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ON A SOVIET ALGORITHM FOR THE LINEAR PROGRAMMING PROBLEM.(U)
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DEPARTMENT of COMPUTER SCIENCE

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On a Soviet Algorithm for the Linear Programming Problem

Science reported ("Mathematicians Amazed by Russian's Discovery", Science 206,545 (1979)) a new algorithm "of great theoretical and probably practical importance" for the solution of linear programming problems. The algorithm was ascribed to a Soviet mathematician L.G. Khachian.

We have received numerous inquiries from people in business, government and universities concerning this algorithm. Here we shall confine ourselves to answering only two questions: What is the practical value of this algorithm and what is its history? We answer each of these in turn.

Our analysis and preliminary experimentation indicate that the method is not of practical interest. The technical basis for this conclusion will be reported elsewhere. Other workers with whom we have discussed this matter, including George Dantzig and Philip Wolfe, have arrived at the same conclusion. For example, Dantzig has told us that problems which arise in energy/economic planning and which are routinely solved at Stanford University in under half an hour by the Simplex Method would take an estimated fifty million years to solve by the new method.

It is possible that the ideas of the Soviet method, in combination with new ideas, will eventually lead to a method competitive with Simplex. But that is not the case today.

We now discuss the history of this method. We believe referring to this as Khachian's method is incorrect. Since, as we shall describe below, five Soviet mathematicians have contributed, we propose naming this the "Soviet Algorithm".

To describe ths history we present the Soviet Algorithm in qualitative terms. For simplicity of exposition, assume a solution exists. A sequence of ellipsoids of decreasing volume is constructed such that each ellipsoid contains a solution. The initial ellipsoid contains a solution and the process is terminated when an ellipsoid is reached with sufficiently small volume.

We show the contributions of various authors towards this result and the year of publication.

In 1965, Levin (1) proposed a construction involving polytopes. He established the geometric convergence of the polytope volumes and that the convergence rate depends only on the problem dimension. He observed that the successive polytopes might have too many faces, making the processes computationally inefficient. Furthermore he observed that the process was potentially infinite.

In 1970, Shor (2) introduced and analyzed a class of "generalized gradient" methods for the solution of linear programming problems. This paper was to form the theoretical foundations for all the later work.

In 1976, Judin and Nemirovsky wrote two related papers (3), (4) primarily devoted to the optimal solution of the linear programming problem. Only a portion of these two papers is relevant to this history. They proposed an algorithm which requires the calculation of a centroid of a body. They briefly mentioned that the calculation is simplified if the body is approximated by an ellipsoid and cite this as a variant of the methods described in (2).

In 1977, Shor (5) studied a particular case of the family of methods he had introduced in (2). He was the first to give a formula for the algorithm. He mentioned that the algorithm could also be obtained from (4). Shor assumed that the initial ellipsoid contains a solution.

In 1979 Khachian (6) analyzed the solution of linear inequalities, which can be used to solve linear programming problems. He presented an algorithm essentially the same as in Shor (5). By assuming integer coefficients he was able to construct a starting ellipsoid which contains a solution. This assumption also implies that the volume of solutions in the initial ellipsoid is positive. Therefore the method terminates after a finite number of steps. Khachian proved that the cost of this algorithm depends polynomially on the number of bits in the integers defining the problem. Therefore in a Turing Machine model of complexity this is a polynomial time algorithm. Incidentally, although Khachian refers to Shor's earlier paper (2) he omits the more relevant (5).

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