

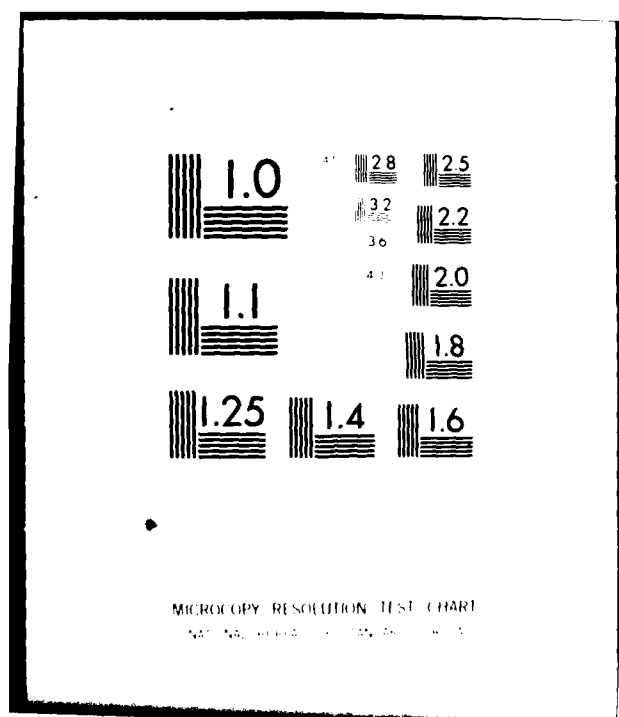
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SCIENTIFIC RESEARCH ASSOCIATES INC GLASTONBURY CT F/G 20/13
COMPUTATION OF DISCRETE HOLE FILM COOLING FLOW USING THE NAVIER--ETC(U)
MAY 80 J P KRESKOVSKY, W R BRILEY, H MCDONALD F49620-78-C-0038
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COMPUTATION OF DISCRETE HOLE FILM COOLING FLOW
USING THE NAVIER-STOKES EQUATIONS

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F49620-78C-0038

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR-80-0687	2. GOVT ACCESSION NO. AD-A088807	3. RECIPIENT'S CATALOG NUMBER 9
4. TITLE (and Subtitle) COMPUTATION OF DISCRETE HOLE FILM COOLING FLOW USING THE NAVIER-STOKES EQUATIONS		5. TYPE OF REPORT & PERIOD COVERED INTERIM (ANNUAL) Rept. 1 Apr 1979 - 31 Mar 1980
6. AUTHOR(s) J. P. KRESKOVSKY W. R. BRILEY H. McDONALD		7. PERFORMING ORG. REPORT NUMBER R80-910002-27
8. PERFORMING ORGANIZATION NAME AND ADDRESS SCIENTIFIC RESEARCH ASSOCIATES, INC. P. O. BOX 498 GLASTONBURY, CT 06033		9. CONTRACT OR GRANT NUMBER(s) F49620-78-C-00384
10. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA BLDG 410 BOLLING AIR FORCE BASE, DC 20332		11. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2307A4 61102F
12. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 24		13. REPORT DATE May 1980
		14. NUMBER OF PAGES 20
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) FILM COOLING NAVIER-STOKES EQUATIONS ZONE EMBEDDING INTERACTIVE BOUNDARY CONDITIONS		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An analysis and computational procedure are described here for predicting flow and heat transfer which results from coolant injection through a single row of round holes oriented normal to a flat surface. The present method solves the compressible Navier-Stokes equations and utilizes "zone embedding", surface-oriented coordinates, interactive boundary conditions, and an efficient split LBI scheme. The approach treats the near-hole flow region where the film cooling flow is initially established. A sample laminar flow calculation is		

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Computation of Discrete Hole Film
Cooling Flow Using the Navier-Stokes Equations

by

J. P. Kreskovsky, W. R. Briley, and H. McDonald

ABSTRACT

An analysis and computational procedure are described here for predicting flow and heat transfer which results from coolant injection through a single row of round holes oriented normal to a flat surface. The present method solves the compressible Navier-Stokes equations and utilizes "zone embedding", surface-oriented coordinates, interactive boundary conditions, and an efficient split LBI scheme. The approach treats the near-hole flow region where the film cooling flow is initially established. A sample laminar flow calculation is presented for a ratio of normal injection to free stream velocity of 0.1. Although present results do not include heat transfer predictions, details of the interaction between injectant and main stream flow near the hole exit are in qualitative agreement with experimental observations for other flow conditions.

INTRODUCTION

To achieve higher turbine efficiencies, designers have been forced to deal with high turbine inlet temperatures which may exceed structural limitations of available materials used for blading unless some method of cooling these blades is introduced. Various cooling schemes have been devised to ensure that blades remain intact when exposed to a high temperature environment for prolonged periods of time. One of the more promising schemes is discrete hole film cooling, wherein cooling air is injected through either a single row or multiple staggered rows of holes, to provide a protective layer of cool air between the blade surface and hot mainstream gases. Discrete hole film cooling is better suited to the high temperatures of advanced high performance engines than the current convectively cooled blades found in commercial engines. A large number of parameters can be varied to obtain a configuration which optimizes heat transfer and aerodynamic characteristics, including hole shapes and patterns, injection angles and rates, surface curvature, coolant temperatures, and mainstream boundary layer characteristics. Since experimental determination of optimal configurations is costly, a computational procedure which could be used to screen alternative configurations without experimental testing would be of considerable value. An analysis and computational procedure is described here for predicting flow and heat transfer which results from coolant injection through a single row of round holes oriented normal to a flat surface. This study provides an improved understanding of the film cooling process, particularly in the near-hole region, and represents a first step toward development of a computer program capable of predicting heat transfer and aerodynamic loss levels associated with discrete hole film cooling on gas turbine blades.

PREVIOUS WORK

Previous experimental work on film cooling flow has consisted mainly of flow visualization studies and measurements of film cooling effectiveness downstream of the coolant injection hole for a variety of flow conditions. For example, Colladay and Russel [1] performed a flow visualization study of discrete hole film cooling in three different hole configurations, to obtain a better understanding of flow behavior associated with coolant injection. They considered injection from discrete holes in a three-row staggered array with five-diameter spacing and three-hole angles: (1) normal to the surface, (2) slanted 30 degrees to the surface but aligned with the free stream, and (3) slanted 30 degrees to the surface and 45 degrees laterally to the free stream. For flow conditions typical of gas turbine applications, Colladay and Russell observed that normal injection is subject to flow separation even at low injection rates, that slanted injection works well except at high injection rates, and that compound slanted-yawed injection is less susceptible to separation but tends to promote entrainment of free stream fluid (which augments heat transfer) and to increase secondary flow losses. These experimental observations emphasize the importance of interaction between the injected fluid and the free stream.

Kadotani and Goldstein [2] have performed experimental measurements for discrete hole film cooling through a row of holes having three-diameter lateral spacing, inclined at 35 degrees to the injection surface, and aligned with the free stream. They considered a variety of flow conditions and varied boundary layer thickness and injection rates, as well as Reynolds number and turbulence properties. They measured lateral and streamwise distributions of film cooling effectiveness downstream of injection, and also measured mean velocity and temperature in a transverse plane normal to the free stream and located at the downstream edge of the injection holes. Kadotani and Goldstein deduced from their measurements that reversed flow is present near the injection hole even for injection rates as low as 0.35, and that the size and strength of the reversed flow increases with increasing boundary layer thickness. They also found that the approaching boundary layer thickness significantly affects the flow behavior and lateral variation of film cooling effectiveness, particularly near the injection hole.

The only previous study known to the authors which treated the discrete hole film cooling problem using a three-dimensional calculation procedure is that of Bergeles, Gosman and Launder [3], who applied the approximate "partially parabolic" calculation procedure of Pratap & Spalding (Ref. 4) for the case of laminar flow. This calculation procedure neglects streamwise diffusion and employs iterated forward marching solution of three approximate momentum equations. The procedure begins with a "guessed" pressure field and performs iterated forward marching sweeps of the three-dimensional flow field, solving the approximate momentum equations and utilizing various strategies to modify or correct the pressure field, so as to improve the continuity balance. Since the calculation procedure is based on forward marching, the flow region within the coolant injection hole is excluded from the computational domain, and instead, the coolant velocity distribution is specified at the hole exit as a two-dimensional uniform stream at a prescribed injection angle. Bergeles, Gosman and Launder made flow calculations for several flow conditions, including different injection angles.

THE PRESENT APPROACH

The present study focuses on the near-hole flow region where the film cooling flow is initially established and where local hot spots may occur. Since the flow field surrounding injection may involve separation and reversed flow relative to the free stream, and since directional and/or other properties of the flow which might permit simplifying assumptions are not clearly in evidence, the present approach is based on numerical solution of the three-dimensional compressible Navier-Stokes equations. The governing equations in general orthogonal coordinates are solved using analytical coordinate data for a polar-cylindrical coordinate system which fits all solid surfaces within the computational domain but is not aligned with the direction of the free stream flow.

In selecting the computational domain, a "zone embedding" approach is adopted whereby attention is focused on a subregion of the overall flow field in the immediate vicinity of the discrete hole coolant injection. At the curved boundaries located within the free stream region, interactive boundary conditions which permit inflow and outflow of shear layers and the compressible inviscid free stream are derived from an assumed flow structure. Since details of the coolant velocity distribution at the hole exit are not known a priori and presumably will depend on an interaction between the coolant and the free stream flow, the computational domain is chosen to include the flow region within the coolant hole as well as that exterior to the hole. Including the flow region within the coolant hole permits the interaction between the coolant and free stream flows and its important influence near the hole to be determined as part of the final solution, without the need for simplifying assumptions. The governing equations are solved by an efficient and noniterative time-dependent "linearized block implicit" (LBI) scheme.

METHOD OF SOLUTION

Zone Embedding and Interactive Boundary Conditions

The geometry, polar-cylindrical coordinate system (r, θ, z) and computational grid used in present calculations is shown in Fig. 1. The polar coordinates r, θ are related to Cartesian coordinates x, y by $x = r \cos \theta$, $y = -r \sin \theta$. The computational domain is defined by $0 \leq r/R \leq 5$, $0 \leq \theta \leq \pi$, $0 \leq z/R \leq 5$ outside the hole and by $0 \leq r/R \leq 1$, $0 \leq \theta \leq \pi$, $-2 \leq z/R \leq 0$ within the hole, where R is the hole radius. Symmetry conditions are applied at $z/R=5$, so that the flow represented is that of opposing cooling holes located on parallel flat plates with spacing $H/R=10$. Velocity boundary conditions at no-slip and symmetry surfaces are straightforward and self-explanatory. The remaining condition at these boundaries is $\partial c_p / \partial n = 0$, where c_p is a pressure coefficient defined by $c_p = 2(p - p_r) / \rho u_r^2$, and n denotes the normal coordinate direction. Here, p_r and u_r are pressure and velocity at free stream reference conditions. The condition $\partial c_p / \partial n = 0$ at a no-slip surface is correct to order Re^{-1} for viscous flow at high Reynolds number.

The treatment of inflow and outflow conditions is the principal obstacle to be overcome within the present zone embedding approach. At curved boundaries located within the free stream region, interactive inflow-outflow boundary conditions are derived from an assumed flow structure and physical approximations. The assumed inflow structure consists of a uniform potential flow core velocity $\bar{U}_I$, two-dimensional estimates of the boundary layer displacement thickness $\delta^*(x)$ on the flat surface, and finally an estimate of the blockage correction factor $B(x)$. The boundary layer displacement thickness $\delta^*(x)$ is estimated from the known development of flow between parallel flat plates (e.g. Kreskovsky and Shamroth [5] and References). Given this estimate of $\delta^*(x)$, the boundary layer thickness is approximated by the Blasius relationship $\delta(x) = 5/1.72 \delta^*(x)$ and the blockage factor is given by $B(x) = [H/2R - \delta^*(x)]^{-1}$. Finally, boundary layer velocity profile shapes $f(z/\delta)$, $0 \leq f \leq 1$ are defined from von-Karman Pohlhausen polynomial profiles and the boundary layer thickness $\delta(x)$. The initial velocity vector $\bar{U}$ at $t=0$ is defined by

$$\bar{U}(r, \theta, z) = \bar{U}_I B(x) f[z/\delta(x)] \quad (1)$$

At the inflow boundary, a "two-layer" boundary condition is devised such that stagnation pressure P_o is fixed at the free stream reference value $(P_o)_r$ in the core flow region ($z > \delta$), and the Cartesian velocity u is set by $u = u_e(x, t) f(z/\delta)$ for $z \leq \delta$. Here, u_e is the local free stream velocity consistent with P_o and the local wall static pressure (assumed constant across the shear layer), which is determined as part of the solution and updated after each time step. The remaining inflow conditions are $v = 0$, $\partial^2 w / \partial n^2 = 0$, and $\partial^2 c_p / \partial n^2 = g(\theta)$ where g is the value of this quantity at $t = 0$ with c_p defined as $(1 - B^2 \bar{U}_I \cdot \bar{U}_I)$, its value from the potential flow corrected for estimated blockage.

For outflow conditions, second normal derivatives of each computed velocity component are set to zero and c_p is imposed and updated after each time step from an assumed form which varies linearly with x from a fixed value at the down stream limit of the flow region ($r/R=5, \theta=0$) to a value at $\theta=\pi/2$ which is consistent with the upstream total pressure and the local free stream dynamic head. At the inflow boundary within the coolant hole, the normal velocity component is given a specified velocity profile distribution with uniform core region, and remaining velocity components are extrapolated using zero normal second derivative. The remaining condition is $\partial^2 c_p / \partial n^2 = 0$.

The foregoing interactive inflow-outflow boundary conditions are designed to permit the mass flux through the computational domain to adjust to both the imposed downstream static pressure and to viscous losses present in the flow, while maintaining a specified flow structure based on physical assumptions consistent with the flow problem under consideration. Various refinements in the interactive boundary conditions are possible, such as including the effect of local pressure gradient on boundary layer growth and profile shape.

Differencing Procedures

The differencing procedures used are a straightforward adaptation of those used by Briley and McDonald [6] in Cartesian coordinates for flow in a straight duct. Although the present procedure can ultimately be used to compute local heat transfer distributions, the present study has focused on methodology for treating the fluid flow near and within an injection hole. Accordingly, for the present, the fluid stagnation enthalpy is assumed constant for reasons of

economy. The definition of stagnation enthalpy and the equation of state for a perfect gas can then be used to eliminate pressure and temperature as dependent variables, and solution of the energy equation is unnecessary. The continuity and three momentum equations are solved with density and the r , θ and z velocity components as dependent variables. Three-point central differences were used for spatial derivatives, and second-order artificial dissipation terms are added as in [6] to prevent spatial oscillations at high cell Reynolds number. This treatment lowers the formal accuracy to first order but does not seriously degrade accuracy in representing viscous terms in thin shear layers. Analytical coordinate transformations were used to provide increased resolution of critical portions of the flow field. These coordinate transformations serve to concentrate grid points near the flat surface, near the hole surface, near the hole exit, and in the region just downstream of the hole. Derivatives of geometric data were determined analytically for use in the difference equations.

Split LBI Algorithm

The numerical algorithm used in the consistently-split "linearized block implicit" (LBI) scheme developed by Briley and McDonald [6, 7] for systematic use in solving systems of nonlinear parabolic-hyperbolic partial differential equations (PDE's). To illustrate the algorithm, let

$$(\phi^{n+1} - \phi^n)/\Delta t = \beta D(\phi^{n+1}) + (1 - \beta) D(\phi^n) \quad (2)$$

approximate a system of time-dependent nonlinear PDE's (centered about $t^n + \theta \Delta t$) for the vector ϕ of dependent variables, where D is a multidimensional vector spatial differential operator, and t is a discretized time variable such that $\Delta t = t^{n+1} - t^n$. A local time linearization (Taylor expansion about ϕ^n) is introduced, and this serves to define a linear differential operator L such that

$$D(\phi^{n+1}) = D(\phi^n) + L^n(\phi^{n+1} - \phi^n) + O(\Delta t^2) \quad (3)$$

Eq. (2) can thus be written as the linear system

$$(I - \beta \Delta t L^n)(\phi^{n+1} - \phi^n) = \Delta t D(\phi^n) \quad (4)$$

The multidimensional operator L is divided into three "one-dimensional" sub-operators $L = L_1 + L_2 + L_3$ (associated here with the three coordinate directions), and Eq. (4) is split as in the scalar development of Douglas & Gunn [8] and is written as

$$(I - \beta \Delta t L_1^n)(\phi^* - \phi^n) = \Delta t D(\phi^n) \quad (5a)$$

$$(I - \beta \Delta t L_2^n)(\phi^{**} - \phi^n) = \phi^* - \phi^n \quad (5b)$$

$$(I - \beta \Delta t L_3^n)(\phi^{***} - \phi^n) = \phi^{**} - \phi^n \quad (5c)$$

$$\phi^{n+1} = \phi^{***} + O(\Delta t^3) \quad (5d)$$

If spatial derivatives appearing in L are replaced by three-point difference formulas, then each step in Eqs. (5a-c) can be solved by a block-tridiagonal "inversion". Eliminating the intermediate steps in Eqs. (5a-d) results in

$$(I - \beta \Delta t L_1^n)(I - \beta \Delta t L_2^n)(I - \beta \Delta t L_3^n)(\phi^{n+1} - \phi^n) = \Delta t D(\phi^n) \quad (6)$$

which approximates Eq. (4) to order Δt^3 . Complete derivations are given in [6, 7]. It is noted that Beam & Warming [9] have reformulated this algorithm as a widely-used "delta form" approximate factorization scheme whose two-level form is identical to Eq. (5).

Finally, it is noted that a special treatment for radial boundary conditions is needed at the polar axis $r=0$ of the coordinate system being used. As a result of the splitting of Eq. (4), a separate (i.e., "multivalued") radial boundary condition is needed for each r-implicit step of the split algorithm. Symmetry conditions are valid at $r=0$ and provide a usable radial boundary condition for the implicit step which corresponds to $\theta=\pi/2$. Consequently, the r-implicit steps are reordered so as to solve the $\theta=\pi/2$ step first, using symmetry conditions. This defines implicit values for ϕ at $r=0$ which in turn are used to specify function boundary conditions for the remaining r-implicit steps ($\theta \neq \pi/2$), whose order is immaterial.

Computed Results

To illustrate the application of the present approach to the discrete hole film cooling problem, a sample calculation has been computed for laminar flow with coolant injected at 90 degrees to the free stream. Although present results are limited to normal injection, the approach can be extended easily to treat turbulent flow and heat transfer with injection through slanted holes at various angles with the cooled surface and either aligned or not aligned with the free stream. In the flow considered, the ratio of average coolant velocity to average main stream velocity is 0.1, the free stream Reynolds number based on duct width is 400, and the reference Mach number is 0.3. The Reynolds number based on hole diameter is 80. The thickness δ/R of the approaching boundary layer is 1.8. The solution converged in about 50 time steps and with a $14 \times 19 \times 15$ grid required about 8 minutes of CDC 7600 run time.

Selected results from this sample calculation are shown in Figs. 2-7. A vector plot of velocity in the vertical symmetry plane ($\theta=0,\pi$) passing through the center of the coolant hole is shown in Fig. 2. It can be seen that most of the coolant flow leaves the hole near its down stream edge, and for this particular set of flow conditions, the coolant flow does not penetrate the main stream boundary layer. Contours of velocity components comprising the velocity vectors in Fig. 2 are shown in Fig. 3. It can be seen that the streamwise component is not greatly disturbed by the coolant injection at this ratio of injectant to free stream velocity (0.1). However, it is clear from the contours of vertical velocity that there is significant local interaction between the

main stream and coolant stream near the hole exit, and that this interaction affects the coolant flow within the hole. These observations are in agreement with experimental observations of Bergeles, Gosman, and Launder [10] for flow at a much higher Reynolds number. The present approach provides a means for detailed prediction of this local interaction, by including a portion of the hole within the computed flow region. It is also noted that the present results did not predict a reversed flow region down stream of the hole, as was observed by Bergeles, Gosman and Launder [10]. Presumably, this reflects either the difference in flow Reynolds number or else may indicate that increased vertical mesh resolution is needed.

A further indication of the coolant injection flow is given in Fig. 4, where contours of the vertical velocity (normal to the flat surface) are shown for selected horizontal planes. Coolant velocity profiles within the hole are shown in Fig. 5 along the vertical symmetry plane, and the development from a uniform profile at the inlet ($z/R=-2$) to a highly skewed profile at the hole exit ($z/R=0$) is again evident. A vector plot of velocity in a horizontal plane one grid point above the hole exit ($z/R=0.12$) is shown in Fig. 5, and this figure shows the extent of lateral flow deflection present in the computed results. Finally, vector plots of secondary velocity in vertical planes perpendicular to the main flow direction are shown in Fig. 7 for several streamwise locations upstream, within, and downstream of the hole. These plots provide an indication of the secondary flow and fluid entrainment induced by the coolant injection.

Summary and Concluding Remarks

A computational procedure has been described for predicting flow and heat transfer which results from coolant injection through round holes oriented normal to a flat surface. The procedure solves the compressible Navier-Stokes equations without simplifying assumptions, and includes portions of the flow field both within the coolant hole and exterior to the hole. Zone embedding and interactive boundary conditions are used to achieve economy by limiting the solution domain to the near-hole flow region, where simplifying assumptions are undesirable because of local interaction and reversed flow. A sample coarse-mesh calculation has been obtained for laminar adiabatic flow with a ratio of normal injection to free stream velocity of 0.1. This calculation

demonstrates the general capability of the present procedure for treating coolant injectant flows without making simplifying assumptions in the near-hole flow region. A more detailed evaluation of the procedure and its capabilities is of interest and would require fine-mesh calculations and consideration of nonadiabatic flows and other coolant velocity ratios. The present procedure can also be extended to treat injection through slanted holes, by using a sheared coordinate system.

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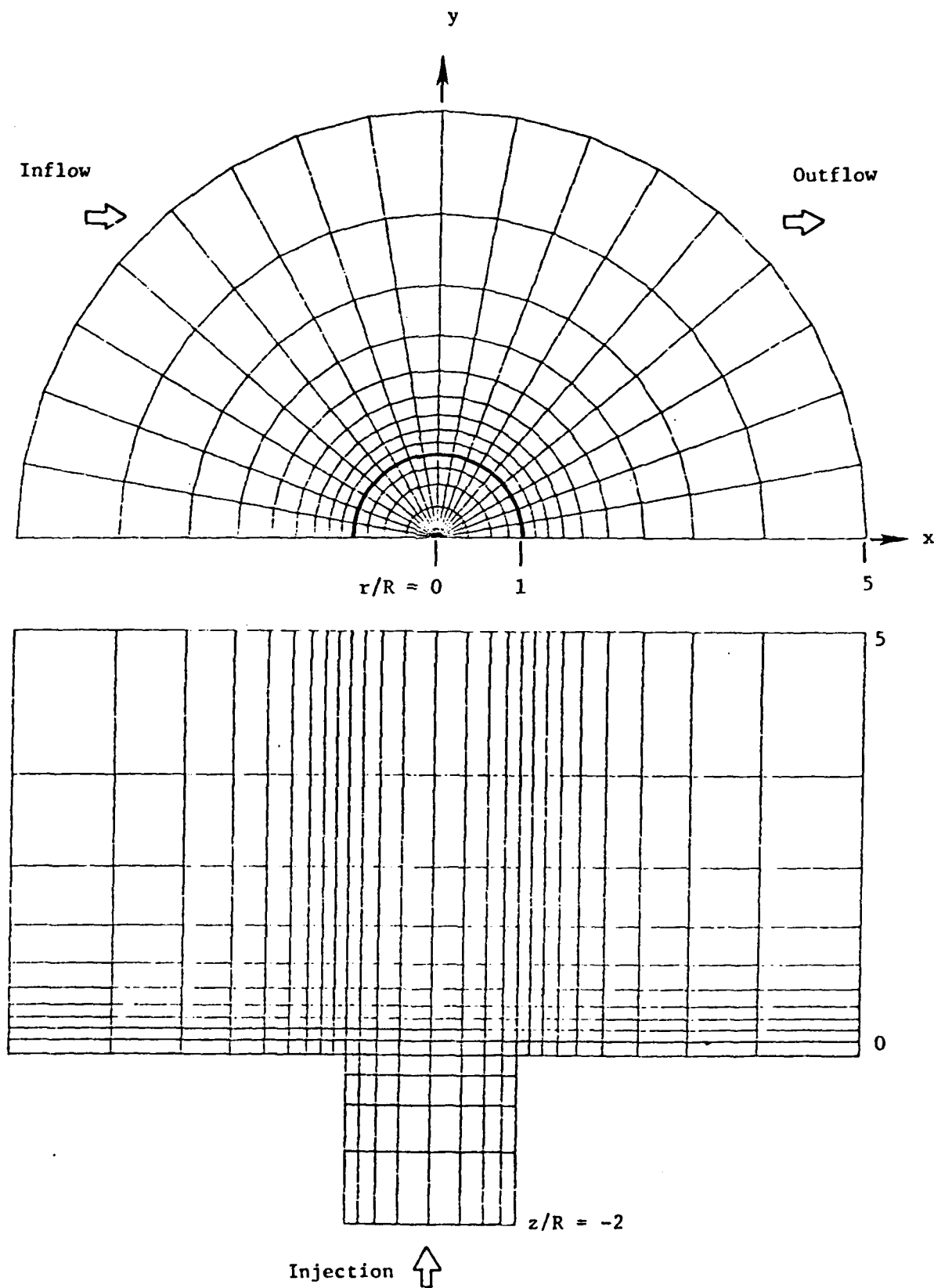


Fig. 1 - Geometry, Coordinate System and Computational Grid.

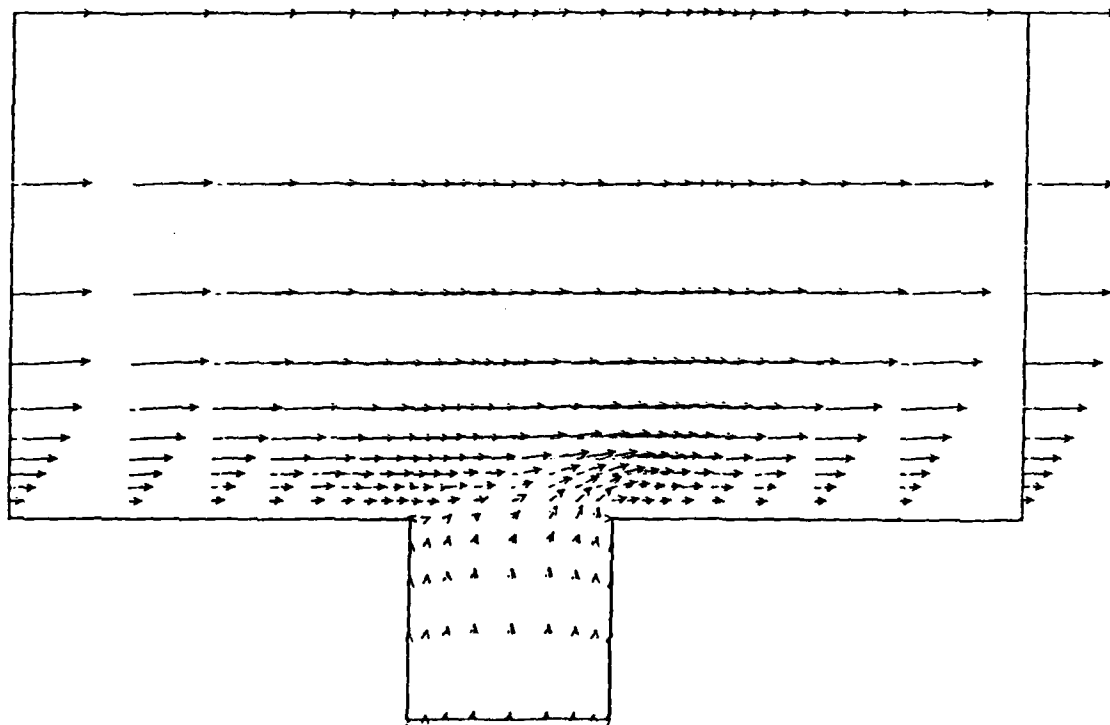
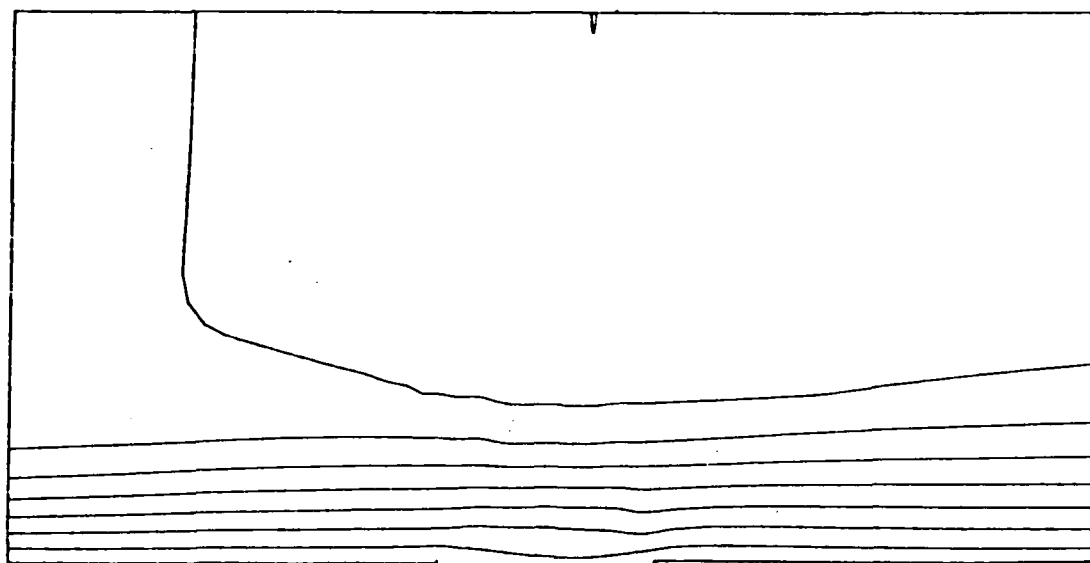
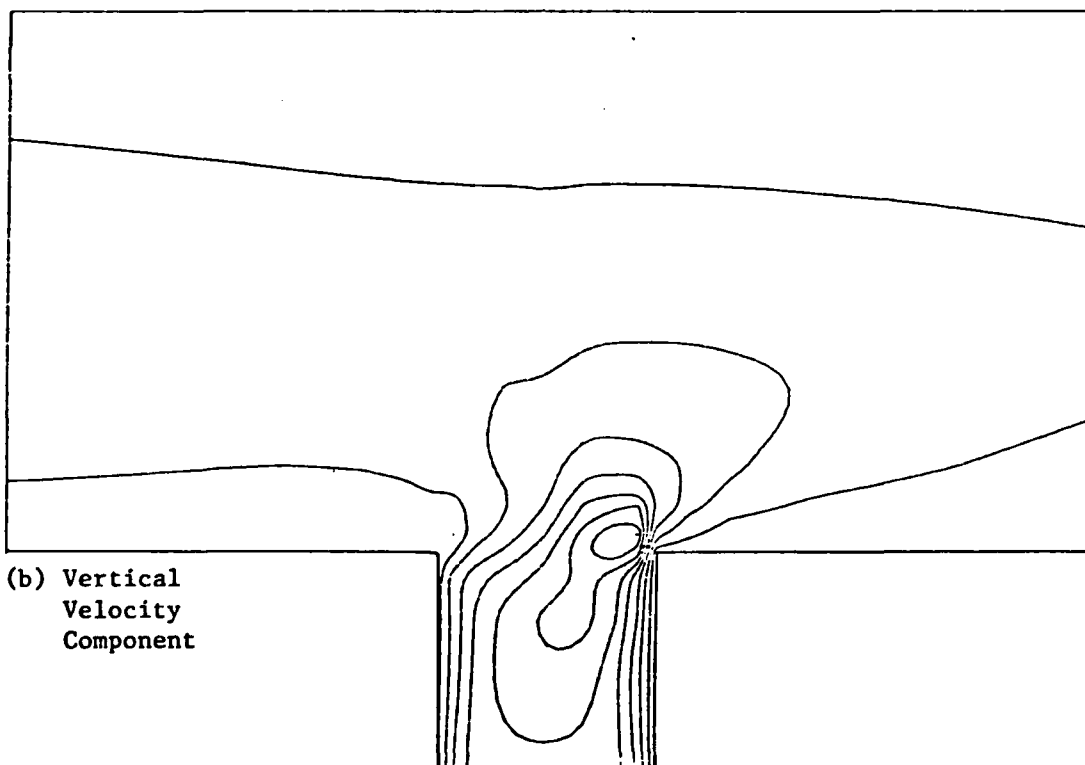


Fig. 2 - Vector Plot of Velocity in Vertical Symmetry Plane
Bisecting Coolant Hole.



(a) Streamwise
Velocity
Component



(b) Vertical
Velocity
Component

Fig. 3 - Contours of Velocity in Vertical Symmetry Plane Bisecting Coolant Hole.

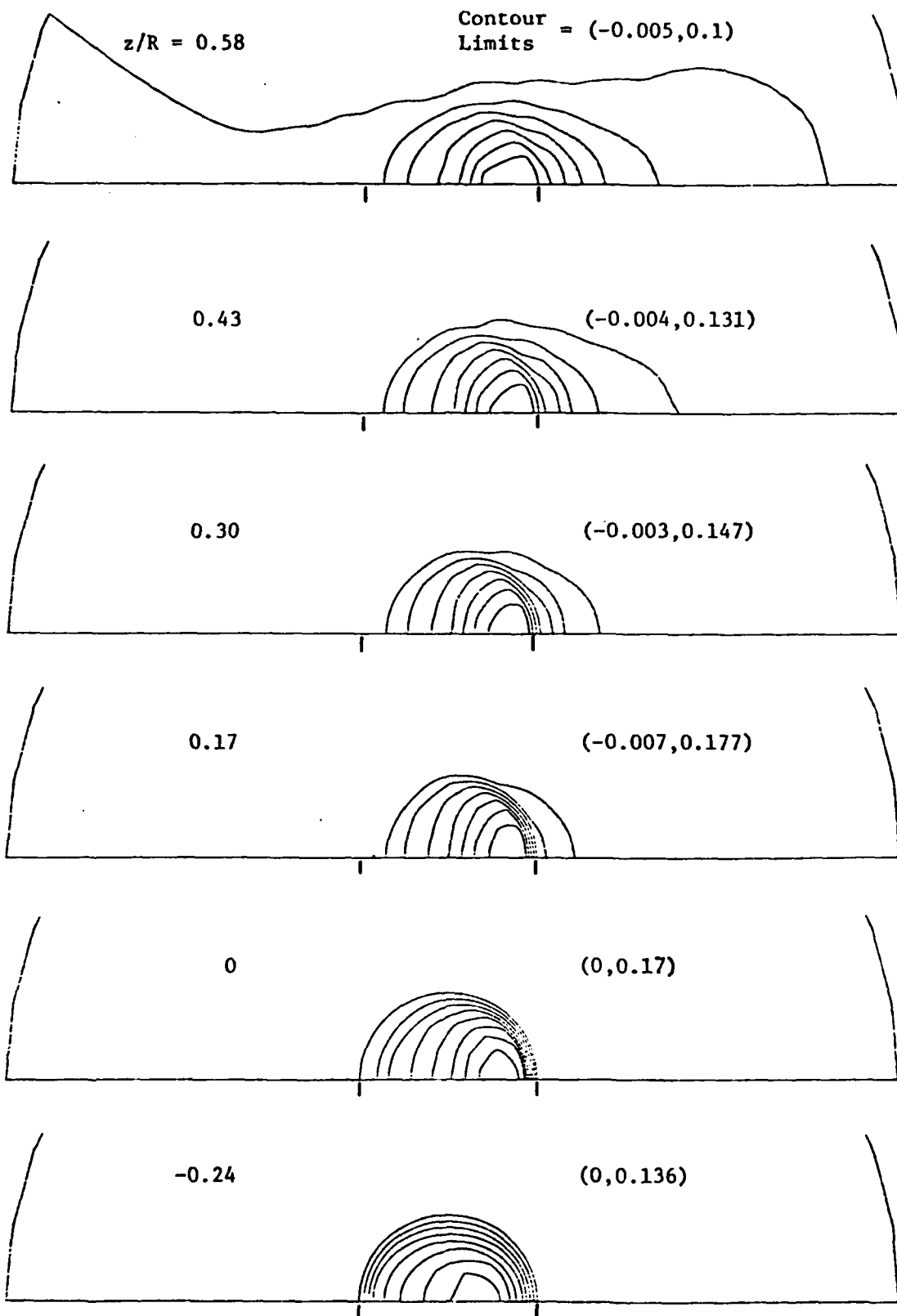


Fig. 4 - Contours of Vertical Velocity at Selected Planes Parallel to the Flat Surface.

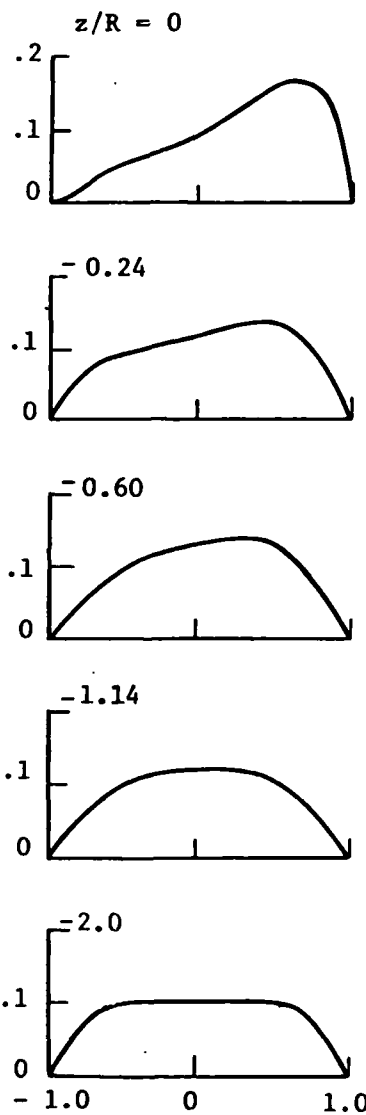


Fig. 5 - Velocity Profiles in Symmetry Plane for Flow Within the Coolant Hole.

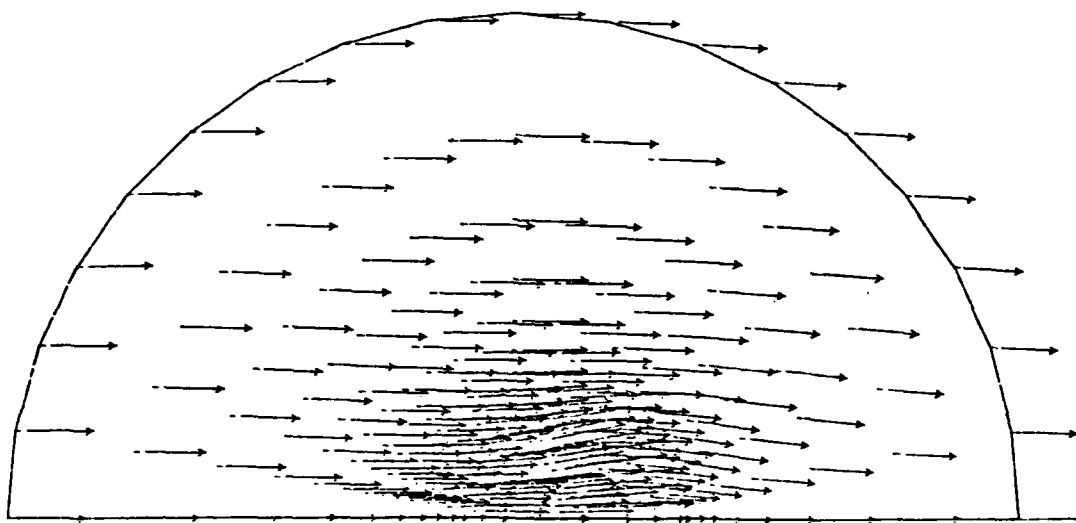


Fig. 6 - Vector Plot of velocity in Horizontal Plane at $z/R = 0.17$.

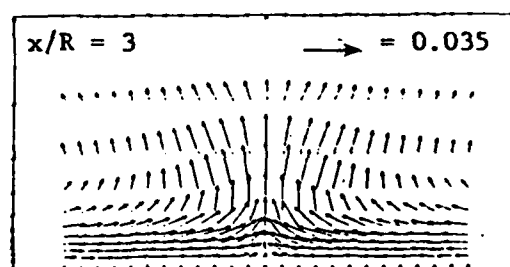
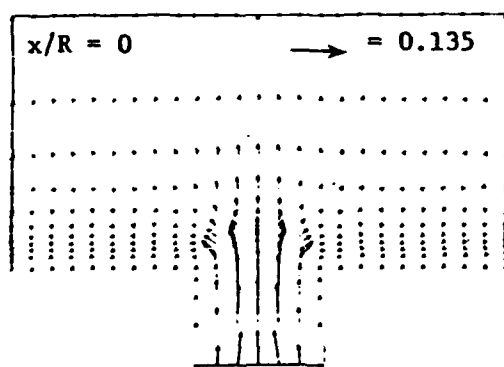
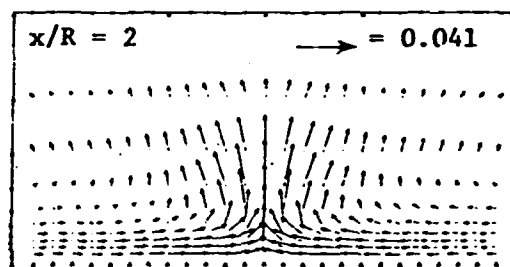
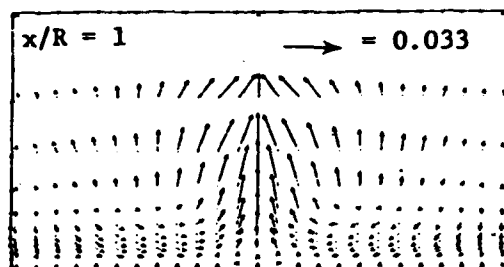
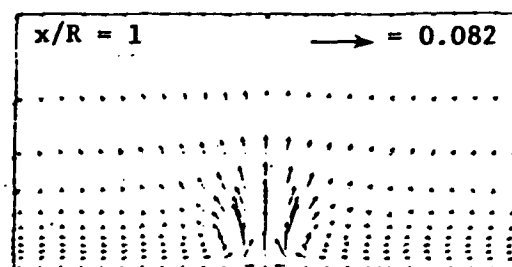
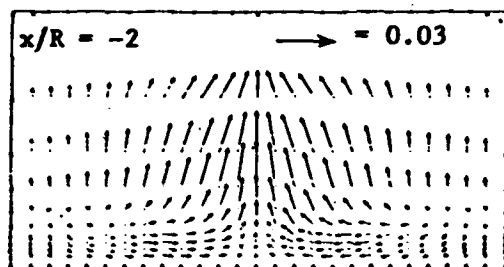


Fig. 7 - Secondary Velocity in Vertical Planes Perpendicular to Main Flow Direction.

