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STUDY OF A CLASS OF

NON-GAUSSIAN SIGNAL PROCESSING PROBLEMS*

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I. INTRODUCTION

Gaussian assumption has been a fundamental one in most statistical signal processing work. The assumption not only simplifies the analytical problems involved but also matches the data characteristics in many cases because of the law of large numbers. In a number of Navy sonar, radar and communications systems, signal processing algorithms must be developed without the Gaussian assumption. The need for non-Gaussian signal processing study was well described in a recent ONR SRO booklet. In our view the following is a partial list of a broad spectrum of correlated problem areas in non-Gaussian signal processing.

1. To investigate non-linear adaptive procedures to assess their detection performance against transient signals.

2. To investigate non-linear spectral analysis techniques such as the Maximum Entropy Spectral Analysis (MESA) and the Maximum Likelihood Method (MLM) to assess their detection performance against transient signals.

3. To investigate techniques for weak signal extraction in the presence of strong interferring signals. As a special case, potential techniques for detecting broad-band (such as spread-spectrum) signals can be explored including the analysis of changes in noise statistics caused by pseudo-noise signals.

4. To explore the use of nonparametric and robust statistical approaches to devise algorithms which perform well under different noise characteristics.

5. To study non-Gaussian models appropriate for reverberation or reverberation-like noise found in shallow water environment.

6. To examine the effects of non-Gaussian sources of impulsive noise interference in time-delay estimation, passive range estimation and related underwater acoustic signal processing problems.

7. To enlarge the basic mathematical and statistical theory of non-Gaussian stochastic processes applicable to both electromagnetic and underwater sound environments .

8. To characterise the non-Gaussian channels and interferences including the multipath, dispersion, doppler effects and the optimal reception under these circumstances.

9. To examine the modifications required in the existing signal processing system hardware for non-Gaussian signal processing.

Obviously good solutions to the above problems require expertise in several areas such as communications and information theory, pattern recognition, wave propagation, digital signal processing, image processing, and mathematical statistics, etc. As the problems are interrelated, they should be treated jointly as much as possible. Problems 1 and 2 are examined in detail in this report. For Problem 3, detection performance of spread-spectrum signals and the extraction of weak signals in Gaussian noise have been considered even though further work is much needed. For a general discussion of spread-spectrum systems, see [1]. Adaptive digital filtering and Kalman filtering are suitable techniques for extraction of broad-band non-Gaussian signals in noises. However analytical work on the detection performance is not available. For Problem 4, some existing analytical work on robust and nonparametric detection of signals in impulsive

noise and random interference (See [2] and the references listed in the paper)
are useful. While general theoretical work based on general assumptions is
very important, special cases should be considered so that the detection pro-
cedures and performances developed will match more closely to specific impul-
sive noise and random interference conditions considered. Proper mathematical
modelling of impulsive noise, random interference and channels is thus required,
which is the Problem 5 stated above.

Time-delay estimation and passive ranging estimation from an array of
sensors are typical sonar signal processing problems (see e.g. [3] [4]). Pro-
blem 6 suggests that the non-Gaussian sources be taken into consideration as
the existing work based on Gaussian assumption is quite restrictive. Problem 7
presents a number of mathematical and statistical topics to be explored. In
fact some theoretical work for Gaussian signals and Gaussian noises can be ex-
tended to non-Gaussian cases [5]. For example, consider the detection and esti-
mation of Gaussian signal in Gaussian noise. The conditional mean of the signal
is the best estimate for a variety of criteria. In the non-Gaussian case it is
the best linear estimate in the sense that mean-squared error is minimized. When
the signal is unknown but non-random, the maximum likelihood estimate of the
signal is the same whether the noise is Gaussian or non-Gaussian. The test sta-
tistics for detecting non-Gaussian signal in Gaussian noise is the same as that
for detecting Gaussian signal in Gaussian noise at small signal condition. Many
other mathematical properties remain to be examined.

Problem 8 has been well explored and reported in publications in communica-
tions, wave propagation and other areas. However, much less amount of work has
been done for underwater transmission media. Although most signal processing

hardware designed for Gaussian signal processing can also be used for non-Gaussian signal processing with non-optimal performance, a modification of the hardware is a worthwhile effort to provide significant performance improvement in many cases.

II. NON-LINEAR ADAPTIVE DETECTION PROCEDURES

One fundamental problem with the non-Gaussian signal processing is that the data is often nonstationary. Adaptive procedures are needed for estimation and detection of transient signals. Starting with the linear adaptive detection procedures, we have extensively explored the feasibility of Adaptive Digital Filtering and Kalman Adaptive Filtering [6] [7]. As the statistical characteristics of the data change with time, filter parameters must be readjusted at periodic intervals. This is considered as a piecewise linear adaptive procedure. If the data statistics are

monitored continuously and the parameters are adjusted with each significant change in data statistics, we will have a nonlinear adaptive procedure. Detection of the desired signal from the filtered output can be performed by the thresholding or correlation operation.

The Adaptive Digital Filter as shown in Figure 1 is a non-recursive digital noise by itself without making Gaussian assumption of the data. Let N be the order of the filter. The input-output relation of the filter is given by

$$q_m = \sum_{n=0}^{N} b_{n,m} f_{m-n} \tag{1}$$

where q_m is the filter output, $b_{n,m}$ is the filter coefficient, and f_n is the filter output. Define the vectors
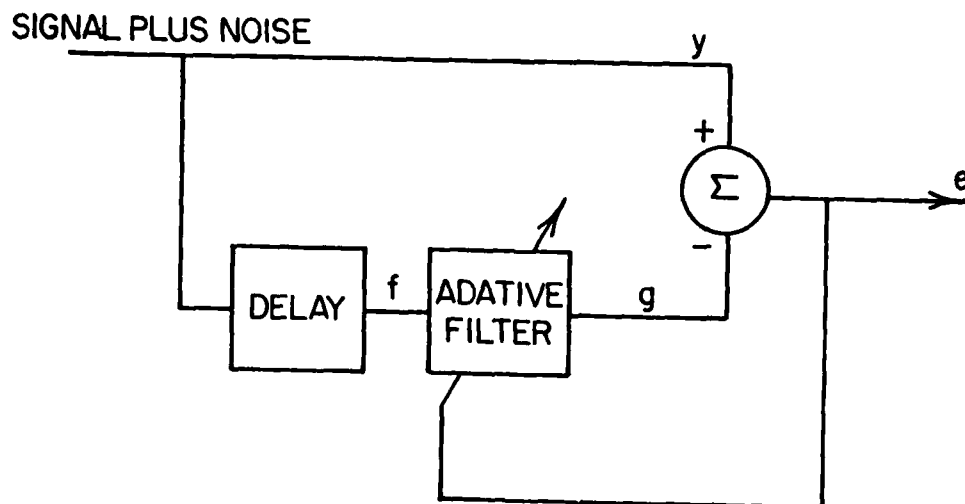
SIGNAL PLUS NOISE
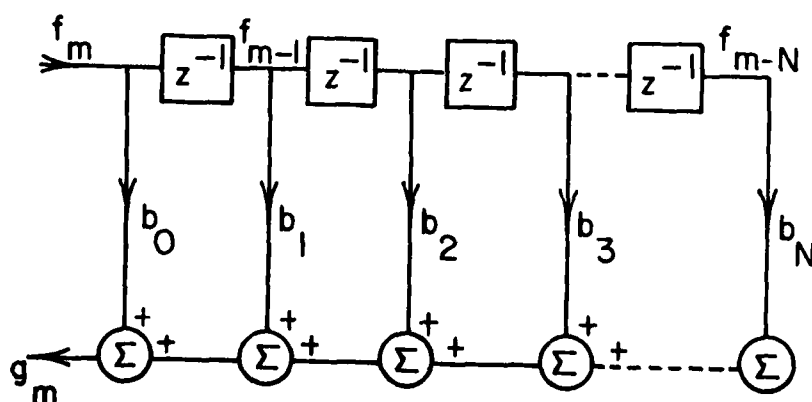
Fig. 1a The Adaptive Digital Filtering System.

Fig. 1b The nonrecursive Adaptive Filter.

$$B_m' = [b_{0,m} \quad b_{1,m} \quad \text{------} \quad b_{N,m}]$$

and

$$F_m' = [f_m \quad f_{m-1} \quad \text{------} \quad f_{m-N}]$$

Equation (1) can be written as

$$q_m = B_m' \, F_m = F_m' B_m \tag{2}$$

By using the Noisy Least-Mean-Square Algorithm [8], the filter coefficients can be determined from the equations

$$\begin{cases} B_{m+1} = B_m + ve_m \, F_m \\[2mm] e_m = y_m - q_m \\[2mm] q_m = B_m' F_m \end{cases} \tag{3}$$

where v is a constant. After adjusting the parameters that include v, N, m, and delay time, we can obtain the desired result with a greatly improved signal-to-noise ratio. Obviously the Adaptive Digital Filter works well with stationary data. Parameter choice may require a lot of trial-and-error even though the filtering procedure is fairly simple.

The signal model of the Kalman Adaptive Filter is given by:

$$x_{k+1} = F_k \, x_k + w_k \tag{4}$$

where x_k is the state variable, w_k is the system noise, v_k is the measurement noise. Note that Gaussian assumption is not required of x_k, v_k and w_k. Given a set of measurements z_k, k = 1, 2, ------, N, we would like to estimate x_k, k = 1, 2, ------ N, based on the state-space signal model given by Eq. (4). Since

$z_k = x_k + v_k$, we have

$$z_{k+1} = x_{k+1} + v_{k+1} = (F_k x_k + w_k) + v_{k+1} \qquad (5)$$

Define an error criterion function,

$$J_{k+1} = (z_{k+1} - (F_k x_k + w_k) - v_{k+1})^2 = L^2 \qquad (6)$$

If z_{k+1} and x_k are known, we determine the values of w_k, v_{k+1} and f_k iteratively such that the state space model is optimum in the miniumum mean square error sense. The method of Steepest Descent can be used in the optimization process [9]. Once the optimum values of w_k, v_{k+1}, and F_k are obtained, we can perform a one-step prediction of x_{k+1} by using the equation

$$x_{k+1} = F_k x_k + w_k \qquad (7)$$

with w_k being the prediction error. The iterative procedure for the optimization at the ith iteration is given by

$$w_{k,i+1} = w_{k,i} + \lambda(2L)$$

$$v_{k+1,i+1} = v_{k+1,i} + \lambda(2L) \qquad (8)$$

$$F_{k,i+1} = F_{k,i} + \lambda(2Lx_k)$$

Here the increment λ and $w_{k,0}$, $v_{k+1,0}$, and $F_{k,0}$ were given initially. In seismic data processing, it is reasonable to assume that

$$F_{k,0} = w_{k,0} = v_{k+1,0} = 0$$

Furthermore, the initial state can be chosen as $x_0 = 0$. These assumptions should be appropriate for many non-Gaussian signal processing problems.

It will be of interest to examine some computer results with the seismic data which is non-Gaussian [7]. Both Adaptive Digital Filtering and Kalman Adap-

tive Filtering can perform well to extract impulse-like signals from a corre-
lated non-Gaussian background noise. With properly selected parameters the
Adaptive Digital Filtering performs nearly the same as the Kalman Adaptive
Filtering. Figure 2 shows the results of Kalman Adaptive Filtering for three
channels of data with 960 data points each. This data segment represents a
section of complete file. The upper part of each photo is the original data
while the lower part is the filtered result. The extracted impulses occur at
nearly the same time instants. The only parameters to be selected are AL
(the increment) and ITR (the number of iteration). An empirical relationship
between AL and ITR is established for each channel as shown in Figure 3. Each
point on the curves represents an optimal combination of AL and ITR. Although,
the curves are somewhat depending on the data especially on the background noise
level, they serve as a useful guideline for the Kalman Adaptive Filtering which
is almost nonparametric and is more robust than the Adaptive Digital Filtering.
If automatic detection is desired, a correlation or a thresholding operation
can be used. Theoretical detection performance, however, is not available.

As the data statistics are generallly nonstationary, the parameters of the
Adaptive Digital Filtering must be adjusted periodically. No such requirement
is needed in Kalman Adaptive Filtering as the final estimate for the previous
data point can be used as initial value for the current estimate. Another
method to incorporate the time-varying statistics in the Kalman filtering is to
use an "on-line" monitoring process to detect the significant change or "jump"
in the state and update the state vector accordingly [10]. This nonlinear method
however is limited to Gaussian statistics. Based on the above discussion the
Kalman Adaptive Filtering method appears to be most suitable for non-Gaussian
signal processing. Experimental study has demonstrated good detection perform-
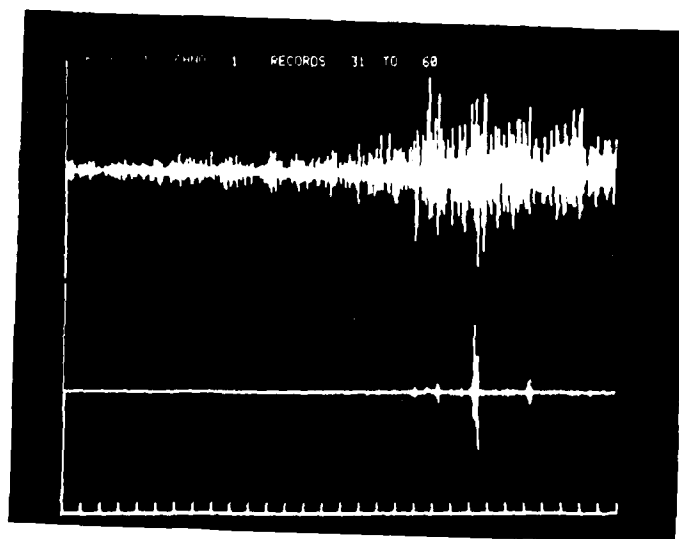ance with the method.

Fig. 2a
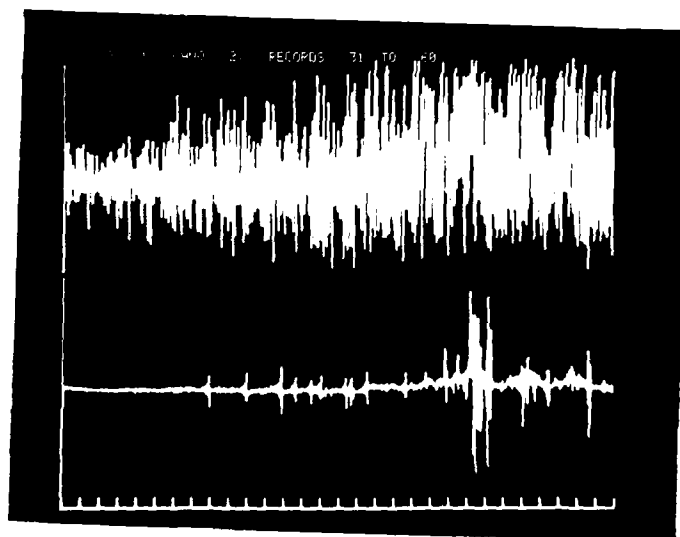Channel 1 and adaptive
Kalman filtered data.
AL=0.0038
ITR=10

Fig. 2b
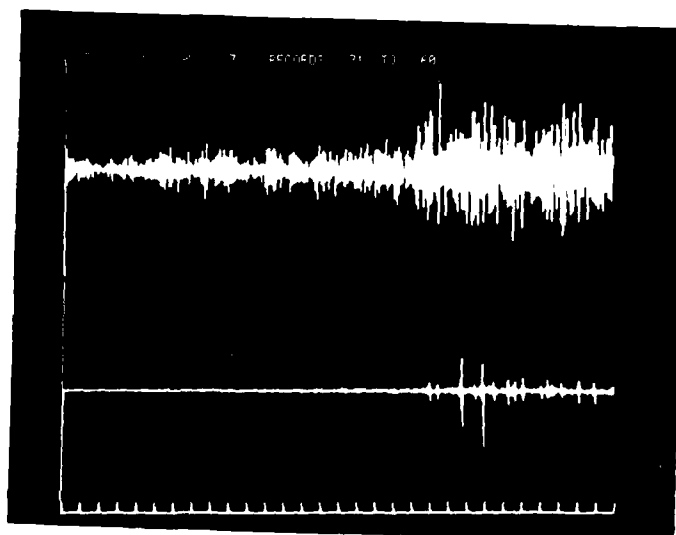Channel 2 and adaptive
Kalman filtered data.
AL=0.0028
ITR=10

Fig. 2c
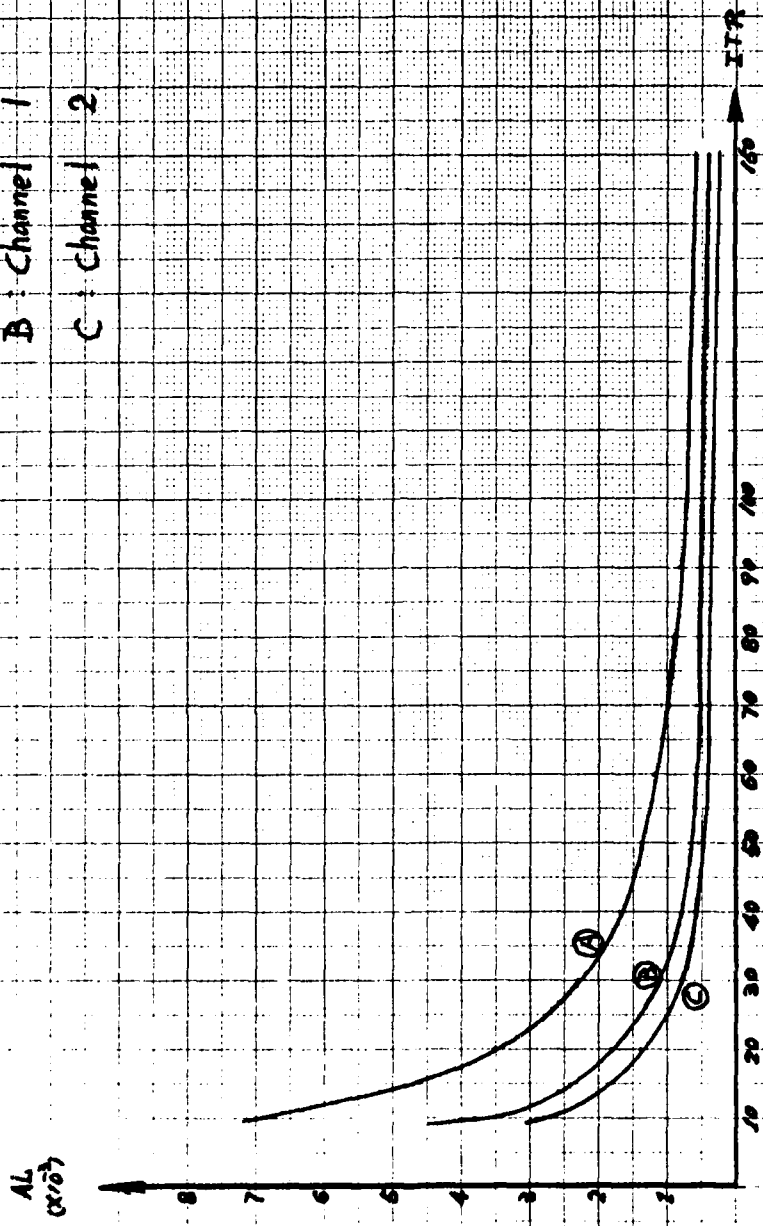Channel 3 and adaptive
Kalman filtered data.
AL=0.007
ITR=10

Figure 3 - Relationship between increment (ΔL) and the number of iterations (ITR) for Kalman filtering of transient signals.

Empirical equation: $\Delta L = f(ITR) = a \exp{-b(ITR)}$

III. Nonlinear Spectral Analysis Techniques

Both the maximum entropy method (MEM) due to Burg and the maximum likelihood method (MLM) for power spectral analysis are considered as non-linear high-resolution spectral analysis techniques suitable for processing transient signals. They both bear a very close relationship to non-linear adaptive array processing techniques [11]. Furthermore the two methods are very closely related [12]. Experimental results have demonstrated that MEM is superior to MLM in spectral resolution (see e.g. [13]). Improvement over the existing Burg's MEM is possible by further reducing the final prediction error with the Fletcher and Powell optimization method as proposed by Fougere [14]. Other algorithms that provide more accurate spectral analysis than the Burg's method have also been proposed (e.g. [15]). The maximum entropy method based on the Fougere's technique has been successfully implemented at the PDP 11/45 minicomputer here at the Southeastern Massachusetts University.

Given n data points x_1, x_2, —, x_n, we define an $(m + 1)$ - point prediction error filter $(1, g_{m1}, g_{m2}, —, g_{mm})$ such that the kth prediction errors are

$$\varepsilon_{1k} = \sum_{i=0}^{m} x_{k+m-i} \, g_{mi}$$

$$\varepsilon_{2k} = \sum_{i=0}^{m} x_{k+1} \, g_{mi} \quad , \qquad k = 1,2,3, —, i-m \tag{9}$$

where $g_{m0} = 1$ and ε_{1k} and ε_{2k} are the forward and backward prediction errors, respectively. The mean square prediction error, or mean error power, in both time directions is

$$P_m = 0.5(n - m)^{-1} \sum_{s=1}^{2} \sum_{k=1}^{n-m} \varepsilon_{sk}^2 \tag{10}$$

If the prediction error filter of all orders 1, 2, ——, m are gathered in one matrix G_m (with the leading "1" suppressed), we may write

$$
G_m = \begin{pmatrix}
g_{11} & & & \\
g_{21} & g_{22} & & \\
& \vdots & & \\
g_{m1} & g_{m2} & - - - - & g_{mm}
\end{pmatrix} \tag{11}
$$

The off-diagonal elements of G_m can be determined from the diagonal elements by using the Levinson recursion,

$$
g_{jk} = g_{j-1,k} + g_{jj} \; g_{j-1,j-k} \tag{12}
$$

The magnitude of the diagonal elements of G_m must be less than 1 in order that the prediction error filter be minimum phase. In order to enforce this condition, we can set

$$
g_{jj} = U \sin \theta_j \tag{13}
$$

where θ_j is any real angle and U is a positive constant slightly less than unity depending on the computer used. For the minicomputer a good choice is U = 0.99999. Once the computation of the Burg's method is completed the gradient of resulting P_m with respect to θ is determined so that the error minimization procedure can be started. After a number of iterations the filter coefficients can be recomputed from which the power spectrum is determined. Although the original software developed by Fougere is not suitable for the minicomputer, proper modification has made it possible to

use this powerful technique in the minicomputer. Figure 4a shows one cycle of sinewave of 1 Hz at sampling rate of 20 samples per second . Figure 4b shows the Burg's spectrum for the 20 data points. There is spectrum splitting and the spectral peak is shifted to 0.92Hz. The Fougere technique after 10 iterations provides a correctly located peak which is 53 times of the peak in the Burg's spectrum (The vertical scales are not the same in the two photos. Every division in the horizontal scale is 0.5Hz).

It is important to note that very high resolution spectrum can be obtained from the minicomputer using the method described above at only slightly increased computational complexity over the Burg's method. No Gaussian assumption is made in the method. As the method is designed for a small number of samples it is particularly suitable for analyzing the spectrum of the transient signal.

With improved computer hardware the method may become useful for near real-time operations. Each spectral peak can have some physical meaning such as in target detection in clutter. With proper trade-off in speed and accuracy the method can prove to be very useful in non-Gaussian signal processing.

IV. Concluding Remarks

Although only two problem areas are considered in this report, non-Gaussian signal processing is a challenging area which requires using knowledge in different disciplines to derive effective solutions. This area also provides an opportunity to integrate or correlate many current signal processing research activities which are developed independently. Continued and strong research effort is needed to pursue new directions and techniques to solve some difficult analytical problems in non-Gaussian signal processing.
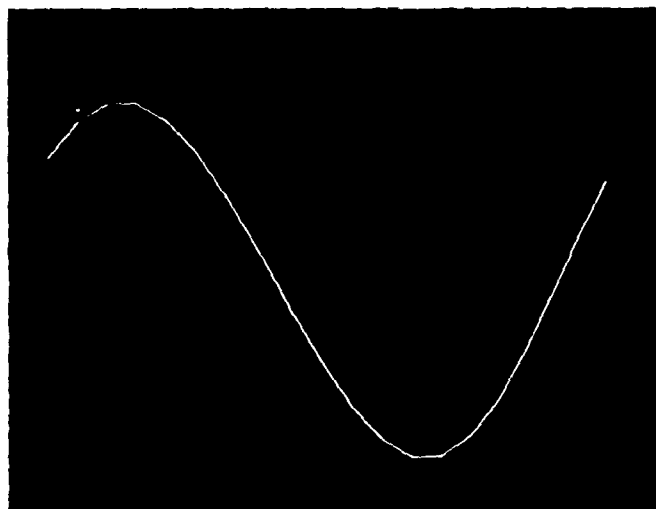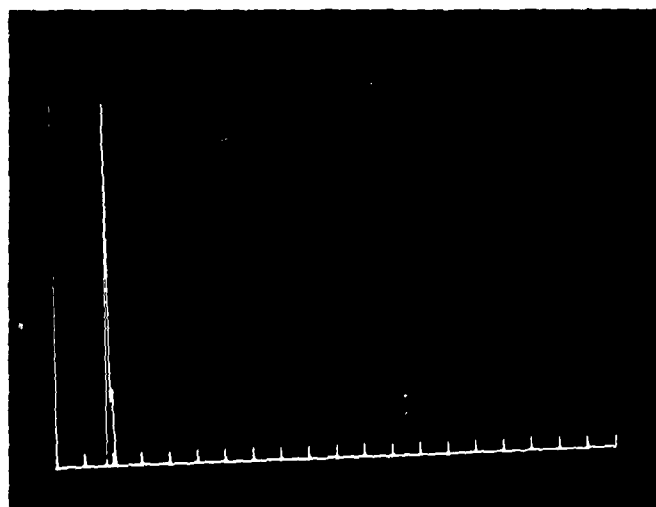
Fig. 4a
One cycle of sinewave
at 1 Hz.

Fig. 4b
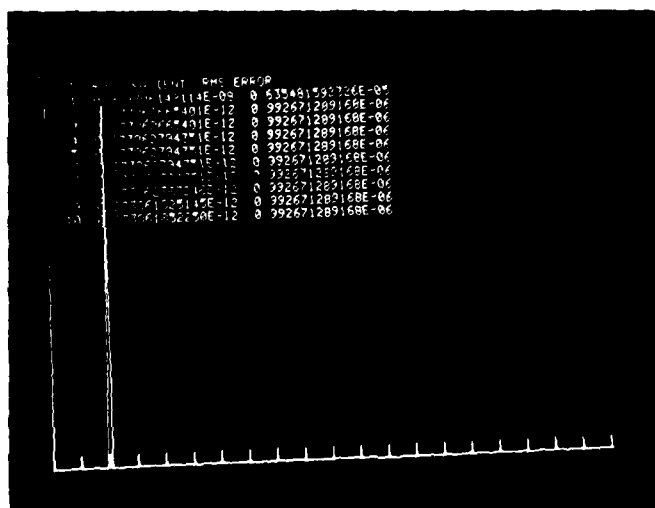Burg's Maximum Entropy
Power Spectrum.

Fig. 4c
Fougere's Maximum Entropy
Power Spectrum.

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

After presenting an overview of important problem areas in non-Gaussian signal processing, the report examines in detail two topics, viz. the non-linear adaptive procedures including the Adaptive Digital Filtering and Kalman Adaptive Filtering for processing non-Gaussian transient signals, and the non-linear spectral analysis techniques for transient signals. A non-linear maximum entropy method is discussed. New computer results on both topics are also presented.

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