on the ASC (the ASC has a 32-bit word). The reason seems to be that the orthogonalization has to be achieved with high precision.
- 2) A simple-minded iterative scheme with single precision worked well only for a low accuracy and relatively small $n \leq 30$. The scheme eventually did converge but with a great number of iterations.

- 3) The use of recursion rather than iteration for the incomplete LU decomposition for a specific approximation did not (essentially) change the convergence rate.
- 4) The use of $\mu = 3, \nu_{(S)} = 2$ compared to $\mu = 2, \nu_{(S)} = 1$ decreases the number of iterations in about the same ratio as the number of operations per point increases. Therefore, the total time remained essentially constant.
- 5) The use of recursion in the second index (allowing partial vectorization on a vector-computer) only decreased slightly the number of iterations. It seems to be more appropriate - at least in the test cases run - to use the same approximation in the x and y directions. There may be asymmetric cases where this will not be true.

Timing

The number of operations per point is:

Scalar products	2
Compute x,r,p	6
q = Mp	9
Total	17
rms error	11

The number of operations for $x = (LL^T)^{-1}$ depends on the approximation.

Let nsca be the equivalent number of vector operations for one scalar operation; then the ratio of operations for the equivalent number for a recursive procedure becomes for $\mu = 2, \nu_{(S)} = 1$, for $\mu = 3, \nu_{(S)} = 2$

$$r_1 = \frac{38}{4nsca + 24}$$

$$r_2 = \frac{58}{4nsca + 26}$$

This gives for $nsca = 20$ (\approx factor for the ASC)

$$r_1 = .37$$

$$r_2 = .55$$

The iterative procedure can be vectorized over the whole array. For the recursive procedure the remaining vectorization can be only achieved for an inner loop, thus increasing the setup times compared to the iterative procedure. Also the more efficient use of two pipes on the ASC machine decreases the ratio further. Test runs have shown for $\mu = 2$, $\nu_S = 1$ an overall savings of about a factor of 5.

The total number of iterations N_i seems to be proportional to

$$N_i \approx (N_x N_y)^{3/2}$$

The dependence of N_i on the error reduction rate seems to be more complicated, roughly speaking 5 iterations per factor 10, such that the convergence factor is $f_{red} = (.1)^{1/5}$. In contrast to ADI and other methods the convergence factor seems to be more or less constant and independent of the error itself.

Imposing the Neumann boundary condition $\frac{\partial f}{\partial y} = 0$ at $y = \pm 1$ introduces an asymmetric matrix M . The number of iterations about the asymmetry is on average two. The program imposes at each iteration an error limit which is about 1/2 the error of the asymmetry. It thereby avoids unnecessary iterations for the symmetric solutions. Test runs have shown that the number of iterations (computing time) increases by about 50%, when compared to the same problem using Dirichlet boundary conditions.

In time dependent problems the computing time depends to a large extent on the guess of f_0 . Crude time dependent calculations where the

center of the exponential function is simply shifted and f_0 is set to the previous solution, gives a reduction of about 3 - 10 over that given by $f_0 = 0$, depending on the required rms error and N_x, N_y . The relationships for N_i and f_{red} still hold approximately. The reduction is about 10 for higher accuracy and lower for a lower accuracy because the change in f is the same.

S. Zalesak has used the scheme extensively in his electrostatic code. For small error (rms = 10^{-3}) and bad approximations the code runs about as fast as a vectorized ADI. For higher accuracy (10^{-4}) and a good approximation the code runs about two times faster than ADI with an accuracy of 10^{-3} . ADI did not converge after a reasonable number of iterations for a rms error of 10^{-4} .

IV. Conclusion

A vectorized incomplete Cholesky description scheme for the solution of large linear systems for sparse matrices has been developed. The vectorization is achieved by systematically replacing recursive formulae by non-recursive expansions in connection strength, both in the incomplete LU decomposition and in applying $(LU)^{-1}$ to a vector. The conjugate gradient method assures convergence. The saving in computing time over the recursive solution on a vector machine is about a factor five.

V. Acknowledgement

The author wishes to thank S. Zalesak for many discussions.

VI. References

1. David S, Kershaw, The incomplete Cholesky - conjugate gradient method for the iterative solution of systems of linear equations, *Journal of Computational Physics* 26, pp. 43-65 (1978).
2. M. Petravac, g. Kuo-Petravac, An ILUUG algorithm which minimizes in the Eulerian norm., *Journal of Computational Physics* 26, pp. 263-269 (1979).

VII. Distribution List

DEPARTMENT OF DEFENSE

ASSISTANT SECRETARY OF DEFENSE
COMM, CMD, CONT & INTELL
WASHINGTON, D.C. 20301
O1CY ATTN J. BABCOCK
O1CY ATTN M. EPSTEIN

ASSISTANT TO THE SECRETARY OF DEFENSE
ATOMIC ENERGY
WASHINGTON, D.C. 20301
O1CY ATTN EXECUTIVE ASSISTANT

DIRECTOR
COMMAND CONTROL TECHNICAL CENTER
PENTAGON RM BE 685
WASHINGTON, D.C. 20301
O1CY ATTN C-650
O1CY ATTN C-312 R. MASON

DIRECTOR
DEFENSE ADVANCED RSCH PROJ AGENCY
ARCHITECT BUILDING
1400 WILSON BLVD.
ARLINGTON, VA. 22209
O1CY ATTN NUCLEAR MONITORING RESEARCH
O1CY ATTN STRATEGIC TECH OFFICE

DEFENSE COMMUNICATION ENGINEER CENTER
1860 WIEHLE AVENUE
RESTON, VA. 22090
O1CY ATTN CODE R820
O1CY ATTN CODE R410 JAMES W. MCLEAN
O1CY ATTN CODE R720 J. WORTHINGTON

DIRECTOR
DEFENSE COMMUNICATIONS AGENCY
WASHINGTON, D.C. 20305
(ADR CNWDI: ATTN CODE 240 FOR)
O1CY ATTN CODE 101B

DEFENSE DOCUMENTATION CENTER
CAMERON STATION
ALEXANDRIA, VA. 22314
(12 COPIES IF OPEN PUBLICATION, OTHERWISE 2 COPIES)
12CY ATTN TC

DIRECTOR
DEFENSE INTELLIGENCE AGENCY
WASHINGTON, D.C. 20301
O1CY ATTN DT-1B
O1CY ATTN DB-4C E. O'FARRELL
O1CY ATTN DIAAP A. WISE
O1CY ATTN DIAST-5
O1CY ATTN DT-1BZ R. MORTON
O1CY ATTN HQ-TR J. STEWART
O1CY ATTN W. WITTIG DC-7D

DIRECTOR
DEFENSE NUCLEAR AGENCY
WASHINGTON, D.C. 20305
O1CY ATTN STVL
O4CY ATTN TITL
O1CY ATTN DOST
O3CY ATTN RAAE

COMMANDER
FIELD COMMAND
DEFENSE NUCLEAR AGENCY
KIRTLAND AFB, NM 87115
O1CY ATTN FCPR

DIRECTOR
INTERSERVICE NUCLEAR WEAPONS SCHOOL
KIRTLAND AFB, NM 87115
O1CY ATTN DOCUMENT CONTROL

JOINT CHIEFS OF STAFF
WASHINGTON, D.C. 20301
O1CY ATTN J-3 WNMCCS EVALUATION OFFICE

DIRECTOR
JOINT STRAT TGT PLANNING STAFF
OFFUTT AFB
OMAHA, NB 68113
O1CY ATTN JLTH-2
O1CY ATTN JPST G. GOETZ

CHIEF
LIVERMORE DIVISION FLD COMMAND DNA
DEPARTMENT OF DEFENSE
LAWRENCE LIVERMORE LABORATORY
P. O. BOX 808
LIVERMORE, CA 94550
O1CY ATTN FCPR

DIRECTOR
NATIONAL SECURITY AGENCY
DEPARTMENT OF DEFENSE
FT. GEORGE G. MEADE, MD 20755
O1CY ATTN JOHN SKILLMAN R52
O1CY ATTN FRANK LEONARD
O1CY ATTN W14 PAT CLARK
O1CY ATTN OLIVER H. BARTLETT W32
O1CY ATTN R5

COMMANDANT
NATO SCHOOL (SHAPE)
APO NEW YORK 09172
O1CY ATTN U.S. DOCUMENTS OFFICER

UNDER SECY OF DEF FOR RSCH & ENGRG
DEPARTMENT OF DEFENSE
WASHINGTON, D.C. 20301
O1CY ATTN STRATEGIC & SPACE SYSTEMS (OS)

WNMCCS SYSTEM ENGINEERING ORG
WASHINGTON, D.C. 20305
O1CY ATTN R. CRAWFORD

COMMANDER/DIRECTOR
ATMOSPHERIC SCIENCES LABORATORY
U.S. ARMY ELECTRONICS COMMAND
WHITE SANDS MISSILE RANGE, NM 88002
O1CY ATTN DELAS-EO F. NILES

DIRECTOR
BMD ADVANCED TECH CTR
MUNTSVILLE OFFICE
P. O. BOX 1500
MUNTSVILLE, AL 35807
O1CY ATTN ATC-T MELVIN T. CAPPS
O1CY ATTN ATC-O W. DAVIES
O1CY ATTN ATC-R DON RUSS

PROGRAM MANAGER
BMD PROGRAM OFFICE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333
O1CY ATTN DACS-BMT J. SHEA

CHIEF C-E SERVICES DIVISION
U.S. ARMY COMMUNICATIONS CMD
PENTAGON RM 18269
WASHINGTON, D.C. 20310
O1CY ATTN C-E-SERVICES DIVISION

COMMANDER
FRADCOM TECHNICAL SUPPORT ACTIVITY
DEPARTMENT OF THE ARMY
FORT MONMOUTH, N.J. 07703
O1CY ATTN DRSEL-NL-RD H. BENNET
O1CY ATTN DRSEL-PL-ENV H. BOMKE
O1CY ATTN J. E. QUIGLEY

COMMANDER
HARRY DIAMOND LABORATORIES
DEPARTMENT OF THE ARMY
2800 POWDER MILL ROAD
ADELPHI, MD 20783
(CNMCI-INNER ENVELOPE: ATTN: DELHD-RBH)
O1CY ATTN DELHD-TI M. WEINER
O1CY ATTN DELHD-RB R. WILLIAMS
O1CY ATTN DELHD-NP F. WIMENITZ
O1CY ATTN DELHD-NP C. MOAZED

COMMANDER
U.S. ARMY COMM-ELEC ENGRG INSTAL AGY
FT. HUACHUCA, AZ 85613
O1CY ATTN CCC-EMEO GEORGE LANE

COMMANDER
U.S. ARMY FOREIGN SCIENCE & TECH CTR
220 7TH STREET, NE
CHARLOTTESVILLE, VA 22901
O1CY ATTN DRXST-SD
O1CY ATTN R. JONES

COMMANDER
U.S. ARMY MATERIEL DEV & READINESS CMD
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333
O1CY ATTN DRCLDC J. A. BENDER

COMMANDER
U.S. ARMY NUCLEAR AND CHEMICAL AGENCY
7500 BACKLICK ROAD
BLDG 2073
SPRINGFIELD, VA 22150
O1CY ATTN LIBRARY

DIRECTOR
U.S. ARMY BALLISTIC RESEARCH LABS
ABERDEEN PROVING GROUND, MD 21005
O1CY ATTN TECH LIB EDWARD BAICY

COMMANDER
U.S. ARMY SATCOM AGENCY
FT. MONMOUTH, NJ 07703
O1CY ATTN DOCUMENT CONTROL

COMMANDER
U.S. ARMY MISSILE INTELLIGENCE AGENCY
REDSTONE ARSENAL, AL 35809
O1CY ATTN JIM GAMBLE

DIRECTOR
U.S. ARMY TRADOC SYSTEMS ANALYSIS ACTIVITY
WHITE SANDS MISSILE RANGE, NM 88002
O1CY ATTN ATAA-SA
O1CY ATTN TCC/F. PAYAN JR.
O1CY ATTN ATAA-TAC LTC J. HESSE

COMMANDER
NAVAL ELECTRONIC SYSTEMS COMMAND
WASHINGTON, D.C. 20360
O1CY ATTN NAVALEX 034 T. HUGHES
O1CY ATTN PME 117
O1CY ATTN PME 117-T
O1CY ATTN CODE 5011

COMMANDING OFFICER
NAVAL INTELLIGENCE SUPPORT CTR
4301 SUITLAND ROAD, BLDG. 5
WASHINGTON, D.C. 20390
O1CY ATTN MR. DUBBIN STIC 12
O1CY ATTN NISC-50
O1CY ATTN CODE 5404 J. GALET

COMMANDER
NAVAL OCEAN SYSTEMS CENTER
SAN DIEGO, CA 92152
O3CY ATTN CODE 532 W. MOLER
O1CY ATTN CODE 0230 C. BAGGETT
O1CY ATTN CODE 81 R. EASTMAN

DIRECTOR
NAVAL RESEARCH LABORATORY
WASHINGTON, D.C. 20375
O1CY ATTN CODE 4700 TIMOTHY P. COFFEY (25 CYS
IF UNCLASS, 1 CY IF CLASS)
O1CY ATTN CODE 4701 JACK D. BROWN
O1CY ATTN CODE 4780 BRANCH HEAD (150 CYS
IF UNCLASS, 1 CY IF CLASS)
O1CY ATTN CODE 7500 HQ COMM DIR BRUCE WALD
O1CY ATTN CODE 7550 J. DAVIS
O1CY ATTN CODE 7580
O1CY ATTN CODE 7551
O1CY ATTN CODE 7555
O1CY ATTN CODE 4730 E. MCLEAN
O1CY ATTN CODE 4127 C. JOHNSON

COMMANDER
NAVAL SEA SYSTEMS COMMAND
WASHINGTON, D.C. 20362
O1CY ATTN CAPT R. PITKIN

COMMANDER
NAVAL SPACE SURVEILLANCE SYSTEM
DAHLGREN, VA 22448
O1CY ATTN CAPT J. H. BURTON

OFFICER-IN-CHARGE
NAVAL SURFACE WEAPONS CENTER
WHITE OAK, SILVER SPRING, MD 20910
O1CY ATTN CODE F31

DIRECTOR
STRATEGIC SYSTEMS PROJECT OFFICE
DEPARTMENT OF THE NAVY
WASHINGTON, D.C. 20376
O1CY ATTN MSP-2141
O1CY ATTN NSSP-2722 FRED WIMBERLY

NAVAL SPACE SYSTEM ACTIVITY
P. O. BOX 92960
WORLDWAY POSTAL CENTER
LOS ANGELES, CALIF. 90009
O1CY ATTN A. B. MAZZARD

COMMANDER
NAVAL SURFACE WEAPONS CENTER
DAHLGREN LABORATORY
DAHLGREN, VA 22448
O1CY ATTN CODE DF-14 R. BUTLER

COMMANDING OFFICER
NAVY SPACE SYSTEMS ACTIVITY
P.O. BOX 92960
WORLDWAY POSTAL CENTER
LOS ANGELES, CA. 90009
O1CY ATTN CODE 52

OFFICE OF NAVAL RESEARCH
ARLINGTON, VA 22217
O1CY ATTN CODE 465
O1CY ATTN CODE 461
O1CY ATTN CODE 402
O1CY ATTN CODE 420
O1CY ATTN CODE 421

COMMANDER
AEROSPACE DEFENSE COMMAND/DC
DEPARTMENT OF THE AIR FORCE
ENT AFB, CO 80912
O1CY ATTN DC MR. LONG

COMMANDER
AEROSPACE DEFENSE COMMAND/XPD
DEPARTMENT OF THE AIR FORCE
ENT AFB, CO 80912
O1CY ATTN XPDQQ
O1CY ATTN XP

AIR FORCE GEOPHYSICS LABORATORY
HANSCOM AFB, MA 01731
O1CY ATTN OPR HAROLD GARDNER
O1CY ATTN OPR-1 JAMES C. ULWICK
O1CY ATTN LKB KENNETH S. W. CHAMPION
O1CY ATTN OPR ALVA T. STAIR
O1CY ATTN PHP JULES AARONS
O1CY ATTN PHD JURGEN BUCHAU
O1CY ATTN PHD JOHN P. MULLEN

AF WEAPONS LABORATORY
KIRTLAND AFB, NM 87117
O1CY ATTN SUL
O1CY ATTN CA ARTHUR H. GUENTHER
O1CY ATTN DYC CAPT J. BARRY
O1CY ATTN DYC JOHN M. KAMM
O1CY ATTN DYT CAPT MARK A. FRY
O1CY ATTN DES MAJ GARY GANONG
O1CY ATTN DYC J. JANNI

AFTAC
PATRICK AFB, FL 32925
O1CY ATTN TF/MAJ WILEY
O1CY ATTN TN

AIR FORCE AVIONICS LABORATORY
WRIGHT-PATTERSON AFB, OH 45433
O1CY ATTN AAD WADE HUNT
O1CY ATTN AAD ALLEN JOHNSON

DEPUTY CHIEF OF STAFF
RESEARCH, DEVELOPMENT, & ACQ
DEPARTMENT OF THE AIR FORCE
WASHINGTON, D.C. 20330
O1CY ATTN AFRDQ

HEADQUARTERS
ELECTRONIC SYSTEMS DIVISION/XR
DEPARTMENT OF THE AIR FORCE
HANSCOM AFB, MA 01731
O1CY ATTN XR J. DEAS

HEADQUARTERS
ELECTRONIC SYSTEMS DIVISION/YSEA
DEPARTMENT OF THE AIR FORCE
HANSCOM AFB, MA 01731
O1CY ATTN YSEA

HEADQUARTERS
ELECTRONIC SYSTEMS DIVISION/DC
DEPARTMENT OF THE AIR FORCE
HANSCOM AFB, MA 01731
O1CY ATTN DCKC MAJ J.C. CLARK

COMMANDER
FOREIGN TECHNOLOGY DIVISION, AFSC
WRIGHT-PATTERSON AFB, OH 45433
O1CY ATTN NICD LIBRARY
O1CY ATTN ETD B. BALLARD

COMMANDER
ROME AIR DEVELOPMENT CENTER, AFSC
GRIFFISS AFB, NY 13441
O1CY ATTN DOC LIBRARY/TSLD
O1CY ATTN OCSE V. COYNE

SAMSO/SZ
POST OFFICE BOX 92960
WORLDWAY POSTAL CENTER
LOS ANGELES, CA 90009
(SPACE DEFENSE SYSTEMS)
O1CY ATTN SZJ

STRATEGIC AIR COMMAND/XPFS
OFFUTT AFB, NB 68113
O1CY ATTN XPFS MAJ B. STEPHAN
O1CY ATTN ADWATE MAJ BRUCE BAUER
O1CY ATTN NRT
O1CY ATTN DOK CHIEF SCIENTIST

SAMSO/YA
P. O. BOX 92960
WORLDWAY POSTAL CENTER
LOS ANGELES, CA 90009
O1CY ATTN YAT CAPT L. BLACKWELDER

SAMSO/SK
P. O. BOX 92960
WORLDWAY POSTAL CENTER
LOS ANGELES, CA 90009
O1CY ATTN SKA (SPACE COMM SYSTEMS) M. CLAVIN

SAMSO/MN
NORTON AFB, CA 92409
(MINUTEMAN)
O1CY ATTN MNML LTC KENNEDY

COMMANDER
ROME AIR DEVELOPMENT CENTER, AFSC
HANSCOM AFB, MA 01731
O1CY ATTN EEP A. LORENTZEN

DEPARTMENT OF ENERGY
ALBUQUERQUE OPERATIONS OFFICE
P. O. BOX 5400
ALBUQUERQUE, NM 87115
O1CY ATTN DOC CON FOR D. SHERWOOD

DEPARTMENT OF ENERGY
LIBRARY ROOM G-042
WASHINGTON, D.C. 20545
O1CY ATTN DOC CON FOR A. LABOWITZ

EG&G, INC.
LOS ALAMOS DIVISION
P. O. BOX 809
LOS ALAMOS, NM 85544
OICY ATTN DOC CON FOR J. BREEDLOVE

UNIVERSITY OF CALIFORNIA
LAWRENCE LIVERMORE LABORATORY
P. O. BOX 808
LIVERMORE, CA 94550
OICY ATTN DOC CON FOR TECH INFO DEPT
OICY ATTN DOC CON FOR L-389 R. OTT
OICY ATTN DOC CON FOR L-31 R. MAGER
OICY ATTN DOC CON FOR L-46 F. SEWARD

LOS ALAMOS SCIENTIFIC LABORATORY
P. O. BOX 1663
LOS ALAMOS, NM 87545
OICY ATTN DOC CON FOR J. WOLCOTT
OICY ATTN DOC CON FOR R. F. TASCHEK
OICY ATTN DOC CON FOR E. JONES
OICY ATTN DOC CON FOR J. MALIK
OICY ATTN DOC CON FOR R. JEFFRIES
OICY ATTN DOC CON FOR J. ZINN
OICY ATTN DOC CON FOR P. KEATON
OICY ATTN DOC CON FOR D. WESTERVELT

SANDIA LABORATORIES
P. O. BOX 5800
ALBUQUERQUE, NM 87115
OICY ATTN DOC CON FOR J. MARTIN
OICY ATTN DOC CON FOR W. BROWN
OICY ATTN DOC CON FOR A. THORNBROUGH
OICY ATTN DOC CON FOR T. WRIGHT
OICY ATTN DOC CON FOR D. DAHLGREN
OICY ATTN DOC CON FOR 3141
OICY ATTN DOC CON FOR SPACE PROJECT DIV

SANDIA LABORATORIES
LIVERMORE LABORATORY
P. O. BOX 969
LIVERMORE, CA 94550
OICY ATTN DOC CON FOR B. MURPHEY
OICY ATTN DOC CON FOR T. COOK

OFFICE OF MILITARY APPLICATION
DEPARTMENT OF ENERGY
WASHINGTON, D.C. 20545
OICY ATTN DOC CON FOR D. GALE

OTHER GOVERNMENT

CENTRAL INTELLIGENCE AGENCY
ATTN RD/51, RM 5G48, HQ BLDG
WASHINGTON, D.C. 20505
OICY ATTN OSI/PS10 RM 5F 19

DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
WASHINGTON, D.C. 20234
(ALL CORRES: ATTN SEC OFFICER FOR)
OICY ATTN R. MOORE

INSTITUTE FOR TELECOM SCIENCES
NATIONAL TELECOMMUNICATIONS & INFO ADMIN
BOULDER, CO 80303
OICY ATTN A. JEAN (UNCLASS ONLY)
OICY ATTN W. UTLAUT
OICY ATTN D. CROMBIE
OICY ATTN L. BERRY

NATIONAL OCEANIC & ATMOSPHERIC ADMIN
ENVIRONMENTAL RESEARCH LABORATORIES
DEPARTMENT OF COMMERCE
BOULDER, CO 80302
OICY ATTN R. GRUBB
OICY ATTN AERONOMY LAB G. REID

DEPARTMENT OF DEFENSE CONTRACTORS

AEROSPACE CORPORATION
P. O. BOX 92957
LOS ANGELES, CA 90009
OICY ATTN I. GARFUNKEL
OICY ATTN T. SALMI
OICY ATTN V. JOSEPHSON
OICY ATTN S. BOWER
OICY ATTN N. STOCKWELL
OICY ATTN D. OLSEN
OICY ATTN J. CARTER
OICY ATTN F. MORSE
OICY ATTN SMFA FOR PMW

ANALYTICAL SYSTEMS ENGINEERING CORP
5 OLD CONCORD ROAD
BURLINGTON, MA 01803
OICY ATTN RADIO SCIENCES

BERKELEY RESEARCH ASSOCIATES, INC.
P. O. BOX 983
BERKELEY, CA 94701
OICY ATTN J. WORKMAN

BOEING COMPANY, THE
P. O. BOX 3707
SEATTLE, WA 98124
OICY ATTN G. KEISTER
OICY ATTN D. MURRAY
OICY ATTN G. HALL
OICY ATTN J. KENNEY

CALIFORNIA AT SAN DIEGO, UNIV OF
IPAPS, B-019
LA JOLLA, CA 92093
OICY ATTN HENRY G. BOOKER

BROWN ENGINEERING COMPANY, INC.
CUMMINGS RESEARCH PARK
HUNTSVILLE, AL 35807
OICY ATTN ROMEO A. DELIBERIS

CHARLES STARK DRAPER LABORATORY, INC.
555 TECHNOLOGY SQUARE
CAMBRIDGE, MA 02139
OICY ATTN D. B. COX
OICY ATTN J. P. GILMORE

COMPUTER SCIENCES CORPORATION
6565 ARLINGTON BLVD
FALLS CHURCH, VA 22046
OICY ATTN H. BLANK
OICY ATTN JOHN SPOOR
OICY ATTN C. NAIL

COMSAT LABORATORIES
LINTHICUM ROAD
CLARKSBURG, MD 20734
OICY ATTN G. HYDE

CORNELL UNIVERSITY
DEPARTMENT OF ELECTRICAL ENGINEERING
ITHACA, NY 14850
OICY ATTN D. T. FARLEY JR

ELECTROSPACE SYSTEMS, INC.
BOX 1359
RICHARDSON, TX 75080
OICY ATTN M. LOGSTON
OICY ATTN SECURITY (PAUL PHILLIPS)

ESL INC.
495 JAVA DRIVE
SUNNYVALE, CA 94086
OICY ATTN J. ROBERTS
OICY ATTN JAMES MARSHALL
OICY ATTN C. W. PRETTIE

FORD AEROSPACE & COMMUNICATIONS CORP
3939 FABIAN WAY
PALO ALTO, CA 94303
OICY ATTN J. T. MATTINGLEY

GENERAL ELECTRIC COMPANY
SPACE DIVISION
VALLEY FORGE SPACE CENTER
GODDARD BLVD KING OF PRUSSIA
P. O. BOX 8555
PHILADELPHIA, PA 19101
OICY ATTN M. H. BORTNER SPACE SCI LAB

GENERAL ELECTRIC COMPANY
P. O. BOX 1122
SYRACUSE, NY 13201
OICY ATTN F. REIBERT

GENERAL ELECTRIC COMPANY
TEMPO-CENTER FOR ADVANCED STUDIES
816 STATE STREET (P.O. DRAWER QQ)
SANTA BARBARA, CA 93102
OICY ATTN DASIAC
OICY ATTN DON CHANDLER
OICY ATTN TOM BARRETT
OICY ATTN TIM STEPHANS
OICY ATTN WARREN S. KNAPP
OICY ATTN WILLIAM MCNAMARA
OICY ATTN B. GAMBILL
OICY ATTN MACK STANTON

GENERAL ELECTRIC TECH SERVICES CO., INC.
HMES
COURT STREET
SYRACUSE, NY 13201
OICY ATTN G. MILLMAN

GENERAL RESEARCH CORPORATION
SANTA BARBARA DIVISION
P. O. BOX 6770
SANTA BARBARA, CA 93111
OICY ATTN JOHN (SE JR)
OICY ATTN JOEL GARBARINO

GEOPHYSICAL INSTITUTE
UNIVERSITY OF ALASKA
FAIRBANKS, AK 99701
(ALL CLASS ATTN: SECURITY OFFICER)
OICY ATTN T. N. DAVIS (UNCL ONLY)
OICY ATTN NEAL BROWN (UNCL ONLY)
OICY ATTN TECHNICAL LIBRARY

GTE SYLVANIA, INC.
ELECTRONICS SYSTEMS GRP-EASTERN DIV
77 A STREET
NEEDHAM, MA 02194
OICY ATTN MARSHAL CROSS

ILLINOIS, UNIVERSITY OF
DEPARTMENT OF ELECTRICAL ENGINEERING
URBANA, IL 61803
OICY ATTN K. YEH

ILLINOIS, UNIVERSITY OF
107 COBLE HALL
801 S. WRIGHT STREET
URBANA, IL 60680
(CALL CORRES ATTN SECURITY SUPERVISOR FOR)
OICY ATTN K. YEH

INSTITUTE FOR DEFENSE ANALYSES
400 ARMY-NAVY DRIVE
ARLINGTON, VA 22202
OICY ATTN J. M. AFIN
OICY ATTN ERNEST BAUER
OICY ATTN HANS WOLFHARD
OICY ATTN JOEL BENGSTON

HSS, INC.
2 ALFRED CIRCLE
BEDFORD, MA 01730
OICY ATTN DONALD HANSEN

INTL TEL & TELEGRAPH CORPORATION
500 WASHINGTON AVENUE
NUTLEY, NJ 07110
OICY ATTN TECHNICAL LIBRARY

JAYCOR
1401 CAMINO DEL MAR
DEL MAR, CA 92014

JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
JOHNS HOPKINS ROAD
LAUREL, MD 20818
OICY ATTN DOCUMENT LIBRARIAN
OICY ATTN THOMAS POTEMRA
OICY ATTN JOHN DASSOULAS

LOCKHEED MISSILES & SPACE CO INC
P. O. BOX 504
SUNNYVALE, CA 94088
OICY ATTN DEPT 60-12
OICY ATTN D. R. CHURCHILL

LOCKHEED MISSILES AND SPACE CO INC
3251 MANOVER STREET
PALO ALTO, CA 94304
OICY ATTN MARTIN WALT DEPT 52-10
OICY ATTN RICHARD G. JOHNSON DEPT 52-12
OICY ATTN W. L. IMMOF DEPT 52-12

KAMAN SCIENCES CORP
P. O. BOX 7465
COLORADO SPRINGS, CO 80933
OICY ATTN T. MEAGHER

LINKABIT CORP
10453 ROSELLE
SAN DIEGO, CA 92121
OICY ATTN IRWIN JACOBS

M.I.T. LINCOLN LABORATORY
P. O. BOX 73
LEXINGTON, MA 02173
OICY ATTN DAVID M. TOWLE
OICY ATTN L. LOUGHLIN

MARTIN MARIETTA CORP
ORLANDO DIVISION
P. O. BOX 5837
ORLANDO, FL 32805
OICY ATTN R. HEFFNER

MCDONNELL DOUGLAS CORPORATION
5301 BOLSA AVENUE
HUNTINGTON BEACH, CA 92647
01CY ATTN N. HARRIS
01CY ATTN J. MOULE
01CY ATTN GEORGE MROZ
01CY ATTN W. OLSON
01CY ATTN R. W. HALPRIN
01CY ATTN TECHNICAL LIBRARY SERVICES

MISSION RESEARCH CORPORATION
735 STATE STREET
SANTA BARBARA, CA 93101
01CY ATTN P. FISCHER
01CY ATTN W. F. CREVIER
01CY ATTN STEVEN L. GUTSCHE
01CY ATTN D. SAPPENFIELD
01CY ATTN R. BOGUSCH
01CY ATTN R. HENDRICK
01CY ATTN RALPH KILB
01CY ATTN DAVE SOWLE
01CY ATTN F. FAJEN
01CY ATTN M. SCHEIBE
01CY ATTN CONRAD L. LONGMIRE
01CY ATTN WARREN A. SCHLUETER

MITRE CORPORATION, THE
P. O. BOX 208
BEDFORD, MA 01730
01CY ATTN JOHN MORGANSTERN
01CY ATTN G. HARDING
01CY ATTN C. E. CALLAHAN

MITRE CORP
WESTGATE RESEARCH PARK
1820 DOLLY MADISON BLVD
MCLEAN, VA 22101
01CY ATTN W. MALL
01CY ATTN W. FOSTER

PACIFIC-SIERRA RESEARCH CORP
1456 CLOVERFIELD BLVD.
SANTA MONICA, CA 90404
01CY ATTN E. C. FIELD JR

PENNSYLVANIA STATE UNIVERSITY
IONOSPHERE RESEARCH LAB
318 ELECTRICAL ENGINEERING EAST
UNIVERSITY PARK, PA 16802
(NO CLASSIFIED TO THIS ADDRESS)
01CY ATTN IONOSPHERIC RESEARCH LAB

PHOTOMETRICS, INC.
442 MARRETT ROAD
LEXINGTON, MA 02173
01CY ATTN IRVING L. KOFSKY

PHYSICAL DYNAMICS INC.
P. O. BOX 3027
BELLEVUE, WA 98009
01CY ATTN E. J. FREMOUW

PHYSICAL DYNAMICS INC.
P. O. BOX 1069
BERKELEY, CA 94701
01CY ATTN A. THOMPSON

R & D ASSOCIATES
P. O. BOX 9695
MARINA DEL REY, CA 90291
01CY ATTN FORREST GILMORE
01CY ATTN BRYAN GABBARD
01CY ATTN WILLIAM B. WRIGHT JR
01CY ATTN ROBERT F. LELEVIER
01CY ATTN WILLIAM J. KARZAS
01CY ATTN H. ORY
01CY ATTN C. MACDONALD
01CY ATTN R. TURCO

RAND CORPORATION, THE
1700 MAIN STREET
SANTA MONICA, CA 90406
01CY ATTN CULLEN CRAIN
01CY ATTN ED BEDROZIAN

RIVERSIDE RESEARCH INSTITUTE
80 WEST END AVENUE
NEW YORK, NY 10023
01CY ATTN VINCE TRAPANI

SCIENCE APPLICATIONS, INC.
P. O. BOX 2351
LA JOLLA, CA 92038
01CY ATTN LEWIS M. LINSON
01CY ATTN DANIEL A. HAMLIN
01CY ATTN D. SACHS
01CY ATTN E. A. STRAKER
01CY ATTN CURTIS A. SMITH
01CY ATTN JACK MCDUGALL

RAYTHEON CO.
528 BOSTON POST ROAD
SUDBURY, MA 01776
01CY ATTN BARBARA ADAMS

SCIENCE APPLICATIONS, INC.
HUNTSVILLE DIVISION
2109 W. CLINTON AVENUE
SUITE 700
HUNTSVILLE, AL 35805
01CY ATTN DALE H. DIVIS

SCIENCE APPLICATIONS, INCORPORATED
8400 WESTPARK DRIVE
MCLEAN, VA 22101
01CY ATTN J. COCKAYNE

SCIENCE APPLICATIONS, INC.
80 MISSION DRIVE
PLEASANTON, CA 94566
01CY ATTN SZ

SRI INTERNATIONAL
333 RAVENSWOOD AVENUE
MENLO PARK, CA 94025
01CY ATTN DONALD NEILSON
01CY ATTN ALAN BURNS
01CY ATTN G. SMITH
01CY ATTN L. L. COBB
01CY ATTN DAVID A. JOHNSON
01CY ATTN WALTER G. CHESNUT
01CY ATTN CHARLES L. RINO
01CY ATTN WALTER JAYE
01CY ATTN M. BARON
01CY ATTN RAY L. LEADABRAND
01CY ATTN G. CARPENTER
01CY ATTN G. PRICE
01CY ATTN J. PETERSON
01CY ATTN R. HAKE, JR.
01CY ATTN V. GONZALES
01CY ATTN D. MCDANIEL

TECHNOLOGY INTERNATIONAL CORP
75 WIGGINS AVENUE
BEDFORD, MA 01730
OICY ATTN W. P. BOQUIST

TRW DEFENSE & SPACE SYS GROUP
ONE SPACE PARK
REDONDO BEACH, CA 90278
OICY ATTN R. K. PLEBUCH
OICY ATTN S. ALTSCHULER
OICY ATTN D. DEE

VISIDYNE, INC.
19 THIRD AVENUE
NORTH WEST INDUSTRIAL PARK
BURLINGTON, MA 01803
OICY ATTN CHARLES HUMPHREY
OICY ATTN J. W. CARPENTER

