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A GENERAL STATISTICAL APPROACH FOR USING AUXILIARY INFORMATION --ETC(U)
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Applied Research in Statistics - Mathematics - Operations Research

A GENERAL STATISTICAL APPROACH FOR USING AUXILIARY INFORMATION IN THE DEVELOPMENT OF AN IMPACT ACCELERATION INJURY PREDICTION MODEL

by

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TECHNICAL REPORT NO. 112-6

August 1980

This study was supported by the Office of Naval Research under Contract No. N00014-79-C-0128, Task No. NR 207-037

TABLE OF CONTENTS

I. INTRODUCTION

The impact acceleration forces imposed upon aircraft occupants during a naval aircraft ditching accident often result in serious injuries or fatalities, even if there is no direct impact with an object. It is important, therefore, to be able to accurately estimate the dynamic and physical parameters that relate these forces to the likelihood of injury. The Naval Biodynamics Laboratory is conducting an impact acceleration research program which is focused on the dynamic response of the human and simian head/neck system as a function of motion and anthropometric parameters.

The accurate estimation of an injury likelihood prediction model from important qualitative and quantitative information should allow reliable inferences to be made concerning injury probability as a function of dynamic and physical variables. Previous technical reports [6, 8] have discussed injury likelihood model estimation procedures based on experimental data. Another technical report [4] described a procedure for incorporating a priori information with empirical data.

This technical report discusses the broader topic of incorporating various types of auxiliary information with the dichotomous "injury" observations to more accurately develop an impact acceleration injury prediction model. The report describes a general approach to the beneficial use of auxiliary information in developing an injury likelihood prediction model via probit analysis. Continuous preinjury auxiliary information is extremely important because most of it can be obtained without injuring experimental subjects. It is hoped that this general approach will enable accurate estimation without requiring an unethically large number of dichotomous injury observations.

II. BACKGROUND

The auxiliary information considered in this technical report can be classified into two main types:

1. prior information
2. auxiliary empirical information.

The prior information can consist of

(i) a priori estimates of some (possibly vector-valued) function of the model parameters

(ii) a priori knowledge in the form of model parameter equality constraints.

In the injury probability prediction model, auxiliary empirical information could be obtained in the form of an experimental "side effect", e.g., evoked potential response data.

The injury likelihood model considered in this report is to be developed under the assumption that the injury tolerance is normally distributed. This assumption seems to hold true in many probability prediction settings. Furthermore the normal distribution provides the most tractable, fundamental distribution model for incorporating auxiliary empirical information. The analysis of dichotomous response prediction problems with normal tolerances is referred to as "probit" analysis. In previous technical reports [3, 4, 5, 6, 7, 8], the tolerance was assumed to have a logistic distribution. This assumption was made for purposes of mathematical tractability since, in those reports, the assumption of normal versus logistic tolerances made little difference in injury probability prediction.

The basic multipredictor probit model is based on the concept of a subject having a "tolerance" for a stimulus or set of stimuli, e.g. peak sled acceleration. The vector of predictor variables is represented by

$$\underline{x}' = (x_1, x_2, \ldots, x_k),$$

where at least one of the elements of $\underline{x}$ represents a particular level of stimulus. Some of the elements of $\underline{x}$ may also represent attributes of the subject exposed to the stimuli.

The probit model is derived from the notion that the stimuli result in some nonobservable real or hypothetical quantitative response, t. The observable dichotomous response, y, (e.g., injury or no injury) is deemed to occur if t falls below some critical value, which without loss of generality may be taken as zero. If the mean of t is a linear function of $\underline{x}$, then we may write

$$t = \underline{x}'\underline{\beta}_T + \tau$$

where τ is a normal random variable with zero mean and standard deviation σ_τ. Here t is the tolerance of the subject, a normally distributed random variable with mean $\underline{x}'\underline{\beta}_T$ and a standard deviation equal to σ_τ. For a given set of stimuli the dichotomous variable y is set equal to 1 if a response occurs and is set equal to 0 otherwise. Thus,

$$\Pr(y = 1|\underline{x}) = \Pr(t < 0|\underline{x}) = \Phi(-\underline{x}'\underline{\beta}_T/\sigma_\tau)$$

where Φ is the standard normal cumulative distribution function.

With the reparameterization

$$\underline{\beta}_1 = -\underline{\beta}_T/\sigma_\tau, \tag{1}$$

the following expression results:

$$\Pr(y = 1|\underline{x}) = \Phi(\underline{x}'\underline{\beta}_1). \tag{2}$$

The equation given by (2) represents the basic multipredictor probit model. For reference purposes, the random variable y will be denoted by y_1 for the remainder of this report. Model (2) can also be written in terms of the actual dichotomous observations as:

$$y_1 = \Phi(\underline{x}'\underline{\beta}_1) + \varepsilon_1 \tag{3}$$

where $E(\varepsilon_1) = 0$, $Var(\varepsilon_1) = p(1 - p)$, $p = \Phi(\underline{x}'\underline{\beta}_1)$.

In this technical report it is assumed that for each vector of stimuli, a corresponding continuous auxiliary response can be measured. This response could be interpreted as a side effect due to the stimuli. It is also assumed that the relationship between the stimuli and the side effect can be explaineed by a linear regression model of the form

$$y_2 = \underline{x}'\underline{\beta}_2 + \varepsilon_2, \tag{4}$$

where y_2 is the observed side effect. Further it is assumed that τ and ε_2 have a bivariate normal distribution with correlation coefficient ρ .

It should be noted that in the continuous preinjury/dichotomous injury setting that ρ is negative. This is so because the subject's tolerance should be inversely related to the level of side effect. In other words, as the levels of stimuli increase, the side effect increases and the subject's tolerance for the increase levels of stimuli decreases. The experimental data is then modeled by:

$$\underline{y} = f(\underline{\beta}) + \underline{\varepsilon} \tag{5}$$

where

$$\underline{y} = \begin{pmatrix} y_{11} \\ y_{21} \\ \hline \vdots \\ \hline y_{1n} \\ y_{2n} \end{pmatrix}, \qquad \underline{\beta} = \begin{pmatrix} \underline{\beta}_1 \\ \underline{\beta}_2 \end{pmatrix},$$

$$f(\underline{\beta}) = \begin{pmatrix} \Phi(\underline{x}_1'\underline{\beta}_1) \\ \underline{x}_1'\underline{\beta}_2 \\ \hline \vdots \\ \hline \Phi(\underline{x}_n'\underline{\beta}_1) \\ \underline{x}_n'\underline{\beta}_2 \end{pmatrix}, \qquad \underline{\varepsilon} = \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{21} \\ \hline \vdots \\ \hline \varepsilon_{1n} \\ \varepsilon_{2n} \end{pmatrix}.$$

The error structure has the following properties:

(a) $\begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \end{pmatrix}$ are independent vectors for $i = 1, \ldots, n$

(b) ε_{1i} and ε_{2i} are correlated for each i, $i = 1, \ldots, n$

(c) $E(\underline{\varepsilon}) = \underline{0}$

$$(d)\quad \mathrm{Var}(\underline{\varepsilon}) = \underline{V} = \begin{bmatrix} \underline{V}_1 & & & 0 \\ & \underline{V}_2 & & \\ & & \ddots & \\ 0 & & & \underline{V}_n \end{bmatrix},$$

$$\text{where } \underline{V}_i = \mathrm{Var}\begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \end{pmatrix} = \begin{bmatrix} p_i(1 - p_i) & c_i \\ c_i & \sigma_2^2 \end{bmatrix} \text{ for } i = 1,\ldots, n ,$$

$$p_i = \Phi(\underline{x}_i'\underline{\beta}_1),$$

$$c_i = -\rho\sigma_2(2\pi)^{-\frac{1}{2}}\exp\{-(\underline{x}_i'\underline{\beta}_1)^2/2\}.$$

(Derivation of the expression for c_i is given in the appendix.)

As an aid to both estimation and computation, a priori information about the parameters in model (5) can be quite useful. Any good prior estimates of the model parameters or some function of them not only help obtain accurate final model parameter estimates but also aid in the convergence of the estimation procedure which must be performed iteratively. Further, prior information in the form of proper parameter constraints can significantly reduce the variance of the constrained parameters. As presented in this report, the prior information about the model parameters need not be complete. In other words, prior information on each individual parameter need not exist.

With a few reasonable assumptions, prior estimates that are in the form of some function of model parameters, can be modeled in a fashion that lends itself to computation by least squares. For an example of this in the linear least squares setting, see [9]. To see how this prior information can be modeled in a general nonlinear least squares setting, denote a set of generic model parameters by a q x 1 vector, $\underline{\alpha}$. Let $\underline{r}$ be a d x 1 vector, ($d \leq q$), that is an unbiased estimate of some function of $\underline{\alpha}$, $\underline{g}(\underline{\alpha})$, say. Then one can write

$$\underline{r} = \underline{g}(\underline{\alpha}) + \underline{\nu} \tag{6}$$

where $E(\underline{\nu}) = 0$.

Assume further that $\text{Var}(\underline{\nu}) = \underline{\Psi}$, where $\underline{\Psi}$ is known or can at least be estimated, and that the prior information embodied in $\underline{r}$ is stochastically independent of $\underline{y}$. Thus it follows, trivially, that $\underline{\nu}$ and $\underline{\varepsilon}$ are stochastically independent and that

$$\text{Var}\begin{pmatrix} \underline{\nu} \\ \underline{\varepsilon} \end{pmatrix} = \begin{bmatrix} \underline{\Psi} & 0 \\ 0 & \underline{V} \end{bmatrix} .$$

Further, if a maximum likelihood estimate of $\underline{\alpha}$ is desired, it is convenient to assume that $\underline{\nu}$ is multivariate normal with zero mean and covariance matrix $\underline{\Psi}$.

One may wish to impose restrictions across the elements of $\underline{\alpha}$. The most elegant way to represent these restrictions is by reparameterization. Let $\underline{\theta}$ be an s x 1 vector of new parameters relating $\underline{\alpha}$ to $\underline{\theta}$ according to

$$\underline{\alpha} = \underline{h}(\underline{\theta}) .$$

It is assumed that $s \leq q$. Thus under these restrictions, the "prior estimate" model in (6) can be written as

$$\underline{r} = \underline{g}(\underline{h}(\underline{\theta})) + \underline{\nu} , \tag{7}$$

and $\underline{h}(\underline{\theta})$ is also substituted for $\underline{\alpha}$ into the model for the empirical data.

Parameter constraints can also occur in the form of inequalities. For example, suppose it is known that

$$
\begin{aligned}
&g_1(\underline{\alpha}) \leq 0 \\
&\vdots \\
&g_j(\underline{\alpha}) \leq 0
\end{aligned}
$$

for some functions $g_1, \ldots, g_j$. Sometimes these inequality constraints can be converted to equality constraints from results of nonlinear programming theory, e.g., the Kuhn-Tucker conditions. (See [1].)

If any of these inequality constraints cannot be removed, then formal techniques of nonlinear programming need to be introduced to estimate the parameters. In such a case the reader is referred to [1]. General nonlinear programming procedures are not considered in this technical report as they could not be directly performed by the basic nonlinear, least squares statistical packages available.

III. ESTIMATION THEORY

For the general estimation problem considered in this report, nonlinear weighted least squares estimates are most often easier to obtain than are the maximum likelihood estimates. This is especially true when there are parameter constraints linking $\underline{\beta}_1$ and $\underline{\beta}_2$. The likelihood function, however, offers an enlightening insight into the relationship between the dichotomous injury data and the continuous preinjury data.

In this report, therefore, a theoretical analysis of the empirical information is presented via the likelihood function. A future report will discuss the nonlinear weighted least squares approach from a computational point of view. No real gaps in this approach to the estimation problem will occur, at least asymptotically, since maximum likelihood estimates and weighted least squares estimates are essentially the same for moderate to large sample sizes if the data are sampled from a distribution belonging to a regular, exponential family as is the case here.

The estimation theory discussed here centers on just the empirical information in order to keep the presentation from becoming too lengthy. Afterwards, the incorporation of the a priori information can be developed in a straightforward manner.

Denote the random sample, conditional or $\underline{x}_1, \ldots, \underline{x}_n$, by

$$\begin{pmatrix} y_{11} \\ y_{21} \\ \underline{x}_1 \end{pmatrix}, \ldots, \begin{pmatrix} y_{1n} \\ y_{2n} \\ \underline{x}_n \end{pmatrix} \qquad (8)$$

where as before

$y_{11}, \ldots, y_{1n}$ are the dichotomous injury observations

$y_{21}, \ldots, y_{2n}$ are the continuous preinjury observations

$\underline{x}_1, \ldots, \underline{x}_n$ are the corresponding predictor variables.

To derive the likelihood of (8), first consider the joint distribution of each

$$\begin{pmatrix} t_i \\ y_{2i} \end{pmatrix} \quad i = 1, \ldots, n \ ,$$

where as before

$t_1, \ldots, t_n$ are the unobservable injury tolerances.

Each $\begin{pmatrix} t_i \\ y_{2i} \end{pmatrix}$ is by assumption, bivariate normal with mean $\begin{pmatrix} \underline{x}_i'\underline{\beta}_T \\ \underline{x}_i'\underline{\beta}_2 \end{pmatrix}$

and covariance matrix

$$\begin{bmatrix} \sigma_\tau^2 & \rho\sigma_\tau\sigma_2 \\ \rho\sigma_\tau\sigma_2 & \sigma_2^2 \end{bmatrix}$$

For each i, let $f_{t,y_2}(t_i, y_{2i})$ be the joint density function of $\begin{pmatrix} t_i \\ y_{2i} \end{pmatrix}$. Then $f_{t,y_2}(t_i, y_{2i})$ can be written as

$$f_t(t_i | y_{2i}) f_{y_2}(y_{2i})$$

where $f_t(t_i | y_{2i})$ is the univariate, normal density of t_i conditional on y_{2i} , and $f_{y_2}(y_{2i})$ is the univariate, normal density of y_{2i}. It follows that from $f_t(t_i|y_{2i})$, the probability distribution of y_{1i} conditional on y_{2i} is obtained. It is generally known from normal distribution theory that $t_i|y_{2i}$ has a mean equal to

$$\underline{x}_i'\underline{\beta}_T + \rho\frac{\sigma_\tau}{\sigma_2}(y_{2i} - \underline{x}_i'\underline{\beta}_2)$$

and a variance of

$$\sigma_\tau^2(1 - \rho^2) \quad .$$

Now

$$\begin{aligned} Pr(y_{1i} = 1|\underline{x}_i, y_{2i}) &= Pr(T_i < 0|\underline{x}_i, y_{2i}) \\ &= \Phi(-[\underline{x}_i'\underline{\beta}_T + \rho\frac{\sigma_\tau}{\sigma_2}(y_{2i} - \underline{x}_i'\underline{\beta}_2)]/\sigma_\tau[1 - \rho^2]^{\frac{1}{2}}) \\ &= \Phi((1- \rho^2)^{-\frac{1}{2}}[\underline{x}_i'(-\underline{\beta}_T/\sigma_\tau) - \rho((y_{2i} - \underline{x}_i'\underline{\beta}_2)/\sigma_2)]). \end{aligned}$$

However, $\underline{\beta}_1 = -\underline{\beta}_T/\sigma_\tau$ from (1), so that

$$Pr(y_{1i} = 1|\underline{x}_i, y_{2i}) = \Phi((1 - \rho^2)^{-\frac{1}{2}}[\underline{x}_i'\underline{\beta}_1 - \rho((y_{2i} - \underline{x}_i'\underline{\beta}_2)/\sigma_2)]).$$

For brevity's sake let

$$p_i = Pr(y_{1i} = 1|\underline{x}_i, y_{2i}) \quad .$$

Then

$$p_i^{y_{1i}}(1 - p_i)^{(1 - y_{1i})}$$

is the probability distribution of y_{1i} conditional on $\underline{x}_i$ and y_{2i}, so that

$$p_i^{y_{1i}}(1 - p_i)^{(1 - y_{1i})}f_{y2}(y_{2i})$$

is the joint distribution function of $\begin{pmatrix} y_{1i} \\ y_{2i} \end{pmatrix}$ conditional on $\underline{x}_i$.

Thus the likelihood of (8) is

$$\prod_{i=1}^{n} p_i^{y_{1i}}(1 - p_i)^{(1 - y_{1i})} f_{y_2}(y_{2i}) \tag{9}$$

where p_i is as previously defined and

$$f_{y_2}(y_{2i}) = (2\pi)^{-\frac{1}{2}}\sigma_2^{-1}\exp[-\tfrac{1}{2}((y_{2i} - \underline{x}_i'\underline{\beta}_2)/\sigma_2)^2].$$

Now after some reparameterization a surprising result emerges. The maximum likelihood estimates of $\underline{\beta}_2$ and σ_2 do not depend upon $y_{11}, \ldots, y_{1n}$. They are exactly the same estimates that would be obtained from computing the maximum likelihood estimates of $\underline{\beta}_2$ and σ_2 based on only the preinjury data. To make this clear, consider the fact that p_i may be expressed as

$$p_i = (\underline{x}_i'\underline{\beta}^* + \gamma y_{2i}) \tag{10}$$

where $\underline{\beta}^* = (1 - \rho^2)^{-\frac{1}{2}}(\underline{\beta}_1 + \rho\underline{\beta}_2/\sigma_2)$

and $\gamma = -(1 - \rho^2)^{-\frac{1}{2}}\rho/\sigma_2$.

Due to the parameters $\underline{\beta}_1$ and ρ in (10), $\underline{\beta}^*$ and γ are not explicit functions of $\underline{\beta}_2$ and σ_2 alone. The likelihood function in (9) then, can be maximized with respect to $\underline{\beta}_2$ and σ_2 independently of any of the $p_i^{y_{1i}}(1 - p_i)^{(1 - y_{1i})}$ functions in which only the $\underline{\beta}^*$ and γ parameters appear. Thus, the maximum likelihood estimates of $\underline{\beta}_2$ and σ_2 do not depend upon the dichotomous injury observations. This means that the $y_{11}, \ldots, y_{1n}$ observations cannot be used to improve the parameter estimates of $\underline{\beta}_2$ and σ_2. This is not too serious however, as these are simply auxiliary parameters.

The maximum likelihood estimates of $\underline{\beta}^*$ and γ though, depend upon both the $y_{11}, \ldots, y_{1n}$ and the $y_{21}, \ldots, y_{2n}$ observations. To see this, let $\hat{\underline{\beta}}^*$, $\hat{\gamma}$, $\hat{\underline{\beta}}_2$ and $\hat{\sigma}_2$ be the maximum likelihood estimates. Then one can use the equations in (10) to solve for the maximum likelihood estimates[1] $\hat{\underline{\beta}}_1$ and $\hat{\rho}$; they are

$$\hat{\rho} = -\hat{\gamma}\hat{\sigma}_2/(\hat{\gamma}^2\hat{\sigma}_2^2 + 1)^{\frac{1}{2}}$$

and $$\hat{\underline{\beta}}_1 = \hat{\underline{\beta}}^*(1 - \hat{\rho}^2)^{\frac{1}{2}} - \hat{\rho}\hat{\underline{\beta}}_2\hat{\sigma}_2^{-1} \quad .$$

A further result of interest is that the maximum likelihood estimates of $\underline{\beta}_1$ and ρ are functions of the $y_{21},\ldots, y_{2n}$ observations only through the standardized residuals from the (maximum likelihood) linear regressions of the y_{2i}'s on the $\underline{x}_i$'s . This is shown as follows. Let

$$s_i = (y_{2i} - \underline{x}_i'\hat{\underline{\beta}}_2)/\hat{\sigma}_2, \quad i = 1,\ldots, n$$

be the standardized residuals. Now consider again the likelihood function expressed by equation (9). For notational brevity write this function as

$$L(\underline{\beta}_1, \rho, \underline{\beta}_2, \sigma_2) \ . \tag{11}$$

The function in (11) can be maximized with respect to $(\underline{\beta}_1, \rho, \underline{\beta}_2, \sigma_2)$ by maximizing the following function of $\underline{\beta}_1$ and ρ :

$$\max_{\underline{\beta}_2,\sigma_2} L(\underline{\beta}_1, \rho, \underline{\beta}_2, \sigma_2) = L(\underline{\beta}_1, \rho, \hat{\underline{\beta}}_2, \hat{\sigma})$$

However,

$$L(\underline{\beta}_1, \rho, \hat{\underline{\beta}}_2, \hat{\sigma}_2) = \prod_{i=1}^{n} \hat{p}_i^{\,y_{1i}}(1 - \hat{p}_i)^{(1 - y_{1i})}\hat{f}_{y_2}(y_{2i}) \tag{12}$$

where $\hat{p}_i = \Phi((1 - \rho^2)^{-\frac{1}{2}}[\underline{x}_i'\underline{\beta}_1 - \rho s_i])$

and $\hat{f}_y(y_{2i}) = (2\pi)^{-\frac{1}{2}}\hat{\sigma}_2^{-1}\exp[-(y_{2i} - \underline{x}_i'\hat{\underline{\beta}}_2)^2/2\hat{\sigma}_2^2]$

---

[1] $\hat{\underline{\beta}}_1$ and $\hat{\rho}$ are the maximum likelihood estimates by the invariance principle of maximum likelihood estimation. (See [2].)

since $s_i = (y_{2i} - \underline{x}_i'\hat{\underline{\beta}}_2)/\hat{\sigma}_2$

Thus, the maximum likelihood estimates of $\underline{\beta}_1$ and ρ can be based upon the observations

$$\begin{pmatrix} y_{11} \\ \underline{x}_1 \\ s_1 \end{pmatrix}, \ldots, \begin{pmatrix} y_{1n} \\ \underline{x}_n \\ s_n \end{pmatrix} \tag{13}$$

through the likelihood function

$$\prod_{i=1}^{n} \hat{p}_i^{\,y_{1i}}(1 - \hat{p}_i)^{(1 - y_{1i})} \tag{14}$$

So it is now evident that the maximum likelihood estimates of $\underline{\beta}_1$ and ρ can be based upon (13) through (14) or based upon

$$\begin{pmatrix} y_{11} \\ \underline{x}_1 \\ y_{21} \end{pmatrix}, \ldots, \begin{pmatrix} y_{1n} \\ \underline{x}_n \\ y_{2n} \end{pmatrix} \tag{15}$$

through the likelihood function

$$\prod_{i=1}^{n} p_i^{\,y_{1i}}(1 - p_i)^{(1 - y_{1i})} \tag{16}$$

where $p_i = \Phi(\underline{x}_i'\underline{\beta}^* + \gamma y_{2i})$, $i = 1, \ldots, n$.

However, the likelihood function in (16) has the disadvantage that the continuous preinjury, y_{2i}, observations are correlated with the predictor variables. This could lead to a problem of multicollinearity in the estimation of $\underline{\beta}^*$ and γ .

Asymptotically, the nonlinear weighted least squares estimates of $\underline{\beta}_1$ and ρ are equivalent to their maximum likelihood estimates. The nonlinear weighted least squares estimates are derived from the model

$$y_{1i} = \Phi[U_i(\underline{\beta}_1, \rho)] + \xi_i, \quad i = 1,\ldots, n \qquad (17)$$

where $U_i(\underline{\beta}_1, \rho) = (1 - \rho^2)^{-\frac{1}{2}}(\underline{x}_i'\underline{\beta}_1 - \rho s_i), \quad i = 1,\ldots, n$

and $E(\xi_i) = 0$, $Var(\xi_i) = p_i(1 - p_i)$, $p_i = \Phi[U_i(\underline{\beta}_1, \rho)]$.

Computational aspects of this model will be addressed in a future report.

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APPENDIX

This appendix contains a detailed derivation of

$$E(\varepsilon_{2i}y_{1i})$$

$$\text{and } Var(\varepsilon_{2i}y_{1i}) \quad i = 1,\ldots, n .$$

First consider the joint distribution of ε_{2i} and t_i. By assumption, ε_{2i} and t_i have a bivariate normal distribution. For brevity, let the mean of t_i be μ_{T_i}. Now $E(\varepsilon_{2i}y_{1i})$ equals

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varepsilon_{2i}I_i(t_i)f_{\varepsilon,t}(\varepsilon_{2i}, t_i)d\varepsilon_{2i}dt_i$$

where $I_i(t_i) = \begin{cases} 0 \text{ if } t_i \geq 0 \\ 1 \text{ if } t_i < 0 \end{cases}$

and $f_{\varepsilon,t}(\varepsilon_{2i}, t_i)$ is the joint density of ε_{2i} and t_i.

Thus,

$$E(\varepsilon_{2i}y_{1i}) = \int_{-\infty}^{0}\int_{-\infty}^{\infty}\varepsilon_{2i}f_{\varepsilon}(\varepsilon_{2i}|t_i)f_t(t_i)d\varepsilon_{2i}dt_i$$

where $f_{\varepsilon}(\varepsilon_{2i}|t_i)$ is the normal density of ε_{2i} conditional on t_i and $f_t(t_i)$ is the marginal density of t_i.

By integration with respect to ε_{2i}, one obtains

$$E(\varepsilon_{2i}y_{1i}) = \int_{-\infty}^{0} E(\varepsilon_{2i}|t_i)f_t(t_i)dt_i. \tag{1}$$

Now, since

$$E(\varepsilon_{2i}|t_i) = \rho\frac{\sigma_2}{\sigma_T}(t_i - \mu_{T_i}) \tag{2}$$

by substitution of (2) into the right hand side of (1), one obtains

$$E(\varepsilon_{2i}y_{1i}) = \rho\sigma_2 \int_{-\infty}^{0} \left(\frac{t_i - \mu_{T_i}}{\sigma_\tau}\right) f_t(t_i)dt_i \ .$$

Thus,

$$E(\varepsilon_{2i}y_{1i}) = \rho\sigma_2 \int_{-\infty}^{-\mu_{T_i}/\sigma_\tau} \exp[-(v_i^2/2)]/[(2\pi)^{\frac{1}{2}} v_i]dv_i$$

$$= -\rho\sigma_2\phi(-\mu_{T_i}/\sigma_\tau)$$

where $v_i = (t_i - \mu_{T_i})/\sigma_\tau$ and $\phi(h) = \exp(-h^2/2)/(2\pi)^{\frac{1}{2}}$.

However, $\mu_{T_i} = \underline{x}_i'\underline{\beta}$ and $\underline{\beta}_1 = -\underline{\beta}/\sigma_\tau$.

Therefore, $E(\varepsilon_{2i}y_{1i}) = -\rho\sigma_2\phi(\underline{x}_i'\underline{\beta}_1)$. Because $E(\varepsilon_{2i}\varepsilon_{1i}) = E(\varepsilon_{2i}y_{1i})$, this is the value of c_i which appears in section II of this report.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 112-6	2. GOVT ACCESSION NO. AD-A087	3. RECIPIENT'S CATALOG NUMBER 810
4. TITLE (and Subtitle) A GENERAL STATISTICAL APPROACH FOR USING AUXILIARY INFORMATION IN THE DEVELOPMENT OF AN IMPACT ACCELERATION INJURY PREDICTION MODEL		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John J. Peterson and Dennis E. Smith		8. CONTRACT OR GRANT NUMBER(s) N00014-79-C-0128
9. PERFORMING ORGANIZATION NAME AND ADDRESS Desmatics, Inc. P. O. Box 618 State College, PA 16801		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 207-037
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, Va 22217		12. REPORT DATE August 1980 13. NUMBER OF PAGES 19
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) TR-112-6		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this report is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Impact Acceleration Injury Prediction
Use of Auxiliary Information
Statistical Model

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report discusses general procedures for simultaneously incorporating various sources of auxiliary information into an impact acceleration injury prediction model. The sources of auxiliary information considered are supplemental continuous empirical data and a priori knowledge in the form of model parameter estimates and constraints.

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