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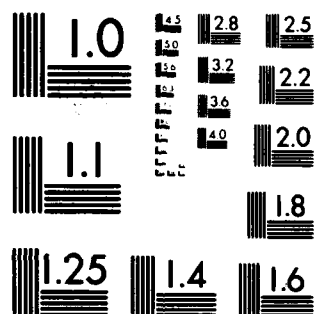
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The Effect of Surface Energy on
Temperature Rise Around a
Fast Running Crack

by

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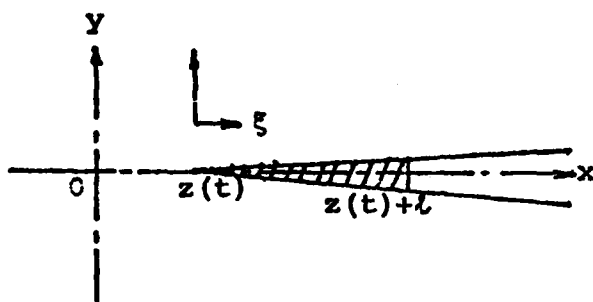
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1. Introduction

Using a local balance law developed by Gurtin [5] for the cohesive zone, we give a system of equations governing the temperature rise around a fast running crack. We note that the effect of surface temperature cannot be neglected for a fast running crack, and we discuss the equations under simplifying assumptions. In particular, assuming that the surface temperature is much higher than the surrounding temperature, we arrive at a simple solution in closed form. This solution agrees with experimental results of Fuller, Fox, and Field [2] for polymethyl methacrylate showing that the temperature rise at the crack tip is independent of crack speed.

2. Theory

Consideration will be restricted to a semi-infinite crack in an infinite plate stressed symmetrically with respect to the plane of the crack.



We assume that the crack is moving with constant velocity v in the direction of negative x . (See the Figure.)

Neglecting thermo-mechanical coupling we have the following energy balance law governing the temperature rise $\theta = \theta(x, y, t)$ away from the crack:

$$\rho c \frac{\partial \theta}{\partial t} = k \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (2.1)$$

where ρ is the density, k conductivity and c specific heat. Let $z(t)$ label the end of cohesive zone and l its length, so that $z(t) + l$ gives the position of the crack tip.

Since the work rate $f(x, t)$ due to plastic work in the cohesive zone will be symmetric about the plane of the crack, there will be no heat conduction across the line $\{y = 0, x < z(t)\}$; hence

$$k \frac{\partial \theta}{\partial y} = 0, \quad y = 0, x < z(t). \quad (2.2)$$

We allow the crack surface to have (surface) temperature rise $\theta(x, t) (\geq 0)$, continuous in $x(> z(t))$.

According to a recent theory of Gurtin [5], the local balance law in the cohesive region is given in the form

$$\dot{\epsilon}_F = h + g \cdot \dot{\delta}, \quad (2.3)$$

where h is the heat flow per unit surface into the crack surface from the body (cf., e.g., (5.6) in [5]). Using (2.3) we have (by routine assumptions and derivations)

$$\frac{\beta}{2} \dot{\phi}(x,t) = h + \frac{f(x,t)}{2}, \quad y = 0, \quad x > z(t), \quad (2.4)$$

$$k \frac{\partial \theta}{\partial y} = h, \quad y = 0^\pm, \quad x > z(t),$$

where β is a constant. We assume that h has the form

$$h = \alpha(\theta - \varphi), \quad (2.5)$$

where α (= constant) is the surface conductivity. We note that, in (2.4)₁ $f(x,t) = 0$ at $x > z(t) + l$, and we neglect heat loss into the surrounding air.¹ In Gurtin [5], the energy of the newly formed free surfaces is assumed to be constant when these surfaces are exposed to a constant environmental temperature. For a fast running crack, however, this assumption is redundant.

Assuming steady-state conditions and introducing the moving coordinate $\xi = x - z(t)$, the equations (2.1)-(2.5) become

¹See Döll [4].

$$\rho c v \frac{\partial \theta}{\partial \xi} = k \left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial y^2} \right), \quad y > 0, \quad -\infty < \xi < \infty$$

$$k \frac{\partial \theta}{\partial y} = 0, \quad y = 0, \quad \xi < 0, \quad (2.6)$$

$$\frac{\beta}{2} v \frac{\partial \theta}{\partial \xi} = \alpha(\theta - \varphi) + \frac{q(\xi)}{2}, \quad y = 0, \quad \xi > 0,$$

$$k \frac{\partial \theta}{\partial y} = \alpha(\theta - \varphi), \quad y = 0^+, \quad \xi > 0,$$

where $g(\xi) := f(\xi + z(t), t)$ is the work rate in the cohesive region. By (2.6)_{3,4}, we have

$$b \frac{\partial \theta}{\partial \xi} = \frac{\partial \theta}{\partial y} + \frac{q(\xi)}{2k}, \quad (2.7)$$

where $b = \frac{\beta v}{2k}$. We note β is the heat required to raise the temperature of a unit surface by one degree ($\text{cal}/\text{cm}^2/^\circ\text{C}$).

We expect β to be negligible small. The dimensionless constant b , however, is approximately

$$6.1 \times 10^4 \times \beta - \text{2024 Aluminium Alloy}$$

$$1.7 \times 10^5 \times \beta - \text{Mild Steel}$$

$$1.4 \times 10^6 \times \beta - \text{6Al-4V Titanium Alloy}$$

$$5.0 \times 10^8 \times \beta - \text{Polymethyl Methacrylate } (k=5 \times 10^{-5}/\text{cm}/^\circ\text{C}/\text{s})$$

for $v = 500$ m/sec. Thus, even for β as small as $10^{-5} \sim 10^{-8} \text{ cal}/\text{cm}^2/^\circ\text{C}$, we may not neglect the term $b \frac{\partial \theta}{\partial \xi}$ compared to $\frac{\partial \theta}{\partial y}$ in

(2.7) if the crack velocity is as large as 500 m/s.

We will discuss the temperature rise around a fast running crack using the above equations, but under certain simplifying assumptions.

3. Analysis and Discussions

Here we assume that $\theta \ll \varphi$ on the crack surface. (Experiments on polymethyl methacrylate show the temperature rise on the crack faces ($\sim 1 \mu\text{m}$ in depth) to be about 500 K throughout the velocity range 200~650 m/s [2], in contrast, the maximum temperature rise at a distance of about 0.2~1 mm from the crack path, is only 0.1~1 K [3].)

Under this assumption the equations (2.7) become

$$\rho c v \frac{\partial \theta}{\partial \xi} = k \left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial y^2} \right), \quad y > 0, \quad -\infty < \xi < \infty,$$

$$k \frac{\partial \theta}{\partial y} = 0, \quad y = 0, \quad \xi < 0$$

$$\frac{\beta}{2} v \frac{\partial \varphi}{\partial \xi} = -\alpha \varphi + \frac{q(\xi)}{2}, \quad y = 0, \quad \xi > 0,$$

$$k \frac{\partial \theta}{\partial y} = -\alpha \varphi, \quad y = 0^+, \quad \xi > 0.$$

We assume that θ and φ tend to zero at infinity. We then have solutions in the form

$$\varphi(\xi) = \begin{cases} \int_0^l \frac{q(t)}{\beta v} e^{-\frac{2\alpha}{\beta v}(\xi-t)} dt & \xi \leq l, \\ \int_0^\xi \frac{q(t)}{\beta v} e^{-\frac{2\alpha}{\beta v}(\xi-t)} dt & 0 \leq \xi < l, \\ 0 & \xi < 0, \end{cases}$$

$$\theta(\xi, y) = \int_0^\infty \frac{\alpha \varphi(t)}{k} K(\xi-t, y) dt,$$

where

$$K(x, y) = \frac{1}{\pi} e^{ax} K_0(a \sqrt{x^2 + y^2}), \quad a = \frac{\rho c v}{2k},$$

with K_0 the modified Bessel function of the second kind.

Using the Dugdale model, the work rate due to plastic work is given in the form¹

$$g(x) = \sigma_0 \frac{d\delta(x)}{dx} v,$$

where σ_0 is the yield stress and δ the separation distance.

Thus the temperature rise $\varphi(\xi)$ for $\xi \geq l$ becomes

$$\begin{aligned} \varphi(\xi) &= \left(e^{-\frac{2\alpha l}{v\beta}} \int_0^l \frac{\sigma_0}{\beta} \frac{d\delta(t)}{dt} e^{\frac{2\alpha t}{v\beta}} dt \right) e^{-\frac{2\alpha}{v\beta}(\xi-l)} \\ &= \left(e^{-\frac{2\alpha l}{v\beta}} \int_0^l \frac{\sigma_0}{\beta} \frac{d\delta(lx)}{dx} e^{\frac{2\alpha l}{v\beta}x} dx \right) e^{-\frac{2\alpha}{v\beta}(\xi-l)} \end{aligned} \quad (3.1)$$

and thus, if $2\alpha l/v\beta$ is negligibly small, (3.1) is approximately,

$$\varphi(\xi) \approx \frac{\sigma_0 \delta(l)}{\beta} e^{-\frac{2\alpha \eta}{\beta v}}, \quad (3.2)$$

where $\eta = \xi - l$.

This result shows, interestingly, that the temperature rise at the crack tip ($\eta = 0$) is proportional to $\sigma_0 \delta(l)$ with constant of proportionality is $1/\beta$.²

¹The work rate $g(\xi)$ for the Dugdale model in small plane strain is computed by Levy and Rice [1] as

$$g(\xi) = \frac{4(1-v^2)\sigma_0^2 v}{E} \ln\left(\frac{1 + \sqrt{E/l}}{1 - \sqrt{E/l}}\right), \quad 0 < \xi < l.$$

²Note that we have assumed plastic work in the cohesive zone is completely converted into heat. (This is generally true of the energy expended in plastically deforming a metal.) For partial conversion g should be scaled down appropriately.

Note that φ depends on v only through a dependence on η/v . For the point $x = y = 0$, say, occupied by the tip at $t = 0$, η will be the distance from the tip at time $t = \eta/v$. Thus, by (3.2), the temperature rise at this point should depend only on time; it should be independent¹ of v . This result is in agreement with experimental result of Fuller, Fox, and Field [2], who found that the temperature rise at a fixed point on the axis of the crack was approximately independent of crack velocity. In fact, they note that "the results combined to give a temperature rise of approximately 500 K throughout the velocity range studied (200~650 m/s)."

For the fixed point $x = y = 0$, (3.2) gives

$$\varphi \approx \frac{\sigma_0 \delta(l)}{\beta} e^{-\frac{2\alpha}{\beta}t}$$

and hence φ has the form

$$\varphi \approx Ae^{-Bt},$$

where $A = \sigma_0 \delta(l)/\beta$ and $B = -2\alpha/\beta$.

The data of [2], when averaged, give

$$\varphi = 457 \text{ K at } t = 10\mu\text{s}$$

$$\varphi = 361 \text{ K at } t = 20\mu\text{s}$$

$$\varphi = 304 \text{ K at } t = 35\mu\text{s}.$$

Using the values at $t = 10\mu\text{s}$ and $t = 35\mu\text{s}$, we find that

$$A = 5.38 \times 10^2 \text{ K},$$

$$B = 1.63 \times 10^4 \text{ sec}^{-1}.$$

¹Here we follow Levy and Rice [1] and assume that $\sigma_0 \delta(l)$ is independent of v . Levy and Rice [1], however, found that this temperature rise is proportional to $\sqrt{v}$.

This gives $\varphi = 388$ K at $t = 20\mu\text{s}$ as compared to the value $\varphi = 361$ K of [2]. Further, we find that $\varphi = 538$ K at $t = 0$; that is, the temperature rise at the crack tip is 538 K. For the values $l = 1$ mm (which we feel is an upper bound for l) and $v = 200$ m/s, we find that the dimensionless constant

$$2\alpha l/v\beta = lB/v,$$

which we neglected in defining (3.2), has the approximate value of 0.08.

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