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INTERMODULATION GENERATION DIAGNOSIS BY ANALYTICAL AND COMPUTER TECHNIQUES

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INTRODUCTION

Recent advances in communications technology suggest that more powerful systematic methods for the reduction of IMG must be developed to keep pace with this technology. In this chapter analytical approaches to IMG diagnosis are reviewed and discussed. It is found that a great deal of information about the nature of the IMG-producing nonlinearity can be deduced from the intermodulation (intermod) spectrum and this can provide important guidelines in identifying the source of the non-linearity. On the other hand it is not possible to identify with certainty a unique physical source of an intermod from the intermod spectrum alone although the reverse is possible. The strengths and limitations of the analytical approach are delineated and the need for the application of high speed computers for Fourier transform analysis of signals containing intermods is demonstrated. Several model computer calculations are carried out and the results are discussed and compared when appropriate to what might be expected from analytical considerations. Insight is provided into such effects as the power of the intermod relative to primary signal power, the surprising decrease in power of certain intermods when additional primary signals are turned on, the relative power of different intermods, etc. The goal is to provide a mathematical and computational basis for predicting the intermodulation spectrum from particular physical models of IMG sources as well as to develop procedures for diagnosing sources of IMG.

ANALYTICAL CONSIDERATIONS

Intermodulation generation occurs when an electronic system contains one or more elements which do not exhibit purely linear current-voltage (i-e) relationships. It is mathematically well-known and obvious how non-linearities give rise to IMG.

Consider the functional relationship

$$e = Ai + Bi^2 \tag{1}$$

where e is the voltage and i is the current across an element in the system.

Then, for a two-carrier signal with frequencies ω_1 and ω_2 and zero relative phase at $t = 0$

$$i = i_1 + i_2 = I_1 \cos\omega_1 t + I_2 \cos\omega_2 t$$

and thus

$$\begin{aligned} e &= A[I_1 \cos\omega_1 t + I_2 \cos\omega_2 t] + B[I_1 \cos\omega_1 t + I_2 \cos\omega_2 t]^2 \\ &= A[I_1 \cos\omega_1 t + I_2 \cos\omega_2 t] + B[I_1^2 \cos^2\omega_1 t + I_2^2 \cos^2\omega_2 t \\ &\quad + 2I_1 I_2 \cos\omega_1 t \cos\omega_2 t] \end{aligned}$$

Recalling that

$$\cos^2 x = 1/2(\cos 2x + 1)$$

$$\cos x \cos y = 1/2[\cos(x + y) + \cos(x - y)] \tag{3}$$

we find

$$e = A\, I_1 \cos\omega_1 t + I_2 \cos\omega_2 t + \frac{1}{2} B\{I_1^2(\cos 2\omega_1 t + 1) + I_2^2(\cos 2\omega_2 t + 1) + 2 I_1 I_2[\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t]\} \tag{4}$$

and the i^2 term is found to give rise to second order intermodulation signals (intermods).

Analogously, if a third order term were also in Eq. (1), Ci^3 say, then additional terms would appear in Eq. (2), to wit

$$\frac{1}{4} C\{I_1^3[\cos 3\omega_1 t + 3\cos\omega_1 t] + I_2^3[\cos 3\omega_2 t + 3\cos\omega_2 t]$$

$$+ 3 I_1 I_2^2 [2\cos\omega_1 t + \cos(\omega_1 + 2\omega_2)t + \cos(\omega_1 - 2\omega_2)t + \cos(\omega_1 - 2\omega_2)t]$$

$$+ 3 I_1^2 I_2 [2\cos\omega_2 t + \cos(2\omega_1 + \omega_2)t + \cos(2\omega_1 - \omega_2)t]\}$$

In principle, this same kind of procedure can be used to carry out an evaluation of the intermodulation signals for an arbitrary number of carriers and for any order term or terms in the current-voltage relationship.

What this implies is that regardless of the functional relationship between voltage and current, if voltage can be expressed as a power series *in the current, i.e. a Taylor series,* then one could apply the technique used in obtaining Eq. (4) to predict the frequencies and amplitudes of the intermods.

A Taylor series expansion would treat the current associated with each primary frequency as an independent variable so that the expansion would appear as

$$e = f(i) = f(0) + \left(\frac{\partial f(0)}{\partial i_1}\right) i_1 + \left(\frac{\partial f(0)}{\partial i_2}\right) i_2 + \ldots + \frac{1}{2!}\left(\frac{\partial^2 f(0)}{\partial i_1^2}\right) i_1^2 + \ldots$$

$$= \left[\sum_{k=1}^{n} i_k \frac{\partial}{\partial i_k}\right] f(0) + \frac{1}{2!}\left[\sum_{k=1}^{n} i_k \frac{\partial}{\partial i_k}\right]^2 f(0) + \frac{1}{3!}\left[\sum_{k=1}^{n} i_k \frac{\partial}{\partial i_k}\right]^3 f(0) + \ldots \tag{5}$$

where $f(0)$ implies evaluation of $f(i)$ where each of the i_k's is zero and n is the number of primary carrier frequencies. Each term in the Taylor series could be treated in the manner leading to Eq. (4) to determine the resultant intermods. Although this procedure would be very long and tedious, in principle it could be carried out just one time for each term in the series and for various numbers of primary carriers and the results tabulated. We have carried out a number of calculations along these lines which have proved to be very important for learning about the relationships between the nonlinearities and the intermods they cause as well as the relationships among different intermods. On the other hand the compilation of the results proves to be rather elaborate and could not include the effects of the relative phases of the primary signals at some particular time. (Consideration of phase will be discussed below.)

More important, however, is the fact that some simple i-e characteristics often found in real systems cannot be expressed in a Taylor series. For example, the simplest rectifier, whose i-e functional dependence is

$$e = Ai \qquad i > 0 \tag{6}$$

$$e = 0 \qquad i < 0$$

where A is some constant, does not fulfill the mathematical requirements for a Taylor series expansion. It can, however, be easily treated using computer techniques and therein lies one major advantage of the computer analysis as compared to the analytical approach.

A number of useful facts for an analytical diagnosis procedure are already obvious from these discussions. *First*, all terms in the *i-e* functional relationship contribute additively to the intermod spectrum. *Second*, the highest order intermod that occurs arises from the highest order (non-zero) term that appears in the Taylor series expansion; i.e. the term $\frac{1}{m!}\left[\sum_{k=1}^{n} i_k \frac{\partial}{\partial i_k}\right]^m f(0)$ will give rise to m^{th} order intermods and in general, all odd or even order intermods lower than m according to whether m is itself odd or even.

[As an aside, we note that if the trigonometric manipulations analogous to those leading up to Eq. 4 were carried out for m^{th} order terms in the Taylor series using exponential rather than conventional forms of the cosine function, one would conclude that only m^{th} order intermods arise from the m^{th} order term in the Taylor series. This would be incorrect. Use of the exponential functions with the taking of the real part at the completion of the calculation is not valid for non-linear problems. The difficulty arises from the difference between the power of the real part of a complex number and the real part of the power of the same number. E.g., $\text{Re}\{(a + ib)^2\} \neq \{\text{Re}(a + ib)\}^2$.]

Third, it is trivial to demonstrate that any particular intermod, say $\cos(2\omega_1 + \omega_2)t$, has the same phase regardless of whether it arises from the third, fifth, seventh, etc. order term in Taylor series. (We are assuming, of course, that the non-linear element(s) giving rise to the intermods are found at a single point in space.) To do this, imagine the quantity $\omega_1 t$ replaced by $(\omega_1 t + \phi)$, and similarly for $\omega_2 t$, $\omega_3 t$, etc. and the result is obvious. On the other hand, given a multicarrier signal, it is likely that two or more different intermods will have the same frequency. For example, if $\omega_1 = 10$, $\omega_2 = 11$ and $\omega_3 = 13$, (in arbitrary units) then $2\omega_2 - \omega_1 = \omega_1 + \omega_3 - \omega_2 = 12$. In this situation there is no reason to expect these two different intermods to have the same or nearly the same phase. This provides an explanation for an often seen phenomenon that is not widely understood; to wit, a given intermod is reduced in amplitude when an additional carrier frequency is turned on. This could occur for example with the signal whose frequency is 12 in the example above when ω_3 is turned on if the $\omega_1 + \omega_3 - \omega_2$ signal is out of phase or nearly out of phase with the $2\omega_2 - \omega_1$ signal.

Fourth, the amplitude or power (power is proportional to the square of the amplitude) dependence of an intermod on the primary signal input amplitude (or power) can be seen from Eq. (5). If an m^{th} intermod arising from the m^{th} order term in the Taylor series expansion has a frequency $(p_1\omega_1 + p_2\omega_2 + p_3\omega_3 + p_4\omega_4 + \ldots) = \sum_i p_i\omega_i$ where the p's are positive or negative integers and $\sum_i |p_i| = m$, then the intermod amplitude will vary as $I_1^{p_1} I_2^{p_2} I_3^{p_3} I_4^{p_4} \ldots = \prod_i I_i^{p_i}$. This dependence of intermod power on primary signal power can be a useful tool in determining the i-e functional relationship. It must be remembered, however, that an m^{th} order term in the Taylor series expansion will also generate, in general, intermods of order $m - 2$, $m - 4$, ... etc. and all of these will also be of m^{th} order in the I's. For these contributions, specific functional dependence of the amplitude of the intermod on the amplitudes of the various primary signals cannot be simply specified. All that can be said without tedious calculation for a specific situation is that if an intermod frequency is $\sum_i p_i\omega_i$, then the amplitude will consist of a sum of terms each of which varies as $\prod_i I_i^{p_i+2n_i}$ where $\sum_i |p_i| = m$ is the intermod order and n_i is an integer equal to or greater than 0. Thus, for example, a three carrier signal composed of frequencies ω_1, ω_2 and ω_3 passing through a nonlinear device may well produce components such as $\cos(2\omega_1 + \omega_2)t$ arising from, say a 5th or 7th order term in the current voltage relationship. Although this intermod does not "contain" an ω_3, it will have an amplitude that does in general depend on I_3. In fact, certain experimental observations, such as the variation of the amplitude of a term like

$\cos(2\omega_1 + \omega_2)t$ with I_3, suggests the presence of 5th or higher order terms. In this regard one should, however, be cautious. This could appear to occur if a different third-order term involving ω_3 happened, by accident, to have a frequency numerically equal to $2\omega_1 + \omega_2$.

Fifth, the relative magnitudes of the intermods as a function of the number of frequencies represented in the intermod, the number of carrier signals, and the order of the nonlinear term are straightforward if tedious to deduce. Suppose the equation describing the nonlinearity is

$$e = Ai + Bi^3 + Ci^5 \tag{7}$$

If one compares a three-carrier signal to a five carrier signal then the Bi^3 term will give rise to the same kind and magnitude of third order terms for both cases. For five carriers, there are simply more third order terms arising from the larger number of combinations of frequencies than for three carriers. On the other hand, the Bi^3 term also gives rise to first order terms and these nonlinear generated first order terms will be larger when there are five carriers than when there are only three.

Analogously, the Ci^5 term will generate more fifth order intermods when five signals are considered than when three are, but their magnitudes will be the same in both cases for "similar" intermods, i.e. $\cos(3\omega_1 - 2\omega_2)t$ is "similar" to $\cos(3\omega_3 + 2\omega_2)t$, but not to either $\cos 5\omega_2 t$ or to $\cos(2\omega_1 + 2\omega_2 + \omega_3)t$, etc. On the other hand, the third-order intermod frequencies that arise from the Ci^5 term do have a magnitude which depends on the number of carrier frequencies.

If the Ci^5 term is the highest order term in the i-e relationship, then the largest fifth order terms will be the so called five carrier fifths, e.g. $\cos(\omega_1 \pm \omega_2 \pm \omega_3 \pm \omega_4 \pm \omega_5)t$. In order of decreasing magnitude will come $\cos(2\omega_1 \pm \omega_2 \pm \omega_3 \pm \omega_4)t$, $\cos(2\omega_1 \pm 2\omega_2 \pm \omega_3)t$, $\cos(3\omega_1 \pm \omega_3)t$, $\cos(3\omega_1 \pm \omega_2)t$, $\cos(4\omega_1 \pm \omega_2)t$, $\cos 5\omega_1 t$. The trend is clear; the more frequencies and the more equally represented in the intermod, the larger that particular intermod will be.

In general, higher order intermods tend to have smaller amplitudes. For example, the Ci^5 term of Eq. (7) will give rise to third order as well as fifth order intermods, and in fact the three carrier third, i.e. $\cos(\omega_1 \pm \omega_2 \pm \omega_3)t$ will have a magnitude somewhat larger than the five carrier fifth mentioned above. It is by no means true, however, that all or even most third order intermods are larger than all the fifth order intermods.

COMPUTER DIAGNOSIS OF NONLINEAR SYSTEMS

Using Equations (2) through (4) is basically a simpler way of obtaining a Fourier transform than the usual analytical procedure. On the other hand, computer based Fourier transform analysis is now a well developed technique. This fact and the widespread availability of high speed computers strongly suggest the application of computers to diagnosing nonlinear systems from the system's intermodulation generation. The advantages of using computers are more than just speed. Unlike the mathematical requirements on the current-voltage relationships that are necessary in the analytical approach discussed above, the computer approach requires no particular restrictive mathematical criteria for the current-voltage dependence. For example, there is no special difficulty in Fourier analyzing a multi-carrier signal imposed on a circuit element with circuit characteristics given by Eq. (6). Thus, given an *i-e* relationship, analytical or otherwise, one can predict the magnitude and relative phase of the intermods.

There are three characteristics of computer based Fourier transforms which should be understood if results are to be properly interpreted. *First*, a computer based Fourier transform (FT) differs from an analytical transform in that, with the former, one works with a discrete (rather than continuous) set of "data" (values of the function being transformed) summed over a finite (rather than infinite) range of the argument of the function. As a consequence of this, the relative phases of the input signals at some fixed time can influence, to some extent, the magnitudes of the resultant intermods. This influence, however, is quite small.

Second, another and much more important consideration associated with signal phase is that all computer results are numerical and there is no way to distinguish between two or more contributing intermods at a given frequency. Further and as mentioned earlier, there is no reason for different intermods of the same frequency to have the same or any other particular phase relative to each other. Thus certain characteristics of particular intermods cannot always be easily sorted out. This corresponds to the actual experimental situation where various intermods can overlap.

Third, a possible major contribution to the overlap of intermods is the phenomenon of aliasing. When N discrete data points are separated from each other by a time interval T, then there is a maximum possible frequency, ω_m, that can be resolved which is approximately $\frac{1}{2T}$ for large N. Frequencies less than ω_m will be correctly given by the computer based Fourier transform. For those frequencies greater than ω_m by $\Delta\omega_m$, the associated computer generated spectrum line will appear at a frequency $(\omega_m - \Delta\omega_m)$ which is, of course, within the range of allowable frequencies. This is the phenomenon known as aliasing. Thus, the probability of signal overlap in the range of allowable frequencies is increased simply because of the apparent increased density of intermod signals. This phenomenon will not usually present too great a problem for two reasons. First, the intermods with frequencies greater than ω_m are usually of higher order and thus can be expected to have very low amplitudes. Second, the value of the intermod frequency can be made to indicate which spectrum lines have appeared by aliasing. For example, if all primary frequencies are whole numbers but ω_{max} is, say, a whole number plus 0.3, then any spectrum line centered on an ω that is not a whole number can be assumed to have arisen from aliasing.

In order to illustrate the nature of the intermod response expected from an i-e relationship not amenable to analytical treatment, a number of computer experiments have been carried out. For this purpose we have worked with linear, quadratic and cubic rectification functions and various combinations thereof. Thus, the i-e relationship is

$$\begin{aligned} e &= ai + bi^2 + ci^3 \quad & i > 0 \\ &= 0 & i < 0 \end{aligned} \tag{8}$$

where any one or two of the coefficients a, b, or c may be zero. For example if $a = b = 0$, $c \neq 0$ then we refer to this as cubic rectification. If only $a \neq 0$, it is linear rectification while if only $b \neq 0$ it is quadratic rectification. Calculations using up to three input frequencies have been carried out with

$$i = I_1 \cos(\omega_1 t + \phi_1) + I_2 \cos(\omega_2 t + \phi_2) + I_3 \cos(\omega_3 t + \phi_3)$$

as the input signal, where $\omega_1 = 10$, $\omega_2 = 11$, and $\omega_3 = 13$ and the phases ϕ_1, ϕ_2 and ϕ_3 could be varied using a random number generator. Amplitudes of signals are in arbitrary units since only ratios of amplitudes are significant.

In the first computer experiment, two of the three primary signals and two second order intermods were examined for several combinations of the coefficients a, b, and c in Eq. (8). The parameters used and results obtained are summarized in Table I. Values of a, b, and c used are shown above the column corresponding to the results. If one or more of these coefficients are not indicated above the columns, the implication is that they are zero. The results themselves are averages of four separate runs using random phases of the input signals with each run. The intent was to minimize the effects of signal phase discussed above.

Table I — Signal amplitudes at various frequencies for the kinds of rectification indicated.

Intermod and Frequency	c=1	b=1	b=c=1	a=1	a=b=c=1
$\omega_1 = 10$	34.571	17.047	51.618	9.265	60.883
$\omega_3 - \omega_1 = 3$	22.835	9.153	31.988	3.299	35.287
$\omega_2 + \omega_3 = 24$	22.697	9.117	31.813	3.036	34.848
$\omega_3 = 13$	34.534	16.695	51.229	9.242	60.470

The most obvious feature of the data in Table I is the equality between the sum of the amplitudes for quadratic ($b = 1$) and cubic ($c = 1$) rectification calculated separately and the result for the sum of quadratic and cubic rectification, column three. The summability is also true for linear rectification. This result is not surprising since it merely implies the obvious summability of spectrum lines from different nonlinearities. A second interesting regularity is that for each type of rectification, "similar" intermods have very nearly the same amplitudes. The extent to which the amplitudes are not identical results from more than one intermod contributing at a given frequency.

The variability of signal amplitude because of multiple intermod contributions at a single frequency, and the manner in which this amplitude can vary with primary signal phase are illustrated in Table II. Here the results of several individual runs with random phases are presented for intermod frequencies of 7 and 12. The reproducibility of the signal whose frequency is 7 is much better than the signal whose frequency is 12 because the two major contributors to the former signal are third order and fifth order intermods. Since the third order intermod should be perfectly reproducible and is much larger than the fifth, whether the fifth adds or subtracts from the third (depending on their relative phase) the amplitude of the net signal is changed only to a limited extent. In contrast, the major contributions to the signal whose frequency is 12 come from two third order intermods whose magnitudes can be expected to be much more nearly alike. Thus, addition or subtraction (constructive or destructive interference) of the two results in large variations in the magnitude of the net signal.

Table II — Signal amplitudes for the $\omega = 7$ and $\omega = 12$ signals for the kinds of rectification indicated.

Intermod and Frequency	run	c=1	b=1	b=c=1	a=1	a=b=c=1
$2\omega_1 - \omega_3 = 7$	1	2.007	.786	2.793	.133	2.876
$3\omega_2 - 2\omega_3 = 7$	2	2.030	.7739	2.803	.137	2.838
	3	2.0412	.7955	2.837	---	2.839
$\omega_1 - \omega_2 + \omega_3 = 12$	1	3.145	1.422	4.563	---	4.590
$2\omega_2 - \omega_1 = 12$	2	4.182	1.813	5.994	.0418	6.030
	3	2.024	1.023	3.047	---	3.022

The surprising thing about Table II is that the sum of the $c = 1$ and $b = 1$ columns equals the $b = c = 1$ column for an intermod frequency of 12. Unlike the results of Table I, the $\omega = 12$ signal shown in Table II is a sum of two different intermods of comparable magnitude. It is easy to show that under these conditions, the phase of the net signal for a particular nonlinearity will not in general be equal to the phase of a signal of the same frequency arising from a second nonlinearity. (The phases would be the same if the same single intermod was the only contribution for both of the nonlinearities as is the case in Table I). Thus the sum of columns $c = 1$ and $b = 1$ would not be expected to equal column $c = b = 1$. Suppose, however, that A_c is the coefficient of the $\omega_1 - \omega_2 + \omega_3$ intermod resulting from $c = 1$ and M_c is the coefficient of the $2\omega_2 - \omega_1$ intermod resulting from $c = 1$ and A_b and M_b are the analogous coefficients for $b = 1$. Then, if $M_c/A_c = M_b/A_b$ it can be shown that the net signal from $c = 1$ and $b = 1$ will have the same phase, and the additivity of the $\omega = 12$ signals in Table II are to be expected.

In Table III a large number of intermod amplitudes are given. It is obvious that the amplitudes of the intermods fall into groupings as shown. Furthermore the relative magnitudes correspond quite well with what we would expect from the analytical discussions earlier in this paper, including the facts that the higher the intermod order, the lower is its amplitude and that similar intermods have similar amplitudes. Thus, it is possible to check the assumption of the previous paragraph that $M_c/A_c = M_b/A_b$. By examining the magnitudes of the intermods $\omega_1 + \omega_2 + \omega_3$ or $\omega_2 + \omega_3 - \omega_1$ and $2\omega_1 + \omega_1$ or $2\omega_2 + \omega_1$, which have frequencies at which no other intermod of comparable order contributes, one can estimate

Table III — Signal amplitudes of the first, second and third order intermods for the kinds of rectification indicated. Intermods at higher order are indicated (in parenthesis) when they overlap a lower order intermod.

Intermod Order	Intermod & frequency	c=1	b=1	a=1
1	$\omega_1 = 10$	34.595	16.984	9.209
1	$\omega_2 = 11$	33.789	16.704	9.007
1	$\omega_3 = 13$	34.589	16.728	9.206
2	$\omega_2 - \omega_1 = 1$	20.044	9.044	3.883
2	$\omega_3 - \omega_2 = 2$	20.999	8.966	3.404
2	$\omega_3 - \omega_1 = 3$	22.789	9.236	3.317
2	$\omega_1 + \omega_2 = 21$	22.350	9.028	3.103
2	$\omega_1 + \omega_3 = 23$	21.961	9.200	3.399
2	$\omega_2 + \omega_3 = 24$	22.579	9.031	3.107
2	$2\omega_1 = 20$			
(4)	$3\omega_2 - \omega_3 = 20$	11.764	4.604	1.745
2	$2\omega_2 = 22$	9.843	4.483	2.219
2	$2\omega_3 = 26$			
(10)	$6\omega_4 - 4\omega_1 = 26$	12.240	4.641	1.647
3	$\omega_1 + \omega_2 - \omega_3 = 8$			
(5)	$3\omega_1 - 2\omega_2 = 8$	13.235	3.899	.193
3	$\omega_1 - \omega_2 + \omega_3 = 12$			
3	$2\omega_2 - \omega_1 = 12$	9.566	2.964	.074
3	$\omega_2 + \omega_3 - \omega_1 = 14$	13.679	4.328	.111
3	$\omega_1 + \omega_2 + \omega_3 = 34$	13.601	3.912	.02
3	$3\omega_1 = 30$	2.422	.789	.189
3	$3\omega_2 = 33$			
3	$2\omega_1 + \omega_3 = 33$	5.481	1.416	----
3	$3\omega_3 = 39$	2.208	.516	.146
3	$2\omega_1 - \omega_2 = 7$			
(5)	$3\omega_2 - 2\omega_3 = 7$	6.923	1.767	.089
3	$2\omega_1 - \omega_2 = 9$			
3	$2\omega_2 - \omega_3 = 9$	4.350	1.545	.109
3	$2\omega_2 - \omega_1 = 12$			
3	$\omega_1 + \omega_3 - \omega_2 = 12$	9.566	2.964	.054
3	$2\omega_3 - \omega_3 = 15$	6.550	1.996	.154
3	$2\omega_3 - \omega_1 = 16$	6.776	1.665	.136
3	$2\omega_1 + \omega_2 = 31$	6.635	1.688	.198
3	$2\omega_2 + \omega_1 = 32$	6.822	1.833	.178
3	$2\omega_1 + \omega_3 = 33$	5.481	1.416	----
3	$2\omega_2 + \omega_3 = 35$	6.649	1.999	.152

closely the magnitude of M_c, A_c, M_b and A_b. It can be seen that the proportion $M_c/A_c = M_b/A_b$ is valid to the expected accuracy. It appears in fact to be valid for all three third order harmonics, but the relationship would not be expected to hold in general.

In Table IV a similar grouping of intermods using only two (different) fundamental frequencies is shown. These data represent averages of only two separate runs but nevertheless result in better reproducibility of intermod amplitude than Table III because of fewer possible intermods having the same frequency.

Table IV — Signal amplitudes of a variety of intermod orders for the kinds of rectification indicated.

Intermod order	Intermod & frequency	c=1	b=1	a=1
1	$\omega_1 = 7$	20.680	13.244	9.191
1	$\omega_2 = 17$	20.687	13.243	9.191
2	$\omega_2 - \omega_1 = 10$	15.950	9.224	4.985
2	$\omega_2 + \omega_1 = 24$	15.702	9.081	4.907
2	$2\omega_1 = 14$	9.457	4.557	1.640
2	$2\omega_2 = 34$	9.406	4.533	1.632
3	$\omega_2 - 2\omega_1 = 3$	6.911	2.656	---
3	$2\omega_2 - \omega_1 = 27$	6.875	2.642	---
3	$\omega_2 + 2\omega_1 = 31$	6.715	2.580	---
3	$3\omega_1 = 21$	2.246	.518	---
4	$2\omega_2 - 2\omega_1 = 20$	2.280	---	.995
4	$3\omega_1 - \omega_2 = 4$	1.353	---	.330
4	$\omega_2 + 3\omega_1 = 38$	1.307	---	.319
5	$2\omega_2 - 3\omega_1 = 13$	---	.380	---
5	$3\omega_2 - 2\omega_1 = 37$	---	.377	---
5	$4\omega_1 - \omega_2 = 11$	---	.074	---
6	$3\omega_2 - 3\omega_1 = 30$	.252	---	.428

A number of other regularities are noticed in Table IV where results for higher order intermods are shown. A cubic rectification ($c = 1$) does not show fifth order intermods, but does generate 3rd order intermods while a quadratic law does not show fourth and sixth order intermods. Linear rectification appears to give only even order intermods (except for the fundamentals) which implies that the "third" order intermods in the last column of Table III probably arise, in fact, from various high order even intermods.

The effect of primary signal amplitude on the amplitude of the intermods for the three kinds of rectification discussed above has also been investigated. To minimize multiple intermod contributions to a single frequency we have used just the two frequencies 7 and 17. To simplify the interpretation, these two frequencies are always taken with equal amplitudes and varied by factors of two.

The amplitude relationships are very obvious (see Table V). For linear rectification, if the imput signals are increased by a factor n, then each intermod is increased by the same factor. For quadratic or cubic rectification, if the imput signals are increased by a factor n, then the intermods are increased by a factor n^2 or n^3 respectively. It is interesting that this is precisely the kind of behavior that is obtained for analytical linear, quadratic and cubic relationships although the particular intermods obtained are not. For example, a cubic i-e relationship (no rectification) would not produce any intermods greater than order three and no even order intermods whatever.

The accuracy of the relationships of the amplitudes discussed in the previous paragraph is extremely high as is the agreement of the amplitudes between different similar intermods (calculated, but not included in Table V). This is a result of only one significant intermod contributing to each frequency. In Table VI the same kind of experiment is carried out except that three frequencies are used and, incidentally, the input signal amplitudes are varied somewhat differently. In spite of the numbers in Table VI being the result of averaging several runs with random phases the variation from the amplitudes expected is somewhat greater than in Table V where a single run is shown.

Table V — Signal amplitudes for a number of intermods for the kinds of rectification indicated. The values of the primary signal amplitudes used are above the columns corresponding to the results.

Intermod order	Intermod & Frequency	1/4	1/2	1	2
		Linear Rectification (a=1)			
1	$\omega_1 = 7$	2.2910	4.5820	9.1637	18.327
2	$\omega_2 + \omega_1 = 24$	1.2157	2.4295	4.8594	9.7181
2	$2\omega_1 = 14$	.4057	.8113	1.6213	3.2433
3	$2\omega_2 - \omega_1 = 27$	----	----	----	----
3	$3\omega_1 = 21$	----	----	----	----
4	$2\omega_2 - 2\omega_1 = 20$	.2510	.4995	.9998	1.9994
4	$3\omega_1 - \omega_2 = 4$	.0807	.1630	.3256	.6514
		Quadratic Rectification (b=1)			
1	$\omega_1 = 7$	.8253	3.3011	13.2044	52.818
2	$\omega_2 + \omega_1 = 24$	.5619	2.2475	8.9899	35.9594
2	$2\omega_1 = 14$	.2817	1.1266	4.5064	18.026
3	$2\omega_2 - \omega_1 = 27$	.1639	.6555	2.6223	10.4893
3	$3\omega_1 = 21$	.0320	.1279	.5120	2.0477
4	$2\omega_2 - 2\omega_1 = 20$	---	----	----	----
4	$3\omega_1 - \omega_2 = 4$	----	----	----	----
		Cubic Rectification (c=1)			
1	$\omega_1 = 7$	.32215	2.5772	20.6175	164.940
2	$\omega_2 + \omega_1 = 24$	.2429	1.9432	15.5454	124.363
2	$2\omega_1 = 14$	.1461	1.1689	9.3512	74.810
3	$2\omega_2 - \omega_1 = 27$	.1066	.8530	6.8239	54.590
3	$3\omega_1 = 21$	.0347	.2775	2.21974	17.758
4	$2\omega_2 - 2\omega_1 = 20$	.0357	.2858	2.28647	18.292
4	$3\omega_1 - \omega_2 = 4$	.0210	.1677	1.3415	10.733

Table VI — Amplitudes as in Table V for three inputs.

Intermod order	Intermod Type	1/2	2/3	1
	Linear Rectification			
1	ω_1 etc	4.59	6.1	9.26
2	$\omega_1 + \omega_2$	1.6	2.25	3.07
3	$\omega_1 + \omega_2 + \omega_3$	.01	.05	.06
3	$2\omega_3 - \omega_1$	.1	.1	.2
	Quadratic Rectification			
1	ω_1	4.20	7.45	16.9
2	$\omega_1 + \omega_2$	2.25	4.05	9.0
3	$\omega_1 + \omega_2 + \omega_3$	.42	.75	1.65
	Cubic Rectification			
1	ω_1	4.32	10.2	34.6
2	$\omega_1 + \omega_2$	2.75	6.65	22.2
3	$\omega_1 + \omega_2 + \omega_3$	1.70	4.05	13.6
3	$2\omega_3 - \omega_1$	.83	1.95	6.85

An additional experiment was carried out to determine the influence of the coefficients, *a*, *b* and *c* on the intermod amplitude. One coefficient was individually varied by a factor of two, while the others were held equal to zero. It was expected that the resulting intermods would vary directly with the magnitude of the coefficient and this is exactly what was found.

CONCLUSION

We have shown here two approaches, the analytical and numerical, for obtaining the intermod spectrum given a specific nonlinearity and given the number of primary frequencies involved. Although one can do much with the analytical approach, we have shown that the computer method is the only way to handle the problem for certain rather common nonlinear systems. Nevertheless it is clear that the IMG diagnosis, that is the deducing of the i-e relationship from the characteristics of the intermod spectrum, cannot be carried out on a "prescription" basis. No routine procedure will quickly and unambiguously provide the i-e functional relationship and thus information leading to identification of the physical origin of the nonlinearity. However, the results of the analytical considerations and the computer method set forth above provide guidance for procedures that can be carried out, on a case by case basis, to obtain the i-e functional form.