

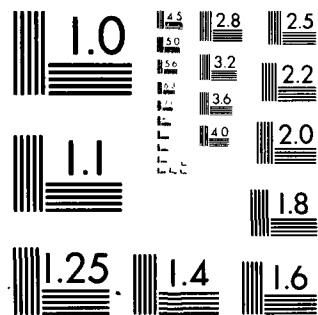
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AN ANALYSIS OF INTERLAMINAR STRESS GRADIENTS AND IMPACT DAMAGE IN GRAPHITE-EPOXY LAMINATES

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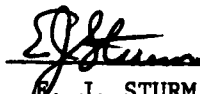
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LIST OF SYMBOLS

<u>Letter</u>	<u>Definition</u>
$(A_n)_{ij}$	Integrated ply property moments; $n = 0,1,2$.
$\bar{B}_{i\alpha}$	Composite stress-curvature stiffness constants from classical 3D lamination theory.
$\bar{C}_{ij}$	Composite stress-strain stiffness constants from classical 3D lamination theory.
$C_{ij}^*(\xi)$	Composite stress-strain parametric stiffness coefficients for 3D finite element laminate modeling.
CCL, etc.	Three character mnemonic defining 3D finite elements where C = cubic, L = linear indicate displacements in the coordinate directions associated with the character position.
$\bar{D}_{\alpha\beta}$	Composite moment-curvature stiffness constants from classical 3D lamination theory.
$\underline{e}_i$	Orthonormal Cartesian bases vectors.
E_{ij}	Orthotropic material extensional moduli.
$F_i(\xi)$	Parametric cubic Hermite functions.
G_{ij}	Orthotropic material shear moduli.
P_{ijk}	Tricubic Hermite coefficients in point format.
R, r	Radial coordinate.
$S_{ij}^*(\xi)$	Composite strain-stress parametric flexibility coefficients for 3D finite element laminate model.
S_m	Cubic Hermite coefficients in algebraic format.
T_{mn}	Point format cubic transformation matrix.
Z_i	Cartesian coordinate.
<u>Greek Letter</u>	<u>Definition</u>
δ_i	Coefficients in the solution of the contact problem for transversely isotropic materials.
ϵ_i^*	Composite strain component.
κ_α	Laminate curvatures in the surface coordinate directions of the composite material.

Greek LetterDefinition ν_{ij}

Orthotropic material Poisson ratios.

 σ_i^*

Composite stress component.

 ξ_i Parametric coordinate $0 \leq \xi_i \leq 1$.

1. INTRODUCTION AND SUMMARY

The sensitivity of laminates to low velocity impact damage is a practical design problem that the ASTM and others have been investigating for several years. One of the difficulties in studying impact response is predicting the complex interlaminar stress gradients that it produces in composite structures. This report presents a systematic approach to interlaminar stress gradient modeling with applications to a graphite-epoxy laminate impacted by a steel sphere and a laminate with an internal disbond. The approach is based on a family of variable property finite elements for laminate models that transition to discrete ply models for regions with interlaminar stress gradients. All 45 elastic constants for a general anisotropic laminate are simulated and considerable error is shown to occur when uniform properties are used with certain laminate models. Careful modeling of the force-deformation behavior is required to predict accurate boundary conditions in the impact region when bending is present. Numerical results are presented for a thin 8 ply laminate and a thick 48 ply laminate.

The report first develops two new finite element modeling tools for interlaminar stress analysis and validates them using control problems with known solutions. The new analysis tools are composite property models that vary through the thickness to correctly model bending and linear element constraints for lamination theory force-deformation modeling. These tools are then used to analyze a thin laminate with an internal disbond and a thick laminate impacted at low velocity by a steel sphere. The disbond produces a locally unbalanced laminate that under biaxial compression warps badly with large out-of-plane displacements, but with very little internal force redistribution. These results were obtained from a model using two planes of symmetry that were found to introduce fictitious constraint forces between 0 and 45 degree plies at the laminate center. Results for the laminate impact problem were obtained from a complete 360 degree model of the impact site. A structural model of the entire laminate was used to obtain displacement boundary conditions for a discrete model of the impact site. These results show large subsurface shears at the perimeter of the contact area in the second ply below the surface. Only the top four plies were modeled individually to reduce costs, but more detailed models were prepared for later solution.

2. INTERLAMINAR STRESS FINITE ELEMENT MODELING

Recently finite element modeling approaches have been suggested for the determination of interlaminar stresses in those problems that cannot be solved using lamination theory [1,2,3]. All of these involve some form of ply-by-ply modeling and an appeal to St. Venants principle to allow a transition to composite laminate modeling away from the region of interlaminar stress gradients. The issues affecting the accuracy and efficiency of computational models for this class of problems are reviewed in this section and a systematic approach is presented for use with PATCHES-III. First a variable property simulation of laminate force-deformation behavior is introduced that can represent all 45 elastic constants necessary to characterize general anisotropic laminate deformations [4]. Next a family of linear constraint finite elements are developed from the PATCHES-III tricubic finite element to model the deformations of general laminates. The final and critical development for interlaminar stress gradient analysis is a procedure to transition from discrete 3D ply modeling to 2D+ composite laminate modeling. The features of this new system are then tested using the edge effect demonstration problem and a collection of balanced and unbalanced laminate problems.

2.1 3D COMPOSITE PROPERTIES FOR LAMINATE FORCE-DEFORMATION BEHAVIOR

Pagano [3] has shown that the forces and moments in a general anisotropic laminate are related to laminate strains and curvatures by 45 elastic constants in the equations

$$\left. \begin{aligned} \sigma_i^* &= \bar{C}_{ij} \epsilon_j^* + \bar{B}_{i\alpha} \kappa_\alpha \\ M_\beta/h^2 &= \bar{B}_{j\beta} \epsilon_j^* + \bar{D}_{\beta\alpha} \kappa_\alpha \end{aligned} \right\} \begin{array}{l} \text{for } 1 \leq i, j \leq 6 \\ \alpha, \beta = 1, 2, 4 \end{array} \quad (2.1)$$

which the PATCHES-III contracted convention is used with ϵ_4 defined as the in-plane shear strain. These equations reduce to the classical plate theory for laminates with monoclinic plies when $\sigma_3 = 0$. Once the volume average strains ϵ_i^* and laminate curvatures κ_α have been found, the individual ply stresses can be determined using ply properties C_{ij} and the relations developed by Pagano [4].

The elastic constants in Equation (2.1) are functions of the integrated moments of the ply properties

$$\left[(A_0)_{ij}, (A_1)_{ij}, (A_2)_{ij} \right] = \int_{-h/2}^{h/2} \left[1, z_3, z_3^2 \right] c_{ij}(z_3) dz_3 \quad (2.2)$$

where the exact expressions for the $\bar{C}_{ij}$, $\bar{B}_{i\alpha}$, $\bar{D}_{\alpha\beta}$ involve transformations arising from the assumption σ_3 , σ_5 , and σ_6 are each constant through the thickness. The approach developed in the present study uses these same moments to define a variable property composite material in the thickness direction with the same force-deformation behavior. When a finite element of this material is subject to the same volume average strains, ϵ_i^* , and curvatures κ_α as a laminate modeled using Equation (2.1), the same stress resultants will be produced. The converse of this will not be true in general unless displacement constraints are introduced in the thickness direction to prevent a pseudo edge effect from warping the cross-section. In this study, a linear displacement constraint in the thickness direction is used with the variable property model to define a CCL finite element that predicts the force-deformation behavior of Equation (2.1).

Consider a parametric cubic model for the property variation in the normal direction. This allows up to a sixth degree polynomial in Z_3 when the thickness coordinate function, $Z_3(\xi)$, is also cubic. However, the use of nonuniform geometry models to aid in composite property modeling while feasible [Ref. 5] is not very convenient as it requires solving nonlinear equations. The use of a uniform (i.e., linear) geometry model for $Z_3(\xi)$,

$$Z_3(\xi) = h(\xi - 1/2) \quad (2.3)$$

and a parametric cubic for each elastic constant in the thickness direction,

$$c_{ij}^*(\xi) = (S_m)_{ij} \xi^{4-m} \quad \text{for } 1 \leq m \leq 4 \quad (2.4)$$

is adequate for most laminates. In some extreme cases, it may be necessary to use nonuniform geometry modeling to avoid unreasonable values of $c_{ij}^*(\xi)$ at local points through the thickness. However, it is always possible using

uniform models to match the integrated moments in Equation (2.2) which is the essential requirement for finite element modeling. Substituting this property model into Equation (2.2) results in four linear equations

$$(A_n)_{ij} = h^{n+1} \int_0^1 (\xi-1/2)^n \xi^{4-m} d\xi (S_m)_{ij}$$

for $0 \leq n \leq 3$
 $1 \leq m \leq 4$ (2.5)

These can be expressed in matrix form

$$A_n = T_{nm} S_m \quad (2.6)$$

for any component of $C_{ij}(\xi)$ where

$$T_{nm} = \begin{bmatrix} h/4 & , & h/3 & , & h/2 & , & h \\ 3h^2/40 & , & h^2/12 & , & h^2/12 & , & 0 \\ 7h^3/240 & , & h^3/30 & , & h^3/24 & , & h^3/12 \\ 13h^4/1120 & , & h^4/80 & , & h^4/80 & , & 0 \end{bmatrix} \quad (2.7)$$

In the special case of a homogeneous laminate, $A_1 = A_3 = 0$ with $A_2 = A_0 h^2/12$, the solution of Equation (2.6) results in simply $C_{ij}^*(\xi) = (A_0)_{ij}/h$ as it must. It is possible to simulate A_0, A_1, A_2 with only a quadratic, but the use of the full cubic reduces the number of pathological cases for which $C_{ij}^*(\xi)$ may not be positive definite at all points through the thickness. It is important to note that this is strictly an interpolation problem that can be avoided at the expense of using more interpolation functions.

At this point, there are two issues to be resolved before the approach can be used. First, does the model accurately represent laminate force-deformation behavior, and secondly is there any significant difference with simple constant property models based on the rule of mixtures? The unbalanced laminate analyzed by Pagano [4] for uniaxial loading is used to answer both questions and to illustrate the practical limits of a uniform PC property model.

Consider the three-ply laminate [60/0/-60] with ply properties

$$\begin{aligned} C_{11} &= 210.35 \text{ GPa} & C_{22} &= C_{33} = 22.30 \text{ GPa} \\ C_{12} &= C_{13} = 7.01 \text{ GPa} & C_{23} &= 5.75 \text{ GPa} \\ C_{44} &= C_{55} = 10.34 \text{ GPa} & C_{66} &= 4.14 \text{ GPa} \end{aligned}$$

as taken from Pagano [2.4] where the msi units of that paper are retained for comparison. These ply properties result in the force-deformation properties shown in Table 1 and their inverse shown in Table 2. Under uniaxial load, for example, the laminate twists and stretches with the deformations given by the corresponding column in Table 2. Consider now the representation of this behavior using $C_{ij}^*(\epsilon)$ determined from Equation (2.6). These property functions in PC point format are displayed in Table 3. Note that the extensional moduli were not determined from Equation (2.6), but are the same constants as in Table 1. The reason for this is evident in Figure 1 which shows the PC function for $C_{11}^*(\epsilon)$. The discrete ply C_{11} properties vary so sharply that their standard deviation, 14.59, is larger than their mean value, 13.66. To represent A_0 , A_1 , and A_2 in this case, a uniform PC function must overshoot zero near the upper and lower surfaces which if used would violate positive definite C_{ij} requirements at these points. To avoid this, only A_0 and A_1 were simulated by the property model in Table 3, and as a result the bending-curvature properties (M_1 , M_2 loading) are not correct. All other force-deformation properties should be exact, and in particular the uniaxial case analyzed by Pagano, $\sigma_1^* = 1$, should be modeled correctly. This was tested using one CCL element in PATCHES-III with Table 3 material properties and loaded by a uniform pressure on opposite faces. The computed laminate strains, ϵ_i^* , and curvatures κ_1 , κ_2 , κ_4 were the same as the Pagano results, Table 4. The results from a unit moment, $M_1 = 1.0$, case are also compared in Table 4, and these deformations are much too low. The bending errors are a direct consequence of the bending stiffness error caused by using constant C_{11} and C_{22} properties from the rule-of-mixtures. In this laminate, [60/0/-60], the stiffness error is over 100 percent, and even in a balanced structural laminate, errors of 10 percent to 20 percent are common.

Consider as an example of this behavior the 48 ply laminate analyzed later in this report. The property distribution (c.f. page 34) shows variable

C14 and C24 coupling through the thickness. This is necessary to account for the bending-shearing coupling $(A_2)_{14}$ and $(A_2)_{24}$ in this laminate which is quite strong. Note that representing this behavior is important to correctly predict the 3D deformation response of the laminate in areas of high local bending.

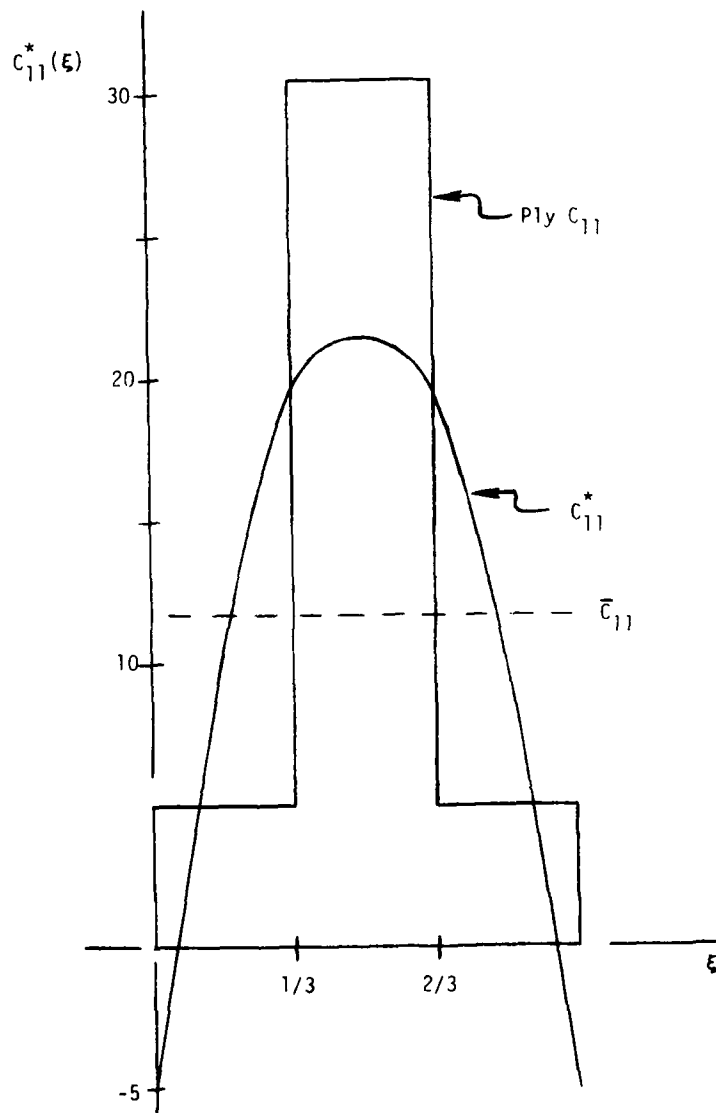


Figure 1. PC Property Modeling Limits

TABLE 1. THREE-PLY LAMINATE FORCE-DEFORMATION PROPERTIES
(10^6 PSI)

	ϵ_1^*	ϵ_2^*	ϵ_3^*	ϵ_4^*	ϵ_5^*	ϵ_6^*	κ_1^*	κ_2^*	κ_4^*
σ_1^*	13.656	4.232	0.925						-0.694
σ_2^*		13.656	0.925						-1.930
σ_3^*			3.234						-0.018
σ_4^*				4.712			-0.689	-1.925	
σ_5^*					0.857				
σ_6^*		(SYM)				0.857			
M_1^*							0.494	0.450	
M_2^*								1.500	
M_4^*									0.512

TABLE 2. THREE-PLY LAMINATE DEFORMATION-FORCE PROPERTIES

	σ_1^*	σ_2^*	σ_3^*	σ_4^*	σ_5^*	σ_6^*	M_1^*	M_2^*	M_4^*
ϵ_1^*	0.082	-.018	-.018						0.042
ϵ_2^*		0.167	-.039						0.602
ϵ_3^*			0.325						0.160
ϵ_4^*				0.453			0.141	0.439	
ϵ_5^*					1.167				
ϵ_6^*		(SYM)				1.167			
κ_1^*							2.829	-.668	
κ_2^*								1.559	
κ_4^*									4.272

TABLE 3. PARAMETRIC CUBIC LAMINATE PROPERTY MODEL*

	$C_{ij}^*(0)$	$C_{ij}^*(1/3)$	$C_{ij}^*(2/3)$	$C_{ij}^*(1)$
C_{11}	13.658	13.658	13.658	13.658
C_{12}	4.231	4.231	4.231	4.231
C_{13}	.926	.926	.926	.926
C_{14}	3.084	1.827	-1.827	-3.084
C_{22}	13.658	13.658	13.658	13.658
C_{23}	.926	.926	.926	.926
C_{24}	8.581	5.085	-5.085	-8.581
C_{33}	3.234	3.234	3.234	3.234
C_{34}	.078	.046	-.046	-.078
C_{44}	8.284	3.523	3.523	8.284
C_{55}	.550	1.217	1.217	.550
C_{56}	.385	.228	-.228	-.385
C_{66}	1.550	.833	.833	1.550

*Only extensional force-deformation properties simulated.

TABLE 4. FORCE-DEFORMATION COMPARISONS

	ϵ_1^*	ϵ_2^*	ϵ_3^*	ϵ_4^*	κ_1	κ_2	κ_4
Exact ($\sigma_1^* = 1$)	.0823	-.0184	-.0184	.0	.0	.0	.0416
CCL ($\sigma_1^* = 1$)	.0822	-.0184	-.0180	.0	.0	.0	.0415
Exact ($M_1^* = 1$)	.0	.0	.0	.141	2.829	-.668	.0
CCL ($M_1^* = 1$)	.0	.0	.0	.065	.978	-.193	.0

*Only extensional force-deformation properties simulated.

2.2 CONSTRAINT FINITE ELEMENTS

The CCL finite element used to model laminate force-deformation behavior is one of a family of linear constraint options developed for the PATCHES-III program. The need for low cost modeling in regions of uniaxial or biaxial strain was noted in an earlier report [6] and the new elements in Table 5 provide this capability. They are based on the linear constraints defined in Equation (2.8)

$$P_i = T_{i\alpha} P_\alpha \quad i = 1, 2, 3, 4$$

$$\alpha = 1, 4 \quad (2.8)$$

where the coefficients $T_{i\alpha}$ are simply

$$T_i = \begin{bmatrix} 1 & , & 0 \\ 2/3 & , & 1/3 \\ 1/3 & , & 2/3 \\ 0 & , & 1 \end{bmatrix} \quad (2.9)$$

The same coefficients apply to all three parametric coordinates and, in general,

$$P_{ijk} = T_{i\alpha} T_{j\beta} T_{k\gamma} P_{\alpha\beta\gamma} \quad (2.10)$$

If constraints are introduced in only two coordinates,

$$P_{ijk} = T_{i\alpha} T_{j\beta} \delta_{kl} P_{\alpha\beta l} \quad (2.11)$$

and for only one constraint

$$P_{ijk} = T_{i\alpha} \delta_{jl} \delta_{km} P_{\alpha lm} \quad (2.12)$$

TABLE 5. PATCHES-III FINITE ELEMENT ADDITIONS

Displacements*	Nodes	Geometry	Properties
LLL	8	CCC	CCC
LLC	16	CCC	CCC
LCC	32	CCC	CCC
CCC	64	CCC	CCC

*Any combination of L and C is available; L = Linear, C = Cubic.

Two key issues affecting the development of the new family of elements are how to efficiently generate their stiffness matrices and how to connect them to each other. After some early confusion, it was determined that all linear constraints can be applied before integration with the same result as when they are applied after integration. This greatly reduces the cost of generating their stiffness matrices. It is simple to demonstrate this equivalence in one dimension where obviously

$$\begin{aligned}
 K_{\alpha\beta}^P &= \int T_{i\alpha} F_i(\xi) F_j(\xi) T_{j\beta} d\xi \\
 &= T_{i\alpha} \int F_i(\xi) F_j(\xi) d\xi T_{j\beta} \\
 &= T_{i\alpha} K_{ij}^P T_{j\beta}
 \end{aligned} \tag{2.13}$$

but in higher dimensions interpolatory quadrature is used in PATCHES-III and this caused some concern at first.

The second issue was resolved by automating the generation of interface constraints between elements of different dimension. This allows the user of PATCHES-III to specify linear constraints on any element or group of elements by simply placing a mnemonic of the type listed in Table 5 on the connectivity card for that element. The program first generates all explicit mesh point constraints and then on a second pass generates all interface or implicit constraints. This requires extensive checking for conflicts, and in order to reduce their incidence, all three displacement components are constrained alike. At every constrained mesh point, one of the Equations (2.10)-(2.12) is automatically generated and applied to all affected matrices.

2.3 INTERLAMINAR MODELING

The use of substructuring to transition from discrete ply molding to composite laminate modeling was recently investigated by Wang and Crossman [2]. The same technique has been used for other composites under the name "minimechanics" [7] and, of course, it has been used for years in aircraft structural analysis. Substructuring in the case of laminates must account for two transition conditions: one along a plane defined by a ply, and the other along a plane normal to the laminate. The first case requires no transition in the shape of the finite element mesh and appears to work well for edge

effects caused by uniaxial loading [2]. Substructuring for the second case will require mesh changes, as well as the transition to composite properties. An approach using the new CCL finite element with laminate properties is evaluated in this section.

Consider the interlaminar stress problem analyzed in the PATCHES-III User's Manual. First, the accuracy of the new constraint finite elements will be demonstrated for the same four element model using the original properties. The two interior elements were constrained to be all linear (LLL) and the two edge elements were constrained linear in the direction of the load (LCC). No significant change in the earlier results should occur because the displacement solution has the same form in these regions. Normal stress comparisons in Table 6 show this to be true even though the model has been reduced to only 92 degrees-of-freedom. In fact, the assumption of linear displacements in the thickness direction (LCL) also was used with very little mid-surface normal stress error as comparisons in Figure 2 show. Interlaminar stress gradients between the 0° and 90° plies, however, are drastically changed in the LCL model as the shear stress comparisons in Table 7 show. Note that the interlaminar shear stress appears to contain a singularity at the free-edge between the 0° and 90° plies, $\sigma_{23}(0,b,h)$. The uniform mesh model gives the mean value of σ_{23} , but one element cannot model this response and return to zero at the free-edge. A nonuniform mesh element does a better job of matching this condition and produces about as much accuracy as can be expected from a single LCC element.

TABLE 6. INTERLAMINAR NORMAL STRESS COMPARISONS*

$\xi/2h$ **	CCC/CCC (428 D.O.F.)	LLL/LCC (92 D.O.F.)	LLL/LCL (44 D.O.F.)
0	+2.95	+2.89	+2.76
1/3	- .26	- .27	- .31
2/3	- .43	- .44	- .25
1	- .16	- .05	- .03
2	- .02	- .03	- .03
3	+ .01	- .01	- .00
4	- .01	+ .02	- .02

*Comparisons at midsurface between 90° plies.

** $Z_2 = b - \xi$; distance from free-edge.

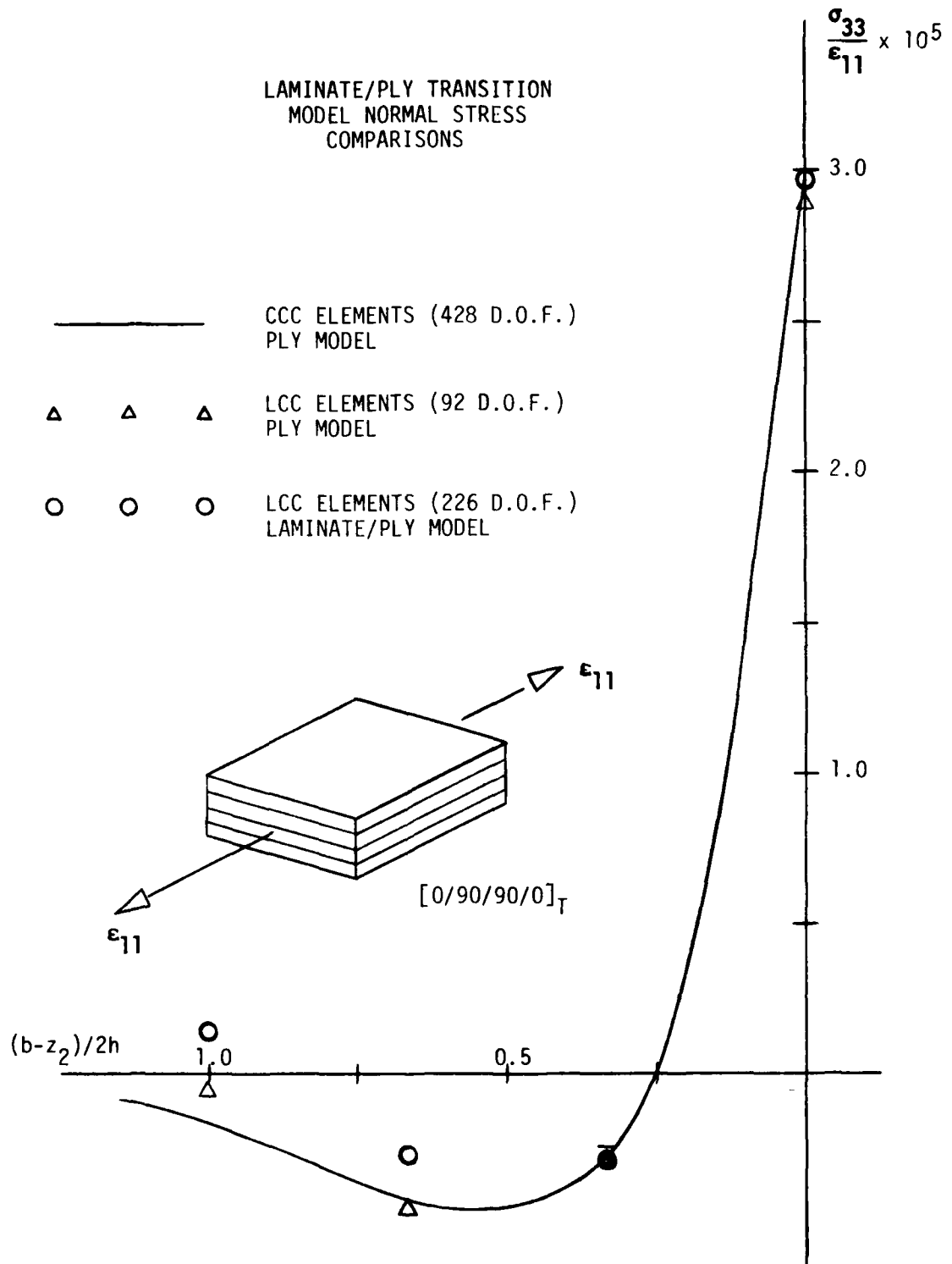


Figure 2. Interlaminar Normal Stress Comparisons

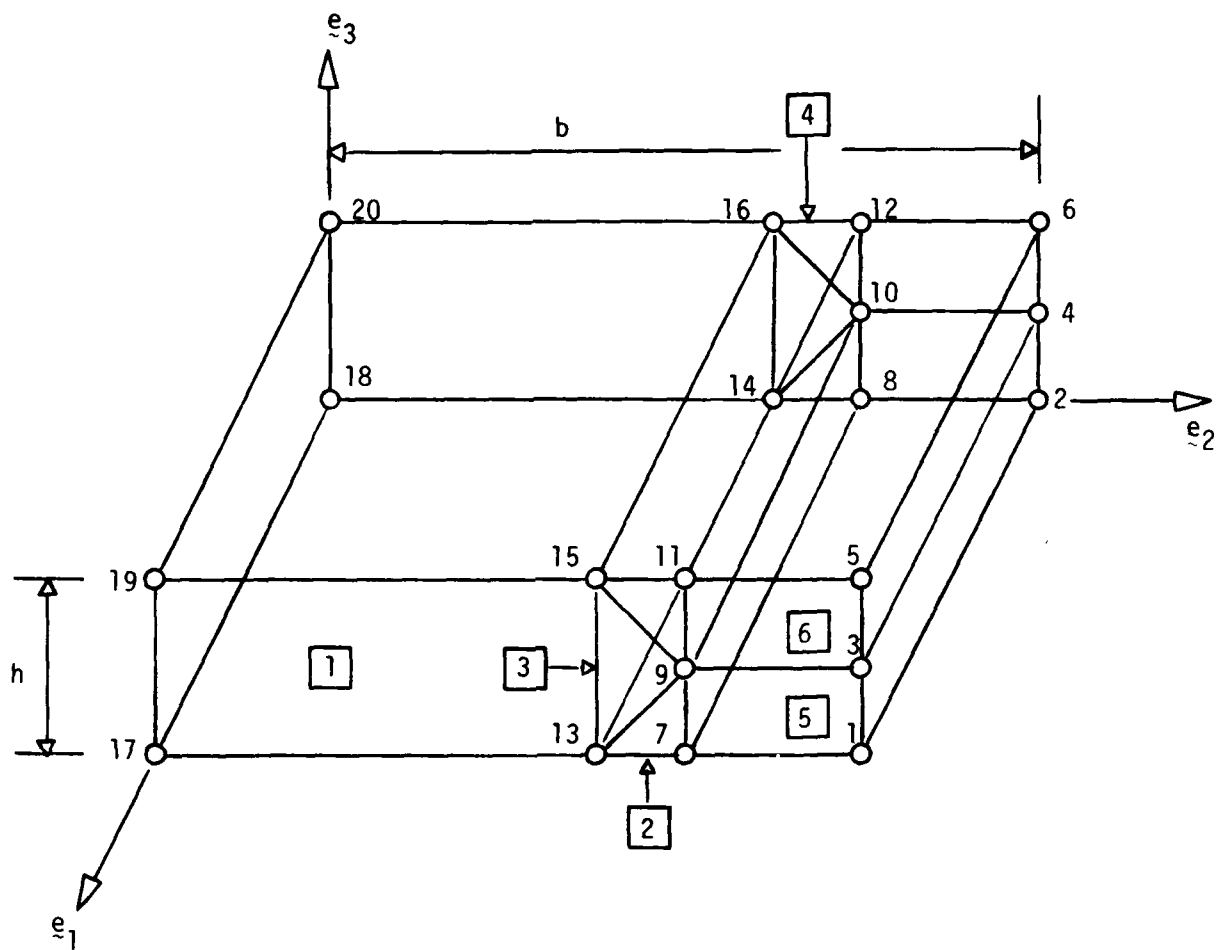
TABLE 7. INTERLAMINAR SHEAR STRESS COMPARISONS*

$\xi/2h$	CCC/CCC Uniform	LLL/LCC Uniform	LLL/LCL Uniform	$\xi/2h$	LLL/LCC Nonuniform**
0	-2.72	-2.72	-.33	0	+ .58
1/3	- .85	- .85	-.47	1/9	-1.94
2/3	- .60	- .60	-.42	4/9	- .67
1	+ .03	+ .03	+.03	1	- .31
2	- .05	- .05	-.04	2	- .03
3	- .02	- .02	-.02	3	- .02
4	.00	.00	.00	4	.00

*Comparisons between 90° and 0° plies.

** $Z_2 = 6 + 4\xi - 2\xi^2$ (edge elements only).

Consider now the transition from discrete ply properties to a composite property model C_{ij}^* away from the free-edge. A six element model shown in Figure 3 was used to determine model accuracy when transitions of this type are made using the present constraint finite elements. Note the two edge elements are exactly as before, and the transition mesh uses three wedge shaped elements. Rule-of-mixture composite properties were used for elements number one and three, and all other elements use local ply properties as in the original model. The interlaminar stresses at the free edge changed very little, Figure 2. At the interface, of course, small local shear stresses occur because of the discrete change in properties. Their maximum value is less than one-half of one percent the applied axial stress. Essentially, the same results were obtained using $C_{ij}^*(\xi)$ variable composite properties. Interestingly, the two ply laminate properties were easier to model with a cubic than the three ply laminate because the $[0/90]$ properties are an odd function. These results indicate the transition from discrete ply modeling to composite laminate modeling can be made using the new constraint elements.



ELEMENT	DISPLACEMENTS	PROPERTIES
1	LCC	LAMINATE , LLC
2	LCC	PLY , 90°
3	LCC	LAMINATE , LLC
4	LCC	PLY , 0°
5	LCC	PLY , 90°
6	LCC	PLY , 0°

Figure 3. PATCHES-III Transition Model for Interlaminar Stress Analysis

3. DISBONDED LAMINATE ANALYSIS

The effects of an unsymmetric disbond on laminate deformations and associated internal load redistributions around the disbond are investigated for a thin laminate. An important aspect of the unsymmetrically disbonded laminate problem not considered in the present investigation is the effect of laminate thickness. There is considerable experimental evidence, Williams [8, 9], that the strength of thick laminates (~50 plies or more) is sensitive to internal disbonds in both unidirectional tape and bidirectional fabric materials of graphite-epoxy. These remarks refer to compressive not tensile loads and to disbonds between plies that occur without fiber damage. Results from the present investigation suggest thin laminates (~10 plies or less) will experience a loss in stiffness caused by out-of-plane bending before any stress critical failure. The laminate in this study is an eight ply graphite-epoxy flat panel with a small circular disbond ($T/R \cong 0.1$) located three plies from the surface and at the center of the panel. A ply-by-ply modeling of this problem is used to study interlaminar force-deformation behavior in the vicinity of the disbond with CCL finite elements. No attempt was made to include in the model any strain singularities that might exist, although this could have been done had fracture mechanics been the focus of the study. The objective was to determine the deformations and internal load redistribution caused by an unsymmetric disbond in a compressively loaded thin laminate.

The disbonded laminate problem was analyzed early in the study using symmetry boundary conditions on two planes. Prototype models indicated a definite skewed displacement response would occur and that the use of symmetry planes would inhibit some response modes. Unfortunately, PATCHES-III was limited to 50 elements at that time and no practical means of avoiding symmetry modeling was available. The resulting symmetry model solution is essentially a 3D laminate solution with interlaminar stress gradients distorted at the origin by the symmetry boundary conditions. However, the force-deformation behavior of the laminate is represented reasonably well and several interesting conclusions can be drawn from the symmetry solution.

3.1 PROTOTYPE MODELS

A schematic of the idealized disbonded laminate analyzed in this study is shown in Figure 4. The disbonded region is at the center of a square

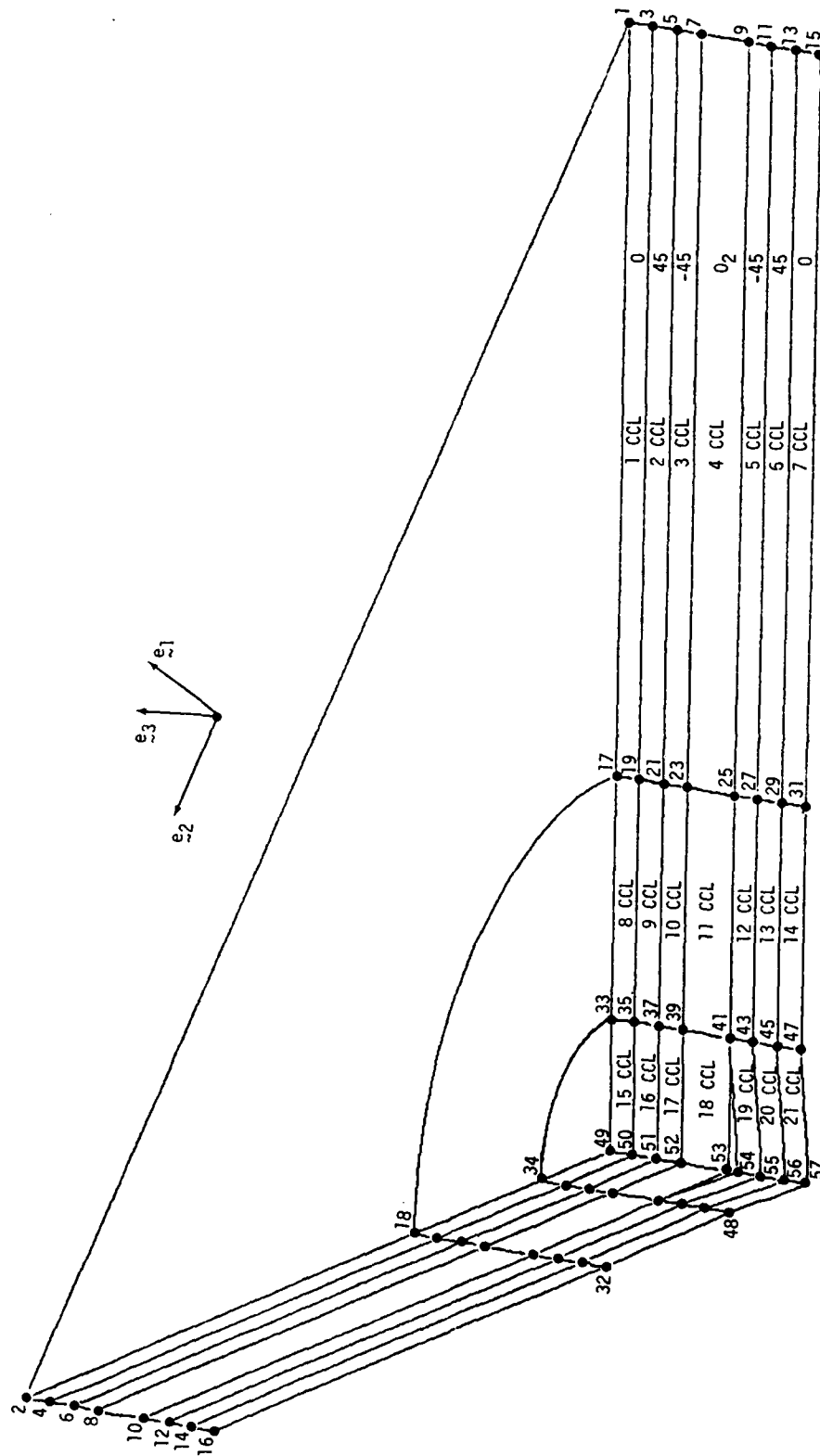


Figure 4. Disbonded Laminate Schematic Using Diagonal Symmetry

laminate between plies five (-45^0) and six (0^0). The region is circular with a bubble shape whose profile is given by

$$Z_3 = t_p(1 + \cos r\pi/R)/2 \quad (3.1)$$

where t_p is a ply thickness and the maximum radius is $R = 1.27$ cm (0.5 in.). Note that this shape is tangent to the adjacent bonded region and corresponds to an elastically warped surface. A catenary shape would correspond to inelastic fiber damage at the boundary of the disbonded region. The laminate is 15.24 cm (6 in.) square and uniformly compressed 0.01524 cm (.006 in.) in both the e_1 and e_2 directions. At the loaded edges, the normal displacement component, u_3 , is restrained to zero. The disbonded plies $[-45/45/0]$ are unbalanced and after their separation, the remaining plies $[0/45/-45/0_2]$ are also unbalanced. As in the unbalanced three-ply laminate analyzed by Pagano, there will be stretching-twisting coupling that must be considered in selecting a symmetry model. It is also important to examine the inplane force changes caused by a disbond to assist in preparation of the ply-by-ply model of the laminate.

A small two element model was used to examine interply force changes when a -45^0 ply and a 0^0 ply disbond locally. The prototype model, Figure 5, consists of a -45^0 ply and a 0^0 ply in one quadrant of the laminate (7.62 cm) loaded by a uniform strain $\epsilon_{11} = 0.001$. To simulate the disbond, the two elements were disconnected at node 8 corresponding to the center of the laminate. The two elements were type LLL and the nodal forces at the interply corners were used to measure the effect of the disbond. These are shown in Figure 6 and indicate only a small constraint force is relaxed by a disbond at node 8. Note, however, that the effect is to increase the transverse load in the 0^0 ply which carries most of the load associated with the applied ϵ_{11} strain. It is obvious that a larger change will occur if either nodes 5 or 7 are cut instead of node 8 which is analogous to a disbond at node 8 in a $+45/0^0$ layup. Results from this case, Figure 7, do show a larger change, but it reduces the transverse load in the 0^0 ply. The directional dependence in stiffness produces a directional dependence in the effect of a disbond under any given load. In this case of a circular disbond, Figure 4, under biaxial load, it appears that interply constraint forces will vary around the perimeter with relative maximums or minimums every 45^0 .

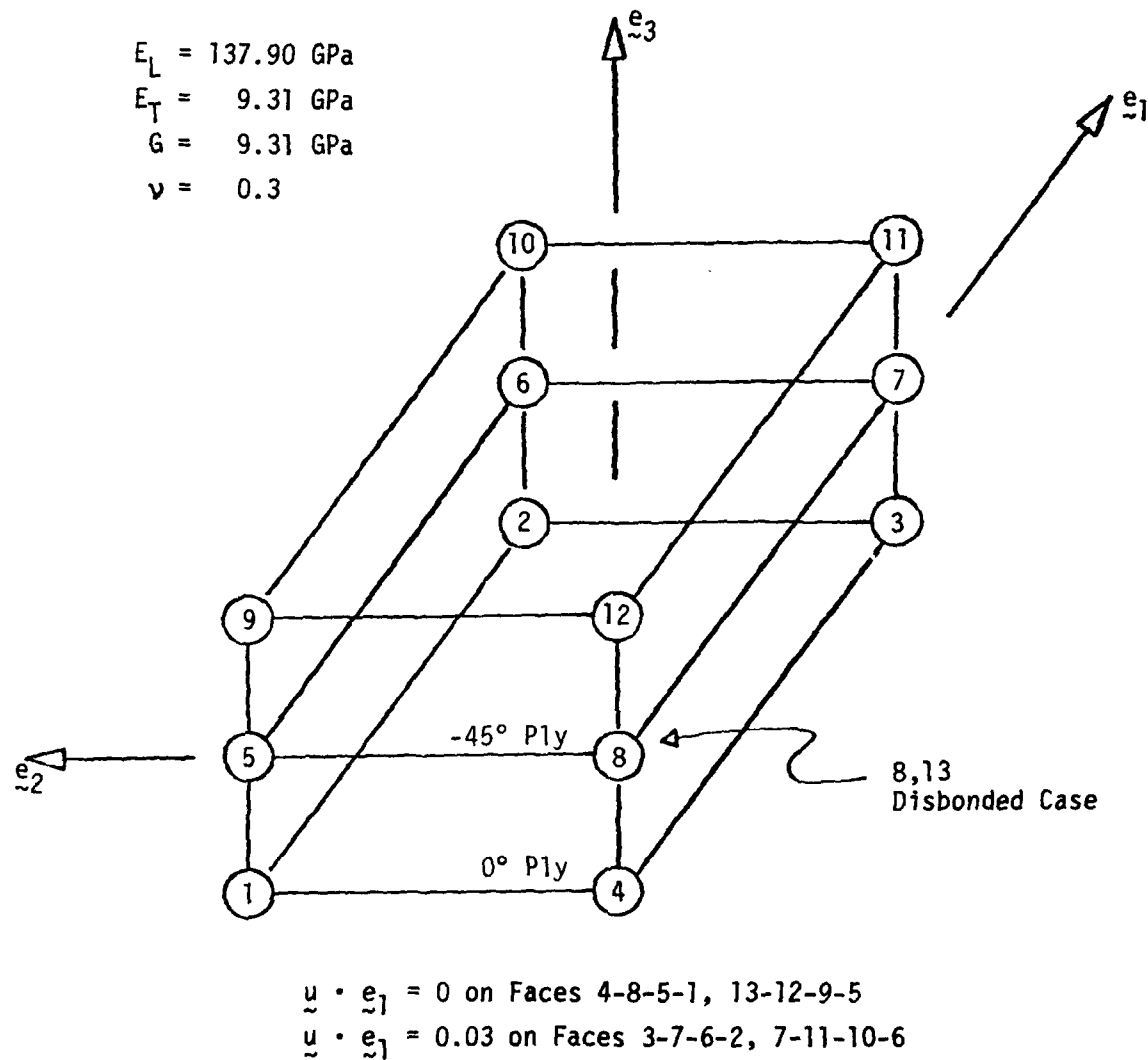


Figure 5. Two-Ply Disbond Prototype Model for Forces

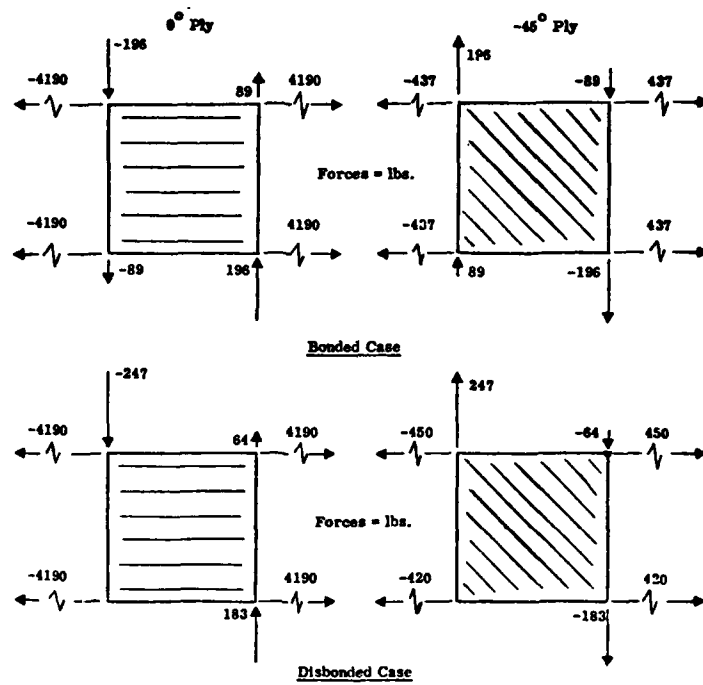


Figure 6. Interply Forces: $[0/-45]$ Uniaxial Load

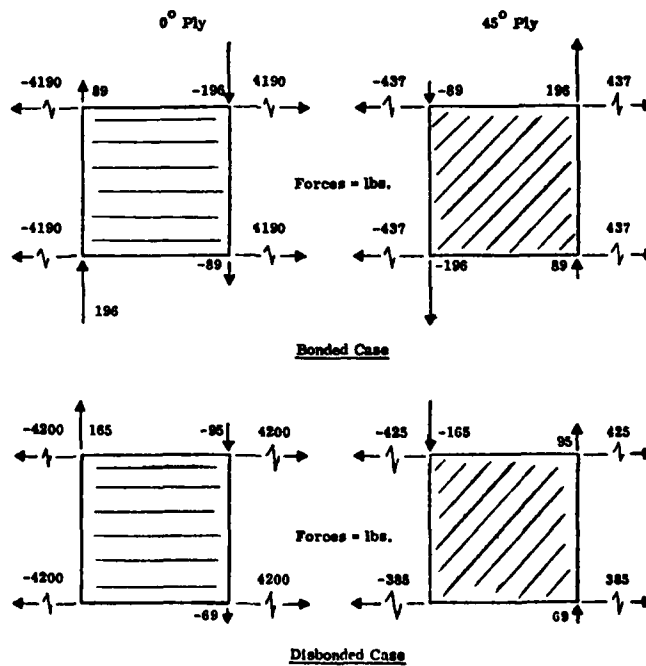


Figure 7. Interply Forces: $[0/45]$ Uniaxial Load

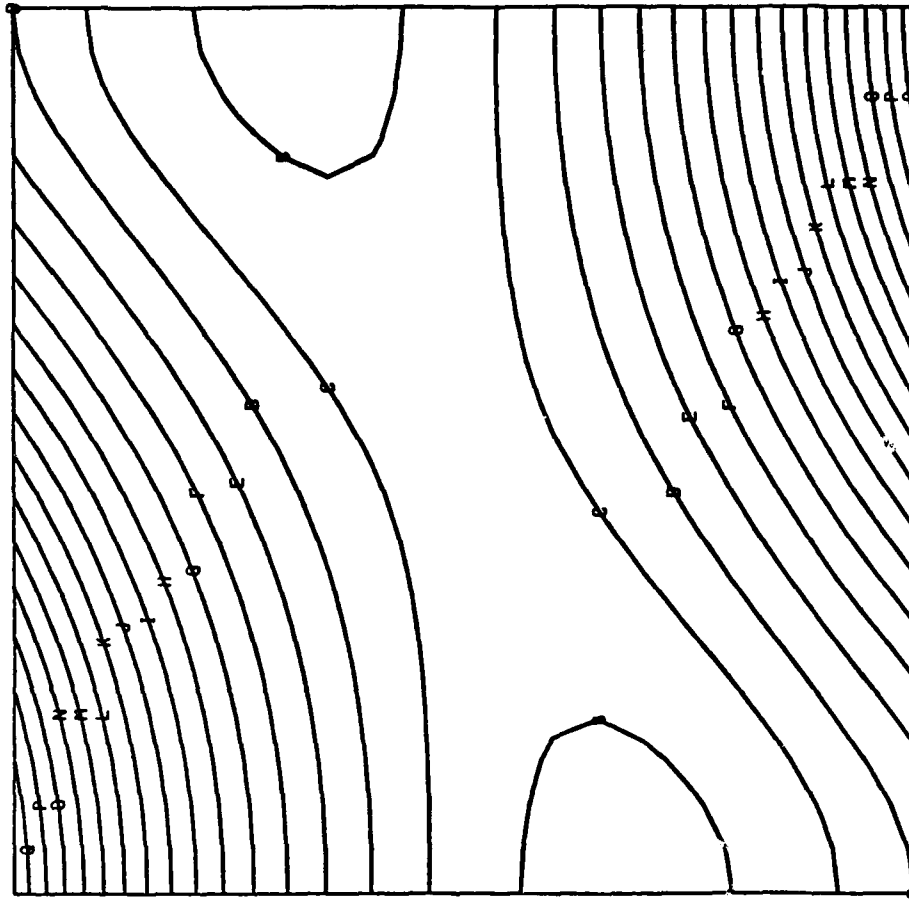
Next, a three-ply [0/45/-45] laminate model was used to examine out-of-plane warping under inplane loading of the unbalanced disbond material. Each ply was modeled with CCL elements, and the dimension of the square model was made equal the radius of the disbond. Under uniaxial load there is elastic coupling between stretching and twisting that caused large out-of-plane displacements because of the low bending stiffness of a three-ply laminate (the plies are only 0.01778 cm thick). The stresses, strains and deformations of this [0/45/-45] laminate are all either asymmetric or symmetric about axes at $\pm 45^\circ$ to axes e_1 , e_2 , Figure 8. These results and the [0/45] constraint force results indicate diagonal symmetry rather than Cartesian symmetry should be used in modeling force-deformation behavior of the disbanded laminate.

Another observation from the unbalanced prototype models was the extremely slow convergence of the conjugate-gradient solution procedure. The membrane response accounts for 99.7 percent of the potential energy and this result is converged in less than N cycles. However, the low energy out-of-plane bending response required almost 4N cycles to converge with little change to principal stresses or strains. Note that the interply normal forces are two orders of magnitude smaller than the inplane forces. The models were changed to CCC elements to check for purely numerical ill-conditioning and the same slow convergence was observed. No solution to the pathological computational behavior caused by weak out-of-plane coupling was found.

3.2 LAMINATE FINITE ELEMENT MODELS

The dimensions of the laminate were specified to insure no coupling between the displacement boundary conditions and deformations in the vicinity of the disbond. The laminate is square of width 15.24 cm (6 inches) with a disbond radius of 1.27 cm (.5-inch) between plies five and six, Figure 4. The plies are graphite-epoxy of thickness 0.01778 cm (0.007-inch) with their nominal elastic properties as given in Table 8. The stacking sequence for the laminate is [0,45,-45,0₂,-45,45,0]_T with the disbond between a 0° ply and a -45° ply. The edges are loaded by imposing a uniform inplane displacement of 0.00762 cm (0.003-inch) which produces biaxial compression.

A control model of the laminate was constructed using the same mesh in the Z_1, Z_2 plane as in Figure 4, but with only four plies [0,45,-45,0]_S through the half thickness. The purpose of this model was to provide before-and-after data for comparison with the disbanded results, in particular the



1 SURFACES OF UNDEFORMED GEOMETRY
 DATA TYPE = DISPL COMPONENT 3
 DATA IS IN THE CARTESIAN FRAME.
 :N INEG = 1 NLINE = 1
 NACCG = 3 NACCD = 3
 VIEW = 0.00 0.00 0.00
 ORIGIN = 0.00 0.00 10.00
 ORTHOGRAPHIC PROJECTION
 SYMMETRY OFF.
 ALL LINES DRAWN

ELEMENT SET= 1
 FACE SET= 6

18 CONTOUR INTERVALS
 -.150000E-01(A) -.100000E-01(B)
 -.500000E-02(C) 0.100000E-01(F)
 .500000E-02(E) .100000E-01(F)
 .150000E-01(G) .200000E-01(H)
 .250000E-01(I) .300000E-01(J)
 .350000E-01(K) .400000E-01(L)
 .450000E-01(M) .500000E-01(N)
 .550000E-01(O) .600000E-01(P)
 .650000E-01(Q) .700000E-01(R)

UNBALANCED LAMINATE

Figure 8. Unbalanced Laminate Bending Displacements

TABLE 8. GRAPHITE-EPOXY NOMINAL ELASTIC PROPERTIES

$E_{11} = 137.90 \text{ GPa (20 msi)}$	$\nu_{12} = 0.3$	$G_{12} = 4.83 \text{ GPa (.7 msi)}$
$E_{22} = 9.31 \text{ GPa (1.35 msi)}$	$\nu_{13} = 0.3$	$G_{13} = 4.83 \text{ GPa (.7 msi)}$
$E_{33} = 9.31 \text{ GPa (1.35 msi)}$	$\nu_{23} = 0.3$	$G_{23} = 4.83 \text{ GPa (.7 msi)}$

internal forces. At the same time the control model shows the magnitude of the interlaminar constraint forces caused by the use of diagonal symmetry, Table 9. The forces F_2 should be exactly zero if the interlaminar deformations are zero at the center of the laminate. Instead, a large self-equilibrating pair of forces occur between the 0° plies and the $\pm 45^\circ$ plies. These forces distort the interlaminar stresses at the origin. It should be noted that F_2 sums to zero between the +45 and -45 and the use of diagonal symmetry for these plies is consistent with material symmetry axes.

TABLE 9. BONDED LAMINATE CONSTRAINT FORCES THROUGH THE THICKNESS AT $r = 0$

	$Z_3 = 4t_p$	$Z_3 = 3t_p$	$Z_3 = 2t_p$	$Z_3 = t_p$	$Z_3 = 0_p$
NODE	49	50	51	52	53
F_1	54.98 N	100.44 N	102.00 N	100.22 N	50.18 N
F_2	0.	43.41 N	0.	-43.41 N	0.

The control model also demonstrates that even the small biaxial strain ($\epsilon \cong 0.1\%$) imposed at the edges of the panel is sufficient to buckle the laminate elastically. An engineering check was made using isotropic simply supported plate formulas with the stiff direction properties. This calculation indicates the laminate will buckle at $P_1 = P_2 \cong 2000 \text{ N}$, while the applied load is $P > 10,000 \text{ N}$. The exact solution for buckling of a laminated anisotropic plate [10] would be lower because the small number of plies accentuates the bending-stretching coupling. It seems likely that the disbond will reduce the buckling load even further or possibly cause a large deflection stability problem in a laminate this thin, $T = 0.142 \text{ cm (0.056-inch)}$.

3.3 DISBONDED LAMINATE RESULTS

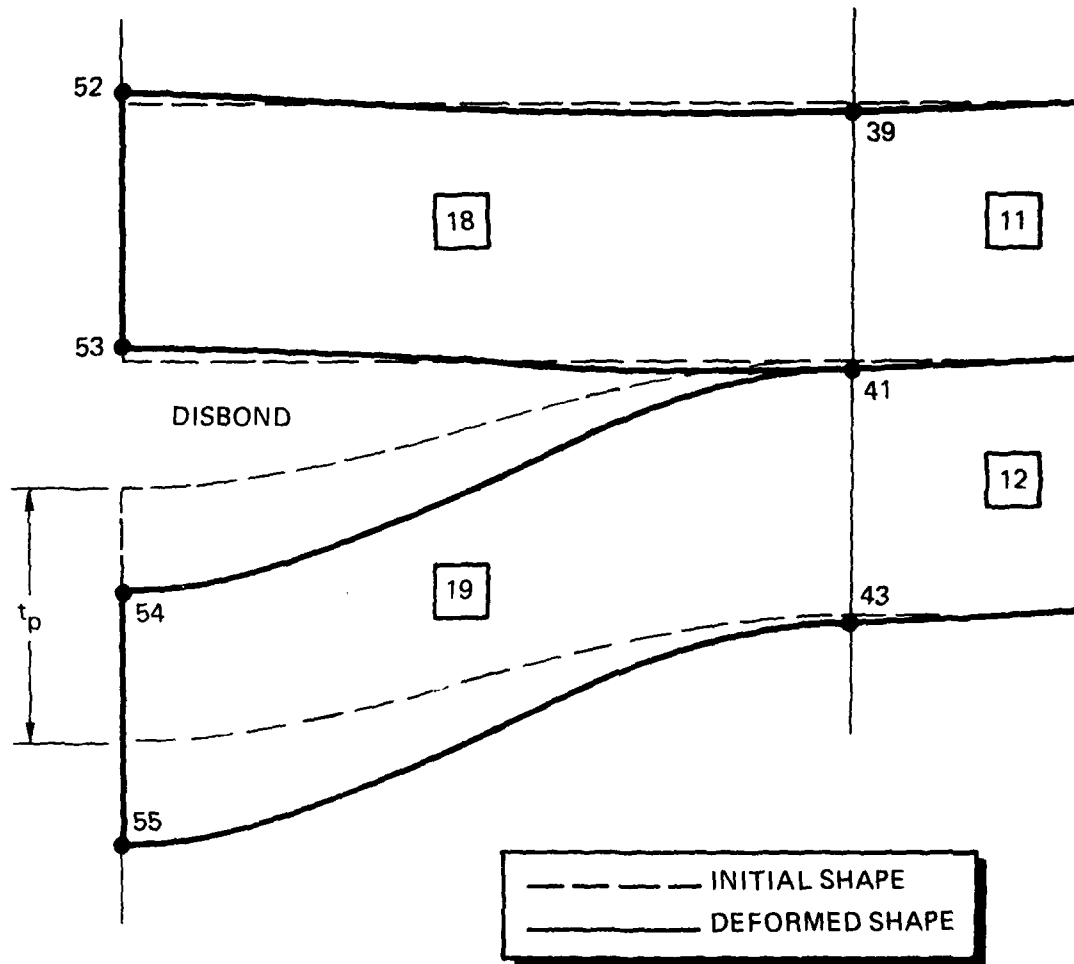
The effect of the disbond on the interlaminar force distribution is quite small and the effect on bending deformation is quite large. The centerpoint initial displacement caused by the disbond is $0.5 t_p$ before any load is applied. The centerpoint elastic displacement after loading to a biaxial strain of only 0.1 percent is an additional $0.4 t_p$, Figure 9. In other words, a unit imposed displacement (biaxial) in the plane of the laminate produces almost a unit normal displacement at the center of the laminate. This result suggests the disbond will initially lead to a stiffness critical failure rather than a stress failure. The reason for this behavior seems to be the extremely low bending stiffness which produces large out-of-plane deformation when the disbond couples the membrane and bending displacement response. It is important to note that the potential energy change between the bonded and disbonded cases is less than one milli Joule, Table 10, which corresponds to an energy release rate of less than 1 N/m. In contrast, the critical energy release rate for an edge delamination, Reference 11, is well over 100 N/m for an 11 ply graphite-epoxy laminate.

TABLE 10. DISBONDED LAMINATE ENERGY CHANGE

Potential Energy		
Bonded	3.958455 Joules	(35.03528 in.-lbs)
Disbonded	3.958069 Joules	(35.03186 in.-lbs)

The small redistribution of internal forces is evident in Table 11 which shows the center point constraint forces through the thickness. Note that the forces at nodes 53 and 54 are summed when the disbond is closed. Note also that the force at node 53 in the bonded model must be added to node 52 for comparison with the disbond results. This is because only one element was used for the two center plies, Figure 4, in the disbond model. A check of the force changes through the thickness was also made at $r = R$, Table 12.

To a large extent the symmetry boundary conditions inhibit in-plane redistribution and to some extent interlaminar normal force changes. Considering the large normal displacements, Figure 9, one might expect an



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Figure 9. Normal Deformations in the Plies Adjacent to the Disbond

TABLE 11. DISBONDED LAMINATE CONSTRAINT FORCE CHANGES THROUGH THE THICKNESS AT $r = 0$

Node	Bonded			Disbonded		
	Z_3	F_1	F_2	Z_3	F_1	F_2
49	$4t_p$	54.98 N	0. N	$4t_p$	44.54 N	1.01 N
50	$3t_p$	100.44 N	43.41 N	$3t_p$	103.68 N	45.75 N
51	$2t_p$	102.00 N	0. N	$2t_p$	105.69 N	-.22 N
52	t_p	100.22 N	-43.41 N	t_p	160.78 N	-46.58 N
53*	0	50.18 N	0. N	$-t_p$	112.41 N	-.90 N
54				$-1.5 t_p$	43.62 N	-41.33 N
55		(Symmetric)		$-2.5 t_p$	90.67 N	3.22 N
56				$-3.5 t_p$	89.49 N	38.57 N
57				$-4.5 t_p$	42.46	1.43 N

*Node 53 is at $Z_3 = 0$ in the bonded laminate case.

interlaminar normal force to occur at $r = R$. As the data in Table 12 show, only a small increase in the small interlaminar normal force occurred at node 41. A possible explanation is that the symmetry boundary conditions at the center prevents nodes 53 and 54 from moving relative to each other in-plane as they probably would in a complete solution. If such deformations occur, they would create transverse shear gradients in-plane and equilibrium would require a normal stress gradient in the thickness direction,

$$\sigma_{31,1} + \sigma_{32,2} + \sigma_{33,3} = 0 \quad (3.2)$$

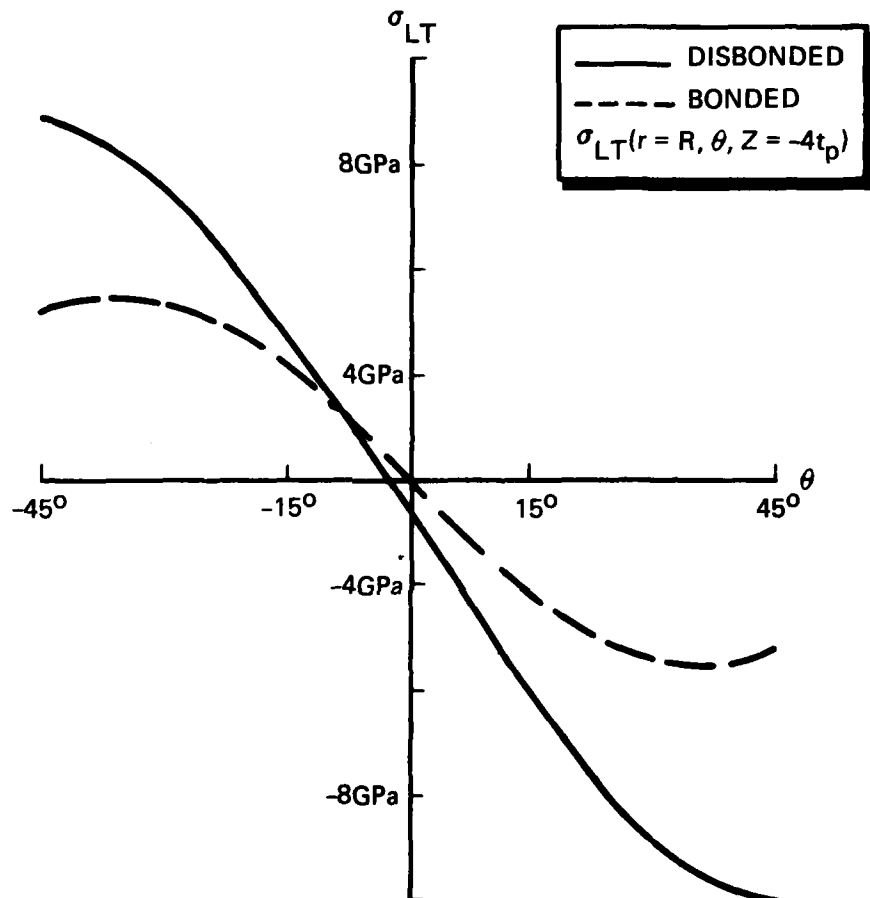
The magnitude of the spurious F_2 force at node 54 suggests this deformation could be significant and would certainly increase the energy release rate. However, it would have to increase by a factor of 100 before the delamination would propagate if the data in Reference 11 apply. This uncertainty can only be resolved by analyzing a complete model without the assumption of symmetry conditions.

TABLE 12. DISBONDED LAMINATE CONSTRAINT FORCE CHANGES THROUGH THE THICKNESS AT $r = R$

Node	Bonded			Disbonded		
	Z_3	$F_1 = F_2$	F_3^*	Z_3	$F_1 = F_2$	F_3
33	$4t_p$	12.18 N	0. N	$4t_p$	14.01 N	0. N
35	$3t_p$	74.54 N	± 0.56 N	$3t_p$	75.17 N	± 0.07 N
37	$2t_p$	68.29 N	± 1.12 N	$2t_p$	69.10 N	± 0.14 N
39	t_p	33.09 N	± 0.84 N	t_p	33.13 N	± 0.93 N
41	$-t_p$	(Symmetric)		$-t_p$	32.16 N	± 1.49 N
43	$-2t_p$			$-2t_p$	67.28 N	± 0.84 N
45	$-3t_p$			$-3t_p$	73.04 N	± 0.34 N
47	$-4t_p$			$-4t_p$	12.19 N	0. N

* F_3 forces are self-equilibrating interlaminar forces.

The interlaminar stress results in keeping with the force results show only small changes in their distribution. On a percentage basis, the largest change occurs in the bottom ply where the small σ_{LT} shear stress is doubled. A plot of this stress component around the disbond at $r = R$, Figure 10, shows relative maximums and minimums at $\pm 45^\circ$ as expected. The use of symmetry boundary conditions also caused local distortions at the origin in the stress results that make their magnitudes questionable. However, these conditions affect the bonded and disbonded cases equally.



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Figure 10. Bottom Ply Shear Stress Changes

4. LAMINATE IMPACT ANALYSES

At low velocities impact damage can be modeled as a quasi-static response problem using Hertzian contact pressures in most cases. This approach is used in the present study, and Greszczuk, Reference 12, gives an excellent account of the method and its rationale. The only analytic results available are for the elastic half space problem which was recently solved for a transversely isotropic material, Reference 13. One effect of transverse isotropy shown graphically in that paper is a shift of the maximum Von Mises stress from the impact centerline to a radius approaching the contact radius. The significance of this stress parameter for a composite is dubious, but it does indicate a trend of increasing transverse shear below the contact radius, which also occurs in the present results.

The disbanded laminate analysis problems caused by symmetry displacement boundary conditions led to the use of a full 360 degree finite element model for the impact problem. Another factor in this decision was the earlier PATCHES-III sandwich panel impact analysis [6] which also showed spurious constraint forces between the 0° and 45° plies on the centerline of a double symmetry model. In order to model a full 360 degrees, ply-by-ply, the program was modified to allow several hundred solid elements. Unfortunately, the computer budget available was sufficient to analyze only a 32 element model after all computational problems were resolved. These results, however, provide the first in-depth picture of the asymmetric interlaminar stress gradients that occur in a thick laminate impacted off-center. More detailed models were prepared for future analysis and these are in an appendix to this report.

4.1 THICK LAMINATE COMPOSITE RESPONSE

A 48 ply graphite-epoxy laminate supported between two rigid spars is shown schematically in Figure 11 being impacted by a steel sphere. The impact point is close to a spar and in this problem a contact force of 4448 N was specified. Considering the large number of plies in this problem, not all plies can be modeled for any given analysis. This practical consideration becomes even more critical when symmetry conditions cannot be used. First a variable property 3D laminate model was used to determine the bending deformations in the vicinity of the impact site. This analysis used a large but finite width strip to avoid boundary condition effects, Figure 12. The 3D

laminates analysis produced large transverse shear stresses at the impact site approaching the bending stresses in magnitude. Laminates force-deformation properties were modeled using the $C_{ij}^*(\xi)$ formulation derived earlier, Table 13. As these data indicate, only the off-diagonal coupling terms vary appreciably through the thickness.

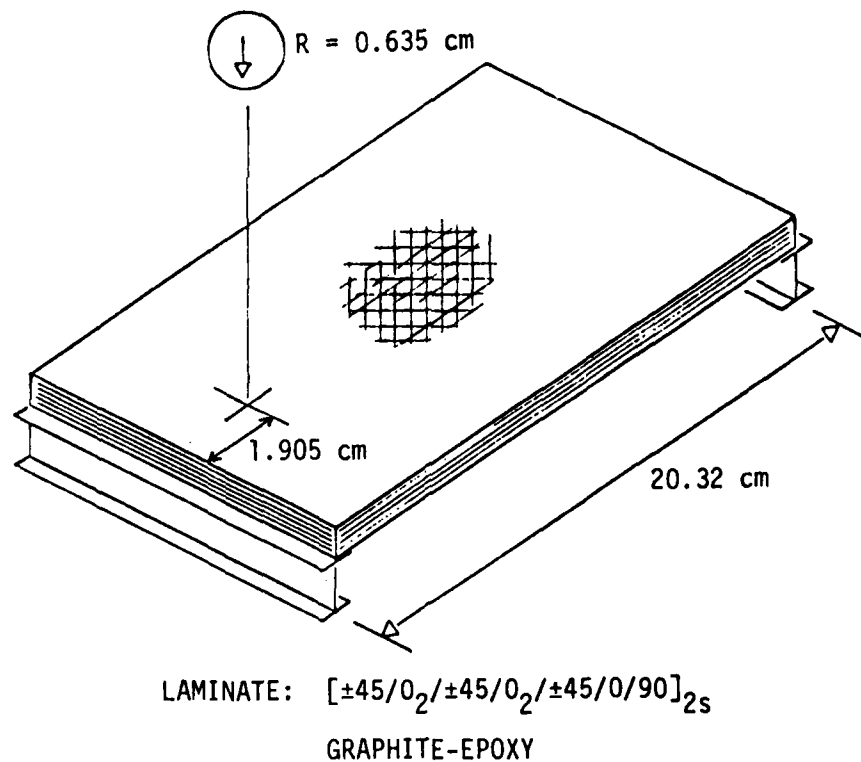


Figure 11. Thick Laminates Impact Schematic

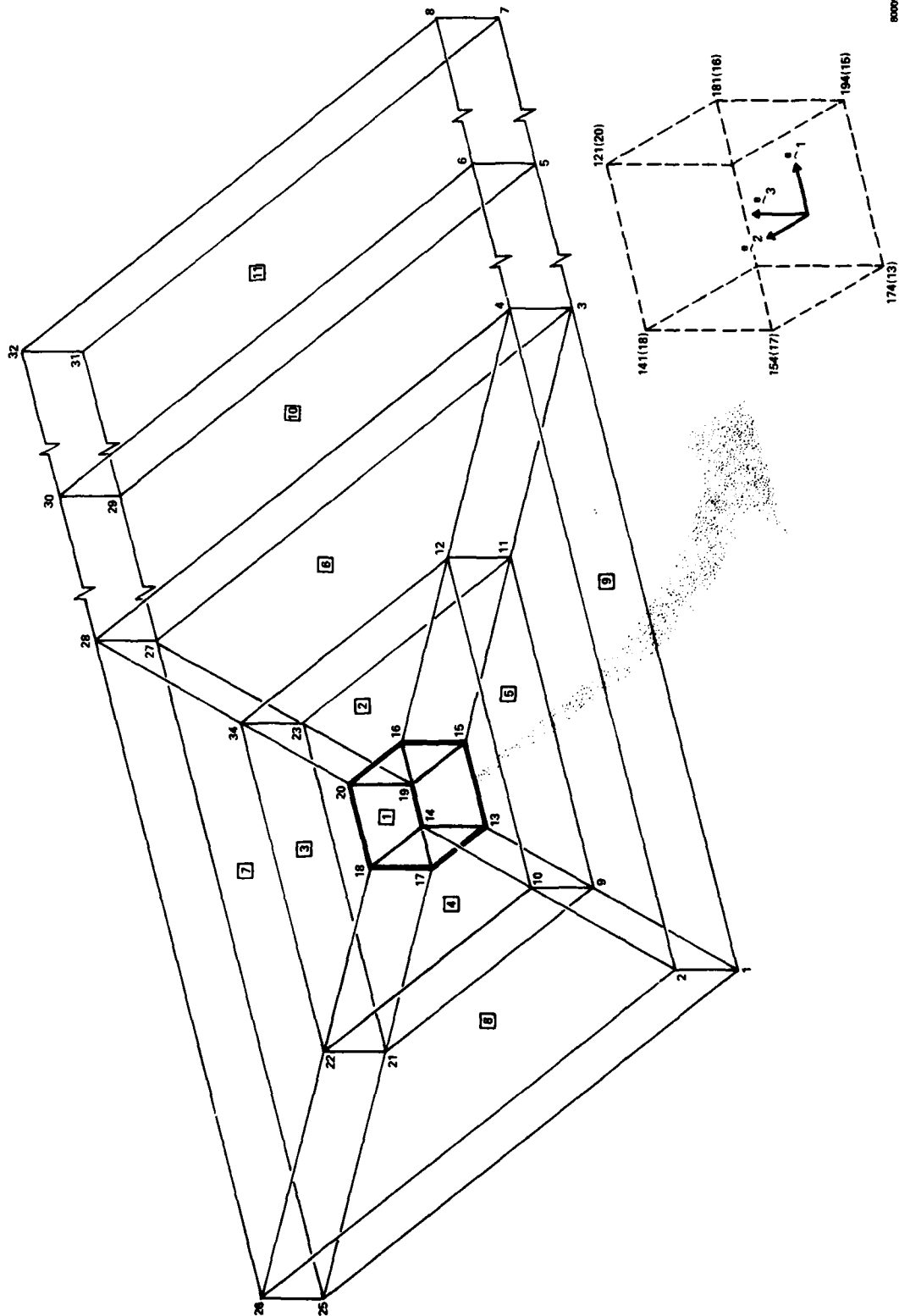


Figure 12. Laminate Structural Response PATCHES-III Model

TABLE 13. PARAMETRIC CUBIC THICK LAMINATE PROPERTY MODEL*

	$C_{ij}^*(0) = C_{ij}^*(1)$	$C_{ij}^*(1/3) = C_{ij}^*(2/3)$
C11	72.622	71.650
C12	21.712	14.906
C13	2.999	2.910
C14	2.427	-0.807
C22	24.173	36.887
C23	2.379	2.468
C24	2.427	-0.807
C33	9.432	9.432
C44	22.091	16.279
C55	3.771	3.771
C66	3.744	3.744

*Stiffness in GPa units (1 Psi = 6894.757 Pascals)

4.2 IMPACT SITE MODEL

The composite laminate displacement response for element number one, Figure 12, provided boundary conditions for the impact site model shown in Figure 13. The contact radius was determined using both the Greszczuk [12] equations and the Dahan and Zarka [13] equations. The latter have typographical errors that were reconciled using the limiting case of an isotropic half-space to yield

$$r_o = \left[\frac{3FR}{2} \left(\frac{\delta_1 - \delta_2}{2} + \frac{1 - \nu^2}{E} \right) \right]^{1/3} \quad (4.1)$$

where δ_1 and δ_2 are lengthy algebraic functions of the half-space flexibilities S_{ij}^* . Substituting the laminate S_{ij}^* in these expressions gave

$$\delta_1 = -8.06 \times 10^{-12} \text{ m}^2/\text{N}$$

$$\delta_2 = -68.09 \times 10^{-12} \text{ m}^2/\text{N}$$

The remaining terms in Equation (4.1) are the force $F = 4448 \text{ N}$, the sphere



Figure 13. Impact Site PATCHES-III Model

radius $R = 6.35$ mm and the elastic constants $E = 206.84$ Gpa and $\nu = 0.3$ for the sphere. These data produced a contact radius of $r = 1.134$ mm. The Greszczuk equation for contact radius

$$r_o = \left[\frac{3\pi}{4} FR (k_1 + k_2) \right]^{1/3} \quad (4.2)$$

also uses the S_{ij}^* to compute related coefficients k_1 and k_2 which are

$$k_1 = 1.4004 \times 10^{-12} \text{ m}^2/\text{N}$$

$$k_2 = 19.8173 \times 10^{-12} \text{ m}^2/\text{N}$$

producing a contact radius of 1.122 mm. The smaller radius was used to model the impact site so that $T/R \cong 5.65$.

Several disastrous attempts at analyzing the impact site model, Figure 13, were made using CCC elements for the upper plies and CCL elements for the subsurface laminate model. These attempts never quite converged and the deformations were highly distorted. After considerable effort it was determined that the model had pathological stiffness discontinuities in the thickness direction that were easily removed once found. At the impact site the elements through the thickness must be either all CCC or all CCL to avoid this condition. When the two element types are mixed under impact loading conditions, the CCC elements deform as if they were pressed against a rigid boundary. After resolving this modeling dilemma, excellent results were obtained using all CCC elements.

4.3 INTERLAMINAR IMPACT STRESS RESULTS

It is important to keep in mind that the present results are for a small sphere impacting a thick laminate, $T/R \cong 5.65$, with sufficient energy to generate a contact force of 4448 N (1000 lbs). The resulting stress-strain gradients are quite high in the upper ply group, Figure 14, especially in material coordinates. Note in particular that the second ply, a - 45, has larger fiber stress gradients in the thickness direction than the top ply, although the magnitudes are slightly lower. The centerline Cartesian shear strains, Figure 15, show rather dramatically that the use of two symmetry planes would have produced large shear errors along the centerline. Interesting, the chordwise shear strain, ϵ_{23} , is very small indicating one plane of symmetry

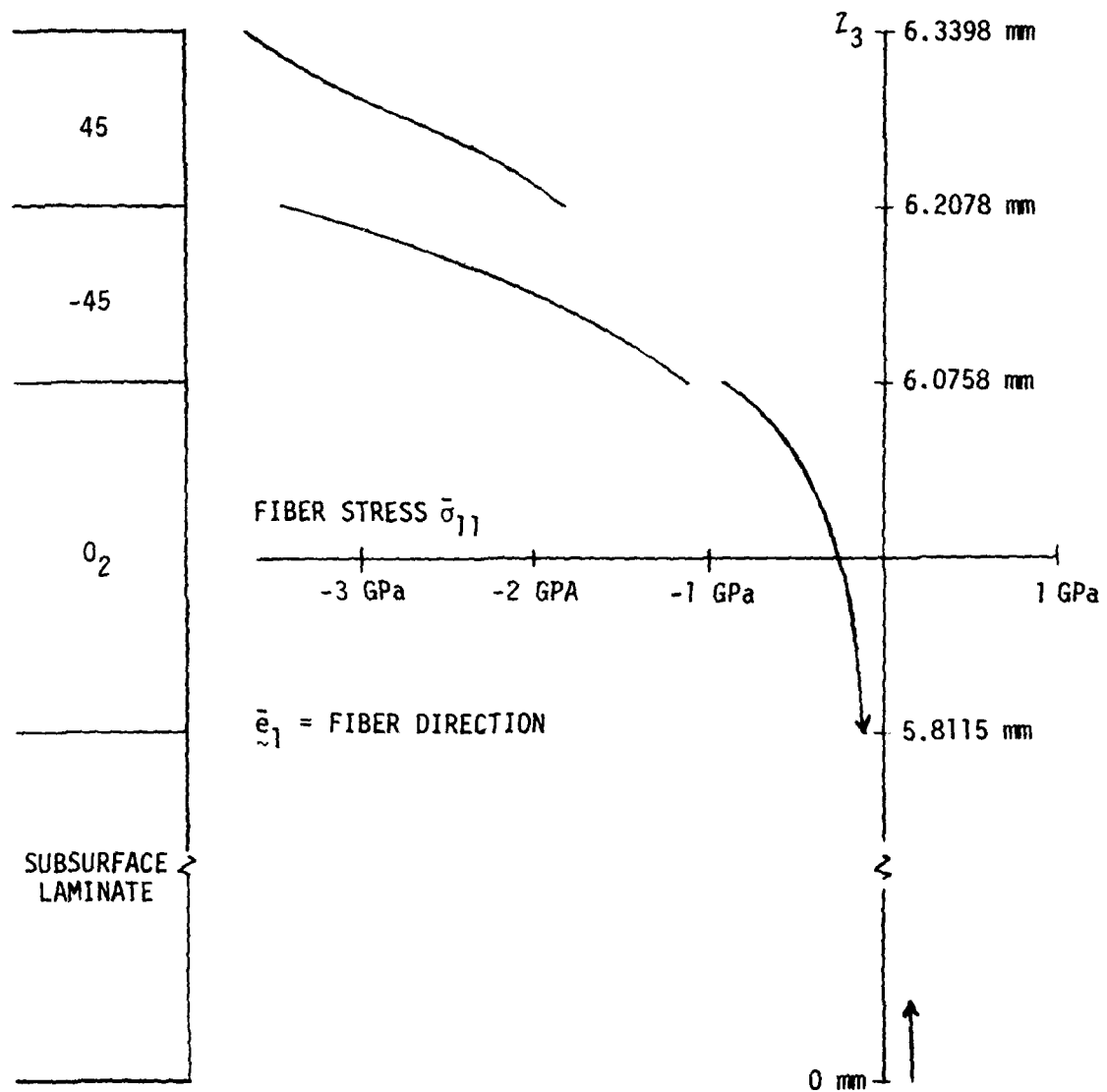


Figure 14. Centerline Fiber Stress

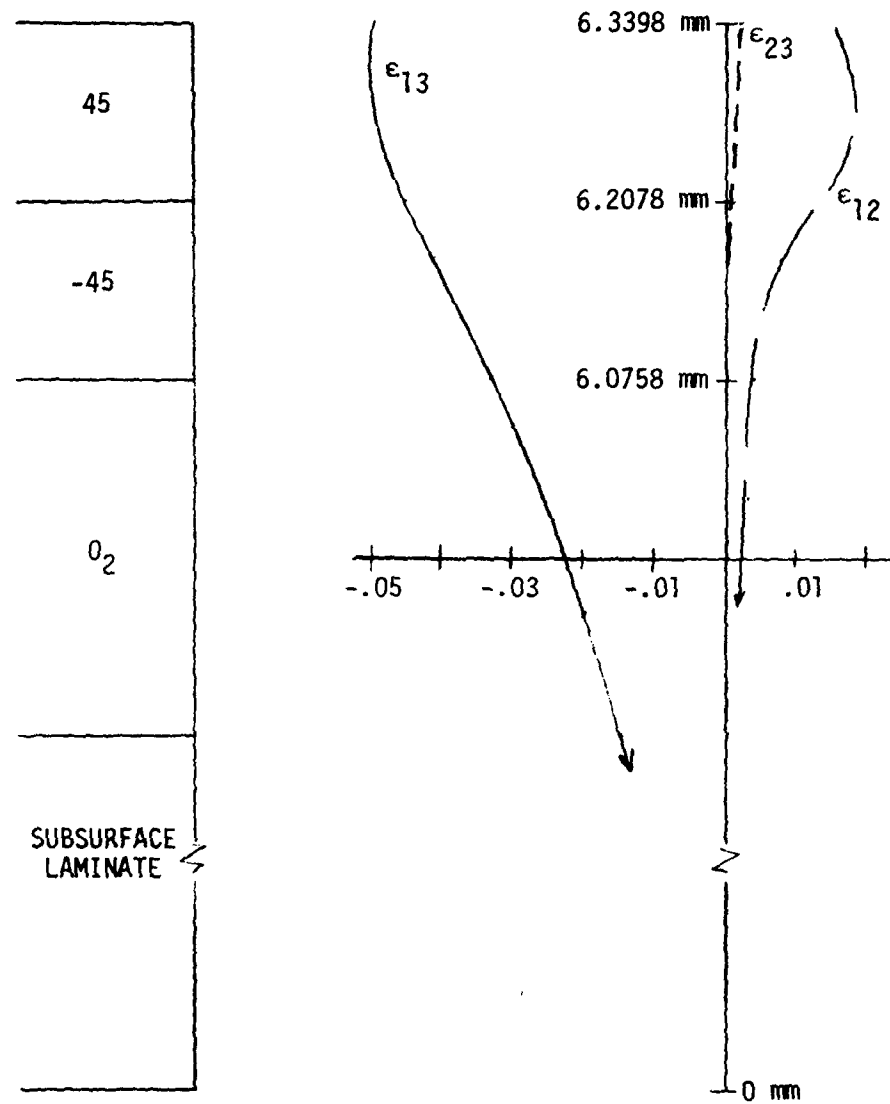


Figure 15. Cartesian Centerline Shear Strains

in the response. If the loading were not in the transverse direction, or if the laminate were unbalanced, even this symmetry condition would not exist.

Consider next the stress distribution around the perimeter of the impact site. The fiber stresses at the top surface as a function of circumferential position, $\bar{\sigma}_{11}(\theta)$, are shown in Figure 16. The tension stresses are confined to a thin layer at the free surface and are probably not a good indicator of actual fiber stresses. When the fiber stresses are averaged through a single ply thickness, Figure 16, the values are all compressive. In either case, the distribution is periodic in θ with a period equal π . The distribution of transverse shear stress in material coordinates is also periodic, Figure 17, but with period equal 2π . The two shear stress components in this figure, $\bar{\sigma}_{13}$ and $\bar{\sigma}_{23}$, were averaged over one ply thickness, the second ply. This ply has the maximum transverse shear in the laminate and it occurs at the impact perimeter not the centerline. Similar behavior was observed by Dahan and Zarka in zinc and cadmium during elastic contact with a steel sphere. These materials also had their low stiffness in the direction normal to the impact surface, but they are not as anisotropic as the laminate material. It is also interesting to note that the two shear stress components $\bar{\sigma}_{13}$ and $\bar{\sigma}_{23}$ are phase shifted by 90° in the circumferential direction. The in-plane ply shear stress $\bar{\sigma}_{12}$ in Figure 18 shows a quasi-periodic distribution of period equal π that is similar to the fiber stress distribution. However, the structural response of the laminate appears to be superimposed on the elastic half-space response. This would explain the $\bar{\sigma}_{12}$ peak nearest the center of the laminate being slightly smaller. There is also a subharmonic component that may be caused by the composite solution boundary conditions applied to the impact site model. To explore this second order effect would require additional analyses. A summary of the ply material stress maximums is provided in Table 14.

It is interesting to observe the similarities and differences in a 3D laminate solution and a 3D elasticity solution in the impact area. The deformations at the top surface are compared in Figure 19 and show very similar shapes. The maximum 3D laminate displacement is about 85 percent of the elasticity result; however, the laminate strains are an order of magnitude too low at the top surface. The elasticity solution using laminate properties was much closer but still low. The normal strain in this case is quite close, but the in-plane strains are one-half to one-third the ply strain results.

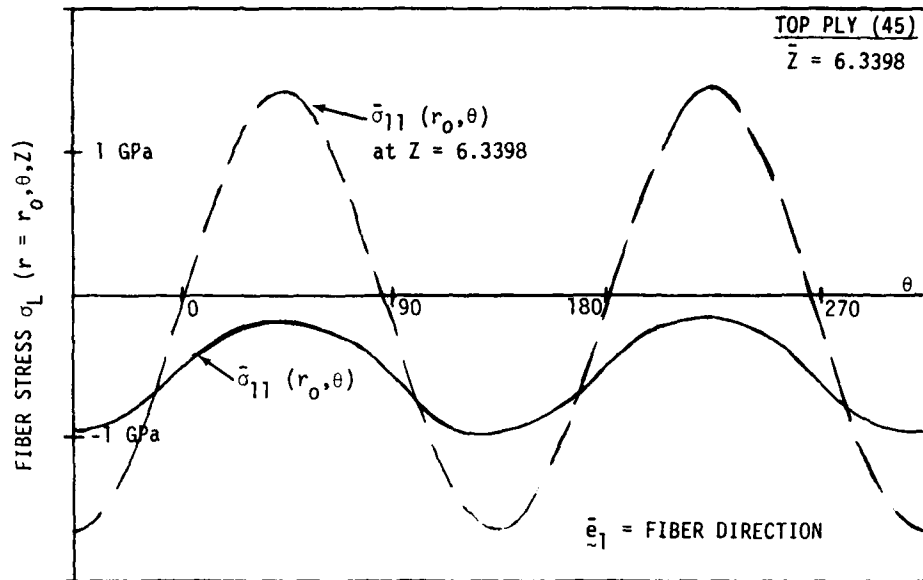


Figure 16. Fiber Stresses Around Impact Perimeter

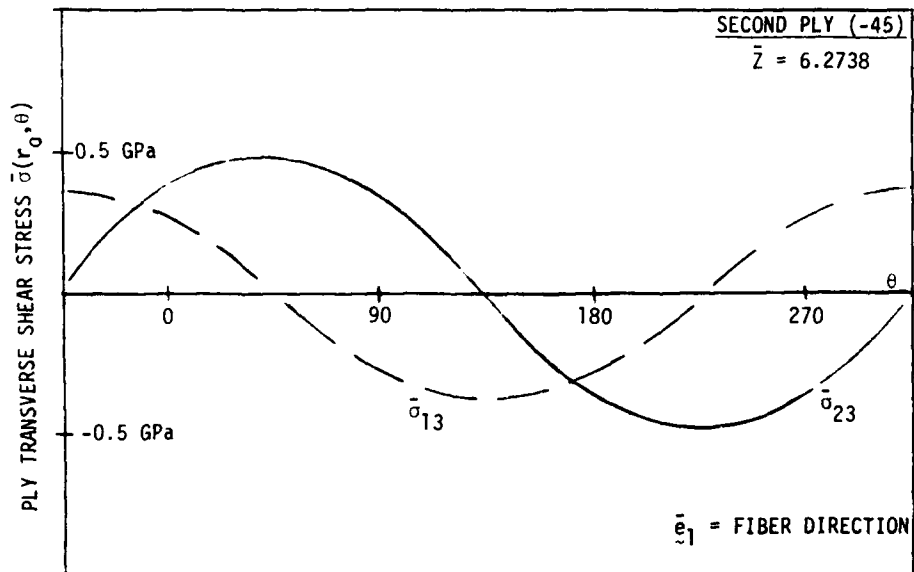


Figure 17. Transverse Shear Stresses Around Impact Perimeter

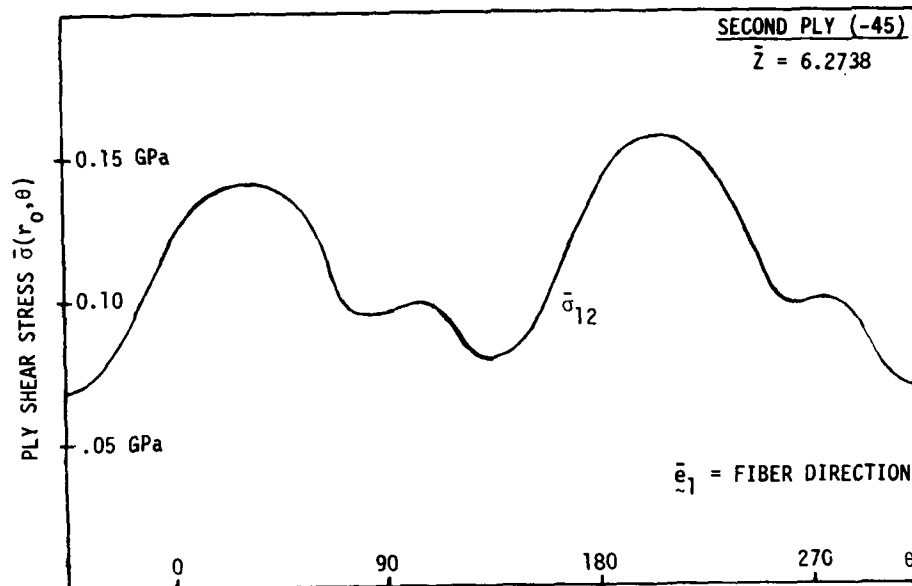


Figure 18. Ply Shear Stress Around Impact Perimeter

TABLE 14. THICK LAMINATE PLY STRESS SUMMARY

	σ_L	σ_T	σ_{LT}	σ_{TT}
σ/P_0	2.05	.99	.28	-.28
Node	1	1	22,62	22,62

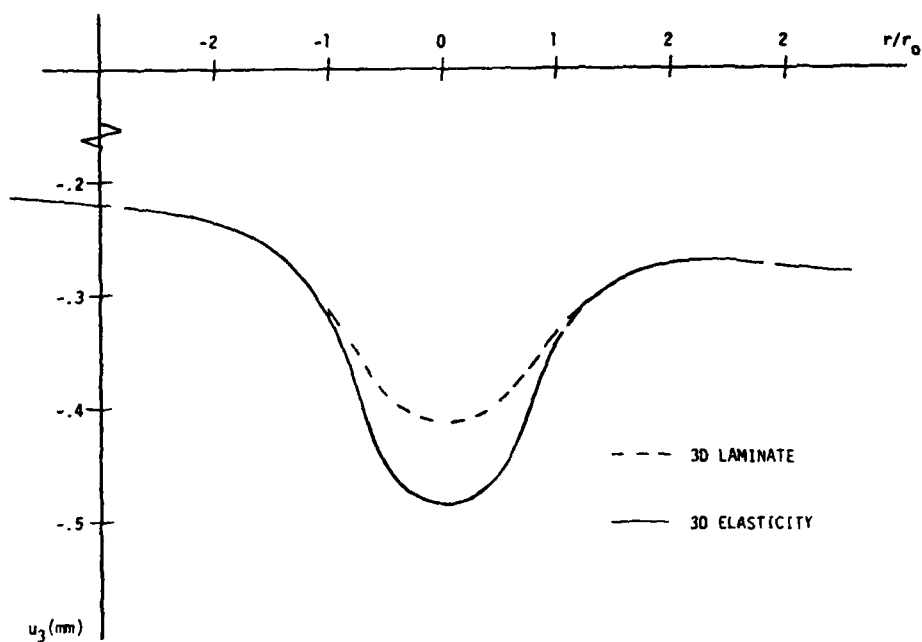


Figure 19. Thick Laminate Local Bending Deformation Comparisons

5. CONCLUSIONS AND RECOMMENDATIONS

A systematic approach to modeling complex interlaminar stress gradients has been developed and applied to two graphite-epoxy laminates. The approach uses variable property 3D finite elements that correctly model force-deformation behavior and constant property 3D finite elements that accurately model ply shear strains. Analyses for this class of problems are difficult, but considerable progress has been made toward solving the computational and modeling issues encountered.

- To model composite laminate force-deformation behavior with 3D finite elements requires, in general, properties that vary through the laminate thickness.
- Interlaminar deformations are not symmetric in a balanced symmetric laminate under symmetric in-plane loads.
- Interlaminar deformations have one plane of symmetry in a balanced symmetric laminate under normal Hertzian contact pressure.

Analysis of the disbanded laminate force deformation behavior indicates a large out-of-plane displacement response occurs without significant internal force redistribution. Comparison with a bonded laminate control model also indicates very little energy is released by the disbond. The new CCL finite element worked well in this application and should be an effective new tool for analyzing interlaminar force-deformation behavior in the vicinity of defects. It is interesting to note that more accurate results can be obtained on reanalysis by simply changing the element specification to CCC as in the P-Version of the finite element method [14].

Analysis of the thick laminate impacted off-center by a small sphere indicates the maximum transverse shear occurs below the surface in the second ply for the $[\pm 45/0_2/\pm 45/0_2/\pm 45/0/90]_{2S}$ layup. This maximum occurs at the contact radius and both transverse shears in material coordinates have a sinusoidal variation around the perimeter. The fiber stresses also vary sinusoidally, but with one-half the period of the transverse shears. Only the in-plane shear stress shows a noticeable effect of the impact site being off-center. The edge closest the spar boundary has slightly higher ply shear. This also occurs in the ply shear strains with the relative increase being

on the order of 10 percent. These results were obtained after several analysis failures caused by mixing CCL and CCC elements in the thickness direction. Based on these results and those from the disbanded laminate the following additional analyses are recommended:

- Analyze the disbanded laminate without any symmetry assumptions.
- Analyze the disbanded laminate with twice as many plies, $[0/\pm 45/0_2/\pm 45/0]_{2S}$, with the disbond in the same location.
- Analyze the impact site with the focus on the bottom ply group.

Now that accurate modeling techniques for finite element analysis of interlaminar behavior have been developed, a combined test/analysis study of the effects of defects is recommended. It would be possible to evaluate energy release rate and possibly other parameters as a measure of how large a delamination can become before unstable propagation takes place. Pre-test analyses could be used to define the critical load conditions and to help locate instrumentation. In order to facilitate this work it would be desirable to interface the PATCHES-III program with a composite material synthesis program [15], [16] for pre- and postprocessing of the finite element results. This would allow basic ply properties and layup data as input and automate the recovery of ply stress-strain data from composite stress-strain results. It would also allow Tsai-Wu, Hill and other failure criteria to be applied to the PATCHES-III results.

6. REFERENCES

1. Spilker, R. L., S. C. Chou, and O. Orringer, "Alternate Hybrid-Stress Elements for Analysis of Multilayer Composite Plates," J. Composite Materials, Vol. 11, pp. 51-70, 1977.
2. Wang, A. S. D. and F. W. Crossman, "Calculation of Edge Stresses in Multi-Layer Laminates by Sub-Structuring," J. Composite Materials, Vol. 12, pp. 76-83, 1978.
3. Pagano, N. J., "Stress Fields in Composite Laminates," AFML TR-77-14, Air Force Materials Laboratory, 1977.
4. Pagano, N. J., "Exact Moduli of Anisotropic Laminates," Composite Materials, Vol. 2, edited by G. P. Sendeckyj, pp. 23-44.
5. Stanton, E. L., "A General Three-Dimensional Computational Model for Nonlinear Composite Structures and Materials," AIAA Paper No. 77-360, in Proceedings AIAA/ASME/SAE 18th SDM Conference, pp. 9-22, 1977.
6. Stanton, E. L. and L. M. Crain, "Application of PATCHES-III to the Three-Dimensional Response of Laminated Composites," NADC Contract N62269-75-C-0438, Final Report, 1975.
7. Kibler, J. J. and S. N. Chatterjee, "Development of Minimechanics Model for 3D Carbon/Carbon Materials," Materials Science Corp., Report TFR 7510, 1975.
8. Williams, J. G., et al., "Recent Developments in the Design Testing and Impact-Damage Tolerance of Stiffened Composite Panels, NASA TM-80077, April 1979.
9. Williams, J. G., et al., "Low-Velocity Impact Damage in Graphite-Fiber Reinforced Epoxy Laminates," Presented at the 34th Annual SAMPE Conference, New Orleans, 1979.
10. Whitney, J. M. and A. W. Leissa, "Analysis of a Simply Supported Laminated Anisotropic Rectangular Plate," AIAA Journal, Vol. 8, pp. 28-38, 1970.
11. Rybicki, E. F., et al., "An Energy Release Rate Approach for Stable Crack Growth in the Free-Edge Effect Delamination Problem," J. Composite Materials, Vol. 11, pp. 470-487, 1977.
12. Greszczuk, L. B., et al., "Investigation of Brittle Fractures in Graphite Epoxy Composites Subjected to Impact," USAAMRDL-TR-75-15 (AD-A012-269), May 1975.
13. Dahan, M. and J. Zarka, "Elastic Contact Between a Sphere and a Semi Infinite Transversely Isotropic Body," Int. J. Solids Structures, Vol. 13, pp. 229-238, 1977.

14. Babuska, I., et al., "The P-Version of the Finite Element Method," Washington University, Report No. WU/CCM-79/1, May 1979.
15. Chamis, C. C., "Computerized Multilevel Analysis for Multilayered Fiber Composites," Computers and Structures, Vol. 3, pp. 467-482, 1973.
16. Stanton, E. L. and L. M. Crain, "Advances in Parametric Cubic Modeling for Finite Element Data Generation," Proceedings Second World Finite Element Conference, Bournemouth, England, pp. 386-496, 1978.

APPENDIX

The PATCHES-III input data for the two laminates analyzed in this report and data modifications necessary to analyze three additional impact cases, Table A-1, are provided in this appendix. The structural response model that provides displacement boundary conditions for the impact site model is included for each case. Also, a schematic of a more detailed model of the impact site that includes the entire 12 plies in the basic repeating ply group is provided.

TABLE A-1. PATCHES-III LAMINATE IMPACT CASES

Case	Impact Site	Sphere Radius	Contact Radius	Status
1	Z1 = 1.91 cm	0.635 cm	1.12 mm	Analyzed
2	Z1 = 1.91 cm	2.540 cm	1.78 mm	Modeled
3	Z1 = 10.16 cm	0.635 cm	1.12 mm	Modeled
4	Z1 = 10.16 cm	2.540 cm	1.78 mm	Modeled

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PATCHES-III INPUT DATA
DISBONDED LAMINATE MODEL

A-3

VERSION 8.2C

W
Z
I

10.32.16. 12/21/77

CASE CONTROL DATA

TITLE = DISORD LAMINATE CASE
SUBTITLE = SYMMETRY BOUNDARY CONDITIONS
TIME = 14
SDC = 10
CHPNT = ELEMENT.CG
OUTPUT
EVERYTHING
BEGIN. ALLK

CAPD FIELD-1, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10

1-	CPDE3	1	19	20	4	3	+C1
2-	+C1	CC1*	17	18	2	1	
3-	CPDE3	2	21	22	6	5	+C2
4-	+C2	CC1*	19	20	4	3	
5-	CPDE3	3	23	24	8	7	+C3
6-	+C3	CC1*	21	22	6	5	
7-	CPDE3	4	25	26	10	9	+C4
8-	+C4	CC1*	23	24	8	7	
9-	CPDE3	5	27	28	12	11	+C5
10-	+C5	CC1*	25	26	10	9	
11-	CPDE3	6	29	30	14	13	+C6
12-	+C6	CC1*	27	28	12	11	
13-	CPDE3	7	31	32	16	15	+C7
14-	+C7	CC1*	29	30	14	13	
15-	CPDE3	8	35	36	20	19	+C8
16-	+C8	CC1*	33	34	18	17	
17-	CPDE3	9	37	38	22	21	+C9
18-	+C9	CC1*	35	36	20	19	
19-	CPDE3	10	39	40	24	23	+C10
20-	+C10	CC1*	37	38	22	21	
21-	CPDE3	11	41	42	26	25	+C11
22-	+C11	CC1*	39	40	24	23	
23-	CPDE3	12	43	44	28	27	+C12
24-	+C12	CC1*	41	42	26	25	
25-	CPDE3	13	45	46	30	29	+C13
26-	+C13	CC1*	43	44	28	27	
27-	CPDE3	14	47	48	32	31	+C14
28-	+C14	CC1*	45	46	30	29	
29-	CPDE3	15	50	50	36	35	+C15
30-	+C15	CC1*	49	49	34	33	
31-	CPDE3	16	51	51	38	37	+C16
32-	+C16	CC1*	50	50	36	35	
33-	CPDE3	17	52	52	40	39	+C17
34-	+C17	CC1*	51	51	38	37	
35-	CPDE3	18	53	53	42	41	+C18
36-	+C18	CC1*	52	52	40	39	
37-	CPDE3	19	55	55	44	43	+C19
38-	+C19	CC1*	54	54	42	41	
39-	CPDE3	20	56	56	46	45	+C20
40-	+C20	CC1*	55	55	44	43	
41-	CPDE3	21	57	57	48	47	+C21
42-	+C21	CC1*	56	56	46	45	
43-	HP2PAT	1	2	1			
44-	HP2PAT	2	3	2			
45-	HP2PAT	3	4	3			
46-	HP2PAT	4	5	4			

FIELD-1, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10
CAPU

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A-6

CARD FIELD-1, FIELD-2, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10

12X-37	-021	-021	-021	+M37
12X-37	-021	-021	-021	+M37

4122-30	-020	-026	+H30
4123	-020	-020	+H30

IRX-61	- .6105	- .007	- .000366	- .0105	- .007	- .000366	- .000366	+M41
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194X-42 -.0175 -.000366-.0175 -.0140 -.000366-.000366+.M41

43	000366-	000366-M43
42	000366-	000366-M42
41	000366-	000366-M41
40	000366-	000366-M40
39	000366-	000366-M39
38	000366-	000366-M38
37	000366-	000366-M37
36	000366-	000366-M36
35	000366-	000366-M35
34	000366-	000366-M34
33	000366-	000366-M33
32	000366-	000366-M32
31	000366-	000366-M31
30	000366-	000366-M30
29	000366-	000366-M29
28	000366-	000366-M28
27	000366-	000366-M27
26	000366-	000366-M26
25	000366-	000366-M25
24	000366-	000366-M24
23	000366-	000366-M23
22	000366-	000366-M22
21	000366-	000366-M21
20	000366-	000366-M20
19	000366-	000366-M19
18	000366-	000366-M18
17	000366-	000366-M17
16	000366-	000366-M16
15	000366-	000366-M15
14	000366-	000366-M14
13	000366-	000366-M13
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9	000366-	000366-M9
8	000366-	000366-M8
7	000366-	000366-M7
6	000366-	000366-M6
5	000366-	000366-M5
4	000366-	000366-M4
3	000366-	000366-M3
2	000366-	000366-M2
1	000366-	000366-M1

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443	44	000366-	0315	0280	000366-	000366+M44

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PAICH	3	8	11	21	33

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PATCH	5	8	11	21	35

PATCH	6	11	21	36
PATCH	7	11	21	37
S	9	11	21	38

PATCH	Q	R	S	T
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PATCH	8	9	12	31
PATCH	8	9	12	31

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94	H	12	22	32

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13	BATCH	8	10	23	36

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24	8	12	22

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BATCH				
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BATCH				

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DATE	TIME	PLACE	REMARKS
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1941	13	13	35
1941	13	13	22
1941	13	13	22
1941	13	13	41

ATCH	17	0	13	23	41
ATCH	20	8	13	23	42
ATCH	21	8	13	23	43

DATE	TIME	LOCATION	WIND	WAVE	SEA	TEMP	WIND	WAVE	SEA	TEMP
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25	13	13	8	13	23	44				
26	13	13	8	13	23	44				
27	13	13	8	13	23	44				
28	13	13	8	13	23	44				
29	13	13	8	13	23	44				
30	13	13	8	13	23	44				
31	13	13	8	13	23	44				

2	1	45.0
2	1	45.0
3	1	-45.0

4	1	0.
4	1	0.
5	1	-45.

7	1	06DE3
7	1	06DE3
6	1	09DE3
6	1	09DE3
12.		
45.		

1	40E3	8	0.
1	00E3	9	45.

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FIELD-1, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10

139-	143F3	10	1	-45.
140-	143F3	11	1	0.
141-	143F3	12	1	-45.
142-	143F3	13	1	45.
143-	143F3	14	1	0.
144-	143F3	15	1	0.
145-	143F3	16	1	45.
146-	143F3	17	1	-45.
147-	143F3	18	1	0.
148-	143F3	19	1	-45.
149-	143F3	20	1	45.
150-	143F3	21	1	0.
151-	143F3	10	3	19
152-	143F3	10	3	20
153-	143F3	10	2	21
154-	143F3	10	2	22
155-	143F3	10	3	23
156-	143F3	10	3	24
157-	143F3	10	3	25
158-	143F3	10	4	26
159-	143F3	10	5	27
160-	143F3	10	5	28
161-	143F3	10	6	29
162-	143F3	10	6	30
163-	143F3	10	7	31
164-	143F3	10	7	32
165-	143F3	10	8	33
166-	143F3	10	8	34
167-	143F3	10	9	35
168-	143F3	10	9	36
169-	143F3	10	10	37
170-	143F3	10	10	38
171-	143F3	10	11	39
172-	143F3	10	11	40
173-	143F3	10	12	41
174-	143F3	10	12	42
175-	143F3	10	12	43
176-	143F3	10	13	44
177-	143F3	10	14	45
178-	143F3	10	14	46
179-	143F3	10	15	47
180-	143F3	10	15	48
181-	143F3	10	16	49
182-	143F3	10	16	50
183-	143F3	10	17	51
184-	143F3	10	17	52

F U L K D A T A C A R D S

PAGE 5

CARD FIELD-1, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10

185-	SDC20	10	18	3	53	41	39	52	
186-	SDC20	10	18	3	53	42	40	52	
187-	SDC20	10	19	3	55	43	41	54	
188-	SDC20	10	19	3	55	44	42	54	
189-	SDC20	10	20	3	56	45	43	55	
190-	SDC20	10	20	3	56	46	44	55	
191-	SDC20	10	21	3	57	47	45	56	
192-	SDC20	10	21	3	57	48	46	56	
193-	SPC2	10	1	411	-.004243	0.	1	+S1	
194-	+S1	7							
195-	SPC2	10	1	421	-.003	0.			
196-	SPC2	10	1	431	-.003	0.			
197-	SPC2	10	1	441	-.004243	0.	1	+S2	
198-	+S2	7							
199-	SPC2	10	1	414	-.004243	0.	1	+S3	
200-	+S3	7							
201-	SPC2	10	1	424	-.003	0.			
202-	SPC2	10	1	434	-.003	0.			
203-	SPC2	10	1	444	-.004243	0.	1	+S4	
204-	+S4	7							
205-	SPC2	10	2	411	-.004243	0.	1	+S5	
206-	+S5	7							
207-	SPC2	10	2	421	-.003	0.			
208-	SPC2	10	2	431	-.003	0.			
209-	SPC2	10	2	441	-.004243	0.	1	+S6	
210-	+S6	7							
211-	SPC2	10	3	411	-.004243	0.	1	+S7	
212-	+S7	7							
213-	SPC2	10	3	421	-.003	0.			
214-	SPC2	10	3	431	-.003	0.			
215-	SPC2	10	3	441	-.004243	0.	1	+S8	
216-	+S8	7							
217-	SPC2	10	4	411	-.004243	0.	1	+S9	
218-	+S9	7							
219-	SPC2	10	4	421	-.003	0.			
220-	SPC2	10	4	431	-.003	0.			
221-	SPC2	10	4	441	-.004243	0.	1	+S10	
222-	+S10	7							
223-	SPC2	10	5	411	-.004243	0.	1	+S11	
224-	+S11	7							
225-	SPC2	10	5	421	-.003	0.			
226-	SPC2	10	5	431	-.003	0.			
227-	SPC2	10	5	441	-.004243	0.	1	+S12	
228-	+S12	7							
229-	SPC2	10	6	411	-.004243	0.	1	+S13	
230-	+S13	7							

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P U L K D A T A C A R D S

CARD FIELD-1, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10

231-	SPC2	10	6	421	-0.003	0.
232-	SPC2	10	6	431	-0.003	0.
233-	SPC2	10	6	441	-0.004243	0.
234-	+S14	7				+S14
235-	SPC2	10	7	411	-0.004243	0.
236-	+S15	7				+S15
237-	SPC2	10	7	421	-0.003	0.
238-	SPC2	10	7	431	-0.003	0.
239-	SPC2	10	7	441	-0.004243	0.
240-	+S16	7				+S16
241-	PARAM	ITER	1400			
242-	END DATA					

PATCHES-III INPUT DATA
LAMINATE IMPACT
STRUCTURAL RESPONSE MODEL

Case 1. (See Computer Listing)

Case 2. Change gridpoints 9, 11, 13, 14, 17, 19, 21, 23.

```
GRID, 9, , -0.356-2, -0.356-2
GRID, 11, , 0.356-2, -0.356-2
GRID, 13, , -0.178-2, -0.178-2
GRID, 15, , 0.178-2, -0.178-2
GRID, 17, , -0.178-2, 0.178-2
GRID, 19, , 0.178-2, 0.178-2
GRID, 21, , -0.356-2, 0.356-2
GRID, 23, , 0.356-2, 0.356-2
```

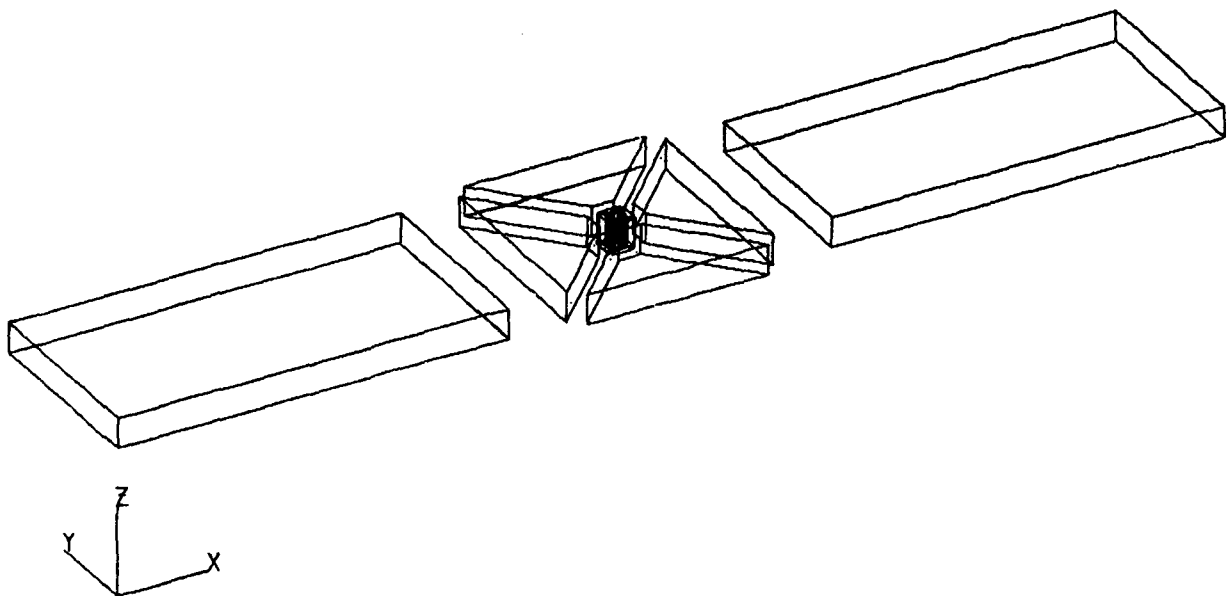
Case 3. Change gridpoints 5, 7, 29, 31 and redefine element number 11 data.

```
GRID, 5, , 10.16-2, -2.0-2
GRID, 7, , -10.16-2, -2.0-2
GRID, 29, , 10.16-2, 2.0-2
GRID, 31, , -10.16-2, 2.0-2
PATCHQ , 11, 7, 31, 25, 1
CPDE3 , 11, 7, 31, 25, 1
+C11 , CCL, 8, 32, 26, 2
SDC10 , 10, 11, 123, 7, 8, 32, 31
SDC10 , 10, 11, 2, 7, 1, 2, 8
SDC10 , 10, 11, 2, 31, 25, 26, 32
SDC10 , 10, 10, 123, 5, 6, 30, 29
```

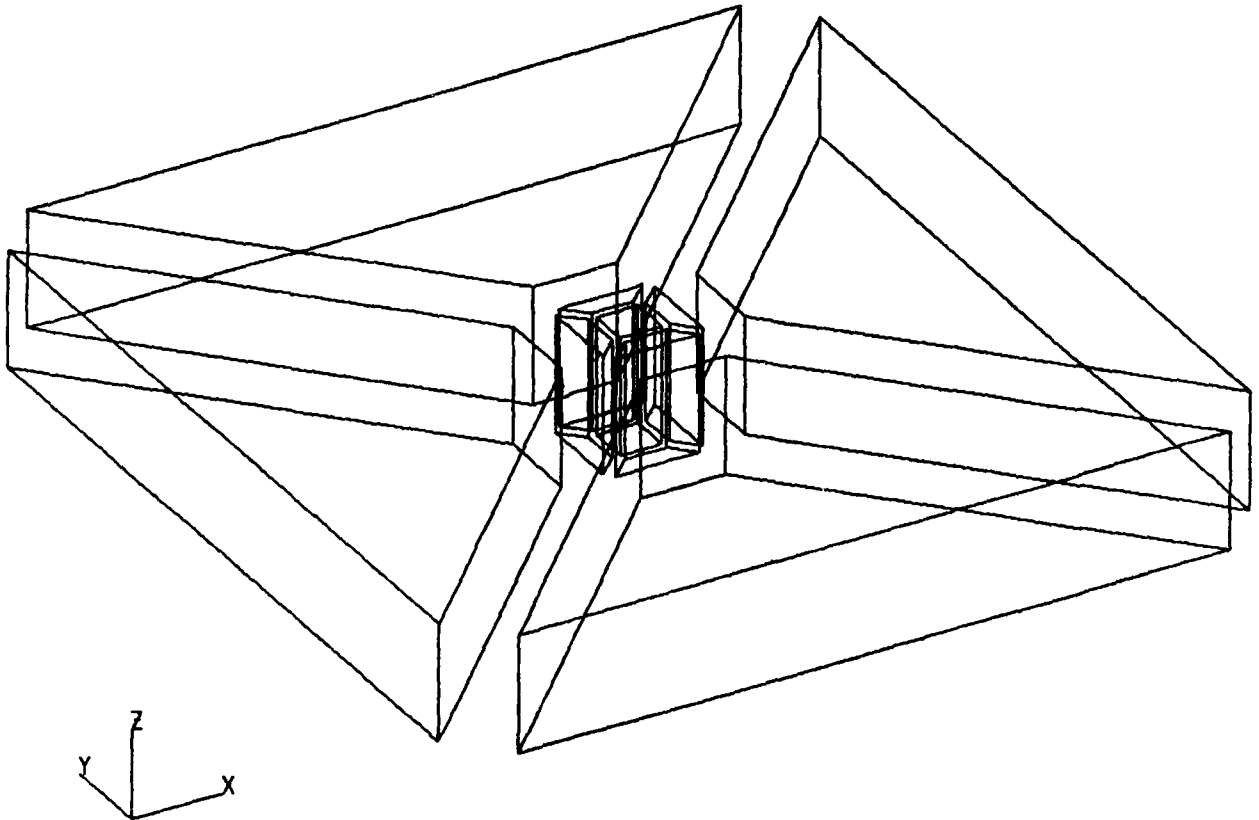
(delete) SDC10 , 10, 8, 123, 1, 2, 26, 25

Case 4. Combine the changes for Cases 2 and 3.

LAMINATE STRUCTURAL RESPONSE MODEL
FOR CASES 3 AND 4



IMPACT SITE STRUCTURAL RESPONSE MODEL
FOR ALL CASES



22.45.06. 78/05/25.

CASE CONTROL DATA

TITLE = NADC LAMINATE IMPACT
 SUBTITLE = 48 PLY MODEL ** VARIABLE PROPERTIES
 SDC = 10
 LOAD = 5
 TIME = 17
 CRPNT = FLEHENT.CG
 OUTPUT
 EVERYTHING
 BEGIN RUN

PAGE 1

R U L K D A T A C A R D S

F1FD=1,F1FD=2,F1FD=3,F1FD=4,F1FD=5,F1FD=6,F1FD=7,F1FD=8,F1FD=9,F1FD=10

CARD

1-	CPDEF 1	13	17	19	15	+C1
2-	+C1 CCL	14	18	20	16	
3-	CPDEF 2	15	19	21	11	+C2
4-	+C2 CCL	16	20	34	12	
5-	CPDEF 3	17	21	23	19	+C3
6-	+C3 CCL	18	22	34	20	
7-	CPDEF 4	9	21	17	13	+C4
8-	+C4 CCL	10	22	18	14	
9-	CPDEF 5	9	13	15	11	+C5
10-	+C5 CCL	10	14	16	12	
11-	CPDEF 6	11	23	27	3	+C6
12-	+C6 CCL	12	34	28	4	
13-	CPDEF 7	21	25	27	23	+C7
14-	+C7 CCL	22	26	28	34	
15-	CPDEF 8	1	25	21	9	+C8
16-	+C8 CCL	2	26	22	10	
17-	CPDEF 9	1	9	11	3	+C9
18-	+C9 CCL	2	10	12	4	
19-	CPDEF 10	3	27	29	5	+C10
20-	+C10 CCL	4	28	30	6	
21-	CPDEF 11	5	29	31	7	+C11
22-	+C11 CCL	6	30	32	8	
23-	GRID 1		-1.905-2-2.0-2			
24-	GRID 3		1.905-2-2.0-2			
25-	GRID 5		8.255-2-2.0-2			
26-	GRID 7		18.415-2-2.0-2			
27-	GRID 9		-.224-2-.224-2			
28-	GRID 11		-.224-2-.224-2			
29-	GRID 13		-.112-2-.112-2			
30-	GRID 15		-.112-2-.112-2			
31-	GRID 17		-.112-2-.112-2			
32-	GRID 19		-.112-2-.112-2			
33-	GRID 21		-.224-2-.224-2			
34-	GRID 23		-.224-2-.224-2			
35-	GRID 25		-1.905-2-2.0-2			
36-	GRID 27		1.905-2-2.0-2			
37-	GRID 29		8.255-2-2.0-2			
38-	GRID 31		18.415-2-2.0-2			
39-	PATCHQ 1	13	17	19	15	
40-	PATCHQ 2	15	19	23	11	
41-	PATCHQ 3	17	21	23	19	
42-	PATCHQ 4	9	21	17	13	
43-	PATCHQ 5	9	13	15	11	
44-	PATCHQ 6	11	23	27	3	
45-	PATCHQ 7	21	25	27	23	
46-	PATCHQ 8	1	25	21	9	

MULLIK DATA CARDS

FIELD=1, FIELD=2, FIELD=3, FIELD=4, FIELD=5, FIELD=6, FIELD=7, FIELD=8, FIELD=9, FIELD=10

[illegible]

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BULK DATA CARDS

FIELD-1, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10

CARD

93-
94-
95-
96-
97-
98-
99-
100-
101-

SNC10 10 7
SNC10 10 8
NPATCH 1 P
MTRX=20 0. 0.
+01 0. 1.
PIAD3 5 1
PARAM ITER 700
PARAM DERIG 1
END DATA

25 27 28 26 25
1 2 26 16
14 20 1.
0. 0. 0.
0. 0. 0.
-1.576+9

+D1
0.
0.

PATCHES-III INPUT DATA

LAMINATE IMPACT

IMPACT SITE MODEL

- Case 1. (See Computer Listing)
- Case 2. (See Computer Listing Page A-29 for gridpoint changes).
- Case 3. Same as Case 1 with new structural response input data.
- Case 4. Same as Case 2 with new structural response input data.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459 460 461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544 545 546 547 548 549 550 551 552 553 554 555 556 557 558 559 560 561 562 563 564 565 566 567 568 569 570 571 572 573 574 575 576 577 578 579 580 581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599 600 601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620 621 622 623 624 625 626 627 628 629 630 631 632 633 634 635 636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651 652 653 654 655 656 657 658 659 660 661 662 663 664 665 666 667 668 669 670 671 672 673 674 675 676 677 678 679 680 681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700 701 702 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719 720 721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739 740 741 742 743 744 745 746 747 748 749 750 751 752 753 754 755 756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780 781 782 783 784 785 786 787 788 789 790 791 792 793 794 795 796 797 798 799 800 801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839 840 841 842 843 844 845 846 847 848 849 850 851 852 853 854 855 856 857 858 859 860 861 862 863 864 865 866 867 868 869 870 871 872 873 874 875 876 877 878 879 880 881 882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899 900 901 902 903 904 905 906 907 908 909 910 911 912 913 914 915 916 917 918 919 920 921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971 972 973 974 975 976 977 978 979 980 981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000 1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020 1021 1022 1023 1024 1025 1026 1027 1028 1029 1030 1031 1032 1033 1034 1035 1036 1037 1038 1039 104

VERSION 8.17C

CASE CONTROL DATA

16.35.07. 78/11709.25

TITLE = INTERPLANAR STRESS GRADIENT ANALYSIS MODEL
 SUBTITLE = 48 PLY LAMINATE

SDC = 10

LOAD = 16

TIME = 30

RSTART = CG

OUTPUT

SET 1 = ALL

VOLUME = 1

FOTSP = 1

ESTRAIN = 1

FSTRESS = 1

ELOAD = 1

EPODCE = 1

MSTRESS = 1

MSTRAIN = 1

BEGIN BULK

QUICK DATA CARDS

FIELD-1, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10

CARD

1-	CPDE3	21	2	2	22	A2	+C1
2-	+C1		1	1	21	81	
3-	CPDE3	41	2	2	42	22	+C2
4-	+C2		1	1	41	21	
5-	CPDE3	61	2	2	62	42	+C3
6-	+C3		1	1	61	41	
7-	CPDE3	81	2	2	82	62	+C4
8-	+C4		1	1	81	61	
9-	CPDE3	121	A2	22	122	1A2	+C5
10-	+C5		A1	21	121	1A1	
11-	CPDE3	141	22	42	142	122	+C6
12-	+C6		21	41	141	121	
13-	CPDE3	161	42	62	162	142	+C7
14-	+C7		41	61	161	141	
15-	CPDE3	181	62	A2	182	162	+C8
16-	+C8		61	A1	181	161	
17-	CPDE3	22	3	3	23	A3	+C9
18-	+C9		2	2	22	A2	
19-	CPDE3	42	3	3	43	23	+C10
20-	+C10		2	2	42	23	
21-	CPDE3	62	3	3	63	43	+C11
22-	+C11		2	2	62	42	
23-	CPDE3	A2	3	3	A3	63	+C12
24-	+C12		2	2	A2	62	
25-	CPDE3	122	83	23	123	1P3	+C13
26-	+C13		A2	22	122	1P2	
27-	CPDE3	142	23	43	143	123	+C14
28-	+C14		22	42	142	122	
29-	CPDE3	162	43	63	163	143	+C15
30-	+C15		42	62	162	142	
31-	CPDE3	182	63	A3	183	163	+C16
32-	+C16		62	A2	182	162	
33-	CPDE3	23	4	4	24	84	+C17
34-	+C17		3	3	23	A3	
35-	CPDE3	43	4	4	44	24	+C18
36-	+C18		3	3	43	23	
37-	CPDE3	63	4	4	64	44	+C19
38-	+C19		3	3	63	43	
39-	CPDE3	A3	4	4	A4	64	+C20
40-	+C20		3	3	A3	63	
41-	CPDE3	123	A4	24	124	184	+C21
42-	+C21		A3	23	123	183	
43-	CPDE3	143	24	44	144	124	+C22
44-	+C22		23	43	143	123	
45-	CPDE3	163	44	64	164	144	+C23
46-	+C23		43	63	163	143	

RULK DATA CARDS

FIELD=1, FIELD=2, FIELD=3, FIELD=4, FIELD=5, FIELD=6, FIELD=7, FIELD=8, FIELD=9, FIELD=10

47-	CPNE3	183	64	84	184	164	+C24
48-	+C24		83	83	183	163	
49-	CPNE3	24	5	5	25	85	+C25
50-	+C25		4	4	24	84	
51-	CPNE3	44	5	5	45	25	+C26
52-	+C26		4	4	44	24	
53-	CPNE3	64	5	5	65	45	+C27
54-	+C27		4	4	64	44	
55-	CPNE3	84	5	5	85	65	+C28
56-	+C28		4	4	84	64	
57-	CPNE3	124	85	25	125	185	+C29
58-	+C29		84	24	124	184	
59-	CPNE3	144	25	45	145	125	+C30
60-	+C30		24	44	144	124	
61-	CPNE3	164	45	65	165	145	+C31
62-	+C31		44	64	164	144	
63-	CPNE3	184	65	85	185	165	+C32
64-	+C32		64	84	184	164	
65-	GRID	1		0	0	63398-2	
66-	GRID	2		0	0	62078-2	
67-	GRID	3		0	0	60758-2	
68-	GRID	4		0	0	58115-2	
69-	GRID	5		0	0	58115-2	
70-	GRID	81		7920-3	7920-3	63398-2	
71-	GRID	82		7920-3	7920-3	62078-2	
72-	GRID	83		7920-3	7920-3	60758-2	
73-	GRID	84		7920-3	7920-3	58115-2	
74-	GRID	85		7920-3	7920-3	58115-2	
75-	GRID	121		2240-2	2240-2	63398-2	
76-	GRID	122		2240-2	2240-2	62078-2	
77-	GRID	123		2240-2	2240-2	60758-2	
78-	GRID	124		2240-2	2240-2	58115-2	
79-	GRID	125		2240-2	2240-2	58115-2	
80-	GRID	141		2240-2	2240-2	63398-2	
81-	GRID	142		2240-2	2240-2	62078-2	
82-	GRID	143		2240-2	2240-2	60758-2	
83-	GRID	144		2240-2	2240-2	58115-2	
84-	GRID	145		2240-2	2240-2	58115-2	
85-	GRID	161		2240-2	2240-2	63398-2	
86-	GRID	162		2240-2	2240-2	62078-2	
87-	GRID	163		2240-2	2240-2	60758-2	
88-	GRID	164		2240-2	2240-2	58115-2	
89-	GRID	165		2240-2	2240-2	58115-2	
90-	GRID	181		2240-2	2240-2	63398-2	
91-	GRID	182		2240-2	2240-2	62078-2	
92-	GRID	183		2240-2	2240-2	60758-2	

R U N K D A T A C A R D S

CARD FIELD-1, FIELD-2, FIELD-3, FIELD-4, FIELD-5, FIELD-6, FIELD-7, FIELD-8, FIELD-9, FIELD-10

.2240-2-.2240-2, 58115-2

.2240-2-.2240-2, 0

GRTO 184

GRTO 184

LINEPC 1 82

LINEPC 2 A3

LINEPC 3 A4

LINEPC 4 A5

PATCHR 21 1

PATCHR 22 2

PATCHR 23 3

PATCHR 24 4

PATCHR 41 1

PATCHR 42 2

PATCHR 43 3

PATCHR 44 4

PATCHR 61 1

PATCHR 62 2

PATCHR 63 3

PATCHR 64 4

PATCHR 81 1

PATCHR 82 2

PATCHR 83 3

PATCHR 84 4

PATCHQ 121 182

PATCHQ 122 183

PATCHQ 123 184

PATCHQ 124 185

PATCHQ 141 122

PATCHQ 142 123

PATCHQ 143 144

PATCHQ 144 145

PATCHQ 145 146

PATCHQ 161 142

PATCHQ 162 143

PATCHQ 163 144

PATCHQ 164 145

PATCHQ 181 162

PATCHQ 182 163

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284-	DPATQ	44	1	165	164	184	185			
285-	DPATQ	144	2	165	164	184	185			
286-	DPATQ	284	3	165	164	184	185			
287-	DPATCH	1	P	10	1	1	21	R1		
288-	DPATCH	2	P	10	1	1	41	21		
289-	DPATCH	3	P	10	1	1	61	41		
290-	DPATCH	4	P	10	1	1	81	61		
291-	MTX-10	1.0		.942809	.745356	0.				
292-	+MX10	1.0		.942809	.745356	0.				+MX10
293-	PLNAD3	16	21	.942809	.745356	0.				
294-	PLNAD3	16	41	2	-1.814+9					
295-	PLNAD3	16	61	3	-1.814+9					
296-	PLNAD3	16	R1	4	-1.814+9					
297-	END DATA									

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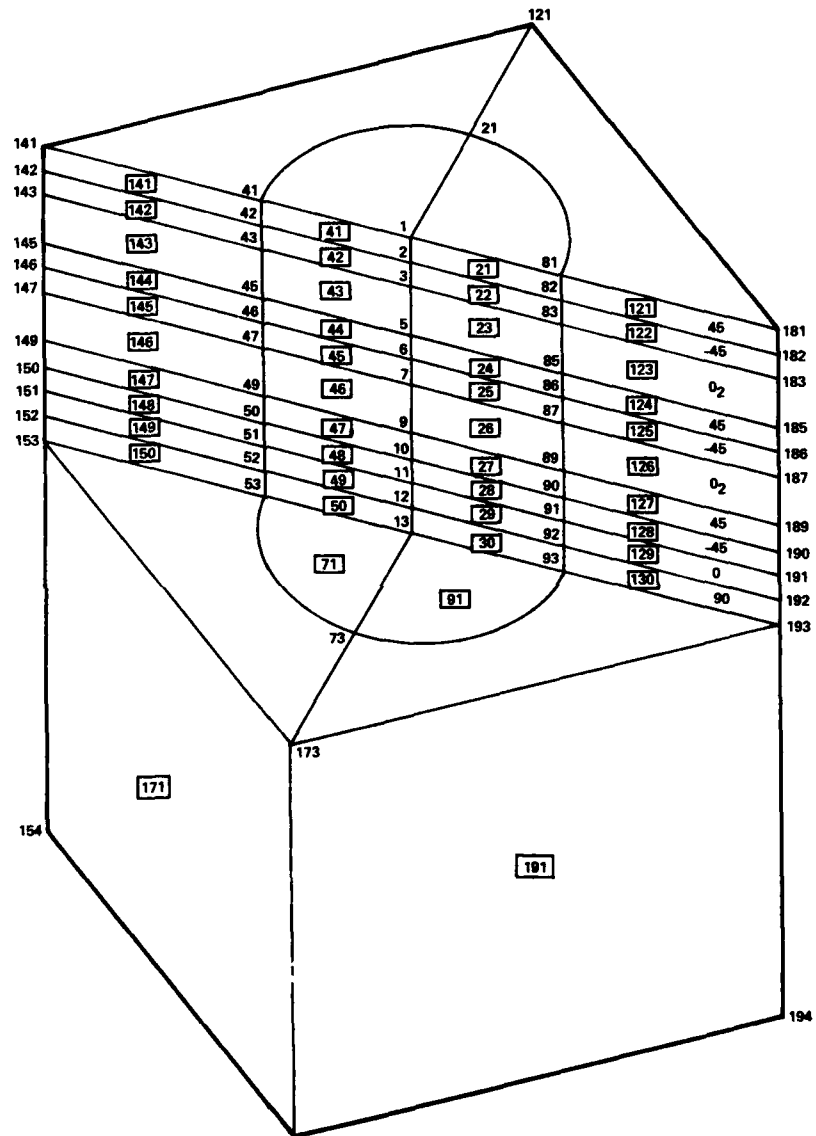
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2.	GRID 2	0.0	0.0	.62078-2
3.	GRID 3	0.0	0.0	.60758-2
4.	GRID 4	0.0	0.0	.58115-2
5.	GRID 5	0.0	0.0	.0
6.	GRID A1	.1259-2	.1259-2	.63398-2
7.	GRID A2	.1259-2	.1259-2	.62078-2
8.	GRID A3	.1259-2	.1259-2	.60758-2
9.	GRID A4	.1259-2	.1259-2	.58115-2
10.	GRID A5	.1259-2	.1259-2	.0
11.	GRID 121	.3560-2	.3560-2	.63398-2
12.	GRID 122	.3560-2	.3560-2	.62078-2
13.	GRID 123	.3560-2	.3560-2	.60758-2
14.	GRID 124	.3560-2	.3560-2	.58115-2
15.	GRID 125	.3560-2	.3560-2	.0
16.	GRID 141	.3560-2	.3560-2	.63398-2
17.	GRID 142	.3560-2	.3560-2	.62078-2
18.	GRID 143	.3560-2	.3560-2	.60758-2
19.	GRID 144	.3560-2	.3560-2	.58115-2
20.	GRID 145	.3560-2	.3560-2	.0
21.	GRID 161	.3560-2	.3560-2	.63398-2
22.	GRID 162	.3560-2	.3560-2	.62078-2
23.	GRID 163	.3560-2	.3560-2	.60758-2
24.	GRID 164	.3560-2	.3560-2	.58115-2
25.	GRID 165	.3560-2	.3560-2	.0
26.	GRID 181	.3560-2	.3560-2	.63398-2
27.	GRID 182	.3560-2	.3560-2	.62078-2
28.	GRID 183	.3560-2	.3560-2	.60758-2
29.	GRID 184	.3560-2	.3560-2	.58115-2
30.	GRID 185	.3560-2	.3560-2	.0
31.	END DATA			

PATCHES-III SCHEMATIC
LAMINATE IMPACT
IMPACT SITE 12 PLY MODEL

The input data for this model are a direct extension of the 4 ply model with 32 finite elements increased to 88 finite elements. If the findings of the present report are used, then one plane of symmetry (e_1, e_3) can be used to reduce the number of elements. In this instance the topologically equivalent model would have 66 finite elements. To reduce this number further will require a change to the basic layout used in this report.

IMPACT SITE 12 PLY MODEL



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