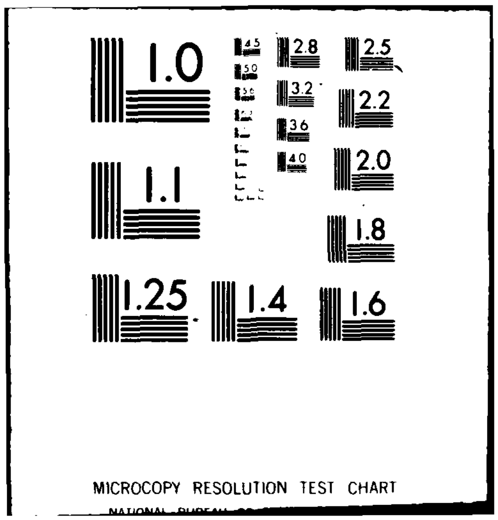
AD-A089 108 TEXAS A AND M UNIV COLLEGE STATION INST OF STATISTICS F/G 12/1
HOW NEAR IS A NEAR-OPTIMAL SOLUTION: CONFIDENCE LIMITS FOR THE --ETC(U)
MAY 80 R L SIELKEN, H M MONROE N00014-78-C-0426
UNCLASSIFIED THEMIS-TR-62 NL

1 of 1
AD A
089108


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
MICROCOPY RESOLUTION TEST CHART

LEVEL II

(12)

AD A089108

TEXAS A&M UNIVERSITY

COLLEGE STATION, TEXAS 77843

THEMIS

(12)

TEXAS A&M UNIVERSITY
PROJECT THEMIS

Technical Report, 62

HOW NEAR IS A NEAR-OPTIMAL SOLUTION :
CONFIDENCE LIMITS FOR THE GLOBAL OPTIMUM

by

Robert L. Sielken, Jr. ~~and~~ Howard M. Monroe

THEMIS-TR-62

Texas A&M University
Office of Naval Research
Contract N00014-78-C-0426
Project NR047-179

ATTACHMENT I

341380

30B

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 62	2. GOVT ACCESSION NO.: AD-A089 108
3. RECIPIENT'S CATALOG NUMBER:	
4. TITLE (and Subtitle): HOW NEAR IS A NEAR-OPTIMAL SOLUTION : CONFIDENCE LIMITS FOR THE GLOBAL OPTIMUM	5. TYPE OF REPORT & PERIOD COVERED: Technical
	6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Robert L. Sielken, Jr. and Howard M. Monroe	8. CONTRACT OR GRANT NUMBER(s): N00014-78-C-0426
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Texas A&M University, Institute of Statistics, College Station, TX 77843	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR 047-179
11. CONTROLLING OFFICE NAME AND ADDRESS:	12. REPORT DATE: May 1980
	13. NUMBER OF PAGES: 87
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): Office of Naval Research	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

1. confidence limits for the global optimum
2. evaluating heuristic solutions
3. estimating a function's maximum or minimum
4. goodness-of-fit criteria
5. quadratic programming

cont.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

For many optimization problems approximate or near-optimal solutions are the only practical solutions available. This paper identifies and compares some procedures which use independent near-optimal solutions to determine a confidence limit such that, with a user specified confidence, the global optimum is between the best near-optimal solution and the confidence limit.

A computer implementation of the procedures is available from the authors.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19. cont.
6. approximating polynomial distribution functions
7. transformations of order statistics
8. computer implementation
9. Monte Carlo study

HOW NEAR IS A NEAR-OPTIMAL SOLUTION :

CONFIDENCE LIMITS FOR THE GLOBAL OPTIMUM

by

Robert L. Sielken, Jr. and Howard M. Monroe

THEMIS OPTIMIZATION RESEARCH PROGRAM
Technical Report No. 62
May, 1980

INSTITUTE OF STATISTICS
Texas A&M University

Research conducted through the
Texas A&M Research Foundation
and sponsored by the
Office of Naval Research
Contract N00014-78-C-0426
Project NR047-179

ATTACHMENT II

ABSTRACT

For many optimization problems approximate or near-optimal solutions are the only practical solutions available. This paper identifies and compares some procedures which use independent near-optimal solutions to determine a confidence limit such that, with a user specified confidence, the global optimum is between the best near-optimal solution and the confidence limit.

A computer implementation of the procedures is available from the authors.

TABLE OF CONTENTS

1. Introduction

Mathematical programming problems have the general form of maximizing a function $g(x)$ with respect to a vector x which is restricted to be in some feasible region R. Let

$$G^* = \max_{x \epsilon R} g(x).$$

Unfortunately, unless g and R are extremely well-behaved it is quite often impractical to find an $x \epsilon R$ with $g(x) = G^*$. Hence the literature abounds with heuristic procedures purporting to provide "good" or "near-optimal" solutions for special cases of g and R. A difficulty with using these procedures in specific problems is that it is hard to determine how close such an approximate solution, say $\hat{x}$, is to being optimal; in particular, how small is $G^* - g(\hat{x})$.

The objective of this paper is to indicate some relatively new statistical procedures for obtaining an upper confidence limit on G^*. Each of these procedures results in a statement that with a chosen confidence (say, a 95% confidence) G^* does not exceed the computed confidence limit, $\hat{G}^*$; that is

$$P(G^* \leq \hat{G}^*) \doteq .95 .$$

The procedures do not indicate a feasible x with $g(x) = G^*$ or even necessarily produce a feasible x with $g(x)$ near G^*. Therefore, a confidence limit procedure is not intended as a replacement for a "solution" finding algorithm, but rather it is intended as a supplement to such an algorithm in the sense that the difference $\hat{G}^* - g(\hat{x})$ provides a needed indication of just how "near" the "near-optimal" solution is to being optimal.

The following situation illustrates one type of setting in which a confidence limit procedure can be used. A steepest ascent procedure is used to find the $x \varepsilon R$ which maximizes g. The procedure requires a feasible x as a starting point. Hence 10 starting points $x_1, \ldots, x_{10}$ are selected at random from R. Then the steepest ascent procedure is carried out beginning at each starting point. The 10 corresponding "near-optimal" solutions $\hat{x}_1, \ldots, \hat{x}_{10}$ had

$$g_1 = g(\hat{x}_1) = 3.7 \quad , \quad g_2 = g(\hat{x}_2) = 7.5 \quad ,$$
$$g_3 = g(\hat{x}_3) = 3.9 \quad , \quad g_4 = g(\hat{x}_4) = 3.6 \quad ,$$
$$g_5 = g(\hat{x}_5) = 9.8 \quad , \quad g_6 = g(\hat{x}_6) = 4.3 \quad ,$$
$$g_7 = g(\hat{x}_7) = 1.1 \quad , \quad g_8 = g(\hat{x}_8) = 7.6 \quad ,$$
$$g_9 = g(\hat{x}_9) = 7.5 \quad , \quad g_{10} = g(\hat{x}_{10}) = 4.1 \quad .$$

Let the ordered values of this independent sample of g values be denoted by $g_{(1)}, g_{(2)}, \ldots, g_{(10)}$ where $g_{(1)} \leq g_{(2)} \leq \cdots \leq g_{(10)}$. Then the confidence limit procedure would use the sample size, n = 10 here, and the ordered values $g_{(1)}, \ldots, g_{(n)}$ to produce an approximate 95% upper confidence limit $\hat{G}^*$ on the unknown G^*. The $\hat{G}^*$ indicates roughly where G^* is and the difference $\hat{G}^* - g_{(10)}$ indicates how near $g_{(10)}$ is to G^*. The chosen confidence level, say 95%, means that approximately 95% of the time when an independent sample of g values $g_1, \ldots, g_n$ are observed and $\hat{G}^*$ computed

$$g_{(n)} \leq G^* \leq \hat{G}^* \ .$$

Integer programming problems are another setting in which a confidence limit procedure can be used. In these problems the solution strategy is usually to implicitly enumerate the feasible integer solutions using a branch-and-bound scheme. In such a scheme the important task is to determine whether for say fixed values of $x_1, \ldots, x_p$ there exist feasible $x_{p+1}, \ldots, x_m$ such that the objective function $g(x_1, \ldots, x_m)$ exceeds the best value of g found so far. If no such completion $x_{p+1}, \ldots, x_m$ exists, then all solutions with these fixed values of $x_1, \ldots, x_p$ have been effectively enumerated. An approximate way of determining whether any such completion $x_{p+1}, \ldots, x_m$ of a partial solution $x_1, \ldots, x_p$ exists is to randomly select n feasible completions, calculate g for each completion, and determine a 95% upper confidence limit $\hat{G}^*$ on the maximum value G^* of g with the given fixed values of $x_1, \ldots, x_p$. If $\hat{G}^*$ doesn't exceed the best value of g found so far, then all solutions with these fixed values of $x_1, \ldots, x_p$ are considered to have been enumerated. Thus, since a relatively small sample of completions can be evaluated much faster than an extensive enumeration of the completions, a confidence limit procedure can greatly facilitate the solution of integer programming problems.

2. Confidence Limit Procedures

Three confidence limit procedures are described in the next three subsections. The author's computational experience with these procedures is described in section 3.

All three procedures determine the confidence limit $\hat{G}^*$ on G^* in basically the same way.

(1) Begin with n independent sample values of g denoted by $g_1, \ldots, g_n$. Denote the ordered values of $g_1, \ldots, g_n$ by $g_{(1)}, \ldots, g_{(n)}$ with $g_{(1)} \leq g_{(2)} \leq \cdots \leq g_{(n)} \leq G^*$.

(2) Assume that the probability $P(g)$ of a sample value of g being less than or equal to say g, ($g_{(1)} \leq g \leq G^*$), is well approximated by an m-th degree polynomial, say

$$F(g\,;\alpha,\,G^*) = \sum_{j=1}^{m} \alpha_j\,(g - G^*)^j + 1 \text{ where } m \text{ is specified but}$$

$\alpha_1, \ldots, \alpha_m$ are unknown.

(3) For a given estimate G of G^* calculate estimates $\hat{\alpha}_1, \ldots, \hat{\alpha}_m$ of $\alpha_1, \ldots, \alpha_m$.

(4) If the estimated distribution function $F(\cdot;\hat{\alpha},G)$ is a "good fit" to the observed $g_1, \ldots, g_n$, increase the estimate G of G^* and repeat (3).

(5) The upper confidence limit $\hat{G}^*$ on G^* is the largest value of G for which a good fit is obtained.

The three procedures differ in the way the $\hat{\alpha}_1, \ldots, \hat{\alpha}_m$ are determined and the criterion used to evaluate the goodness of the fit of $F(\cdot;\hat{\alpha},G)$ to $g_1, \ldots, g_n$.

The execution of the confidence limit procedures does not depend upon the specific way in which $g_1, \ldots, g_n$ are obtained.

The magnitude of $|\hat{G}^* - G^*|$ and hence the usefulness of $\hat{G}^*$ depends upon

(1) the confidence level chosen (95% in the discussion below) ;

(2) the shape of $P(g)$ and hence the way in which the sample values $g_1, \ldots, g_n$ are obtained; and

(3) how close $P(g)$ is to being an m-th order polynomial.

2.1 A Confidence Limit Procedure based on Sample Proportions

Before the sample $g_1, \ldots, g_n$ is observed, choose values $G_1, \ldots, G_k$ such that

$$G_1 < G_2 < \ldots . < G_k \le G^*$$

and

$$0 < P(G_1).$$

Let

$$np_i = \# \text{ of } g_1, \ldots, g_n \le G_i \;, \quad i = 1, \ldots, k.$$

Then $p_1, \ldots, p_k$ are sample proportions having a multinomial distribution with (symmetric) covariance matrix $V_1 = (v_{1ij})$

where

$$v_{1ij} = P(G_i)\,[1-P(G_j)]/n \text{ for } i \le j.$$

If $V_1^{-1} = (v_{1ij}^{(-1)})$, then a goodness-of-fit criterion for the P_i's and their expectations (the $P(G_i)$'s) is the quadratic form

$$Q_{1k} = \sum_{i=1}^{k} \sum_{j=1}^{k} v_{1ij}^{(-1)}\,[p_i - P(G_i)][p_j - P(G_j)]$$

which has approximately a chi-squared distribution with k degrees of freedom (see e.g. Kendall and Stuart (1960) and Watson (1959)). Furthermore, if $\hat{V}_1 = (\hat{v}_{1ij})$ where

$$\hat{v}_{1ij} = p_i[1-p_j]/n \quad \text{for } i \le j$$

and if $\hat{V}_1^{-1} = (\hat{v}_{1ij}^{(-1)})$, then

$$\bar{Q}_{1k} = \sum_{i=1}^{k} \sum_{j=1}^{k} \hat{v}_{1ij}^{(-1)}\,[p_i - F(G_i;\alpha,G^*)][p_j - F(G_j;\alpha,G^*)]$$

has approximately a chi-squared distribution with k degrees of freedom.

For a given estimate G of G^* determine $\hat{\alpha}$ by minimizing

$$\hat{Q}_{1k} = \sum_{i=1}^{k} \sum_{j-1}^{k} \hat{v}_{1ij}^{(-1)}[p_i - F(G_i;\hat{\alpha},G)][p_j - F(G_j;\hat{\alpha},G)]$$

subject to the linear restrictions

$$\left.\frac{dF(g';\hat{\alpha},G)}{dg'}\right|_{g'=g_s'} = \sum_{j=1}^{m} j\hat{\alpha}_j\,(g_s' - G)^{j-1} \geq 0, \qquad (2.1)$$

$$s = 1, \ldots, S ,$$

(since $F(\cdot;\hat{\alpha},G)$ should be a distribution function) where $g_1', \ldots, g_S'$ are a relative dense set of grid points with $g_{(1)} \leq g_1' \leq \ldots \leq g_S' \leq G$.

Having solved this quadratic programming problem using any of several available algorithms (e.g. Wolfe (1959)), accept $F(\cdot;\hat{\alpha},G)$ as being a "good fit" to $g_1, \ldots, g_n$ if and only if $\hat{Q}_{1k}$ doesn't exceed the 95-th percentile of a chi-squared distribution with k degrees of freedom. Let the confidence limit $\hat{G}_1^*$ be the largest G for which $F(\cdot;\hat{\alpha},G)$ is accepted as being a "good fit" to $g_1, \ldots, g_n$.

This confidence limit procedure was first reported by Hartley and Pfaffenberger (1969).

2.2 A Confidence Limit Procedure based on the Largest Order Statistics

If $P(g) = F(g\,;\alpha,G^*)$, then the transformed variables

$$z_i = F(g_{(i)};\alpha,G^*) \ , \ i = n-k+1, \ldots, n \ , \qquad (2.2)$$

would have the distribution of the k largest order statistics from a uniform distribution. Hence, $z_{n-k+1}, \ldots, z_n$ would have

$$E[z_i] = i \,/\, n+1 \ , \quad i = n-k+1, \ldots, n \ ,$$

and (symmetric) covariance matrix $V_2 = (v_{2ij})$ where

$$v_{2ij} = i(n-j+1)/(n+1)^2(n+2) \text{ for } i \leq j \ .$$

A goodness-of-fit criterion for the z_i's and their expectations is the quadratic form

$$Q_{2k} = \sum_{i=n-k+1}^{n} \sum_{j=n-k+1}^{n} (z_i - i/n+1)(z_j - j/n+1)\, v_{2ij}^{(-1)} \tag{2.3}$$

where $V_2^{-1} = (v_{2ij}^{(-1)})$ with

$$v_{2ii}^{(-1)} = (n-k+2)(n+1)(n+2)/(n-k+1)\ , \quad i = n-k+1$$

$$v_{2ii}^{(-1)} = 2(n+1)(n+2)\ , \quad i = n-k+2, \ldots, n$$

$$v_{2ij}^{(-1)} = -(n+1)(n+2) \quad \text{if } |i-j| = 1\ ,$$

$$v_{2ij}^{(-1)} = 0 \quad \text{otherwise}\ .$$

Lurie (1971) and Hartley and Pfaffenberger (1972) provided tables of the relevant percentiles of Q_{2k} , and these are reproduced in Appendix F.

For a given estimate G of G^* let

$$\hat{z}_i = F(g_{(i)};\hat{\alpha},G)$$

where $\hat{\alpha}$ is determined by minimizing

$$\hat{Q}_{2k} = \sum_{i=n-k+1}^{n} \sum_{j=n-k+1}^{n} (\hat{z}_i - i/n+1)(\hat{z}_j - j/n+1) v_{2ij}^{(-1)} \tag{2.4}$$

subject to the linear restrictions (2.1) with $g_{(n-k+1)} \leq g_1' \leq \cdots \leq g_S' \leq G$.

Having solved this quadratic programming problem, accept $F(\cdot;\hat{\alpha},G)$ as being a "good fit" to $g_1, \ldots, g_n$ if and only if the observed value of $\hat{Q}_{2k}$ doesn't exceed the tabled 95-th percentile of $\hat{Q}_{2k}$ given in Appendix F. Let the confidence limit $\hat{G}_2^*$ be the largest G for which $F(\cdot;\hat{\alpha},G)$ is accepted as being a "good fit" to $g_1, \ldots, g_n$.

Unfortunately, the distribution of $\hat{Q}_{2k}$ is very different from that of Q_{2k}. In fact, for n = 40, if

$$Q_{2k}' = Q_{2k} \text{ with } z_{n-k+1} = \cdots = z_n = 1,$$

and

$$Q''_{2k} = \text{95-th percentile of } Q_{2k} \text{ from Table F.1,}$$

then

$$\hat{Q}_{2k} \leq Q'_{2k} = 5.83 \leq Q''_{2k} = 15.52 \quad \text{for } k = 5,$$

$$\hat{Q}_{2k} \leq Q'_{2k} = 13.54 \leq Q''_{2k} = 25 \quad \text{for } k = 10,$$

and

$$\hat{Q}_{2k} \leq Q'_{2k} = 24.23 \leq Q''_{2k} = 32.7 \quad \text{for } k = 15.$$

Thus, it is fruitless to compare $\hat{Q}_{2k}$ with the percentiles of Q_{2k} given in Tables F.1, F.2, and F.3. Although the distribution of Q_{2k} doesn't depend upon P(g), the distribution of $\hat{Q}_{2k}$ probably does. Nevertheless, the empirical evidence reported in section 3 suggests that $\hat{Q}_{2k}$ can be fruitfully compared to the Monte Carlo generated percentiles of $\hat{Q}_{2k}$ given in Appendix F (Tables F.4, F.5, ...). These percentiles were generated by simulating $z_{n-k+1}, \ldots, z_n$ directly from the uniform distribution, determining $\hat{\alpha}$, and calculating $\hat{Q}_{2k}$. Each tabled percentile was based on 1500 repetitions.

2.3 A Confidence Limit Procedure based on Derivatives

Liau, Hartley, and Sielken (1973) proposed a modification to the confidence limit procedure described in section 2.2. The proposal uses the transformed variables $z_1, \ldots, z_n$ and uses Q_{2k} as its goodness-of-fit criterion for the z_i's and their expectations. However, for a given estimate G of G^* the z_i's are estimated by

$$\tilde{z}_i = \sum_{j=1}^{n} \hat{\alpha}_j (g_{(i)} - G)^j + 1 + \hat{\alpha}_0$$

where the $\hat{\alpha}_0, \ldots, \hat{\alpha}_m$ are determined by doing an unrestricted minimization of

$$\sum_{i=n-k+1}^{n} \sum_{j=n-k+1}^{n} (\tilde{z}_i - i/n+1)(\tilde{z}_j - i/n+1) v_{2ij}^{(-1)} \quad . \qquad (2.5)$$

The difference between $\hat{\alpha}_0$ and 0 is an additional indication of the suitability of G as an estimate of G^*.

Since the $\hat{\alpha}$'s are unrestricted in (2.5), (2.5) corresponds to a weighted least squares fit of the $\tilde{z}_i$'s to their expectations. Therefore,

$$\hat{\alpha} = (X_G^T V_2^{-1} X_G)^{-1} X_G^T V_2^{-1} Y$$

where

$$Y = \left\{ [(n-k+1)/(n+1)]^{-1}, \ldots, [n/(n+1)]^{-1} \right\}^T$$

and

$$X_G = \begin{bmatrix} 1 & (g_{(n-k+1)}^{\,G}) & \cdots & (g_{(n-k+1)} - G)^m \\ \vdots & \vdots & & \vdots \\ 1 & (g_{(n)} - G) & \cdots & (g_{(n)} - G)^m \end{bmatrix}.$$

Since the $\hat{\alpha}$ in

$$\tilde{F}(g\ ; \hat{\alpha}, G) = \sum_{j=1}^{m} \hat{\alpha}_j\ (g - G)^j + 1 + \hat{\alpha}_0$$

was determined without any restriction on $\hat{\alpha}_0$ and the derivatives of $\tilde{F}$, the goodness of the fit of $\tilde{F}$ relative to $g_1, \ldots, g_n$ is evaluated in terms of $\hat{\alpha}_0$ and the derivative of $\tilde{F}$ between $g_{(n)}$ and G. In particular, defining

$$g_{(n+i)} = g_{(n)} + \frac{i}{m}\ (G - g_{(n)}), \quad i = 1, \ldots, m ,$$

and

$$\hat{\lambda}_0 = \hat{\alpha}_0 ,$$

$$\hat{\lambda}_i = \frac{d\tilde{F}}{dg'}\ (g'; \hat{\alpha}, G) \Bigg|_{g' = g_{(n+i)}} = \sum_{j=1}^{m} j\ \hat{\alpha}_j (g_{(n+i)} - G)^{j-1} , \quad i = 1, \ldots, m ,$$

then $\hat{\lambda}_0$ should be approximating zero and the $\hat{\lambda}_1, \ldots, \hat{\lambda}_m$ should be approximating nonnegative derivatives. Since

$$\hat{\lambda} = (\hat{\lambda}_0, \hat{\lambda}_1, \ldots, \hat{\lambda}_m)^T = H\hat{\alpha}$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 2(g_{(n+1)}-G) & \cdots & m(g_{(n+1)}-G)^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 1 & 2(g_{(n+m)}-G) & \cdots & m(g_{(n+m)}-G)^{m-1} \end{bmatrix},$$

$\hat{\lambda}$ is approximately a multivariate normal vector with covariance matrix $\Sigma = H(X_G^T V_2^{-1} X_G)^{-1} H^T$. Thus, for some $\lambda = (0, \lambda_1, \ldots, \lambda_m)^T$ with $\lambda_i \geq 0$, $i = 1, \ldots, m$, the $Q_{2k} \equiv (\hat{\lambda}-\lambda)^T \Sigma^{-1} (\hat{\lambda}-\lambda)$ should have approximately a chi-squared distribution with m+1 degrees of freedom (see, e.g. Kshirsagar(1972)). Hence, if $\chi^2_{m+1,.95}$ is the 95-th percentile of a chi-squared distribution with m+1 degrees of freedom, G is accepted if and only if

$$\hat{Q}_{3k} \equiv \min_{\lambda = (0,\lambda_1, \ldots, \lambda_m)^T \geq 0} (\hat{\lambda}-\lambda)\, \Sigma^{-1}\, (\hat{\lambda}-\lambda) \leq \chi^2_{m+1,.95}.$$

The confidence limit $\hat{G}_3^*$ is the largest G accepted.

Of course,

$$\min_{\lambda = (0,\lambda_1, \ldots, \lambda_m) \geq 0} (\hat{\lambda}-\lambda)\, \Sigma^{-1}\, (\hat{\lambda}-\lambda)$$

is a quadratic programming problem.

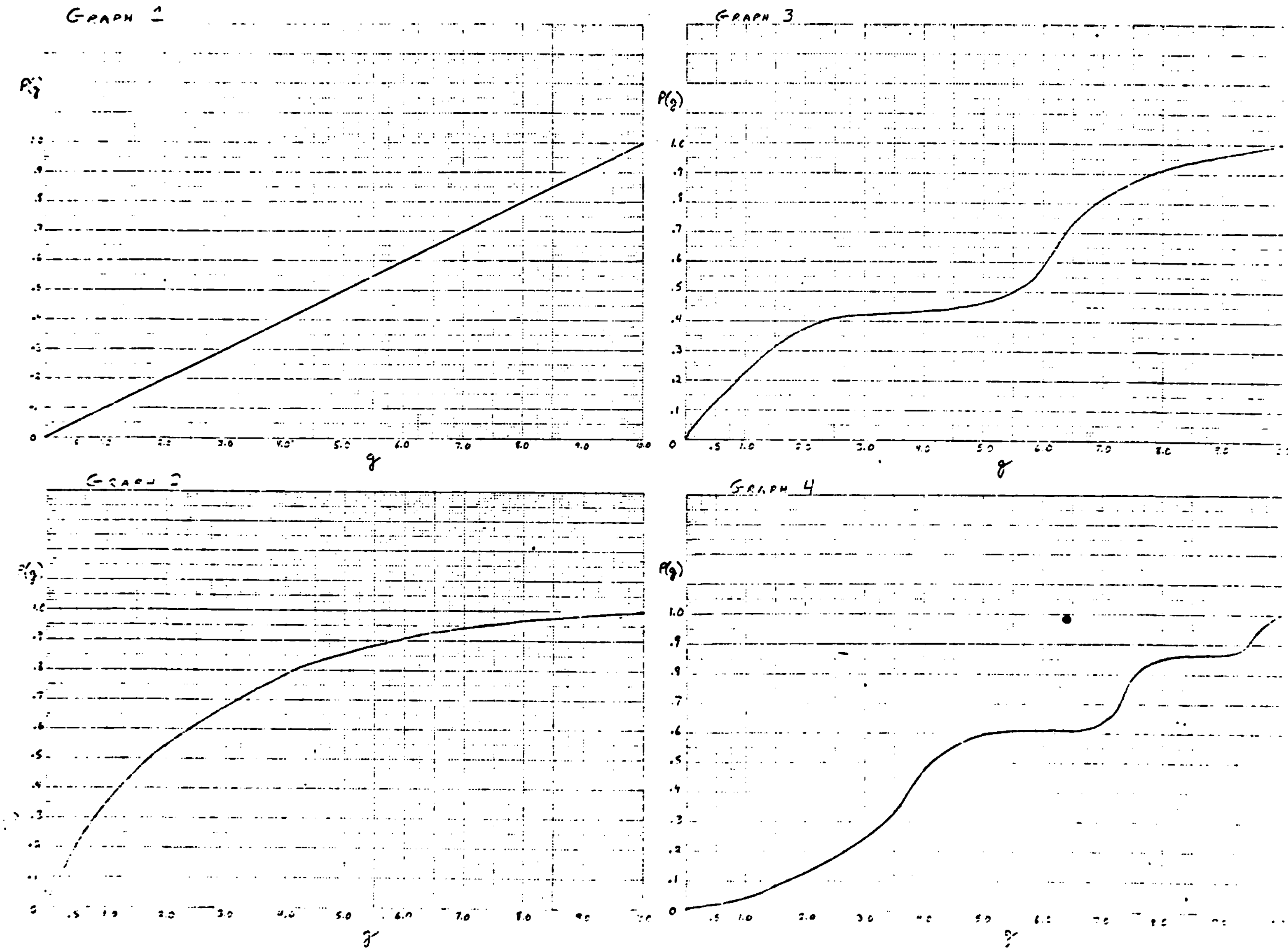
3. Observed Empirical Behavior of the Confidence Limits

The usefulness of Q_{1k} as a goodness-of-fit statistic for a completely specified distribution P(g) is well understood. In addition

Hartley and Pfaffenberger (1972) and Lurie (1971) have shown that Q_{2k} can be a very powerful goodness-of-fit statistic for a completely specified distribution P(g). However, the distributions of the goodness-of-fit statistics $\hat{Q}_{1k}$, $\hat{Q}_{2k}$, and $\hat{Q}_{3k}$ which essentially represent the minimums over the parameter spaces of Q_{1k}, Q_{2k}, Q_{3k} respectively are mostly unknown. The original hope was that the distribution of the $\hat{Q}_k$'s would be sufficiently close to that of the Q_k's to make the $\hat{Q}_k$'s useful in determining goodness-to-fit for not completely specified distributions. (The $F(\cdot;\alpha,G^*)$ are, of course, not completely specified since α and G^* are both unknown.) Unfortunately, in a recent Monte Carlo study it has become apparent that this hope has not been fully realized. The relatively large difference between the distributions of Q_{2k} and $\hat{Q}_{2k}$ has already been noted and at least partially compensated for via the Monte Carlo sample percentiles for $\hat{Q}_{2k}$ tabled in Appendix F.

The Monte Carlo study was itself quite reasonable although small. Each of the four distributions, P(g), illustrated in Figure 1 was considered. In each case the range of the distribution is from 0 to 10 with $G^* = 10$. Three independent samples of size n = 40 were selected from each P(g) and 95% confidence limits $\hat{G}_1^*$, $\hat{G}_2^*$, $\hat{G}_3^*$ determined. The values k = 5, 10, 20 were considered with m = 1 and 2 for each k. When determining $\hat{G}_1^*$, the G_1, ..., G_k were 2, 4, 6, 8, 10 for k = 5, and 1, 2, ..., 10 for k = 10, and .5, 1.0, 1.5, ..., 10.0 for k = 20. When determining $\hat{G}_1^*$ and $\hat{G}_2^*$, the number of grid points g_1', ..., g_S' at which the derivative of the estimated distribution was required to be nonnegative was S = 10, and the g_1', ..., g_{10}' were evenly spaced between $g_{(1)}$ and G for $\hat{G}_1^*$ and evenly spaced between $g_{(n-k+1)}$ and G for $\hat{G}_2^*$. The sequence of estimates G of G^* considered was 10.0, 10.1, 10.2, 10.3, 10.4, 10.5, 10.6, 10.8, 11.0,

Figure 1. Graphs of the distribution functions P(g) sampled in the Monte Carlo Study



11.2, 11.4, 11.6, 11.8, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5. The last estimate G of G^* corresponding to a "good fit" was taken to be the 95% confidence limit $\hat{G}^*$. These confidence limits are given in Table 1.

The Monte Carlo study indicated several interesting characteristics of the confidence limit procedures.

All three confidence limits $\hat{G}_1^*$, $\hat{G}_2^*$, $\hat{G}_3^*$ did better in terms of $|\hat{G}^* - G^*|$ for m = 1 than they for m = 2. Some additional experimentation with m = 3 supports the idea that m = 1 is probably best.

Three practical limitations to the confidence limit $\hat{G}_1^*$ are

(i) the necessity of specifying apriori $G_1 < G_2 < \ldots < G_k < G^*$ with $0 < P(G_1) < P(G_2) < \ldots < P(G_k) < 1$,

(ii) the singularity of V_1 if $P(G_1) = 0$ or $G_i \geq G^*$ for any i, and

(iii) the frequent singularity of $\hat{V}_1$ if k is large relative to n so that two or more sample proportions (p_i) are equal.

This last limitation made $\hat{G}_1^*$ useless most of the time when k = 10 or 20 and n = 40.

The confidence limits $\hat{G}_2^*$ and $\hat{G}_3^*$ tended to improve as k increased. Although the four P(g) graphs in Figure 1 are certainly not all linear, they do not depart "drastically" from linearity, so it is entirely possible that $\hat{G}_2^*$ and $\hat{G}_3^*$ might not always improve with k for other shapes of P(g).

Unfortunately, for small k (k = 5, 10) the distribution of $\hat{Q}_{3k}$ seems to not be close enough to a chi-squared distribution with m + 1 degrees of freedom to make $\hat{G}_3^*$ very useful. From a theoretical viewpoint the problem is in the "approximate" normality of $\hat{\alpha} = (X_G^T V_2^{-1} X_G)^{-1} X_G^T V_2^{-1} Y$ since it is the X_G which is random and not the Y. From a practical viewpoint the problem is that almost any reasonable $\hat{\alpha}$ has $\hat{\lambda}_i \geq 0$, i = 1, ..., m. Hence Q_{3k} becomes almost entirely based on $|\hat{\alpha}_0 - 0|$.

Table 1. Approximate 95% confidence limits for G^* = 10 based on 40 observations from the P(g) distributions in graphs 1, 2, 3, 4*

Confidence Limit	m	k	Graph 1 Trial: 1	2	3	Graph 2 1	2	3	Graph 3 1	2	3	Graph 4 1	2	3
$\hat{G}_1^*$	1	5	10.2	10.1	10.1	10.0	10.0	10.3	10.1	10.0	10.1	10.0	10.1	10.1
	1	10	10.2	10.1	10.2	10.0	--	--	--	--	10.2	--	--	--
	1	20	--	--	--	--	--	--	--	--	--	--	--	--
	2	5	13.5	13.5	13.5	11.0	--	14.5	10.2	10.0	10.1	13.0	13.5	13.0
	2	10	--	13.5	13.5	10.3	--	--	--	--	--	--	--	--
	2	20	--	--	--	--	--	--	--	--	--	--	--	--
$\hat{G}_2^*$	1	5	11.6	11.0	11.2	15.0	--	--	10.5	--	14.0	10.0	10.0	10.1
	1	10	13.0	11.6	11.2	11.2	11.6	14.5	12.5	13.0	13.0	10.0	10.0	10.3
	1	20	13.0	11.2	11.4	10.0	10.5	10.0	10.0	11.0	11.0	10.0	10.0	10.0
	2	5	13.5	12.0	13.0	--	--	--	10.0	--	--	--	10.0	10.3
	2	10	--	11.6	14.0	--	--	--	15.0	--	--	10.0	10.0	10.1
	2	20	--	15.0	--	--	--	--	10.0	--	15.0	10.0	10.0	10.0
$\hat{G}_3^*$	1	5	--	--	--	--	--	--	--	--	--	--	--	--
	1	10	15.0	--	--	--	--	--	--	--	--	--	--	--
	1	20	11.8	11.6	11.2	11.4	12.5	14.5	13.0	12.5	11.2	--	--	14.5
	2	5	--	--	--	--	--	--	--	--	--	--	--	--
	2	10	--	--	--	--	--	--	--	--	--	--	--	--
	2	20	--	--	--	--	--	--	--	14.5	14.5	--	--	--

*A -- indicates that none of the trial estimates of G^* yielded a goodness-of-fit index exceeding its critical 95-th percentile.

But even concentrating on $\hat{\alpha}_0$ and evaluating an estimate G of G^* on the basis of the "approximate" normality of $(\hat{\alpha}_0 - 0)/[(X_G^T V_2^{-1} X_G)^{-1}_{1,1}]^{1/2}$ doesn't yield a $\hat{G}^*$ close enough to G^* often enough to be useful.

The champion among $\hat{G}_1^*$, $\hat{G}_2^*$, and $\hat{G}_3^*$ appears to be $\hat{G}_2^*$. The confidence limit $\hat{G}_2^*$ probably has the strongest intuitive appeal, and the usefulness of Q_{2k} as a goodness-of-fit index for a completely specified P(g) also makes $\hat{G}_2^*$ attractive.

As one might expect, the more nonlinear P(g) is near G^* and the nearer $\left.\frac{dP(g)}{dg}\right|_{g=G^*}$ is to zero the tougher it is to estimate G^*. In fact, the ranking for the slopes of P(g) near G^* for graphs 1, 2, 3, and 4 was 4, 1, 3, 2 with 4 being the largest and the ranking of the performance of $\hat{G}_2^*$ was the same 4, 1, 3, 2 with $\hat{G}_2^*$ having the best performance for graph 4. This is particularly heartening from a practical viewpoint since graph 4 corresponds to a g(x) with local optimum for $x \varepsilon R$ - a situation which seems to occur frequently in practical mathematical programming problems.

The behavior of $\hat{G}^*$ appears to be best when the P(g) distribution is linear with positive slope for $g < G^*$ in a neighborhood of G^*. A way to transform the underlying distribution P(g) into a distribution having these characteristics more closely is to perform the confidence limit procedures on

$$\tilde{g}_i = g_i - u_i , \quad i = 1, \ldots, n,$$

where $u_1, \ldots, u_n$ are independent uniform deviates with say

$$\begin{aligned} P(u \le t) &= 0 , & t < 0 \\ &= \frac{1}{U^*} , & 0 \le t \le U^* \\ &= 1 & t \ge U^* \end{aligned}$$

and U^* a suitably chosen constant. Of course, the least upper bound on the $\tilde{g}$'s is still G^*. At first glance it seems counter-productive to introduce noise (the u's) into the observations analyzed. However, when $P(g)$ is nonlinear with slope near 0 at G^*, the additional noise in the $\tilde{g}$'s may be more than offset by the relative linearity of $P(\tilde{g})$. To illustrate these ideas, the above Monte Carlo study was essentially repeated with the g_i's transformed to $\tilde{g}$'s using u's that were uniformly distributed on $[0, U^*]$ with $U^* = .5$, 1.0, 2.0, and 5.0. The results are indicated in Tables 2, 3, 4 and 5 respectively. (Since $\hat{G}_3^*$ appeared to do better for k large than for k small, k = 5 was replaced by k = 40 in this study.)

Several interesting observations can be made concerning the outcome of the Monte Carlo study of the uniform deviate transformation. Several of the tentative conclusions based on the original Monte Carlo study were confirmed; namely,

(i) m = 1 tends to be best,

(ii) $\hat{G}_1^*$ requires too much apriori information and usually can't be determined for k large relative to n (k = 10, 20 here),

(iii) $\hat{G}_2^*$ and $\hat{G}_3^*$ tend to improve as k increase, and

(iv) $\hat{G}_2^*$ tends to perform the best overall.

As expected, the best value for U^* depended on the graph of $P(g)$. Tables 6 and 7 indicate the relative behavior of $\hat{G}_2^*$ for $U^* = 0.0$ (the original Monte Carlo study), 0.5, 1.0, 2.0, and 5.0. In graphs 1, 2, 3, and 4 the relative magnitudes of the slope of $P(g)$ near G^* is roughly the same as the relative magnitudes of $P(9.0 \leq g \leq 10.0)$. Hence the following comparison is revealing

Graph	$P(9.0 \le g \le 10)$	Best U^*
4	.15	0
1	.10	1 or 2
3	.05	1 or 2
2	.01	5 .

Thus, it appears that the greater the probability of g being close to G^* the smaller U^* should be. A conclusion which agrees with most people's intuition.

Table 2. Approximate 95% confidence limits for $G^* = 10$ based on 40 transformed observations from the P(g) distributions in graphs 1, 2, 3, and 4 with the transformation being the subtraction of a uniform deviate with range [0, .5]*

Confidence Limit	m	k	Graph 1			Graph 2			Graph 3			Graph 4		
		Trial:	1	2	3	1	2	3	1	2	3	1	2	3
$\hat{G}_1^*$	1	5	10.3	10.2	10.2	--	--	10.6	10.0	--	10.2	10.0	10.0	10.2
	1	10	--	10.2	--	--	--	--	--	--	--	--	10.1	--
	1	20	--	--	--	--	--	--	--	--	--	--	--	--
	2	5	14.0	13.5	13.5	--	--	15.0	10.2	--	10.2	13.0	13.5	10.2
	2	10	--	13.5	--	--	--	--	--	--	--	--	13.5	--
	2	20	--	--	--	--	--	--	--	--	--	--	--	--
$\hat{G}_2^*$	1	5	11.4	11.4	11.2	14.5	--	--	11.4	--	14.0	14.5	15.0	10.3
	1	10	12.5	11.6	11.0	12.5	14.5	12.5	12.5	12.5	12.5	10.0	10.0	10.2
	1	20	13.0	11.4	11.2	10.0	10.8	10.0	11.2	10.0	10.8	10.0	10.0	10.0
	2	5	13.0	13.0	12.5	--	--	--	13.0	--	--	--	--	10.8
	2	10	--	11.6	11.0	--	--	--	--	--	--	10.0	10.0	11.2
	2	20	--	15.5	15.5	15.0	--	--	10.5	15.5	14.0	10.0	10.0	10.0
$\hat{G}_3^*$	1	10	13.0	--	--	--	--	--	--	--	15.0	--	--	14.0
	1	20	11.8	11.6	11.2	11.4	11.8	14.0	12.5	12.5	11.6	--	13.5	12.5
	1	40	11.0	10.8	10.6	10.0	10.0	11.0	11.2	11.6	11.0	11.4	11.2	11.2
	2	10	--	--	--	--	--	--	--	--	--	--	--	--
	2	20	--	--	15.5	--	--	--	--	15.0	15.5	--	--	--
	2	40	15.0	14.0	11.4	10.8	12.5	10.3	15.5	--	15.0	10.0	15.5	15.5

*A -- indicates that none of the trial estimates of G^* yielded a goodness-of-fit index exceeding its critical 95-th percentile.

Table 3. Approximate 95% confidence limits for $G^* = 10$ based on 40 transformed observations from the P(g) distributions in graphs 1, 2, 3, and 4 with the transformation being the subtraction of a uniform deviate with range [0, 1.0]*

Confidence Limit				Graph 1			Graph 2			Graph 3			Graph 4		
	m	k	Trial:	1	2	3	1	2	3	1	2	3	1	2	3
$\hat{G}_1^*$	1	5		10.3	10.2	10.2	--	--	10.8	10.3	--	10.2	10.2	10.0	10.0
	1	10		10.4	--	10.2	--	--	--	--	--	--	--	--	--
	1	20		--	--	--	--	--	--	--	--	--	--	--	--
	2	5		14.0	13.5	13.5	--	--	15.5	10.3	--	10.6	13.5	13.5	10.0
	2	10		14.5	--	13.5	--	--	--	--	--	--	--	--	--
	2	20		--	--	--	--	--	--	--	--	--	--	--	--
$\hat{G}_2^*$	1	5		11.0	11.4	10.8	14.0	--	--	11.6	--	13.5	14.0	15.0	10.5
	1	10		11.8	11.6	11.0	10.8	11.8	14.0	12.5	12.5	12.5	10.0	11.8	10.1
	1	20		12.5	11.2	11.0	10.2	10.6	11.4	11.0	10.0	10.5	10.0	12.5	10.0
	2	5		12.5	13.0	11.8	--	--	--	13.5	--	--	--	--	11.4
	2	10		15.5	14.0	13.5	--	--	--	--	--	--	10.0	15.5	11.2
	2	20		--	15.5	15.0	--	--	--	10.5	15.0	14.5	10.0	--	10.0
$\hat{G}_3^*$	1	10		--	--	14.0	--	--	--	--	--	--	--	--	10.6
	1	20		11.8	11.4	11.0	10.2	11.2	12.5	13.0	13.0	11.6	14.5	12.0	11.4
	1	40		10.8	10.8	10.4	10.2	10.0	10.4	11.2	10.0	10.8	10.8	10.8	11.0
	2	10		--	--	--	--	--	--	--	--	--	--	--	13.0
	2	20		--	--	15.0	15.5	--	--	--	15.0	13.5	15.5	--	15.5
	2	40		15.0	12.5	11.0	11.4	13.0	12.5	15.5	--	15.0	14.5	14.5	12.0

*A -- indicates that none of the trial estimates of G^* yielded a goodness-of-fit index exceeding its critical 95-th percentile.

Table 4. Approximate 95% confidence limits for G^* = 10 based on 40 transformed observations from the P(g) distributions in graphs 1, 2, 3, and 4 with the transformation being the subtraction of a uniform deviate with range [0, 2.0]*

Confidence Limit	m	k	Graph 1 Trial: 1	2	3	Graph 2 1	2	3	Graph 3 1	2	3	Graph 4 1	2	3
$\hat{G}_1^*$	1	5	10.3	10.2	10.3	--	--	--	10.2	10.0	10.0	10.3	10.0	10.2
	1	10	--	10.3	--	--	--	--	--	--	--	10.2	--	10.2
	1	20	--	--	--	--	--	--	--	--	--	--	--	--
	2	5	14.0	13.5	14.0	--	--	--	10.3	10.0	13.5	13.5	14.0	10.2
	2	10	--	14.0	--	--	--	--	--	--	--	14.0	--	13.5
	2	20	--	--	--	--	--	--	--	--	--	--	--	--
$\hat{G}_2^*$	1	5	10.0	14.0	11.8	--	13.5	--	15.0	14.5	14.5	14.0	14.0	11.0
	1	10	10.2	13.0	10.5	11.2	11.0	15.0	13.0	10.8	12.5	12.5	10.0	11.6
	1	20	10.0	12.0	10.4	10.0	10.0	11.0	12.0	10.0	10.8	12.0	11.2	11.4
	2	5	10.0	--	14.0	--	--	--	--	--	--	--	15.0	12.0
	2	10	10.2	--	13.5	--	--	--	--	14.5	--	--	10.0	14.5
	2	20	10.0	--	14.5	--	--	--	--	14.0	15.0	--	--	15.5
$\hat{G}_3^*$	1	10	--	15.0	13.5	--	12.5	--	--	--	15.0	--	--	14.0
	1	20	13.5	11.8	10.4	10.4	10.0	14.0	12.5	10.6	11.2	12.0	13.5	11.2
	1	40	10.6	11.0	10.0	10.0	10.0	11.2	11.0	10.0	10.6	10.6	11.4	10.5
	2	10	--	--	--	--	--	--	--	--	--	--	--	--
	2	20	--	--	14.5	13.5	15.0	--	--	14.5	14.5	--	--	14.5
	2	40	14.0	14.5	10.2	10.0	12.0	10.0	15.5	14.0	11.6	14.5	13.0	13.5

*A -- indicates that none of the trial estimates of G^* yielded a goodness-of-fit index exceeding its critical 95-th percentile.

Table 5. Approximate 95% confidence limits for $G^* = 10$ based on 40 transformed observations from the P(g) distributions in graphs 1, 2, 3, and 4 with the transformation being the subtraction of a uniform deviate with range [0, 5.0]*

Confidence Limit	m	k	Graph 1 Trial: 1	Graph 1 Trial: 2	Graph 1 Trial: 3	Graph 2 Trial: 1	Graph 2 Trial: 2	Graph 2 Trial: 3	Graph 3 Trial: 1	Graph 3 Trial: 2	Graph 3 Trial: 3	Graph 4 Trial: 1	Graph 4 Trial: 2	Graph 4 Trial: 3
$\hat{G}_1^*$	1	5	--	10.3	10.3	--	--	--	10.5	--	--	--	--	10.3
	1	10	--	--	--	--	--	--	10.5	--	--	--	--	10.3
	1	20	--	--	--	--	--	--	--	--	--	--	--	--
	2	5	--	15.0	15.0	--	--	--	15.5	--	--	--	--	14.0
	2	10	--	--	--	--	--	--	15.5	--	--	--	--	14.0
	2	20	--	--	--	--	--	--	--	--	--	--	--	--
$\hat{G}_2^*$	1	5	14.0	12.5	11.4	11.4	13.5	10.8	14.5	--	10.0	10.0	13.5	12.5
	1	10	14.0	11.0	12.5	10.0	10.0	11.0	13.5	13.5	10.5	10.0	11.6	12.0
	1	20	12.5	11.0	11.4	10.0	10.0	10.0	13.5	11.4	10.0	10.0	10.1	11.6
	2	5	--	15.5	13.5	--	--	14.5	--	--	11.8	11.6	--	14.5
	2	10	--	15.5	--	15.5	15.5	--	--	--	14.5	11.6	--	15.5
	2	20	--	--	--	14.0	14.5	15.0	--	--	15.5	13.0	--	--
$\hat{G}_3^*$	1	10	--	13.0	15.5	11.4	--	14.5	--	--	14.0	14.0	--	--
	1	20	11.8	10.6	11.0	10.0	10.3	10.0	12.5	13.0	10.0	10.0	10.4	11.8
	1	40	10.5	10.0	10.3	10.0	10.0	10.0	11.4	11.6	10.0	10.0	10.0	11.0
	2	10	--	--	--	--	--	--	--	--	--	--	--	--
	2	20	--	15.5	--	13.0	13.5	14.5	--	15.5	14.5	13.5	15.0	--
	2	40	--	10.0	10.6	11.0	12.5	11.4	--	--	10.0	11.4	13.5	15.0

*A -- indicates that none of the trial estimates of G^* yielded a goodness-of-fit index exceeding its critical 95-th percentile.

Table 6. $\hat{G}_2^*$ based on 40 observations from the P(g) distributions in graphs 1, 2, 3, 4 when $G^* = 10$ and a uniform 0, U^* deviates are subtracted from the observed g's*

				Graph 1			Graph 2			Graph 3			Graph 4		
U^*	m	k	Trial:	1	2	3	1	2	3	1	2	3	1	2	3
0.0	1	5		11.6	11.0	11.2	15.0	--	--	10.5	--	14.0	10.0	10.0	10.1
0.5	1	5		11.4	11.4	11.2	14.5	--	--	11.4	--	14.0	14.5	15.0	10.3
1.0	1	5		11.0	11.4	10.8	14.0	--	--	11.6	--	13.5	14.0	15.0	10.5
2.0	1	5		10.0	14.0	11.8	--	13.5	--	15.0	14.5	14.5	14.0	14.0	11.0
5.0	1	5		14.0	12.5	11.4	11.4	13.5	10.8	14.5	--	10.0	10.0	13.5	12.5
0.0	1	10		13.0	11.6	11.2	11.2	11.6	14.5	12.5	13.0	13.0	10.0	10.0	10.3
0.5	1	10		12.5	11.6	11.0	12.5	14.5	12.5	12.5	12.5	12.5	10.0	10.0	10.2
1.0	1	10		11.8	11.6	11.0	10.8	11.8	14.0	12.5	12.5	12.5	10.0	11.8	10.1
2.0	1	10		10.2	13.0	10.5	11.2	11.0	15.0	13.0	10.8	12.5	12.5	10.0	11.6
5.0	1	10		14.0	11.0	12.5	10.0	10.0	11.0	13.5	13.5	10.5	10.0	11.0	12.0
0.0	1	20		13.0	11.2	11.4	10.0	10.5	10.0	10.0	11.0	11.0	10.0	10.0	10.0
0.5	1	20		13.0	11.4	11.2	10.0	10.8	10.0	11.2	10.0	10.8	10.0	10.0	10.0
1.0	1	20		12.5	11.2	11.0	10.2	10.6	11.4	11.0	10.0	10.5	10.0	12.5	10.0
2.0	1	20		10.0	12.0	10.4	10.0	10.0	11.0	12.0	10.0	10.8	12.0	11.2	11.4
5.0	1	20		12.5	11.0	11.4	10.0	10.0	10.0	13.5	11.4	10.1	10.0	10.1	11.6

*A -- indicates that none of the trial estimates of G^* yielded a goodness-of-fit index exceeding its critical 95-th percentile.

Table 7. Relative behavior of $\hat{G}_2^*$ in Monte Carlo study of the uniform $[0, U^*]$ deviate transformation*

		Average $\hat{G}_2^*$				
	U^*:	0.0	0.5	1.0	2.0	5.0
Graph	k					
1	5	11.3	11.3	11.1	11.9	12.6
1	10	11.9	11.7	11.5	11.2	12.5
1	20	11.9	11.9	11.6	10.8	11.6
2	5	--	--	--	--	11.9
2	10	12.4	13.2	12.2	12.4	10.3
2	20	10.2	10.3	10.7	10.3	10.0
3	5	--	--	--	14.7	--
3	10	12.8	12.5	12.5	12.1	12.5
3	20	10.7	10.7	10.5	10.9	11.7
4	5	10.0	13.3	13.2	13.0	12.0
4	10	10.1	10.1	10.6	11.4	11.0
4	20	10.0	10.0	10.8	11.5	10.6

*A -- indicates that in at least one trial none of the trial estimates G of $G^* = 10$ yielded a goodness-of-fit index exceeding its critical 95-th percentile.

4. Concluding Remarks

Certainly the final word on a practical confidence limit $\hat{G}^*$ has not been written. In this paper 3 confidence limit procedures based upon 3 goodness-of-fit statistics have been considered. Of these the procedure based upon Q_{2k}, m = 1, and the approximate percentiles of $\hat{Q}_{2k}$ has had the best empirical behavior.

The goodness-of-fit index Q_{2k} given in (2.3) can be rewritten in the form

$$Q_{2k} = (n+1)(n+2)\left\{\frac{z_{n-k+1}^2}{n-k+1} + \sum_{t=n-k+2}^{n}(z_t - z_{t-1})^2 + (1-z_n)^2\right\} - (n+2)$$

(See Hartley and Pfaffenberger (1972)). For m = 1

$$\begin{aligned} z_i &= F(g_{(i)};\alpha,G^*) \\ &= 1 + \alpha\ (g_{(i)} - G^*) \end{aligned}$$

and the $\hat{\alpha}$ which yields $\hat{Q}_{2k}$ for a given estimate G of G^* is

$$\hat{\alpha} = (G - g_{(n-k+1)})/[(n-k+1)T]$$

with

$$T = (G-g_{(n)})^2 + \sum_{t=n-k+2}^{n}(g_{(t)} - g_{(t-1)})^2 + (G-g_{(n-k+1)})^2/(n-k+1)$$

and

$$\hat{Q}_{2k} = (n+1)(n+2)\left\{(n-k+1)^{-1} + 2\hat{\alpha}(g_{(n-k+1)} - G)(n-k+1)^{-1} + \hat{\alpha}^2\ T\right\} - (n+2).$$

Thus, for m = 1, $\hat{Q}_{2k}$ and $\hat{G}_2^*$ are easy to evaluate.

The confidence limit procedures considered herein are based on a goodness-of-fit evaluation of $F(\cdot;\hat{\alpha},G)$. An alternative approach taken by several authors is to base $\hat{G}^*$ on the limiting distribution of order statistics. For example, Boender et. al. (1980) use a method based upon the two largest order statistics and a result due to De Haan (1979). Robson and Whitlock (1964) also base their confidence limit on the distribution of the two largest order statistics, but they assume that the underlying distribution is uniform. Clough (1969) assumes that the g_i's themselves implicitly act like largest order statistics having a limiting distribution of the exponential form and determines his confidence limit on G^* using parameter estimation techniques appropriate for exponential distributions. Golden and Alt (1979) generalize Clough's approach by assuming that the limiting distribution is of the Weibull form. The crucial issue concerning these alternative approaches is whether or not in a particular problem the sample sizes (explicit or implicit) are sufficiently large for the limiting distributions to apply.

The "optimal" transformation $\tilde{g}$ of g has not been identified for confidence limits based either on goodness-of-fit criteria or on limiting distributions. The usefulness of transformations is a topic worthy of further research.

Although neither the use of statistical techniques in mathematical programming nor the use of confidence limits in steps of various algorithms are new (see, e.g. Graves and Whinston (1968), (1970)), such usage is certainly not widespread. Hopefully, improved confidence limit procedures will greatly increase their usage.

The author gratefully acknowledges the support of the Office of Naval Research.

APPENDIX A

Specific Input Instructions for the Computer Implementation of the Procedures based on Q_{1k}, Q_{2k}, and Q_{3k}.

Card A. Title.

Columns 1-80: Any identifying title.

Card B. Procedures to be used.

Col. 1: IQ1K

IQ1K = 1 if the procedure based on Q_{1k} and the sample proportions is to be performed.

IQ1K = 0 otherwise.

Col. 2: IQ2K

IQ2K = 1 if the procedure based on Q_{2k} and the k largest order statistics is to be performed.

IQ2K = 0 otherwise.

Col. 3: IQ3K

IQ3K = 1 if the procedure based on Q_{3k} and the estimated derivations is to be performed.

IQ3K = 0 otherwise.

Card C. Controlling the amount of printed output.

Col. 1: IWRIT1

IWRIT1 = 1 implies that the sample proportions, the estimated alphas and distributed function resulting from the procedure based on Q_{1k} will be printed.

IWRIT1 = 0 otherwise.

Col. 2: IWRIT2

IWRIT2 = 1 implies that the estimated alphas and distribution function resulting from the procedure based on Q_{2k} will be printed.

IWRIT2 = 0 otherwise.

Col. 3: IWRIT3

IWRIT3 = 1 implies that the estimated alphas, lambdas, and distribution function resulting from the procedure based on Q_{3k} will be printed.

IWRIT3 = 0 otherwise.

Card D. Sample size.

Col. 1-3: n = the number of independent observations on g. (Format I3; i.e. a three digit integer right justified.) $1 \leq n \leq 100$.

Card E. Observations in the random sample.

Col. 1-80: g_i, $i = 1, \ldots, n$.

g_i = i-th independent observation on g.

(Format 8F10.0; i.e., eight observations per card with the first observation in columns 1-10 either right justified or with the decimal point included, the second observation in columns 11-20, etc.)

Card F. Transformation. (See Appendix D for a discussion of transformations.)

Col. 1-3: L (Format I3)

L = 0 if the g_i's are <u>not</u> to be transformed.

$L > 0$ implies that L non-random estimates of G^* (namely, $g_1^*, \ldots, g_L^*$ to be read in on Card G) will be used to transform each independent observation g_i to $\tilde{g}_i$ where

$$\tilde{g}_i = WWg_i + \sum_{j=1}^{L} WT_j \, g_{(j)}^* \quad \text{if } g_i < g_{(1)}^*$$

$$= WWg_i + \sum_{j=h+1}^{L-1} WT_{j-h} \, g_{(j)}^* + g_{(L)}^* \sum_{j=L-h}^{L} WT_j$$

$$\text{if } g_{(h)}^* \leq g_i < g_{(h+1)}^*$$

$$= g_i \quad \text{if } g_i \geq g_{(L)}^*$$

where $g_{(1)}^* < \ldots < g_{(L)}^*$ are the ordered values of $g_1^*, \ldots, g_L^*$ and the WW, $WT_1, \ldots, WT_L$ are the user specified weights (see Card H and possibly Cards I and J). If $L > 0$, then the procedures based on Q_{1k}, Q_{2k}, and Q_{3k} are performed on the $\tilde{g}_1, \ldots, \tilde{g}_n$ instead of the $g_1, \ldots, g_n$.

$0 \leq L \leq 10$.

Card G. Transformation. If L = 0, Card G is omitted.

Col. 1-80: g_j^*, $j = 1, \ldots, L$. (Format 8F10.0)

g_j^* = j-th non-random estimate of G^*; $g_j^* \leq G^*$.

Card F. Transformation. If L = 0, Card F is omitted.

Col. 1: INDEXW

INDEXW = 0 implies that the weights WW and $WT_1, \ldots, WT_L$ are to be read in.

Note: $WW + \sum_{j=1}^{L} WT_j = 1$.

INDEXW = 1 implies WW = 1/(L + 1) and

$WT_i = 1/(L + 1)$.

INDEXW = 2 implies WW = 1/[(L + 2)(L + 1)/2] and

$WT_i = (i + 1)/[(L + 2)(L + 1)/2]$.

Card H. Transformation weight. If INDEXW>0, then Card H is omitted.

Col. 1-10: WW, the weight on g_i. (Format F10.0)

Card I. Transformation weight. If INDEXW>0, then Card I is omitted.

Col. 1-80: WT_j, j = 1, ..., L. (Format 8F10.0)

WT_j is proportional to the weight placed on the j-th ordered non-random estimate $g^*_{(j)}$ in constructing the $\tilde{g}_1, \ldots, \tilde{g}_n$.

In the instructions below, read $\tilde{g}_1, \ldots, \tilde{g}_n$ in place of $g_1, \ldots, g_n$ if L > 0.

Card J. Transformation.

Col. 1: ISUBU

ISUBU = 1 if uniform random variables $U_1, \ldots, U_n$ on the range $[0,U^*]$ are to be subtracted from $g_1, \ldots, g_n$ respectively. The resulting observations will also be denoted by $g_1, \ldots, g_n$ herein. Note that G^* remains unchanged.

ISUBU = 0 if the observed $g_1, \ldots, g_n$ are <u>not</u> to be transformed by subtracting uniform deviates.

Col. 6-15: U^*. (Format F10.0) This value is ignored if ISUBU = 0.

Col. 16-25: NSEED1. (Format I10) Ignored if ISUBU = 0

Col. 26-35: NSEED2. (Format I10) Ignored if ISUBU = 0

NSEED1, NSEED2 are any 2 ten digit odd integers. They are used to initialize the composite uniform random deviate generator.

NSEED1, NSEED2 $\leq$ 2, 147, 483, 647.

Card K. Nonnegative derivatives. If the procedures based on Q_{1k} and Q_{2k} are neither one to be used, Card K is omitted.

Col. 1-2: S. (Format I2)

S = the number of relatively dense grid points, $g_1', \ldots, g_S'$, at which the derivative of the estimated distribution function $F(\cdot; \hat{\alpha}, G)$ is explicitly required to be nonnegative. For the procedure based on Q_{1k}

$$g_i' = g_{(1)} + (i-1)[(G-g_{(1)})/S]$$

for i = 1, ..., S. For the procedure based on Q_{2k}

$$g_i' = g_{(n-k+1)} + (i-1)[(G-g_{(n-k+1)})/S]$$

for i = 1, ..., S.

$1 \leq S \leq 40$.

Card L. Trial estimates of G^*.

Col. 1-3: NTRIAL. (Format I3)

NTRIAL = the maximum number of trial estimates, GTRIAL, of G^* to be evaluated

$1 \leq$ NTRIAL ≤ 100.

Card M. Trial estimates of G^*.

Col. 1: INDEXG

INDEXG = 0 implies that the NTRIAL trial estimates of G^* are being read in.

INDEXG = 1 implies that the trial estimates of G^* are to be GTRIAL, GTRIAL + GINC, GTRIAL + 2*GINC, ..., GTRIAL + (NTRIAL-1) *GINC.

Card N. Trial estimates of G^*. Only if INDEXG = 0, is Card N included.

Col. 1-80: $GTRIAL_1$, ..., $GTRIAL_{NTRIAL}$. (Format 8F10.0)

Note $GTRIAL_1 < \ldots < GTRIAL_{NTRIAL}$.

Card O. Trial estimates of G^*. Only if INDEXG = 1, is Card O included.

Col. 1-10: GTRIAL. (Format F10.0)

Col. 11-20: GINC. (Format F10.0)

Card P. Stopping rules.

Col. 1: ISTOP

ISTOP = 0 implies that all NTRIAL trial estimates of G^* will be considered.

ISTOP = 1 implies that the estimation of G^* will stop as soon as a trial estimate of G^* is found with a goodness-of-fit index greater than its critical value.

Card Q. Using Q_{3k}. Include Card Q only if IQ3K = 1.

Col. 1-8: XNORM. (Format F8.0)

XNORM = the 100 * γ -th percentile of the standard normal distribution function (N(0,1)) when 100 * γ % confidence limits on G^* based on the approximate normality of $\hat{\alpha}_0/(X_G^T V^{-1} X_G)_{1,1}^{1/2}$ are desired. For a 95% confidence limit XNORM = 1.645. (See Appendix G for a table of XNORM values)

Card R. Using Q_{3k}. Include Card R only if IQ3K = 1.

Col. 1-40: $\chi_{m+1,\gamma}$, m = 1, ..., 5. (Format 5F8.0)

$\chi_{m+1,\gamma}$ = the 100 * γ -th percentile of a chi-squared distribution with m + 1 degrees of freedom when 100 * γ % confidence limits on G^* based on Q_{3k} are desired. (See Appendix H for a table of percentiles of chi-squared distributions.)

Card S. Degree of F.

Col. 1: IREADM

IREADM = 0 implies that the degree, m, of the approximating distribution function F is to be determined using the procedure described in Appendix E.

IREADM = 1 implies that m is to be read in. $1 \leq m \leq 5$.

Col. 2: ICHKM

ICHKM = 0 implies that the degree of F will not be evaluated using the procedure described in Appendix E.

ICHKM = 1 implies that a value for m will be computed using the procedure described in Appendix E. (If IREADM = 0, then ICHKM is automatically set to 1.) If IREADM = 1, the value for m read in is used in the procedures based on Q_{1k}, Q_{2k}, and Q_{3k}; the computed value of m is only for information and is not used in the procedures involving Q_{1k}, Q_{2k}, and Q_{3k}.

Card T. Degree of F. Include Card T only if IREADM = 1.

Col. 1: m, the degree of the approximating distribution function F.

Card U. Using Q_{1k}. Include Card U only if IQ1K = 1.

Col. 1-2: k. (Format I2)

k = the number of sample proportions to be used in the confidence limit procedure based on Q_{1k}. $1 \leq k \leq 20$.

Col. 3-12: $\chi^2_{k,\gamma}$. (Format F10.0)

$\chi^2_{k,\gamma}$ = the 100 $\times$ γ -th percentile of a chi-squared distribution with k degrees of freedom when a 100 $\times$ γ% confidence limit on G^* is desired. (See Appendix H for a table of percentiles of chi-squared distributions.)

Card V. Using Q_{1k}. Include Card V only if IQ1K = 1.

Col. 1-80: $G_1, \ldots, G_k$. (Format 8F10.0)

G_i = the i-th scale point used in the procedure based on Q_{1k}.

Card W. Using Q_{2k}. Include Card W only if IQ2K = 1.

Col. 1-2: k. (Format I2)

The k largest order statistics $g_{(n-k+1)}, \ldots, g_{(n)}$ will be used to estimate the upper tail of the distribution F in the procedure based on Q_{2k}. $1 \leq k \leq n \leq 100$.

Card X. Using Q_{2k}. Include Card X only if IQ2k = 1

Col. 1-30: CRITL(1), CRITL(2). (Format 2F10.0)

These are the $100 \times \gamma$ -th percentiles of $\hat{Q}_{2k}$ for m = 1,2, and 3 respectively. These percentiles can be found in Tables F.4, F.5, ... in Appendix F. If $m \geq 3$ is specified, then CRITL(2) is used as an approximate $100 \times \gamma$ -th percentile of $\hat{Q}_{2k}$. This level $100 \times \gamma$ should correspond to the desired confidence level in the $100 \times \gamma$ % confidence limit on G^*.

Card Y. Using Q_{3k}. Include Card Y only if IQ3K = 1.

Col. 1-2: k. (Format I2)

The k largest order statistics $g_{(n-k+1)}, \cdots, g_{(n)}$ will be used in the procedure based on Q_{3k} and derivatives.

$1 \leq k \leq n \leq 100$.

APPENDIX B

Sample Input for

Computer Implementation

```
              GRAPH 1     TRIAL 2
111
111
040
1.50     4.65     4.83     9.30     3.99     0.49     7.29     9.19
1.43     3.68     6.95     4.09     9.39     6.11     9.73     1.27
2.13     5.40     5.39     9.76     9.12     5.84     3.23     2.70
3.30     7.22     2.05     5.73     0.42     2.64     0.47     6.98
6.57     5.79     8.34     4.25     5.63     1.85     8.96     6.27
0
0
10
 21
0
10.0     10.1     10.2     10.3     10.4     10.5     10.6     10.8
11.0     11.2     11.4     11.6     11.8     12.0     12.5     13.0
13.5     14.0     14.5     15.0     15.5
1
1.645
5.99    7.81    9.49    11.1    12.6
10
1
 5
11.1
2.0      4.0      6.0      8.0      10.0
10
7.5151   6.7973
20
```

APPENDIX C

Sample Output from Computer Implementation

full output

```
GRAPH 1    TRIAL 2

THE INDEPENDENT OBSERVATIONS ON G(X) ARE:
        1-TH OBSERVATION ON G(X) =    0.1500000000D+01
        2-TH OBSERVATION ON G(X) =    0.4650000000D+01
        3-TH OBSERVATION ON G(X) =    0.4830000000D+01
        4-TH OBSERVATION ON G(X) =    0.9300000000D+01
        5-TH OBSERVATION ON G(X) =    0.3990000000D+01
        6-TH OBSERVATION ON G(X) =    0.6900000000D+00
        7-TH OBSERVATION ON G(X) =    0.7290000000D+01
        8-TH OBSERVATION ON G(X) =    0.9190000000D+01
        9-TH OBSERVATION ON G(X) =    0.1430000000D+01
       10-TH OBSERVATION ON G(X) =    0.3680000000D+01
       11-TH OBSERVATION ON G(X) =    0.6950000000D+01
       12-TH OBSERVATION ON G(X) =    0.4090000000D+01
       13-TH OBSERVATION ON G(X) =    0.9390000000D+01
       14-TH OBSERVATION ON G(X) =    0.6110000000D+01
       15-TH OBSERVATION ON G(X) =    0.9730000000D+01
       16-TH OBSERVATION ON G(X) =    0.1270000000D+01
       17-TH OBSERVATION ON G(X) =    0.2130000000D+01
       18-TH OBSERVATION ON G(X) =    0.5400000000D+01
       19-TH OBSERVATION ON G(X) =    0.5390000000D+01
       20-TH OBSERVATION ON G(X) =    0.9760000000D+01
       21-TH OBSERVATION ON G(X) =    0.9120000000D+01
       22-TH OBSERVATION ON G(X) =    0.5840000000D+01
       23-TH OBSERVATION ON G(X) =    0.3230000000D+01
       24-TH OBSERVATION ON G(X) =    0.2700000000D+01
       25-TH OBSERVATION ON G(X) =    0.3300000000D+01
       26-TH OBSERVATION ON G(X) =    0.7220000000D+01
       27-TH OBSERVATION ON G(X) =    0.2050000000D+01
       28-TH OBSERVATION ON G(X) =    0.5730000000D+01
       29-TH OBSERVATION ON G(X) =    0.4200000000D+00
       30-TH OBSERVATION ON G(X) =    0.2640000000D+01
       31-TH OBSERVATION ON G(X) =    0.4700000000D+00
       32-TH OBSERVATION ON G(X) =    0.6980000000D+01
       33-TH OBSERVATION ON G(X) =    0.6570000000D+01
       34-TH OBSERVATION ON G(X) =    0.5790000000D+01
       35-TH OBSERVATION ON G(X) =    0.8340000000D+01
       36-TH OBSERVATION ON G(X) =    0.4250000000D+01
       37-TH OBSERVATION ON G(X) =    0.5630000000D+01
       38-TH OBSERVATION ON G(X) =    0.1850000000D+01
       39-TH OBSERVATION ON G(X) =    0.8960000000D+01
       40-TH OBSERVATION ON G(X) =    0.6270000000D+01

THERE ARE   0 NONRANDOM INITIAL ESTIMATES OF G*

THE ORDERED VALUES OF THE OBSERVED G'S ARE:
        1-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.4200000000D+00
        2-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.4700000000D+00
        3-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.6900000000D+00
        4-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.1270000000D+01
        5-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.1430000000D+01
        6-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.1500000000D+01
        7-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.1850000000D+01
        8-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.2050000000D+01
        9-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.2130000000D+01
       10-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.2640000000D+01
       11-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.2700000000D+01
       12-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.3230000000D+01
       13-TH SMALLEST VALUE OF THE OBSERVED G'S =    0.3300000000D+01
```

```
     14-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.36800000000D+01
     15-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.39900000000D+01
     16-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.40900000000D+01
     17-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.42500000000D+01
     18-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.46500000000D+01
     19-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.48300000000D+01
     20-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.53900000000D+01
     21-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.54000000000D+01
     22-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.56300000000D+01
     23-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.57300000000D+01
     24-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.57900000000D+01
     25-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.58400000000D+01
     26-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.61100000000D+01
     27-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.62700000000D+01
     28-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.65700000000D+01
     29-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.69500000000D+01
     30-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.69800000000D+01
     31-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.72200000000D+01
     32-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.72900000000D+01
     33-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.83400000000D+01
     34-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.89600000000D+01
     35-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.91200000000D+01
     36-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.91900000000D+01
     37-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.93000000000D+01
     38-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.93900000000D+01
     39-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.97300000000D+01
     40-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.97600000000D+01

THE TRIAL ESTIMATES OF G0 ARE:
      1-TH TRIAL ESTIMATE OF G0 =     0.10000000000D+02
      2-TH TRIAL ESTIMATE OF G0 =     0.10100000000D+02
      3-TH TRIAL ESTIMATE OF G0 =     0.10200000000D+02
      4-TH TRIAL ESTIMATE OF G0 =     0.10300000000D+02
      5-TH TRIAL ESTIMATE OF G0 =     0.10400000000D+02
      6-TH TRIAL ESTIMATE OF G0 =     0.10500000000D+02
      7-TH TRIAL ESTIMATE OF G0 =     0.10600000000D+02
      8-TH TRIAL ESTIMATE OF G0 =     0.10800000000D+02
      9-TH TRIAL ESTIMATE OF G0 =     0.11000000000D+02
     10-TH TRIAL ESTIMATE OF G0 =     0.11200000000D+02
     11-TH TRIAL ESTIMATE OF G0 =     0.11400000000D+02
     12-TH TRIAL ESTIMATE OF G0 =     0.11600000000D+02
     13-TH TRIAL ESTIMATE OF G0 =     0.11800000000D+02
     14-TH TRIAL ESTIMATE OF G0 =     0.12000000000D+02
     15-TH TRIAL ESTIMATE OF G0 =     0.12500000000D+02
     16-TH TRIAL ESTIMATE OF G0 =     0.13000000000D+02
     17-TH TRIAL ESTIMATE OF G0 =     0.13500000000D+02
     18-TH TRIAL ESTIMATE OF G0 =     0.14000000000D+02
     19-TH TRIAL ESTIMATE OF G0 =     0.14500000000D+02
     20-TH TRIAL ESTIMATE OF G0 =     0.15000000000D+02
     21-TH TRIAL ESTIMATE OF G0 =     0.15500000000D+02

THE ESTIMATION PROCEDURE WILL STOP AS SOON AS A TRIAL ESTIMATE OF G0 HAS A GOODNESS-OF-FIT INDEX
GREATER THAN ITS CRITICAL VALUE.
```

```
THE CONFIDENCE LIMIT BASED ON THE SAMPLE PROPORTIONS WILL BE CONSIDERED.      (OIKHAT)

     THE SCALE POINTS USED TO CREATE THE SAMPLE PROPORTIONS WERE AS FOLLOWS:
          0.20000D+01          0.40000D+01          0.60000D+01          0.80000D+01          0.10000D+02

     THE NUMBER OF SAMPLE PROPORTIONS : K =    5

A  1-TH DEGREE POLYNOMIAL WILL BE USED TO APPROXIMATE THE FINITE RANGE DISTRIBUTION NEAR G0.

     THE OBSERVED SAMPLE PROPORTIONS :

        THE    1-TH SAMPLE PROPORTION =    0.17500000D+00
        THE    2-TH SAMPLE PROPORTION =    0.37500000D+00
        THE    3-TH SAMPLE PROPORTION =    0.62500000D+00
        THE    4-TH SAMPLE PROPORTION =    0.80000000D+00
        THE    5-TH SAMPLE PROPORTION =    0.99900000D+00

GTRIAL =  0.10000D+02; THE GOODNESS-OF-FIT INDEX  OIKHAT   =  0.52601D+00; THE CRITICAL CHI-SQUARED PERCENTILE =   11.10

     THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

        ESTIMATED ALPHA(1) =  0.10280452D+00

     FITTED CDF EVALUATED AT K SCALE POINTS:

        FHAT( 1) =  0.17756381D+00              SCALE POINT( 1) =  0.20000000D+01
        FHAT( 2) =  0.38317286D+00              SCALE POINT( 2) =  0.40000000D+01
        FHAT( 3) =  0.58878191D+00              SCALE POINT( 3) =  0.60000000D+01
        FHAT( 4) =  0.79439095D+00              SCALE POINT( 4) =  0.80000000D+01
        FHAT( 5) =  0.10000000D+01              SCALE POINT( 5) =  0.10000000D+02

GTRIAL =  0.10100D+02; THE GOODNESS-OF-FIT INDEX  OIKHAT   =  0.38336D+01; THE CRITICAL CHI-SQUARED PERCENTILE =   11.10

     THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

        ESTIMATED ALPHA(1) =  0.99781208D-01

     FITTED CDF EVALUATED AT K SCALE POINTS:

        FHAT( 1) =  0.19177222D+00              SCALE POINT( 1) =  0.20000000D+01
        FHAT( 2) =  0.39133463D+00              SCALE POINT( 2) =  0.40000000D+01
        FHAT( 3) =  0.59089705D+00              SCALE POINT( 3) =  0.60000000D+01
        FHAT( 4) =  0.79045946D+00              SCALE POINT( 4) =  0.80000000D+01
        FHAT( 5) =  0.99002188D+00              SCALE POINT( 5) =  0.10000000D+02

GTRIAL =  0.10200D+02; THE GOODNESS-OF-FIT INDEX  OIKHAT   =  0.14036D+02; THE CRITICAL CHI-SQUARED PERCENTILE =   11.10

     THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

        ESTIMATED ALPHA(1) =  0.93120784D-01

     FITTED CDF EVALUATED AT K SCALE POINTS:

        FHAT( 1) =  0.23640957D+00              SCALE POINT( 1) =  0.20000000D+01
        FHAT( 2) =  0.42265114D+00              SCALE POINT( 2) =  0.40000000D+01
        FHAT( 3) =  0.60889271D+00              SCALE POINT( 3) =  0.60000000D+01
        FHAT( 4) =  0.79513428D+00              SCALE POINT( 4) =  0.80000000D+01
        FHAT( 5) =  0.98137584D+00              SCALE POINT( 5) =  0.10000000D+02
```

THE CONFIDENCE LIMIT BASED ON THE 10 LARGEST ORDER STATISTICS WILL BE CONSIDERED. (Q2KHAT)

A 1-TH DEGREE POLYNOMIAL WILL BE USED TO APPROXIMATE THE FINITE RANGE DISTRIBUTION NEAR G*.

GTRIAL = 0.10000D+02; THE GOODNESS-OF-FIT INDEX Q2KHAT = 0.65041D+01; THE CRITICAL PERCENTILE = 7.51510

THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

ESTIMATED ALPHA(1) = 0.45616397D-01

THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

FHAT(31) =	0.87318642D+00	E(31-TH LARGEST UNIFORM ORDER STATISTIC) =	0.75609756D+00
FHAT(32) =	0.87637956D+00	E(32-TH LARGEST UNIFORM ORDER STATISTIC) =	0.78048778D+00
FHAT(33) =	0.92427678D+00	E(33-TH LARGEST UNIFORM ORDER STATISTIC) =	0.80487800D+00
FHAT(34) =	0.95255895D+00	E(34-TH LARGEST UNIFORM ORDER STATISTIC) =	0.82926828D+00
FHAT(35) =	0.95985757D+00	E(35-TH LARGEST UNIFORM ORDER STATISTIC) =	0.85365850D+00
FHAT(36) =	0.96305072D+00	E(36-TH LARGEST UNIFORM ORDER STATISTIC) =	0.87804878D+00
FHAT(37) =	0.96806852D+00	E(37-TH LARGEST UNIFORM ORDER STATISTIC) =	0.90243900D+00
FHAT(38) =	0.97217400D+00	E(38-TH LARGEST UNIFORM ORDER STATISTIC) =	0.92682922D+00
FHAT(39) =	0.98768357D+00	E(39-TH LARGEST UNIFORM ORDER STATISTIC) =	0.95121950D+00
FHAT(40) =	0.98905206D+00	E(40-TH LARGEST UNIFORM ORDER STATISTIC) =	0.97560972D+00

GTRIAL = 0.10100D+02; THE GOODNESS-OF-FIT INDEX Q2KHAT = 0.62705D+01; THE CRITICAL PERCENTILE = 7.51510

THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

ESTIMATED ALPHA(1) = 0.45492603D-01

THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

FHAT(31) =	0.86898130D+00	E(31-TH LARGEST UNIFORM ORDER STATISTIC) =	0.75609756D+00
FHAT(32) =	0.87216579D+00	E(32-TH LARGEST UNIFORM ORDER STATISTIC) =	0.78048778D+00
FHAT(33) =	0.91993302D+00	E(33-TH LARGEST UNIFORM ORDER STATISTIC) =	0.80487800D+00
FHAT(34) =	0.94813843D+00	E(34-TH LARGEST UNIFORM ORDER STATISTIC) =	0.82926828D+00
FHAT(35) =	0.95541725D+00	E(35-TH LARGEST UNIFORM ORDER STATISTIC) =	0.85365850D+00
FHAT(36) =	0.95860173D+00	E(36-TH LARGEST UNIFORM ORDER STATISTIC) =	0.87804878D+00
FHAT(37) =	0.96360592D+00	E(37-TH LARGEST UNIFORM ORDER STATISTIC) =	0.90243900D+00
FHAT(38) =	0.96770025D+00	E(38-TH LARGEST UNIFORM ORDER STATISTIC) =	0.92682922D+00
FHAT(39) =	0.98316774D+00	E(39-TH LARGEST UNIFORM ORDER STATISTIC) =	0.95121950D+00
FHAT(40) =	0.98453252D+00	E(40-TH LARGEST UNIFORM ORDER STATISTIC) =	0.97560972D+00

GTRIAL = 0.10200D+02; THE GOODNESS-OF-FIT INDEX Q2KHAT = 0.61093D+01; THE CRITICAL PERCENTILE = 7.51510

THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

ESTIMATED ALPHA(1) = 0.44939758D-01

THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

FHAT(31) =	0.86607952D+00	E(31-TH LARGEST UNIFORM ORDER STATISTIC) =	0.75609756D+00
FHAT(32) =	0.86922530D+00	E(32-TH LARGEST UNIFORM ORDER STATISTIC) =	0.78048778D+00
FHAT(33) =	0.91641205D+00	E(33-TH LARGEST UNIFORM ORDER STATISTIC) =	0.80487800D+00
FHAT(34) =	0.94427470D+00	E(34-TH LARGEST UNIFORM ORDER STATISTIC) =	0.82926828D+00
FHAT(35) =	0.95146506D+00	E(35-TH LARGEST UNIFORM ORDER STATISTIC) =	0.85365850D+00
FHAT(36) =	0.95461084D+00	E(36-TH LARGEST UNIFORM ORDER STATISTIC) =	0.87804878D+00
FHAT(37) =	0.95955422D+00	E(37-TH LARGEST UNIFORM ORDER STATISTIC) =	0.90243900D+00
FHAT(38) =	0.96359880D+00	E(38-TH LARGEST UNIFORM ORDER STATISTIC) =	0.92682922D+00
FHAT(39) =	0.97887831D+00	E(39-TH LARGEST UNIFORM ORDER STATISTIC) =	0.95121950D+00
FHAT(40) =	0.98022651D+00	E(40-TH LARGEST UNIFORM ORDER STATISTIC) =	0.97560972D+00

```
GTRIAL =  0.10300D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.60156D+01; THE CRITICAL PERCENTILE =    7.51510

     THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

         ESTIMATED ALPHA(1) =  0.44028310D-01

     THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

         FHAT(31) =  0.864392810D+00              E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.756097560D+00
         FHAT(32) =  0.867474790D+00              E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.780487780D+00
         FHAT(33) =  0.913704510D+00              E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.804878000D+00
         FHAT(34) =  0.941002060D+00              E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.829268280D+00
         FHAT(35) =  0.948046590D+00              E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.853658500D+00
         FHAT(36) =  0.951128580D+00              E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.878048780D+00
         FHAT(37) =  0.955971690D+00              E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.902439000D+00
         FHAT(38) =  0.959934240D+00              E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.926829220D+00
         FHAT(39) =  0.974903860D+00              E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.951219500D+00
         FHAT(40) =  0.976224710D+00              E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.975609720D+00
GTRIAL =  0.10400D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.59819D+01; THE CRITICAL PERCENTILE =    7.51510

     THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

         ESTIMATED ALPHA(1) =  0.42834636D-01

     THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

         FHAT(31) =  0.863785860D+00              E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.756097560D+00
         FHAT(32) =  0.866784280D+00              E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.780487780D+00
         FHAT(33) =  0.911760650D+00              E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.804878000D+00
         FHAT(34) =  0.938318120D+00              E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.829268280D+00
         FHAT(35) =  0.945171670D+00              E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.853658500D+00
         FHAT(36) =  0.948170090D+00              E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.878048780D+00
         FHAT(37) =  0.952881900D+00              E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.902439000D+00
         FHAT(38) =  0.956737020D+00              E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.926829220D+00
         FHAT(39) =  0.971300790D+00              E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.951219500D+00
         FHAT(40) =  0.972585830D+00              E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.975609720D+00
GTRIAL =  0.10500D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.59993D+01; THE CRITICAL PERCENTILE =    7.51510

     THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

         ESTIMATED ALPHA(1) =  0.41433506D-01

     THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

         FHAT(31) =  0.864098100D+00              E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.756097560D+00
         FHAT(32) =  0.866998450D+00              E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.780487780D+00
         FHAT(33) =  0.910503630D+00              E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.804878000D+00
         FHAT(34) =  0.936192400D+00              E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.829268280D+00
         FHAT(35) =  0.942821760D+00              E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.853658500D+00
         FHAT(36) =  0.945722110D+00              E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.878048780D+00
         FHAT(37) =  0.950279790D+00              E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.902439000D+00
         FHAT(38) =  0.954008810D+00              E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.926829220D+00
         FHAT(39) =  0.968096200D+00              E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.951219500D+00
         FHAT(40) =  0.969339210D+00              E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.975609720D+00
GTRIAL =  0.10600D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.60584D+01; THE CRITICAL PERCENTILE =    7.51510

     THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:
```

```
      ESTIMATED ALPHA(1) =  0.39892841D-01

   THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

      FHAT(31) =  0.865162280D+00        E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.756097560D+00
      FHAT(32) =  0.867954780D+00        E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.780487780D+00
      FHAT(33) =  0.909842180D+00        E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.804878000D+00
      FHAT(34) =  0.934575740D+00        E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.829268280D+00
      FHAT(35) =  0.940958660D+00        E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.853658500D+00
      FHAT(36) =  0.943751090D+00        E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.878048780D+00
      FHAT(37) =  0.948139310D+00        E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.902439000D+00
      FHAT(38) =  0.951729660D+00        E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.926829220D+00
      FHAT(39) =  0.965293230D+00        E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.951219500D+00
      FHAT(40) =  0.966490010D+00        E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.975609720D+00

GTRIAL =  0.10800D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.62671D+01; THE CRITICAL PERCENTILE =    7.51510

   THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

      ESTIMATED ALPHA(1) =  0.36614686D-01

   THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

      FHAT(31) =  0.868919420D+00        E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.756097560D+00
      FHAT(32) =  0.871482450D+00        E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.780487780D+00
      FHAT(33) =  0.909927870D+00        E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.804878000D+00
      FHAT(34) =  0.932628980D+00        E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.829268280D+00
      FHAT(35) =  0.938487330D+00        E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.853658500D+00
      FHAT(36) =  0.941050360D+00        E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.878048780D+00
      FHAT(37) =  0.945077970D+00        E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.902439000D+00
      FHAT(38) =  0.948373290D+00        E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.926829220D+00
      FHAT(39) =  0.960822290D+00        E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.951219500D+00
      FHAT(40) =  0.961920730D+00        E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.975609720D+00

GTRIAL =  0.11000D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.65482D+01; THE CRITICAL PERCENTILE =    7.51510

   THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

      ESTIMATED ALPHA(1) =  0.33338341D-01

   THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

      FHAT(31) =  0.873981070D+00        E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.756097560D+00
      FHAT(32) =  0.876314760D+00        E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.780487780D+00
      FHAT(33) =  0.911320010D+00        E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.804878000D+00
      FHAT(34) =  0.931989780D+00        E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.829268280D+00
      FHAT(35) =  0.937323920D+00        E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.853658500D+00
      FHAT(36) =  0.939657600D+00        E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.878048780D+00
      FHAT(37) =  0.943324820D+00        E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.902439000D+00
      FHAT(38) =  0.946325270D+00        E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.926829220D+00
      FHAT(39) =  0.957660310D+00        E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.951219500D+00
      FHAT(40) =  0.958660480D+00        E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.975609720D+00

GTRIAL =  0.11200D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.68597D+01; THE CRITICAL PERCENTILE =    7.51510

   THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

      ESTIMATED ALPHA(1) =  0.30254436D-01

   THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:
```

```
        FHAT(31) =  0.87958735D+00            E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.75609756D+00
        FHAT(32) =  0.88170516D+00            E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.78048778D+00
        FHAT(33) =  0.91347231D+00            E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.80487800D+00
        FHAT(34) =  0.93223006D+00            E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.82926828D+00
        FHAT(35) =  0.93707077D+00            E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.85365850D+00
        FHAT(36) =  0.93918858D+00            E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.87804878D+00
        FHAT(37) =  0.94251657D+00            E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.90243900D+00
        FHAT(38) =  0.94523947D+00            E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.92682922D+00
        FHAT(39) =  0.95552598D+00            E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.95121950D+00
        FHAT(40) =  0.95643361D+00            E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.97560972D+00

GTRIAL =  0.11400D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.71748D+01; THE CRITICAL PERCENTILE =    7.51510

    THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

        ESTIMATED ALPHA(1) =  0.27449625D-01

    THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

        FHAT(31) =  0.88526057D+00            E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.75609756D+00
        FHAT(32) =  0.88718204D+00            E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.78048778D+00
        FHAT(33) =  0.91600415D+00            E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.80487800D+00
        FHAT(34) =  0.93302291D+00            E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.82926828D+00
        FHAT(35) =  0.93741485D+00            E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.85365850D+00
        FHAT(36) =  0.93933633D+00            E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.87804878D+00
        FHAT(37) =  0.94235579D+00            E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.90243900D+00
        FHAT(38) =  0.94482625D+00            E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.92682922D+00
        FHAT(39) =  0.95415913D+00            E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.95121950D+00
        FHAT(40) =  0.95498261D+00            E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.97560972D+00

GTRIAL =  0.11600D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.74785D+01; THE CRITICAL PERCENTILE =    7.51510

    THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

        ESTIMATED ALPHA(1) =  0.24947750D-01

    THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

        FHAT(31) =  0.89072885D+00            E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.75609756D+00
        FHAT(32) =  0.89247520D+00            E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.78048778D+00
        FHAT(33) =  0.91867033D+00            E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.80487800D+00
        FHAT(34) =  0.93413794D+00            E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.82926828D+00
        FHAT(35) =  0.93812958D+00            E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.85365850D+00
        FHAT(36) =  0.93987592D+00            E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.87804878D+00
        FHAT(37) =  0.94262017D+00            E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.90243900D+00
        FHAT(38) =  0.94486547D+00            E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.92682922D+00
        FHAT(39) =  0.95334771D+00            E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.95121950D+00
        FHAT(40) =  0.95409614D+00            E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.97560972D+00

GTRIAL =  0.11800D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.77633D+01; THE CRITICAL PERCENTILE =    7.51510

    THE ALPHA VECTOR THAT MINIMIZED THE QUADRATIC FORM:

        ESTIMATED ALPHA(1) =  0.22739130D-01

    THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS:

        FHAT(31) =  0.89585478D+00            E( 31-TH LARGEST UNIFORM ORDER STATISTIC) =  0.75609756D+00
        FHAT(32) =  0.89744652D+00            E( 32-TH LARGEST UNIFORM ORDER STATISTIC) =  0.78048778D+00
        FHAT(33) =  0.92132261D+00            E( 33-TH LARGEST UNIFORM ORDER STATISTIC) =  0.80487800D+00
        FHAT(34) =  0.93542087D+00            E( 34-TH LARGEST UNIFORM ORDER STATISTIC) =  0.82926828D+00
```

```
FHAT(35) =  0.93905913D+00        E( 35-TH LARGEST UNIFORM ORDER STATISTIC) =  0.85365850D+00
FHAT(36) =  0.94065087D+00        E( 36-TH LARGEST UNIFORM ORDER STATISTIC) =  0.87804878D+00
FHAT(37) =  0.94315218D+00        E( 37-TH LARGEST UNIFORM ORDER STATISTIC) =  0.90243900D+00
FHAT(38) =  0.94519970D+00        E( 38-TH LARGEST UNIFORM ORDER STATISTIC) =  0.92682922D+00
FHAT(39) =  0.95293000D+00        E( 39-TH LARGEST UNIFORM ORDER STATISTIC) =  0.95121950D+00
FHAT(40) =  0.95361217D+00        E( 40-TH LARGEST UNIFORM ORDER STATISTIC) =  0.97560972D+00
```

```
THE CONFIDENCE LIMIT BASED ON DERIVATIVES WILL BE CONSIDERED.      (Q3KHAT)

ONLY THE K =  20 LARGEST ORDER STATISTICS WILL  BE USED.

A  1-TH DEGREE POLYNOMIAL WILL BE USED TO APPROXIMATE THE FINITE RANGE DISTRIBUTION NEAR G*.

GTRIAL =  0.10000D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.45166D+00; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

     ALPHA(0) = -0.16347670D-01
     ALPHA(1) =  0.66426560D-01

THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

     FHAT( 0.54000000D+01) =  0.67809015D+00          EXPECTED VALUE =  0.51219511D+00
     FHAT( 0.56300000D+01) =  0.69336826D+00          EXPECTED VALUE =  0.53658533D+00
     FHAT( 0.57300000D+01) =  0.70001092D+00          EXPECTED VALUE =  0.56097555D+00
     FHAT( 0.57900000D+01) =  0.70399651D+00          EXPECTED VALUE =  0.58536583D+00
     FHAT( 0.58400000D+01) =  0.70731784D+00          EXPECTED VALUE =  0.60975605D+00
     FHAT( 0.61100000D+01) =  0.72525301D+00          EXPECTED VALUE =  0.63414632D+00
     FHAT( 0.62700000D+01) =  0.73588126D+00          EXPECTED VALUE =  0.65853655D+00
     FHAT( 0.65700000D+01) =  0.75580923D+00          EXPECTED VALUE =  0.68292677D+00
     FHAT( 0.69500000D+01) =  0.78105132D+00          EXPECTED VALUE =  0.70731705D+00
     FHAT( 0.69900000D+01) =  0.78304412D+00          EXPECTED VALUE =  0.73170727D+00
     FHAT( 0.72200000D+01) =  0.79898649D+00          EXPECTED VALUE =  0.75609756D+00
     FHAT( 0.72900000D+01) =  0.80363635D+00          EXPECTED VALUE =  0.78048778D+00
     FHAT( 0.83400000D+01) =  0.87338424D+00          EXPECTED VALUE =  0.80487800D+00
     FHAT( 0.89600000D+01) =  0.91456871D+00          EXPECTED VALUE =  0.82926828D+00
     FHAT( 0.91200000D+01) =  0.92519696D+00          EXPECTED VALUE =  0.85365850D+00
     FHAT( 0.91900000D+01) =  0.92984682D+00          EXPECTED VALUE =  0.87804878D+00
     FHAT( 0.93000000D+01) =  0.93715374D+00          EXPECTED VALUE =  0.90243900D+00
     FHAT( 0.93900000D+01) =  0.94313213D+00          EXPECTED VALUE =  0.92682922D+00
     FHAT( 0.97300000D+01) =  0.96571716D+00          EXPECTED VALUE =  0.95121950D+00
     FHAT( 0.97600000D+01) =  0.96770996D+00          EXPECTED VALUE =  0.97560972D+00

THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

     LAMBDA HAT(0) =  0.0
     LAMBDA HAT(1) = -0.16347670D-01

THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

     LAMBDA(1) =  0.0

  SINCE   -0.67205750D+00  DOES NOT EXCEED THE N(0,1) CRITICAL VALUE OF     0.16450000D+01  THEN THE CURRENT TRIAL
 ESTIMATE  OF G*, NAMELY     0.10000000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.10100D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.15423D+00; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

     ALPHA(0) = -0.97050139D-02
     ALPHA(1) =  0.66426560D-01

THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

     FHAT( 0.54000000D+01) =  0.67809015D+00          EXPECTED VALUE =  0.51219511D+00
     FHAT( 0.56300000D+01) =  0.69336826D+00          EXPECTED VALUE =  0.53658533D+00
```

```
        FHAT( 0.57300000D+01) =  0.70001092D+00          EXPECTED VALUE =  0.56097555D+00
        FHAT( 0.57900000D+01) =  0.70399651D+00          EXPECTED VALUE =  0.58536583D+00
        FHAT( 0.58400000D+01) =  0.70731784D+00          EXPECTED VALUE =  0.60975605D+00
        FHAT( 0.61100000D+01) =  0.72525301D+00          EXPECTED VALUE =  0.63414633D+00
        FHAT( 0.62700000D+01) =  0.73588126D+00          EXPECTED VALUE =  0.65853655D+00
        FHAT( 0.65700000D+01) =  0.75580923D+00          EXPECTED VALUE =  0.68292677D+00
        FHAT( 0.69500000D+01) =  0.78105132D+00          EXPECTED VALUE =  0.70731705D+00
        FHAT( 0.69800000D+01) =  0.78304412D+00          EXPECTED VALUE =  0.73170727D+00
        FHAT( 0.72200000D+01) =  0.79898649D+00          EXPECTED VALUE =  0.75609756D+00
        FHAT( 0.72900000D+01) =  0.80363635D+00          EXPECTED VALUE =  0.78048778D+00
        FHAT( 0.83400000D+01) =  0.87338424D+00          EXPECTED VALUE =  0.80487800D+00
        FHAT( 0.89600000D+01) =  0.91456871D+00          EXPECTED VALUE =  0.82926828D+00
        FHAT( 0.91200000D+01) =  0.92519696D+00          EXPECTED VALUE =  0.85365850D+00
        FHAT( 0.91900000D+01) =  0.92984682D+00          EXPECTED VALUE =  0.87804878D+00
        FHAT( 0.93000000D+01) =  0.93715374D+00          EXPECTED VALUE =  0.90243900D+00
        FHAT( 0.93900000D+01) =  0.94313213D+00          EXPECTED VALUE =  0.92682922D+00
        FHAT( 0.97300000D+01) =  0.96571716D+00          EXPECTED VALUE =  0.95121950D+00
        FHAT( 0.97600000D+01) =  0.96770996D+00          EXPECTED VALUE =  0.97560972D+00

     THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

        LAMBDA HAT(0) =  0.0
        LAMBDA HAT(1) = -0.67050139D-02

     THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

        LAMBDA(1) =  0.0

   SINCE   -0.39271900D+00  DOES NOT EXCEED THE N(0,1) CRITICAL VALUE OF    0.16450000D+01  THEN THE CURRENT TRIAL
  ESTIMATE  OF G0, NAMELY    0.10100000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.10200D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.14801D-01; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

     THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

        ALPHA(0) = -0.30623579D-02
        ALPHA(1) =  0.66426560D-01

     THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

        FHAT( 0.54000000D+01) =  0.67809015D+00          EXPECTED VALUE =  0.51219511D+00
        FHAT( 0.56300000D+01) =  0.69336826D+00          EXPECTED VALUE =  0.53658533D+00
        FHAT( 0.57300000D+01) =  0.70001092D+00          EXPECTED VALUE =  0.56097555D+00
        FHAT( 0.57900000D+01) =  0.70399651D+00          EXPECTED VALUE =  0.58536583D+00
        FHAT( 0.58400000D+01) =  0.70731784D+00          EXPECTED VALUE =  0.60975605D+00
        FHAT( 0.61100000D+01) =  0.72525301D+00          EXPECTED VALUE =  0.63414633D+00
        FHAT( 0.62700000D+01) =  0.73588126D+00          EXPECTED VALUE =  0.65853655D+00
        FHAT( 0.65700000D+01) =  0.75580923D+00          EXPECTED VALUE =  0.68292677D+00
        FHAT( 0.69500000D+01) =  0.78105132D+00          EXPECTED VALUE =  0.70731705D+00
        FHAT( 0.69800000D+01) =  0.78304412D+00          EXPECTED VALUE =  0.73170727D+00
        FHAT( 0.72200000D+01) =  0.79898649D+00          EXPECTED VALUE =  0.75609756D+00
        FHAT( 0.72900000D+01) =  0.80363635D+00          EXPECTED VALUE =  0.78048778D+00
        FHAT( 0.83400000D+01) =  0.87338424D+00          EXPECTED VALUE =  0.80487800D+00
        FHAT( 0.89600000D+01) =  0.91456871D+00          EXPECTED VALUE =  0.82926828D+00
        FHAT( 0.91200000D+01) =  0.92519696D+00          EXPECTED VALUE =  0.85365850D+00
        FHAT( 0.91900000D+01) =  0.92984682D+00          EXPECTED VALUE =  0.87804878D+00
        FHAT( 0.93000000D+01) =  0.93715374D+00          EXPECTED VALUE =  0.90243900D+00
        FHAT( 0.93900000D+01) =  0.94313213D+00          EXPECTED VALUE =  0.92682922D+00
        FHAT( 0.97300000D+01) =  0.96571716D+00          EXPECTED VALUE =  0.95121950D+00
        FHAT( 0.97600000D+01) =  0.96770996D+00          EXPECTED VALUE =  0.97560972D+00
```

```
        THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

            LAMBDA HAT(0) =  0.0
            LAMBDA HAT(1) = -0.30623579D-02

        THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

            LAMBDA(1) =  0.0

   SINCE    -0.12166603D+00  DOES NOT EXCEED THE N(0,1) CRITICAL VALUE OF     0.16450000D+01   THEN THE CURRENT TRIAL
  ESTIMATE  OF G*, NAMELY     0.10200000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.10300D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT   =  0.19411D-01; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

        THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

            ALPHA(0) =  0.35802981D-02
            ALPHA(1) =  0.66426560D-01

        THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

            FHAT( 0.54000000D+01) =  0.67809015D+00            EXPECTED VALUE =  0.51219511D+00
            FHAT( 0.56300000D+01) =  0.69336826D+00            EXPECTED VALUE =  0.53658533D+00
            FHAT( 0.57300000D+01) =  0.70001092D+00            EXPECTED VALUE =  0.56097555D+00
            FHAT( 0.57900000D+01) =  0.70399651D+00            EXPECTED VALUE =  0.58536583D+00
            FHAT( 0.58400000D+01) =  0.70731784D+00            EXPECTED VALUE =  0.60975605D+00
            FHAT( 0.61100000D+01) =  0.72525301D+00            EXPECTED VALUE =  0.63414633D+00
            FHAT( 0.62700000D+01) =  0.73588126D+00            EXPECTED VALUE =  0.65853655D+00
            FHAT( 0.65700000D+01) =  0.75580923D+00            EXPECTED VALUE =  0.68292677D+00
            FHAT( 0.69500000D+01) =  0.78105132D+00            EXPECTED VALUE =  0.70731705D+00
            FHAT( 0.69800000D+01) =  0.78304412D+00            EXPECTED VALUE =  0.73170727D+00
            FHAT( 0.72200000D+01) =  0.79898649D+00            EXPECTED VALUE =  0.75609756D+00
            FHAT( 0.72900000D+01) =  0.80363635D+00            EXPECTED VALUE =  0.78048778D+00
            FHAT( 0.83400000D+01) =  0.87338424D+00            EXPECTED VALUE =  0.80487800D+00
            FHAT( 0.89600000D+01) =  0.91456871D+00            EXPECTED VALUE =  0.82926828D+00
            FHAT( 0.91200000D+01) =  0.92519696D+00            EXPECTED VALUE =  0.85365850D+00
            FHAT( 0.91500000D+01) =  0.92984682D+00            EXPECTED VALUE =  0.87804878D+00
            FHAT( 0.93000000D+01) =  0.93715374D+00            EXPECTED VALUE =  0.90243900D+00
            FHAT( 0.93500000D+01) =  0.94313213D+00            EXPECTED VALUE =  0.92682922D+00
            FHAT( 0.97300000D+01) =  0.96571716D+00            EXPECTED VALUE =  0.95121950D+00
            FHAT( 0.97600000D+01) =  0.96770996D+00            EXPECTED VALUE =  0.97560972D+00

        THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

            LAMBDA HAT(0) =  0.0
            LAMBDA HAT(1) =  0.35802981D-02

        THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

            LAMBDA(1) =  0.0

   SINCE     0.13932190D+00  DOES NOT EXCEED THE N(0,1) CRITICAL VALUE OF     0.16450000D+01   THEN THE CURRENT TRIAL
  ESTIMATE  OF G*, NAMELY     0.10300000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.10400D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT   =  0.15123D+00; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

        THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

            ALPHA(0) =  0.10222954D-01
            ALPHA(1) =  0.66426560D-01
```

```
THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

    FHAT( 0.54000000D+01) =  0.67809015D+00          EXPECTED VALUE =  0.51219511D+00
    FHAT( 0.56300000D+01) =  0.69336826D+00          EXPECTED VALUE =  0.53658533D+00
    FHAT( 0.57300000D+01) =  0.70001092D+00          EXPECTED VALUE =  0.56097555D+00
    FHAT( 0.57900000D+01) =  0.70399651D+00          EXPECTED VALUE =  0.58536583D+00
    FHAT( 0.58400000D+01) =  0.70731784D+00          EXPECTED VALUE =  0.60975605D+00
    FHAT( 0.61100000D+01) =  0.72525301D+00          EXPECTED VALUE =  0.63414633D+00
    FHAT( 0.62700000D+01) =  0.73588126D+00          EXPECTED VALUE =  0.65853655D+00
    FHAT( 0.65700000D+01) =  0.75580923D+00          EXPECTED VALUE =  0.68292677D+00
    FHAT( 0.69500000D+01) =  0.78105132D+00          EXPECTED VALUE =  0.70731705D+00
    FHAT( 0.69800000D+01) =  0.78304412D+00          EXPECTED VALUE =  0.73170727D+00
    FHAT( 0.72200000D+01) =  0.79898649D+00          EXPECTED VALUE =  0.75609756D+00
    FHAT( 0.72900000D+01) =  0.80363635D+00          EXPECTED VALUE =  0.78048778D+00
    FHAT( 0.83400000D+01) =  0.87338424D+00          EXPECTED VALUE =  0.80487800D+00
    FHAT( 0.89600000D+01) =  0.91456871D+00          EXPECTED VALUE =  0.82926828D+00
    FHAT( 0.91200000D+01) =  0.92519696D+00          EXPECTED VALUE =  0.85365850D+00
    FHAT( 0.91900000D+01) =  0.92984682D+00          EXPECTED VALUE =  0.87804878D+00
    FHAT( 0.93000000D+01) =  0.93715374D+00          EXPECTED VALUE =  0.90243900D+00
    FHAT( 0.93900000D+01) =  0.94313213D+00          EXPECTED VALUE =  0.92682922D+00
    FHAT( 0.97300000D+01) =  0.96571716D+00          EXPECTED VALUE =  0.95121950D+00
    FHAT( 0.97600000D+01) =  0.96770996D+00          EXPECTED VALUE =  0.97560972D+00

THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

    LAMBDA HAT(0) =  0.0
    LAMBDA HAT(1) =  0.10222954D-01

THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

    LAMBDA(1) =  0.0

  SINCE    0.38987930D+00  DOES NOT EXCEED THE N(0,1) CRITICAL VALUE OF    0.16450000D+01  THEN THE CURRENT TRIAL
 ESTIMATE  OF G0, NAMELY    0.10400000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.10500D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.39190D+00; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

    ALPHA(0) =  0.16885610D-01
    ALPHA(1) =  0.66426560D-01

THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

    FHAT( 0.54000000D+01) =  0.67809015D+00          EXPECTED VALUE =  0.51219511D+00
    FHAT( 0.56300000D+01) =  0.69336826D+00          EXPECTED VALUE =  0.53658533D+00
    FHAT( 0.57300000D+01) =  0.70001092D+00          EXPECTED VALUE =  0.56097555D+00
    FHAT( 0.57900000D+01) =  0.70399651D+00          EXPECTED VALUE =  0.58536583D+00
    FHAT( 0.58400000D+01) =  0.70731784D+00          EXPECTED VALUE =  0.60975605D+00
    FHAT( 0.61100000D+01) =  0.72525301D+00          EXPECTED VALUE =  0.63414633D+00
    FHAT( 0.62700000D+01) =  0.73588126D+00          EXPECTED VALUE =  0.65853655D+00
    FHAT( 0.65700000D+01) =  0.75580923D+00          EXPECTED VALUE =  0.68292677D+00
    FHAT( 0.69500000D+01) =  0.78105132D+00          EXPECTED VALUE =  0.70731705D+00
    FHAT( 0.69800000D+01) =  0.78304412D+00          EXPECTED VALUE =  0.73170727D+00
    FHAT( 0.72200000D+01) =  0.79898649D+00          EXPECTED VALUE =  0.75609756D+00
    FHAT( 0.72900000D+01) =  0.80363635D+00          EXPECTED VALUE =  0.78048778D+00
    FHAT( 0.83400000D+01) =  0.87338424D+00          EXPECTED VALUE =  0.80487800D+00
    FHAT( 0.89600000D+01) =  0.91456871D+00          EXPECTED VALUE =  0.82926828D+00
    FHAT( 0.91200000D+01) =  0.92519696D+00          EXPECTED VALUE =  0.85365850D+00
    FHAT( 0.91900000D+01) =  0.92984682D+00          EXPECTED VALUE =  0.87804878D+00
    FHAT( 0.93000000D+01) =  0.93715374D+00          EXPECTED VALUE =  0.90243900D+00
```

```
        FHAT( 0.93900000D+01) =  0.94313213D+00            EXPECTED VALUE =  0.92682922D+00
        FHAT( 0.97300000D+01) =  0.96571716D+00            EXPECTED VALUE =  0.95121950D+00
        FHAT( 0.97600000D+01) =  0.96770996D+00            EXPECTED VALUE =  0.97560972D+00

     THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

        LAMBDA HAT(0) =  0.0
        LAMBDA HAT(1) =  0.16865610D-01

     THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

        LAMBDA(1) =  0.0

   SINCE    0.62609330+00  DOES NOT EXCEED THE N(0,1) CRITICAL VALUE OF    0.16450000D+01  THEN THE CURRENT TRIAL
 ESTIMATE  OF G0, NAMELY    0.10500000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.10600D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.72324D+00; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

     THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

        ALPHA(0) =  0.23508266D-01
        ALPHA(1) =  0.66426560D-01

     THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

        FHAT( 0.54000000D+01) =  0.67809015D+00            EXPECTED VALUE =  0.51219511D+00
        FHAT( 0.56300000D+01) =  0.69336826D+00            EXPECTED VALUE =  0.53658533D+00
        FHAT( 0.57300000D+01) =  0.70001092D+00            EXPECTED VALUE =  0.56097555D+00
        FHAT( 0.57900000D+01) =  0.70399651D+00            EXPECTED VALUE =  0.58536583D+00
        FHAT( 0.58400000D+01) =  0.70731784D+00            EXPECTED VALUE =  0.60975605D+00
        FHAT( 0.61100000D+01) =  0.72525301D+00            EXPECTED VALUE =  0.63414633D+00
        FHAT( 0.62700000D+01) =  0.73588126D+00            EXPECTED VALUE =  0.65853655D+00
        FHAT( 0.65700000D+01) =  0.75580923D+00            EXPECTED VALUE =  0.68292677D+00
        FHAT( 0.69500000D+01) =  0.78105132D+00            EXPECTED VALUE =  0.70731705D+00
        FHAT( 0.69800000D+01) =  0.78304412D+00            EXPECTED VALUE =  0.73170727D+00
        FHAT( 0.72200000D+01) =  0.79898649D+00            EXPECTED VALUE =  0.75609756D+00
        FHAT( 0.72900000D+01) =  0.80363635D+00            EXPECTED VALUE =  0.78048778D+00
        FHAT( 0.83400000D+01) =  0.87338424D+00            EXPECTED VALUE =  0.80487800D+00
        FHAT( 0.89600000D+01) =  0.91456871D+00            EXPECTED VALUE =  0.82926828D+00
        FHAT( 0.91200000D+01) =  0.92519696D+00            EXPECTED VALUE =  0.85365850D+00
        FHAT( 0.91900000D+01) =  0.92984682D+00            EXPECTED VALUE =  0.87804878D+00
        FHAT( 0.93000000D+01) =  0.93715374D+00            EXPECTED VALUE =  0.90243900D+00
        FHAT( 0.93900000D+01) =  0.94313213D+00            EXPECTED VALUE =  0.92682922D+00
        FHAT( 0.97300000D+01) =  0.96571716D+00            EXPECTED VALUE =  0.95121950D+00
        FHAT( 0.97600000D+01) =  0.96770996D+00            EXPECTED VALUE =  0.97560972D+00

     THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

        LAMBDA HAT(0) =  0.0
        LAMBDA HAT(1) =  0.23508266D-01

     THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

        LAMBDA(1) =  0.0

   SINCE    0.95043410+00  DOES NOT EXCEED THE N(0,1) CRITICAL VALUE OF    0.16450000D+01  THEN THE CURRENT TRIAL
 ESTIMATE  OF G0, NAMELY    0.10600000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.10800D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.15876D+01; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

     THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :
```

```
      ALPHA(0) =  0.36793578D-01
      ALPHA(1) =  0.66426560D-01

   THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

      FHAT( 0.54000000D+01) =  0.67809015D+00            EXPECTED VALUE =  0.51219511C+00
      FHAT( 0.56300000C+01) =  0.69336826D+00            EXPECTED VALUE =  0.53658533D+00
      FHAT( 0.57300000D+01) =  0.70001092D+00            EXPECTED VALUE =  0.56097555D+00
      FHAT( 0.57900000D+01) =  0.70399651D+00            EXPECTED VALUE =  0.58536583D+00
      FHAT( 0.58400000D+01) =  0.70731784D+00            EXPECTED VALUE =  0.60975605D+00
      FHAT( 0.61100000D+01) =  0.72525301D+00            EXPECTED VALUE =  0.63414633D+00
      FHAT( 0.62700000C+01) =  0.73588126D+00            EXPECTED VALUE =  0.65853655C+00
      FHAT( 0.65700000D+01) =  0.75580923D+00            EXPECTED VALUE =  0.68292677D+00
      FHAT( 0.69500000C+01) =  0.78105132D+00            EXPECTED VALUE =  0.70731705C+00
      FHAT( 0.69800000C+01) =  0.78304412D+00            EXPECTED VALUE =  0.73170727C+00
      FHAT( 0.72200000D+01) =  0.79898649D+00            EXPECTED VALUE =  0.75609756D+00
      FHAT( 0.72400000D+01) =  0.80363635D+00            EXPECTED VALUE =  0.78048778C+00
      FHAT( 0.83400000D+01) =  0.87338424D+00            EXPECTED VALUE =  0.80487800D+00
      FHAT( 0.89600000D+01) =  0.91456871D+00            EXPECTED VALUE =  0.82926828D+00
      FHAT( 0.91200000D+01) =  0.92519696D+00            EXPECTED VALUE =  0.85365850C+00
      FHAT( 0.91900000D+01) =  0.92984682D+00            EXPECTED VALUE =  0.87804878C+00
      FHAT( 0.93000000D+01) =  0.93715374D+00            EXPECTED VALUE =  0.90243900C+00
      FHAT( 0.93900000D+01) =  0.94313213D+00            EXPECTED VALUE =  0.92682922C+00
      FHAT( 0.97300000D+01) =  0.96571716D+00            EXPECTED VALUE =  0.95121950D+00
      FHAT( 0.97600000D+01) =  0.96770996D+00            EXPECTED VALUE =  0.97560972C+00

   THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

      LAMBDA HAT(0) =  0.0
      LAMBDA HAT(1) =  0.36793578D-01

   THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

      LAMBDA(1) =  0.0

  SINCE    0.12599085D+01  DOES NOT EXCEED THE N(0,1) CRITICAL VALUE OF    0.16450000D+01  THEN THE CURRENT TRIAL
 ESTIMATE  OF G*, NAMELY    0.10800000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.11000D+02; THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.26201D+01! THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

   THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

      ALPHA(0) =  0.50078890D-01
      ALPHA(1) =  0.66426560D-01

   THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

      FHAT( 0.54000000D+01) =  0.67809015D+00            EXPECTED VALUE =  0.51219511C+00
      FHAT( 0.56300000D+01) =  0.69336826D+00            EXPECTED VALUE =  0.53658533D+00
      FHAT( 0.57300000D+01) =  0.70001092D+00            EXPECTED VALUE =  0.56097555D+00
      FHAT( 0.57900000C+01) =  0.70399651D+00            EXPECTED VALUE =  0.58536583D+00
      FHAT( 0.58400000D+01) =  0.70731784D+00            EXPECTED VALUE =  0.60975605D+00
      FHAT( 0.61100000D+01) =  0.72525301D+00            EXPECTED VALUE =  0.63414633C+00
      FHAT( 0.62700000C+01) =  0.73588126D+00            EXPECTED VALUE =  0.65853655D+00
      FHAT( 0.65700000D+01) =  0.75580923D+00            EXPECTED VALUE =  0.68292677C+00
      FHAT( 0.69500000D+01) =  0.78105132D+00            EXPECTED VALUE =  0.70731705C+00
      FHAT( 0.69800000C+01) =  0.78304412D+00            EXPECTED VALUE =  0.73170727D+00
      FHAT( 0.72200000D+01) =  0.79898649D+00            EXPECTED VALUE =  0.75609756C+00
      FHAT( 0.72400000D+01) =  0.80363635D+00            EXPECTED VALUE =  0.78048778D+00
      FHAT( 0.83400000D+01) =  0.87338424D+00            EXPECTED VALUE =  0.80487800D+00
```

```
            FHAT( 0.89600000D+01) =  0.91456871D+00              EXPECTED VALUE =  0.82926828D+00
            FHAT( 0.91200000D+01) =  0.92519696D+00              EXPECTED VALUE =  0.85365850D+00
            FHAT( 0.91900000D+01) =  0.92984682D+00              EXPECTED VALUE =  0.87804878D+00
            FHAT( 0.93000000D+01) =  0.93715374D+00              EXPECTED VALUE =  0.90243900D+00
            FHAT( 0.93600000D+01) =  0.94313213D+00              EXPECTED VALUE =  0.92682922D+00
            FHAT( 0.97300000D+01) =  0.96571716D+00              EXPECTED VALUE =  0.95121950D+00
            FHAT( 0.97600000D+01) =  0.96770996D+00              EXPECTED VALUE =  0.97560972D+00

         THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

            LAMBDA HAT(0) =  0.0
            LAMBDA HAT(1) =  0.50078890D-01

         THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMING ALGORITHM :

            LAMBDA(1) =  0.0

      SINCE     0.16188860D+01  DOES NOT EXCEED THE N(0.1) CRITICAL VALUE OF     0.16450000D+01  THEN THE CURRENT TRIAL
     ESTIMATE  OF G0, NAMELY     0.11000000D+02  IS NOT TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

 GTRIAL =  0.11200D+02: THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.37291E+01: THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

         THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SCUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

            ALPHA(0) =  0.63364202D-01
            ALPHA(1) =  0.66426560D-01

         THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

            FHAT( 0.54000000D+01) =  0.67809015D+00              EXPECTED VALUE =  0.51219511D+00
            FHAT( 0.56300000D+01) =  0.69336826D+00              EXPECTED VALUE =  0.53658533D+00
            FHAT( 0.57300000D+01) =  0.70001092D+00              EXPECTED VALUE =  0.56097555D+00
            FHAT( 0.57900000D+01) =  0.70399651D+00              EXPECTED VALUE =  0.58536583D+00
            FHAT( 0.58400000D+01) =  0.70731784D+00              EXPECTED VALUE =  0.60975605D+00
            FHAT( 0.61100000D+01) =  0.72525301D+00              EXPECTED VALUE =  0.63414633D+00
            FHAT( 0.62700000D+01) =  0.73588126D+00              EXPECTED VALUE =  0.65853655D+00
            FHAT( 0.65700000D+01) =  0.75580923D+00              EXPECTED VALUE =  0.68292677D+00
            FHAT( 0.69500000D+01) =  0.78105132D+00              EXPECTED VALUE =  0.70731705D+00
            FHAT( 0.69800000D+01) =  0.78304412D+00              EXPECTED VALUE =  0.73170727D+00
            FHAT( 0.72200000D+01) =  0.79898649D+00              EXPECTED VALUE =  0.75609756D+00
            FHAT( 0.72900000D+01) =  0.80363635D+00              EXPECTED VALUE =  0.78048778D+00
            FHAT( 0.83400000D+01) =  0.87338424D+00              EXPECTED VALUE =  0.80487800D+00
            FHAT( 0.89600000D+01) =  0.91456871D+00              EXPECTED VALUE =  0.82926828D+00
            FHAT( 0.91200000D+01) =  0.92519696D+00              EXPECTED VALUE =  0.85365850D+00
            FHAT( 0.91900000D+01) =  0.92984682D+00              EXPECTED VALUE =  0.87804878D+00
            FHAT( 0.93000000D+01) =  0.93715374D+00              EXPECTED VALUE =  0.90243900D+00
            FHAT( 0.93900000D+01) =  0.94313213D+00              EXPECTED VALUE =  0.92682922D+00
            FHAT( 0.97300000D+01) =  0.96571716D+00              EXPECTED VALUE =  0.95121950D+00
            FHAT( 0.97600000D+01) =  0.96770996D+00              EXPECTED VALUE =  0.97560972D+00

         THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

            LAMBDA HAT(0) =  0.0
            LAMBDA HAT(1) =  0.63364202D-01

         THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMING ALGORITHM :

            LAMBDA(1) =  0.0

      SINCE     0.19310790D+01  DOES  EXCEED THE N(0.1) CRITICAL VALUE OF     0.16450000D+01  THEN THE CURRENT TRIAL
     ESTIMATE  OF G0, NAMELY     0.11200000D+02  IS TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)
```

```
GTRIAL =  0.11400D+02: THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.484743D+01: THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

        THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

            ALPHA(0) =  0.766495140D-01
            ALPHA(1) =  0.664265600D-01

        THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

            FHAT( 0.540000000D+01) =  0.678090150D+00            EXPECTED VALUE =  0.512195110D+00
            FHAT( 0.563000000D+01) =  0.693368260D+00            EXPECTED VALUE =  0.536585330D+00
            FHAT( 0.573000000D+01) =  0.700010920D+00            EXPECTED VALUE =  0.560975550D+00
            FHAT( 0.579000000D+01) =  0.703996510D+00            EXPECTED VALUE =  0.585365830D+00
            FHAT( 0.584000000D+01) =  0.707317840D+00            EXPECTED VALUE =  0.609756050D+00
            FHAT( 0.611000000D+01) =  0.725253010D+00            EXPECTED VALUE =  0.634146330D+00
            FHAT( 0.627000000D+01) =  0.735881260D+00            EXPECTED VALUE =  0.658536550D+00
            FHAT( 0.657000000D+01) =  0.755809230D+00            EXPECTED VALUE =  0.682926770D+00
            FHAT( 0.695000000D+01) =  0.781051320D+00            EXPECTED VALUE =  0.707317050D+00
            FHAT( 0.698000000D+01) =  0.783044120D+00            EXPECTED VALUE =  0.731707270D+00
            FHAT( 0.722000000D+01) =  0.798986490D+00            EXPECTED VALUE =  0.756097560D+00
            FHAT( 0.725000000D+01) =  0.803636350D+00            EXPECTED VALUE =  0.780487780D+00
            FHAT( 0.834000000D+01) =  0.873384240D+00            EXPECTED VALUE =  0.804878000D+00
            FHAT( 0.896000000D+01) =  0.914568710D+00            EXPECTED VALUE =  0.829268220D+00
            FHAT( 0.912000000D+01) =  0.925196960D+00            EXPECTED VALUE =  0.853658500D+00
            FHAT( 0.919000000D+01) =  0.929846020D+00            EXPECTED VALUE =  0.878048780D+00
            FHAT( 0.930000000D+01) =  0.937153740D+00            EXPECTED VALUE =  0.902439000D+00
            FHAT( 0.939000000D+01) =  0.943132130D+00            EXPECTED VALUE =  0.926829220D+00
            FHAT( 0.973000000D+01) =  0.965717160D+00            EXPECTED VALUE =  0.951219500D+00
            FHAT( 0.976000000D+01) =  0.967709960D+00            EXPECTED VALUE =  0.975609720D+00

        THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

            LAMBDA HAT(0) =  0.0
            LAMBDA HAT(1) =  0.766495140D-01

        THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

            LAMBDA(1) =  0.0

     SINCE    0.220168700+01  DOES  EXCEED THE N(0,1) CRITICAL VALUE OF     0.164500000+01  THEN THE CURRENT TRIAL
    ESTIMATE  OF G0, NAMELY     0.114000000+02  IS TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)
GTRIAL =  0.11600D+02: THE GOODNESS-OF-FIT INDEX  Q3KHAT  =  0.593450D+01: THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

        THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

            ALPHA(0) =  0.899348260D-01
            ALPHA(1) =  0.664265600D-01

        THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

            FHAT( 0.540000000D+01) =  0.678090150D+00            EXPECTED VALUE =  0.512195110D+00
            FHAT( 0.563000000D+01) =  0.693368260D+00            EXPECTED VALUE =  0.536585330D+00
            FHAT( 0.573000000D+01) =  0.700010920D+00            EXPECTED VALUE =  0.560975550D+00
            FHAT( 0.579000000D+01) =  0.703996510D+00            EXPECTED VALUE =  0.585365830D+00
            FHAT( 0.584000000D+01) =  0.707317840D+00            EXPECTED VALUE =  0.609756050D+00
            FHAT( 0.611000000D+01) =  0.725253010D+00            EXPECTED VALUE =  0.634146330D+00
            FHAT( 0.627000000D+01) =  0.735881260D+00            EXPECTED VALUE =  0.658536550D+00
            FHAT( 0.657000000D+01) =  0.755809230D+00            EXPECTED VALUE =  0.682926770D+00
            FHAT( 0.695000000D+01) =  0.781051320D+00            EXPECTED VALUE =  0.707317050D+00
            FHAT( 0.698000000D+01) =  0.783044120D+00            EXPECTED VALUE =  0.731707270D+00
```

```
     FHAT( 0.72200000D+01) =  0.79898649D+00            EXPECTED VALUE =  0.75609756D+00
     FHAT( 0.72900000D+01) =  0.80363635D+00            EXPECTED VALUE =  0.78048778D+00
     FHAT( 0.83400000D+01) =  0.87338424D+00            EXPECTED VALUE =  0.80487800D+00
     FHAT( 0.89600000D+01) =  0.91456871D+00            EXPECTED VALUE =  0.82926828D+00
     FHAT( 0.91200000D+01) =  0.92519696D+00            EXPECTED VALUE =  0.85365850D+00
     FHAT( 0.91900000D+01) =  0.92984682D+00            EXPECTED VALUE =  0.87804878D+00
     FHAT( 0.93000000D+01) =  0.93715374D+00            EXPECTED VALUE =  0.90243900D+00
     FHAT( 0.93900000D+01) =  0.94313213D+00            EXPECTED VALUE =  0.92682922D+00
     FHAT( 0.97300000D+01) =  0.96571716D+00            EXPECTED VALUE =  0.95121950D+00
     FHAT( 0.97600000D+01) =  0.96770996D+00            EXPECTED VALUE =  0.97560972D+00

   THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

     LAMBDA HAT(0) =  0.0
     LAMBDA HAT(1) =  0.89934826D-01

   THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

     LAMBDA(1) =  0.0

   SINCE    0.24360880D+01  DOES  EXCEED THE N(0.1) CRITICAL VALUE OF    0.16450000D+01  THEN THE CURRENT TRIAL
 ESTIMATE  OF G*, NAMELY    0.11600000D+02  IS TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)

GTRIAL =  0.11800D+02; THE GOODNESS-OF-FIT INDEX  Q3KMAT  =  0.69664D+01; THE CRITICAL CHI-SQUARED PERCENTILE =    5.99

   THE ALPHA VECTOR THAT MINIMIZED THE WEIGHTED LEAST SQUARES FIT OF THE UNIFORM ORDER STATISTICS TO THEIR EXPECTATIONS :

     ALPHA(0) =  0.10322014D+00
     ALPHA(1) =  0.66426560D-01

   THE FITTED CDF EVALUATED AT THE K LARGEST ORDER STATISTICS :

     FHAT( 0.54000000D+01) =  0.67809015D+00            EXPECTED VALUE =  0.51219511D+00
     FHAT( 0.56300000D+01) =  0.69336826D+00            EXPECTED VALUE =  0.53658533D+00
     FHAT( 0.57300000D+01) =  0.70001092D+00            EXPECTED VALUE =  0.56097555D+00
     FHAT( 0.57900000D+01) =  0.70399651D+00            EXPECTED VALUE =  0.58536583D+00
     FHAT( 0.58400000D+01) =  0.70731784D+00            EXPECTED VALUE =  0.60975605D+00
     FHAT( 0.61100000D+01) =  0.72525301D+00            EXPECTED VALUE =  0.63414633D+00
     FHAT( 0.62700000D+01) =  0.73588126D+00            EXPECTED VALUE =  0.65853655D+00
     FHAT( 0.65700000D+01) =  0.75580923D+00            EXPECTED VALUE =  0.68292677D+00
     FHAT( 0.69500000D+01) =  0.78105132D+00            EXPECTED VALUE =  0.70731705D+00
     FHAT( 0.69800000D+01) =  0.78304412D+00            EXPECTED VALUE =  0.73170727D+00
     FHAT( 0.72200000D+01) =  0.79898649D+00            EXPECTED VALUE =  0.75609756D+00
     FHAT( 0.72900000D+01) =  0.80363635D+00            EXPECTED VALUE =  0.78048778D+00
     FHAT( 0.83400000D+01) =  0.87338424D+00            EXPECTED VALUE =  0.80487800D+00
     FHAT( 0.89600000D+01) =  0.91456871D+00            EXPECTED VALUE =  0.82926828D+00
     FHAT( 0.91200000D+01) =  0.92519696D+00            EXPECTED VALUE =  0.85365850D+00
     FHAT( 0.91900000D+01) =  0.92984682D+00            EXPECTED VALUE =  0.87804878D+00
     FHAT( 0.93000000D+01) =  0.93715374D+00            EXPECTED VALUE =  0.90243900D+00
     FHAT( 0.93900000D+01) =  0.94313213D+00            EXPECTED VALUE =  0.92682922D+00
     FHAT( 0.97300000D+01) =  0.96571716D+00            EXPECTED VALUE =  0.95121950D+00
     FHAT( 0.97600000D+01) =  0.96770996D+00            EXPECTED VALUE =  0.97560972D+00

   THE LAMBDA HAT VECTOR CORRESPONDING TO THE ESTIMATED ALPHA VECTOR :

     LAMBDA HAT(0) =  0.0
     LAMBDA HAT(1) =  0.10322014D+00

   THE LAMBDA VECTOR DETERMINED BY THE QUADRATIC PROGRAMMING ALGORITHM :

     LAMBDA(1) =  0.0
```

```
SINCE    0.2639398D+01  DOES  EXCEED THE N(0,1) CRITICAL VALUE OF    0.1645000D+01  THEN THE CURRENT TRIAL
ESTIMATE  OF G0, NAMELY    0.1100000D+02  IS TOO LARGE (BASED ON THE APPROXIMATE NORMAL PROCEDURE)
```

Short form
for output

```
GRAPH 1     TRIAL 2

THE INDEPENDENT OBSERVATIONS ON G(X) ARE:
         1-TH OBSERVATION ON G(X) =     0.1500000000D+01
         2-TH OBSERVATION ON G(X) =     0.4650000000D+01
         3-TH OBSERVATION ON G(X) =     0.4830000000D+01
         4-TH OBSERVATION ON G(X) =     0.9300000000D+01
         5-TH OBSERVATION ON G(X) =     0.3990000000D+01
         6-TH OBSERVATION ON G(X) =     0.6900000000D+00
         7-TH OBSERVATION ON G(X) =     0.7290000000D+01
         8-TH OBSERVATION ON G(X) =     0.9190000000D+01
         9-TH OBSERVATION ON G(X) =     0.1430000000D+01
        10-TH OBSERVATION ON G(X) =     0.3680000000D+01
        11-TH OBSERVATION ON G(X) =     0.6950000000D+01
        12-TH OBSERVATION ON G(X) =     0.4090000000D+01
        13-TH OBSERVATION ON G(X) =     0.9390000000D+01
        14-TH OBSERVATION ON G(X) =     0.6110000000D+01
        15-TH OBSERVATION ON G(X) =     0.9730000000D+01
        16-TH OBSERVATION ON G(X) =     0.1270000000D+01
        17-TH OBSERVATION ON G(X) =     0.2130000000D+01
        18-TH OBSERVATION ON G(X) =     0.5400000000D+01
        19-TH OBSERVATION ON G(X) =     0.5393000000D+01
        20-TH OBSERVATION ON G(X) =     0.9760000000D+01
        21-TH OBSERVATION ON G(X) =     0.9120000000D+01
        22-TH OBSERVATION ON G(X) =     0.5840000000D+01
        23-TH OBSERVATION ON G(X) =     0.3230000000D+01
        24-TH OBSERVATION ON G(X) =     0.2700000000D+01
        25-TH OBSERVATION ON G(X) =     0.3300000000D+01
        26-TH OBSERVATION ON G(X) =     0.7220000000D+01
        27-TH OBSERVATION ON G(X) =     0.2050000000D+01
        28-TH OBSERVATION ON G(X) =     0.5730000000D+01
        29-TH OBSERVATION ON G(X) =     0.4200000000D+00
        30-TH OBSERVATION ON G(X) =     0.2640000000D+01
        31-TH OBSERVATION ON G(X) =     0.4700000000D+00
        32-TH OBSERVATION ON G(X) =     0.6980000000D+01
        33-TH OBSERVATION ON G(X) =     0.6570000000D+01
        34-TH OBSERVATION ON G(X) =     0.5790000000D+01
        35-TH OBSERVATION ON G(X) =     0.8340000000D+01
        36-TH OBSERVATION ON G(X) =     0.4250000000D+01
        37-TH OBSERVATION ON G(X) =     0.5610000000D+01
        38-TH OBSERVATION ON G(X) =     0.1850000000D+01
        39-TH OBSERVATION ON G(X) =     0.8960000000D+01
        40-TH OBSERVATION ON G(X) =     0.6270000000D+01

THERE ARE    0 NONRANDOM INITIAL ESTIMATES OF G*

THE ORDERED VALUES OF THE OBSERVED G'S ARE:
         1-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.4200000000D+00
         2-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.4700000000D+00
         3-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.6900000000D+00
         4-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.1270000000D+01
         5-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.1430000000D+01
         6-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.1500000000D+01
         7-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.1850000000D+01
         8-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.2050000000D+01
         9-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.2130000000D+01
        10-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.2640000000D+01
        11-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.2700000000D+01
        12-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.3230000000D+01
        13-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.3300000000D+01
```

```
    14-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.36800000000D+01
    15-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.39900000000D+01
    16-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.40900000000D+01
    17-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.42500000000D+01
    18-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.46500000000D+01
    19-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.48300000000D+01
    20-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.53900000000D+01
    21-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.54000000000D+01
    22-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.56300000000D+01
    23-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.57300000000D+01
    24-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.57900000000D+01
    25-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.58400000000D+01
    26-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.61100000000D+01
    27-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.62700000000D+01
    28-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.65700000000D+01
    29-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.69500000000D+01
    30-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.69800000000D+01
    31-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.72200000000D+01
    32-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.72500000000D+01
    33-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.83400000000D+01
    34-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.89600000000D+01
    35-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.91200000000D+01
    36-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.91900000000D+01
    37-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.93000000000D+01
    38-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.93900000000D+01
    39-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.97300000000D+01
    40-TH SMALLEST VALUE OF THE OBSERVED G'S =     0.97600000000D+01

THE TRIAL ESTIMATES OF G* ARE:
     1-TH TRIAL ESTIMATE OF G* =     0.10000000000D+02
     2-TH TRIAL ESTIMATE OF G* =     0.10100000000D+02
     3-TH TRIAL ESTIMATE OF G* =     0.10200000000D+02
     4-TH TRIAL ESTIMATE OF G* =     0.10300000000D+02
     5-TH TRIAL ESTIMATE OF G* =     0.10400000000D+02
     6-TH TRIAL ESTIMATE OF G* =     0.10500000000D+02
     7-TH TRIAL ESTIMATE OF G* =     0.10600000000D+02
     8-TH TRIAL ESTIMATE OF G* =     0.10800000000D+02
     9-TH TRIAL ESTIMATE OF G* =     0.11000000000D+02
    10-TH TRIAL ESTIMATE OF G* =     0.11200000000D+02
    11-TH TRIAL ESTIMATE OF G* =     0.11400000000D+02
    12-TH TRIAL ESTIMATE OF G* =     0.11600000000D+02
    13-TH TRIAL ESTIMATE OF G* =     0.11800000000D+02
    14-TH TRIAL ESTIMATE OF G* =     0.12000000000D+02
    15-TH TRIAL ESTIMATE OF G* =     0.12500000000D+02
    16-TH TRIAL ESTIMATE OF G* =     0.13000000000D+02
    17-TH TRIAL ESTIMATE OF G* =     0.13500000000D+02
    18-TH TRIAL ESTIMATE OF G* =     0.14000000000D+02
    19-TH TRIAL ESTIMATE OF G* =     0.14500000000D+02
    20-TH TRIAL ESTIMATE OF G* =     0.15000000000D+02
    21-TH TRIAL ESTIMATE OF G* =     0.15500000000D+02

THE ESTIMATION PROCEDURE WILL STOP AS SOON AS A TRIAL ESTIMATE OF G* HAS A GOODNESS-OF-FIT INDEX
GREATER THAN ITS CRITICAL VALUE.
```

```
THE CONFIDENCE LIMIT BASED ON THE SAMPLE PROPORTIONS WILL BE CONSIDERED.      (Q1KHAT)

     THE SCALE POINTS USED TO CREATE THE SAMPLE PROPORTIONS WERE AS FOLLOWS:
             0.200000D+01         0.400000D+01         0.600000D+01         0.800000D+01         0.100000D+02

     THE NUMBER OF SAMPLE PROPORTIONS : K =    5

A  1-TH DEGREE POLYNOMIAL WILL BE USED TO APPROXIMATE THE FINITE RANGE DISTRIBUTION NEAR G*.

   GTRIAL =  0.100000D+02; THE GOODNESS-OF-FIT INDEX  Q1KHAT  =  0.586010D+00; THE CRITICAL CHI-SQUARED PERCENTILE =   11.10

   GTRIAL =  0.101000D+02; THE GOODNESS-OF-FIT INDEX  Q1KHAT  =  0.383360D+01; THE CRITICAL CHI-SQUARED PERCENTILE =   11.10

   GTRIAL =  0.102000D+02; THE GOODNESS-OF-FIT INDEX  Q1KHAT  =  0.140360D+02; THE CRITICAL CHI-SQUARED PERCENTILE =   11.10
```

```
THE CONFIDENCE LIMIT BASED ON THE  10 LARGEST ORDER STATISTICS WILL BE CONSIDERED.    (Q2KHAT)
A  1-TH DEGREE POLYNOMIAL WILL BE USED TO APPROXIMATE THE FINITE RANGE DISTRIBUTION NEAR G0.
     GTRIAL =  0.10000D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.65041D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.10100D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.62705D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.10200D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.61093D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.10300D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.60156D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.10400D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.59819D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.10500D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.59993D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.10600D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.60584D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.10800D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.62671D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.11000D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.65482D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.11200D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.68597D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.11400D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.71748D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.11600D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.74785D+01; THE CRITICAL PERCENTILE =   7.51510
     GTRIAL =  0.11800D+02; THE GOODNESS-OF-FIT INDEX  Q2KHAT  =  0.77633D+01; THE CRITICAL PERCENTILE =   7.51510
```

```
THE CONFIDENCE LIMIT BASED ON DERIVATIVES WILL BE CONSIDERED.      (O3KHAT)
ONLY THE K =  20 LARGEST ORDER STATISTICS WILL  BE USED.
A  1-TH DEGREE POLYNOMIAL WILL BE USED TO APPROXIMATE THE FINITE RANGE DISTRIBUTION NEAR G*.

GTRIAL =  0.10000D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.45146D+00; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.10100D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.15423D+00; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.10200D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.14801D-01; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.10300D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.19411D-01; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.10400D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.15173D+00; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.10500D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.39159D+00; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.10600D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.72324D+00; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.10800D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.15876D+01; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.11000D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.26208D+01; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.11200D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.37291D+01; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.11400D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.48474D+01; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.11600D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.59345D+01; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
GTRIAL =  0.11800D+02; THE GOODNESS-OF-FIT INDEX  O3KHAT  =  0.69664D+01; THE CRITICAL CHI-SQUARED PERCENTILE =   5.99
```

APPENDIX D

Transformations of the Initial Random Sample $g_1, \ldots, g_n$

The confidence limit procedures described in section 2 assume only that

(i) $g_1, \ldots, g_n$ are n independent observations on a random variable g,

(ii) the least upper bound on g is G^*, and

(iii) the distribution function, $P(g)$, of g is well approximated by a polynomial of degree at most m.

The only characteristic of $P(g)$ of real interest is G^*. Hence, if

$\tilde{g}_i$ = transformation of g_i,

$\tilde{g}_1, \ldots, \tilde{g}_n$ are independent,

G^* = least upper bound on $\tilde{g}$, and

$P(\tilde{g})$ = distribution function of $\tilde{g}$,

the sample $\tilde{g}_1, \ldots, \tilde{g}_n$ could also be analyzed by the confidence limit procedures and provide the desired information on G^*. In fact, if $\tilde{P}(g)$ is more benign than $P(g)$, the analysis of $\tilde{g}_1, \ldots, \tilde{g}_n$ may provide a better confidence limit on G^* than that based on $g_1, \ldots, g_n$.

At this point in time the most benign form for $\tilde{P}(g)$ is unknown. It is suspected that a helpful form for $\tilde{P}(g)$ is roughly linear for $g < G^*$ in a neighborhood of G^* with the slope of $\tilde{P}(g)$ near G^* being neither too near 0 nor too large.

Frequently when

$$G^* = \max_{x \in R} g(x) \quad ,$$

some "good" values for x with "large" values of g(x) are known or can be guessed apriori. Such x values (say, $x_1, \ldots, x_L$) and there associated g values (say $g_1^* = g(x_1), \ldots, g_L^* = g(x_L)$) are usually known or guessed from the physical characteristics of the problem. The collection $g_1^*, \ldots, g_L^*$ represents L nonrandom estimates of G^*. Since they are nonrandom, the confidence limit procedures should not be applied directly to $g_1^*, \ldots, g_L^*$. However, the $g_1^*, \ldots, g_L^*$ can be used to "upgrade" a random sample $g_1, \ldots, g_n$. The upgraded random sample could be generated by the following general transformation

$$\begin{aligned} \tilde{g}_i &= WWg_i + \sum_{j=1}^{L} WTg^*_{(j)} \text{ if } g_i < g^*_{(1)} \\ &= WWg_i + \sum_{j=h+1}^{L} WT_{j-h}\, g^*_{(j)} + g^*_{(L)} \sum_{j=L-h}^{L} WT_j \text{ if } g^*_{(h)} \leq g_i < g^*_{(h+1)} \\ &= g_i \text{ if } g_i \geq g^*_{(L)} \end{aligned}$$

where $g^*_{(1)} < \ldots < g^*_{(L)}$ are the ordered values of $g_1^*, \ldots, g_L^*$ and $WW, WT_1, \ldots, WT_L$ are nonnegative weights with

$$1 = WW + \sum_{j=1}^{L} WT_j \;.$$

Here $\tilde{g}_i$ is a weighted average of g_i and those g^*'s $> g_i$. The computer implementation of the confidence limit procedures allows the user to specify

$$WW = WT_1 = \ldots = WT_L = 1/(L+1)$$

or

$$WW = 1/[(L+2)(L+1)/2]$$

$$WT_i = (i+1)/[(L+2)(L+1)/2]$$

or read in any values for $WW > 0, WT_1, \ldots, WT_L$. At this time it is not apparent what "optimal" weights are.

Of course, it is entirely feasible to create the observations to be analyzed by first transforming $g_1, \ldots, g_n$ using L nonrandom estimates of G^* and then subtracting random deviates $u_1, \ldots, u_n$ from these transformed variables.

As discussed in section 3 an alternative transformation of $g_1, \ldots, g_n$ is

$$\tilde{g}_i = g_i - u_i \,, \quad i = 1, \ldots, n$$

where $u_1, \ldots, u_n$ are independent nonnegative random variables with greatest lower bound zero. In particular u being a uniform random variable on $[0, U^*]$ seems like a reasonable choice since this tends to make $\tilde{P}(g)$ linear near G^*. Currently, the "optimal" magnitude of U^* relative to the range of g is unknown.

APPENDIX E

An Automated Procedure for Determining the Degree of the Approximating Polynomial Distribution Function

Sometimes an investigation of the degree of the approximating polynomial distribution function is of interest. If

$$F(g;\alpha,G^*) = \sum_{j=1}^{m} \alpha_j (g - G^*)^j + 1$$

is rewritten as

$$F(g;\alpha,G^*) = \sum_{j=0}^{m} \beta_j g^j ,$$

then the smallest integer $m \geq 2$, say m^*, such that the estimate of α_m is not significantly different from zero can be roughly determined. This investigation takes the same form for the procedures based on Q_{2k} and Q_{3k} but has a different form for the procedure based on Q_{1k}. These two forms are discussed below. If the user of the computer implementation so requests, the value of m^* for any or all of the confidence limit procedures will be determined. The user may also specify whether or not m is to be set equal to $m^* - 1$ in each of the confidence limit procedures.

For the confidence limit procedure based on Q_{1k} the determination of m^* is as follows: As described in section 2.1, determine the sample proportions $p_1, \ldots, p_k$ where

$$np_i = \text{\# of } g_1, \ldots, g_n \leq G_i, \quad i = 1, \ldots, k.$$

Then the vector $(p_1, \ldots, p_k)$ has approximately a multivariate normal distribution with mean vector $\left(\sum_{j=0}^{m} \beta_j G_1^{\,j}, \ldots, \sum_{j=0}^{m} \beta_j G_k^{\,j}\right)$

and symmetric covariance matrix $V_1 = (v_{1ij})$ with

$$v_{1ij} = \left(\sum_{h=0}^{m} \beta_h G_i^{\,h}\right)\left(1 - \sum_{h=0}^{m} \beta_h G_j^{\,h}\right)/n \text{ for } i \le j \,.$$

Thus, the weighted least squares estimate of $\beta = (\beta_1, \ldots, \beta_m)^T$ is

$$\hat{\beta} = (X^T\hat{V}_1^{-1}X)^{-1} \; X^T\hat{V}_1^{-1}\, Y$$

which has approximately a multivariate normal distribution with mean vector β and symmetric covariance matrix $X^T V_1^{-1} X$

where

$$\hat{\beta} = (\hat{\beta}_1, \ldots, \hat{\beta}_m)^T \,,$$

$X = (x_{ij})$ with $x_{ij} = G_i^{\,j}$ for $i = 1, \ldots, k$ and $j = 0, \ldots, m$,

$\hat{V}_1 = (\hat{v}_{1ij})$ with $\hat{v}_{1ij} = p_i(1-p_j)/n$ for $i = 1, \ldots, k$ and $j = 1, \ldots, k$, and

$Y = (p_1, \ldots, p_k)^T$.

Furthermore, if β_m is 0, then $\beta_m / \left[(X^T\hat{V}_1^{-1}X)^{-1}_{m,m}\right]^{\frac{1}{2}}$ has approximately a t distribution with k-(m + 1) degrees of freedom. Since $\beta_m = \alpha_m$, the value of α_m is assumed to be zero if

$$\left|\beta_m / \left[(X^T\hat{V}_1^{-1}X)^{-1}_{m,m}\right]^{\frac{1}{2}}\right| \le t_{k-m-1,.95}$$

where $t_{k-m-1,.95}$ is the 95-th percentile of a t distribution with k-(m+1) degrees of freedom. This is a test of the hypothesis $\alpha_m = 0$ with a significance level of approximately .10. The value of m^* is the smallest integer $m \geq 2$ for which α_m can be assumed to be zero on the basis of the above t test.

For the confidence limit procedures based on either Q_{2k} or Q_{3k} the determination of m^* is as follows: If

$$g_{(n-k+1)} < \cdots < g_{(n)}$$

are the k largest order statistics from

$$F(g) = \sum_{j=0}^{m} \beta_j \, g^j ,$$

then

$$z_i = F(g_{(i)}) \quad , \quad i = n - k + 1, \ldots, n$$

are distributed like the k largest order statistics from a uniform distribution. Analogously to section 2.2,

$$Q_{2k,m}^{(\beta)} = \sum_{i=n-k+1}^{n} \sum_{j=n-k+1}^{n} \left[\left(\sum_{h=0}^{m} \beta_h g_{(i)}^h\right) - (i/n+1)\right]\left[\left(\sum_{h=0}^{m} \beta_h g_{(j)}^h\right) - j/n+1\right] v_{2ij}^{(-1)}$$

is minimized with respect to β. Letting $Q_{2k,m}^{(\hat{\beta})}$ represent this minimum, the value of m^*-1 is taken to the smallest integer in such that $Q_{2k,m}^{(\hat{\beta})}$ is less than the 90-th percentile of $\hat{Q}_{2k}$ for m = 1 given in Appendix F (tables F.4 - F.9).

The computer implementation considers only $m \leq 5$. If m^* seems to be ≥ 6, the value of m^* is set equal to 6. This corresponds to a maximum allowable value of m in the computer implementation being m = 5.

APPENDIX F

Percentiles of Q_{2k} and $\hat{Q}_{2k}$

Table F.1 95-th percentile of the goodness-of-fit criterion Q_{2k} based on a Monte Carlo simulation involving 10,000 samples.*

n	k = 5	10	15	20	25	30	35	40	45	50
10	13.46									
15	14.42	21.85								
20	14.87	23.12	29.38							
25	15.14	23.88	30.72	36.52						
30	15.31	24.37	31.60	37.84	43.41					
35	15.43	24.74	32.22	38.78	44.68					
40	15.52	25.00	32.70	39.47	45.64	51.34				
45	15.60	25.20	33.05	40.01	46.39	52.31	57.85			
50	15.65	25.35	33.34	40.43	46.97	53.06	58.82			
60	15.72	25.61	33.76	41.07	47.82	54.18	60.20	65.99		
70	15.78	25.77	34.07	41.51	48.43	54.97	61.19	67.17	72.92	
80	15.83	25.89	34.29	41.85	48.88	55.55	61.91	68.05	73.96	79.76
90	15.86	26.00	34.46	42.10	49.22	56.00	62.48	68.73	74.79	80.65
100	15.88	26.07	34.59	42.31	49.51	56.36	62.91	69.26	75.40	81.42

*Information taken from Lurie (1971).

Table F.2 99-th percentiles of the goodness-of-fit criterion Q_{2k} based on a Monte Carlo simulation involving 10,000 samples.*

n	k = 5	10	15	20	25	30	35	40	45	50
10	21.47									
15	24.21	31.99								
20	25.69	34.89	41.20							
25	26.60	36.72	43.94	49.70						
30	27.23	37.99	45.83	52.26	57.75					
35	27.66	38.90	47.22	54.12	60.12					
40	28.00	39.59	48.27	55.54	61.95	67.88				
45	28.27	40.14	49.10	56.67	63.38	69.44	75.01			
50	28.48	40.59	49.77	57.57	64.53	70.86	76.70			
60	28.81	41.25	50.77	58.95	66.28	73.02	79.26	85.17		
70	29.04	41.73	51.52	59.92	67.54	74.56	81.12	87.32	93.21	
80	29.21	42.09	52.06	60.69	68.49	75.71	82.50	88.94	95.11	101.00
90	29.35	42.36	52.49	61.27	69.23	76.61	83.60	90.20	96.58	102.69
100	29.46	42.60	52.84	61.74	69.83	77.36	84.45	91.24	97.71	103.98

*Information taken from Lurie (1971).

Table F.3 Upper percentage points of Q_{2n} computed from (a) a Pearson Type V approximation, (b) The exact recurrence formula using numerical integration and (c) Monte Carlo based on 10,000 samples *

	10%			5%			1%		
n	(*a*)	(*b*)	(*c*)	(*a*)	(*b*)	(*c*)	(*a*)	(*b*)	(*c*)
3	5·75	5·92	—	6·98	7·39	—	9·89	10·19	—
4	7·40	7·50	—	8·98	9·27	—	12·78	13·19	—
5	9·00	9·08	—	10·89	11·14	—	15·51	16·02	—
6	10·56	10·61	—	12·74	12·96	—	18·09	18·69	—
7	12·08	12·13	—	14·52	14·71	—	20·55	21·11	—
8	13·57	13·59	—	16·26	16·44	—	22·91	23·33	—
9	15·03	15·05	—	17·95	18·11	—	25·18	25·69	—
10	16·47	16·48	16·46	19·61	19·75	19·83	27·37	27·79	27·36
11	17·90	17·90	—	21·23	21·35	—	29·50	29·88	—
12	19·30	19·32	—	22·83	22·94	—	31·56	31·89	—
13	20·70	20·69	—	24·41	24·48	—	33·58	33·86	—
14	22·08	22·09	—	25·96	26·05	—	35·54	35·73	—
15	23·44	23·47	22·96	27·49	27·56	26·93	37·46	37·53	37·00
16	24·80	24·83	—	29·00	29·02	—	39·35	39·22	—
17	26·14	—	—	30·50	—	—	41·19	—	—
18	27·48	—	—	31·98	—	—	43·01	—	—
19	28·80	—	—	33·44	—	—	44·80	—	—

	10%		5%		1%	
n	(*a*)	(*c*)	(*a*)	(*c*)	(*a*)	(*c*)
20	30·12	29·77	34·90	34·50	46·56	45·98
21	31·43	—	36·34	—	48·30	—
22	32·74	—	37·77	—	50·01	—
23	34·03	—	39·19	—	51·70	—
24	35·32	—	40·60	—	53·38	—
25	36·61	—	42·00	—	55·03	—
26	37·89	—	43·39	—	56·66	—
27	39·16	—	44·77	—	58·28	—
28	40·43	—	46·15	—	59·89	—
29	41·70	—	47·52	—	61·48	—
30	42·96	42·77	48·88	48·72	63·05	62·64
35	49·20	—	55·59	—	70·75	—
40	55·35	55·19	62·16	61·74	78·20	79·55
45	61·43	—	68·63	—	85·44	—
50	67·45	67·21	75·00	75·01	92·53	93·60
55	73·42	—	81·30	—	99·48	—
60	79·34	—	87·53	—	106·31	—
65	85·22	—	93·71	—	113·05	—
70	91·07	—	99·83	—	119·70	—
75	96·88	96·73	105·90	105·68	126·28	125·61
80	102·67	—	111·94	—	132·70	—
85	108·43	—	117·94	—	139·24	—
90	114·16	—	123·91	—	145·64	—
95	119·88	—	129·84	—	152·00	—
100	125·57	125·97	135·75	135·15	158·31	157·59
500	558·42	—	578·11	—	618·11	—

* Information taken from Hartley and Pfaffenberger (1972).

Table F.4 Observed percentiles of $\hat{Q}_{2k}$ for m = 1 and k = 5 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10	0.7950	1.0949	2.3042	3.3690	3.9015	4.5410	5.3294
15	0.6751	0.9775	2.1596	3.0425	3.5542	3.9302	4.4962
20	0.6877	0.9782	2.0696	2.9158	3.3288	3.6932	4.2915
25	0.6893	0.9596	1.9700	2.7530	3.2200	3.5032	4.0544
30	0.7273	0.9544	1.9289	2.7361	3.1056	3.3973	3.9597
35	0.6550	0.9029	1.9346	2.6579	3.1028	3.4212	3.9295
40	0.7358	0.9684	1.9094	2.6982	3.0836	3.4030	3.8690
45	0.6800	0.8878	1.8490	2.6046	3.0072	3.2672	3.8270
50	0.7206	0.9589	1.9374	2.6762	3.0377	3.2993	3.7678
60	0.6842	0.9035	1.8724	2.5782	2.9308	3.2392	3.6676
70	0.6255	0.8787	1.8664	2.5900	2.9160	3.1548	3.6470
80	0.6601	0.9105	1.8391	2.4997	2.8721	3.1398	3.5502
90	0.7128	0.9529	1.8747	2.6185	2.9622	3.2496	3.5794
100	0.6180	0.8571	1.8293	2.5376	2.8690	3.1759	3.6190

Table F.5 Observed percentiles of $\hat{Q}_{2k}$ for m = 1 and k = 10 based on a Monte Carlo simulation involving 1500 samples

Percentile	5	10	50	80	90	95	99
n							
10	3.3711	4.0552	7.3316	10.6786	12.8215	15.1200	20.8051
15	2.8167	3.4484	5.8781	8.1584	9.3515	10.6328	13.1150
20	2.8702	3.3154	5.4976	7.3223	8.3734	9.2977	11.2154
25	2.7925	3.1467	5.2457	6.7980	7.7112	8.5010	9.6934
30	2.5955	3.0817	5.0570	6.5695	7.3799	7.9939	9.4950
35	2.6555	3.1218	4.9751	6.2796	7.1655	7.8611	8.7823
40	2.6070	3.0222	4.7909	6.1524	6.9258	7.5151	8.8534
45	2.6782	3.0145	4.7416	6.0532	6.8252	7.3820	8.4659
50	2.4862	2.9308	4.6528	5.9321	6.6772	7.2681	8.2132
60	2.5016	2.9023	4.5525	5.8266	6.5429	7.0334	7.9444
70	2.5207	2.9203	4.4808	5.6674	6.3457	6.8675	7.6635
80	2.5231	2.9628	4.5102	5.6608	6.3043	6.7866	7.7215
90	2.4011	2.8398	4.4074	5.6483	6.3077	6.7621	7.6957
100	2.4607	2.8555	4.4198	5.6185	6.2526	6.8083	7.6995

Table F.6 Observed percentiles of $\hat{Q}_{2k}$ for m = 1 and k = 15 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10							
15	6.0914	7.1359	12.2815	16.6570	20.2154	23.8590	33.7135
20	5.6776	6.4080	9.9970	13.2789	15.5312	17.4707	21.1083
25	5.3608	6.1082	9.3787	11.9973	13.5924	15.0445	17.7235
30	5.2068	5.9049	8.7984	11.0806	12.3676	13.4961	15.2003
35	4.8240	5.5523	8.3797	10.4859	11.6225	12.6532	14.6437
40	4.7368	5.4552	8.0681	10.0152	11.1980	12.3028	14.1008
45	4.6838	5.4340	7.9397	9.8499	10.9455	11.9427	13.4631
50	4.7923	5.3627	7.8080	9.6826	10.6161	11.5213	12.8710
60	4.7227	5.2296	7.5711	9.2368	10.1833	11.2696	12.6117
70	4.5246	5.0369	7.5225	9.1451	10.1479	10.8993	12.3907
80	4.5285	5.1607	7.2436	8.9037	9.8969	10.5539	12.1855
90	4.5712	5.1717	7.2649	8.8319	9.6632	10.3896	11.8508
100	4.5097	5.0009	7.2393	8.7609	9.5403	10.2972	11.6714

Table F.7 Observed percentiles of $\hat{Q}_{2k}$ for m = 1 and k = 20 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10							
15							
20	9.2579	10.5217	16.5988	22.7815	26.9844	30.6751	38.3473
25	8.5401	9.5732	14.4469	19.0561	21.8008	24.3088	29.6780
30	8.0960	8.9815	13.4853	17.0045	19.0373	21.2573	24.8389
35	7.8583	8.7390	12.5412	15.8006	17.7042	19.5008	22.8097
40	7.6267	8.5997	12.0263	14.8637	16.5321	17.6523	20.5317
45	7.4320	8.3000	11.7349	14.3124	15.8265	17.0063	19.8576
50	7.2564	7.9921	11.3406	13.7053	15.1775	16.5539	18.8688
60	7.0809	7.8149	10.8401	13.2074	14.5522	15.4355	18.5607
70	6.9626	7.8092	10.5761	12.7222	13.8563	14.9032	16.8736
80	6.8612	7.6857	10.3791	12.3866	13.4785	14.3257	16.4240
90	6.7622	7.5427	10.1685	12.3459	13.3713	14.4311	16.4125
100	6.9108	7.5716	10.2337	12.1787	13.1416	14.2235	16.1448

Table F.8 Observed percentiles of $\hat{Q}_{2k}$ for m = 1 and k = 25 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10							
15							
20							
25	13.0864	14.5874	22.0860	29.4558	33.8175	38.7396	53.8266
30	11.6170	13.0592	19.0359	24.5862	27.9174	30.9565	38.9233
35	11.2338	12.4910	17.7130	22.1414	24.4157	27.2240	31.7071
40	10.7206	11.8601	16.6420	20.5820	22.9580	25.3261	30.1502
45	10.4122	11.7226	15.8099	19.2329	21.2110	23.1475	26.9785
50	10.1611	11.0480	15.2611	18.3234	20.4047	21.9522	24.9177
60	9.9086	10.7442	14.6107	17.7812	19.2960	20.9158	23.2428
70	9.3952	10.4303	14.0350	16.7457	18.1994	19.6407	22.7552
80	9.3390	10.1804	13.6569	16.2853	17.8386	19.1040	22.1676
90	9.1780	9.9792	13.4492	15.8638	17.2472	18.4011	20.6108
100	8.8945	9.9902	13.1934	15.6186	16.8663	18.0363	20.4707

Table F.9 Observed percentiles of $\hat{Q}_{2k}$ for m = 1 and k = 30 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10							
15							
20							
25							
30	15.9556	17.8899	26.3351	34.5368	40.0028	45.4640	59.5800
35	15.0606	16.6132	23.7549	29.7509	33.5835	37.1985	44.5941
40	14.6048	16.1073	21.9367	27.1967	29.9356	32.6829	39.0256
45	13.8515	15.1399	20.7218	25.0228	27.8213	30.6415	36.2437
50	13.1543	14.4307	19.7203	24.0512	26.4301	28.8160	33.0465
60	12.6673	13.8159	18.4865	22.1482	24.2446	26.3576	29.1589
70	12.3299	13.5556	17.7348	21.0749	22.7931	24.4371	28.0009
80	11.9518	12.9899	17.1229	20.1220	21.9159	23.5572	26.7444
90	12.0188	12.8475	16.7569	19.7140	21.3977	22.7690	25.3704
100	11.6842	12.5434	16.4005	19.1245	20.5820	22.0335	25.0725

Table F.10 Observed percentiles of $\hat{Q}_{2k}$ for m = 2 and k = 5 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10	0.4613	0.6782	1.7502	2.7411	3.3214	3.8755	4.6372
15	0.4501	0.6765	1.6995	2.4604	2.9661	3.3802	4.0154
20	0.4174	0.6645	1.6416	2.4443	2.8460	3.1589	3.7249
25	0.4632	0.6502	1.6027	2.3419	2.7016	3.0544	3.7034
30	0.4344	0.6789	1.6016	2.2395	2.5904	2.8859	3.4380
35	0.3967	0.6054	1.5620	2.2351	2.6275	2.9467	3.5036
40	0.4358	0.6133	1.5123	2.2375	2.5901	2.8693	3.5123
45	0.4004	0.6090	1.5335	2.1455	2.5141	2.8219	3.2461
50	0.3893	0.6231	1.5539	2.2010	2.5994	2.8621	3.3066
60	0.4146	0.6190	1.5171	2.1555	2.5063	2.7982	3.3952
70	0.4869	0.6595	1.5227	2.1448	2.5051	2.7531	3.1990
80	0.3795	0.6008	1.4664	2.0944	2.4239	2.7459	3.2971
90	0.4492	0.6889	1.4855	2.0973	2.4535	2.7124	3.3451
100	0.4193	0.6387	1.4665	2.1283	2.4971	2.7880	3.1881

Table F.11 Observed percentiles of $\hat{Q}_{2k}$ for m = 2 and k = 10 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10	2.7735	3.4664	6.3194	9.0145	11.0287	12.5317	16.6531
15	2.4236	2.9753	5.1627	7.1074	8.4276	9.4173	11.2169
20	2.4617	2.8621	4.8016	6.4501	7.3404	8.0265	9.4346
25	2.2138	2.6814	4.5273	5.9655	6.7507	7.5323	8.8709
30	2.2791	2.6962	4.4365	5.7928	6.5952	7.2239	8.7974
35	2.2573	2.6721	4.2996	5.6440	6.3629	6.9724	7.9919
40	2.2259	2.6656	4.2738	5.5084	6.2222	6.7973	7.9441
45	2.1947	2.5782	4.2543	5.4622	6.1280	6.8178	7.8115
50	2.1860	2.5545	4.1348	5.3299	6.0032	6.5015	7.6552
60	2.1459	2.5384	4.0685	5.2358	5.7932	6.3799	7.2054
70	2.0723	2.4636	4.0438	5.1125	5.7523	6.2126	6.9384
80	2.0873	2.4621	3.9945	5.1090	5.7458	6.2483	7.2358
90	2.1828	2.5129	3.9601	5.0589	5.6437	6.1858	7.0296
100	2.1810	2.5089	3.9695	5.0775	5.5919	6.0206	6.7823

Table F.12 Observed percentiles of $\hat{Q}_{2k}$ for m = 2 and k = 15 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10							
15	5.4998	6.4797	10.5848	14.2243	16.9938	20.2635	26.6316
20	5.1906	5.8855	9.2666	12.1887	14.1269	15.7781	19.5845
25	4.7518	5.4342	8.3780	10.7441	12.1953	13.5227	15.9983
30	4.3919	5.0953	7.8047	9.9288	11.1551	12.2020	14.3209
35	4.4576	4.9903	7.5455	9.4857	10.5866	11.4383	13.8742
40	4.3089	5.0156	7.3988	9.2471	10.4213	11.2213	12.9824
45	4.2903	4.8581	7.2590	8.9729	10.0068	10.9088	12.7422
50	4.4317	4.9606	7.1508	8.9278	9.9109	10.7047	12.5022
60	4.1014	4.6746	6.9142	8.4749	9.4833	10.3124	11.7445
70	4.1753	4.6893	6.7713	8.4153	9.2361	10.0144	11.4842
80	4.1534	4.6429	6.7791	8.3617	9.1988	9.7937	11.1319
90	4.1527	4.6438	6.7590	8.1571	8.9440	9.6718	10.7496
100	4.0700	4.6384	6.5378	8.0925	8.8913	9.6060	10.8851

Table F.13 Observed percentiles of $\hat{Q}_{2k}$ for m = 2 and k = 20 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10							
15							
20	8.5894	9.7797	14.9154	20.1513	23.5669	25.8466	33.6911
25	7.9831	8.9248	13.2781	16.9533	19.1330	21.4918	26.6315
30	7.5345	8.4257	12.1439	15.3297	17.1796	18.8577	22.9272
35	7.3227	8.2013	11.7392	14.5209	16.1844	17.8072	21.3554
40	7.0572	7.8364	11.1554	13.6066	15.0583	16.4068	19.1958
45	6.6492	7.5481	10.7026	13.1403	14.4691	15.5158	18.1505
50	6.7204	7.4707	10.5959	12.7150	14.0975	15.1015	17.0252
60	6.6992	7.2944	10.1405	12.2380	13.5588	14.3838	16.4226
70	6.4482	7.1605	10.0402	12.0705	13.1646	14.0537	16.1698
80	6.3564	7.0138	9.6327	11.5153	12.5613	13.4435	15.2592
90	6.2466	6.8562	9.5335	11.4035	12.4791	13.1713	14.8876
100	6.4103	6.9463	9.3183	11.3160	12.3835	13.3123	14.4369

Table F.14 Observed percentiles of $\hat{Q}_{2k}$ for m = 2 and k = 25 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10							
15							
20							
25	11.6522	13.1403	19.6254	25.5485	29.0540	33.0684	41.8129
30	11.2810	12.2345	17.6469	22.3011	24.6344	27.5356	33.3442
35	10.3227	11.4571	16.2680	20.0283	22.4879	24.8890	30.7655
40	9.9597	10.9959	15.4268	18.9114	20.9758	23.0225	25.9440
45	9.7321	10.6639	14.6991	18.1540	20.0668	21.5444	25.6196
50	9.4702	10.4074	14.1786	17.4987	19.3772	20.7472	23.5817
60	9.1478	10.0422	13.6210	16.3821	17.9238	19.2319	21.5942
70	8.8462	9.8860	13.1055	15.5060	16.9985	18.1400	20.2148
80	8.8309	9.6249	12.7348	15.2548	16.4483	17.5788	20.3181
90	8.6677	9.4741	12.6442	14.9047	16.2174	17.1329	18.9588
100	8.6958	9.5213	12.5288	14.7986	15.9483	17.0049	18.8317

Table F.15 Observed percentiles of $\hat{Q}_{2k}$ for m = 2 and k = 30 based on a Monte Carlo simulation involving 1500 samples.

Percentile	5	10	50	80	90	95	99
n							
10							
15							
20							
25							
30	15.0876	16.9618	24.5298	30.7084	34.6084	39.1290	48.3428
35	14.4291	15.9360	22.1135	27.8407	31.6929	35.4093	42.3229
40	13.3173	14.6457	20.0889	24.4914	27.5545	30.1500	35.3132
45	13.3775	14.4755	19.5084	23.7013	26.4497	28.5206	32.8791
50	12.2556	13.3886	18.7108	22.5190	24.8608	26.6890	30.5328
60	12.1600	13.1408	17.6514	20.8903	23.0409	24.8029	27.5976
70	11.9319	12.9337	16.8397	19.8259	21.5450	22.9388	25.5781
80	11.2707	12.3369	16.2566	18.9258	20.4412	21.8323	25.2829
90	11.1630	12.0334	15.7477	18.7200	20.5235	21.9124	24.9271
100	11.2597	12.1157	15.6233	18.2995	19.8963	21.1062	23.8996

APPENDIX G

XNORM Values

Confidence level	XNORM value
99.9%	3.09
99.5%	2.58
99.0%	2.33
98.0%	2.05
97.0%	1.88
96.0%	1.75
95.0%	1.64
90.0%	1.28
85.0%	1.04
80.0%	.84
70.0%	.52
60.0%	.25
50.0%	.00

APPENDIX H

Percentiles of Chi-Squared Distributions

*D.F	$\chi^2_{.005}$	$\chi^2_{.01}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.10}$	$\chi^2_{.20}$	$\chi^2_{.30}$	$\chi^2_{.50}$	$\chi^2_{.70}$	$\chi^2_{.80}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$
1	.000	.000	.001	.004	.016	.064	.148	.455	1.07	1.64	2.71	3.84	5.02	6.63	7.88
2	.010	.020	.051	.103	.211	.446	.713	1.39	2.41	3.22	4.61	5.99	7.38	9.21	10.6
3	.072	.115	.216	.352	.584	1.00	1.42	2.37	3.66	4.64	6.25	7.81	9.35	11.3	12.8
4	.207	.297	.484	.711	1.06	1.65	2.20	3.36	4.88	5.99	7.78	9.49	11.1	13.3	14.9
5	.412	.554	.831	1.15	1.61	2.34	3.00	4.35	6.06	7.29	9.24	11.1	12.8	15.1	16.7
6	.676	.872	1.24	1.64	2.20	3.07	3.83	5.35	7.23	8.56	10.6	12.6	14.4	16.8	18.5
7	.989	1.24	1.69	2.17	2.83	3.82	4.67	6.35	8.38	9.80	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	4.59	5.53	7.34	9.52	11.0	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.38	6.39	8.34	10.7	12.2	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.18	7.27	9.34	11.8	13.4	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	6.99	8.15	10.3	12.9	14.6	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	7.81	9.03	11.3	14.0	15.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	8.63	9.93	12.3	15.1	17.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	9.47	10.8	13.3	16.2	18.2	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	10.3	11.7	14.3	17.3	19.3	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.2	12.6	15.3	18.4	20.5	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.0	13.5	16.3	19.5	21.6	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	12.9	14.4	17.3	20.6	22.8	26.0	28.9	31.5	34.8	37.2
19	6.83	7.63	8.91	10.1	11.7	13.7	15.4	18.3	21.7	23.9	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	14.6	16.3	19.3	22.8	25.0	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	15.4	17.2	20.3	23.9	26.2	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	16.3	18.1	21.3	24.9	27.3	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	17.2	19.0	22.3	26.0	28.4	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	18.1	19.9	23.3	27.1	29.6	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	18.9	20.9	24.3	28.2	30.7	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	19.8	21.8	25.3	29.2	31.8	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	20.7	22.7	26.3	30.3	32.9	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	21.6	23.6	27.3	31.4	34.0	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	22.5	24.6	28.3	32.5	35.1	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	23.4	25.5	29.3	33.5	36.2	40.3	43.8	47.0	50.9	53.7
40	20.7	22.1	24.4	26.5	29.0	32.3	34.9	39.3	44.2	47.3	51.8	55.8	59.3	63.7	66.8
50	28.0	29.7	32.3	34.8	37.7	41.4	44.3	49.3	54.7	58.2	63.2	67.5	71.4	76.2	79.5
60	35.5	37.5	40.5	43.2	46.5	50.6	53.8	59.3	65.2	69.0	74.4	79.1	83.3	88.4	92.0

*D.F. = Degrees of Freedom

References

Boender, C. G. E., Rinnooy Kan, A. H. G., Stougie, L. and Timmer, G. T. (1980). "A stochastic method for global optimization." Technical Report 8001/0, Econometric Institute, Erasmus University, Rotterdam.

Clough, D. J. (1969). "An asymptotic extreme value sampling theory for the estimation of a global maximum." Candian Operational Research Society Journal, 7, 102-115.

Golden, B. L. and Alt, F. B. (1979). "Interval estimation of a global optimum for large combinatorial problems." Naval Research Logistics Quarterly, 26, 69-77.

Graves, G. W. and Whinston, A. B. (1968). "A new approach to discrete mathematical programming." Management Science, 15, 117-190.

Graves, G. W. and Whinston, A. B. (1970). "An algorithm for the quadratic assignment problem." Management Science, 17, 453-471.

De Haan, L. (1979). "Estimation of the minimum of a function using order statistics." Technical Report, Erasmus University, Rotterdam.

Hartley, H. O. and Pfaffenberger, R. C. (1969). "Statistical control for non-linear optimization." Themis Technical Report No. 18, Institute of Statistics, Texas A&M University.

Hartley, H. O. and Pfaffenberger, R. C. (1971). "Statistical control of optimization." Optimizing Methods in Statistics. Academic Press, New York.

Hartley, H. O. and Pfaffenberger, R. C. (1972). "Quadratic forms in order statistics as goodness-of-fit criteria." Biometrika, 59, 605-611.

Kendall, M. G. and Stuart, A. (1960). The Advanced Theory of Statistics. Hafner Publishing Company, New York.

Kshirsagar, A. M. (1972). Multivariate Analysis. Marcel Dekker, New York.

Liau, T., Hartley, H. O., and Sielken, R. L. Jr. (1973). "Confidence limits for global optima." Themis Technical Report No. 43, Institute of Statistics, Texas A&M University.

Lurie, D. (1971). A Goodness of Fit Test Based on the Spacing of Selected Order Statistics. Ph.D. Dissertation, Institute of Statistics, Texas A&M University.

Robson, D. and Whitlock, J. (1964). "Estimation of a truncation point." Biometrika, 51, 33-39.

Watson, G. S. (1959). "Some recent results in chi-square goodness-of-fit tests." Biometrics, 15.

Wolfe, P. (1959). "The simplex method for quadratic programming." Econometrika, 29, 382-398.

6 April 1979

BASIC DISTRIBUTION LIST FOR
UNCLASSIFIED TECHNICAL REPORTS
OPERATIONS RESEARCH PROGRAM
ONR CODE 434

Operations Reserach Program (Code 434) (3)
Office of Naval Research
Arlington, VA 22217

Defense Technical Information Center (12)
Cameron Station
Alexandria, VA 22314

Defense Logistics Studies (1)
Information Exchange
Army Logistics Management Center
Fort Lee, VA 23801

Office of Naval Research Branch Office (1)
New York Area Office
715 Broadway - 5th Floor
New York, NY 10003

Office of Naval Research Branch Office (1)
Bldg. 114, Section D
666 Summer Street
Boston, MA 02210

Office of Naval Research Branch Office (1)
1030 East Green Street
Pasadena, CA 91106

(1)

Office of Naval Research Branch Office (1)
536 South Clark Street
Chicago, IL 60605

Professor Martin Shubik (1)
Department of Economics
Yale University
New Haven, CT 06520

Professor Abraham Charnes (1)
Department of Mathematics
The University of Texas
Austin, TX 78712

Professor Gerald L. Thompson (1)
Graduate School of Industrial Adm.
Carnegie-Mellon University
Pittsburgh, PA 15213

Professor George B. Dantzig (1)
Department of Operations Research
Stanford University
Stanford, CA 94305

Professor Ronald W. Shephard (1)
Operations Research Center
University of California
Berkeley, CA 94720

Mr. Harvey Paige (1)
Maritime Transportation Research Board
National Academy of Sciences
Washington, D.C. 20418

Professor William F. Lucas (1)
Department of Operations Research
Cornell University
Ithaca, NY 14850

Professor Arthur M. Geoffrion (1)
Graduate School of Business Adm.
University of California
Los Angeles, CA 90024

Dr. Richard Hatch (1)
Decision Systems Associates, Inc.
350 Fortune Terrace, 2nd Floor
Rockville, MD 20854

Professor H. Donald Ratliff (1)
School of Industrial &
Systems Engineering
Georgia Institute of Technology
Atlanta, GA 30332

Professor Robert M. Stark (1)
Department of Statistics &
Computer Sciences
University of Delaware
Newark, DE 19711

Professor George S. Fishman (1)
Curriculum in Operations Research
& Systems Analysis
University of North Carolina
Chapel Hill, NC 27514

Prof. Harvey M. Wagner (1)
School of Business Administration
University of North Carolina
Chapel Hill, NC 27514

The George Washington University (1)
Logistics Research Project
707 22nd Street, N.W.
Washington, D.C. 20037

Professor Averill M. Law (1)
Department of Industrial Engineering
University of Wisconsin
Madison, WI 53706

Professor Marshall Fisher (1)
Decision Sciences Department
The Wharton School
University of Pennsylvania
Philadelphia, PA 19174

Dr. George E. Pugh (1)
Decision-Science Applications, Inc.
1500 Wilson Blvd.
Arlington, VA 22209

Professor Harvey M. Salkin (1)
Department of Operations Research
Case Western Reserve University
Cleveland, OH 44106

Professor Darwin Klingman (1)
Department of Operations Research
& Computer Sciences
University of Texas
Austin, TX 78765

Professor Douglas Montgomery (1)
School of Industrial & Systems Eng.
Georgia Institute of Technology
Atlanta, GA 30332

Dr. R. L. Sielken (1)
Institute of Statistics
Texas A&M University
College Station, TX 77843

Dr. Joseph Augusta (1)
MATHTECH, Inc.
1401 Wilson Boulevard
Arlington, VA 22209

Professor Gerald J. Lieberman (1)
Department of Operations Research
Stanford University
Stanford, CA 94305

Professor Cyrus Derman (1)
Department of Civil Engineering
and Engineering Mechanics
Columbia University
New York, NY 10027

Professor K. T. Wallenius (1)
Department of Mathematical Sciences
Clemson University
Clemson, SC 29631

Professor M. L. Shooman (1)
Department of Electrical Engineering
Polytechnic Institute of New York
Brooklyn, NY 11201

Dr. Nancy Mann (1)
Rockwell International Corporation
Science Center
P.O. Box 1085
Thousand Oaks, CA 91360

Professor Wallace R. Blischke (1)
Dept. of Quantitative Business Analysis
University of Southern California
Los Angeles, CA 90007

Professor R. S. Leavenworth (1)
Department of Industrial
& Systems Engineering
University of Florida
Gainesville, FL 32611

Professor M. Zia Hassan (1)
Department of Industrial &
Systems Engineering
Illinois Institute of Technology
Chicago, IL 60616

Dr. Paul Arvis (1)
DRXMC-PRO
Army Logistics Management Center
Fort Lee, VA 23801

LCOL Daniel E. Strayer, USAF (1)
Executive Director
Air Force Business Research
Management Center/LAPB
Wright-Patterson AFB, OH 45433

Mr. L. G. LaMarca (1)
Program Director for Sea Control
Studies, Code 127
Naval Weapons Center
China Lake, CA 93555

Dr. Wilhelm Bortels (1)
Naval Underwater Systems Center (Code 214)
New London, CT 06320

Dr. Glen E. Hornbaker (1)
Armaments Development Department
Naval Surface Weapons Center
Dahlgren, VA 22448

Mr. Charles M. Merrow (1)
Operations Analysis Group (Code 121B)
Naval Ocean Systems Center
San Diego, CA 92152

Mr. Thomas E. Willey (1)
Chief of Planning, Systems Analysis
& Engineering Department
Naval Air Development Center
Warminster, PA 18974

Mr. Ted C. Buckley (1)
Analysis & Intelligence Office (Code 53C)
Naval Coastal Systems Laboratory
Panama City, FL 32401

Mr. Marshall J. Tino (1)
Ordnance Systems Assessment Division
Naval Surface Weapons Center
White Oak
Silver Spring, MD 20910

Mr. Russell Richards (1)
Naval Postgraduate School (Code 55)
Monterey, CA 93940

Naval Postgraduate School (1)
Department of Operations Research
Monterey, CA 93940

Naval Postgraduate School (1)
Library (Code 0212)
Monterey, CA 93940

Dr. A. L. Slafkosky (1)
Scientific Advisory
Commandant Marine Corps (Code AX)
Washington, D.C. 20380

Assistant Chief for Technology (1)
Office of Naval Research, Code 200
Arlington, VA 22217

Dr. Joseph Bram (1)
Directorate of Mathematical
& Information Sciences
Air Force Office of Scientific Research/N
Bolling Air Force Base
Washington, D.C. 20032

Applied Mathematics Laboratory (1)
Attn: Mr. Gene Gleissner
Naval Ship Research & Development Center
Washington, D.C. 20007

Army War College (1)
Attn: Library
Carlisle Barracks, PA 17013

Naval War College (1)
Attn: Library
Newport, RI 02840

Captain R. E. Helmes, Jr. (1)
Naval Operations (Code 964)
Pentagon 4A538
Washington, D.C. 20350

Dr. Paul Boggs (1)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709

Professor Stephen E. Jacobsen (1)
Department of System Science
School of Engineering &
Applied Science
University of California
Los Angeles, CA 90024

END