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A MINIMUM EXCURSION-TIME CONTROL SYSTEM

B. GROMEK and T.G. RYALL

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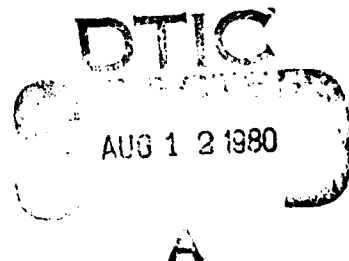
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A MINIMUM EXCURSION-TIME CONTROL SYSTEM,

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SUMMARY

A significant reduction in total test times of computer-controlled fatigue tests has been achieved by the design of a minimum excursion-time control system. The computer calculates in real time the minimum-time path between two excursion points, subject to variable velocity and acceleration constraints. This technique has been extended to multi-channel systems.

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16. ABSTRACT.

A significant reduction in total test times of computer-controlled fatigue tests has been achieved by the design of a minimum excursion-time control system. The computer calculates in real time the minimum-time path between two excursion points, subject to variable velocity and acceleration constraints. This technique has been extended to multi-channel systems.

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NOTATION

Single Channel System

$x(t)$ time path

T total excursion time

L total excursion distance

V maximum allowable velocity

A maximum allowable acceleration

Multi Channel System

$x_i(t)$ time path for the ith channel

$x(t)$ normalised single channel time path

T total excursion time

L_i total excursion distance for the ith channel

V_i maximum allowable velocity for the ith channel

A_i maximum allowable acceleration for the ith channel

N the number of channels with non zero excursion distances

L^* $\min \{L_i; \ L_i \neq 0; \ i = 1,N\}$

V^* $\min \{\frac{L^*}{L_i} \times V_i; \ i = 1,N, \ L_i \neq 0\}$

A^* $\min \{\frac{L^*}{L_i} \times A_i; \ i = 1,N, \ L_i \neq 0\}$

1. INTRODUCTION

A minimum excursion-time control system has been implemented in order to obtain a significant reduction in total test time of computer-controlled fatigue tests, as well as achieving greater flexibility and control over the application of the loads.

Frequently, load spectra used for fatigue tests are specified as a series of turning point values. Loads are applied to specimens by generating smooth waveforms to join successive turning points.

Traditionally, the method of waveform generation has involved the scaling of a constant tabular function, e.g. a sine function, which has a fixed number of steps.

There are several disadvantages with this method:

(i) a constant tabular function leads to a waveform which is not optimal for excursions between most turning points,

(ii) a fixed number of steps between turning points causes load cycles of small amplitude to require a disproportionate amount of time,

(iii) there is no independent control of acceleration.

The need for an optimal excursion generator becomes apparent in a situation where a wide range of load spectra is applied to specimens in tests of long duration, ranging from several weeks to several years.

The system described below has the following features, which overcome the above disadvantages:

(i) All calculations are performed in real-time, and no stored function table is required.

(ii) Full control by the operator of the velocity and acceleration of each individual channel exists (subject to physical limits).

(iii) The excursion time between any two successive turning points is always a minimum (subject to velocity and acceleration constraints).

2. ANALYSIS

2.1 Single Channel System

The mathematical problem may be stated in the following manner:

What is the minimum-time path joining a source and destination such that

(i) the absolute velocity and absolute acceleration are bounded,

(ii) the initial and terminal displacements are given,

(iii) the initial and terminal velocities are equal to zero.

The problem stated mathematically is

$$\text{Min } \{T\}$$

such that $|\dot{x}(t)| \leqslant V; \ |\ddot{x}(t)| \leqslant A; \ t \ \epsilon \ \left[0, T\right]$

$$x(o) = 0; \ \dot{x}(o) = 0$$

$$x(T) = L; \ \dot{x}(T) = 0 \ (L > 0)$$

The calculus of variations may be applied to this problem to prove that the acceleration may take only one of three values at any time. These values are +A, 0, -A. Furthermore it may be shown that the acceleration can only change sign once. As this result is intuitively clear the proof has been omitted.

There are two possible solutions to this problem according to the relative sizes of L,V,A. The first solution has the acceleration taking on the values +A, 0, -A in sequence. The second solution has the acceleration taking on the values +A, -A in sequence. Satisfying continuity of displacement and velocity as well as initial and final conditions implies the following solutions.

1st Solution $\dfrac{L}{V} > \dfrac{V}{A}$

$$x(t) = \tfrac{1}{2} At^2 \quad \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \quad 0 \leqslant t \leqslant \dfrac{V}{A}$$

$$\dot{x}(t) = At$$

$$\ddot{x}(t) = A$$

$$x(t) = -\tfrac{1}{2}\frac{V^2}{A} + Vt$$

$$\dot{x}(t) = V$$

$$\ddot{x}(t) = 0 \qquad \left.\right) \quad \frac{V}{A} < t < \frac{L}{V}$$

$$x(t) = L - \tfrac{1}{2}A(t - \frac{L}{V} - \frac{V}{A})^2$$

$$\dot{x}(t) = -A(t - \frac{L}{V}\frac{V}{A}) \qquad \left.\right) \quad \frac{L}{V} < t < \frac{L}{V} + \frac{V}{A}$$

$$\ddot{x}(t) = -A$$

$$T = \frac{L}{V} + \frac{V}{A}$$

2nd Solution $\frac{L}{V} < \frac{V}{A}$

$$x(t) = \tfrac{1}{2}At^2$$

$$\dot{x}(t) = At \qquad \left.\right) \quad 0 < t < \sqrt{\frac{L}{A}}$$

$$\ddot{x}(t) = A$$

$$x(t) = L - \tfrac{1}{2}A(t - 2\sqrt{\frac{L}{A}})^2$$

$$\dot{x}(t) = -A(t - 2\sqrt{\frac{L}{A}}) \qquad \left.\right) \quad \sqrt{\frac{L}{A}} < t < 2\sqrt{\frac{L}{A}}$$

$$\ddot{x}(t) = -A$$

$$T = 2\sqrt{\frac{L}{A}}$$

2.2   Multichannel System

In the multichannel system each hydraulic actuator must move in phase with every other actuator. It follows that there is an equivalent single channel system.

Since $x_i(t) = \frac{L_i}{L}x(t); \quad x(T) = L^*, \quad i = 1,N$

the constraints $|\dot{x}_i(t)| < V_i \qquad i = 1,N$

$$|\ddot{x}_i(t)| < A_i \qquad i = 1,N$$

imply $|\dot{x}(t)| \leqslant \min (L^* \cdot \dfrac{V_i}{L_i}; \; i = 1,N)$

$$= V^*$$

and $|\ddot{x}(t)| \leqslant \min (L^* \cdot \dfrac{A_i}{L_i}; \; i = 1,N)$

$$= A^*$$

L^* could be any reference length however due to scaling considerations in the computer implementation L^* is defined as $\min (L_i; \; i = 1,N)$. Substituting these values of L^*, V^*, A^* for L,V,A in the single channel system determines $x(t)$, $x_i(t)$ is then determined from the relation

$$x_i(t) = \frac{L_i}{L^*} x(t)$$

3. COMPUTER IMPLEMENTATION

The system described above has been implemented on a PDP-11 computer using the MACRO-11 assembly language.

The required load spectrum is stored on a magnetic tape in the form of sets of turning point values for each actuator, and read into memory in blocks as required.

The normalised single channel time path $x(t)$ is used to generate a voltage excursion in the range -10 volts to +10 volts, which is output via 12-bit digital to analogue converters. The signal is ultimately transmitted via servo-hydraulic actuators to produce the required load spectrum on the test specimen, which may range from a simple linear bar to a complete airframe.

A programmable real-time clock interrupt is generated every millisecond, and this is used as the time increment at which each new value of $x(t)$ is calculated. The number of discrete values of $x(t)$ thus varies for each excursion according to the values of L, V and A. However, during any particular excursion, the same number of discrete values of $x(t)$ is calculated for each actuator to ensure that they all reach their destinations simultaneously.

Due to mechanical and hydraulic limitations, it has been found necessary to reduce the velocity and acceleration of the actuators if the excursion involves a zero-crossing, i.e. goes from tension into compression or vice-versa. Facilities to do this have been incorporated

into the software by allowing operator entry of maximum allowable velocities and accelerations for both zero-crossing and non-zero crossing excursions.

As floating point hardware is not available in our system, it has been necessary to use integer arithmetic throughout, to allow the calculations to be performed in real time. However a hardware multiply/divide and multiple shift unit is available, without which the system could not have been implemented.

For example, a fast square root routine was developed, based on the well-known Newton-Raphson iterative method, viz to calculate $X = Sq.Rt.(Z)$, then make a guess for X and call it X_1, then repeatedly calculate a new approximation X_{n+1} according to the relation

$$X_{n+1} = \tfrac{1}{2} (X_n + \frac{Z}{X_n})$$

until $(X_{n+1} - X_n)$ is less than some acceptable limit ϵ. The technique used in the current implementation is to count the number of shifts required to normalize Z, i.e. the integer part of log base 2 of Z, (this takes only 1 instruction on the PDP-11) and use this as an index into a table of pre-calculated first approximations, i.e. X_1's for the Newton-Raphson method. The correct (integer) result is then guaranteed in 2 iterations for any 16-bit integer.

The algorithm implemented in the excursion generator is outlined below:

(i)   Input the destination, i.e. next turning point, values for each actuator.

(ii) Calculate $L^* = \min \{L_i; i = 1, N\}$ where

L_i = total excursion distance of the ith actuator.

If the destination value is equal to the source value for any actuator, then $L_i = 0$ and no excursion is required for that actuator.

(iii) For each actuator determine whether or not a zero-crossing occurred, and set up maximum allowable velocities and accelerations accordingly.

(iv) Determine the excursion direction for each actuator.

(v)   Calculate $V = \min \{\frac{L^*}{L_i} \times V_i; \ i = 1, \ N; \ L_i \neq 0\}$

(vi)   Calculate $A = \min \{\frac{L^*}{L_i} \times A_i; \ i = 1, \ N, \ L_i \neq 0\}$

(vii)   Calculate L/V and V/A and determine which condition is satisfied by this case, i.e. if $\frac{L}{V} > \frac{V}{A}$ then solution 1 is required

         if $\frac{L}{V} \leq \frac{V}{A}$ then solution 2 is required.

(viii)   A clock generating interrupts each millisecond is then used to time the excursion from source to destination, with t starting from 0 at the start of each excursion.

(ix)   The appropriate equation for x(t), depending on t and whether solution 1 or solution 2 is required, as described in the analysis section, is then solved.

Starting at each millisecond interrupt, and before the next interrupt occurs, the following actions must be carried out, until t has reached $(\frac{L}{V} + \frac{V}{A})$ (case 1) or $2 \sqrt{\frac{L}{A}}$ (case 2):

(x)   Solve the appropriate equation for x(t).

(xi)   Multiply x(t) by $\frac{L_i}{L^*}$ to regain the correct scaling for each actuator.

(xii)   Convert the scaled x(t) values for each actuator to units suitable for output to the respective digital to analogue converters, with appropriate magnitude and direction information included.

(xiii)   Add the base (source) value to the above value for each actuator.

(xiv)   Output the final values to the corresponding D/A converters for each actuator.

Thus all actuators perform smooth excursions, reaching their destinations together, independent of individual magnitudes and directions.

The excursion generator described above requires approximately 500 16-bit words to implement, and is incorporated in a large general-purpose fatigue testing program currently running on several PDP-11's at ARL.

4. CONCLUSION

A minimum excursion-time control system has been implemented for the application of load spectra to fatigue specimens.

The system was realized using a single-channel servo-hydraulic actuator controlled by a PDP-11 computer.

The time taken to complete a typical test was reduced from 6 weeks to 1 week, i.e. a time saving of approximately 85%.

While this problem has been discussed in relation to fatigue testing, the method described has wider applications.

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