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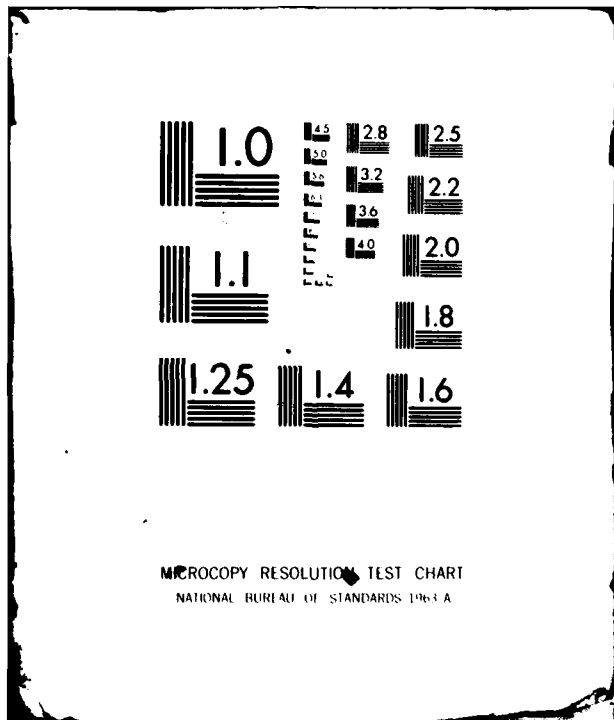
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Numerical Modeling of Dynamic Crack Propagation in Finite Bodies,

By Moving Singular Elements - Part I. Formulation

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Abstract

→ An efficient numerical (finite element) method is presented for the dynamic analysis of rapidly propagating cracks in finite bodies, of arbitrary shape, wherein linear-elastic material behavior and two-dimensional conditions prevail. Procedures to embed analytical asymptotic solutions for singularities in stresses/strains near the propagating crack-tip, to account for the spatial movement of these singularities along with the crack-tip, and to directly compute the dynamic stress-intensity factor, are presented. Numerical solutions of several problems and pertinent discussions are presented in Part II of this paper.

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Introduction

A concise summary of the present status of the theories of dynamic crack propagation can be found in a recent article by Freund [1]. Several analytical solutions of the linear elasto-dynamic equations for crack propagation in unbounded plane bodies have appeared earlier. These include the works of: Yoffé; Cragg; Broberg; and Baker, for Mode I (plane-strain opening mode) crack propagation; and the works of: Eshelby; and Achenbach, for Mode III crack extension. All the above works are summarized and referenced in a paper by Freund [2], who considered the problem of a half-plane crack, in an elastic solid subject to time-independent loading, which is initially at rest and, at a certain instant, begins to move with either a constant velocity [2] or a non-uniform velocity [3]. The studies in [2,3] were later extended [4] to consider stress-wave loading. However, as is usually the case, to study dynamic crack propagation in finite bodies of arbitrary geometry, it is necessary to formulate consistent numerical methods, which may capitalize on the insights, into the field behaviour near propagating crack-tips, gained through the analytical solutions. A critical appraisal of several and varied numerical solution techniques in dynamic fracture mechanics was made in a 1978 paper by Kanninen [5]. Most of the dynamic finite element methods, for fast crack-propagation analysis, reviewed in [5] use the conventional finite elements with simple polynomials for assumed displacements, and do not account for the singularity in strains near the crack-tip. Further, in these methods, the dynamic crack propagation was simulated by a "gradual" release of the restraining nodal force at a finite element node which represents the "current" crack-tip. The dynamic stress-intensity factor is then extracted from the displacement field or from the work done in releasing the nodal force. It was concluded in [5] that the above "node-release" techniques were not sufficiently accurate.

Since the appearance of [5], Bazant et al [6] have presented a calibrated, non-singular, crack-tip element procedure for the dynamic analysis of running cracks. In the procedure of [6], the finite-element grid moves undeformed with the crack-tip. However, the procedure of [6] has two serious limitations: (i) it is restricted to finite bodies whose surfaces and/or bimaterial interfaces are parallel to the direction of crack propagation; and (ii) more importantly, it cannot be applied to bodies having finite dimensions in the direction of crack propagation. On the other hand, Aoki et al [7] presented a finite element procedure wherein the singular nature of stress/strain near the propagating crack-tip is accounted for a priori. However, in [7], only when the crack-tip has reached close to the boundary of the singular element, the entire singular element is shifted, as a rigid body, to a new location. The numerical details of the procedures are still somewhat sketchy in [7]. Finally, King and Malluck [8] reported a procedure of simulating crack-propagation similar to that in [7], except that the singular-element used in [8] has, built within it, a large number of eigen-function solutions corresponding to a stationary crack. In an attempted simulation of the well-known problem of Baker, the procedure in [8] produced spurious oscillations, of large amplitude, in the solution for dynamic stress-intensity factor, as compared to the analytical solution. Based on these results, it is suggested in [8] that the procedure in [8] may not be feasible for simulating large scale fast fracture.

In Part I of the present paper, a "moving singular-element" procedure is presented for the dynamic analysis of fast crack-propagation problems in arbitrary shaped finite bodies. In the present procedure a singular-element, within which a large number of analytical eigen functions corresponding to a propagating crack are used as basis functions for displacements, may move by an arbitrary amount $\Delta \Sigma$ in each time-increment Δt of the numerical time-integration

procedure (as opposed several time steps, say 6 to 8, per increment of crack growth, used in the procedures reviewed in [5]). The moving singular-element, within which the crack-tip always has a fixed location, retains its shape at all times, but the mesh of "regular" (isoparametric) finite elements, surrounding the moving singular-element, deforms accordingly. An energy-consistent variational statement is first developed, as a basis for the above "moving singular-element" finite-element method of dynamic crack growth analysis. The present procedure leads to a direct evaluation of dynamic-stress intensity factor(s), since they are unknown parameters in the assumed basis functions for the singular-element.

In Part II of the paper, several numerical results for cracks propagating in finite bodies are presented and discussed.

In the following we discuss the details of formulation of a moving-singularity finite element formulation for analyzing dynamic crack propagation.

I. Basis Functions for a Moving Singular-Element

We consider Mode I-type dynamic crack propagation in two-dimensional (plane strain) linear elastic isotropic bodies of finite geometry. Let x_α ($\alpha = 1,2$) be fixed cartesian coordinates in the plane of the body, and x_3 be the thickness coordinate of the body such that $x_2 = 0$ defines the plane of the crack. In the context of the present numerical method, without loss of generality, we consider the case when the crack-tip is moving along x_1 axis at a constant speed v . We introduce the coordinate system (ξ, x_2) which remains fixed with respect to the moving crack-tip, such that $\xi = x_1 - vt$. Let ϕ and ψ be the dilatational and shear wave potentials, respectively; and let C_d and C_s be the corresponding wave speeds. It can then be shown [2] that ϕ is governed by the equation:

$$[1 - (v/C_d)^2] \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial x_2^2} = -(2v/C_d^2) \frac{\partial^2 \phi}{\partial t \partial \xi} + (1/C_d^2) \frac{\partial^2 \phi}{\partial t^2} \quad (I.1)$$

and that Ψ is governed by a similar equation, except that C_d is to be replaced by C_s . Consider the "steady-state" solution to the homogeneous part of the above equation, that is, the solution which appears time-invariant to an observer moving with the crack-tip. This eigen-function solution which satisfies the traction-free condition on the crack face ($\xi < 0$, $x_2 = +0$), can be derived easily, as for instance in [9,10], and is given in Appendix A for the sake of completeness.

In the present procedure, a finite region (which, for convenience, is taken to be rectangular in shape) near the moving crack-tip is modeled by one finite-element, in which the displacement field is assumed to be a linear-superposition of a finite number of the above-discussed eigen-functions. However, since the solution, in general, will also explicitly depend on time, the undetermined parameters, β_n , are taken to be functions of time. Thus, in the singular-element, we assume,

$$u_1(\xi, x_2, t) = \sum_n u_{1n}^*(\xi, x_2, v) \beta_n(t) + \text{Rigid body modes} \quad (I.2)$$

$$u_2(\xi, x_2, t) = \sum_n u_{2n}^*(\xi, x_2, v) \beta_n(t) + \text{Rigid body modes} \quad (I.3)$$

where u_{2n}^* , u_{1n}^* are given in Appendix A, and, in particular, $\beta_1(t)$ is identified as the mode I dynamic stress-intensity factor. It can then be seen that, in the present finite element procedure, the dynamic stress-intensity factor is an unknown parameter in the element basis-functions, and thus can be calculated directly. Representing the above Eqs. (I.2,3) in the familiar matrix notation,

$$\underline{u}^s(\xi, x_2, t) = \underline{U}(\xi, x_2, v) \underline{\beta}(t) \quad (I.4)$$

where (\sim) and (\simeq) under a symbol denote a column vector and a matrix, respectively; and \underline{u}^s denotes the vector of displacements in the singular-element. We note that the total velocity and acceleration of a material point in the singular-element are given by:

$$\dot{\underline{u}}^s = \underline{U} \dot{\underline{\beta}} - v(\underline{U})_{,\xi} \underline{\beta} \quad (I.5)$$

and

$$\ddot{\underline{u}}^s = \underline{U} \ddot{\underline{\beta}} - 2v(\underline{U})_{,\xi} \dot{\underline{\beta}} + v^2(\underline{U})_{,\xi\xi} \underline{\beta} \quad (I.6)$$

where, a $(\dot{\quad})$ denotes a total derivative with respect to time t , and $(\quad)_{,\xi}$ denotes a partial derivative with respect to ξ .

Let the domain of the singular element in the present procedure be V_s and its boundary be ∂V_s ; and let ρ_s be that part of ∂V_s where the usual isoparametric finite elements adjoin. In order that convergence of the present finite element method may be achieved, compatibility of displacements, velocities, and accelerations between the singular elements and surrounding regular elements, i.e., at ρ_s , is maintained in a least squares sense as described below. Let the displacement, velocity and acceleration assumption for the regular element, at ρ_s , be taken, respectively, as:

$$\underline{u}^R = \underline{N} \underline{q}_s \quad ; \quad \dot{\underline{u}}^R = \underline{N} \dot{\underline{q}}_s \quad ; \quad \ddot{\underline{u}}^R = \underline{N} \ddot{\underline{q}}_s \quad (I.7a,b,c)$$

where \underline{N} are functions of the boundary coordinate $\eta(x_0)$ at ∂V_s , and \underline{q}_s is the vector of displacements at nodes at ρ_s . The parameters $\underline{\beta}$, $\dot{\underline{\beta}}$ and $\ddot{\underline{\beta}}$ are so chosen that they minimize the error functionals:

$$I_1 = \int_{\rho_s} (\underline{u}^s - \underline{u}^R)^2 d\rho \quad ; \quad I_2 = \int_{\rho_s} (\dot{\underline{u}}^s - \dot{\underline{u}}^R)^2 d\rho \quad ; \quad I_3 = \int_{\rho_s} (\ddot{\underline{u}}^s - \ddot{\underline{u}}^R)^2 d\rho \quad (I.8,9,10)$$

Using Eqs. (I.4,5,6 and 7) in (I.8-10), and minimizing I_1 , I_2 and I_3 successively with respect to $\underline{\beta}$, $\dot{\underline{\beta}}$ and $\ddot{\underline{\beta}}$ it can be shown that,

$$\underline{\underline{\beta}} = \underline{\underline{A}} \underline{\underline{q}}_s ; \quad \underline{\underline{\dot{\beta}}} = \underline{\underline{A}} \underline{\underline{\dot{q}}}_s + \underline{\underline{B}} \underline{\underline{q}}_s ; \quad \underline{\underline{\ddot{\beta}}} = \underline{\underline{A}} \underline{\underline{\ddot{q}}}_s + 2 \underline{\underline{B}} \underline{\underline{\dot{q}}}_s + \underline{\underline{C}} \underline{\underline{q}}_s \quad (\text{I.11,12,13})$$

where:

$$\underline{\underline{A}} = \underline{\underline{H}}^{-1} \underline{\underline{G}} ; \quad \underline{\underline{B}} = (\underline{\underline{v}}) \underline{\underline{H}}^{-1} \underline{\underline{E}} \underline{\underline{A}} \quad (\text{I.14,15})$$

$$\underline{\underline{C}} = 2(\underline{\underline{v}}) \underline{\underline{H}}^{-1} \underline{\underline{E}} \underline{\underline{B}} - (\underline{\underline{v}}^2) \underline{\underline{H}}^{-1} \underline{\underline{F}} \underline{\underline{A}} \quad (\text{I.16})$$

$$\underline{\underline{H}} = \int_{\rho_s} \underline{\underline{U}}^T \underline{\underline{U}} d\rho ; \quad \underline{\underline{G}} = \int_{\rho_s} \underline{\underline{U}}^T \underline{\underline{N}} d\rho \quad (\text{I.17a,b})$$

$$\underline{\underline{E}} = \int_{\rho_s} \underline{\underline{U}}^T (\underline{\underline{U}})_{,\xi} d\rho ; \quad \underline{\underline{F}} = \int_{\rho_s} \underline{\underline{U}}^T (\underline{\underline{U}})_{,\xi\xi} d\rho \quad (\text{I.17c,d})$$

Thus, Eqs. (I.4,5,6) together with (I.11,12,13) represent the displacements, total velocities and total accelerations in the singular element, in terms of its nodal displacements, velocities, and accelerations, $\underline{\underline{q}}_s$, $\underline{\underline{\dot{q}}}_s$ and $\underline{\underline{\ddot{q}}}_s$, respectively. Thus, if $\underline{\underline{q}}_s$ at ρ_s is determined, then $\underline{\underline{\beta}}$ (and especially the mode-I stress intensity factor β_1), can be determined directly. Finally, it is noted that the above Eqs. (I.4,5,6) and (I.11,12,13) represent the assumptions for the relevant field variables in the singular element at any generic time t .

Now we consider the problem of dynamic crack propagation within a time increment Δt between two generic times t_1 and t_2 .

I.B. Variational Principle for Dynamic Crack Propagation Analysis

In the following, we present a variational statement for dynamically growing cracks in linear elastic solids. Consider two instants of time t_1 and t_2 ($= t_1 + \Delta t$) at which the variables of the problem are denoted by superscripts 1 and 2, respectively. At time t_1 , let the volume of the solid be V_1 , the external boundary of the solid where tractions \bar{T}_1^1 are prescribed, be S_{01} ; and let Σ_1^+ and Σ_1^- be, respectively, the two surfaces of the crack. Also, let \bar{F}_1^2 be body forces per unit volume in the body at time t_2 .

We assume that between time t_1 and t_2 , the crack surfaces change by $\Delta\Sigma$. The orientation of $\Delta\Sigma$ to Σ , can be determined by some crack-growth direction criterion; however, for pure Mode I, self-similar growth is assumed. The newly created crack-surfaces can be traction free, but, for the sake of generality, assume that new tractions \bar{T}_1^{2+} and \bar{T}_1^{2-} are applied on the new crack-faces $\Delta\Sigma^+$ and $\Delta\Sigma^-$, respectively; likewise, let new tractions \bar{T}_1^2 act at S_{σ_2} . The principle of virtual work applied at t_2 can be written as:

$$\begin{aligned}
0 = & \int_{V_2} (\sigma_{ij}^2 \delta \epsilon_{ij}^2 + \rho \ddot{u}_i^2 \delta u_i^2) dv - \int_{V_2} \bar{F}_1^2 \delta u_i^2 - \int_{S_{\sigma_2}} \bar{T}_1^2 \delta u_i^2 ds \\
& - \int_{\Sigma_1^+} (\bar{T}_1^2)^+ (\delta u_i^2)^+ ds - \int_{\Sigma_1^-} (\bar{T}_1^2)^- (\delta u_i^2)^- ds \quad (I.18) \\
& - \int_{\Delta\Sigma^+} (\bar{T}_1^2)^+ (\delta u_i^2)^+ ds - \int_{\Delta\Sigma^-} (\bar{T}_1^2)^- (\delta u_i^2)^- ds
\end{aligned}$$

However, for the case of cracked structures, the changes in volume and external surfaces between times t_1 and t_2 , due to a change in the crack-surface by $\Delta\Sigma$ alone, can be assumed to be negligible, i.e., $V_1 \approx V_2$ and $S_{\sigma_1} \approx S_{\sigma_2}$. It is important to note in Eq. (I.18) that $(u_i^2)^+ \neq (u_i^2)^-$ [or $(\delta u_i^2)^+ \neq (\delta u_i^2)^-$] at the initial crack surfaces Σ_1^+ and Σ_1^- , nor, more importantly, for the newly created crack faces $\Delta\Sigma^+$ and $\Delta\Sigma^-$ during the time interval $t_2 - t_1 (= \Delta t)$. If similar virtual displacements (δu_i^2) , such that $(\delta u_i^2)^+ \neq (\delta u_i^2)^-$ either on Σ , or on $\Delta\Sigma$, are considered in the statement of virtual work at time t_1 (prior to the creation of new crack faces $\Delta\Sigma$); this statement can be written as:

*It is noted that the element basis functions assumed in Eqs. (I.2) and (I.3) satisfy only the traction-free conditions on the crack-face. It is, however, easy to accommodate non-zero traction conditions on the crack-face by introducing appropriate additional terms in Eqs. (I.2 and I.3). These additional terms are so chosen that they satisfy the non-zero crack-face traction conditions either exactly or in an average sense.

$$\begin{aligned}
0 = & \int_{V_2} (\sigma_{ij}^1 \delta \epsilon_{ij}^2 + \rho \ddot{u}_i^1 \delta u_i^2) dv - \int_{V_2} \bar{F}_i^1 \delta u_i^2 - \int_{S_{\sigma_2}} \bar{T}_i^1 \delta u_i^2 ds \\
& - \int_{\Sigma_1^+} (\bar{T}_i^1)^+ (\delta u_i^2)^+ ds - \int_{\Sigma_1^-} (\bar{T}_i^1)^- (\delta u_i^2)^- ds - \int_{\Delta \Sigma^+} (\sigma_{ij}^1 v_j^1)^+ (\delta u_i^2)^+ ds \\
& - \int_{\Delta \Sigma^-} (\sigma_{ij}^1 v_j^1)^- (\delta u_i^2)^- ds
\end{aligned} \tag{I.19}$$

wherein the approximations $V_2 \approx V_1$; $S_{\sigma_2} \approx S_{\sigma_1}$ are used and v_j^1 is a unit normal to Σ_1 . Adding Eqs. (I.18) and (I.19), the virtual work principle governing dynamic crack propagation between times t_1 and t_2 can be written as:

$$\begin{aligned}
& \int_{V_2} \{ (\sigma_{ij}^2 + \sigma_{ij}^1) \delta \epsilon_{ij}^2 + \rho (\ddot{u}_i^2 + \ddot{u}_i^1) \delta u_i^2 - (\bar{F}_i^2 + \bar{F}_i^1) \delta u_i^2 \} dv \\
& = \int_{S_{\sigma_2}} (\bar{T}_i^1 + \bar{T}_i^2) \delta u_i^2 ds + \int_{\Sigma_1^+} (\bar{T}_i^2 + \bar{T}_i^1)^+ (\delta u_i^2)^+ ds \\
& + \int_{\Sigma_1^-} (\bar{T}_i^2 + \bar{T}_i^1)^- (\delta u_i^2)^- ds + \int_{\Delta \Sigma^+} (\bar{T}_i^2 + \sigma_{ij}^1 v_j^1)^+ (\delta u_i^2)^+ ds \\
& + \int_{\Delta \Sigma^-} (\bar{T}_i^2 + \sigma_{ij}^1 v_j^1)^- (\delta u_i^2)^- ds
\end{aligned} \tag{I.20}$$

In the finite element development, the domain V_2 can be considered to be broken into: a singular element V_{2s} surrounding the crack tip (See Fig. 1), and a number, N , of regular elements V_{2Rn} ($n=1..N$) (thus, $V_2 = V_{2s} + \sum_n V_{2Rn}$); likewise $S_{\sigma_2} = \sum_n S_{\sigma_2 Rn}$. Also, as seen from Fig. 1, $\Sigma_1^+ = \Sigma_{s1}^+ + \sum_n \Sigma_{1Rn}^+$. Henceforth, for simplicity, we use symbols V_s , V_{Rn} , S_{σ_n} , Σ_s^+ , and Σ_{Rn}^+ instead of V_{2s} , V_{2Rn} , $S_{\sigma_2 Rn}$, Σ_{s1}^+ , and Σ_{1Rn}^+ , respectively. We now restrict our attention to the mode-I case only, i.e., when the applied loading is in a direction normal to the crack plane and is symmetric with respect to the crack plane for all times t . Thus, for the mode I case, using the above notation, the virtual work equation as applicable to a system of finite elements may be written as:

$$\begin{aligned}
& \int_{V_{Rn}} \left\{ (\sigma_{ij}^2 + \sigma_{ij}^1) \delta \varepsilon_{ij}^2 + \rho (\ddot{u}_i^2 + \ddot{u}_i^1) \delta u_i^2 - (\bar{F}_i^2 + \bar{F}_i^1) \delta u_i^2 \right\} dv \\
& - \int_{S_{on}} (\bar{T}_i^2 + \bar{T}_i^1) \delta u_i^2 ds - \int_{\Sigma_n^+} (\bar{T}_i^2 + \bar{T}_i^1)^+ \delta u_i^{2+} d\Sigma \left] + \int_{V_s} \left\{ (\sigma_{ij}^2 + \sigma_{ij}^1) \delta \varepsilon_{ij}^2 \right. \right. \\
& \left. \left. + \rho (\ddot{u}_i^2 + \ddot{u}_i^1) \delta u_i^2 \right\} dv - \int_{\Sigma_s^+} (\bar{T}_i^2 + \bar{T}_i^1)^+ \delta u_i^{2+} d\Sigma \right. \\
& \left. - \int_{\Delta \Sigma^+} (\bar{T}_i^2 + \sigma_{ij}^1 v_j^1)^+ \delta u_i^{2+} ds = 0 \quad (I.21)
\end{aligned}$$

Assuming that crack growth occurs between times t_1 and t_2 (which can be determined by an appropriate criterion, in the so-called "application" calculations using the given material dynamic fracture toughness as an input; or is known, a priori, in the so-called "generation phase", i.e., in the case of simulation of known crack-tip time history data), the singular-element is translated, in the mode I case, along the original crack axis, by an appropriate distance $\Delta \Sigma$ from its location at time t_1 , as shown in Fig. 1.

It is important to note that in the present procedure, this amount $\Delta \Sigma$ is not, in any way, related to the distance between any two adjacent finite element nodes at time t_1 ; as is the case with most common finite element methods which use the "node-release" technique in the simulation of dynamic crack propagation. As can be seen from Fig. 1, as the singular-element is translated by $\Delta \Sigma$ between t_1 and t_2 , the nodal pattern of the surrounding regular elements also changes between t_1 and t_2 . It is to this readjusted finite element mesh at time t_2 that the virtual work equation in Eq. (I.21) is understood to be applied. However, it is also noted that only the nodes of the elements immediately surrounding the singular-element are readjusted due to crack-growth of amount $\Delta \Sigma$ between t_2 and t_1 . Thus, one has to obtain data, such as displacements, velocities, and accelerations, at time t_1 , at the new

nodes of the regular elements, which are indicated by solid circles in Fig. 1. This data can be determined, using elementary interpolation techniques, from the known data, at time t_1 , at the 'old' nodes at time t_1 , which are indicated by open circles in Fig. 1. The details of these interpolation techniques are omitted for simplicity and will be reported elsewhere. Thus, one is in a position to know the relevant data at time t_1 , at new nodes and (hence new elements) corresponding to the mesh in t_2 ; and to assume the appropriate basis functions for the relevant variables at time t_2 for the mesh at time t_2 , as follows:

Known at t_1 for the mesh at t_2 :

$$\text{in } V_{Rn}: \underline{u}_1 = \underline{N} \underline{q}_1 ; \underline{\epsilon}_1 = \underline{B} \underline{q}_1 ; \underline{\sigma}_1 = \underline{E} \underline{B} \underline{q}_1 \quad (\text{I.22-24})$$

$$\dot{\underline{u}}_1 = \underline{N} \dot{\underline{q}}_1 ; \ddot{\underline{u}}_1 = \underline{N} \ddot{\underline{q}}_1 \quad (\text{I.25,26})$$

$$\text{in } V_s: \underline{u}_1 = \underline{U}_1 \underline{\beta}_1 ; \dot{\underline{u}}_1 = \underline{U}_1 \dot{\underline{\beta}}_1 - v_1 \underline{U}_{1,\xi} \underline{\beta}_1 \quad (\text{I.27,28})$$

$$\ddot{\underline{u}}_1 = \underline{U}_1 \ddot{\underline{\beta}}_1 - 2 v_1 \underline{U}_{1,\xi} \dot{\underline{\beta}}_1 + v_1^2 \underline{U}_{1,\xi\xi} \underline{\beta}_1 \quad (\text{I.29})$$

$$\underline{\epsilon}_1 = \underline{S}_1 \underline{\beta}_1 ; \underline{\sigma}_1 = \underline{P}_1 \underline{\beta}_1 ; \underline{T}_1 = \underline{R}_1 \underline{\beta}_1 \quad (\text{I.30-32})$$

Assumed at time t_2 for the mesh at time t_2 :

$$\text{in } V_{Rn}: \underline{u}_2 = \underline{N} \underline{q}_2 ; \underline{\epsilon}_2 = \underline{B} \underline{q}_2 ; \underline{\sigma}_2 = \underline{E} \underline{B} \underline{q}_2 \quad (\text{I.33-35})$$

$$\dot{\underline{u}}_2 = \underline{N} \dot{\underline{q}}_2 ; \ddot{\underline{u}}_2 = \underline{N} \ddot{\underline{q}}_2 \quad (\text{I.36,37})$$

$$\text{in } V_s: \underline{u}_2 = \underline{U}_2 \underline{\beta}_2 ; \dot{\underline{u}}_2 = \underline{U}_2 \dot{\underline{\beta}}_2 - v_2 \underline{U}_{2,\xi} \underline{\beta}_2 \quad (\text{I.38,40})$$

$$\ddot{\underline{u}}_2 = \underline{U}_2 \ddot{\underline{\beta}}_2 - 2 v_2 \underline{U}_{2,\xi} \dot{\underline{\beta}}_2 + v_2^2 \underline{U}_{2,\xi\xi} \underline{\beta}_2 \quad (\text{I.41})$$

$$\underline{\epsilon}_2 = \underline{S}_2 \underline{\beta}_2 ; \underline{\sigma}_2 = \underline{P}_2 \underline{\beta}_2 ; \underline{T}_2 = \underline{R}_2 \underline{\beta}_2 \quad (\text{I.42-44})$$

where the familiar vector representations for displacements, strains, stresses, and tractions, are employed as \underline{u} , $\underline{\epsilon}$, $\underline{\sigma}$ and \underline{T} respectively. Also, v_1 and v_2 are velocities of the crack-tip at times t_1 and t_2 respectively, and the eigen functions \underline{U}_1 and \underline{U}_2 depend on v_1 and v_2 respectively.

Using Eqs. (I.22-44) in Eq. (I.21), the finite element equations, for arbitrary variations δq_2 and $\delta \beta_2$ can be written, as shown in Appendix B, as:

$$\underline{\underline{K}} \underline{\underline{q}}_2 + \underline{\underline{m}} \underline{\underline{\ddot{q}}}_2 = \underline{\underline{Q}}_2 + \underline{\underline{Q}}_1 - \underline{\underline{K}} \underline{\underline{q}}_1 - \underline{\underline{m}} \underline{\underline{\ddot{q}}}_1 \text{ for } V_{Rn} \text{ in } V_2 - V_s \quad (\text{I.45})$$

$$\underline{\underline{K}}_s^* \underline{\underline{q}}_{s2} + \underline{\underline{D}}_s^* \underline{\underline{\dot{q}}}_{s2} + \underline{\underline{m}}_s^* \underline{\underline{\ddot{q}}}_{s2} = \underline{\underline{Q}}_s^* \text{ for } V_s \quad (\text{I.46})$$

where $\underline{\underline{K}}$, $\underline{\underline{m}}$, $\underline{\underline{Q}}_2$, $\underline{\underline{Q}}_1$, $\underline{\underline{K}}_s^*$, $\underline{\underline{D}}_s^*$, and $\underline{\underline{m}}_s^*$ are defined in Appendix B, from which it can be seen that the metrics $\underline{\underline{K}}_s^*$ and $\underline{\underline{D}}_s^*$ are, unfortunately, unsymmetric, while the others are all symmetric. In Eq. (I.45), $\underline{\underline{q}}_2$ and $\underline{\underline{\ddot{q}}}_2$ are displacements and accelerations at t_2 at nodes everywhere in and at the boundary of the region ($V_2 - V_s$); whereas, $\underline{\underline{q}}_{s2}$, $\underline{\underline{\dot{q}}}_{s2}$, and $\underline{\underline{\ddot{q}}}_{s2}$ are displacements, velocities, and acceleration at t_2 at nodes along the boundary ∂V_s of the singular element. When Eqs. (I.45,46) are assembled, it can be seen that the resulting global "stiffness" and "damping" (which, however, is not a physical damping term) matrices have only a "small" degree of unsymmetry, confined to those rows and columns corresponding to nodes around the singular element. We can use the common time-integration schemes to integrate Eqs. (I.45-46). In particular, we use the Newmark's method which can be characterized by the approximations:

$$\underline{\underline{\dot{q}}}_2 = C_1 (\underline{\underline{q}}_2 - \underline{\underline{q}}_1) - C_2 \underline{\underline{\dot{q}}}_1 - C_3 \underline{\underline{\ddot{q}}}_1 \quad (\text{I.47})$$

$$\underline{\underline{\ddot{q}}}_2 = C_4 (\underline{\underline{q}}_2 - \underline{\underline{q}}_1) - C_5 \underline{\underline{\dot{q}}}_1 - C_6 \underline{\underline{\ddot{q}}}_1 \quad (\text{I.48})$$

$$\text{where } C_1 = (\delta/\gamma \Delta t) \quad ; \quad C_2 = (\delta/\gamma) - 1 \quad ; \quad C_3 = \left(\frac{\Delta t}{2}\right) [(\delta/\gamma) - 2]$$

$$C_4 = 1/\gamma (\Delta t)^2 \quad ; \quad C_5 = 1/(\gamma \Delta t) \quad ; \quad C_6 = (\delta/\gamma) - 1 \quad (\text{I.49})$$

where, in the present calculations, $\gamma = \frac{1}{4}$, $\delta = \frac{1}{2}$ are used. With the difference approximations in Eq. (I.47,48), and similar ones for $\underline{\underline{\dot{q}}}_{s2}$ and $\underline{\underline{\ddot{q}}}_{s2}$, we reduce Eqs. (I.45,46) to:

$$\hat{\underline{\underline{K}}}_{\underline{\underline{2}}} \underline{\underline{q}}_2 = \hat{\underline{\underline{Q}}} \text{ for } V_2 - V_s \quad (\text{I.50})$$

$$\hat{\underline{\underline{K}}}_{\underline{\underline{s}}} \underline{\underline{q}}_{s2} = \hat{\underline{\underline{Q}}}_s \text{ for } V_s \quad (\text{I.51})$$

where

$$\hat{\underline{\underline{K}}}_{\underline{\underline{2}}} = \underline{\underline{K}}_{\underline{\underline{2}}} + C_4 \underline{\underline{m}}_{\underline{\underline{2}}} \quad (\text{I.52})$$

$$\hat{\underline{\underline{Q}}} = \underline{\underline{Q}}_2 + \underline{\underline{Q}}_1 - \underline{\underline{K}}_{\underline{\underline{2}}} \underline{\underline{q}}_1 - \underline{\underline{m}}_{\underline{\underline{2}}} \underline{\underline{q}}_1 + \underline{\underline{m}}_{\underline{\underline{2}}} (C_4 \underline{\underline{q}}_1 + C_5 \underline{\underline{\dot{q}}}_1 + C_6 \underline{\underline{\ddot{q}}}_1) \quad (\text{I.53})$$

$$\hat{\underline{\underline{K}}}_{\underline{\underline{s}}} = \underline{\underline{K}}_{\underline{\underline{s}}}^* + C_4 \underline{\underline{m}}_{\underline{\underline{s}}}^* + C_1 \underline{\underline{D}}_{\underline{\underline{s}}}^* \quad (\text{I.54})$$

$$\hat{\underline{\underline{Q}}}_s = \underline{\underline{Q}}_s^* + \underline{\underline{m}}_{\underline{\underline{s}}}^* (C_4 \underline{\underline{q}}_{s1} + C_5 \underline{\underline{\dot{q}}}_{s1} + C_6 \underline{\underline{\ddot{q}}}_{s1}) + \underline{\underline{D}}_{\underline{\underline{s}}}^* (C_1 \underline{\underline{q}}_{s1} + C_2 \underline{\underline{\dot{q}}}_{s1} + C_3 \underline{\underline{\ddot{q}}}_{s1}) \quad (\text{I.55})$$

where $\hat{\underline{\underline{K}}}_{\underline{\underline{2}}}$ is symmetric; however, $\hat{\underline{\underline{K}}}_{\underline{\underline{s}}}$ is unsymmetric. When Eqs. (I.50,51) are assembled, we obtain, the final algebraic equations:

$$[\hat{\underline{\underline{K}}}_{\underline{\underline{2}}}^*] \{ \underline{\underline{q}}_2^* \} = \{ \underline{\underline{Q}}^* \} \quad (\text{I.56})$$

where the stiffness matrix in Eq. (I.56) is, in general, unsymmetric, but the unsymmetry is confined mainly to the rows and columns corresponding to nodes around V_s . A rather simple technique of iterative solution of the above equation, based on the decomposition of the stiffness matrix into symmetric and skew-symmetric parts, as below, was used.

$$\frac{1}{2} [\hat{\underline{\underline{K}}}_{\underline{\underline{2}}}^* + \hat{\underline{\underline{K}}}_{\underline{\underline{2}}}^{*T}] \{ \underline{\underline{q}}_2^{*p} \} = \{ \underline{\underline{Q}}^* \} - \frac{1}{2} [\hat{\underline{\underline{K}}}_{\underline{\underline{2}}}^* - \hat{\underline{\underline{K}}}_{\underline{\underline{2}}}^{*T}] \{ \underline{\underline{q}}_2^{*(p-1)} \} \quad (\text{I.57})$$

for any pth-iteration. In all the solutions obtained, only two iterations were found to adequate. Once $\underline{\underline{q}}_2^*$ is computed from Eq. (I.56), the solution for time $t_2 + \Delta t$ can be repeated, with the approximations for the initial values $\underline{\underline{q}}_2^*$ and $\underline{\underline{\dot{q}}}_2^*$ as:

$$\underline{\underline{\ddot{q}}}_2^* = C_4 [\underline{\underline{q}}_2^* - \underline{\underline{q}}_1^*] - C_5 \underline{\underline{\dot{q}}}_1^* - C_6 \underline{\underline{\ddot{q}}}_1^* \quad (\text{I.58})$$

$$\underline{\underline{\dot{q}}}_2^* = \underline{\underline{\dot{q}}}_1^* + C_7 \underline{\underline{\ddot{q}}}_1^* + C_8 \underline{\underline{\ddot{\ddot{q}}}}_2^* \quad (\text{I.59})$$

where C_4, C_5, C_6 are defined earlier, and $C_7 = \Delta t (1-\delta)$; and $C_8 = \delta \Delta t$ (where a value of $\delta = \frac{1}{2}$ is used presently).

Once the nodal displacements q_2^* (and hence the corresponding displacements at the nodes of the singular-element), at time t_2 , are computed from Eq. (I.57), the unknown parameters β (and hence the dynamic stress-intensity factor β_1) in the singular-element can be computed from Eq. (I.11).

Using Eqs. (I.58 and 59) as initial data, the time-integration between the time steps t_2 and t_3 ($t_2 + \Delta t$) can be carried out and, thus, the process can be repeated for all subsequent time intervals. The successive growth of the crack, for a representative problem is schematically illustrated in Fig. 2.

From the example given in Fig. 2, it is seen that the singular-element (A) remains its shape at all times but the regular-elements (B) in the "immediate surrounding" of the singular-element continually distort. However, in the above example, at $t = 2.0 \mu\text{sec}$, elements B have distorted sufficiently so that the use of isoparametric approximations in these elements may introduce spurious numerical errors. For this reason, as typified by the above example, at $t = 2.0 \mu\text{s}$, the regular elements B are re-adjusted as shown in Fig. 2. This involves a simple reinterpolation of data, in 'B' type elements from $t = 2.0 - 0 \mu\text{s}$ to $t = 2.0 + 0 \mu\text{s}$, the details of which are omitted for brevity. Finite element calculations detailed earlier can be repeated for the readjusted mesh at $t = 2.0 + 0 \mu\text{s}$ until the B type elements become so distorted that another readjustment may be warranted. These mesh readjustments were found to be easy to accomplish in the computer coding based on the present approach.

Finally, it may be of interest to note that in the present singular element, 19 eigen functions^{*} for a propagating crack (See Appendix A) were used

^{*}The number of eigen functions plus the number of rigid modes must be greater than or equal to the number of degrees of freedom at the boundary. A study of the effect of the number of eigen functions used, on the results was conducted, by varying this number from 17 to 25. The results varied only insignificantly (i.e., less than 0.4%), and the number of eigen functions was chosen to be 19 in all subsequent computations.

along with a rigid-body translation mode in x_1 direction; whereas, there are 18 degrees of freedom along the boundary ρ_s of the singular element. The regular elements were of the common 8-noded isoparametric type.

It should be remarked that the problems dealt with in the present paper are limited to the case of determining the stress-intensity factor at the crack-tip which is propagating with a prescribed velocity-time history. Thus the presently treated problem may be considered to fall in the category of "generation phase calculations" in the sense defined in [5]. The present procedure may be used to simulate the experimentally determined crack-velocity-time history in test specimens, such as the double-cantilever-beam (DCB) specimen [11], to determine the velocity dependent dynamic fracture toughness. Using this as input data, the problem of determining the crack-tip motion in plane elastic-bodies subject to Mode-I type dynamic transient loading may be treated. This second phase of research, which is the so-called "application phase" in the sense defined in [5], is currently being completed, and will be the basis of a forthcoming paper.

Finally, we wish to note that once the basic features of the procedure based on the present moving singular-element, with embedded propagating-crack eigenfunctions, are well understood, the numerical procedure can be further simplified. This can be accomplished, for instance, by using the well-known distorted isoparametric elements (the so-called "quarter-point elements") [12] in place of the present singular element. Eventhough the results from the use of a quarter-point element are not expected to be as accurate as from the use of the present singular-element; such results, with a suitable calibration, may be used in analyzing large-scale fast fracture

in practical situations. The results from the use of a quarter-point element, and their comparison with those reported in Part II of this paper (using the present singular-element), will be reported on shortly. Also, since it is known [10] that the eigen-functions for a crack propagating at constant velocity differ significantly in their behavior from those for stationary crack only at very high speeds ($v \approx C_s$) of propagation, the present procedure can be simplified, for practical purposes, by using the stationary-crack eigen-functions in the singular-element. The results from this modifications, are also to be reported shortly.

Closure

In this paper we have presented a new translating-singularity finite element procedure, wherein use is made of analytical eigen-functions for a two-dimensional crack whose tip propagates at a constant velocity. The procedure is capable of modeling large-scale fast crack propagation in finite two-dimensional bodies of arbitrary shape. However, the type of problems considered is limited to the case of determining the dynamic stress-intensity factor at the crack-tip which is propagating with a prescribed velocity-time history.

Implementation of the present approach and numerical example are discussed in an accompanying Part II of the paper.

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Appendix A

Details of basis functions for the singular element, for the mode I case, are given here. The eigen-functions given here are solutions to the following equations for wave potentials ϕ and Ψ :

$$[1 - (v/C_d)^2] \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial x_2^2} = 0 \quad (\text{A.1})$$

with a similar equation for Ψ when C_d is replaced by C_s . For any non-zero, constant, speed of propagation, the eigen functions can be derived to be:

$$u_1 = \sum_n u_{1n}^* \beta_n ; u_2 = \sum_n u_{2n}^* \beta_n ; \sigma_{\alpha\beta} = \sum_n \sigma_{\alpha\beta n} \beta_n ; \alpha, \beta = 1, 2 \quad (\text{A.2,3,4})$$

where

$$u_{1n}^* = \frac{1}{\mu} F(\alpha_s, \alpha_d) [(n/2) + 1] \{ r_1^{n/2} \cos(n \theta_1/2) - (\frac{1}{2}) g(n) r_2^{n/2} \cos(n \theta_2/2) \} \quad (\text{A.5})$$

$$u_{2n}^* = \frac{1}{\mu} F(\alpha_s, \alpha_d) [(n/2) + 1] \{ -\alpha_d r_1^{n/2} \sin(n \theta_1/2) + (\frac{1}{2}) [g(n)/\alpha_s] r_2^{n/2} \sin(n \theta_2/2) \} \quad (\text{A.6})$$

$$\sigma_{11n} = F(\alpha_s, \alpha_d) (n/2) [(n/2) + 1] \{ (2\alpha_d^2 - \alpha_s^2 + 1) r_1^{[(n/2) - 1]} \times \cos [((n/2) - 1) \theta_1] - g(n) r_2^{[(n/2) - 1]} \cos [((n/2) - 1) \theta_2] \} \quad (\text{A.7})$$

$$\sigma_{22n} = F(\alpha_s, \alpha_d) (n/2) [(n/2) + 1] \{ -(1 + \alpha_s^2) r_1^{[(n/2) - 1]} \times \cos [((n/2) - 1) \theta_1] + g(n) r_2^{[(n/2) - 1]} \cos [((n/2) - 1) \theta_2] \} \quad (\text{A.8})$$

$$\sigma_{12n} = F(\alpha_s, \alpha_d) (n/2) [(n/2) + 1] \{ -2 \alpha_d r_1^{[(n/2) - 1]} \times \sin [((n/2) - 1) \theta_1] + (\frac{1}{2}) [(1 + \alpha_s^2)/\alpha_s] g(n) r_2^{[(n/2) - 1]} \times \sin [((n/2) - 1) \theta_2] \} \quad (\text{A.9})$$

where λ , μ are Lamé's constants; C_d and C_s respectively the dilatational and shear wave speeds $C_d = [(\lambda+2\mu)/\rho]^{1/2}$; $C_s = (\mu/\rho)^{1/2}$; and the various parameters appearing above are defined as:

$$\alpha_d^2 = [1 - (v/C_d)^2] \quad ; \quad \alpha_s^2 = [1 - (v/C_s)^2] \quad (\text{A.10})$$

$$F(\alpha_d, \alpha_s) = \frac{4}{3(2\pi)^{1/2}} \frac{(1 + \alpha_s^2)}{4\alpha_s\alpha_d - (1 + \alpha_s^2)^2} \quad (\text{A.11})$$

$$\begin{aligned} g(n) &= (4\alpha_d\alpha_s)/(1 + \alpha_s^2) \text{ when } n \text{ is odd} \\ &= [1 + \alpha_s^2] \text{ when } n \text{ is even} \end{aligned} \quad (\text{A.12})$$

$$r_1 e^{i\theta} = \xi + i \alpha_d x_2 \quad (\text{A.13})$$

$$r_2 e^{i\theta} = \xi + i \alpha_s x_2 \quad (\text{A.14})$$

when $v = 0$, the above functions can be reduced to the usual Williams' [22] eigen functions.

It is interesting to note that the stress field $\sigma_{\alpha\beta}(x_\mu)$ [$\alpha, \beta, \mu = 1, 2$], should in general case, satisfy the equations:

$$\sigma_{\alpha\beta,\beta} = \left(\frac{\partial^2 u_\alpha}{\partial t^2} - 2v \frac{\partial^2 u_\alpha}{\partial \xi \partial t} + v^2 \frac{\partial^2 u_\alpha}{\partial \xi^2} \right) \rho \quad (\text{A.15})$$

However, it can be seen that the special eigen functions given in (A.7-9), corresponding to the solution of Eq. (A.1), satisfy only the equations:

$$\sigma_{\alpha\beta,\beta} - \rho v^2 \frac{\partial^2 u_\alpha}{\partial \xi^2} = 0 \quad (\text{A.16})$$

for all values of v ; thus, when $v = 0$, the correspondingly reduced eigen functions in Eq. (A.7-9), which coincide with the well-known Williams' eigen-function, needless to say, satisfy the static equations of equilibrium, $\sigma_{\alpha\beta,\beta} = 0$.

Appendix B

DETAILS OF FINITE ELEMENT EQUATION DEVELOPMENT FOR DYNAMICALLY PROPAGATING CRACKS

Upon substitution of Eqs. (I.22 to 44), into (I.21), we obtain,

$$\begin{aligned}
 0 = & \sum_{\bar{n}} \{ [(q_2^T K_{\bar{n}}^T + \ddot{q}_2^T m_{\bar{n}}^T - \bar{Q}_2^T + q_1^T K_{\bar{n}}^T + \bar{q}_1^T m_{\bar{n}}^T - \bar{Q}_1^T) \delta \ddot{q}_2] \\
 & + (\beta_2^T K_{\bar{n}s2}^T + \beta_2^T D_{\bar{n}s2}^T + \beta_2^T m_{\bar{n}s2}^T - Q_{s2}^T + \beta_1^T K_{\bar{n}s1}^T \\
 & + \beta_1^T D_{\bar{n}s1}^T + \beta_1^T m_{\bar{n}s1}^T - \bar{Q}_{s1}^T) \delta \beta_2
 \end{aligned} \tag{B.1}$$

where,

$$K_{\bar{n}} = \int_{V_{Rn}} B_{\bar{n}}^T D B_{\bar{n}} dv ; m_{\bar{n}} = \int_{V_{Rn}} \rho N_{\bar{n}}^T N_{\bar{n}} dv \tag{B.2,3}$$

$$Q_2 = \int_{V_{Rn}} N_{\bar{n}}^T \bar{F}_2 dv + \int_{S_{On}} N_{\bar{n}}^T \bar{T}_2 ds \tag{B.4}$$

$$Q_1 = \int_{V_{Rn}} N_{\bar{n}}^T \bar{F}_1 dv + \int_{S_{On}} N_{\bar{n}}^T \bar{T}_1 ds \tag{B.5}$$

$$K_{\bar{n}s2} = \int_{V_s} S_{\bar{n}s2}^T P_{\bar{n}s2} dv + \rho v_2^2 \int_{V_s} U_{\bar{n}s2}^T (U_2)_{,\xi\xi} dv \tag{B.6}$$

$$K_{\bar{n}s1} = \int_{V_s} S_{\bar{n}s1}^T P_{\bar{n}s1} dv + \rho v_1^2 \int_{V_s} U_{\bar{n}s1}^T (U_1)_{,\xi\xi} dv - \int_{\Delta\Sigma} U_{\bar{n}s1}^T R_{\bar{n}s1} ds \tag{B.7}$$

$$m_{\bar{n}s2} = \rho \int_{V_s} U_{\bar{n}s2}^T U_2 dv ; m_{\bar{n}s1} = \rho \int_{V_s} U_{\bar{n}s1}^T U_1 dv \tag{B.8}$$

$$D_{\bar{n}s2} = -2\rho v_2 \int_{V_s} U_{\bar{n}s2}^T (U_2)_{,\xi} dv \tag{B.9}$$

$$D_{\bar{n}s1} = -2\rho v_1 \int_{V_s} U_{\bar{n}s1}^T (U_1)_{,\xi} dv \tag{B.10}$$

$$Q_{s2} = \int_{V_s} U_{\bar{n}s2}^T \bar{F}_2 dv + \int_{\Sigma^+} U_{\bar{n}s2}^T \bar{T}_2 ds \tag{B.11}$$

$$Q_{s1} = \int_{V_s} U_{\bar{n}s1}^T \bar{F}_1 dv + \int_{\Sigma^+} U_{\bar{n}s1}^T \bar{T}_1 ds$$

Now, the conditions of "least-squares" matching of displacements, velocities, and accelerations between the singular element and the surrounding regular elements, i.e., Eqs. (I.11,12,13) are used to express $\underline{\beta}_2$, $\dot{\underline{\beta}}_2$ and $\ddot{\underline{\beta}}_2$ in terms of the respective values \underline{q}_{s2} , $\dot{\underline{q}}_{s2}$, $\ddot{\underline{q}}_{s2}$ at nodes along the boundary of V_s . Thus,

$$\underline{\beta}_1 = A_{\underline{z}1} \underline{q}_{s1} \quad ; \quad \dot{\underline{\beta}}_1 = A_{\underline{z}1} \dot{\underline{q}}_{s1} + B_{\underline{z}1} \underline{q}_{s1} \quad ; \quad \ddot{\underline{\beta}}_1 = A_{\underline{z}1} \ddot{\underline{q}}_{s1} + 2 B_{\underline{z}1} \dot{\underline{q}}_{s1} + C_{\underline{z}1} \underline{q}_{s1} \quad (\text{B.12})$$

$$\underline{\beta}_2 = A_{\underline{z}2} \underline{q}_{s2} \quad ; \quad \dot{\underline{\beta}}_2 = A_{\underline{z}2} \dot{\underline{q}}_{s2} + B_{\underline{z}2} \underline{q}_{s2} \quad ; \quad \ddot{\underline{\beta}}_2 = A_{\underline{z}2} \ddot{\underline{q}}_{s2} + 2 B_{\underline{z}2} \dot{\underline{q}}_{s2} + C_{\underline{z}2} \underline{q}_{s2} \quad (\text{B.13})$$

We note that $(A_{\underline{z}1}, B_{\underline{z}1}$ and $C_{\underline{z}1})$ and $(A_{\underline{z}2}, B_{\underline{z}2}$ and $C_{\underline{z}2})$ are dependent on velocities of crack propagation v_1 and v_2 respectively. When Eqs. (B.12,13) are used, Eq. (B.1) can be rewritten as:

$$0 = \sum_n \{ (q_{\underline{z}2}^T K_{\underline{z}2}^T + \dot{q}_{\underline{z}2}^T m_{\underline{z}2}^T - Q_{\underline{z}2}^T + q_{\underline{z}1}^T K_{\underline{z}1}^T + \dot{q}_{\underline{z}1}^T m_{\underline{z}1}^T - \bar{Q}_{\underline{z}1}^T) \delta q_{\underline{z}2} \} \\ + [q_{\underline{z}s2}^T K_{\underline{z}s}^{*T} + \dot{q}_{\underline{z}s2}^T D_{\underline{z}s}^{*T} + \ddot{q}_{\underline{z}s2}^T m_{\underline{z}s}^{*T} - Q_{\underline{z}s}^{*T}] \delta q_{\underline{z}s2} \quad (\text{B.14})$$

where

$$K_{\underline{z}s}^* = [A_{\underline{z}2}^T K_{\underline{z}s2} A_{\underline{z}2} + A_{\underline{z}2}^T D_{\underline{z}s2} B_{\underline{z}2} + A_{\underline{z}2}^T m_{\underline{z}s2} C_{\underline{z}2}] \quad (\text{B.15})$$

$$D_{\underline{z}s}^* = [A_{\underline{z}2}^T D_{\underline{z}s2} A_{\underline{z}2} + 2 A_{\underline{z}2}^T m_{\underline{z}s2} B_{\underline{z}2}] \quad (\text{B.16})$$

$$m_{\underline{z}s}^* = A_{\underline{z}2}^T m_{\underline{z}s2} A_{\underline{z}2} \quad (\text{B.17})$$

$$Q_{\underline{z}s}^* = A_{\underline{z}2}^T (-K_{\underline{z}s1} \beta_1 - D_{\underline{z}s1} \dot{\beta}_1 - m_{\underline{z}s1} \ddot{\beta}_1 + \bar{Q}_{\underline{z}s2} + \bar{Q}_{\underline{z}s1}) \quad (\text{B.18})$$

From Eqs. (B.14), Eqs. (I.45,46) were derived. It can now be seen that both the singular-element matrices K_s^* and D_s^* are unsymmetric. The "damping" matrix D_s^* is a result of the fact that the total accelerations of a material point in the singular element depend on $\dot{\beta}_2$.

It may be of interest to note that in the evaluation of $K_{\underline{z}s2}$ of Eq. (B.6),

the integrand will have a singularity of the type $(1/r_1)$ and $(1/r_2)$. Special numerical integration schemes to evaluate this domain integral of Eq. (B.6) directly, can be developed. Alternatively, one can use the observation that, by definition, from Eqs. (I.21, B.1)

$$\beta_{\approx 2}^T K_{\approx s 2}^T \delta \beta_{\approx 2} \equiv \int_{V_s} (\sigma_{ij}^2 \delta \epsilon_{ij}^2 + \rho(v_2)^2 \frac{\partial^2 u_\alpha^2}{\partial \xi^2} \delta u_\alpha^2) dv \quad (B.19)$$

Using the divergence theorem, Eq. (B.19) can be rewritten as:

$$\beta_{\approx 2}^T K_{\approx s 2}^T \delta \beta_{\approx 2} = \int_{\partial V_s} \sigma_{ij}^2 v_j^2 \delta u_i^2 ds + \int_{V_s} (-\sigma_{ij,j}^2 + \rho(v_2)^2 \frac{\partial^2 u_i^2}{\partial \xi^2}) \delta u_i^2 dv \quad (B.20)$$

the second integral on the right hand side of Eq. (B.20) vanishes due to the special property of the eigen functions embedded in the singular element, as explained in Eq. (A.16). Thus, one can write alternatively,

$$K_{\approx s 2}^T = \int_{\partial V_s} R_{\approx 2}^T U_{\approx 2} ds \quad (B.21)$$

wherein, the integrand is non-singular along V_s , and no special integration schemes are necessary.

Likewise, it is seen that,

$$\beta_{\approx 1}^T K_{\approx s 1}^T \delta \beta_{\approx 2} = \int_{V_s} (\sigma_{ij}^1 \delta \epsilon_{ij}^2 + \rho(v_1)^2 \frac{\partial^2 u_\alpha^1}{\partial \xi^2} \delta u_\alpha^2) dv - \int_{\Delta \Sigma} \sigma_{ij}^1 v_j^1 \delta u_i^2 ds \quad (B.22)$$

once again, using the property, as given in Eq. (A.16), of the eigen functions σ_{ij}^1 in V_1 , and using the divergence theorem, we write

$$\beta_{\approx 1}^T K_{\approx s 1}^T \delta \beta_{\approx 2} = \int_{\partial V_s} \sigma_{ij}^1 v_j^1 \delta u_i^2 ds - \int_{\Delta \Sigma} \sigma_{ij}^1 v_j^1 \delta u_i^2 ds \quad (B.23)$$

It can easily be seen that the above equation can be simplified to:

$$\beta_{\approx 1}^T K_{\approx s 1}^T \delta \beta_{\approx 2} = \int_{\rho_s} \sigma_{ij}^1 v_j^1 \delta u_i^2 ds \quad (B.24)$$

The above simplification is possible because: $\partial V_s = \rho_s + \Sigma_1 + \Delta\Sigma + S_{u2}$, where ρ_s is the interface of the singular element with surrounding regular elements, and Σ_1 is assumed, without loss of generality, to be free of any applied tractions at all times, and S_{u2} is the ligament ahead of the crack-tip (along x_1 axis) in the singular element, where, for mode I problems, $T_1^1 = 0$, and $u_2^1 = 0$. The boundary integration as indicated by Eq. (B.24) to evaluate K_{s1} may be more convenient than to directly apply Eqs. (B.22) or (B.23).

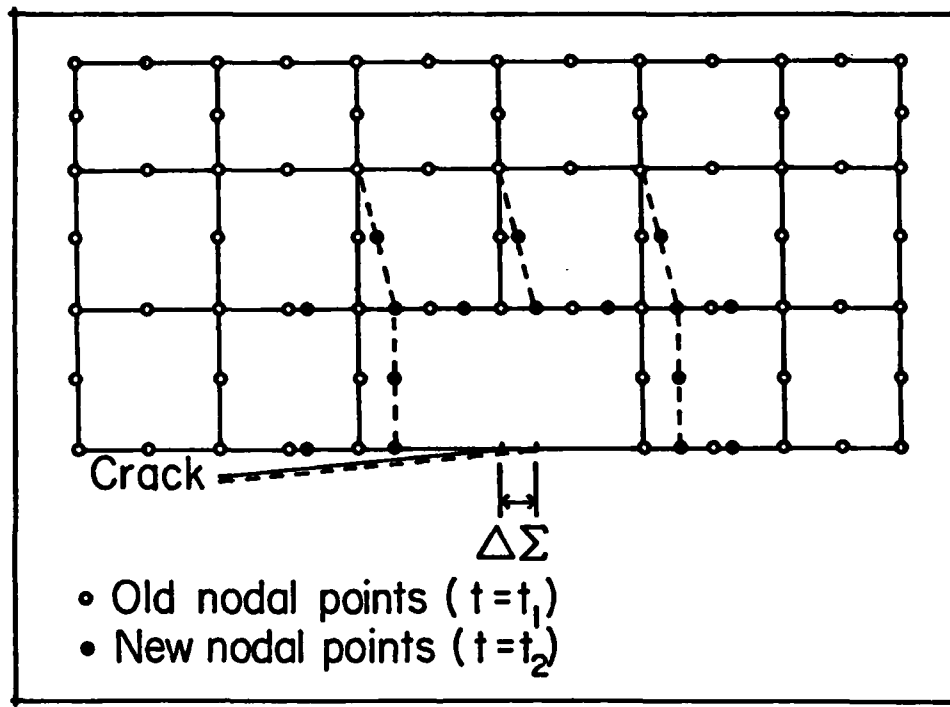
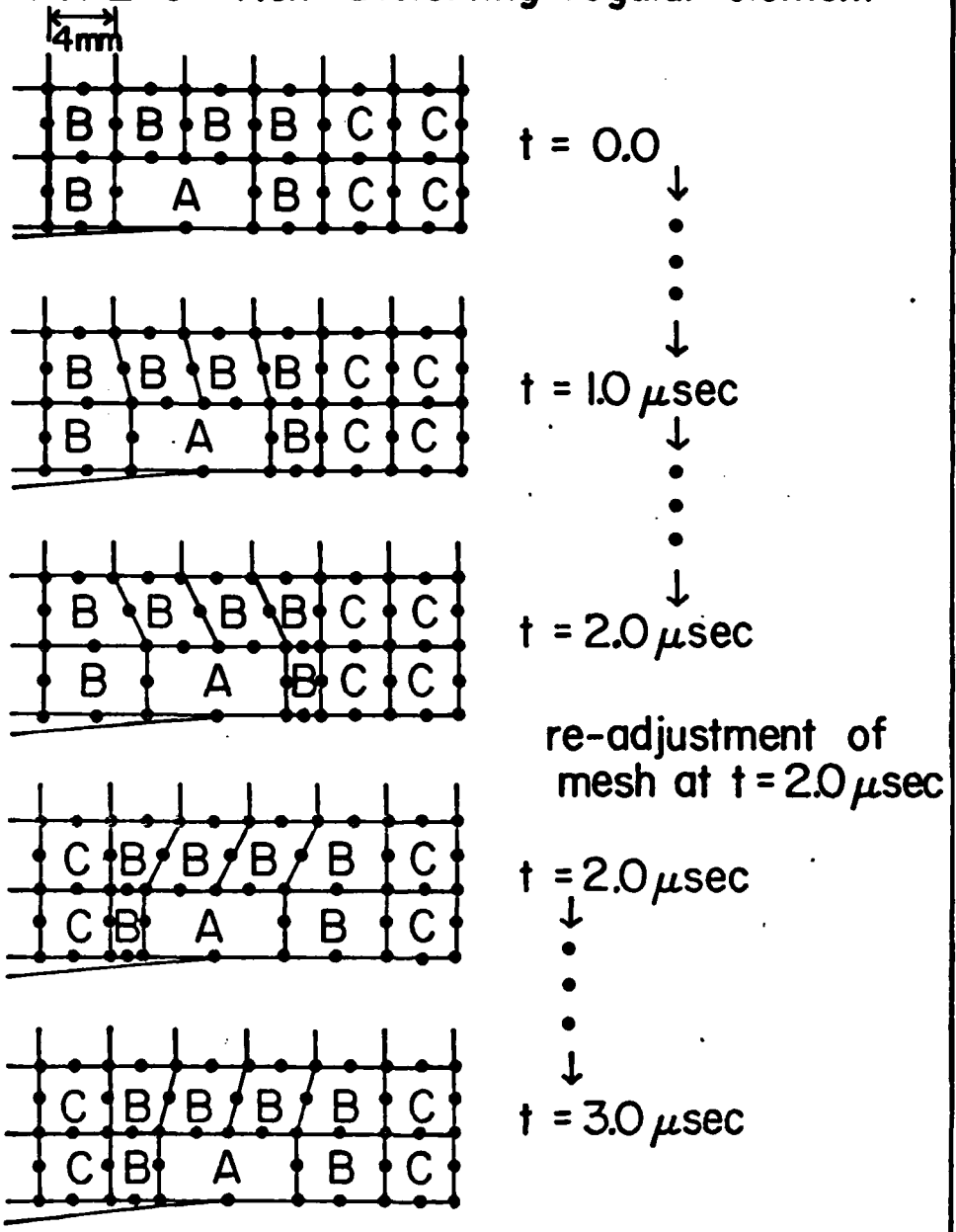


Fig. 1

TYPE A : Moving singular element
 TYPE B : Distorting regular element
 TYPE C : Non-Distorting regular element



EXAMPLE : $v = 1000 \text{ m/sec}$
 $\Delta t = 0.2 \mu\text{sec}$
 $\Delta \Sigma = 0.2 \text{ mm}$

Fig. 2

Figure Captions

Fig. 1 Schematic Representation of the Movement of the "Singular-Element"

Fig. 2 Schematic Representation of Crack Growth in a Typical Problem:
Constant Crack Velocity $v = 1000$ m/s; $\Delta t = 0.2\mu\text{s}$; $\Delta\Sigma = 0.2$ mm.
The mesh of regular elements around the singular element is readjusted
at $t = 2.0\mu\text{s}$.

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