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INFORMATION ANALYSIS OF DYNAMICAL RADAR PARAMETERS.(U)

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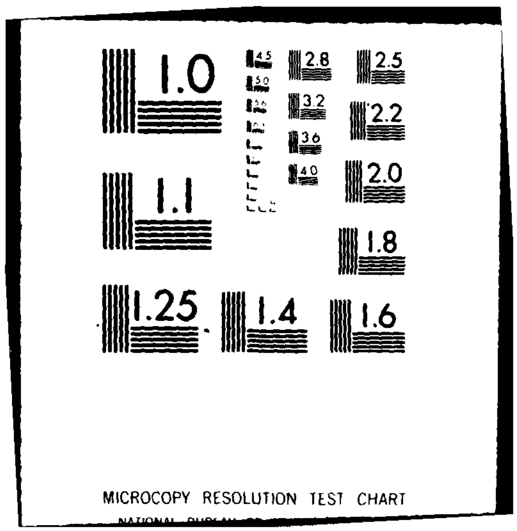
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INFORMATION ANALYSIS OF DYNAMICAL RADAR PARAMETERS

AD A088195

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1 April 1980

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35809

Prepared for:

Advanced Sensors Directorate
U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TR-RE-CR-80-12	2. GOVT ACCESSION NO. AD-A088 195	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) INFORMATION ANALYSIS OF DYNAMICAL RADAR PARAMETERS,		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER 80-036
7. AUTHOR(s) Dr. H. Vincent Poor		8. CONTRACT OR GRANT NUMBER(s) DAAK40-79-D-0018
9. PERFORMING ORGANIZATION NAME AND ADDRESS M&S Computing, Inc. P.O. Box 5183 Huntsville, Alabama 35805		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Order 006
11. CONTROLLING OFFICE NAME AND ADDRESS Commander U. S. Army Missile Command DRSMI-RPT Redstone Arsenal, Alabama 35809		12. REPORT DATE 1 April 1980
		13. NUMBER OF PAGES 20 (12) 23
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Commander U. S. Army Missile Command Attn: DRSMI-REO Redstone Arsenal, Alabama 35809		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Millimeter Wave Radar/Sensors
Signal Modeling
Target Recognition System

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The extension of the classical information analysis of radar systems to more general situations arising in modern radar practice is considered in this report. In particular, the introduction of dynamics into parameter modeling is considered in an information theoretical context. Such a treatment is motivated by the need to consider parameter dynamics in discrimination-mode radar signal processor design. Expected applications of the proposed techniques include the development of theoretical performance

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limitations which can be used as benchmarks to which the performance of practical systems can be compared.

PREFACE

This report presents an extension of the classical information analysis of radar systems to the more general situations arising in modern radars. In particular, the introduction of dynamics into parameter modeling is considered. This effort was performed for the Advanced Sensors Directorate, U. S. Army Missile Technology Laboratory, U. S. Army Missile Command, Redstone Arsenal, Alabama, under Contract No. DAAK40-79-D-0018, Work Order 006.

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Dr. H. Vincent Poor

Approved by:

Glenn D. Weathers

Dr. Glenn D. Weathers

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I. INTRODUCTION

Information analysis is a useful tool in the characterization of radar system performance. This type of analysis was first applied by Woodward and Davies in their now classical study of the information-gathering capabilities of matched-filter radar receivers with respect to the range parameters.[1,2] In modern radars, target discrimination plays an important role, and thus the consideration of other target signature parameters is necessary. A recent study of the extension of the Woodward-Davies approach to more general parameter sets is reported in Poor.[3] An important aspect of this problem which has not been considered previously is the treatment of time-varying parameters. Since many of the parameters of interest in the discrimination mode of a radar are dynamical in nature, the study of such parameters in that context is of interest.

In this report we consider the application of the notions of continuous-time information theory to the analysis of the information-gathering capabilities of radars with respect to dynamical parameters. Section II presents the basic definitions of interest in this problem and includes a discussion of the earlier studies of radar information analysis for nondynamical signature parameters. In Section III the analysis of dynamical signature parameters when observed in white Gaussian noise is considered. Results are presented for a general model which can be applied to parameters such as target vibration. Section IV considers the problem of characterizing the rate at which radar observations generate information about parameters, and it is seen that, within mild restrictions, this rate can be determined from the spectral of the signature parameters. A discussion of several extensions of this work is included in Section V.

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1. P. M. Woodward and I. L. Davies, "A Theory of Radar Information," *Phil. Mag.*, Vol. 41, 1950, pp. 1001-1017.

2. P. M. Woodward, *Probability and Information Theory, with Applications to Radar*, McGraw-Hill, New York, 1955.

3. H. V. Poor, *Information and Ambiguity in Millimeter-Wave Radar: Characterization and Signal Modeling*, U.S. Army Missile Command, Contractor Report TR-RE-CR-80-11, M&S Computing, Inc., Huntsville, Alabama, January 1980.

II. BACKGROUND

A. The Information Theoretical Model

As a general model, consider the configuration depicted in Figure 1. This model consists of an "observation" Y which is the output of the "channel" resulting from the input "message" X. Associated with the message X is a quantity of information denoted by H(X) and termed the entropy of X. Also associated with X and Y is a quantity of mutual information, denoted by I(Y;X), which is the amount of information about X contained in Y. This latter quantity is also known as the information gain of the channel and is a measure of the information-gathering capability of the channel with respect to X. Thus we have a perfect channel if I(Y;X) = H(X) and a useless channel if I(Y;X) = 0.

As an example, consider the finite-alphabet case in which X can take one of n possible values $\{x_1, x_2, \ldots, x_n\}$ and Y can take one of m possible values $\{y_1, y_2, \ldots, y_m\}$. The probabilistic nature of X and Y can be determined completely in this case by their joint probability mass function P_{XY} defined by

$$P_{XY}(x_i, y_j) = \text{Prob}\ \{X = x_i \text{ and } Y = y_j\}; \quad \begin{array}{l} i = i,2,\ldots,n \\ j = i,2,\ldots,m. \end{array} \tag{1}$$

Similarly, X can be described by its marginal probability mass function P_X given by

$$P_X(x_i) = \text{Prob}\ \{X = x_i\} = \sum_{j=1}^{m} P_{XY}(x_i, y_j); \quad i = 1,2,\ldots,n, \tag{2}$$

and Y by its marginal

$$P_Y(Y_j)\ \text{Prob}\ \{Y = y_j\} = \sum_{i=1}^{n} P_{XY}(x_i, y_j); \quad j = 1,2\ldots,m \quad . \tag{3}$$

For this situation the entropy of X is given by

$$H(X) = -\sum_{i=1}^{n} P_X(x_i) \log\ [P_X(x_i)], \tag{4}$$

and the mutual information of X and Y is given by

$$I(Y;X) = \sum_{i=1}^{n} \sum_{j=1}^{m} P_{XY}(x_i, y_j) \log \left[\frac{P_{XY}(x_i, y_j)}{P_X(x_i)\ P_Y(y_j)} \right] \quad . \tag{5}$$

Figure 1. General information theoretical configuration.

Similarly, for X and Y continuous-valued or vector-valued, H(X) and I(Y;X) can be defined in terms of the joint and marginal statistical properties of X and Y. A thorough treatment of these ideas is found in Gallagher.[4]

B. Information Analysis of Radar Systems

To apply the model of Figure 1 to the radar problem, we consider the "message" X to be the radar signature of a target of interest and the observation Y to be either (a) the radar return, or (b) the output of a radar signal processor. Thus, in case (a), the mutual information I(Y;X) determines the total amount of information available about the radar signature; and, in case (b), I(Y;X) measures the ability of the radar to gather this information.

The first application of these ideas to the radar problem is due to Woodward and Davies[1] in which the quantity X is the range delay of a target and the observation Y is the output from a matched-filter receiver. Assuming that the radar return is observed in additive white Gaussian noise and that the range delay is uniformly distributed over the delay-gate interval [0,T], Woodward and Davies showed that

$$I(Y;X) \cong \begin{cases} \log\left(\rho\beta T/\sqrt{2\pi e}\right); & \text{if } A \cong 0 \\ (\rho^2+1)/2 - \log\left(\rho\sqrt{2\pi}\right); & \text{if } A \cong 1 \end{cases}$$

where ρ is the signal-to-noise ratio, β is the signal bandwidth and A is an ambiguity parameter.

Two extensions of the Woodward and Davies approach are necessary in order to apply this analysis to more general radar problems. The first is to apply this approach to more general parameters and parameter sets, and the second is to generalize the approach to the dynamical case in which the parameters are time varying within the observation interval. The first of these two extensions is discussed in a recent report.[3] In the following sections the second extension for general dynamical parameter models is considered.

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4. R. G. Gallagher, *Information Theory and Reliable Communication*, John Wiley & Sons, New York, 1968.

III. INFORMATION GAIN FOR DYNAMICAL RADAR PARAMETERS

A. I(Y;X) for Noisy Observation of a Random Process

In the discrimination mode of a radar, many signature parameters are time-varying within the observation interval. Since the dynamics of most parameters are target dependent, this dynamical behavior might be exploited to improve discrimination performance. Thus, we would like to generalize the adynamical approach discussed in Section II to the dynamical case. To do so we assume now that the signature parameter X is a random process, i.e., that

$$X = \{X(t);\ 0 \le t \le T\} \tag{6}$$

where $[0,T]$ is the observation interval. We also assume that the observation Y consists of a noisy observation of X; i.e., we have $Y = \{Y(t);\ 0 \le t \le T\}$ where

$$Y(t) = X(t) + N(t);\ 0 \le t \le T, \tag{7}$$

and where $\{N(t);\ 0 \le t \le T\}$ represents Gaussian white noise, independent of X, with spectral height N_0.

For the model of Equation (7) we would like to determine the mutual information I(Y;X). However, the determination (or even the definition) of this quantity is not as simple as for the finite-alphabet case noted in subsection IIA. The treatment of information theoretical quantities for continuous-time random processes is discussed in detail in the book by Pinsker.[5] The reader is referred to this reference for details of this analysis. The model of Equation (7) has been studied by Duncan,[6] and it can be shown that for this case

$$I(X;Y) = \frac{1}{2N_0} E \left\{ \int_0^T |X(t) - \hat{X}(t)|^2 \, dt \right\} \tag{8}$$

where $E\{\cdot\}$ denotes statistical expectation and where $\hat{X}(t)$ is the minimum-mean-square-error (MMSE) estimate of X(t) based on observing $\{Y(\tau);\ 0 \le \tau \le t\}$. Note that it can be shown that (see, for example, Wong[7])

$$\hat{X}(t) = E\{X(t) | Y(\tau);\ 0 \le \tau \le t\} \tag{9}$$

where $E\{\cdot|\cdot\}$ denotes conditional expectation.

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5. M. S. Pinsker, *Information and Information Stability of Random Variables and Processes*, Holden-Day, San Francisco, 1964.

6. T. E. Duncan, "On the Calculation of Mutual Information," *SIAM J. Appl. Math.*, Vol. 19, 1970, pp. 215-220.

7. E. Wong, *Random Processes in Information and Dynamical Systems*, John Wiley & Sons, New York, 1980.

Equation (8) indicates that, for the model of Equation (7), the mutual information $I(Y;X)$ is one-half of the normalized mean-integrated-square error associated with the MMSE estimate of $X(t)$ based on $\{Y(\tau);\ 0 \leq \tau \leq t\}$. The problem of determining these latter quantities has been studied extensively for the situation in which the process X is generated by a stochastic dynamical equation. A summary of these techniques is found in Gelb, et al.[8] In the following subsection we consider the application of the formula of Equation (8) to the specific case of target vibration.

B. Information Gain for the Vibration Signature

An important discriminant for surface-to-surface radar application is the vibration signature of the target. It has been demonstrated that the primary effect of target vibration on the radar return is to produce a frequency modulation.[9] Thus, we consider the observation configuration depicted in Figure 2. Recall that a discriminator is a device which extracts frequency information from a waveform. Because of imperfect operation of the discriminator, thermal noise, and other channel noise, we may assume that the discriminator output $Y = \{Y(t);\ 0 \leq t \leq T\}$ is a noisy version of the target vibration process $X = \{X(t);\ 0 \leq t \leq T\}$. Assuming the white-noise model of Equation (7) we can thus compute $I(Y;X)$ for the configuration of Figure 2.

The vibration process X can be modeled as a superposition of the effects of the various vibrational modes of the target resulting from excitation by the vibrating source (i.e., the motor). Thus, we can write

$$X(t) = \sum_{i=1}^{n} c_i\, Z_i(t);\ 0 \leq t \leq T \tag{10}$$

where $Z_i(t);\ i=1,2,\ldots,n$ are components due to the vibrational resonant modes of the target, and the $c_i;\ i=1,2,\ldots,n$ are weights reflecting the relative effects of the various vibrational modes on the waveform modulation. The generating mechanism of the vibrational modes can be modeled as a linear stochastic multivariable dynamical system; that is, $\underline{Z}(t) = [Z_1(t), Z_2(t),\ldots,Z_n(t)]^T$ satisfies the vector differential equation*

8. A. Gelb, et al., *Applied Optimal Estimation*, MIT Press, Cambridge, Massachusetts, 1974.

9. P. M. Alexander, *A Theoretical Analysis of Characteristic Radar Signals from Vibrating Targets*, Technical Note No. T-79-14, Advanced Sensors Directorate, U.S. Army Missile Research and Development Command, Redstone Arsenal, Alabama, April 1979.

* Note that a superscript T denotes transposition.

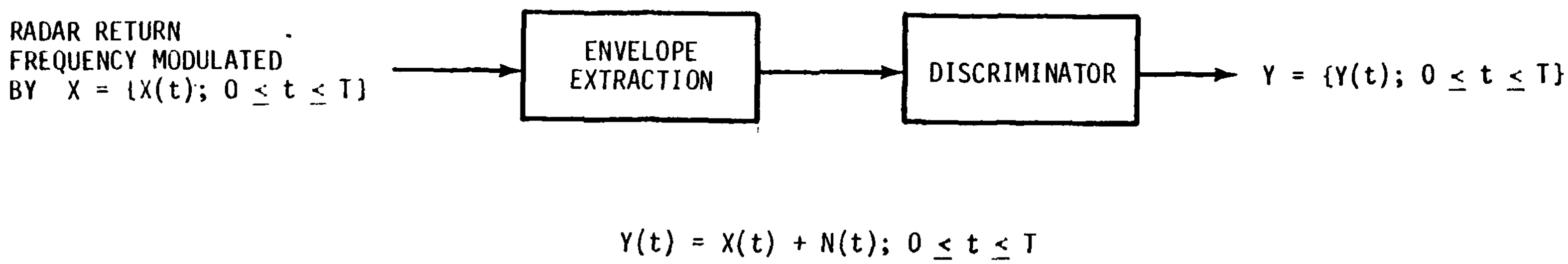

Figure 2. Observation model for target vibration.

$$\frac{d\underline{Z}(t)}{dt} = \underset{\sim}{A}\ \underline{Z}\ (t) + \underline{u}(t);\ 0 \leq t \leq T \tag{11}$$

where $\underline{u}(t)$ is the vibration driving process, and the system matrix $\underset{\sim}{A}$ represents the model response of the target. It can be assumed that the driving process $\{\underline{u}(t),\ 0 \leq t \leq T\}$ is a vector white Gaussian process with zero mean and autocorrelation matrix

$$E\ \{\underline{u}(t)\ \underline{u}^T\ (s)\} = \underset{\sim}{Q}\ \delta(t-s) \tag{12}$$

where $\underset{\sim}{Q}$ is an n x n covariance matrix, and $\delta(t-s)$ denotes the Dirac delta function. We may also assume that the initial condition vector $\underline{Z}(0)$ is Gaussian.

Note that our observation model is given by

$$Y(t) = \underline{c}^T\ \underline{Z}(t) + N(t);\ 0 \leq t \leq T \tag{13}$$

where $\underline{c}^T = [c_1, c_2, \ldots, c_n]$. Thus since $X(t) = \underline{c}^T\underline{Z}(t)$, we have the MMSE estimate,

$$\begin{aligned} \hat{X}(t) &= E\ \{X(t)\,|\,Y(\tau);\ 0 \leq \tau \leq t\} \\ &= E\ \{\underline{c}^T\underline{Z}(t)\,|\,Y(\tau);\ 0 \leq \tau \leq t\} \\ &= \underline{c}^T\ E\ \{\underline{Z}(t)\,|\,Y(\tau);\ 0 \leq \tau \leq t\} \end{aligned} \tag{14}$$

where we have used linearity of the expectation operator. We note now that the quantity

$$\hat{\underline{Z}}(t) = E\ \{\underline{Z}(t)\,|\,Y(\tau);\ 0 \leq \tau \leq t\} \tag{15}$$

is the minimum-mean-norm-error estimate of $\underline{Z}(t)$ based on $\{Y(\tau);\ 0 \leq \tau \leq t\}$, which for this model [Equations (11) and (13)] is given by the Kalman-Bucy filter.[10]

Referring to Equation (8), we have

$$\begin{aligned} I(Y;X) &= \frac{1}{2\ N_0}\ E\left\{\int_0^T |\ X(t) - \hat{X}(t)\,|^2\ dt\right\} \\ &= \frac{1}{2\ N_0}\ E\left\{\int_0^T |\underline{c}^T\ \underline{Z}\ (t) - \underline{c}^T\ \hat{\underline{Z}}(t)\,|^2\ dt\right\} \\ &= \frac{1}{2\ N_0}\ E\left\{\int_0^T \underline{c}^T\ [\underline{Z}(t) - \hat{\underline{Z}}(t)]\,[\underline{Z}(t) - \hat{\underline{Z}}(t)]^T\underline{c}\ dt\right\} \end{aligned}$$

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10. R. E. Kalman and R. Bucy, "New Results in Linear Filtering and Prediction," *ASME J. Basic Engineering*, Vol. 83D, 1961, pp. 95-108.

$$= \frac{1}{2 N_0} \int_0^T \underline{c}^T \underset{\sim}{P}(t) \, \underline{c} \, dt, \qquad (16)$$

where

$$\underline{P}(t) = E \{ [\underline{Z}(t) - \hat{\underline{Z}}(t)] [\underline{Z}(t) - \hat{\underline{Z}}(t)]^T \}. \qquad (17)$$

Note that $\underset{\sim}{P}(t)$ is the error covariance matrix for the Kalman-Bucy filter and thus satisfies the matrix Riccati equation [10]

$$\frac{d\underset{\sim}{P}(t)}{dt} = \underset{\sim}{A} \, \underset{\sim}{P}(t) + \underset{\sim}{P}(t)\underset{\sim}{A}^T + \underset{\sim}{Q} - N_0^{-1} \underset{\sim}{P}(t)\underline{c} \, \underline{c}^T \underset{\sim}{P}(t); \; 0 \leq t \leq T \qquad (18)$$

with initial condition $\underset{\sim}{P}(0)$ equal to the convariance matrix of $\underline{Z}(0)$. Note that $\underset{\sim}{A}$ is from Equation (11), $\underset{\sim}{Q}$ is from Equation (12), and N_0 is the spectral height of the white observation noise. The solution to the matrix Riccati equation of Equation (18) is discussed in Reference 8, pp. 136-142. Thus we see that Equations (16) and (18) allow for the computation of $I(Y;X)$ for the vibration signature model of Equations (11) and (13).

C. Information Gains for Other Dynamical Parameters

Note that the procedure outlined in subsection IIIB can be applied to any parameter which has the dynamical structure described by Equation (11). Signature parameters are normally present in the form of modulations on the radar return and dynamical models such as Equation (11) for waveform modulation have been studied extensively (see, for example, Snyder[11]). Note also that vector dynamical parameters can also be treated in this manner, as follows.

Suppose that $X = \{\underline{X}(t);\ 0 \leq t \leq T\}$ is a vector random process (say, m - dimensional) and $\overline{Y}$ is a vector observation process given by

$$\underline{Y}(t) = \underline{X}(t) + \underline{N}(t); \; 0 \leq t \leq T \qquad (19)$$

where $\underline{N}(t)$ is a vector Gaussian white noise (independent of X) with autocorrelation matrix

$$E \{\underline{N}(t) \, \underline{N}^T(s)\} = N_0 \, \underset{\sim}{I} \, \delta(t-s) \qquad (20)$$

where $\underset{\sim}{I}$ is the n x n identity matrix. Then it can be shown that (see Reference 6)

11. D. L. Snyder, *The State-Variable Approach to Continuous Estimation*, MIT Press, Cambridge, Massachusetts, 1969.

$$I(Y;X) = \frac{1}{2N_0} E \left\{ \int_0^T [\underline{X}(t) - \hat{\underline{X}}(t)]^T [\underline{X}(t) - \hat{\underline{X}}(t)]\, dt \right\} \tag{21}$$

where

$$\hat{\underline{X}}(t) = E\{\underline{X}(t) | \underline{Y}(\tau);\ 0 \leq \tau \leq t\}. \tag{22}$$

If we have

$$\underline{X}(t) = \underset{\sim}{C}\, \underline{Z}(t);\ 0 \leq t \leq T \tag{23}$$

where $\underline{Z}(t)$ is generated as in Equation (11) and where $\underset{\sim}{C}$ is an m x n matrix, then it is straightforward to show that I(Y;X) will be given by

$$I(Y;X) = \frac{1}{2N_0} \int_0^T \mathrm{tr}\, \{\underset{\sim}{C}\, \underset{\sim}{P}(t) \underset{\sim}{C}^T\}\, dt \tag{24}$$

where $\underset{\sim}{P}(t)$ is defined by Equation (17) and tr $\{\cdot\}$ denotes the trace operator. For this case $\underset{\sim}{P}(t)$ will be given by Equation (18), with the final term replaced by

$$N_0^{-1}\, \underset{\sim}{P}(t)\, \underset{\sim}{C}^T\, \underset{\sim}{C}\, \underset{\sim}{P}(t) \tag{25}$$

where N_0 is the scale of the observation correlation matrix from Equation (20).

The observation model of Equation (7) can be also generalized to include parameter extraction systems with feedback. In particular, we can consider the model

$$Y(t) = \phi[Y_0^{\ t};\ X\ ;\ t] + N(t);\ 0 \leq t \leq T, \tag{26}$$

where $Y_0^{\ t} = \{Y(\tau);\ 0 \leq \tau \leq t\}$ and where $\phi[\cdot;\cdot;\cdot]$ is an arbitrary parameter extraction system. The presence of $Y_0^{\ t}$ allows feedback systems (such as phase-lock loops) to be included in the model. Note also that ϕ can be a nonlinear functional of X and can change with time. Here, as before, $\{N(t);\ 0 \leq t \leq T\}$ is white Gaussian noise. The mutual information or information gain for this extraction system can be shown to be [12]

$$I(Y;X) = \frac{1}{2N_0} E\cdot \left\{ \int_0^T [\phi\ [Y_0^{\ s};\ X;\ s] - \hat{\phi}\ [Y_0^{\ s};\ s]]^2\, ds \right\} \tag{27}$$

where

$$\hat{\phi}\ [Y_0^{\ s};\ s] = E\{\phi[Y_0^{\ s};\ X; s] | Y_0^{\ s}\}. \tag{28}$$

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12. T. T. Kadota, M. Zakai, and J. Ziv, "Mutual Information of the White Gaussian Channel With and Without Feedback," *IEEE Trans. Inform. Theory*, Vol. IT-17, 1971, pp. 368-371.

Note that this result is much more general than that of Equation (8). It should be noted, however, that the exact computation of Equation (27) may be difficult for nonlinear problems. However, if X is generated by a dynamical system (linear or nonlinear), the expression of Equation (27) can be approximated by considering the extended Kalman filter.[11]

IV. THE RATE AT WHICH INFORMATION IS PRODUCED BY RADAR OBSERVATIONS

A. Introduction

Suppose again that the signature parameter X is a random process $\{X(t);\ 0 \leq t \leq T\}$ and that the observation Y is a random process $\{Y(t);\ 0 \leq t \leq T\}$. The quantity $I(Y;X)$ is a measure of how much information Y contains about X. Thus, the rate at which this information is produced is given by $[I(Y;X)/T]$. To get a single number for the rate, independent of the length T of the observation interval, we can consider the following definition for information rate:

$$R(Y;X) = \lim_{T\to\infty} \left[\frac{I(Y;X)}{T}\right] \quad . \tag{29}$$

Thus $R(Y;X)$ is a measure of the rate at which the radar observation Y produces information about the signature parameter X and will give us an idea of how long we must observe a target to get a given amount of information about X. In this section we consider the computation of $R(Y;X)$ for some general observation models.

B. Information Rate for the Vibration Signature

Before considering the information rate for general models, we first will consider the vibration signature model discussed in subsection IB. Using Equations (16) and (29) we see that

$$R(Y;X) = \lim_{T\to\infty} \left\{\frac{1}{2\ N_0\ T} \int_0^T \underline{c}^T\ \underset{\sim}{P}(t)\ \underline{c}\ dt\right\}. \tag{30}$$

Since the vibrational mode matrix $\underset{\sim}{A}$ of Equation (11) represents a passive mechanical system (the source enters the system through $\{\underline{u}(t);\ 0 \leq t \leq T\}$), the vibrational modes $\{\underline{Z}\ (t);\ 0 \leq t \leq T\}$ will achieve a statistical steady state as $T\to\infty$, and the error covariance matrix $\underset{\sim}{P}(t)$ will become constant as $T\to\infty$. In particular, we will have

$$\lim_{T\to\infty} \underset{\sim}{P}(t) = \underset{\sim}{P} \tag{31}$$

where $\underset{\sim}{P}$ is the solution to the steady-state Riccati equation,

$$\underset{\sim}{0} = \underset{\sim}{A}\ \underset{\sim}{P} + \underset{\sim}{P}\ \underset{\sim}{A}^T + \underset{\sim}{Q} - N_0^{-1}\ \underset{\sim}{P}\ \underset{\sim}{C}\ \underset{\sim}{C}^T\ \underset{\sim}{P}\ . \tag{32}$$

Here $\underset{\sim}{0}$ denotes the n x n matrix with all zero entries.

Using Equations (30) and (31), we then have that

$$R(Y;X) = \frac{1}{2\ N_0}\ \underline{c}^T\ \underset{\sim}{P}\ \underline{c} \tag{33}$$

where $\underset{\sim}{P}$ is from Equation (32). Equation (33) can be written in a more useful form. In particular, we note that Equation (33) represents one-half of the normalized steady-state Wiener filtering error for the model of Equation (7). Expressions for this error have been derived by several authors, and the error is given for this case by (see, for example, Reference 11, p. 43)

$$\underline{c}^T \underset{\sim}{P} \underline{c} = \frac{N_0}{2\pi} \int_{-\infty}^{\infty} \log\, [1 + S_{XX}\, (\omega)/N_0]\, d\omega \tag{34}$$

where $S_{XX}(\omega)$ is the power spectral density of the parameter process X; that is, $S_{XX}(\omega)$ is the vibration spectrum of the target. Since we must have $S_{XX}(\omega) = S_{XX}(-\omega)$, Equations (33) and (34) give

$$R(Y;X) = \frac{1}{2\pi} \int_{0}^{\infty} \log\, [1 + S_{XX}(\omega)/N_0]\, d\omega. \tag{35}$$

Thus the rate at which the configuration of Figure 2 produces information about the target vibration signature is given in terms of the vibration spectrum by Equation (35). Note that the measurement of vibration spectra of radar targets has been studied by Webb, Emmons, and Curtis.[13]

C. A General Formula for Information Rate

Equation (35) gives a useful formula for the rate at which the system of Figure 2 produces information about the vibration signature of a target. The general theory of the information rates of random processes is treated in detail by Pinsker.[5] More general observation models than that of Equation (7) can be considered in this context, and in this subsection we present a generalization of Equation (35) for such models.

Assume that the signature and observation processes $\{X(t);\ 0 \leq t \leq T\}$ and $\{Y(t);\ 0 \leq t \leq T\}$ are jointly Gaussian and are jointly and individually stationary. Let $S_{XY}(\omega)$ denote the cross power spectrum of X and Y, $S_{XX}(\omega)$ denote the power spectrum of X, and $S_{YY}(\omega)$ denote the power spectrum of Y. A spectrum is said to be <u>rational</u> if it can be written as the ratio of two polynomials in ω. Processes with rational spectra arise naturally from the conventional finite-state linear models for dynamical parameters. If the spectra S_{XY}, S_{XX} and S_{YY} are rational, then the rate R(Y;X) is given by (see Reference 5, pp. 181-182)

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13. W. E. Webb, G. A. Emmons, and R. A. Custis, *Measurement of Vibration Signatures by Means of a CO_2 Laser Radar*, Technical Report RE-77-2, Advanced Sensors Directorate, U.S. Army Missile Command, Redstone Arsenal, Alabama, October 1976.

$$I(Y;X) = \frac{1}{2\pi} \int_0^\infty \log \left[1 - |r_{XY}(\omega)|^2\right] d\omega \qquad (36)$$

where

$$|r_{XY}(\omega)|^2 = \frac{|S_{XY}(\omega)|^2}{S_{XX}(\omega)\ S_{YY}(\omega)} \ . \qquad (37)$$

The expression of Equation (36) also holds when the condition of rational spectra is relaxed to a more general condition (see Reference 5, pp. 182).

Equations (36) and (37) give a formula for the rate at which Y produces information about X in terms of the spectral properties of X and Y. To illustrate this result, let us consider the observation model of Equation (7); that is,

$$Y(t) = X(t) + N(t);\ 0 \le t \le T \qquad (38)$$

where $\{X(t);\ 0 \le t \le T\}$ and $\{N(t);\ 0 \le t \le T\}$ are orthogonal. In this case, we have

$$|S_{XY}(\omega)|^2 = S_{XX}{}^2(\omega) \qquad (39)$$

and

$$S_{YY}(\omega) = S_{XX}(\omega) + S_{NN}(\omega), \qquad (40)$$

where $S_{NN}(\omega)$ is the spectrum of the noise process $\{N(t)\ ;\ 0 \le t \le T\}$. Equation (36) becomes

$$I(Y;X) = \frac{1}{2\pi} \int_0^\infty \log \left[1 + \frac{S_{XX}(\omega)}{S_{NN}(\omega)}\right] d\omega, \qquad (41)$$

which generalizes Equation (35).

V. SUMMARY AND DISCUSSION

In this report we have considered the generalization of the Woodward-Davies information analysis to the case of general target signature parameters exhibiting dynamical behavior. We see from the results of Section III that the information gained by radar observation of a dynamical parameter can be computed from the underlying dynamical structure of the parameter. Similarly, it was demonstrated in Section IV that the rate at which radar observations produce information about a dynamical parameter is determined by the spectral properties of the parameter and of the noise introduced by the channel.

A number of potentially useful extensions of the ideas are presented in this report. For example, we can also consider measures of the value of a given parameter as a discriminant between targets. Information theoretical measures of discrimination ability (e.g., the I-divergence) can be computed using techniques similar to those used for computing mutual information. Previous work relevant to this aspect of information analysis is found in two papers by Schweppe.[14,15] Again, the dynamical and spectral properties of the parameters can be exploited.

Other useful topics of interest include the design and implementation of experiments for the determination of parameter dynamics and spectral properties and the study of the signal processing required to achieve performance near the fundamental limits.

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