Use of a Subjective Prior Distribution for the Reliability of Computer Software*

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In the development of large-scale computer software and in the management of the development process, it is often useful to model the reliability and the cost of development of these software packages. There have been many papers that develop models and show their usefulness as management tools. The models that use Bayesian methodology assume that a prior distribution is given.

Our paper offers a methodology of assessing a prior distribution subjectively. Two computer programs have been developed for this particular purpose: One assesses a subjective prior distribution and the other suggests a family of probability functions.

The importance of consistent prior distributions is twofold. First, these distributions reflect consistent initial predictions because they are developed by a structured process. Second, these distributions are the starting point for applying Bayes' theorem to develop the posterior distribution by modifying the prior distribution with actual data available later.

INTRODUCTION

During the past decade, several probability distributions have been used in modeling the reliability of computer software. Among the models proposed are the exponential distribution, the Rayleigh distribution, and the Poisson distribution [1–5]. Recently, Bayesian methodology has been proposed [6–9]. To apply this method, a prior distribution is necessary.

In this paper, we offer a structured approach in

subjective assessment of the probability distribution. Once this has been done, the general shape of the distribution can be ascertained. Then, the search for the mathematical form is greatly simplified. For instance, the probability distribution may be skewed, not exist for negative values of the random variable. This would eliminate a class of probability models like the normal distribution and give rise to a host of others. Although it is still possible to select a model from many available basic models, the selection process is at least based upon some evidence, namely, the opinion of the experts in charge of developing the software package.

Two computer programs were written:

1. The first assesses a subjective prior distribution by eliciting answers to questions on a cathode ray tube (crt). The answers to these questions are used to plot the distribution function as well as the density function.
2. From the general shape of these functions, the second suggests a family of probability functions. For instance, an inverted gamma distribution, a beta distribution, or a lognormal distribution might by hypothesized. Some of the summary outputs of the first program become inputs for finding the parameters of the assumed distributions.

An example of a lognormal distribution is used, but other families of distributions could have been selected as well.

This paper does not deal explicitly with the derivation of the posterior distribution which is found via Bayes' theorem in conjunction with incoming data. The prior distribution, however, is an essential part of finding the posterior distribution. If the prior distribution found is integrated with test information as data become available, then obviously this is more complete information than test information alone.

PREVIOUS WORK IN PROBABILITY ASSESSMENT

More recently, decision theory has been considered as a general framework for logical analysis of a decision problem under uncertainty. As such, considerable attention has been given to problem formulation and methods for the assessment of a prior distribution. For example, Schlaifer's book [10], is largely devoted to the formulation and prior analysis of decision problems: posterior analysis is discussed only in the last part of the text. Howard and his associates (see, for example, [11,12]), have emphasized the application of decision theory to complex, dynamic, and uncertain decision problems. In dealing with these problems, they have explicitly included the problem formulation phase in the decision analysis cycle.

Decision theory, either concerned with specific models or general frameworks, treats uncertainty through subjective probability and treats attitude toward risk through utility theory. Regardless of whether the decision maker is concerned with prior or posterior analysis, the prior probability distribution, reflecting his quantified judgments about uncertainty, is an indispensable input to the analysis.

One difficulty associated with probability assessment is the assessor's inconsistencies which often occur in formulating a prior distribution. The question of how to discover and remove inconsistencies is of general interest to decision analysts. Another question of interest is how to fit a probability distribution using the assessed fractile in order to make the subsequent analysis more tractable. Both of these questions are addressed in this paper. The paper offers two computer programs. The first program allows a person to interact with the computer via a graphical device [cathode ray tube (crt)] during the course of establishing a subjective distribution. The second program fits a lognormal distribution to the subjective distribution.

During recent years, subjective probability has been studied by researchers in various disciplines such as psychology, mathematics, statistics, engineering, and business administration (as evidenced by the references at the end of the paper). While some of these studies are mainly theoretical or philosophical, others are experimental.

In their text [13], Pratt et al. present the method of equally likely subintervals. Subsequently, Raiffa [14] illustrates this method in detail by providing a dialogue between a decision analyst and his client. Schlaifer [10] advocates this method and offers a computer program for fitting a cumulative function through assessed fractiles.

For his experimental study, Winkler [15] developed a questionnaire using four assessment techniques:

1. cumulative distribution function—assessment of fractiles by means of equally likely subintervals or direct questions regarding fractiles,
2. hypothetical future samples,
3. equivalent prior sample information, and
4. probability density function.

He used this questionnaire to elicit prior distributions from 38 selected subjects involved in his study.

The use of penalty functions, or scoring methods, has been discussed by several researchers as means of encouraging honest assessments. Specifically, de Finetti [16] presents the quadratic scoring rule. Savage [17] derives the general class of strictly proper scoring rules by considering probabilities as special cases of rates of substitutions. Winkler discusses the use of scoring rules and other payoff schemes [18] and reports his experimental results [19].

Staël von Holstein and his associates [12,20] focus on the subject of eliciting the opinions of experts in practical situations rather than laboratory experiments. They discuss probability encoding in the context of decision analysis and propose the use of a probability wheel to facilitate the encoding process.

At the Reliability Conference in 1970, Lin and Schick [1] presented the use of an on-line computer system to assist a person in developing a prior distribution to represent his beliefs. Although the console-aided procedure is illustrated by a problem in the reliability field, this procedure is applicable to assessment of any prior distribution. Since then, considerable experience with this procedure has been gained from experiments involving students in several statistics and decision theory classes at the University of Southern California.

The present paper results from the authors' continued effort in making the probability assessment more practical by using modern electronic computers. This paper offers a newly designed computer program which has incorporated the experience gained from the use of the previous program. To simplify the assessment procedure, the new program

1. reduces the number of questions significantly (from 12 to 6),
2. is highly conversational and interactive,
3. checks for consistency as the user answers question by question,
4. uses graphical display rather than the typewriter terminal to help the user visualize the assessment process as well as to greatly increase the speed of drawing the assessed probability curves, and

5. plots not only the cumulative function but also the density function.

Once a subjective distribution has been determined, a second computer program will fit a lognormal distribution to the subjective distribution to make the subsequent analysis of maintainability problems more tractable mathematically.

METHOD OF ASSESSMENT

Several methods have been suggested for estimating prior distributions (see, for example [13,15,21]). Our computer program makes use of the method of equally likely subintervals, which perhaps is the most commonly used approach. The basic idea of this method is to ask the decision maker, at any stage, to divide a given interval into two judgmentally equally likely subintervals.

To begin with, the interval covering all possible values of an uncertain quantity (usually called a random variable) is split into two subintervals and the decision maker is asked to choose which subinterval to bet on. The dividing point is then changed until a point of indifference as to betting on one or the other subinterval is reached. When this point is reached, the decision maker feels that it is equally likely that the actual value of the uncertain quantity will fall above (to the right of) or below (to the left of) this point. The indifference point, which divides the entire interval into two subintervals with equal probabilities, is the median. Next, the decision maker is asked to specify a point that will further divide the subinterval to the left of the median into two equally likely parts. This new point is the first quartile. Similarly, the subinterval to the right of the median may be further divided into two equally likely parts. The decision maker may proceed in this manner to divide any given interval (generated previously) into two equally likely subintervals.

Suppose we let x_k designate the kth fractile of the uncertain quantity $\dot{x}$; i.e..

$$P(\dot{x} < x_k) = k, \qquad 0 \leq k \leq 1.$$

Then, using the method of equally likely subintervals, the decision maker is asked to respond to a series of questions that will lead to a determination of x_k values for such k as 0.5, 0.25, 0.75, etc.

COMPUTER PROGRAM

The program stores a set of questions for the method of equally likely subintervals. These questions are displayed successively on a crt: the user responds to

the questions by typing answers on a teletype. The response to each of the questions is processed immediately and checked for logical consistency.

Assuming you are the user of the program, the first question calls for the lower limit of the probability distribution by asking you to:

Specify the largest value such that you feel virtually certain that the actual value of the uncertain quantity will fall above this value.

The second question, on the other hand, calls for the upper limit of the distribution by asking you to:

Specify the smallest value such that you feel virtually certain that the actual value of the uncertain quantity will fall below this value.

In terms of the fractile notation described earlier, the first question asks for x_0 and the second question asks for x_1. The program will check to see if x_0 is less than x_1 and if you feel virtually certain that the actual value of the uncertain quantity will lie in between x_0 and x_1.

The third question asks you to divide the interval defined by the limits x_0 and x_1 into two equally likely subintervals. The question says:

Specify the value such that you feel it is equally likely that the actual value of the uncertain quantity will fall above or below this value.

The answer to this question yields $x_{0.5}$, which should lie in between x_0 and x_1.

The fourth question, which calls for $x_{0.25}$, is as follows:

Suppose you were told that actual value is less than $x_{0.5}$. Specify the value such that it is equally likely that the actual value of the uncertain quantity is either above or below this value.

The program will check to see if this answer lies in between x_0 and $x_{0.5}$.

The fifth question, which calls for $x_{0.75}$, is the following:

Suppose you were told the actual value is greater than $x_{0.5}$. Specify the value such that it is equally likely that the actual value of the uncertain quantity is either above or below this value.

This answer is checked to see if it lies in between $x_{0.5}$ and x_1.

At this point, the program further checks for consistency. Specifically, it asks:

Now, do you feel it is equally likely that the actual value

of the uncertain quantity will lie within the interval between $x_{0.25}$ and $x_{0.75}$ or outside of this interval?

If the check is not met, the program will direct you to review and revise each of your previous answers. Otherwise, the program will proceed to ask you to specify the most likely value (the mode).

The assessments thus obtained are summarized on the crt. The program then fits a smooth cumulative distribution function through the assessed fractiles. At your request, it will plot the cumulative curve and the corresponding density curve. If these graphs do not seem to reflect your judgments about the uncertain quantity, you will be guided by the program to revise your previous responses. Whenever you are satisfied with the assessed distribution, the mean and the standard deviation are computed. In addition, you may ask for 0.005, 0.015, 0.025, . . . , 0.995 fractiles of the distribution.

COMPUTER OUTPUT

To illustrate the computerized method of probability assessment discussed above, the computer output of an example is presented. In this example, the expert (italics) is asked to quantify judgments concerning the debugging hours for a particular job. As we can see from this output, the expert violates some of the probability axioms and is asked to revise his responses several times.

THIS PROGRAM IS DESIGNED TO ASSIST YOU IN (A) QUANTIFYING YOUR PROBABILITY JUDGMENTS CONCERNING AN UNCERTAIN QUANTITY. (B) CALCULATING THE MEAN AND STANDARD DEVIATION OF THE PROBABILITY DISTRIBUTION OBTAINED FROM THIS QUANTIFICATION, AND (C) FITTING THE ASSESSED DISTRIBUTION TO A THEORETICAL DISTRIBUTION.

WHAT IS THE UNCERTAIN QUANTITY OF YOUR CONCERN NOW?

NUMBER OF DEBUGGING HOURS
PLEASE RESPOND TO THE FOLLOWING QUESTIONS WITH YOUR CAREFUL JUDGMENTS:

(1) SPECIFY THE LARGEST VALUE SUCH THAT YOU FEEL VIRTUALLY CERTAIN THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL FALL ABOVE THIS VALUE.

650

(2) SPECIFY THE SMALLEST VALUE SUCH THAT YOU FEEL VIRTUALLY CERTAIN THAT THE

ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL FALL BELOW THIS VALUE.

100

SORRY, YOU HAVE MISINTERPRETED THESE TWO QUESTIONS. QUESTION (1) ASKS FOR THE LOWER LIMIT OF THE UNCERTAIN QUANTITY. PLEASE GIVE YOUR NEW ANSWER TO (1).

100

QUESTION (2) ASKS FOR THE UPPER LIMIT OF THE UNCERTAIN QUANTITY. PLEASE GIVE YOUR NEW ANSWER TO (2).

650

YOUR ANSWERS TO THESE TWO QUESTIONS IMPLY THAT YOU FEEL VIRTUALLY CERTAIN THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL LIE IN BETWEEN 100 AND 650. DO YOU AGREE?

NO

THEN YOU MUST REVISE YOUR ANSWER(S).
PLEASE TYPE YOUR NEW ANSWER TO (1).

50

PLEASE TYPE YOUR NEW ANSWER TO (2).

650

(3) SPECIFY THE VALUE SUCH THAT YOU FEEL IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL FALL ABOVE OR BELOW THIS VALUE.

250

(4) SUPPOSE YOU WERE TOLD THAT THE ACTUAL VALUE IS LESS THAN 250. SPECIFY THE VALUE SUCH THAT IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY IS EITHER ABOVE OR BELOW THIS VALUE.

310

I BELIEVE YOU DIDN'T ANSWER THIS QUESTION CAREFULLY. PLEASE GIVE IT SOME THOUGHT.
DO YOU NOW REALIZE WHAT YOU DID WRONG?

NO

YOUR ANSWER TO (4) SHOULD HAVE BEEN IN BETWEEN 50 AND 250. PLEASE TYPE YOUR NEW ANSWER.

130

(5) SUPPOSE YOU WERE TOLD THAT THE ACTUAL VALUE IS GREATER THAN 250. SPECIFY THE VALUE SUCH THAT IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY IS EITHER ABOVE OR BELOW THIS VALUE.

400

(6) NOW, DO YOU FEEL IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL LIE WITHIN THE INTERVAL BETWEEN 130 AND 400 OR OUTSIDE OF THIS INTERVAL?

NO

SORRY, YOUR NO ANSWER INDICATES THAT YOUR ANSWERS TO QUESTIONS (3) THROUGH (5) MAY NOT REFLECT YOUR PROBABILITY JUDGMENTS. SO, LET'S REVIEW EACH OF THESE ANSWERS.

(3A) ACCORDING TO YOUR ANSWER TO QUESTION (3), YOU FEEL IT IS EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL FALL ABOVE OR BELOW 250. DO YOU AGREE?

YES

(4A) WE NOW LOOK AT YOUR ANSWER TO QUESTION (4). IF THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WERE LESS THAN 250, WOULD YOU FEEL EQUALLY LIKELY THAT THE ACTUAL VALUE IS EITHER ABOVE OR BELOW 130?

NO

PLEASE TYPE YOUR NEW VALUE FOR WHICH YOU WOULD FEEL EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL FALL ABOVE OR BELOW THIS VALUE.

160

(5A) FINALLY, YOUR ANSWER TO QUESTION (5) IMPLIED THAT IF THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WERE GREATER THAN 250 YOU WOULD FEEL EQUALLY LIKELY THAT THE ACTUAL VALUE IS EITHER ABOVE OR BELOW 400. DO YOU STILL AGREE?

YES

(6A) NOW, LET'S CHECK THE CONSISTENCY OF YOUR REVISED ANSWERS. THE LOGICAL CONSISTENCY REQUIRES YOU TO FEEL EQUALLY LIKELY THAT THE ACTUAL VALUE OF THE UNCERTAIN QUANTITY WILL LIE WITHIN THE INTERVAL BETWEEN 160 AND 400 OR OUTSIDE OF THIS INTERVAL. DO YOU FEEL THAT WAY?

YES

(7) SPECIFY THE MOST LIKELY VALUE (THE MODE). [see Table 1].

200

Table 1

Cum. pros.	Value	Corresp. question
0.00	50	1
0.25	160	4
0.50	250	3
0.75	400	5
1.00	650	2
Mode	200	7

(8) GREAT, YOU NOW HAVE DONE YOUR ASSESSMENTS AS SUMMARIZED.

DO YOU WISH TO REVISE YOUR ANSWERS?

NO

FROM THESE ASSESSED POINTS, A SMOOTHED CUMULATIVE DISTRIBUTION WILL BE OBTAINED. WOULD YOU LIKE TO SEE THE GRAPHS FOR THE DISTRIBUTION FUNCTION AND THE DENSITY FUNCTION?

YES [see Figures 1 and 2]

(9) WOULD YOU ACCEPT THESE GRAPHS AS REASONABLY ACCURATE EXPRESSIONS OF YOUR JUDGMENTS CONCERNING THE UNCERTAIN QUANTITY?

YES

(10) SUMMARY MEASURES OF THIS DISTRIBUTION ARE

MEAN   284.9094
STANDARD DEVIATION   155.4427

(11) DO YOU WANT TO SEE THE VARIOUS FRACTILES?

YES [see Table 2]

(12) DO YOU WANT TO FIT THE ASSESSED DISTRIBUTION TO A THEORETICAL DISTRIBUTION?

NO

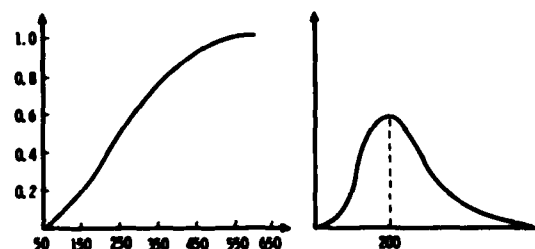Figure 1. Distribution and density functions for debugging hours.

Table 2

0.005	54.896	··	0.255	161.796	··	0.505	252.420	··	0.755	403.608
0.015	59.664	··	0.265	165.364	··	0.515	257.380	··	0.765	410.872
0.025	64.400	··	0.275	168.900	··	0.525	262.500	··	0.775	418.200
0.035	69.104	··	0.285	172.404	··	0.535	267.780	··	0.785	425.592
0.045	73.776	··	0.295	175.876	··	0.545	273.220	··	0.795	433.048
0.055	78.416	··	0.305	179.316	··	0.555	278.820	··	0.805	440.568
0.065	83.024	··	0.315	182.724	··	0.565	284.576	··	0.815	448.152
0.075	87.600	··	0.325	186.100	··	0.575	290.400	··	0.825	455.800
0.085	92.144	··	0.335	189.444	··	0.585	296.256	··	0.835	463.714
0.095	96.656	··	0.345	192.756	··	0.595	302.144	··	0.845	472.090
0.105	101.136	··	0.355	196.036	··	0.605	308.064	··	0.855	480.930
0.115	105.584	··	0.365	199.284	··	0.615	314.016	··	0.865	490.234
0.125	110.000	··	0.375	200.000	··	0.625	320.000	··	0.875	500.000
0.135	114.368	··	0.385	203.616	··	0.635	326.016	··	0.885	510.080
0.145	118.672	··	0.395	207.264	··	0.645	332.064	··	0.895	520.320
0.155	122.912	··	0.405	210.944	··	0.655	338.144	··	0.905	530.720
0.165	127.088	··	0.415	214.656	··	0.665	344.256	··	0.915	541.280
0.175	131.200	··	0.425	218.400	··	0.675	350.400	··	0.925	552.000
0.185	135.248	··	0.435	222.176	··	0.685	356.576	··	0.935	562.880
0.195	139.232	··	0.445	226.020	··	0.695	362.820	··	0.945	573.920
0.205	143.152	··	0.455	230.020	··	0.705	369.220	··	0.955	585.120
0.215	147.008	··	0.465	234.180	··	0.715	375.780	··	0.965	596.480
0.225	150.800	··	0.475	238.500	··	0.725	382.500	··	0.975	606.000
0.235	154.528	··	0.485	242.980	··	0.735	389.380	··	0.985	619.680
0.245	158.192	··	0.495	247.620	··	0.745	396.420	··	0.995	631.500

(13) DO YOU WISH TO QUANTIFY YOUR JUDG-
MENTS CONCERNING ANY OTHER UNCER-
TAIN QUANTITY?

NO

THANK YOU FOR YOUR COOPERATION. GOOD-
BYE.

AN APPLICATION FROM PROGRAM VERIFICATION

From the assessment procedure given earlier, several
fractile points, the mean, and the standard deviation
are available in the summary output of the computer
program. Any two fractile points, or a fractile point
and the mean, or a fractile point and the standard de-
viation, etc., can be used to determine the parameters
of the lognormal distribution. This distribution plays
an important role in the field of maintainability. A new

Figure 2. Cumulative distribution.

program was developed that allows some 20 different
input combination pairs in the procedure for deter-
mining the parameters of the lognormal distribution.
The density function of the lognormal distribution is
given by:

$$f(x) = \frac{1}{\beta\sqrt{2\pi}} x^{-1} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \alpha}{\beta}\right)^2\right], \quad \begin{array}{l} x > 0 \\ \beta > 0. \end{array} \quad (1)$$

where α and β are the parameters of the lognormal
distribution.

It is well known that the mean $E(x)$ and the variance
$V(x)$ are given by:

$$E(x) = \mu = \exp(\alpha + [\tfrac{1}{2}]\beta^2),$$
$$V(x) = \sigma^2 = \mu^2\left[(\exp \beta^2) - 1\right].$$

The mode of this distribution is at

$$\text{mode} = \exp(\alpha - \beta^2),$$

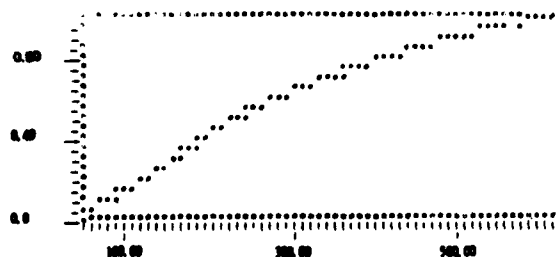whereas the median or 50th percentile P_{50} is at

$$P_{50} = e^\alpha.$$

By letting $y = \ln x - \alpha/\beta$ in (1) and using standard
normal tables, the 90th percentile was found to be

$$P_{90} = \exp(1.282\beta + \alpha).$$

Other fractile points can be found in a similar fashion.

As we have seen the lognormal distribution has two
parameters α and β. Thus to fit a lognormal distri-
bution to the subjectively derived distribution we only

have to specify two values such as P_{50} and P_{90}, or the mean and the standard deviation. For the following example the mode = 200 and the median = 250 are used. The program output includes a distribution function and a density function. The latter is given in Figure 3.

LOG NORMAL DISTRIBUTION

DO YOU NEED THE COMBINATION PRINTOUT?
YES=1, NO=0 ?0

WHAT IS THE INPUT COMBINATION NUMBER ?13
MEDIAN = ?250
MODE = ?200

ALPHA	BETA	MEDIAN	MEAN
5.5215	0.4724	250.0000	279.5085

STD DEV	MODE	90TH PCTLE TIME
139.7542	200.0000	458.0842

DO YOU WISH TO INTEGRATE-NO=0, YES=1,
RETURN = 2 ?0

DO YOU WISH TO PRINT X AND Y-NO=0, YES = 1,
RETURN=2 ?1
WHAT IS XMIN, XMAX, DELX
$100,650,20

X-VALUES	Y-VALUES	X-VALUES	Y-VALUES
100	1.28704E-03	400	1.28704E-03
120	2.10476E-03	420	1.10016E-03
140	2.84011E-03	440	9.37937E-04
160	3.37814E-03	460	7.98064E-04
180	3.68409E-03	480	6.78084E-04
200	3.77688E-03	500	5.75580E-04
220	3.7078E-03	520	4.88276E-04
240	3.50578E-03	540	4.14090E-04
260	3.23704E-03	560	3.51159E-04
280	2.93064E-03	580	2.97842E-04
300	2.61306E-03	600	2.52707E-04
320	2.30232E-03	620	2.14516E-04
360	1.74150E-03	640	1.82209E-04
380	1.50047E-03		

DO YOU WISH TO PLOT X AND Y; NO=0, YES=1 ?1 [See Figure 3.]

Now the distribution function or density function can be visually compared with the subjectively derived prior distribution using the questionnaire involving the debugging hours. If "reasonable" agreement has been achieved, the mathematical form of the density has been found. Several combinations of input values might have to be examined in order to achieve the "best" fit. This form is important in order to establish the posterior distribution using incoming data and the likelihood function according to Bayes' theorem. On the other hand, if "reasonable" agreement between the two distribution functions has not been achieved, a new family of distributions may be tried and/or the empirical distribution might be ques-
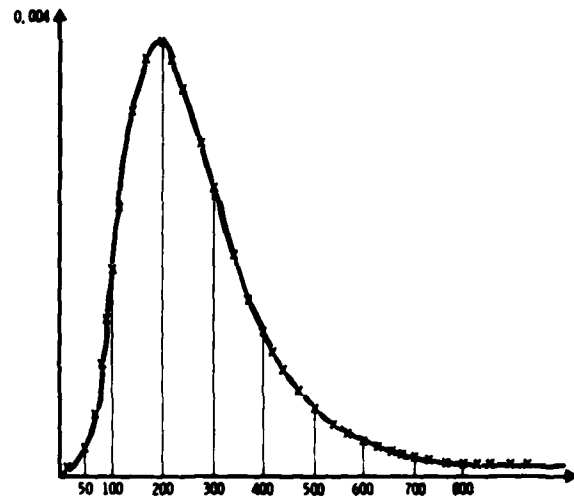

Figure 3. Lognormal density function with median = 250 and mode = 200

tioned. Ultimately, agreement will be found unless the lognormal distribution is not a valid model describing debugging hours.

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In the development of large-scale computer software and in the management of the development process, it is often useful to model the reliability and the cost of development of these software packages. There have been many papers that develop models and show their usefulness as management tools. The models that use Bayesian methodology assume that a prior distribution is given.

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Our paper offers a methodology of assessing a prior distribution subjectively. Two computer programs have been developed for this particular purpose: One assesses a subjective prior distribution and the other suggests a family of probability functions.

The importance of consistent prior distributions is twofold. First, these distributions reflect consistent initial predictions because they are developed by a structured process. Second, these distributions are the starting point for applying Bayes' theorem to develop the posterior distribution by modifying the prior distribution with actual data available later.

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