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PARAMETRIC INSTABILITIES OF ELECTRON CYCLOTRON WAVES IN PLASMAS--ETC(U)
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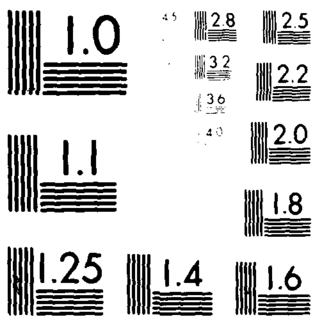

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V. K. Tripathi and C. S. Liu

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UNIVERSITY OF MARYLAND
DEPARTMENT OF PHYSICS AND ASTRONOMY
COLLEGE PARK, MARYLAND

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PARAMETRIC INSTABILITIES OF

ELECTRON CYCLOTRON WAVES IN PLASMAS

V. K. Tripathi and C. S. Liu
Department of Physics and Astronomy,
University of Maryland,
College Park, Maryland 20742

ABSTRACT

We have studied the parametric instabilities of electron cyclotron waves in I) the EBT ring and II) the large tokamaks eg. PLT. In the EBT, the electron cyclotron pump of finite wavenumber k_o decays into two Bernstein modes at the second harmonic cyclotron layer and can account for the heating of the ring in the initial phase. The coupling coefficient for this decay vanishes for a dipole pump, whereas the convective threshold with finite k_o is $\sim$200 W/cm^2. For large tokamaks, the convective threshold for various decay channels turns out to be $\gtrsim$200 KW/cm^2 at 3mm wavelength.

I. INTRODUCTION

The recent developments of powerful microwave sources in the millimeter range have aroused substantial efforts on the electron cyclotron heating of large devices, eg., Elmo bumpy torus, tokamak and mirrors. In the Elmo bumpy torus[1] (EBT) the microwaves are substantially absorbed at the second harmonic cyclotron layer ($\omega_o \simeq 2\omega_c$) and a hot electron ring with several keV temperature is formed. No satisfactory explanation of how the ring is formed is available so far. Here we examine a possible parametric instability at the second harmonic which may play a role in the acceleration of the high energy electrons.

In the case of tokamak[2,3], the experiments on electron cyclotron heating of ISXB, TM-3, Tuman-2 and other devices have proved promising with almost a hundred percent power absorption efficiency. Ott et al.[4] and Eldridge et al.[5] have predicted the absorption efficienty of 100% for the ordinary and extraordinary modes in PLT on the basis of linear theory of cyclotron absorption. The presently employed power densities for tokamak heating ($\lesssim$ 1 KW/cm^2) at electron cyclotron frequency are far too low to excite any parametric processes. Nevertheless the use of higher power dnesities in future experiments might initiate such processes. An estimate of these effects is necessary for the prediction of the heating rate.

Ott et al.[4] have recently studied the parametric instabilities of the ordinary pump wave in PLT on the low magnetic field side of the torus; the parametric instabilities of the extraordinary mode have been omitted on the ground that on the inner side of the torus, from where this mode is launched, the electrostatic high frequency modes possess frequencies greater than that of the pump. The two channels of decay viz, resonant decay into upper hybrid and ion cyclotron waves and nonresonant decay into electron Bernstein mode have been studied. The threshold powers for these channels are greater than 200 KW/cm^2, hence they are unimportant. Elder and Perkins[6] have investigated the parametric decay of the extraordinary mode, launched from the outside of the torus, into ion acoustic and Langmuir waves. The threshold power for this decay is very large: powers of the order of 350 MW are required for its onset in PDX device.

In this paper, we have investigated the parametric instabilities of electron cyclotron waves I) at the second harmonic cyclotron layer in EBT and II) at the cyclotron harmonic in PLT. In the EBT, the pump wave could decay into two Bernstein waves which possess slow group velocities and the decay channel should have lowest convective threshold. In the case of PLT, we shall match the frequency of the pump to the cyclotron frequency at the center of the torus (x=0) and would be interested in the parametric instabilities in the vicinity of the center. The extraordinary mode launched from the inside of the torus ($x=-r_o$) could decay into i) two lower hybrid waves, ii) lower hybrid and ion cyclotron waves (for $\omega_c^2 < \omega_p^2 < 2\omega_c^2$) and iii) lower hybrid and quasimodes (for $\omega_c^2 < \omega_p^2 < 2\omega_c^2$) on the innerside of the torus. The ordinary mode launched from the outside of the torus ($x=r_o$) could decay into i) two lower hybrid waves, ii) Bernstein and quasimodes, iii) upper hybrid and ion cyclotron modes and quasimodes (for $\omega_p^2 < \omega_c^2$). The last channel is, however, possible in the low density region hence we would ignore it.

In Section II we have obtained the coupling coefficient for the decay of a finite wavenumber ($k_o \neq 0$) electron cyclotron pump around $\omega_o \simeq 2\omega_c$ into two Bernstein modes in EBT by solving the Vlasov equation for electrons in the guiding center coordinates. The coupling coefficient vanishes for $k_o=0$. Following Liu[7], the convective threshold for the instability has been obtained. In Section III, we have studied the decay instability of electron cyclotron pump ($\omega_o \simeq \omega_c$) in a tokamak by employing fluid theory, which is applicable for long wavelength decay modes. A discussion of results is given in Section IV.

II. TWO BERNSTEIN MODE DECAY IN EBT

We consider the propagation of an extraordinary pump wave

$$\vec{E}_o \exp - i(\omega_o t - k_o x) \quad (1)$$

into the mirror midplane of EBT, transverse to the direction of static magnetic field $\vec{B}_s \parallel \hat{z}$; $E_{ox} = -i\omega_c \omega_p^2 E_{oy}/\omega_o(\omega_o^2 - \omega_c^2)$, $E_{oz} = 0$, $B_{oz} = k_o c E_{oy}/\omega_o$, $B_{ox} = B_{oy} = 0$,

$$k_o = \frac{\omega_o}{c}\left[1 - \frac{\omega_p^2}{\omega_o^2}\,\frac{\omega_o^2 - \omega_p^2}{\omega_o^2 - \omega_p^2 - \omega_c^2}\right]^{1/2} \quad (2)$$

and ω_p and ω_c are the electron plasma and cyclotron frequencies. The density gradient is along x axis and the pump frequency is close to $2\omega_c$. The pump wave decays into two electrostatic waves $\vec{E}(\omega,\underline{k}) = -\underline{\nabla}[\phi \exp - i(\omega t - \vec{k}\cdot\vec{x})]$ and $\vec{E}_1(\omega_1,\underline{k}_1) = -\underline{\nabla}[\phi_1 \exp - i(\omega_1 t - \vec{k}_1\cdot\vec{x})]$ where $\omega_1 = \omega - \omega_o$, $\vec{k}_1 = \vec{k} - \vec{k}_o$. The response of electrons to these fields is governed by the Vlasov equation[8]

$$\frac{\partial f}{\partial t} + \frac{p_z}{m}\frac{\partial f}{\partial z} + \dot{\theta}\frac{\partial f}{\partial \theta} + \dot{\mu}\frac{\partial f}{\partial \mu} - eE_z^T\frac{\partial f}{\partial p_z} + \dot{x}_g\frac{\partial f}{\partial x_g} + \dot{y}_g\frac{\partial f}{\partial y_g} = 0 \quad (3)$$

where $\mu = mv_\perp^2/2\omega_c$ is the magnetic moment, θ is the gyrophase angle, $v_\perp$ is the perpendicular velocity, -e and m are the electronic change and mass, respectively, $\omega_c = eBs/mc$ and $\vec{x}_g = \vec{x} - \vec{v}_\perp \times \vec{\omega}_c/\omega_c^2$ are the guiding center coordinates. Using the equation of motion, one can write

$$\dot{\mu} = - e\vec{E}_\perp^T \cdot \vec{v}_\perp/\omega_c$$

$$\dot{\theta} = \omega_c + \frac{eB_{oz}}{mc} + \frac{e}{mv_\perp}(E_x^T \sin\theta - E_y^T \cos\theta)$$

$$\dot{\vec{x}}_g = \frac{e}{m\omega_c^2} \vec{E}_\perp^T \times \vec{\omega}_c - \frac{eE_{oy}k_o}{m\omega_o\omega_c} \vec{v}$$

where the superscript T refers to the total field. Now we express the total electric field and the distribution function as

$$\begin{aligned} \vec{E}^T &= \vec{E}_o\, e^{-i(\omega_o t - k_o x_g)} \sum_n e^{in\theta} J_n^o \\ &- i\vec{k}\phi e^{-i(\omega t - \vec{k}\cdot\vec{x}_g)} \sum_n e^{in(\theta-\delta)} J_n \\ &- i\vec{k}_1\phi_1 e^{-i(\omega_1 t - \vec{k}_1\cdot\vec{x}_g)} \sum_n e^{in(\theta-\delta_1)} J_n^1 , \end{aligned} \quad (4)$$

$$\begin{aligned} f &= f_o^o + e^{-i(\omega_o t - k_o x_g)} \sum_n e^{in\theta} f_n^o \\ &+ e^{-i(\omega t - \vec{k}\cdot\vec{x}_g)} \sum_n e^{in(\theta-\delta)} f_n \\ &+ e^{-i(\omega_1 t - \vec{k}_1\cdot\vec{x}_g)} \sum_n e^{in(\theta-\delta_1)} f_n^1 \end{aligned} \quad (5)$$

and also

$$\begin{aligned} \dot{\mu} &= -\frac{ev_\perp E_{o\perp}}{2\omega_c} e^{-i(\omega_o t - k_o x_g)} \sum_n e^{in\theta} (J_{n-1}^o e^{-i\delta_o} \\ &+ J_{n+1}^o e^{i\delta_o}) + ie\phi\, e^{-i(\omega t - \vec{k}\cdot\vec{x}_g)} \sum_n n\, e^{in(\theta-\delta)} J_n \\ &+ ie\phi_1\, e^{-i(\omega_1 t - \vec{k}_1\cdot\vec{x}_g)} \sum_n n\, e^{in(\theta-\delta_1)} J_n^1 , \end{aligned} \quad (6)$$

$$\begin{aligned} \dot{\theta} &= \omega_c + \frac{eE_{o\perp}}{2imv_\perp} \sum_n e^{in\theta} (J_{n-1}^o e^{-i\delta_o} - J_{n+1}^o e^{i\delta_o}) \\ &\cdot e^{-i(\omega_o t - k_o x_g)} - \frac{e\phi k_\perp}{mv_\perp} e^{-i(\omega t - \vec{k}\cdot\vec{x}_g)} \sum_n e^{in(\theta-\delta)} J_n' \end{aligned}$$

$$- \frac{e\phi_1 k_{1\perp}}{mv_\perp} e^{-i(\omega_1 t - \vec{k}_1 \cdot \vec{x}_g)} \sum_n e^{in(\theta - \delta_1)} J_n^{1'}$$

$$+ \frac{ek_o E_{oy}}{m\omega_o} e^{-i(\omega_o t - k_o x_g)} \sum_n e^{in\theta} J_n^o \tag{7}$$

$$\dot{\vec{x}}_g = \frac{e}{m\omega_c^2} \vec{E}_\perp^T \times \vec{\omega}_c - \frac{eE_{oy}}{m\omega_o} e^{-i(\omega_o t - k_o x_g)}$$

$$\cdot \sum_n e^{in\theta} (\hat{x}\, n\, J_n^o - i\, \hat{y}\, k_o \rho\, J_n^{o'}), \tag{8}$$

where $J_n \equiv J_n(k_\perp \rho)$, $J_n^o(k_o \rho)$, $J_n^1 = J_n(k_{1\perp}\rho)$, $\rho = v_\perp/\omega_c$, prime over the Bessel function denotes the derivative with respect to its argument, δ_o is the angle between $\vec{E}_{o\perp}$ and x axis, δ between $\vec{k}_\perp$ and x axis and δ_1 between $\vec{k}_{1\perp}$ and x axis. Now onwards we shall assume that $k_\parallel = 0$ for the Bernstein modes. Using Eqs. (4) - (8) in Eq. (3), we obtain the linear response,

$$f_n^o = \frac{ev_\perp E_{o\perp} f_o^o}{2Ti(\omega_o - n\omega_c)} [J_{n-1}^o e^{-i\delta_o} + J_{n+1}^o e^{i\delta_o}]$$

$$f_n = - \frac{e\phi\, n\omega_c\, f_o^o}{T(\omega - n\omega_c)} J_n$$

$$f_n^1 = - \frac{e\phi_1\, n\omega_c\, f_o^o J_n^1}{T(\omega_1 - n\omega_c)} \tag{9}$$

where we have assumed f_o^o to be Maxwellian at temperature T. For the nonlinear response we get,

$$i(\omega - n\omega_c) f_n^{NL} = \frac{ei}{2m\omega_c} \sum_\ell \, [-i\, k_{1y}\, k_o\, \phi_1\, J_\ell^1$$

$$f_{n-\ell}^o + (E_{oy}\, k_{1x} - k_y\, E_{ox}) J_{n-\ell}^o\, f_\ell^1]\, e^{-i\ell\delta_1}$$

$$- \frac{ev_\perp E_{o\perp}}{4\omega_c} \sum_\ell (J_{\ell-1}^o e^{-i\delta_o} + J_{\ell+1}^o e^{i\delta_o}) \frac{\partial f_{n-\ell}^1}{\partial \mu}$$

$$e^{-i(n-\ell)\delta_1} + \frac{ie\phi_1}{2} \sum_\ell \ell J_\ell^1 \frac{\partial f^o_{n-\ell}}{\partial\mu} e^{-i\delta_1\ell}$$

$$+ \frac{eE_{o\perp}}{4imv_\perp} \sum_\ell (J^o_{\ell-1} e^{-i\delta_o} - J^o_{\ell+1} e^{i\delta_o}) \, i(n-\ell) \, f^1_{n-\ell}$$

$$e^{-i\overline{n-\ell}\,\delta_1} - \frac{e\phi_1 k_{1\perp}}{2mv_\perp} \sum_\ell J_\ell^{1'} e^{-i\ell\delta_1} f^o_{n-\ell} \, i(n-\ell)$$

$$- \frac{eE_{oy}}{2m\omega_o} i \sum_\ell (\ell J^o_\ell k_{1x} - i \, k_o \, \rho \, k_y \, J_\ell^{o'}) \, e^{-i(n-\ell)\delta_1}$$

$$f^1_{n-\ell} + \frac{ek_o E_{oy}}{2m\omega_o} \sum_\ell J^o_\ell \, i(n-\ell) \, e^{-i(n-\ell)\delta_1} f^1_{n-\ell} \quad . \tag{10}$$

First, let us mention here that in the limit of $k_o = 0$ (dipole pump), f_n^{NL} is greatly simplified. However, the nonlinear density perturbation n^{NL} obtained from this after carrying out the $v_\perp$ integration, vanishes, identically. In the next approximation we retain terms that go linearly with k_o and possess resonant denominators, then we obtain

$$n^{NL} = \sum_n 2\pi \int_0^\infty v_\perp \, dv_\perp \, J_n \, e^{in\delta} f_n^{NL}$$
$$\simeq n^o_o \frac{e^2 E_{o\perp} \, k_o \, \phi_1 \, e^{i(\delta - \delta_o + \delta_1)}}{8iTm(\omega - \omega_c)^2} \frac{d}{db} (bI_1 e^{-b}) \ , \tag{11}$$

where n_o^o is the unperturbed electron density, $b = k_\perp^2 v_e^2/2\omega_c^2$, $v_e = (2T/m)^{1/2}$ and I_1 is the modified Bessel function of argument b. Similarly, one obtains,

$$n_1^{NL} = - n^o_o \frac{e^2 E_{o\perp} \, k_o \, \phi \, e^{-i(\delta - \delta_o + \delta_1)}}{8iTm(\omega - \omega_c)^2} \frac{d}{db} (bI_1 e^{-b}) \ . \tag{12}$$

Using Eqs. (11) and (12) in the Poisson's equation, we get the following nonlinear dispersion relation

$$1 = \mu/\varepsilon\ \varepsilon_1 , \tag{13}$$

where μ is coupling coefficient and ε, ε_1 are the dielectric functions at $(\omega,\vec{k})$, $(\omega_1,\vec{k}_1)$;

$$\mu = \left(\frac{\omega_p^2}{k^2 v_e^2}\right)^2 \frac{e^2 E_{o\perp}^2\ k_o^2}{16m^2(\omega - \omega_c)^4} \left[\frac{d}{db}\ (bI_1 e^{-b})\right]^2 , \tag{14}$$

$$\varepsilon = 1 + \frac{2\,\omega_p^2}{k^2 v_e^2}\ \left[1 - I_o e^{-b} - \frac{\omega I_1 e^{-b}}{\omega - \omega_c}\right] . \tag{15}$$

Using the linear dispersion relation ($\varepsilon = 0$) for the Bernstein mode, one may take

$$\omega - \omega_c \simeq \omega_c I_1 e^{-b}/(1 - I_o e^{-b} + b\omega_c^2/\omega_p^2) \simeq \omega_p^2\ I_1 e^{-b}/b\omega_c$$

and hence

$$\mu \simeq \frac{u^2}{4c^2}\ \frac{\omega_c^4}{\omega_p^4}\ \pi\ (b/2)^3 ; \tag{16}$$

$u = eE_{o\perp}/m\omega_c$. In writing Eq. (16) we have assumed $b > 1$. Following Liu, the convective amplification of the decay waves from Eq. (13) can be obtained as

$$A = \mu/\left[\frac{\partial\varepsilon}{\partial k_x}\ \frac{\partial\varepsilon_1}{\partial k_{1x}}\ \frac{\partial}{\partial x}\ (k_x - k_o - k_{1x})\right] \simeq \frac{u^2}{4c^2}\ \frac{\omega_c^4}{\omega_p^4}\ \frac{k^2 L_n}{k_o}\ \pi(b/2)^3$$

Taking $k_\perp v_e/\omega_c \sim 3$, $v_e/\omega_c \sim 10^{-2}$, $k^2 L_n/k_o \sim 10^5$, $k \sim 3 \times 10^2$ and expressing p in KW/cm^2, we get

$$A \simeq 0.3\left(\frac{\omega_c}{\omega_p}\right)^4\ p$$

Thus, the threshold power for two Bernstein mode decay is $\approx 200\ W/cm^2$ for $(\omega_c/\omega_p) \simeq 2$. For shorter wavelength modes (i.e. larger $k_\perp$ and larger b) the threshold power can be further reduced. However, one can not reduce the wavelength to very small values without considering the collisional effects.

III. DECAY INSTABILITY IN PLT

The decay modes for a pump around $\omega_o \sim \omega_c$ possess long wavelengths $k_\perp \rho_e < 1$ (except for the case of decay into a Bernstein mode and a low frequency quasimode which has been studied by Ott et al.[4] in the dipole approximation) and hence, fluid theory can be employed to obtain the nonlinear response of electrons. Again, we consider the propagation of the extraordinary pump wave [cf. Eqs. (1) and (2)] along x asix in the tokamak, with a static magnetic field in the z direction. The linear response of the pump may be written as

$$v_{ox} = -u$$

$$v_{oy} = -iu \frac{\omega_o^2 - \omega_p^2}{\omega_c \omega_o}$$

$$u = \frac{e\, \omega_c\, E_{oy}}{m(\omega_o^2 - \omega_u^2)}$$

$$n_o = n_o^o \frac{k_o\, v_{ox}}{\omega_o}\ , \quad B_{oz} = u\, k_o \frac{\omega_o^2 - \omega_u^2}{\omega_c^2 \omega_o}\, B_s$$

$$\omega_u^2 = \omega_p^2 + \omega_c^2\ . \tag{17}$$

The pump decays into two electrostatic waves $\phi(\omega, \vec{k})$ and $\phi_1(\omega_1, \vec{k}_1)$. The response of electrons to these fields is governed by the equations of motion and continuity,

$$\frac{\partial n}{\partial t} + \underline{\nabla} \cdot (n\vec{v}) = 0 \tag{18}$$

$$\frac{\partial \underline{v}}{\partial t} + \vec{v} \cdot \underline{\nabla}\vec{v} = -\frac{e\vec{E}}{m} - \frac{e\vec{V} \times \vec{B}}{mc} - \frac{T}{n} \underline{\nabla} n\ . \tag{19}$$

Expanding

$$n = n_o^o + n_o(\omega_o,\vec{k}_o) + n(\omega,\vec{k}) + n_1(\omega_1,\vec{k}_1)$$

$$\vec{v} = \vec{v}_o + \vec{v} + \vec{v}_1$$

$$\vec{B} = (B_s + B_o)\hat{z}$$

the linear response at $\omega_1,\underline{k}_1$ may be written as

$$\vec{v}_{1\perp} = \frac{e\phi_1(\omega_1\vec{k}_{1\perp} - i\vec{k}_{1\perp} \times \vec{\omega}_c)}{m(\omega_c^2 - \omega_1^2)}$$

$$v_{1z} = -\frac{e\phi_1 k_{1z}}{m\omega_1}$$

$$n_1 = -\frac{n_o^o e\phi_1}{m}\left[\frac{k_{1z}^2}{\omega_1^2} + \frac{k_{1\perp}^2}{\omega_1^2 - \omega_c^2}\right]. \tag{20}$$

To obtain the nonlinear response we consider two cases.

A) $\underline{\omega \lesssim k_z v_e}$

In this limit, the low frequency nonlinearity arises through the parallel ponderomotive force $F_{pz} = -iek_z\phi_p$,

$$\phi_p = -\frac{m}{2}\vec{v}_o \cdot \underline{\nabla}_\perp v_{1\parallel}$$

$$\approx \frac{\phi_1}{2\omega_1}\vec{k}_{1\perp} \cdot \vec{v}_{o\perp} \tag{21}$$

and hence the low frequency density perturbation comes out to be

$$n = \frac{k^2}{4\pi e} \chi_e(\phi + \phi_p) \tag{22}$$

where $\chi_e = \frac{2\omega_p^2}{k^2 v_e^2} [1 + \frac{\omega}{k_z v_e} Z(\frac{\omega}{k_z v_e})]$. Employing Eqs. (17) and (22), the nonlinear contribution to the high frequency density perturbation turns out to be

$$\begin{aligned} n_1^{NL} &= \frac{\vec{k}_{1\perp} \cdot \vec{v}_o^*}{2\omega_1} n \\ &= \frac{k^2 \chi_e}{8\pi e\, \omega_1} \vec{k}_{1\perp} \cdot \vec{v}_o^* (\phi + \phi_p) \end{aligned} \tag{23}$$

where * denotes the complex conjugate. Using Eqs. (22) and (23) in the Poisson's equation, we obtain Eq. (13) as the nonlinear dispersion relation with the following coupling coefficient,

$$\mu = -\chi_e(1 + \chi_i) \frac{k^2}{4k_1^2\omega_1^2} |\vec{k}_{1\perp} \cdot \vec{v}_o|^2 \; ; \tag{24}$$

χ_i is the ion susceptibility at $(\omega, \underline{k})$.

B) $\underline{\omega > k_z v_e}$

In this limit, ω and ω_1 may be comparable and the nonlinearity in the response at both the frequencies arises through the equation of continuity as well as the ponderomotive force. On writing equations similar to Eqs. (20) for the linear response at $(\omega, \vec{k})$ evaluating the nonlinear density perturbations and employing the Poisson's equation, we obtain

$$\mu = \frac{\omega_p^4}{k^2 k_1^2} \alpha_1 \alpha_2 \, uu^* \quad ,$$

$$\alpha_1 = \frac{k_z^2}{2\omega\omega_1} \left[\frac{k_{1x}}{\omega} + \frac{k_o}{\omega_o} + \frac{k_x}{\omega_1} + \frac{i(\omega_o^2 - \omega_p^2)(2\omega - \omega_o)}{\omega_c\omega_o\omega\omega_1} \; k_y \right.$$

$$- \frac{1}{2\omega(\omega_c^2 - \omega_1^2)} \left[\frac{k_o}{\omega_o} (\omega \vec{k}_\perp \cdot \vec{k}_{1\perp} - i\omega_c k_o k_y) \right.$$

$$\left. \left. + k_{1\perp}^2 (k_x + i \frac{\omega_o^2 - \omega_p^2}{\omega_c \omega_o} k_y) \right] \right] - \frac{1}{(\omega_c^2 - \omega_1^2)(\omega_c^2 - \omega^2)}$$

$$\cdot \left\{ \frac{\omega_o^2 - \omega_u^2}{2\omega_c\omega_o} k_o \left[\frac{2\omega - \omega_o}{\omega} \omega_c \vec{k}_\perp \cdot \vec{k}_{1\perp} + i(\omega_1 + \frac{\omega_c^2}{\omega}) k_o k_y \right] \right.$$

$$+ \frac{1}{2} \left[k_x + i \frac{\omega_o^2 - \omega_p^2}{\omega_c\omega_o} (k_y + i \frac{\omega_c}{\omega} k_x) - \frac{i\omega_c}{\omega} k_y \right]$$

$$\cdot (\omega_1 k_o k_{1x} - i k_o k_y \omega_c) + \frac{1}{2\omega} (k_{1x} + i \frac{\omega_o^2 - \omega_p^2}{\omega_c\omega_o} k_y)$$

$$\left. \cdot [\vec{k}_\perp \cdot \vec{k}_{1\perp} (\omega\omega_1 - \omega_c^2) - i\omega_o\omega_c k_y k_o] \right\} ,$$

$$\alpha_2 = \alpha_1(\omega_1, \vec{k}_1 \leftrightarrow \omega, \vec{k};\ \omega_o, \vec{k}_o \leftrightarrow -\omega_o, -\vec{k}_o) \ . \tag{25}$$

The dispersion relation is again given by Eq. (13). Now we discuss some specific channels of decay.

i) Two Lower Hybrid Waves.

The linear dispersion relation for lower hybrid waves, viz.,

$$\omega^2 = \frac{1}{2} \omega_u^2 - \frac{1}{2} [\omega_u^4 - 4\omega_{pi}^2 \omega_c^2 (1 + \frac{k_z^2 m_i}{k^2 m})]^{1/2}$$

tells that $\omega \lesssim \min(\omega_c, \omega_p)$, hence the channel of decay involving two lower hybrid waves is possible for $2\omega_p > \omega_o \simeq \omega_c$. The decay waves can have either $k \gg k_o$ or $k \lesssim k_o$. In the former case $\omega \simeq \omega_o/2$,

$$\mu \simeq 4 \frac{k_o^2 \, uu^*}{\omega_o^2} \frac{\omega_p^4}{\omega_o^4} \frac{k_z^4}{k^4}$$

and the convective amplification is

$$A \simeq 0.1 \frac{u^2}{c^2} \frac{k^2}{k_x^2} \frac{k}{k_o} kL_n$$

where we have assumed $\omega_o \sim \omega_c$ (which corresponds to $k_z^2/k^2 \simeq 1/3$ for a lower hybrid wave of $\omega = \omega_c/2$. For PLT parameters, $B_o = 35$ kG, $L_n \sim 10$ cm, $k_o = 18\ \text{cm}^{-1}$, $k = 20\ k_o$, $k/k_x \simeq 10$, $A \simeq 8 \times 10^5 u^2/c^2 \simeq 1.6$ at 200 KW/cm^2. Thus the threshold for this decay is $\sim$150 KW/cm^2.

For $k \lesssim k_o$, the decay wave frequencies may be considerably different from each other. The high frequency lower hybrid wave $(\omega_1, \underline{k}_1)$ has to be close to ω_o, i.e., $\omega_p \gtrsim \omega_c$, $k_{1z}^2 >> k_{1\perp}^2$; the latter inequality is satisfied for $k_1 \simeq k_x \simeq k_o$. The amplification comes out to be

$$A \simeq \frac{\omega_p^4}{k^2 k_1^2} \frac{uu^*}{\omega_o^2} \frac{k_z^4 k_o^2}{\omega_p^4} \frac{kL_n}{8} \simeq 0.1 \frac{u^2}{c^2} kL_n .$$

This requires threshold powers above 500 KW/cm^2 for PLT parameters.

The decay instability of the ordinary pump wave possess almost the same amplification.

ii) Lower Hybrid and Ion Cyclotron Waves.

The ion cyclotron wave,

$$\omega \simeq \omega_{ci}(1 + \frac{I_{1i} e^{-bi}}{1 + T_i/T_e})$$

has $\omega \simeq \omega_{ci}$, $\omega - \omega_{ci} > k_z v_i$, $\omega < k_z v_e$, $k_1 \rho_e < 1$ and this decay channel is possible only for an extraordinary pump in between $-x_o < x < 0$ (where $\omega_c > \omega_o$) and when $\omega_p^2 \gtrsim \omega_o^2$. The resonance condition gives $k_\perp >> k_z >> k_{1i}$ i.e., $k_x \simeq k_o$. The convective amplification turns out to be

$$A \simeq \frac{u^2}{16v_e^2} k_y L_n$$

and the threshold power $\gtrsim$200 KW/cm^2 for PLT parameters.

iii) Lower Hybrid Mode and Quasimode.

When the parallel phase velocity of the lower frequency lower hybrid wave equals the electron thermal speed, $\omega \simeq k_z v_e$, it is strongly Landau damped; the higher frequency lower hybrid wave is weakly damped. Solving Eq. (13) with $\omega = \omega + i\gamma$, $\varepsilon_1 \simeq i\gamma \frac{\partial\varepsilon_1}{\partial\omega_1}$ the homogeneous medium growth rate turns out to be

$$\gamma_o = \frac{I_m(\mu/\varepsilon)}{\partial\varepsilon_1/\partial\omega_1}$$

$$\simeq 3 \times 10^{-3}\, \omega_o \frac{u^2}{c^2}$$

which is $\sim 10^4$ at 200 KW/cm^2 and PLT parameters. This growth rate is extremely low to substantially emplify the daughter wave in one pass through the region of parametric resonance.

iv) Filamentation Instability.

The analysis developed here is applicable only to three wave decay processes. However, one could initially include the upper sideband $(\omega + \omega_o, \vec{k} + \vec{k})$ in the analysis to study four wave processes. The filamentation instability of the ordinary mode in tokamak, with the perturbation along z axis $(\vec{k} \,||\, \hat{z})$, is not to be affected by the magnetic field and the amplification factor turns out to be

$$A \sim \frac{\omega_p}{\omega_o} \frac{u}{v_e} kL_n$$

which again requires threshold power $\sim$200 KW/cm^2 for PLT parameters. The filamentation and modulation instabilities of the extraordinary mode require much higher power densities.

IV. DISCUSSION

The presently employed microwave power densities ($\gtrsim$ 200 Watt/cm^2) in EBT seem to be sufficient to parametrically excite two electron Bernstein modes at the second harmonic cyclotron layer, as long as $T_e \lesssim$ 200 eV. The Bernstein modes are strongly absorbed through cyclotron absorption and may account for the formation of the hot electron annulus. The coupling coefficient for this decay vanishes in the dipole approximation.

The situation in large tokamaks where $T_e \gtrsim$ 1 keV and $B_o \gtrsim$ 30 kG is quite unfavorable for parametric instabilities. The most prominant channel of decay seems to be the one involving two lower hybrid waves of short wavelengths ($k \gg k_o$, $\omega \simeq \omega_o/2$), however, the convective threshold for this channel is $\sim$150 KW/cm^2 which is orders of magnitude higher than the presently employed power densities.

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REFERENCES

1. R. A. Dandl, H. O. Eason and H. Ikegarni, ORNL report TM-6703 (1979).

2. V. V. Alikaev, G. A. Bobrovskii, V. I. Pozynyak, K. A. Razumova, V. V. Sanikov, Y. A. Sokolov and A. A. Shmarin, Sov. J. Plasma Phys. 2, 212 (1976); See also the references cited therein.

3. A. G. Litvak et al., Nucl. Fusion 17, 659 (1977).

4. E. Ott, B. Hui and K. R. Chu, NRL report No. 4028 (1979).

5. O. Eldridge, W. Namkung and A. C. England, ORNL report No. TM-6052 (1979).

6. G. Elder and F. W. Perkins, PPPL-1578 (1979).

7. C. S. Liu, in Advances in Plasma Physics, edited by A. Simon and W. B. Thompson (Wiley, N. Y. 1976) Vol. 6, p. 121.

8. V. K. Tripathi, C. S. Liu and C. Grebogi, Phys. Fluids 22, 301 (1979).