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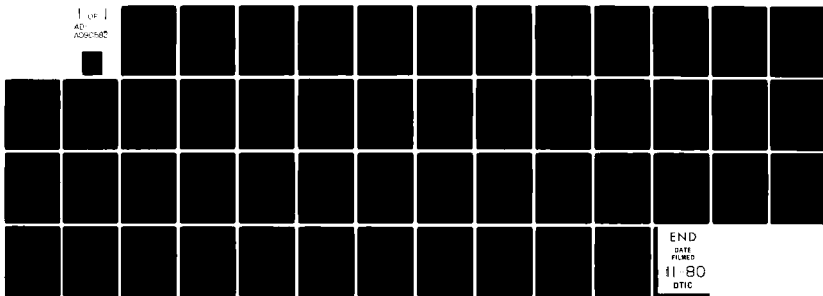
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ELECTROMAGNETIC WAVES SCATTERING BY
BURIED DIELECTRIC OBJECTS.

FINAL

LEVEL II

(10) S. K. Chang ~~and~~ K. K. Mei

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ABSTRACT

Time harmonic modal electromagnetic fields in two-medium half spaces are investigated. For practical and numerical considerations, the primary sources of the modal fields are chosen to be the spherical multipoles, and the potential vectors are z-directed. It is shown that modal fields of such combination are not able to represent a conventional spherical modal field. The horizontal rotating potentials are added to ensure proper representation and fast convergence.

The recurrence relations which transform the spherical Hankel-Legendre functions into the Fourier-Bessel integrals are derived. The secondary fields of the Sommerfeld's type are obtained for all spherical multipole sources, and the added horizontally rotating potentials. The combination of the modal fields are capable of representing arbitrary electromagnetic fields resulting from radiation and scattering problems.

Results are given for scattered surface fields of a buried finite dielectric cylinder.

I. INTRODUCTION

Modal expansions of electromagnetic fields are frequently a mathematical necessity in classical solutions of electromagnetic problems. Recent development of the unimoment method of computation has also made extensive use of modal fields, which satisfy the radiation conditions outside spherical surfaces [1], [2]. Indeed, the application of the modal fields is essential in terminating the finite difference or the finite element equations at a spherical surface. It is quite evident that the unimoment method can readily be applied to scattering or radiation problems involving air-ground half-spaces, if the modal fields satisfying the air-ground interface boundary conditions are available.

The integrals which represent the potentials of a dipole near an air-ground interface are known as Sommerfeld integrals, which were first derived by Sommerfeld in 1909 [3] and 1926 [4]. They have been the foundations of research in electromagnetic radiation by sources near a lossy ground ever since. The objective of this paper is to investigate the generalization of Sommerfeld integrals for the modal fields so that they can represent arbitrary electromagnetic fields near an air-ground interface.

The generalization of Sommerfeld integrals has been given by C.T. Tai [5], whose systematic approach to Dyadic Green's function very elegantly arrives at modal fields of Sommerfeld type. However, Tai's generalization entails cylindrical sources, i.e., each of the exterior modes is singular along the z-axis. Using Tai's modal fields to terminate the finite element mesh, one would have to use an infinite cylinder as a terminating surface, such as shown in Fig. 1a. The matching points between the finite element solutions and the modal fields would spread fairly far along the cylinder, and the numerical objective would be finding a continuous function $f(\lambda)$ for the values of λ along the integration path. The above particulars are not attractive features for computational purposes. The ideal matching surface should be finite and the ideal numerical objective should be a discrete set of coefficients. In this paper we shall attempt to generalize Sommerfeld integrals based on spherical multipole expansions. The numerical application of such generalized Sommerfeld integrals should result in spherical matching surfaces, as shown in Fig. 1b, and discrete numerical objectives.

The fields concerned in the following discussions are all assumed to have the $\exp(j\omega t)$ harmonic time dependence.

II. MULTIPOLE EXPANSIONS AND VERTICAL VECTOR POTENTIALS

The first step in deriving Sommerfeld integrals is to decide what type of primary sources are to be used. The electromagnetic fields in a homogeneous isotropic medium can usually be derived from the electric and magnetic vector potentials, $\vec{A}_e$ and $\vec{A}_m$, as follows

$$\vec{E} = \nabla \times \vec{A}_m + \frac{1}{j\omega\epsilon} \nabla \times \nabla \times \vec{A}_e \quad (1a)$$

$$\vec{H} = \nabla \times \vec{A}_e - \frac{1}{j\omega\mu} \nabla \times \nabla \times \vec{A}_m \quad (1b)$$

where ϵ and μ are the permittivity and the permeability of the medium, ω is the angular frequency and the potentials $\vec{A}_e$ and $\vec{A}_m$ both satisfy the vector Helmholtz equations. The above potential representations are complete in homogeneous source free medium where the fields derived from the gradient of a potential vanishes [6].

Our intention to obtain spherical matching surfaces in numerical application limits our option to spherical multipoles, i.e., the primary fields will be derived from

$$\begin{pmatrix} \vec{A}_e \\ \vec{A}_m \end{pmatrix} = \hat{z} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \begin{pmatrix} \alpha_{m,n} \\ \beta_{m,n} \end{pmatrix} \frac{1}{k} h_n^{(2)}(kr) P_n^m(\cos \theta) e^{jm\phi} \quad (2)$$

where $k = \omega\sqrt{\mu\epsilon}$ is the wave number and $\hat{z}$ is a unit vector normal to the ground plane. It is noticed that the vector potentials are z-directed rather than the conventional radial vector. The direction of the potential vector is dictated by the need to satisfy the continuity conditions of the planar air ground interface for the total fields.

The use of the conventional spherical potentials using radial vectors does not lead to expressions which are compatible with air-earth interface field matching. This particular combination of spherical harmonics and

z-directed potential vectors is not new. The complete expression has been given by Tai [5 Appendix B]; however, the modal fields derived from it have never been applied to solve specific problems. For the convenience of the further discussions, the fields derived from (2) will be named as $TM_{m,n}^{(z)}$ and $TE_{m,n}^{(z)}$ modes for $\vec{A}_e$ and $\vec{A}_m$ respectively.

The modal electromagnetic fields derived from (2) are denoted by $\vec{e}_{m,n}^{TM(z)}$, $\vec{h}_{m,n}^{TM(z)}$ and $\vec{e}_{m,n}^{TE(z)}$, $\vec{h}_{m,n}^{TE(z)}$. The definitions of these notations are listed in Table 1.

One of the most important questions about the modal fields derived from (2) is whether the expansions can represent arbitrary radiating fields. The following derivations will show that the modal expansions of a class of radiating fields by $TM_{m,n}^{(z)}$ and $TE_{m,n}^{(z)}$ result in diverging series. Such a class of radiating fields include the conventional spherical vector wave modes. It is found that proper additional terms may be added to (2) to make the modal expansion series convergent.

For clarity in the ensuing discussions, we should examine the conventional curvilinear spherical vector waves obtained from

$$\begin{pmatrix} \vec{A}_e \\ \vec{A}_m \end{pmatrix} = \vec{r} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \begin{pmatrix} a_{m,n} \\ b_{m,n} \end{pmatrix} h_n^{(2)}(kr) P_n^m(\cos \theta) e^{jm\phi} \quad (3)$$

where $\vec{r}$ is a radial vector. These expansions were proved

to be complete in the homogenous source free region where the divergence of $\vec{E}$ and $\vec{H}$ vanishes [6] and have been applied to numerous practical applications. We shall name the expansion modes of (3) as $TM_{m,n}^{(r)}$ and $TE_{m,n}^{(r)}$ for $\vec{A}_e$ and $\vec{A}_m$ respectively. The modal fields are denoted by $\vec{e}_{m,n}^{TM(r)}$, $\vec{h}_{m,n}^{TM(r)}$, and $\vec{e}_{m,n}^{TE(r)}$, $\vec{h}_{m,n}^{TE(r)}$ as shown in Table 1.

We first study the singularities of $TE_{m,n}^{(r)}$, $TM_{m,n}^{(r)}$ and $TE_{m,n}^{(z)}$, $TM_{m,n}^{(z)}$ fields as $r \rightarrow 0$. The spherical Hankel function and its first derivative have singularities at $r \rightarrow 0$

$$\lim_{r \rightarrow 0} h_n^{(2)}(kr) \rightarrow (kr)^{-n}$$

$$\lim_{r \rightarrow 0} \frac{d h_n^{(2)}(kr)}{d kr} \rightarrow -j(kr)^{-(n+1)}$$

Hence the modal electric fields have the singularities as follows:

$$\begin{cases} \lim_{r \rightarrow 0} \vec{e}_{m,n}^{TE(z)} \rightarrow r^{-(n+1)} \\ \lim_{r \rightarrow 0} \vec{e}_{m,n}^{TM(z)} \rightarrow r^{-(n+2)} \end{cases}$$

$$\begin{cases} \lim_{r \rightarrow 0} \vec{e}_{m,n}^{TE(r)} \rightarrow r^{-n} \\ \lim_{r \rightarrow 0} \vec{e}_{m,n}^{TM(r)} \rightarrow r^{-(n+1)} \end{cases}$$

The expansions of different numbers of m are decoupled because the Fourier series modes are mutually orthogonal. For a particular value of m , we found that as $r \rightarrow 0$ $TE_{m,n}^{(z)}$ and $TM_{m,n}^{(z)}$ have a singularity sequence of $r^{-(|m|+1)}$, $r^{-(|m|+2)}$, ... whereas $TE_{m,n}^{(r)}$ and $TM_{m,n}^{(r)}$ have $r^{-|m|}$, $r^{-(|m|+1)}$, ... Therefore the lowest singularity term $r^{-|m|}$ is missing for the $TE_{m,n}^{(z)}$, $TM_{m,n}^{(z)}$ modes. This suggests that additional terms of proper singularity must be added to $TE_{m,n}^{(z)}$ and $TM_{m,n}^{(z)}$ modes to represent arbitrary radiating fields.

To show that the $TE_{m,n}^{(z)}$ and $TM_{m,n}^{(z)}$ modes do not yield converging series when expanding a certain class of radiating fields, let us take a simple example of the fields radiated by a horizontal rotating dipole. The vector potentials of the dipole can be written as

$$\vec{A}_e = (\hat{x} + j\hat{y}) h_0^{(2)}(kr) \quad (4)$$

The electromagnetic fields generated by (4) are found to be exactly the same as those of the $TM_{1,1}^{(r)}$ mode. We shall find the coefficients α_n , β_n in the following expansion

$$\vec{e}_{1,1}^{TM(r)} = \sum_{n=1}^{\infty} \left[\alpha_n \vec{e}_{1,n}^{TM(z)} + \beta_n \cdot jk \cdot \vec{e}_{1,n}^{TE(z)} \right] \quad (5)$$

It should be noted that neither $\vec{e}_{m,n}^{TM(z)}$ nor $\vec{e}_{m,n}^{TE(z)}$ modes are orthogonal on the surface of a sphere.

By using the relations given by Tai [5, p. 228], such as

$$\vec{e}_{1,n}^{TE(z)} = j \frac{1}{n(n+1)} \frac{1}{k} \vec{e}_{1,n}^{TM(r)} + \frac{n}{(2n+1)(n+1)} \vec{e}_{1,n+1}^{TE(r)} + \frac{n+1}{(2n+1)n} \vec{e}_{1,n-1}^{TE(r)} \quad (6a)$$

$$\vec{e}_{1,n}^{TM(z)} = j \frac{1}{n(n+1)} \cdot k \cdot \vec{e}_{1,n}^{TE(r)} + \frac{n}{(2n+1)(n+1)} \vec{e}_{1,n+1}^{TM(r)} + \frac{n+1}{(2n+1)n} \vec{e}_{1,n-1}^{TM(r)} \quad (6b)$$

and substituting (5) into (6a) and (6b), one can solve α_n and β_n via a set of linear equations involving a tri-diagonal matrix. The results are

$$\alpha_n = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^{\frac{n}{2} + 1} \cdot \frac{(2n+1)}{n(n+1)} & \text{if } n \text{ is even} \end{cases} \quad (7a)$$

$$\beta_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ (-1)^{\frac{n-1}{2}} \cdot \frac{(2n+1)}{n(n+1)} & \text{if } n \text{ is odd} \end{cases} \quad (7b)$$

Examining (6a) and (6b) one finds that α_n and β_n have asymptotic behavior of $O(n^{-1})$ for large orders of n . The asymptotic form of the spherical Hankel function for large

n [7, eq. 37] is

$$\lim_{n \rightarrow \infty} h_n^{(2)}(kr) \approx j \sqrt{\frac{1}{(kr)(n+\frac{1}{2})}} \cdot \left(\frac{2(n+\frac{1}{2})}{e \cdot (kr)} \right)^{(n+\frac{1}{2})}$$

where e is the base number of the natural logarithm. Combining the slowly convergence of α_n and β_n and the rapidly increasing of $h_n^{(2)}(kr)$ at $(n + \frac{1}{2})^n$, it is thus shown that the expansion of (5) diverges for all kr . Similar conclusions can also be obtained for other $TM_{m,n}^{(r)}$ modes.

The fact that $TM_{m,n}^{(z)}$ and $TE_{m,n}^{(z)}$ yield diverging series when expanding the field of a rotating dipole and the $TM_{m,n}^{(r)}$ and $TE_{m,n}^{(r)}$ modal fields has made the expansion of (2) unusable for general applications in scattering problems. To remedy this situation, additional terms of proper orders of singularities must be included in (2). It was found that the field of a rotating dipole given by (4) can provide the singularity needed for $m=1$. Hence, we suggest adding two circularly polarized vector potentials, e.g.,

$$\begin{pmatrix} \vec{A}_e \\ \vec{A}_m \end{pmatrix} = (\hat{x} \pm j\hat{y}) \sum_{m=0}^{\infty} \begin{pmatrix} \gamma_{\pm m} \\ \eta_{\pm m} \end{pmatrix} \frac{1}{k} h_m^{(2)}(kr) P_m^m(\cos\theta) e^{\pm jm\phi} \quad (8)$$

where $\hat{x}$ and $\hat{y}$ are unit horizontal vectors on the rectangular coordinates. These rotating modes will be named $RTM_{\pm(m+1)}$ and $RTE_{\pm(m+1)}$ for $\vec{A}_e$ and $\vec{A}_m$ respectively. The modal fields will be denoted by $\vec{e}_{\pm(m+1)}^{RTM}$, $\vec{h}_{\pm(m+1)}^{RTM}$ and $\vec{e}_{\pm(m+1)}^{RTE}$,

$\vec{h}_{\pm(m+1)}^{RTE}$. Indeed, these rotating modal fields are similar to the lowest order modes for each m of the conventional spherical vector waves. That is, they are similar to the modal fields of $TM_{\pm m, m}^{(r)}$ and $TE_{\pm m, m}^{(r)}$.

The proper general solution of Maxwell's equations outside a sphere in terms of (2) and (8) is thus

$$\begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} = \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{m,n} \begin{pmatrix} \vec{e}_{m,n}^{TE(z)} \\ \vec{h}_{m,n}^{TE(z)} \end{pmatrix} + \beta_{m,n} \begin{pmatrix} \vec{e}_{m,n}^{TM(z)} \\ \vec{h}_{m,n}^{TM(z)} \end{pmatrix} \right] \\ + \sum_{m=1}^{\infty} \left[\gamma_m \begin{pmatrix} \vec{e}_m^{RTE} \\ \vec{h}_m^{RTE} \end{pmatrix} + \eta_m \begin{pmatrix} \vec{e}_m^{RTM} \\ \vec{h}_m^{RTM} \end{pmatrix} \right] \\ + \sum_{m=1}^{\infty} \left[\gamma_{-m} \begin{pmatrix} \vec{e}_{-m}^{RTE} \\ \vec{h}_{-m}^{RTE} \end{pmatrix} + \eta_{-m} \begin{pmatrix} \vec{e}_{-m}^{RTM} \\ \vec{h}_{-m}^{RTM} \end{pmatrix} \right] \quad (9)$$

where $\alpha_{m,n}$, $\beta_{m,n}$, γ_m , and η_m

are the expansion coefficients. The modal fields of the expansion are defined in Table 1.

It can be shown [8] that any one of the conventional spherical vector modal fields can be expanded in a finite number of terms by using (9). The convergence rate of (9) is the same as that using $TM_{m,n}^{(r)}$ and $TE_{m,n}^{(r)}$ modes when solving the same problem.

Table 1. The Definition of the Primary Modal Fields

MODES	VECTOR POTENTIALS	MODAL FIELDS
$TE_{\pm m, n}^{(z)}$	$\vec{A}_m = \hat{z} \frac{1}{k} h_n^{(2)}(kr) P_n^m(\cos\theta) e^{\pm jm\phi}$	$\vec{e}_{\pm m, n}^{TE(z)} = \nabla \times \vec{A}_m$ $\vec{h}_{\pm m, n}^{TE(z)} = j \frac{1}{kZ} \nabla \times \nabla \times \vec{A}_m$
$TM_{\pm m, n}^{(z)}$	$\vec{A}_e = \hat{z} \frac{1}{k} h_n^{(2)}(kr) P_n^m(\cos\theta) e^{jm\phi}$	$\vec{e}_{\pm m, n}^{TM(z)} = -j \frac{Z}{k} \nabla \times \nabla \times \vec{A}_e = -Z^2 \vec{h}_{\pm m, n}^{TE(z)}$ $\vec{h}_{\pm m, n}^{TM(z)} = \nabla \times \vec{A}_e = \vec{e}_{\pm m, n}^{TE(z)}$
$RTE_{\pm m}$	$\vec{A}_m = (\hat{x} \pm j\hat{y}) \frac{1}{k} h_{m-1}^{(2)}(kr) P_{m-1}^{m-1}(\cos\theta) e^{\pm j(m-1)\phi}$	$\vec{e}_{\pm m}^{RTE} = \nabla \times \vec{A}_m$ $\vec{h}_{\pm m}^{RTE} = j \frac{1}{kZ} \nabla \times \nabla \times \vec{A}_m$
$RTM_{\pm m}$	$\vec{A}_e = (\hat{x} \pm j\hat{y}) \frac{1}{k} h_{m-1}^{(2)}(kr) P_{m-1}^{m-1}(\cos\theta) e^{\pm j(m-1)\phi}$	$\vec{e}_{\pm m}^{RTM} = -j \frac{Z}{k} \nabla \times \nabla \times \vec{A}_e = -Z^2 \vec{h}_{\pm m}^{RTE}$ $\vec{h}_{\pm m}^{RTM} = \nabla \times \vec{A}_e = \vec{e}_{\pm m}^{RTE}$
$TE_{\pm m, n}^{(r)}$	$\vec{A}_m = \hat{r} h_n^{(2)}(kr) P_n^m(\cos\theta) e^{\pm jm\phi}$	$\vec{e}_{\pm m, n}^{TE(r)} = \nabla \times \vec{A}_m$ $\vec{h}_{\pm m, n}^{TE(r)} = j \frac{1}{kZ} \nabla \times \nabla \times \vec{A}_m$
$TM_{\pm m, n}^{(r)}$	$\vec{A}_e = \hat{r} h_n^{(2)}(kr) P_n^m(\cos\theta) e^{\pm jm\phi}$	$\vec{e}_{\pm m, n}^{TM(r)} = j \frac{Z}{k} \nabla \times \nabla \times \vec{A}_e = -Z^2 \vec{h}_{\pm m, n}^{TE(r)}$ $\vec{h}_{\pm m, n}^{TM(r)} = \nabla \times \vec{A}_e = \vec{e}_{\pm m, n}^{TE(r)}$

NOTE: $Z = \sqrt{\mu/\epsilon}$

III. THE FOURIER-BESSEL INTEGRALS OF $h_n^{(2)}(kr) P_n^m(\cos\theta)$

The multipole expansions discussed in the last section are for the primary fields, which are valid in an infinite uniform space. To find the fields of the multipoles near an air-ground interface, the multipole fields must be expressed in cylindrical harmonics so that the secondary waves can be derived. We shall derive the Fourier-Bessel integrals for the spherical Hankel-Legendre functions in this section.

Since both $h_n^{(2)}(kr) P_n^m(\cos\theta) e^{\pm jm\phi}$ and $J_m(\lambda\rho) e^{-\sqrt{\lambda^2 - k^2}|z|} e^{\pm jm\phi}$ are solutions of the same scalar Helmholtz equation

$$\nabla^2 \psi + k^2 \psi = 0 \quad (10)$$

one may represent the spherical harmonics by the superposition of a complete cylindrical eigenfunction [6, 9]. That is

$$h_n^{(2)}(kr) P_n^m(\cos\theta) = [\text{sgn}(z)]^{n-m} \int_0^\infty f_{m,n}(\lambda) J_m(\lambda\rho) e^{-u|z|} d\lambda \quad (11)$$

where $u = \sqrt{\lambda^2 - k^2}$, $\text{Real}(u) \geq 0$, and $\text{sgn}(z) = 1$ for positive z and -1 for negative z . The function $f_{m,n}(\lambda)$ is an amplitude function for the transformation. The first special case of the Fourier-Bessel transform in (11) is for $m=n=0$ of a dipole, as shown by Sommerfeld in [9].

$$h_0^{(2)}(kr) = j \frac{e^{-jkr}}{kr} = \int_0^\infty j \frac{\lambda}{ku} J_0(\lambda\rho) e^{-u|z|} d\lambda \quad (12)$$

Hence the function $f_{0,0}(\lambda)$ is

$$f_{0,0}(\lambda) = j \frac{\lambda}{ku} \quad (13)$$

The functions for other orders of m and n can be obtained by recurrence relations.

The following two recurrence relations are proved in Appendices A and B, respectively.

Recurrence Relation 1.

$$\begin{aligned} h_{m+1}^{(2)}(kr) P_{m+1}^m(\cos \theta) &= -(2m+1) \frac{\partial}{\partial k\rho} [h_m^{(2)}(kr) P_m^m(\cos \theta)] \\ &+ \frac{m(2m+1)}{kr \sin \theta} [h_m^{(2)}(kr) P_m^m(\cos \theta)] \end{aligned} \quad (14)$$

Recurrence Relation 2.

$$\begin{aligned} \frac{\partial}{\partial kz} [h_n^{(2)}(kr) P_n^m(\cos \theta)] &= \frac{(n+m)}{(2n+1)} h_{n-1}^{(2)}(kr) P_{n-1}^m(\cos \theta) \\ &- \frac{(n-m+1)}{(2n+1)} h_{n+1}^{(2)}(kr) P_{n+1}^m(\cos \theta) \end{aligned} \quad (15)$$

The above recurrence formulas are unique in that they relate spherical modal potentials rather than single variable special functions. The physical interpretations of these formulas are also of interest, but we shall not attempt to discuss these here.

It is noted that the first recurrence relation raises the order of m , and the second raises n for a fixed value of m .

The formulation of $f_{m+1,m+1}(\lambda)$ is related to $f_{m,m}(\lambda)$ by substituting the integral of (11) into the first recurrence relation (14). That is

$$\begin{aligned} & \int_0^{\infty} f_{m+1,m+1}(\lambda) J_{m+1}(\lambda \rho) e^{-u|z|} d\lambda \\ &= -(2m+1) \int_0^{\infty} f_{m,m}(\lambda) \left[\frac{\partial J_m(\lambda \rho)}{\partial k \rho} - \frac{m}{k \rho} J_m(\lambda \rho) \right] e^{-u|z|} d\lambda \\ &= (2m+1) \int_0^{\infty} f_{m,m}(\lambda) \cdot \frac{\lambda}{k} J_{m+1}(\lambda \rho) e^{-u|z|} d\lambda \end{aligned} \quad (16)$$

This relation holds for all values of ρ and z , hence

$$f_{m+1,m+1}(\lambda) = (2m+1) \frac{\lambda}{k} f_{m,m}(\lambda) \quad (17)$$

Using the initial formula of $f_{0,0}(\lambda)$ in (13) and the relation (17), one has a general expression of $f_{m,m}(\lambda)$ as follows:

$$f_{m,m}(\lambda) = j \left(\frac{\lambda}{k} \right)^{m+1} \frac{1}{u} P_m^m(0) \quad (18)$$

for $m = 0, 1, 2, \dots$. Because $P_{m-1}^m(\cos \theta) \equiv 0$, we have

$$f_{m,m-1}(\lambda) \equiv 0 \quad (19)$$

Similarly, by substituting the integral of (11), the second recurrence relation (15) leads to the following relation:

Recurrence Relation 3.

$$f_{m,n+1}(\lambda) = \frac{(2n+1)}{(n-m+1)} \left[\frac{u}{k} f_{m,n}(\lambda) + \frac{(n+m)}{(2n+1)} f_{m,n-1}(\lambda) \right] \quad (20)$$

A complete formulation of $f_{m,n}(\lambda)$ can readily be obtained by (18), (19) and the third recurrence relation (20). The recurrence relations (17) and (20) resemble those for the associated Legendre functions. We found that $f_{m,n}(\lambda)$ can be written in a closed form to be

$$f_{m,n}(\lambda) = j^{(n-m+1)} \frac{\lambda}{ku} P_n^m \left(-j \frac{u}{k} \right) \quad (21)$$

Although (21) may also be obtained by other approaches, they have not been explicitly shown in open literature. The approach we have presented here is more direct and computationally appealing than integration of plane waves in the complex plane.

It is worth noting that the Fourier-Bessel integral is equivalent to the Fourier-Hankel integral [9].

$$h_n^{(2)}(kr) P_n^m(\cos \theta) = [\text{sgn}(z)]^{n-m} \cdot \frac{1}{2} \int_{-\infty}^{\infty} f_{m,n}(\lambda) H_m^{(2)}(\lambda \rho) e^{-u|z|} d\lambda \quad (22)$$

The integration paths and branch cuts in the complex λ -plane for (11) and (22) are shown in Figure 2. The path P_1 is a permissible integration path of the Fourier-Bessel integral (21), and path $P_2 + P_1$ is for the Fourier-Hankel integral (22).

IV. THE GENERALIZED SOMMERFELD INTEGRALS

Let the multipoles be at a depth d underground. The coordinates in the meridional plane are shown in Figure 3. The relative dielectric constants of regions I and II are ϵ_1 and ϵ_2 respectively. We assume the same magnetic permeability in both regions, although it is straightforward to consider the case of different magnetic permeabilities.

The vector potentials in regions I and II are $\vec{A}_m^I$ (or $\vec{A}_e^I$) and $\vec{A}_m^{II}$ (or $\vec{A}_e^{II}$). The fields in region II are decomposed into primary and secondary waves as follows:

$$\vec{A}_m^{II} = \vec{A}_m^{Pri} + \vec{A}_m^{Sec} \quad (23.a)$$

$$\vec{A}_e^{II} = \vec{A}_e^{Pri} + \vec{A}_e^{Sec} \quad (23.b)$$

The primary waves are the multipole fields in an infinite space, which have been discussed in previous sections. The secondary waves are the fields due to the air-ground interface.

Owing to their basic differences in the formulation, we shall discuss the derivations for TE(z), TM(z), RTE, and RTM fields separately.

1. The Vertical Magnetic Multipoles, TE(z)_{±m,n} Fields

The primary field for TE(z)_{±m,n} in region II are

$$\vec{A}_m^{Pri} = A_m^{Pri} \hat{z} = \hat{z} h_n^{(2)}(k_2 r) P_n^m(\cos \theta) e^{\pm j m \phi} \quad (24)$$

Using the Fourier-Bessel integral (11) derived in Section III, the primary fields become

$$A_m^{\text{Pri}} = e^{\pm jm\phi} \int_0^\infty f_{m,n}(\lambda) J_m(\lambda\rho) e^{-u_2|z|} d\lambda \quad (25)$$

where $f_{m,n}(\lambda)$ is given in Section III, and $u_2 = \sqrt{\lambda^2 - k_2^2}$, Real (u_2) ≥ 0 . Note that we shall use (25) only when $z > 0$ in the following derivations.

The secondary field in region II can be represented by the following complete cylindrical field integral

$$\vec{A}_m^{\text{Sec}} = A_m^{\text{Sec}} \vec{z} = \vec{z} e^{\pm jm\phi} \int_0^\infty g(\lambda) J_m(\lambda\rho) e^{u_2(z-d)} d\lambda \quad (26)$$

The field in region I is represented by

$$\vec{A}_m^{\text{I}} = A_m^{\text{I}} \vec{z} = \vec{z} e^{\pm jm\phi} \int_0^\infty h(\lambda) J_m(\lambda\rho) e^{-u_1(z-d)} d\lambda \quad (27)$$

where $u_1 = \sqrt{\lambda^2 - k_1^2}$, Real (u_1) ≥ 0 .

The functions $g(\lambda)$ and $h(\lambda)$ are obtained by employing the boundary conditions on the air-ground interface. The corresponding electric and magnetic fields from the TE(z) $_{\pm m,n}$ vector potentials are

$$\vec{E} = \pm \rho \frac{jm}{\rho} A_m + \hat{\phi} \frac{\partial A_m}{\partial \rho} \quad (28.a)$$

$$\vec{H} = \frac{-1}{j\omega\mu} \left[\hat{\rho} \frac{\partial^2 A_m}{\partial \rho \partial z} \pm \hat{\phi} \frac{jm}{\rho} \frac{\partial A_m}{\partial z} + \vec{z} \left(k^2 A_m + \frac{\partial^2 A_m}{\partial z^2} \right) \right] \quad (28.b)$$

The sufficient conditions for the continuity of tangential electric and magnetic fields (E_ρ , E_ϕ , H_ρ , and H_ϕ) on the interface between regions I and II are

$$\left[A_m^I \right]_{z=d} = \left[A_m^{II} \right]_{z=d} \quad (29.a)$$

and

$$\left[\frac{\partial A_m^I}{\partial z} \right]_{z=d} = \left[\frac{\partial A_m^{II}}{\partial z} \right]_{z=d} \quad (29.b)$$

Substituting (23.a), (24), and (27) into (29.a) and (29.b), we have

$$e^{\pm j m \phi} \int_0^\infty \left[f_{m,n}(\lambda) e^{-u_2 d} + g(\lambda) - h(\lambda) \right] J_m(\lambda \rho) d\lambda = 0 \quad (30.a)$$

$$e^{\pm j m \phi} \int_0^\infty \left[u_2 f_{m,n}(\lambda) e^{-u_2 d} - u_2 g(\lambda) - u_1 h(\lambda) \right] J_m(\lambda \rho) d\lambda = 0 \quad (30.b)$$

The conditions of (30.a) and (30.b) must be true for all ϕ and ρ , hence

$$f_{m,n}(\lambda) e^{-u_2 d} + g(\lambda) - h(\lambda) = 0 \quad (31.a)$$

$$u_2 f_{m,n}(\lambda) e^{-u_2 d} - u_2 g(\lambda) - u_1 h(\lambda) = 0 \quad (31.b)$$

The functions $g(\lambda)$ and $h(\lambda)$ are obtained by solving (31.a) and (31.b).

$$g(\lambda) = \frac{u_2 - u_1}{u_2 + u_1} f_{m,n}(\lambda) e^{-u_2 d} \quad (32.a)$$

$$h(\lambda) = \frac{2u_2}{u_2 + u_1} f_{m,n}(\lambda) e^{-u_2 d} \quad (32.b)$$

Replacing $g(\lambda)$ and $h(\lambda)$ in (26) and (27) by the above expressions, we obtain the solution of $TE(z)_{\pm m,n}$ fields for $z > d$ and $z < d$ as follows

$$\vec{A}_m^I = \hat{z} e^{\pm j m \phi} \int_0^\infty \frac{2u_2}{u_2 + u_1} f_{m,n}(\lambda) J_m(\lambda \rho) e^{-u_2 d - u_1(z-d)} d\lambda \quad (33.a)$$

$$\begin{aligned} \vec{A}_m^{II} = \hat{z} e^{\pm j m \phi} & \left[h_n^{(2)}(k_2 r) P_n^m(\cos \theta) \right. \\ & \left. + \int_0^\infty \frac{u_2 - u_1}{u_2 + u_1} f_{m,n}(\lambda) J_m(\lambda \rho) e^{-u_2(2d-z)} d\lambda \right] \end{aligned} \quad (33.b)$$

Taking $m = 0$ and $n = 0$, one finds that the solution given in (33.a) and (33.b) for $TE(z)_{0,0}$ is exactly the same as that of the Sommerfeld integrals for a vertical magnetic dipole buried underground at a depth of d . Note that the Hertz potentials, $\vec{\pi}_e$ and $\vec{\pi}_m$, as used by Sommerfeld and the vector potentials used here differ by constant multipliers as

$$\vec{A}_e = j\omega\epsilon\vec{\pi}_e \quad (34)$$

$$\vec{A}_m = j\omega\mu\vec{\pi}_m$$

2. The Vertical Electric Multipoles $TM(z)_{\pm m, n}$ Fields

The general expressions for fields of $TM(z)_{\pm m, n}$ in regions I and II are identical to Eqs. (23), (24) and (25) by replacing A_m with A_e .

The electric and magnetic fields corresponding to the $TM(z)_{\pm m, n}$ vector potentials are

$$\vec{E} = \frac{1}{j\omega\epsilon} \left[\hat{\rho} \frac{\partial^2 A_e}{\partial z^2} \pm \hat{\phi} \frac{j\omega}{\rho} \frac{\partial A_e}{\partial z} + \hat{z} \left(k^2 A_e + \frac{\partial^2 A_e}{\partial z^2} \right) \right] \quad (35.a)$$

$$\vec{H} = \pm \hat{\rho} \frac{j\omega}{\rho} A_e + \hat{\phi} \frac{\partial A_e}{\partial \rho} \quad (35.b)$$

The sufficient conditions that the tangential components of electric and magnetic fields be continuous on the interface between region I and II are

$$\frac{1}{\epsilon_1} \left[\frac{\partial A_e^I}{\partial z} \right]_{z=d} = \frac{1}{\epsilon_2} \left[\frac{\partial A_e^{II}}{\partial z} \right]_{z=d} \quad (36.a)$$

$$\left[A_e^I \right]_{z=d} = \left[A_e^{II} \right]_{z=d} \quad (36.b)$$

Enforcing the boundary conditions at the air-ground interface, we have

$$e^{\pm jm\phi} \int_0^\infty \left[\frac{u_2}{\epsilon_2} f_{m,n}(\lambda) e^{-u_2 d} - \frac{u_2}{\epsilon_2} g(\lambda) - \frac{u_1}{\epsilon_1} h(\lambda) \right] J_m(\lambda \rho) d\lambda = 0 \quad (37.a)$$

$$e^{\pm jm\phi} \int_0^\infty \left[f_{m,n}(\lambda) e^{-u_2 d} + g(\lambda) - h(\lambda) \right] J_m(\lambda \rho) d\lambda = 0 \quad (37.b)$$

Equations (37.a) and (37.b) must be true for all ρ and ϕ , so that the integrands are zero, i.e.,

$$\frac{u_2}{\epsilon_2} f_{m,n}(\lambda) e^{-u_2 d} - \frac{u_2}{\epsilon_2} g(\lambda) - \frac{u_1}{\epsilon_1} h(\lambda) = 0 \quad (38.a)$$

$$f_{m,n}(\lambda) e^{-u_2 d} + g(\lambda) - h(\lambda) = 0 \quad (38.b)$$

Solving for $g(\lambda)$ and $h(\lambda)$, one obtains

$$g(\lambda) = \frac{\epsilon_1 u_2 - \epsilon_2 u_1}{\epsilon_1 u_2 + \epsilon_2 u_1} f_{m,n}(\lambda) e^{-u_2 d} \quad (39.a)$$

$$h(\lambda) = \frac{2\epsilon_1 u_2}{\epsilon_1 u_2 + \epsilon_2 u_1} f_{m,n}(\lambda) e^{-u_2 d} \quad (39.b)$$

The general solution of $TM(z)_{\pm m,n}$ fields for $z > d$ and $z < d$ are obtained by replacing $g(\lambda)$, $h(\lambda)$ in (33), (34) with the above formulas. We then have

$$\vec{A}_e^I = \hat{z} e^{\pm jm\phi} \int_0^\infty \frac{2\epsilon_1 u_2}{\epsilon_1 u_2 + \epsilon_2 u_1} f_{m,n}(\lambda) J_m(\lambda \rho) e^{-u_2 d - u_1(z-d)} d\lambda \quad (40.a)$$

$$\begin{aligned} \vec{A}_e^{II} = \hat{z} e^{\pm jm\phi} & \left[h_n^{(2)}(k_2 r) P_n^m(\cos \theta) \right. \\ & \left. + \int_0^\infty \frac{\epsilon_1 u_2 - \epsilon_2 u_1}{\epsilon_1 u_2 + \epsilon_2 u_1} f_{m,n}(\lambda) J_m(\lambda \rho) e^{-u_2(2d-z)} d\lambda \right] \quad (40.b) \end{aligned}$$

The solutions given in (40.a) and (40.b) for $TM(z)_{0,0}$ are exactly the same as those of the Sommerfeld integrals for a vertical electric dipole buried underground at a depth d , except for the differences between the vector potentials and the Hertz potentials, as given by (34).

3. The Rotating Magnetic Multipoles, $RTE_{\pm m}$ Fields

The primary fields of $RTE_{\pm m}$ for $m = 1, 2, 3, \dots$ in region II are

$$\begin{aligned} \vec{A}_m^{\text{Pri}} &= A_m^{\text{Pri}} (\hat{x} \pm j\hat{y}) = (\hat{x} \pm j\hat{y}) h_{m-1}^{(2)}(k_2 r) p_{m-1}^{m-1} (\cos \theta) e^{\pm j(m-1)\phi} \\ &= (\hat{\rho} \pm j\hat{\phi}) e^{\pm jm\phi} \int_0^\infty f_{m-1,m-1}(\lambda) J_{m-1}(\lambda \rho) e^{-u_2 |z|} d\lambda \end{aligned} \quad (41)$$

For the same reason pointed out by Sommerfeld for a horizontal dipole, it is required that the vector potentials of the secondary fields have a z -component in order to satisfy the boundary conditions of the air-ground interface. Therefore we assume the secondary fields in region II and the total field in region I as

$$\vec{A}_m^{\text{Sec}} = (\hat{\rho} \pm j\hat{\phi}) A_m^r + A_{mz}^r \hat{z} \quad (42.a)$$

$$\vec{A}_m^{\text{I}} = (\hat{\rho} \pm j\hat{\phi}) A_m^t + A_{mz}^t \hat{z} \quad (42.b)$$

The superscripts "r" and "t" in the above equations denote the reflected and the transmitted waves due to the lossy-ground surface.

The potential components can be represented by the following integrals of the cylindrical eigenfunctions:

$$A_m^r = e^{\pm jm\phi} \int_0^\infty g(\lambda) J_{m-1}(\lambda\rho) e^{-u_2(d-z)} d\lambda \quad (43.a)$$

$$A_{mz}^r = e^{\pm jm\phi} \int_0^\infty g_z(\lambda) J_m(\lambda\rho) e^{-u_2(d-z)} d\lambda \quad (43.b)$$

$$A_m^t = e^{\pm jm\phi} \int_0^\infty h(\lambda) J_{m-1}(\lambda\rho) e^{-u_1(z-d)} d\lambda \quad (43.c)$$

$$A_{mz}^t = e^{\pm jm\phi} \int_0^\infty h_z(\lambda) J_m(\lambda\rho) e^{-u_1(z-d)} d\lambda \quad (43.d)$$

The corresponding electric and magnetic fields for the RTE_{±m} modes are

$$\vec{E} = \pm j \frac{\partial A_m}{\partial z} \hat{\rho} + \frac{\partial A_m}{\partial z} \hat{\phi} \pm j \left[\frac{\partial A_m}{\partial \rho} - \frac{(m-1)}{\rho} A_m \right] \hat{z} \pm j \frac{m}{\rho} A_{mz} \hat{\rho} - \frac{\partial A_{mz}}{\partial \rho} \hat{\phi} \quad (44.a)$$

$$\begin{aligned} \vec{H} = j \frac{1}{\omega\mu} \left\{ \left[k^2 A_m + \frac{\partial^2 A_m}{\partial \rho^2} - \frac{(m-1)}{\rho} \frac{\partial A_m}{\partial \rho} + \frac{(m-1)}{\rho^2} A_m \right] \hat{\rho} + \frac{\partial^2 A_{mz}}{\partial \rho \partial z} \hat{\rho} \right. \\ \left. \pm j \left[k^2 A_m + \frac{m}{\rho} \frac{\partial A_m}{\partial \rho} - \frac{m(m-1)}{\rho^2} A_m \right] \hat{\phi} \pm j \frac{m}{\rho} \frac{\partial A_{mz}}{\partial z} \hat{\phi} \right. \\ \left. + \left[\frac{\partial^2 A_m}{\partial z \partial \rho} - \frac{(m-1)}{\rho} \frac{\partial A_m}{\partial z} \right] \hat{z} + \left[k^2 A_{mz} + \frac{\partial^2 A_{mz}}{\partial z^2} \right] \hat{z} \right\} \quad (44.b) \end{aligned}$$

The sufficient conditions for the continuity of the tangential electric field at $z = d$ are

$$\left[\frac{\partial A_m^{\text{Pri}}}{\partial z} + \frac{\partial A_m^{\text{r}}}{\partial z} \right]_{z=d} = \left[\frac{\partial A_m^{\text{t}}}{\partial z} \right]_{z=d} \quad (45.a)$$

$$\left[A_{mz}^{\text{r}} \right]_{z=d} = \left[A_{mz}^{\text{t}} \right]_{z=d} \quad (45.b)$$

The conditions for the continuity of the tangential magnetic field at $z = d$ are

$$\epsilon_2 \left[A_m^{\text{Pri}} + A_m^{\text{r}} \right]_{z=d} = \epsilon_1 \left[A_m^{\text{t}} \right]_{z=d} \quad (45.c)$$

$$\begin{aligned} \left[\frac{\partial A_{mz}^{\text{r}}}{\partial z} - \frac{\partial A_{mz}^{\text{t}}}{\partial z} \right]_{z=d} &= \left[\frac{\partial A_m^{\text{t}}}{\partial \rho} - \frac{(m-1)}{\rho} A_m^{\text{t}} \right]_{z=d} \\ &- \left[\frac{\partial A_m^{\text{Pri}}}{\partial \rho} + \frac{\partial A_m^{\text{r}}}{\partial \rho} - \frac{(m-1)}{\rho} A_m^{\text{Pri}} - \frac{(m-1)}{\rho} A_m^{\text{r}} \right]_{z=d} \end{aligned} \quad (45.d)$$

Substituting (43.a) and (43.c) into (45.a) and (45.c), one can solve for $g(\lambda)$ and $h(\lambda)$.

$$g(\lambda) = \frac{\epsilon_1 u_2 - \epsilon_2 u_1}{\epsilon_1 u_2 + \epsilon_2 u_1} f_{m-1, m-1}(\lambda) e^{-u_2 d} \quad (46.a)$$

$$h(\lambda) = \frac{2\epsilon_2 u_2}{\epsilon_1 u_2 + \epsilon_2 u_1} f_{m-1, m-1}(\lambda) e^{-u_2 d} \quad (46.b)$$

The functions $g_z(\lambda)$ and $h_z(\lambda)$ are derived from (45.b) and (43.d).

$$g_z(\lambda) = h_z(\lambda) = \frac{(\epsilon_1 - \epsilon_2) 2u_2 \lambda}{(u_1 + u_2)(\epsilon_1 u_2 + \epsilon_2 u_1)} f_{m-1, m-1}(\lambda) e^{-u_2 d} \quad (46.c)$$

The complete vector potentials of RTE_{±m} fields for $z > d$ and $z < d$ are readily obtained as follows:

$$\begin{aligned} \vec{A}_m^I = & (\hat{x} + j\hat{y}) e^{\pm j(m-1)\phi} \int_0^\infty \frac{2\varepsilon_2 u_2}{\varepsilon_2 u_1 + \varepsilon_1 u_2} f_{m-1,m-1}(\lambda) J_{m-1}(\lambda\rho) \\ & e^{-u_2 d - u_1(z-d)} d\lambda \\ & + \hat{z} e^{\pm jm\phi} \int_0^\infty \frac{(\varepsilon_1 - \varepsilon_2) 2u_2 \lambda}{(u_1 + u_2)(\varepsilon_2 u_1 + \varepsilon_1 u_2)} f_{m-1,m-1}(\lambda) J_m(\lambda\rho) \\ & e^{-u_2 d - u_1(z-d)} d\lambda \end{aligned} \quad (47.a)$$

$$\begin{aligned} \vec{A}_m^{II} = & (\hat{x} \pm j\hat{y}) h_{m-1}^{(2)}(k_2 r) P_{m-1}^{m-1}(\cos \theta) e^{\pm j(m-1)\phi} \\ & + (\hat{x} \pm j\hat{y}) e^{\pm j(m-1)\phi} \int_0^\infty \frac{\varepsilon_1 u_2 - \varepsilon_2 u_1}{\varepsilon_2 u_1 + \varepsilon_1 u_2} f_{m-1,m-1}(\lambda) \\ & J_{m-1}(\lambda\rho) e^{-u_2(2d-z)} d\lambda \\ & + \hat{z} e^{\pm jm\phi} \int_0^\infty \frac{(\varepsilon_1 - \varepsilon_2) 2u_2 \lambda}{(u_1 + u_2)(\varepsilon_2 u_1 + \varepsilon_1 u_2)} f_{m-1,m-1}(\lambda) \\ & J_m(\lambda\rho) e^{-u_2(2d-z)} d\lambda \end{aligned} \quad (47.b)$$

The solutions given in (47.a) and (47.b) for RTE_{±1} are the same as the Sommerfeld integrals of a buried horizontal magnetic dipole at a depth d rotating counter-clockwise (for RTE₊₁) or clockwise (for RTE₋₁).

4. The Rotating Electric Multipoles, $RTM_{\pm m}$ Fields

The general expressions for the primary fields of $RTM_{\pm m}$ for $m = 1, 2, 3, \dots$ in region II are identical to Eqs. (41), (42) and (43) by replacing A_m with A_e and A_{mz} with A_{ez} . The electric and magnetic fields for the $RTM_{\pm m}$ modes are

$$\begin{aligned} \vec{E} = -j \frac{1}{\omega \epsilon} \left\{ \left[k^2 A_e + \frac{\partial^2 A_e}{\partial \rho^2} - \frac{(m-1)}{\rho} \frac{\partial A_e}{\partial \rho} + \frac{(m-1)}{\rho^2} A_e \right] \hat{\rho} + \frac{\partial^2 A_{ez}}{\partial \rho \partial z} \hat{\rho} \right. \\ \left. \pm j \left[k^2 A_e + \frac{m}{\rho} \frac{\partial A_e}{\partial \rho} - \frac{m(m-1)}{\rho} A_e \right] \hat{\phi} \pm j \frac{m}{\rho} \frac{\partial A_{mz}}{\partial z} \hat{\phi} \right. \\ \left. + \left[\frac{\partial^2 A_e}{\partial z \partial \rho} - \frac{(m-1)}{\rho} \frac{\partial A_e}{\partial z} \right] \hat{z} + \left[k^2 A_{mz} + \frac{\partial^2 A_{mz}}{\partial z^2} \right] \hat{z} \right\} \quad (48.a) \end{aligned}$$

$$\begin{aligned} \vec{H} = \pm j \frac{\partial A_e}{\partial z} \hat{\rho} + \frac{\partial A_e}{\partial z} \hat{\phi} \pm j \left[\frac{\partial A_e}{\partial \rho} - \frac{(m-1)}{\rho} A_e \right] \hat{z} \\ \pm j \frac{m}{\rho} A_{ez} \hat{\rho} - \frac{\partial A_{ez}}{\partial \rho} \hat{\phi} \quad (48.b) \end{aligned}$$

The conditions for the continuity of the tangential electric field at $z = d$ are

$$\left[\frac{\partial A_e^{Pri}}{\partial z} + \frac{\partial A_e^r}{\partial z} \right]_{z=d} = \left[\frac{\partial A_e^t}{\partial z} \right]_{z=d} \quad (49.a)$$

$$\left[A_{ez}^r \right]_{z=d} = \left[A_{ez}^t \right]_{z=d} \quad (49.b)$$

The conditions for the continuity of the tangential magnetic field at $z = d$ are

$$\left[A_e^{\text{Pri}} + A_e^r \right]_{z=d} = \left[A_e^t \right]_{z=d} \quad (49.c)$$

$$\begin{aligned} \frac{1}{\epsilon_2} \left[\frac{\partial A_e^r}{\partial z} \right]_{z=d} - \frac{1}{\epsilon_1} \left[\frac{\partial A_e^t}{\partial z} \right]_{z=d} &= \frac{1}{\epsilon_1} \left[\frac{\partial A_e^t}{\partial \rho} - \frac{(m-1)}{\rho} A_e^t \right]_{z=d} \\ &- \frac{1}{\epsilon_2} \left[\frac{\partial A_e^{\text{Pri}}}{\partial \rho} + \frac{\partial A_e^r}{\partial \rho} - \frac{(m-1)}{\rho} A_e^{\text{Pri}} - \frac{(m-1)}{\rho} A_e^r \right]_{z=d} \end{aligned} \quad (49.d)$$

Enforcing the boundary conditions at the air-ground interface, we have

$$g(\lambda) = \frac{u_2 - u_1}{u_2 + u_1} f_{m-1, m-1}(\lambda) e^{-u_2 d} \quad (50.a)$$

$$h(\lambda) = \frac{2u_2}{u_2 + u_1} f_{m-1, m-1}(\lambda) e^{-u_2 d} \quad (50.b)$$

The functions $g_z(\lambda)$ and $h_z(\lambda)$ are obtained from (49.b) and (49.c).

$$g_z(\lambda) = h_z(\lambda) = \frac{(\epsilon_1 - \epsilon_2) 2u_2 \lambda}{(\epsilon_1 u_2 + \epsilon_2 u_1)(u_1 + u_2)} f_{m-1, m-1}(\lambda) e^{-u_2 d} \quad (50.c)$$

The complete vector potentials of $\text{RTM}_{\pm m}$ fields for $z > d$ and $z < d$ are obtained as follows:

$$\begin{aligned}
\vec{A}_e^I = & (x \pm jy) e^{\pm j(m-1)\phi} \int_0^\infty \frac{2u_2}{u_2 + u_1} f_{m-1,m-1}(\lambda) J_{m-1}(\lambda\rho) \\
& e^{-u_2 d - u_1(z-d)} d\lambda \\
& + \hat{z} e^{\pm jm\phi} \int_0^\infty \frac{(\epsilon_1 - \epsilon_2) 2u_2 \lambda}{(u_1 + u_2)(\epsilon_2 u_1 + \epsilon_1 u_2)} f_{m-1,m-1}(\lambda) J_m(\lambda\rho) \\
& e^{-u_2 d - u_1(z-d)} d\lambda \quad (51.a)
\end{aligned}$$

$$\begin{aligned}
\vec{A}_e^{II} = & (\hat{x} \pm j\hat{y}) h_{m-1}^{(2)}(k_2 r) p_{m-1}^{m-1}(\cos \theta) e^{\pm j(m-1)\phi} \\
& + (\hat{x} \pm j\hat{y}) e^{\pm j(m-1)\phi} \int_0^\infty \frac{u_2 - u_1}{u_2 + u_1} f_{m-1,m-1}(\lambda) J_{m-1}(\lambda\rho) \\
& e^{-u_2(2d-z)} d\lambda \\
& + \hat{z} e^{\pm jm\phi} \int_0^\infty \frac{(\epsilon_1 - \epsilon_2) 2u_2 \lambda}{(u_1 + u_2)(\epsilon_2 u_1 + \epsilon_1 u_2)} f_{m-1,m-1}(\lambda) J_m(\lambda\rho) \\
& e^{-u_2(2d-z)} d\lambda \quad (51.b)
\end{aligned}$$

The solutions given in (51.a) and (51.b) for $RTM_{\pm 1}$ are the same as the Sommerfeld integrals of a horizontal electric dipole, buried at a depth d , rotating counter-clockwise (for RTM_{+1}) or clockwise (for RTM_{-1}).

V. RESULTS

Utilizing the exterior modal fields generated by the Generalized Sommerfeld's Integrals, we are able to compute the scattered surface fields of a buried dielectric finite cylinder. The scattering configuration is shown in Fig. 4. Figure 5 shows the surface area where the fields will be computed. Figures 6-10 display the 3-D and contour plots of the scattered fields for 700 MHz and 1000 MHz.

VI. CONCLUSION

There are several numerical ways to solve radiation or scattering of electromagnetic fields near a lossy half space; notably, the method of moments [11], the extended boundary condition method [12], and the unimoment method [1]. All the methods in one way or another make use of the celebrated Sommerfeld integrals. The problem may also be considered as a limiting case of scattering by two non-concentric dielectric spheres with one imbedded in the other [13]. But that special approach entails such complex addition theorems and integral expressions that only the zeroth order azimuthal mode has been obtained.

In this paper, we have presented the groundwork for the application of the unimoment method to solve the scattering by buried obstacles, which requires the generalization of the Sommerfeld integrals for multipole sources. For successful numerical applications, we find it necessary to use spherical harmonics combined with z-directed potential vectors. The modal fields so obtained, however, appear to be inadequate in representing a conventional spherical mode. While the situation is duly corrected by addition of two horizontally rotating potential vectors for the lowest order azimuthal modes, further theoretical investigation of this unexpected discrepancy is definitely warranted.

The differences between the generalized Sommerfeld integrals we have put forth and those given by C.T. Tai have been discussed. It is interesting to note that should we use C.T. Tai's formulas in the unimoment method, the numerical objective would be to find $f(\lambda)$ in the integrals, such as,

$$\int_0^{\infty} f(\lambda) \frac{u_2 - u_1}{u_2 + u_1} J_m(\lambda \rho) e^{-u_2(2d-z)} d\lambda \quad (52)$$

And, using the formulas we derived, the numerical objective is to find A_n in the summation of the type

$$\sum_{n=m}^{\infty} A_n \int_0^{\infty} \frac{\lambda}{k u_2} P_n^m \left(-j \frac{u_2}{k_2} \right) \frac{u_2 - u_1}{u_2 + u_1} J_m(\lambda \rho) e^{-u_2(2d-z)} d\lambda \quad (53)$$

which is numerically more preferable than (52).

All the formulas presented have been numerically tested and found to be applicable to solve scattering by buried obstacles.

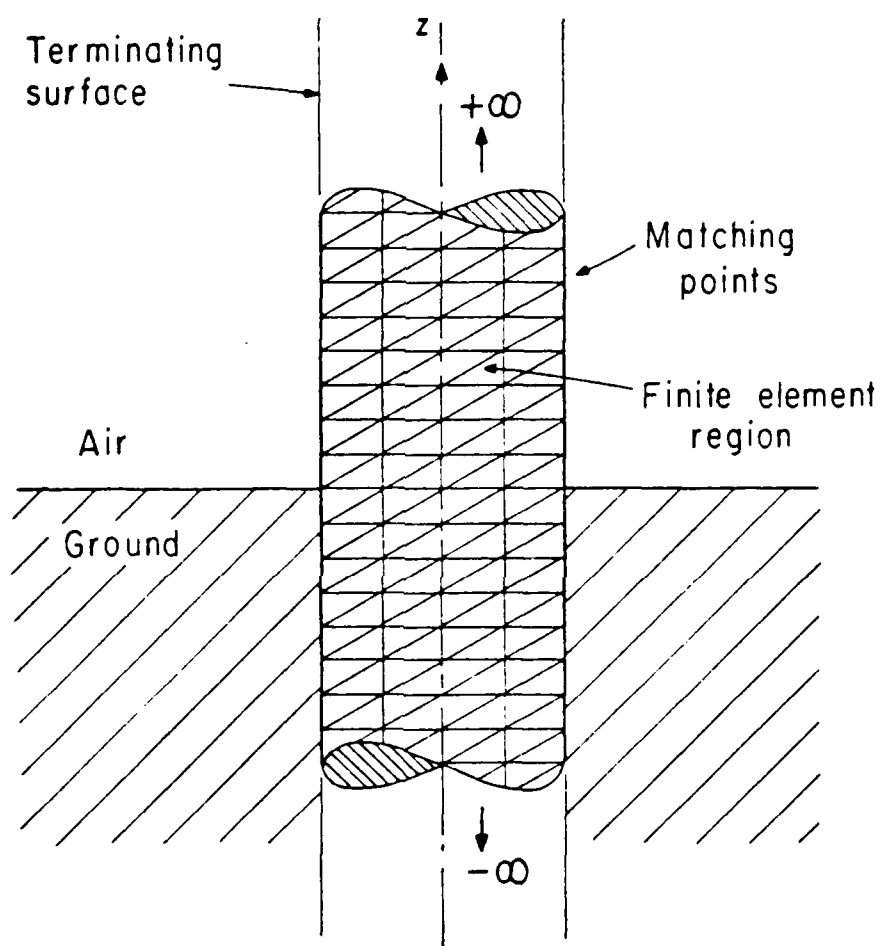


Figure 1a. The terminating surface of the finite element method using Tai's modal fields.

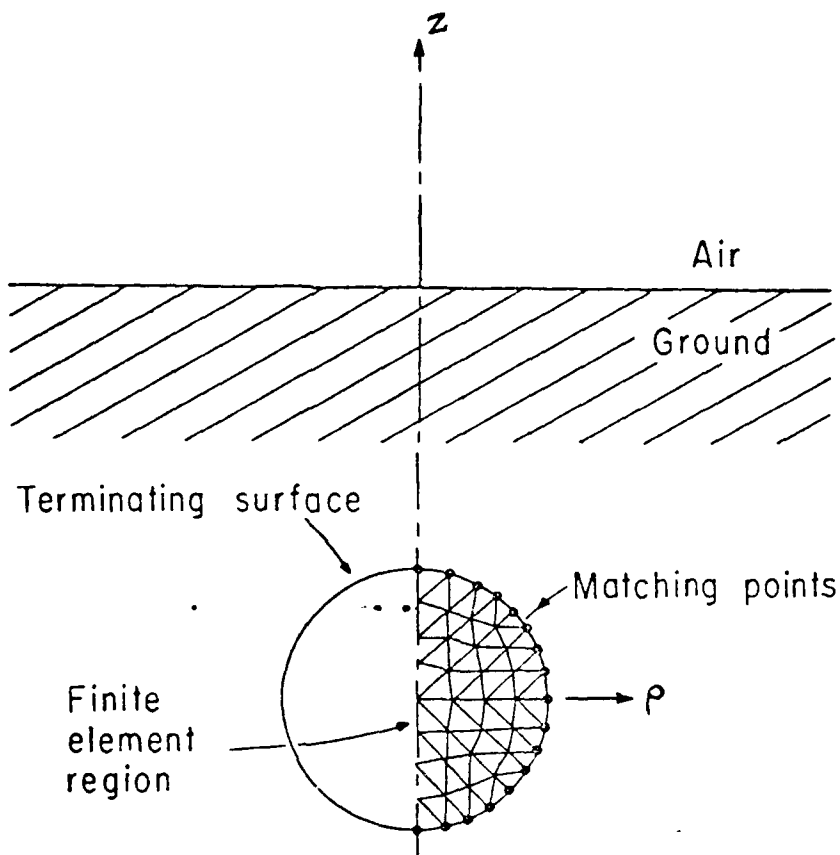


Figure 1b. The desired terminating surface of the finite element method using proper modal fields.

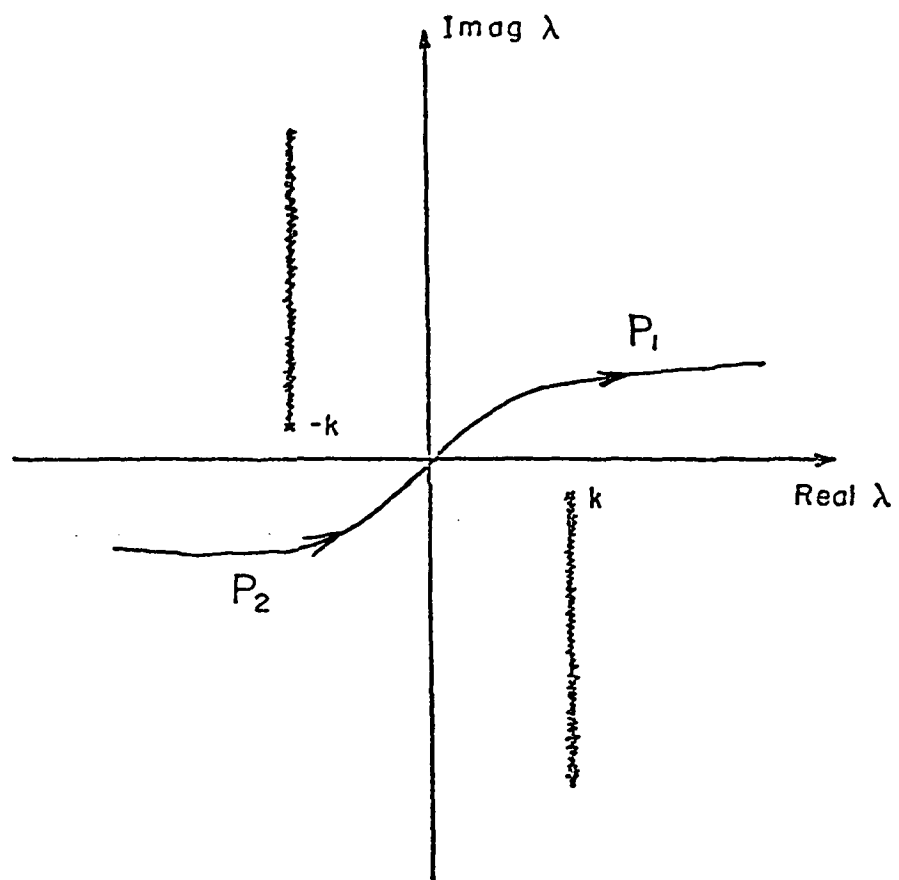


Figure 2. Branch Cuts and Permissible Paths of Integration.

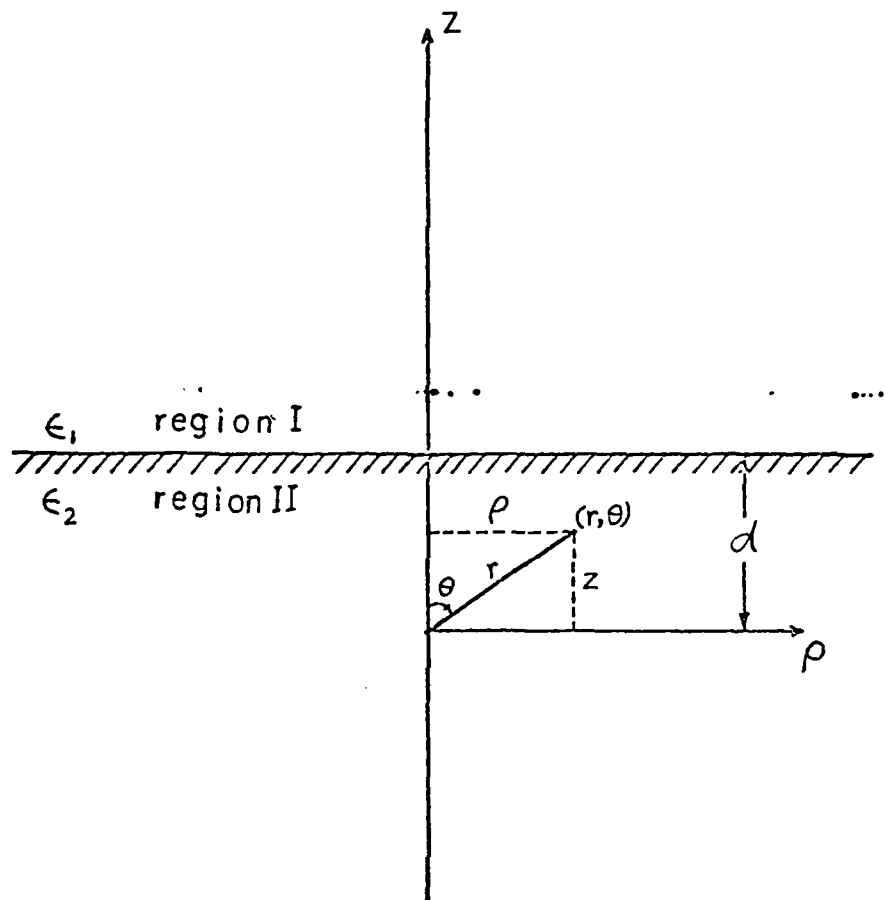
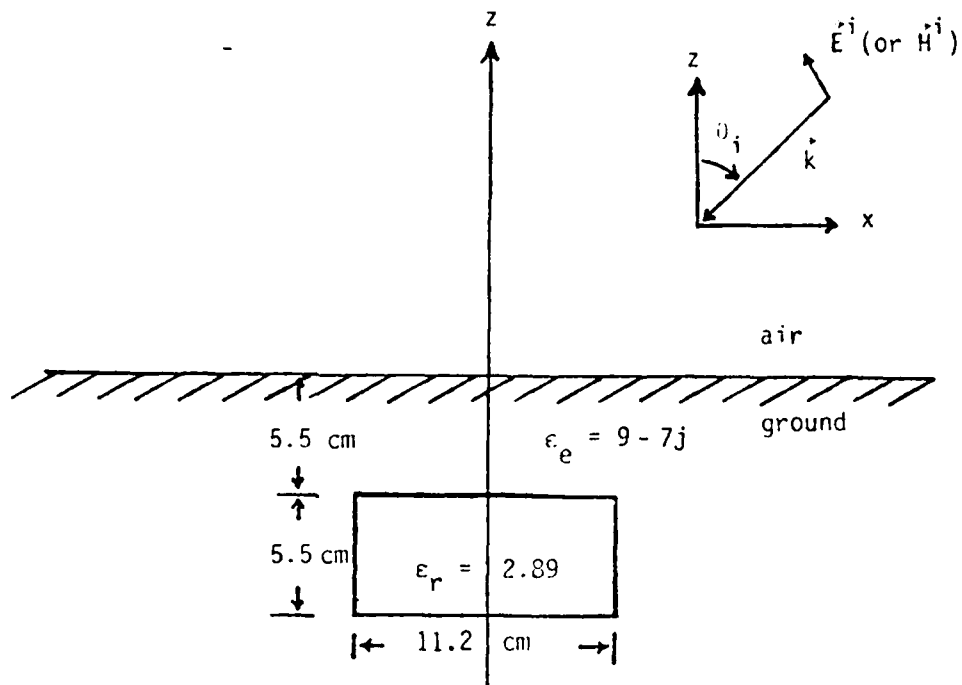


Figure 3. Coordinates in the Meridional Plane
 $(\phi=0 \text{ and } \phi=\pi)$.



Incident angles $\theta_i = 0^\circ, 45^\circ$

Incident polarizations = $\vec{E}^i$ or $\vec{H}^i$

Frequencies $f = 700, 1000$ MHz

Figure 4. scattering configurations and computational parameters

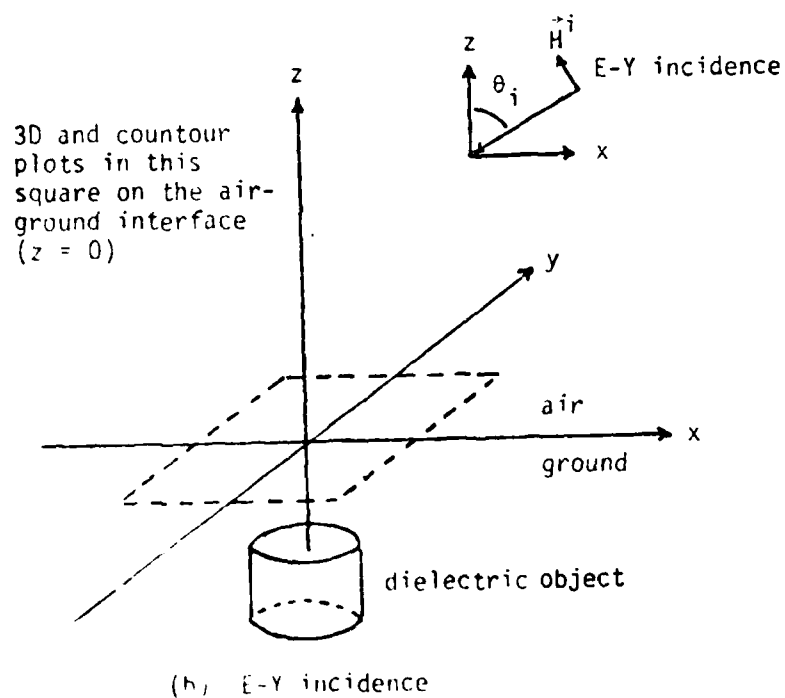
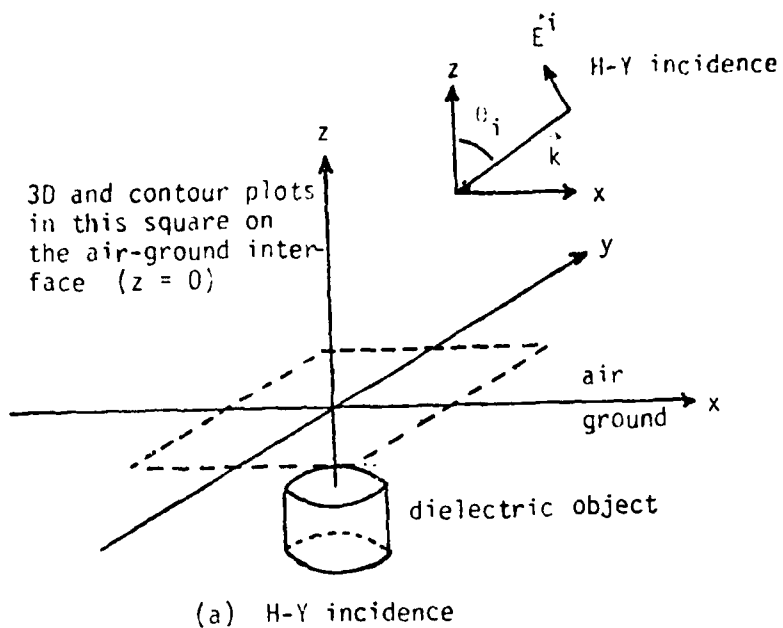
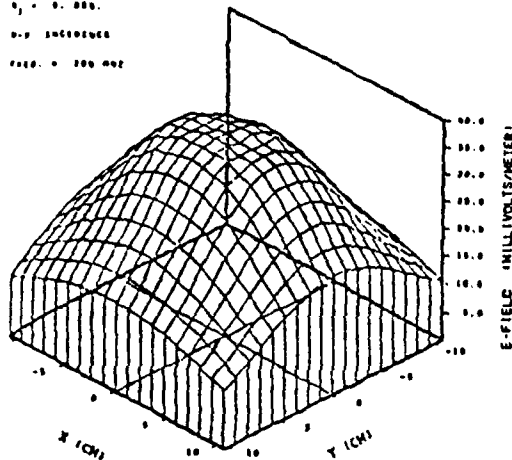


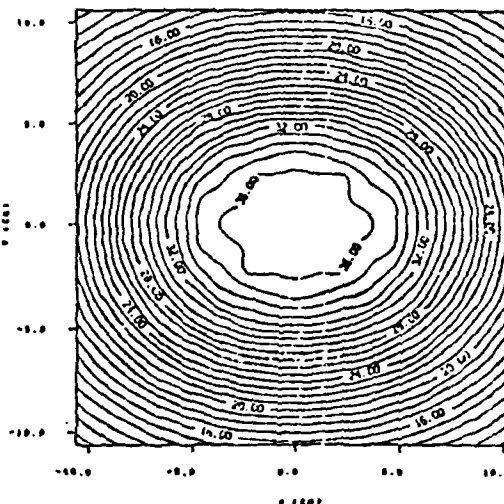
Figure 5. The square on the ground plane in which the 3D and contour plots of the scattered electric fields will be shown

$\theta_i = 0.000$
H-Y INCIDENCE
FREQ. = 700 MHz



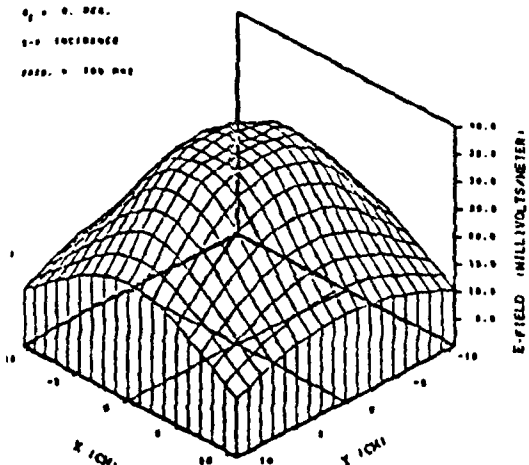
(a) 3-D plot for H-Y incidence

$\theta_i = 0.000$ H-Y INCIDENCE FREQ. = 700 MHz



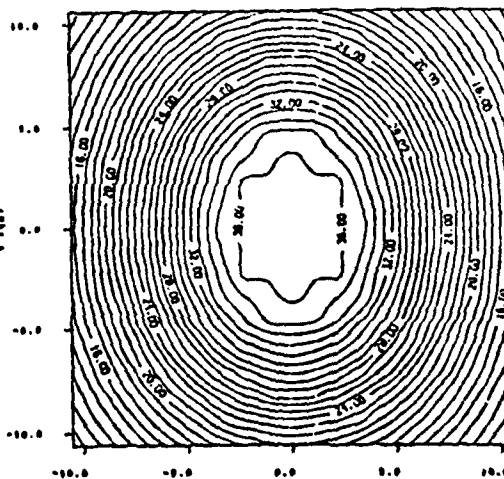
(b) Contour plot for H-Y incidence

$\theta_i = 0.000$
E-Y INCIDENCE
FREQ. = 700 MHz



(a) 3-D plot for E-Y incidence

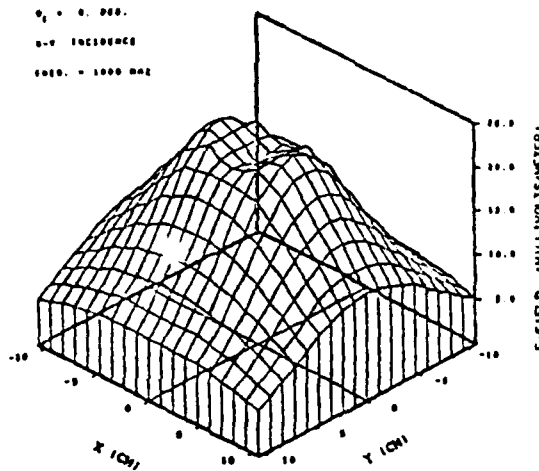
$\theta_i = 0.000$ E-Y INCIDENCE FREQ. = 700 MHz



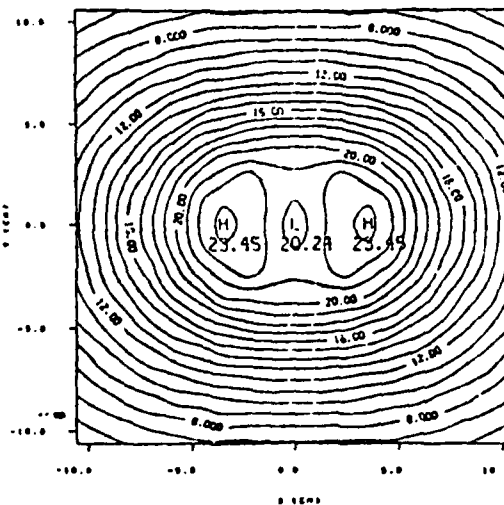
(b) Contour plot for E-Y incidence

Figure 6. 3-D and contour plots of scattered E-field amplitude on the earth surface ($\theta_i = 0^\circ$, frequency = 700 MHz)

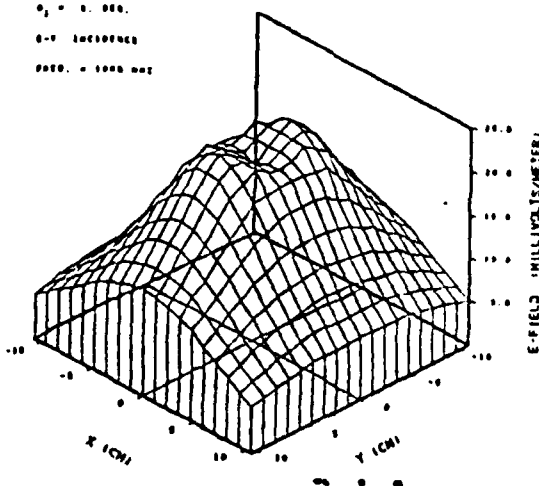
$\theta_i = 0.000$
 0-0 INCIDENCE
 FREQ. = 1000 MHz



$\theta_i = 0.000$ 0-0 INCIDENCE FREQ. = 1000 MHz



$\theta_i = 0.000$
 0-0 INCIDENCE
 FREQ. = 1000 MHz



$\theta_i = 0.000$ 0-0 INCIDENCE FREQ. = 1000 MHz

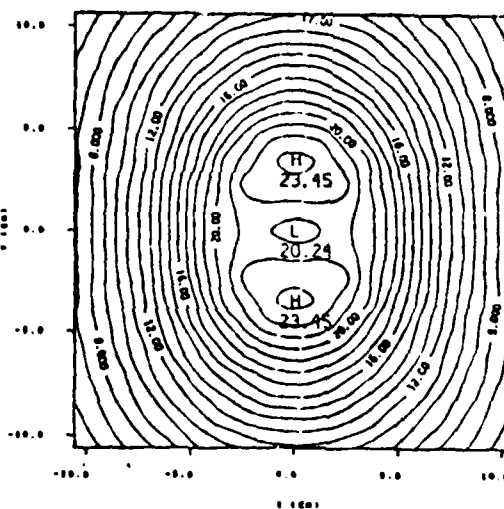


Figure 7. 3-D and contour plots of scattered E-field amplitude on the earth surface ($\theta_i = 0^\circ$, frequency = 1000 MHz)

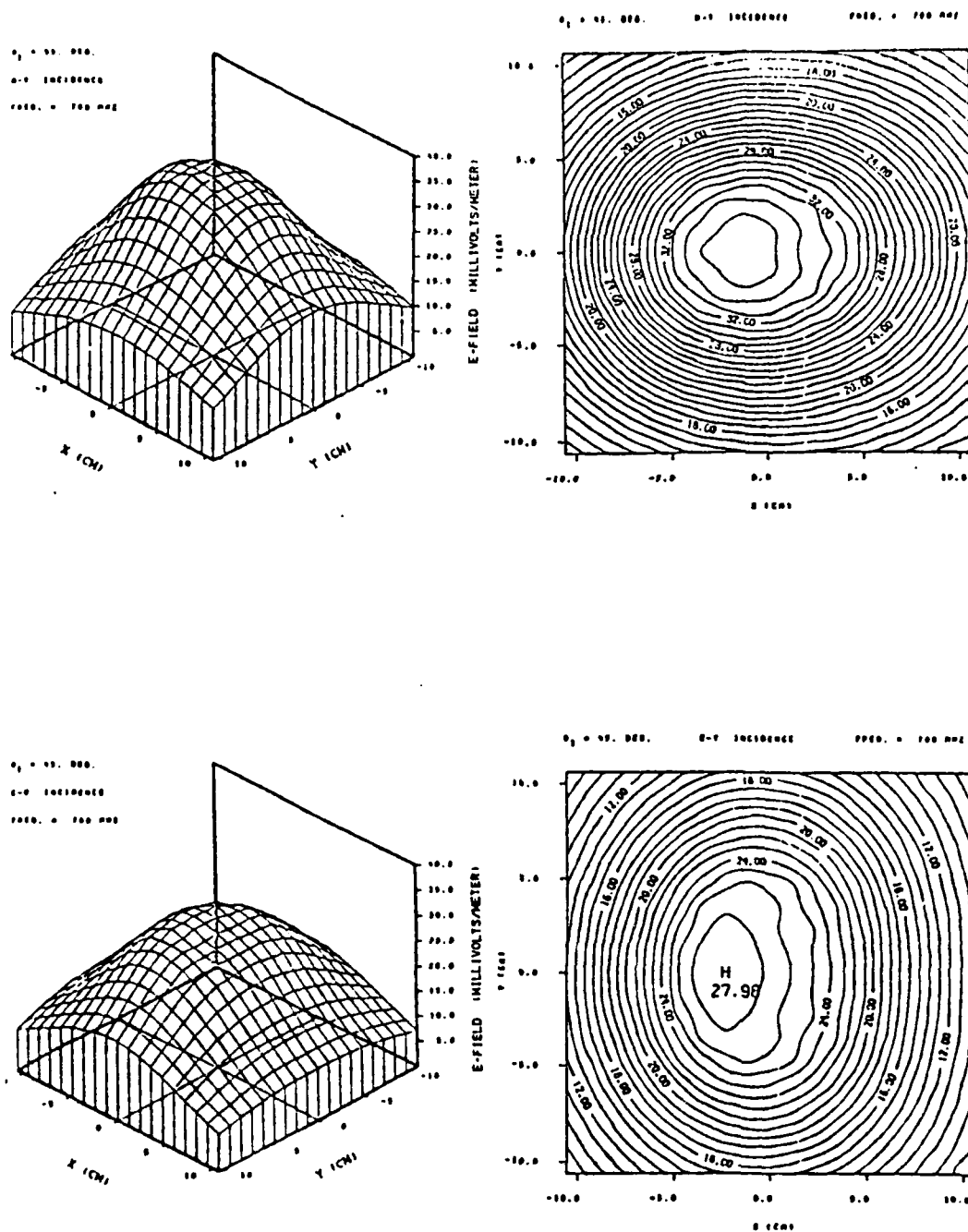


Figure 8. 3-D and contour plots of scattered E-field amplitude on the earth surface ($\theta_i = 45^\circ$, frequency = 700 MHz)

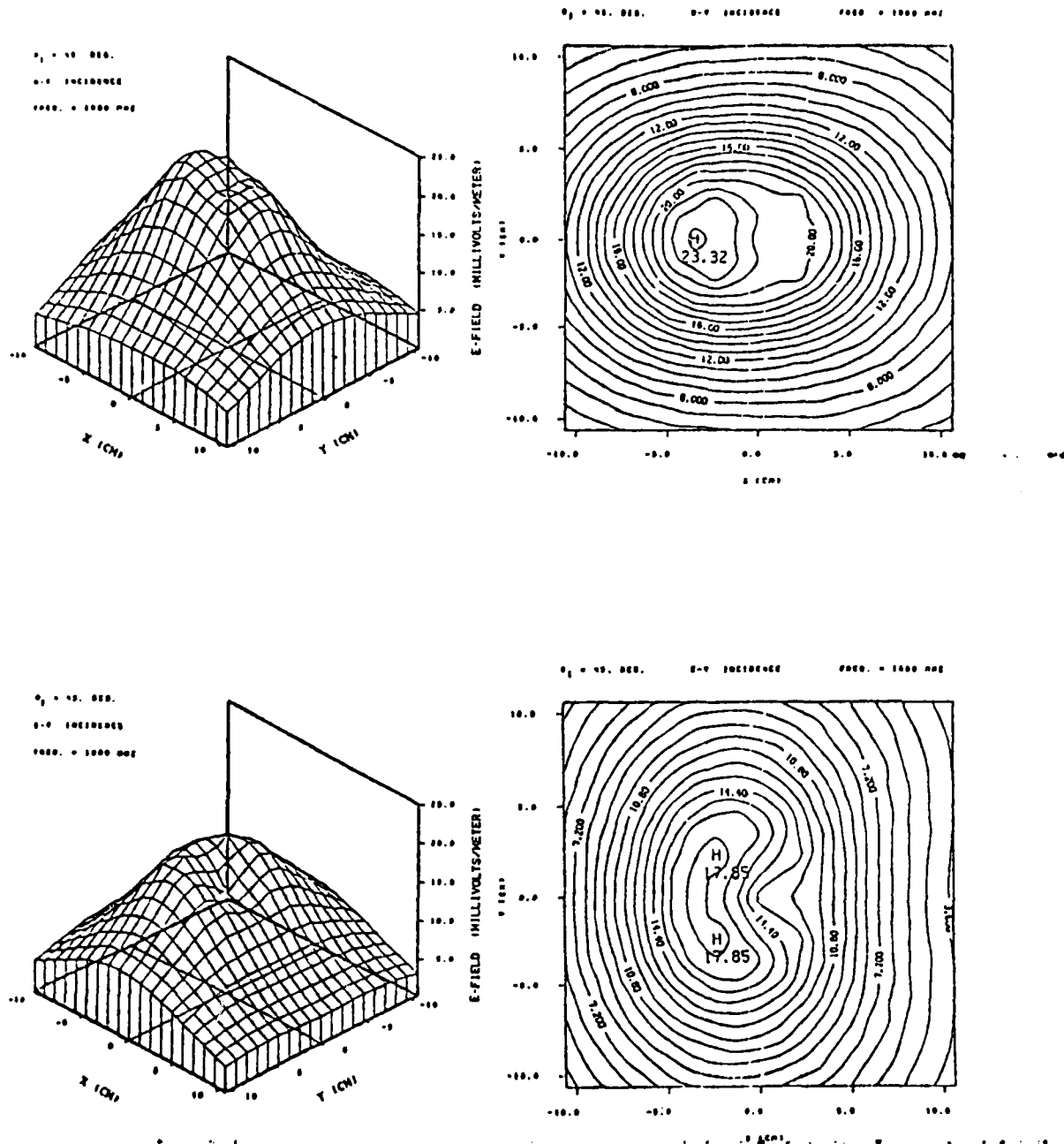


Figure 9. 3-D and contour plots of scattered E-field amplitude on the earth surface ($\theta_i = 45^\circ$, frequency = 1000 MHz)

APPENDIX A. Proof of Recurrence Relation 1

$$\begin{aligned}
 \frac{\partial}{\partial k\rho} [h_m^{(2)}(kr) P_m^m(\cos \theta)] &= \frac{dh_m^{(2)}(kr)}{dkr} P_m^m(\cos \theta) \sin \theta \\
 &- \frac{1}{kr} h_m^{(2)}(kr) \sin \theta \cos \theta \frac{dP_m^m(\cos \theta)}{d \cos \theta} \\
 &= \left[\frac{m}{kr} h_m^{(2)}(kr) - h_{m+1}^{(2)}(kr) \right] \cdot \frac{1}{(2m+1)} P_{m+1}^{m+1}(\cos \theta) \\
 &- \frac{1}{kr} h_m^{(2)}(kr) \cdot \left[\frac{m}{2m+1} P_{m+1}^{m+1}(\cos \theta) - \frac{m}{\sin \theta} P_m^m(\cos \theta) \right] \\
 &= \frac{m}{kr \sin \theta} h_m^{(2)}(kr) P_m^m(\cos \theta) - \frac{1}{(2m+1)} h_{m+1}^{(2)}(kr) P_{m+1}^{m+1}(\cos \theta)
 \end{aligned}$$

Recurrence Relation 1 is obtained by multiplying both sides of the above equation by $-(2m+1)$.

APPENDIX B. Proof of Recurrence Relation 2

$$\begin{aligned}
 \frac{\partial}{\partial kz} [h_n^{(2)}(kr) P_n^m(\cos \theta)] &= \frac{dh_n^{(2)}(kr)}{dkr} \cos \theta P_n^m(\cos \theta) \\
 &+ \frac{1}{kr} h_n^{(2)}(kr) \sin^2 \theta \frac{dP_n^m(\cos \theta)}{d \cos \theta} \\
 &= \left[\frac{n}{kr} h_n^{(2)}(kr) - h_{n+1}^{(2)}(kr) \right] \cos \theta P_n^m(\cos \theta) \\
 &+ \frac{1}{kr} h_n^{(2)}(kr) \cdot [(n+1) \cos \theta P_n^m(\cos \theta) - (n-m+1) P_{n+1}^m(\cos \theta)] \\
 &= -h_{n+1}^{(2)}(kr) \cdot \cos \theta P_n^m(\cos \theta) + \frac{(n+m)}{kr} h_n^{(2)}(kr) P_{n-1}^m(\cos \theta)
 \end{aligned}$$

$$\begin{aligned}
&= - \frac{(n-m+1)}{(2n+1)} h_{n+1}^{(2)}(kr) P_{n+1}^m(\cos \theta) \\
&\quad + \frac{(n+m)}{(2n+1)} \left[\frac{(2n+1)}{kr} h_n^{(2)}(kr) - h_{n+1}^{(2)}(kr) \right] P_{n-1}^m(\cos \theta) \\
&= - \frac{(n-m+1)}{(2n+1)} h_{n+1}^{(2)}(kr) P_{n+1}^m(\cos \theta) \\
&\quad + \frac{(n+m)}{(2n+1)} h_{n-1}^{(2)}(kr) P_{n-1}^m(\cos \theta)
\end{aligned}$$

Hence Recurrence Relation 2 is proved.

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