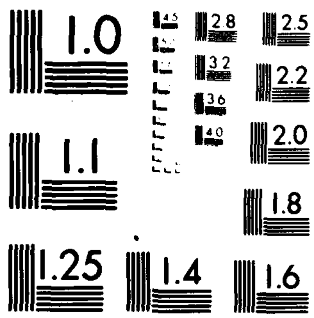

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A ZERO-ONE DICHOTOMY THEOREM FOR r-SEMI-STABLE LAWS ON INFINITE DIMENSIONAL LINEAR SPACES

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ABSTRACT

Let μ be an r-semistable probability measure on a real linear space E. It is shown that the μ-measure of any translate of an arbitrary measurable linear subspace over certain countable subfield of reals is 0 or 1. This result yields immediately the 0 - 1 laws for stable measures of Dudley-Kanter (Proc. Amer. Math. Soc., 45(1974), 245 - 252) and also a more recent 0 - 1 law of Fernique for quasi-stable measures which is included in his ISI lectures of September, 1978. It is also shown that r-semi-stable measures - like stable ones - are continuous, i.e., they assign zero mass to singletons.

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I. INTRODUCTION

Let $(E, \mathfrak{F})$ be a measurable vector space in the sense of [2], and μ a stable probability measure (p. m.) on E. Recently, Dudley-Kanter [2] have shown that the μ-measure of certain measurable subspaces of E is 0 or 1. More recently Fernique exhibited a similar 0 - 1 law for what he calls quasi-stable p. measures. A natural and nontrivial generalization of stable p. measures is the class of r-semi-stable p. measures, which was first introduced and studied on the real line R by P. Lévy [6]. Later Kruglov [3] obtained a quite explicit form of the characteristic function of r semistable p. measures on R and showed that this class have many properties similar to those exhibited by stable probability measures. (This in Hilbert space setting is also shown in Kruglov [4] and Kumar [5]). Partly motivated from these papers we raised and completely answered the question whether r-semi-stable p. measures share with stable measures the 0 - 1 dichotomy results obtained in [2]. Explicitly we prove that if $(E, \mathfrak{F})$ is a measurable vector space over R, μ a r-semi-stable p.m. (see §2) on $(E, \mathfrak{F})$ and G a measurable subspace over the field Q(c), the smallest subfield containing Q, the rationals, and $c = c(r)$, then $\mu(G - z) = 0$ or 1, for every $z \in E$ (Theorem 3.1). This result includes and, in fact, extends the 0 - 1 theorems for stable p. measures obtained in [2] (Corollary 3.2); also the method of proof of the result includes a recent 0 - 1 dichotomy theorem of Fernique (ISI Calcutta, Lectures '78) for quasi-stable p. measures (Corollary 3.3). Further, we also show that, like stable p. measures, non-degenerate r-semistable p. measures are continuous; that is, they assign zero mass to singletons (Corollary 3.4). Our proof of the 0 - 1 dichotomy theorem seems new as well as simpler than those in [2] (we use only the definition of canvolution and Fubini's theorem); in particular, we do not require any number theory results which was not the case in the proofs of [2].

2. PRELIMINARIES

Let $(E, \mathfrak{F})$ be a measurable vector space and μ be a p.m. on $\mathfrak{F}$. Let $r \in (0, 1)$; then μ is called r-semistable if there is a constant $c(r) = c$ with $0 < c \neq 1$ and a semigroup $\{\mu^s; s > 0\}$ of p. measures on $\mathfrak{F}$ and a sequence $\{x_m\}$ in E such that the following hold

$$\mu^1 = \mu \tag{2.1}$$

$$\mu^{r^m} = T_{c^m} \mu * \delta_{x_m} , \tag{2.2}$$

for each $m = 1, 2, \ldots$, where for $a > 0$, $T_a \mu$ denotes the measure $T_a \mu(B) = \mu(a^{-1} B)$, for every $B \in \mathfrak{F}$ and $*$ denotes the usual canvolution.

The above definition is motivated from a characterization of a class of measures also called r-semistable on locally convex topological vector spaces (LCTVS) obtained in [1]. It follows from [1] that our results are applicable for r-semistable (and hence stable and Gaussian) measures studied in [1].

3. 0 - 1 DICHOTOMY THEOREM FOR r-SEMI-STABLE MEASURES

The main result we propose to prove is the following:

Theorem 3.1. Let μ be a r-semistable p.m. on a measurable vector space $(E, \mathcal{F})$ over R and let G be a subspace over the subfield $Q[c]$ such that $G \in \mathcal{F}$ (c is the constant appearing in (2.2)). Then $\mu(G - z) = 0$ or 1, for all $z \in E$.

Proof. Let $z_1 \in E$ and assume that $\mu(G-z_1) > 0$. We will show that $\mu(G-z_1) = 1$. Choose an integer n_r so that $0 < 1/n_r < 1-r$. Let

$$\mathcal{X} = \{G - x \mid \mu(G-x) > 0 \text{ or } \mu^{1/n_r}(G-x) > 0\} \subseteq E/G,$$

$$<\mathcal{X}> = \text{linear span of } \mathcal{X} \text{ in } E/G \text{ over the field } Q[c], \text{ and}$$

$$G_0 = \text{inverse image of } <\mathcal{X}> \text{ under natural projection} = \bigcup <\mathcal{X}>.$$

Then G_0 is a vector subspace of E over $Q[c]$ and clearly, $G_0 \in \mathcal{F}$, since G_0 is a countable union of sets in $\mathcal{F}$.

For the sake of clairity, the remainder of the proof will be divided into seven parts.

(i) $\mu^{1-r} * \delta_{x(1)}(G_0) = 1$.

Proof of (i). Observe that $\mu(G_0 - r^{-1/\alpha} y) = 0$, for all $y \in G_0^c$, and that $\mu = \mu^r * \mu^{1-r} = T_c \mu * \mu^{1-r} * \delta_{x(1)}$.

Thus

$$0 < \mu(G_0) = \int_E T_c \mu (G_0 - y) \mu^{1-r} * \delta_{x(1)} (dy)$$

$$= \int_{G_0} \mu(G_0 - c^{-1} y) \mu^{1-r} * \delta_{x(1)} (dy)$$

$$= \mu(G_0) \mu^{1-r} * \delta_{x(1)} (G_0).$$

Consequently, $\mu^{1-r} * \delta_{x(1)} (G_0) = 1$.

(ii) $\mu^{1/n_r}(G_o) = 1$.

Proof of (ii). Since $\mu = \mu^{1/n_r} * (\mu^{1/n_r})^{*(n_r-1)}$, we have

$$0 < \mu(G-z_1) = \int_E \mu^{1/n_r}(G - z_1 - y)\,(\mu^{1/n_r})^{*(n_r-1)}(dy).$$

Thus there exists $y \in E$ so that $\mu^{1/n_r}(G-z_1-y) > 0$, and hence $\mu^{1/n_r}(G_o) > 0$.

Now $\mu^{1-r} * \delta_{x(1)} = \mu^{1/n_r} * \mu^{1-r-1/n_r} * \delta_{x(1)}$, and so, from (i),

$$1 = \mu^{1-r} * \delta_{x(1)}(G_o) = \int_E \mu^{1-r-1/n_r} * \delta_{x(1)}(G_o-y)\mu^{1/n_r}(dy),$$

which implies that $\mu^{1-r-1/n_r} * \delta_{x(1)}(G_o-y) = 1$ a.s. $[\mu^{1/n_r}]$. Since $\mu^{1/n_r}(G_o) > 0$, it follows that $\mu^{1-r-1/n_r} * \delta_{x(1)}(G_o) = 1$.

Consequently,

$$\begin{aligned} 1 = \mu^{1-r} * \delta_{x(1)}(G_o) &= \int_{G_o} \mu^{1/n_r}(G_o-y)\,\mu^{1-r-1/n_r} * \delta_{x(1)}(dy) \\ &= \mu^{1/n_r}(G_o)\,\mu^{1-r-1/n_r} * \delta_{x(1)}(G_o) \\ &= \mu^{1/n_r}(G_o). \end{aligned}$$

(iii) $\mu(G_o) = 1$.

Proof of (iii). It follows from (ii) that

$$\begin{aligned} \mu(G_o) &= \int_{G_o} (\mu^{1/n_r})^{*(n_r-1)}(G_o-y)\mu^{1/n_r}(dy) \\ &= (\mu^{1/n_r})^{*(n_r-1)}(G_o)\,\mu^{1/n_r}(G_o) \\ &= (\mu^{1/n_r})^{*(n_r-1)}(G_o) \\ &= (\mu^{1/n_r}(G_o))^{n_r-1} \\ &= 1. \end{aligned}$$

We will use the fact that $\mu(G_o) = 1$ to conclude that $\mu(G-z_1) = 1$ (see (vii)). To this end, we proceed.

Recall that G_o is a countable (possibly finite) union of disjoint cosets of G. Let $\{x_1, x_2, \ldots\}$ be a sequence of distinct points in E so that

$G_0 = \dot{\bigcup}_k G-x_k$ (disjoint union). Clearly, we may assume, without loss of generality, that $\mu(G-x_1) \geq \mu(G-x_2) \geq \cdots$. Let N_1 be the largest integer so that $\mu(G-x_1) = \mu(G-x_{N_1})$. For the sake of simplicity of notation, let $t = t(m) = c^m$, $m = 1,2,\ldots$ and let $\nu_t = \mu^{1-r^m} * \delta_{x(m)}$. Then $\mu = T_t\mu*\nu_t$, for any t.

(iv) For each t, $\nu_t(\dot{\bigcup}_{k=1}^{N_1} G-x_n+tx_k) = 1$, for $1 \leq n \leq N_1$.

<u>Proof of (iv)</u>. Observe that if $y \in G-x_k$, then $G-x_n-ty = G-x_n+tx_k$, for all n and k. Thus.

$$\begin{aligned}\mu(G-x_n) &= \int_{G_0}\nu_t(G-x_n-ty)\mu(dy)\\ &= \sum_k \nu_t(G-x_n+tx_k)\mu(G-x_k), \quad (3.1)\end{aligned}$$

for n = 1, 2, ... Now, for $1 < n < N_1$, we have

$$\begin{aligned}\mu(G-x_n) &= \sum_k \nu_t(G-x_n+tx_k)\mu(G-x_k)\\ &\leq \mu(G-x_n)\sum_k \nu_t(G-x_n+tx_k)\\ &= \mu(G-x_n)\nu_t(\dot{\bigcup}_k G-x_n+tx_k)\\ &\leq \mu(G-x_n).\end{aligned}$$

Thus

$$\mu(G-x_n)\nu_t(G-x_n+tx_k) = \mu(G-x_k)\nu_t(G-x_n+tx_k),$$

for $1 \leq n \leq N_1$ and any k, which implies that $\nu_t(G-x_n+tx_k) = 0$ for $1 \leq n \leq N_1$ and $k > N_1$.

Thus

$$\begin{aligned}\mu(G-x_n) &= \sum_{k=1}^{N_1} \nu_t(G-x_n+tx_k)\mu(G-x_k),\\ &= \mu(G-x_n)\sum_{k=1}^{N_1} \nu_t(G-x_n+tx_k)\\ &= \mu(G-x_n)\nu_t(\dot{\bigcup}_{k=1}^{N_1} G-x_n+tx_k),\end{aligned}$$

for $1 \leq n \leq N_1$.

Hence

$$1 = \nu_t\left(\bigcup_{k=1}^{N_1} G-x_n+tx_k\right),$$

for $1 \leq n \leq N_1$, since $\mu(G-x_n) = \mu(G-x_1) > 0$ for $1 \leq n \leq N_1$.

(v) $N_1 = 1$ or, equivalently, $(G-x_1) > \mu(G-x_k)$, for all $k > 1$.

Proof of (v). Suppose $N_1 \geq 2$ and consider the $2xN_1$ array M_1:

$$\begin{array}{ccccc} G-x_1+tx_1 & G-x_1+tx_2 & G-x_1+tx_3 & \ldots & G-x_1+tx_{N_1} \\ G-x_2+tx_1 & G-x_2+tx_2 & G-x_2+tx_3 & \ldots & G-x_2+tx_{N_1} \end{array}.$$

By (iv), the ν_t-measure of row 1 of M_1 is 1. Thus there is an integer k_1, $1 \leq k_1 \leq N_1$, so that $\nu_t(G-x_1+tx_{k_1}) > 0$, for infinitely many values of t. Now, the ν_t-measure of row 2 is also 1 (by (iv) again), which implies that $G-x_1+tx_{k_1}$ intersects row 2, for infinitely many values of t. Thus there is an integer k_2, $1 \leq k_2 \leq N_1$, so that $G-x_1+tx_{k_1} = G-x_2+tx_{k_2}$, for infinitely many values of t. Consequently, there are integers k_1 and k_2, $1 \leq k_1 \leq N_1$ $1 \leq k_2 \leq N_1$, so that

$$G-x_1+x_2 = G-t(x_{k_2}-x_{k_1}), \quad (3.2)$$

for infinitely many values of t. In particular, these exist t_1 and t_2, $t_1 \neq t_2$, so that $G-t_1(x_{k_2}-x_{k_1}) = G - t_2(x_{k_2}-x_{k_1})$ which implies that $G = G+(t_1-t_2)(x_{k_2}-x_{k_1})$ and so, $(t_1-t_2)(x_{k_2}-x_{k_1}) \in G$ from which it follows that $G-x_{k_1} = G-x_{k_2}$. Consequently, since G_0 is a disjoint union, we have $k_1 = k_2$ which implies, from (3.2), that $G-x_1 = G-x_2$. But $G-x_1 \neq G-x_2$. Hence (v)follows.

(vi) For each t, $\nu_t(G-x_1+tx_1) = 1$.

Proof of (vi). This is immediate from (iv) and (v)

(vii) $\mu(G-z_1) = \mu(G_0)$.

Proof of (vii). Suppose $\mu(G-z_1) < \mu(G_0)$

Then $\mu(G-x_2) > 0$. Let N_2 be the largest integer so that $\mu(G-x_2) = \mu(G-x_{N_2})$.

Observe that, by (vi), we have that for each t, $\nu_t(G-x_n+tx_1) = 0$, for all $n \geq 2$; otherwise, we get $G-x_n = G-x_1$, for some $n \geq 2$.

Thus, by (3.1), for $2 \leq n \leq N_2$,

$$\begin{aligned}\mu(G-x_n) &= \nu_t(G-x_n+tx_1) + \sum_{k\geq 2} \nu_t(G-x_n+tx_k)\mu(G-x_k)\\ &= \sum_{k\geq 2}\nu_t(G-x_n+tx_k)\overline{\mu}(G-x_k)\\ &\leq \mu(G-x_n)\sum_{k\geq 2}\nu_t(G-x_n+tx_k)\\ &= \mu(G-x_n)\nu_t(\bigcup_{k\geq 2} G-x_n+tx_k)\\ &\leq \mu(G-x_n).\end{aligned}$$

It follows that

$\mu(G-x_n)\nu_t(G-x_n+tx_k) = \mu(G-x_k)\nu_t(G-x_n+tx_k)$, for $2 \leq n \leq N_2$ and any $k \geq 2$, which implies that $\nu_t(G-x_n+tx_k) = 0$, for $2 \leq n \leq N_2$ and $k > N_2$.

Consequently,

$$\begin{aligned}\mu(G-x_n) &= \sum_{k=2}^{N_2}\nu_t(G-x_n+tx_k)\mu(G-x_k)\\ &= \mu(G-x_n)\sum_{k=2}^{N_2}\nu_t(G-x_n+tx_k)\\ &= \mu(G-x_n)\ \nu_t\ (\bigcup_{k=2}^{N_2} G-x_n+tx_k)\ ,\end{aligned}$$

for $2 \leq n \leq N_2$.

Hence, for all t,

$$1 = \nu_t(\bigcup_{k=2}^{N_2} G-x_n+tx_k), \quad (3.3)$$

for $2 \leq n \leq N_2$, since $\mu(G-x_n) = \mu(G-x_2) > 0$, for $2 \leq n \leq N_2$.

Observe that, by (vi), $\nu_t(G-x_2+tx_2) = 0$; otherwise, $G-x_2+tx_2 = G-x_1+tx_1$ which implies that $G-x_1 = G-x_2$. Consequently, from (3.3), $N_2 \geq 3$, and so, G_0 contains at least three disjoint cosets of G. Now consider the $2x(N_2-1)$ array M_2:

$$G-x_2+tx_2 \quad G-x_2+tx_3 \quad G-x_2+tx_4 \quad \ldots \quad G-x_2+tx_{N_2}$$

$$G-x_3+tx_2 \quad G-x_3+tx_3 \quad G-x_3+tx_4 \quad \ldots \quad G-x_3+tx_{N_2}.$$

Observe that the ν_t-measure of each row of M_2 is equal to 1. Now proceed, as in (v), to show that there exist integers k_1 and k_2, $2 \leq k_1 \leq N_2$, $2 \leq k_2 \leq N_2$, $k_1 \neq k_2$, so that

$$G-x_2+x_3 = G-t(x_{k_2}-x_{k_1}), \quad (3.4)$$

for infinitely many values of t. It follows, from (3.4), like in (v), that $k_1 = k_2$. Consequently, by (3.4), $G-x_2 = G-x_3$. This is a contradiction! Hence our initial assumption must be false and it follows that $\mu(G-z_1) = \mu(G_0)$.

To complete the proof of the theorem, observe that, by (iii) and (vii), we have $\mu(G-z_1) = \mu(G_0) = 1$.

In view of the last sentence of the previous section, we have the analogue of Theorem 3.1 for stable and Gaussian measures if the measures are K-regular and are defined on the Borel σ-algebra of a complete LCTVS. In the following corollary, we show, however, that the same result can be recovered from Theorem 3.1 even if the stable measures μ is defined on a measurable vector space $(E, \mathfrak{F})$ provided μ has the index; i.e. there exists an $\alpha > 0$ such that for every $a > 0$, $b > 0$, $T_a u * T_b u = T_{(a^\alpha + b^\alpha)^{1/\alpha}} \mu * \delta_x$, for some $x \in E$. This corollary contains and extends various results of [2]; we do not, however, deal with 0 - 1 laws when G belongs to the completed σ-algebra.

Corollary 3.2: Let $(E, \mathcal{F})$ be a measurable vector space and let G be a rational subspace of E, $G \in \mathcal{F}$. Then

(i) If μ is a strictly stable p.m. of index α on $(E, \mathcal{F})$, then for all $z \in E$, $\mu(G - z) = 0$ or 1.

(ii) If μ is a stable p.m. of index α on $(E, \mathcal{F})$, then $\mu(G) = 0$ or 1.

Proof: (i) Assume μ is strictly stable of index α and set $\mu^s = T_{s^{1/\alpha}} \mu$. Then $\{\mu^s | s > 0\}$ is a semigroup with $\mu^1 = \mu$ and (2.1), (2.2) are satisfied for all $r > 0$, with $x(m) = \theta$, and $c = s^{1/\alpha}$. Then, it is easy to see that μ is a r-semistable p.m. for all $0 < r < 1$. Choose r_0, $0 < r_0 < 1$, so that $r_0^{1/\alpha}$ is rational. Then $Q(r_0^{1/\alpha}) = Q$. Now apply Theorem 3.1 to obtain the desired result.

(ii) Let μ be a stable p.m. of index α and assume that $\mu(G) > 0$. Let $\nu = \mu * T_{-1} \mu$ be the symmetrization of μ. Then ν is a strictly stable p.m. of index α. Observe that

$$\begin{aligned} \nu(G) &= \int_E \mu(G + y)\mu(dy) \\ &\geq \int_G \mu(G + y)\, \mu(dy) \\ &= (\mu(G))^2 > 0 . \end{aligned}$$

Thus, by (i), $\nu(G) = 1$, and so $\mu(G + y) = 1$ a.s. (μ) which implies that $\mu(G) = 1$.

The following corollary shows that the method of proof of Theorem 3.1 also yields the 0 - 1 dichotomy theorem for quasi-stable measures recently obtained by Fernique who uses a non-trivial inequality of Kantor for his proof. Our proof, as we noted earlier, uses only elementary facts about convolution. Now we recall the definition of quasi-stable as introduced by Fernique. Let μ be a p. measure on a measurable vector space $(E, \mathfrak{F})$, then μ is said to be quasi-stable if $\mu^{*2} = T_c\mu$, for some $c > 0$, $c \neq 1$.

<u>Corollary 3.3</u>: Let $(E, \mathfrak{F})$ be a measurable vector space and μ be quasi-stable on E. Let G be $Q(c)$ vector space which belongs to $\mathfrak{F}$. Then $\mu(G - z) = 0$ or 1, for every $z \in E$.

<u>Proof</u>: Let $\mu(G - z_1) > 0$ and let $\mathcal{H}' = \{G - x: \mu(G - x) > 0\}$ and define G_0 as in the beginning of the proof of Theorem 3.1 with $\mathcal{H}$ replaced by $\mathcal{H}'$. Since

$$0 < \mu(G_0) = T_c\mu(G_0) = \mu^{*2}(G_0) = \int_{G_0} \mu(G_0 - x)\, \mu(dx)$$

(as $x \in G_0^c$ implies $\mu(G_0 - x) = 0$), we have $\mu(G_0) = 1$. Now the definition of quasi-stability implies $\mu^{*2^m} = T_{c^m}\mu$; hence $\mu = T_{(1/c)^m}\, \mu^{*2^m} = T_{(1/c)^m}\, \mu^{*2^m-1} * T_{(1/c)^m}\, \mu$. Setting $(1/c)^m = t(m)$ and $T_{(1/c)^m}\, \mu^{*2^m-1} = \nu_t$, we see that $\mu = \nu_t * T_t\mu$. Now repeating the proof of (iv) to (vii) of Theorem 3.1 without any change at all, one shows $\mu(G - z_1) = 1$. Completing the proof.

The following corollary shows that nondegenerate r-semistable p. measures cannot have positive point mass.

Corollary 3.4: Let μ be a nondegenerate r-semistable measure of index α on a measurable vector space $(E, \mathfrak{F})$. Assume that $\{x\} \in \mathfrak{F}$, for all $x \in E$. Then $\mu\{x\} = 0$, for all $x \in E$.

Proof: Let $G = \{\theta\}$ and $x \in E$. If $\mu\{G + x\} = \mu\{x\} > 0$, then, by Theorem 3.1, $\mu\{x\} = 1$. Hence μ is degenerate, a contradiction.

REFERENCES

1. D.M. Chung, B.S. Rajput, and Albert Tortrat, Semistable laws on topological vector space, to appear.

2. R.M. Dudley and M. Kanter, Zero-one laws for stable measures, Proc. Amer. Math. Soc., 45(1974), 245 - 252.

3. V.M. Kruglov, On the extension of the class of stable distributions, Theory Prob. Applications, 17(1972), 685 - 694.

4. ________, On a class of limit laws in a Hilbert space, Lit. Mat. Sbornik, 12(1972), 85 - 88 (in Russian).

5. A. Kumar, Semistable probability measures on Hilbert spaces, J. Multivariate Anal., 6(1976), 309 - 318.

6. P. Lévy, Theorie de l'Addition Variables Aleatories, Bautier-Villars, Paris, France, 1937.

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LET μ BE AN r-SEMISTABLE PROBABILITY MEASURE ON A REAL LINEAR SPACE E. IT IS SHOWN THAT THE μ-MEASURE OF ANY TRANSLATE OF AN ARBITRARY MEASURABLE LINEAR SUBSPACE OVER CERTAIN COUNTABLE SUBFIELD OF REALS IS 0 OR 1. THIS RESULT YIELDS DUDLEY-KANTOR 0-1 LAWS FOR STABLE LAWS AND FERNIQUE 0-1 LAW FOR QUASISTABLE LAWS. IT IS ALSO SHOWN THAT THE r-SEMISTABLE LAWS - LIKE STABLE ONES - ARE CONTINUOUS: i.e. THEY ASSIGN ZERO MASS TO SINGLETONS.

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