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MOTION OF TRAPPED ELECTRONS AND PROTONS IN
SATURN'S INNER MAGNETOSPHERE

by

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ABSTRACT

A summary is given of basic formulas for the guiding center
motion of energetic charged particles trapped in a dipolar mag-
netic field. These formulas for longitudinal drift rates,
latitudinal bounce periods, equatorial gyroradii, and equatorial
gyroperiods are then stated in convenient numerical form for
electrons and protons as functions of kinetic energy E, magnetic
shell parameter L, and equatorial pitch angle α_g for a slightly
simplified model of the observed magnetic field of Saturn. To
aid in the study of the interaction of charged particles with the
rings and inner satellites of Saturn, additional formulas are
given for the time interval between successive encounters of
charged particles with a satellite in a circular prograde orbit
and for the energies of electrons whose longitudinal angular
velocity is resonant, or synchronous, with the Keplerian
angular velocity of such a satellite.

INTRODUCTION

The purpose of this short paper is to summarize useful parameters which characterize the motion of charged particles in Saturn's inner magnetosphere. Such motion is a combination of gyration about a magnetic field line, latitudinal bounce along the field line, and longitudinal drift perpendicular to the field line. The longitudinal drift arises from the presence of a (corotational) electric field and from the curvature and gradient of the magnetic field. The adopted magnetic field model is that of a point dipole of moment $M = 0.20$ gauss R_s^3 $= 4.32 \times 10^{28}$ gauss cm^3 (equatorial radius of the planet $= 1 R_s = 6 \times 10^9$ cm), located at the geometrical center of the planet with $\vec{M}$ parallel to and in the same sense as $\vec{\Omega}$, the rotational angular velocity of the planet. This model is a slight simplification of the observed field [Smith et al., 1980; Acuna and Ness, 1980]. Parameters related to the motion of electrons and protons are presented as functions of kinetic energy E, magnetic shell parameter L, and equatorial pitch angle α_o. We adopt the convention that the longitudinal angular velocity is positive for eastward drift (same sense as the rotation of the planet) and negative for westward drift.

Unless otherwise stated, all basic formulas are in gaussian c.g.s. units and all numerical formulas give angular velocities in radian s^{-1}, distances in cm and periods in seconds when kinetic energy E is expressed in MeV. A short table of exemplary values is included. Extensive tables are contained in University of Iowa 80-25 of July 1980, available on request to the authors.

SYMBOLS AND CONSTANTS

c = speed of light = 2.997925×10^{10} cm s^{-1}

mc^2 = rest energy of the particle
= 0.511 MeV for electron.
= 938.3 MeV for proton

E = kinetic energy of particle

v = speed of particle

β = v/c

γ = $(1 - \beta^2)^{-\frac{1}{2}}$

q = charge of particle = 4.80325×10^{-10} e.s.u. per unit charge

R = equatorial radius of planet; 1 R_s = 6×10^9 cm for Saturn

B_o = equatorial surface field of planetary dipole moment
= 0.20 gauss for Saturn

M = dipole moment of planet = 4.32×10^{28} gauss cm^3 for Saturn

r = radial distance from center of the planet

α_o = equatorial pitch angle of a particle

L = equatorial crossing distance of a dipolar magnetic field line in units of R

λ = magnetic latitude

λ_m = magnetic latitude of a particle's mirror point

$B(\lambda)$, $B(\lambda_m)$
= magnetic field strength at λ, λ_m

GM = gravitational constant of planet
= 3.79311×10^{22} cm^3 s^{-2} for Saturn

J_2 = second zonal harmonic coefficient of planet's gravitational potential = 1667×10^{-5} for Saturn

a = semimajor axis of a satellite orbit

Other symbols are defined as they appear in the text.

GRADIENT AND CURVATURE DRIFT

The bounce-averaged longitudinal drift of the guiding center
of a particle trapped in a dipolar magnetic field due to the
curvature and gradient of the field has been studied by Hamlin
et al. [1961], Lew [1961] and subsequent authors. Following
Hamlin et al. [1961], Lew [1961] found that the bounce-averaged
angular velocity can be expressed as

$$\omega_D = \frac{3mc^3\beta^2\gamma L}{2qB_oR^2} \frac{F(\lambda_m)}{G(\lambda_m)} ,\qquad (1)$$

where

$$F(\lambda_m) = \oint \frac{(1-\sin^4\lambda)}{(1+3\sin^2\lambda)^{\frac{3}{2}}} \cdot \frac{2-B(\lambda)/B(\lambda_m)}{[1-B(\lambda)/B(\lambda_m)]^{\frac{1}{2}}} \cos\lambda\, d\lambda$$

$$G(\lambda_m) = \oint \frac{(1+3\sin^2\lambda)^{\frac{1}{2}}}{[1-B(\lambda)/B(\lambda_m)]^{\frac{1}{2}}} \cos\lambda\, d\lambda .$$

The integrals are carried out over a complete latitudinal bounce
period. For Saturn, Equation (1) becomes

$$\omega_D = \pm 2.083 \times 10^{-5} \, LE \left(\frac{E + 2mc^2}{E + mc^2} \right) \left(\frac{F}{G} \right). \qquad (2)$$

In (2) the + sign applies to protons and the - sign to electrons. Lew [1961] further presented an analytical approximation to the ratio $F(\lambda_m)/G(\lambda_m)$ which is good to within one part in 10^3.

$$(F/G)^{-1} = [A + B \sin^2 \lambda_m - C \exp(-k \sin^2 \lambda_m)] \qquad (3)$$

with

$$A = 1.04675$$
$$B = 0.45333$$
$$C = 0.04675$$
$$k = 6.34568$$

Values of λ_m and F/G for several values of the particle's equatorial pitch angle α_o are given in Table 1.

It is essential to note that ω_D is the longitudinal angular velocity of particle drift in the reference frame rotating with the planet.

COROTATIONAL DRIFT

In addition to longitudinal drift due to the curvature and gradient of the magnetic field, charged particles also drift in the presence of an electric field perpendicular to the magnetic field [Jackson, 1962]. Referenced to an inertial coordinate system, an electric field of this nature exists in the magnetosphere of a rotating planet if the planetary ionosphere is electrically coupled to the magnetosphere, as is the case in the presence of the ionospheric and magnetospheric plasmas which make the electrical conductivity parallel to magnetic field lines essentially infinite. Davis [1948] showed that charge flows in such a plasma surrounding a rotating, conducting magnetized sphere until the electric field

$$\vec{E} = -\frac{1}{c} (\vec{\Omega} \times \vec{r}) \times \vec{B} \qquad (4)$$

is established in the plasma. This electric field results in a longitudinal drift angular velocity equal to the planet's angular velocity Ω and in the same sense [Birmingham and Northrop, 1979]. The value adopted herein for Ω, the angular velocity of Saturn's rotation, is 1.637×10^{-4} radian s^{-1}, which corresponds to the rotational period 10 hours 39.9 ± 0.3 minutes determined recently from radio observations of Saturn by Voyager instruments [Kaiser et al., 1980].

ANGULAR VELOCITY IN AN INERTIAL FRAME

The corotational angular velocity is additive to the magnetic gradient and curvature drift angular velocity. The algebraic sum of these two angular velocities is the longitudinal angular velocity of a particle relative to an inertial frame. Thus

$$\omega_I = \Omega + \omega_D . \tag{5}$$

KEPLERIAN ORBITAL ANGULAR VELOCITY

From Aksnes [1977], the mean angular velocity of a neutral body in gravitational orbit about Saturn is

$$\omega_k = \left(\frac{GM}{a^3} \right)^{\frac{1}{2}} \left(1 - \frac{3J_2 R^2}{2a^2} \right)^{-\frac{1}{2}} . \tag{6}$$

With a in units of R,

$$\omega_k = \frac{4.1905 \times 10^{-4}}{a^{\frac{3}{2}}} \left(1 + \frac{1.25 \times 10^{-2}}{a^2} \right) . \tag{7}$$

For $r = 1.871\ R_s$, $\Omega = \omega_k$; for $r > 1.871\ R_s$, $\Omega > \omega_k$; and for $r < 1.871\ R_s$, $\Omega < \omega_k$.

TIME INTERVAL BETWEEN SUCCESSIVE ENCOUNTERS OF A CHARGED PARTICLE WITH A SATELLITE

The angular velocity of a charged particle relative to a satellite in a circular prograde orbit in the equatorial plane at the same radial distance is

$$(\omega_I - \omega_k) = (\Omega + \omega_D - \omega_k) \tag{8}$$

and the interval between encounters is

$$T_E = 2\pi/(|\omega_I - \omega_k|) . \tag{9}$$

Note that $(\omega_I - \omega_k)$ may be positive (eastward), zero (resonant), or negative (westward).

RESONANT ENERGY

The value of E such that T_E is infinite is called the resonant (or synchronous) energy E_R and is given by

$$\Omega + \omega_D - \omega_k = 0, \tag{10}$$

a quadratic equation in E_R, whose solution for electrons is

$$E_R = 0.5 \left[E' - 1.022 + (E'^2 + 1.0445)^{\frac{1}{2}} \right], \tag{11}$$

where

$$E' = 4.800 \times 10^4 \, (\Omega - \omega_k) / (LF/G). \tag{12}$$

Only for $r < 1.871 \, R_s$ can protons be resonant with a satellite; inasmuch as this dividing radius is near the outer edge of Ring B there is no evident interest in proton resonances in Saturn's magnetosphere.

BOUNCE PERIOD

The latitudinal bounce period of a particle oscillating between mirror points on a given magnetic field line is

$$T_B = \oint \frac{ds}{v_\parallel}, \tag{13}$$

where the integral is carried out over arc length s along the field line for a complete (round trip) bounce between mirror points. For a dipole field,

$$T_B = \frac{4RL}{v} \int_0^{\lambda_m} \frac{\cos \lambda \ (1+3 \sin^2 \lambda)^{\frac{1}{2}}}{[1-B(\lambda)/B(\lambda_m)]^{\frac{1}{2}}} \, d\lambda \quad . \tag{14}$$

$$T_B = \frac{4RL}{\beta c} \ H(\alpha_0) \tag{15}$$

$$T_B = 0.8006 \ \frac{L}{\beta} H(\alpha_0) \tag{16}$$

or

$$T_B = 0.8006 \ \frac{L(E+mc^2)}{[E(E+2mc^2)]^{\frac{1}{2}}} \ H(\alpha_0) \quad . \tag{17}$$

The integral $H(\alpha_0)$ has been approximated by Lenchek et al. [1961] as

$$H(\alpha_0) \approx 1.38 - 0.32 \ (\sin \alpha_0 + \sin^{\frac{1}{2}} \alpha_0) \ . \tag{18}$$

Sample values of $H(\alpha_0)$ are given in Table 1.

EQUATORIAL GYROPERIOD

The relativistically correct angular gyrofrequency
[Jackson, 1962] of a charged particle is given by

$$\omega_g = \frac{qB}{\gamma m c} = \frac{q B c}{E + m c^2} \quad . \tag{19}$$

In the equatorial plane of the Saturnian dipolar magnetic field,

$$\omega_g = \frac{1.798 \times 10^6}{L^3 (E + mc^2)} \quad . \tag{20}$$

The corresponding equatorial gyroperiod $T_g = 2\pi/\omega_g$.

$$T_g = 3.495 \times 10^{-6} L^3 (E + mc^2) \quad . \tag{21}$$

EQUATORIAL GYRORADIUS

The equatorial gyroradius (radius of the cylindrical surface on which the particle's helical trajectory lies),

$$r_g = \frac{\beta c \sin \alpha_o}{\omega_g} \tag{22}$$

or

$$r_g = 1.667 \times 10^4 L^3 \sin \alpha_o [E(E + 2mc^2)]^{\frac{1}{2}} \quad . \tag{23}$$

APPLICABILITY OF FORMULAS

The formulas given in the foregoing sections are useful for any values of E, L, and α_o for which the assumptions are applicable. For $L \geqslant 7$, the adopted model of Saturn's magnetic field becomes progressively less accurate because of magneto-spheric current systems and for $L \geqslant 13$ the formulas are of very limited value. Also, they are not applicable to particles whose energies exceed the stable trapping limit [Van Allen, 1962].

L VALUES OF SPECIAL INTEREST

A particular purpose of this paper is to aid the study of the interaction of trapped particles with the rings and inner satellites of Saturn. L values of special interest are given in Table 2. In refined considerations, a full set of orbital elements, including eccentricity and inclination, is required [Explanatory Supplement, 1961] as are diameters of the satellites [Cruikshank, 1978].

EXEMPLARY TABLE

In Table 3, we give exemplary values of the parameters discussed herein for L = 3.092. Note that in this table we depart from the system of units in which the numerical formulas are given.

ACKNOWLEDGMENTS

Table 1

α_o	λ_m	F/G	H
90°	0:0	1.000	0.740
80	4.7	0.995	0.747
70	9.6	0.980	0.769
60	14.7	0.957	0.805
50	20.2	0.927	0.855
40	26.3	0.891	0.918
30	33.2	0.851	0.994
20	41.4	0.805	1.083
10	52.5	0.751	1.191

Table 2

L Values of Special Interest

Satellite	L	References
1979 S5/1979 S6	2.347	Van Allen et al. [1980b]
1979 S2/1979 S4	2.528	Van Allen et al. [1980b]
1979 S3	2.840	Van Allen et al. [1980a; 1980b]
SI Mimas	3.092	Null and Lieske [1980]; Explanatory Supplement to the Astronomical Ephemeris and Nautical Almanac [1961]
SII Enceladus	3.968	Explanatory Supplement ... [1961]
SIII Tethys	4.913	Explanatory Supplement ... [1961]
SIV Dione	6.292	Explanatory Supplement ... [1961]
SV Rhea	8.787	Explanatory Supplement ... [1961]

Table 3

Exemplary Values of Trapped Particle Parameters for L = 3.092

All angular velocities in radian s^{-1}, T_E in hr, T_B in s, T_g in s and r_g in km.

$\omega_K = 7.717 \, E - 5$, $\Omega = 1.637 \, E - 4$

(a) Electrons

E (MeV)	α_o	ω_D	ω_I	$\omega_I - \omega_K$	T_E	T_B	T_g	r_g
0.1	90°	- 1.18 E - 5	1.52 E - 4	7.47 E - 5	23.4	3.34	6.31 E - 5	1.65
0.5	90	- 4.85 E - 5	1.15 E - 4	3.81 E - 5	45.9	2.12	1.04 E - 4	4.30
0.9	90	- 7.90 E - 5	8.47 E - 5	7.57 E - 6	230.	1.97	1.46 E - 4	6.48
1.005	90	- 8.65 E - 5	7.72 E - 5	Resonant	—	1.95	1.57 E - 4	7.03
1.059	60	- 8.65 E - 5	7.72 E - 5	Resonant	—	2.11	1.62 E - 4	6.34
1.218	30	- 8.65 E - 5	7.72 E - 5	Resonant	—	2.58	1.79 E - 4	4.07
1.1	90	- 9.33 E - 5	7.04 E - 5	- 6.79 E - 6	257.	1.93	1.66 E - 4	7.53
5.	90	- 3.52 E - 4	- 1.88 E - 4	- 2.65 E - 4	6.58	1.84	5.69 E - 4	27.0
10.	90	- 6.75 E - 4	- 5.12 E - 4	- 5.89 E - 4	2.96	1.83	1.09 E - 3	51.7

(b) Protons

E (MeV)	α_o	ω_D	ω_I	$\omega_I - \omega_K$	T_E	T_B	T_g	r_g
0.1	90	1.29 E - 5	1.77 E - 4	9.94 E - 5	17.6	125.	9.70 E - 2	67.5
0.5	90	6.44 E - 5	2.28 E - 4	1.51 E - 4	11.6	56.1	9.70 E - 2	151.
1.0	90	1.29 E - 4	2.92 E - 4	2.15 E - 4	8.11	39.7	9.70 E - 2	214.
5.	90	6.42 E - 4	8.06 E - 4	7.29 E - 4	2.39	17.8	9.75 E - 2	478.
10.	90	1.28 E - 3	1.45 E - 3	1.37 E - 3	1.28	12.6	9.80 E - 2	677.
50.	90	6.28 E - 3	6.44 E - 3	6.36 E - 3	0.274	5.83	1.02 E - 1	1,530.
100.	90	1.23 E - 2	1.24 E - 2	1.23 E - 2	0.141	4.28	1.07 E - 1	2,190.

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