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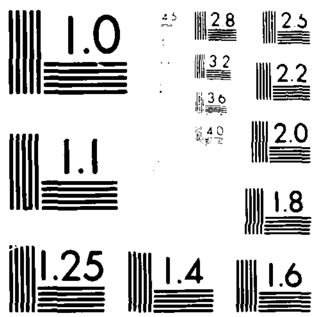
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EXPERIMENTAL RANDOM FATIGUE IN ELASTIC RANGE - FIRST ORDER MODELS

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EXPERIMENTAL RANDOM FATIGUE IN ELASTIC RANGE-FIRST ORDER MODELS

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Abstract

An experiment program, based on probabilistic parameters and experiment design, was conducted for the fatigue life under random vibrations. A batch of 24 specimens were tested to study the effects of 8 probabilistic parameters on the fatigue life in 3 designs. The response of each tested specimen was fitted with time series models from which the spectrum and spectral moments were computed. The magnitude levels of all 8 probabilistic parameters for every specimen were computed from its spectral moments and were coded. The first order models to predict fatigue life were obtained by regressing the log of fatigue life on coded probabilistic parameters. From the table of analysis of variance the F-ratio is computed to check if the model is acceptable. The tables of predicted lives together with residuals and 95% confidence intervals are also given for each model. The percent deviation from the actual life of the predicted life obtained from the first order models of life predicting equations were in general comparable with or better than the conventional deterministic fatigue tests. Furthermore, this method is in contrast with the existing method which is based on linear cumulative damage and cycle counting and normally involves several hundred percent error. Therefore the existing method remains academic as it is not suitable for application.

INTRODUCTION

The principle of linear damage accumulation based on equilvalent cycle counting has been repeatedly proved to be unreliable to predict fatigue life under random loading [1-12]. These references [1-12] are a selected group which have been discussed in [13]. In applying this linear cumulative damage principle, the random loading is evaluated in terms of equivalent deterministic cycles. The equivalent cycles for a random loading have been obtained by several investigators using different methods of cycle counting [13,14]. Each equivalent cycle contributes its share to damage based on deterministic tests of that amplitude. In the principle of linear damage accumulation, the shares of damage due to all the equivalent cycles are sumed up. The fatigue failure is predicted when the damage sum reaches a value of 1.0. The life is considered to be overestimated if the actual sum remains below 1.0 and is underestimated if the actual sum exceeds 1.0 at failure.

The above mentioned representative references dealing with the linear damage accumulation principle have reported deviations as follows. The factor of overestimation of the fatigue life using the peak counting method ranged from 1.5 to 3.5 in [1], from 2.0 to 3.5 in [2], and reached a value of 10.0 in [3]. Kowalewski [4], Locock and Williams [5], and Brown and Ikegemi [6] have used the mean crossing peak count method and have found that the fatigue life was overestimated by a factor which went up to 3.0 in [4], ranged from -5.0 to 15.0 in [5], and varied between 2.0 to 3.0 in [6]. Dowling [7] used the rainflow counting method and reported that the predicted fatigue life in some cases was as high as 3 times the actual life. The following investigators have used counting methods to evaluate the fatigue loading in equivalent cycles but the methods were neither mentioned nor described. Schultz [8] reported that the fatigue life was overestimated by a factor up to 10.0, and Buch [9] found that the life was overestimated by a factor ranging between ~2.95 to 6.0. In Smith and Malme [10] and Swanson [11] it was mentioned that the fatigue lives were overestimated but the factors of overestimation were not given. On the basis of their study Ford, et. al. [12] mentioned that the failure that most frequently occurs under a general loading spectrum is not predictable from the constant amplitude tests. The counting methods and the factors of overestimation obtained by several investigators using various counting methods are summarized in tabular form in [13].

The review above shows that the principle of linear damage accumulation is not accurate to predict the fatigue life under random loading and is therefore not reliable for industrial use. Recently a new history-dependent stochastic model of cumulative damage is being developed by Bogdanoff [15,16,17] by taking a comprehensive view of the entire failure process. The model includes major sources of variability encountered in the complex fatigue process by acquiring data of a specific type in addition to the data on time to failure. This data is essential for improvement in predictive accuracy. It also implies that the frequent inaccuracies encountered in life predictions when using current cumulative damage models cannot be substantially reduced using such models. In this report an entirely different phenomenological approach has been undertaken and an experiment program based on 8 probabilistic parameters was conducted using experiment design technique [18]. The models were developed which would produce a reliable estimate of fatigue life for the given levels of the probabilistic parameters.

Since the purpose of this research is to develop a novel methodology for random fatigue based on the probabilistic parameters and experiment design, the choice of the specimen materials, types of loadings (axial, shear or bending), and range of parameters is irrelevant. Therefore, they are arbitrarily chosen.

A comprehensive collection of references related to random fatigue prior to 1968 was provided by Swanson [19]. Bogdanoff [15,16,17] has highlighted many references particularly in the probabilistic and statistical aspects of cumulative damage in developing his new cumulative damage model.

I. EXPERIMENTS, PARAMETERS, DESIGNS AND MODELS

1. Experiment

The experiment program was conducted with 24 specimens numbered from 1-24 in 3 designs. A bending specimen of aluminum alloy 6061-T6 was chosen for the experiment. The same specimen and the experimental setup were used in a preliminary investigation of fatigue failure of materials under narrow band random vibrations [20,21]. In [20] the specimen and the experimental setup are described and pictorially illustrated. The dimensions of the specimen are so designed that a constant strain is obtained all along the test length under any bending load. The dimensions of the testing section of the specimen are shown in Fig. 1.

2. Parameters

Fig. 2 shows a typical random response signal in elastic range which is characterized by 8 probabilistic parameters. They are mean, variance, zero upcrossings, ε_f level upcrossings, duration of excursion above zero level, duration of excursion above ε_f level, band width, and average amplitude above ε_f level. In these variables ε_f is the strain level corresponding to a material life of 10^7 cycles in the deterministic fatigue tests, and ε_y is the yield point strain of the materials. A mathematical analysis of these 8 probabilistic parameters is given in [22]. Hereafter these probabilistic parameters will be simply referred to as variables as well.

3. Designs

The first design of the experiment consisted of 10 tests and formed a full factorial design with 2 center points. The second design, a central composite design with four center points, was formed by adding 8 more tests to the first design. The third design was formed by adding to the second design random replications of six tests from the first and the second designs.

a. Range The first design was based on the three different magnitude levels of the three variables; namely, mean, variance and zero upcrossings. Two more levels from each of the 3 variables were added in the second design so that one was lower and the other one was higher than the 3 previous levels. In the third design, 6 tests were replicated without introducing new levels of the variables. Thus, the second and the third designs had 5 levels from each of the above 3 variables. The lowest and the highest of the 5 levels represent the upper and the lower bounds of the range of variables.

The upper bounds of the mean and variance are limited such that the response remains within the elastic range. Otherwise the range of the variables can be arbitrarily chosen. The ranges of the mean, variance and zero upcrossings based on 5 levels are 110-1800 micro inches/inch, 733,373 - 2,250,955 (micro inches/inch)^2, and 7.0 - 13.8, respectively.

b. Code The above 3 variables which were controlled in the experiment were coded. The magnitude levels of the remaining 5 variables which could not be controlled in the experiment were computed from the response of each test specimen. The response of each test specimen was digitized and recorded. The time series models were fitted to the recorded response of every test specimen and the spectrum and spectral moments were computed from the time series model parameters [23]. The actual magnitude levels of the variables, were computed and their magnitude levels were coded. The variables and the actual levels, and coded variables and coded levels are shown in Table 1. Tables 2 and 3 show the actual levels and the corresponding coded levels, respectively, of all variables for all 24 tests.

4. Models

The first-order regression models were obtained for all 3 designs by regressing the log of actual fatigue lives, y, of the specimens on corresponding coded levels of variables. The regression models, which in our case represent life-predicting equations, predict log of life, $\hat{y}$, of the materials for any coded levels of variables. The predicted fatigue life, $\hat{T}$, was computed by taking antilog of the $\hat{y}$.

Two first order, life predicting equations were obtained for each of the 3 designs. The first was obtained by regressing y on 3 coded variables, and the second one was obtained by regressing all 8 coded variables. For each equation the tables of analysis of variance, and of predicted lives together with residuals and 95% confidence intervals, were constructed. In the analysis of variance the F-ratio was computed for every life predicting equation in order to determine whether the regression was effective and the model was acceptable. The confidence intervals were computed using the standard deviations of $\hat{y}$ and the t value from the t-table with degrees of freedom equal to that of residuals. The distribution of residuals for every equation were carefully studied for any trend or pattern present in the residuals. In case of any trend or pattern in the residuals the model was considered inadequate.

II. FULL FACTORIAL DESIGN WITH 2 CENTER POINTS

The two life predicting equations are obtained, using first the ten tests of the experiment, by regressing the log of life, y, on the coded variables under consideration. These ten tests form a full factorial design with 2 center points. The first equation consists of three variables which are mean, variance, and zero upcrossings. The second equation consists of all 8 variables. The tables of analysis of variance, and of predicted lives with 95% confidence interval are constructed for each equation.

1. Three Variables

The life predicting equation is obtained as

$$\hat{y} = 5.94 - 0.224x_1 - 0.690x_2 - 0.080x_3 \qquad (1)$$

The analysis of variance is given in Table 4. The F-ratio of 21.65

with 3 and 6 degrees of freedom is obtained. The corresponding F value from the F-table at 95% significance level is 4.76, which shows that the regression is effective and that the model is acceptable. The residual sum of squares is 0.4897 in comparison to a total of 5.7888, a 8.5%. The other 92.5% of the total sum of square is due to the regression. Analysis of variance indicates that the zero upcrossings have no significant effect on the fatigue life.

The predicted lives, together with residuals and 95% confidence intervals, are given in Table 5. A few of the confidence intervals are quite wide because of the small number of tests. The residuals appear to have a sinusoidal pattern. This indicates that some variables which may have significant effect on fatigue life are missing from the equation.

2. <u>All 8 Variables</u>

The life-predicting equation is obtained as

$$\hat{y} = 5.90 - 0.226x_1 - 0.687x_2 + 0.519x_3 + 0.261x_4 + 0.378x_5 - 0.382x_6 - 0.285x_7 - 0.509x_8 \qquad (2)$$

The analysis of variance is given in Table 6. The F-ratio for this equation is 4.796 with 8 and 1 degree of freedom. The corresponding F-value from the F-table at 95% significance level is 239. The comparison of the two F-values indicates that the regression above is not effective, even though 97.6% of the total sum of squares is due to regression. This is because the number of variables is too large in comparison to the number of tests.

The predicted lives, together with residuals and 95% confidence intervals, are given in Table 7. Most of the confidence intervals are fairly wide because the value of t used in computing the intervals is very high for 1 degree of freedom. The sinusoidal pattern in residuals has disappeared here.

III. CENTRAL COMPOSITE DESIGN WITH 4 CENTER POINTS

The full factorial design was augmented by 8 more test runs to make it a central composite design with 4 center points. This design has a total of 18 tests. The two life predicting equations obtained using 18 tests are given below.

1. Three Variables

The life predicting equation is obtained as

$$\hat{y} = 5.9008 - 0.1117x_1 - 0.7291x_2 - 0.0124x_3$$

The analysis of variance of the equation is given in Table 8. The F-ratio with 3 and 14 degrees of freedom was found to be 33.86. The corresponding F-value from the F-table of 95% significance level is 3.34. The comparison of two F-values indicates that the regression is effective and that the model is acceptable. In this case the effect of zero upcrossings on the fatigue life is also observed to be insignificant. The residual sum of squares is 1.1841 in comparison to a total of 9.7843, a 12.1%. The other 87.9% of the total sum of squares is due to the regression.

The predicted lives, together with residuals and 95% confidence intervals, are given in Table 9. The actual fatigue lives of a couple of specimens are out of the 95% confidence interval. This indicates that more variables are needed to improve the prediction.

2. All 8 Variables

The life-predicting equation is obtained as

$$\hat{y} = 5.823 + 0.039x_1 - 0.9040x_2 - 0.0817x_3 + 0.1267x_4 - 0.2310x_5 + 0.1000x_6 - 0.1118x_7 + 0.0145x_8 \quad (4)$$

The analysis of variance of the above equation is given in Table 10. The F-ratio for this equation is computed as 19.29 with 8 and 9 degrees of freedom. The corresponding value of F from the F-table at 95% significance level is 3.28. The comparison of the two F-values indicates that the regression is effective and that the model is acceptable. The residual sum of squares is 0.5394, in comparison to a total of 9.7843, a 5.5%. The other 94.5% of the total sum of squares is due to the regression. The analysis of variance indicates that zero upcrossings, duration of excursion above zero, and average amplitude above ε_f level have negligible effectson the fatigue life.

The predicted lives together with residuals and 95% confidence intervals are given in Table 11. In this case all the actual lives are within the predicted confidence interval. The confidence intervals are relatively wider in this case in comparison to the case of 3 variables because the number of variables is larger. Consequently, the number of degrees of freedom associated with t is small, which results in a large value of t.

IV. CENTRAL COMPOSITE DESIGN WITH 4 CENTER POINTS AND 6 REPLICATIONS

Six more tests are added to the central composite design by randomly replicating six points on the design. All test numbers and test points are shown in Fig. 3. Adding these six runs resulted in a total of 24 tests. Two life predicting equations were obtained for this design also. These equations and their analyses are given below:

1. Three Variables

The life-predicting equation is obtained as

$$\hat{y} = 5.8545 - 0.1205x_1 - 0.7595x_2 - 0.0063x_3 \qquad (5)$$

The analysis of variance of this equation is presented in Table 12. The computed F-ratio is 52.48 with 3 and 20 degrees of freedom. The corresponding table value of F from the F-table at 95% significance level is 3.10, and a comparison of the two F-values indicates that the regression is effective and the model is acceptable. The residual sum of squares is 1.4781 in comparison to a total of 13.1131, a 11.3%. The other 88.7% of the total sum of squares is due to the regression. Zero upcrossings were found to have an insignificant effect on the fatigue life.

The predicted lives, together with residuals and 95% confidence interval, are given in Table 13. There are six actual fatigue lives which fall out of the predicted confidence intervals. This observation also indicates a need for some more variables to be introduced to improve prediction, but it should be noticed that the confidence intervals are relatively narrow.

2. All 8 Variables

The life predicting equation is obtained as

$$\hat{y} = 5.826 - 0.037x_1 - 0.9194x_2 - 0.0268x_3 + 0.0782x_4$$
$$- 0.1390x_5 + 0.0530x_6 - 0.0830x_7 + 0.0437x_8 \qquad (6)$$

The analysis of variance of this equation is given in Table 14. The computed F-ratio is 23.78 with 8 and 15 degrees of freedom in comparison to F value of 2.64 from the F-table corresponding to the same degrees of freedom at 95% significance level. This shows that the regression

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The analysis of variance of this equation is given in Table 14. The computed F-ratio is 23.78 with 8 and 15 degrees of freedom in comparison to F value of 2.64 from the F-table corresponding to the same degrees of freedom at 95% significance level. This shows that the regression is effective and that the model is acceptable. The residual sum of squares is 0.9585 in comparison to a total of 13.1131, a 7.3%. The other 92.7% of the total sum of squares is due to the regression. In this case the 3 variables, zero upcrossings, the duration of excursion above zero, and the average amplitude above ε_f were also found to have an insignificant effect on the fatigue life.

The predicted lives, together with residuals and 95% confidence intervals, are given in Table 15. It is seen that one of the fatigue lives is out of the confidence interval and that the two others are on the border. The confidence intervals are fairly narrow. This is found to be the statistically best model out of all six models.

V. DISCUSSIONS AND CONCLUSIONS

An experiment program based on 8 probabilistic parameters and experiment design has been performed to predict fatigue life under random loading. The regression models of these probabilistic parameters for all 3 designs of 10, 18 and 24 tests have been developed to predict fatigue life under random vibrations. Unlike the linear damage accumulation theory, the approach described in this report is proved to be reliable and accurate.

It appears from the analysis of all the models that the following 4 variables, mean; variance; duration of excursion above ε_f level, and the band width showed significant effects on the fatigue life. Among these 4 variables the variance has the most significant effect, followed by band width and mean. The duration of excursion above ε_f level has the least. The regression on 4 variables reduced the residual sum of squares by 5.6% for the 18 tests and 3.6% for the 24 tests designs in comparison to the one obtained by regressing on variance alone as shown in Table 16. Any reduction in the residual sum of squares results in a corresponding improvement in the predicted lives, $\hat{y}$. Consequently, even a small improvement in $\hat{y}$ would produce a large improvement in predicted fatigue life, $\hat{T}$, as the models predict the log of fatigue life, $\hat{y}$.

A comparison with respect to the deviations of the predicted lives and residual sum of squares of all the models investigated in this report is given in Table 17. This table shows that the model of all 8 probabilistic parameters gives a lower residual sum of squares as compared to the one of 3 parameters for each of the 3 designs. The model of 8 probabilistic parameters for the first design of 10 tests is found inadequate as it did not pass the F-test. The better predicted lives and a lower residual sum of squares based on 24 tests is obtained for model of equation (6). This model was considered to be the best fit model. The predicted lives of the best fit model deviate from the actual lives within a range of -59.6% to 24.1%, with an average deviations of 17.4% on the negative side and 16.0 on the positive side. These deviations are small in comparison to the several hundred percent obtained in the linear damage accumulation theory.

The actual lives of almost all the tests is higher than the lower level of the 95% confidence interval of the best fit model. So the lower level can be considered to be a conservative estimate of the fatigue life. The predicted life, $\hat{T}$, by the model is the mean expected fatigue life of the material for the given levels of the variables.

The variables which showed significant effects on the fatigue life will be referred to as significant variables and considered for further studies. The analysis of a few more first order and some second-order models involving significant varibles will be re-posted in [24].

VI. SUMMARY

(1) An experiment based on 8 probabilistic parameters and experiment design has been performed to predict fatigue life under random loading. This method has been proved to be a reliable and accurate approach in contrast to the linear damage accumulation theory.

(2) The experiment was conducted in 3 designs and for each design 2 first order life predicting equations are obtained. The tables of analysis of variance, and of the predicted lives together with residuals and 95% confidence intervals, are constructed for each equation.

(3) From the table of analysis of variance, the F-ratio is computed to determine whether the regression is effective and the model acceptable.

(4) Four out of 8 variables have been found to have significant effects on the fatigue life. These 4 variables are the mean; variance; duration of excursion above ε_f level and the band width.

(5) For each of the 3 designs, the models of all 8 probabilistic parameters were found to have lower residual sum of squares and lower percent deviations in comparison to the models of 3 parameters.

(6) The model of 8 probabilistic parameters, based on 24 tests, was considered to be the best fit model. The predicted lives obtained by this model deviate from the actual lives within a range of -59.6% to 24.1%, with an average deviation of 17.4% on the negative side and 16.0% on the positive side.

ACKNOWLEDGMENT

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Table 1 Actual Levels, Coded Levels and Coded Variables

No.	Variables	Actual Level			Coded Level			Coded Variables
		Low	Center	High	Low	Center	High	
1	Mean	200.00	447.21	1000.0	-1	0	+1	x_1
2	Variance	929500.00	1284831.00	1776000.0	-1	0	+1	x_2
3	Zero Upcrossings	7.00	8.97	11.50	-1	0	+1	x_3
4	ε_f Level Upcrossings	3.00	4.24	6.0	-1	0	+1	x_4
5	Duration of Excursion Above Zero Level	0.65	0.743	0.85	-1	0	+1	x_5
6	Duration of Excursion Above ε_f Level	0.20	0.346	0.60	-1	0	+1	x_6
7	Band Width	0.935	0.95	0.965	-1	0	+1	x_7
8	Average Amplitude Above ε_f Level	2250.00	2395.75	2600.00	-1	0	+1	x_8

Table 2 Test Numbers and Actual Levels of Variables

Test No.	Actual Levels of Variables 1	2	3	4	5	6	7	8
1	218.29	880780.86	6.94	2.43	0.57	0.10	0.9189	2011.31
2	210.37	930122.76	14.00	1.00	0.56	0.09	0.9552	2104.36
3	196.42	1769478.37	11.66	6.91	0.58	0.18	0.9730	2386.04
4	1039.41	934007.27	7.30	4.29	0.85	0.38	0.9735	2333.94
5	999.22	1780656.42	5.52	4.01	0.77	0.75	0.9192	2712.85
6	1044.67	1820240.94	9.93	7.29	0.78	0.40	0.9674	2729.46
7	963.97	897936.15	4.40	2.37	0.84	0.31	0.9300	2313.13
8	210.12	1792791.17	7.07	4.15	0.57	0.16	0.9236	2413.17
9	447.64	1288513.58	9.09	4.01	0.65	0.18	0.9698	2183.95
10	436.46	1274963.37	8.92	4.12	0.63	0.21	0.9494	2228.45
11	445.35	1296591.74	9.26	4.65	0.64	0.20	0.9548	2259.02
12	465.25	1299401.32	9.39	4.68	0.65	0.20	0.9484	2193.31
13	1800.23	1255893.53	2.81	4.33	0.94	0.62	0.9472	3303.22
14	109.18	1287049.27	9.25	4.39	0.53	0.13	0.9559	2283.18
15	460.22	2371494.62	9.14	4.73	0.61	0.29	0.9602	2692.97
16	454.59	716608.00	8.63	4.44	0.67	0.15	0.9517	1949.01
17	452.13	1322376.67	3.37	6.31	0.71	0.11	0.9556	1973.89
18	453.28	1394215.78	13.16	2.36	0.63	0.21	0.9543	1931.09
19	112.24	1333860.83	10.06	6.37	0.53	0.15	0.9859	2193.51
20	202.90	1768328.57	11.40	6.84	0.58	0.18	0.9719	2316.86
21	999.67	1761848.86	9.47	7.16	0.77	0.39	0.9772	2814.93
22	199.47	1763345.15	6.99	4.00	0.56	0.18	0.9246	2359.52
23	998.08	950678.53	7.42	4.28	0.85	0.35	0.9757	2330.46
24	1008.71	1755965.70	5.27	3.96	0.76	0.38	0.9258	2773.35

Table 3 Test Number and Coded Levels of Coded Variables

Test No.	Coded Levels of Probabilistic Parameters							
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	-.891	-1.166	-1.035	-1.609	-1.979	-2.355	-2.100	-2.551
2	.937	-.998	1.792	-.161	-2.111	-2.454	1.314	1.486
3	1.022	.989	1.056	1.409	-1.849	-1.192	-1.136	-.188
4	1.048	-.985	-.831	.034	1.000	.168	1.555	-.493
5	0.999	1.027	-1.957	-.162	.234	1.406	-2.079	1.588
6	1.054	1.076	.409	1.562	.388	.262	1.157	1.672
7	.954	-1.107	-2.871	-1.678	.938	-.202	-1.339	-.617
8	-.939	1.029	-.960	-.066	1.979	-1.406	-1.777	.032
9	.001	.009	.053	-.087	-1.000	-1.192	-.032	-1.412
10	-.031	-.024	-.023	.263	.867	.911	.327	-.768
11	.005	.028	.127	.283	-1.116	-.973	.0988	-.945
12	.049	.035	.183	.057	-.966	-1.000	-.179	1.353
13	1.731	-.070	-4.677	.098	1.798	1.060	.400	4.311
14	1.752	.005	.127	.313	-2.522	-1.784	.6842	-.795
15	-.036	1.893	.075	1.145	-1.474	-.032	.380	1.486
16	.020	-1.803	-.157	-1.687	-.774	-1.524	.293	-2.986
17	.014	.089	-3.940	-4.158	-.342	-2.088	.354	-2.811
18	.017	.252	1.543	1.173	.213	-.885	2.357	-3.114
19	-1.718	.116	.461	.584	2.564	-1.560	.121	-1.352
20	.982	.987	.965	1.376	1.894	-1.222	1.451	.595
21	1.000	.975	.218	1.511	.292	.202	1.796	2.099
22	-1.003	.978	-1.006	.171	-2.718	-1.243	-1.708	-.343
23	.998	-.930	-.765	.024	.956	-.007	1.698	-.514
24	1.010	.965	-2.144	.199	.166	.158	-1.626	1.893

Table 4. Analysis of Variance of 10 Tests
First Order Model of 3 Variables
Life predicting equation:

$$\hat{y} = 5.94 - 0.224x_1 - 0.690x_2 - 0.0600x_3$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.3337	1	0.3337	
Due to Variance	4.8717	1	4.8717	
Due to Zero Upcrossings	0.0937	1	0.0937	
Due to Regression	5.2990	3	1.7663	21.65
Residuals	0.4897	6	0.0816	
Total	5.7888	9		

F-ratio is greater than the table value 4.76 with 3 and 6 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 5. Results of 10 Tests, First Order Model of 3 Variables
Life predicting equation:

$$\hat{y} = 5.94 - 0.224x_1 - 0.690x_2 - 0.0800x_3$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
1	1363.48	7.218	7.029	1128.90	0.188	6.573	7.218	715.50	2599.36
2	938.83	6.845	6.698	810.73	0.147	5.834	7.562	341.81	1923.17
3	165.08	5.106	5.404	222.29	-0.298	4.770	6.638	117.93	419.01
4	391.97	5.971	6.454	635.24	-0.483	5.831	7.707	340.76	1184.14
5	156.20	5.051	4.854	128.25	0.197	4.094	5.614	59.98	274.22
6	160.83	5.080	4.931	138.18	0.149	4.286	5.576	72.67	264.02
7	1011.42	6.919	6.723	831.31	0.196	5.918	7.527	371.88	1858.30
8	259.08	5.557	5.519	249.39	0.038	4.703	6.335	110.33	563.71
9	347.50	5.851	5.932	376.91	-0.081	5.595	6.269	268.99	528.13
10	370.33	5.914	5.968	390.72	-0.053	5.631	6.305	278.85	547.49

Table 6. Analysis of Variance of 10 Tests
First Order Model of 8 Variables
Life predicting equation:

$$\hat{y} = 5.90 - 0.226x_1 - 0.687x_2 + 0.519x_3 + 0.261x_4 + 0.378x_5 - 0.382x_6 - 0.285x_7 - 0.509x_8$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.334	1	0.334	
Due to Variance	4.872	1	4.872	
Due to Zero Upcrossings	0.094	1	0.044	
Due to Level Upcrossings Above ε_f	0.203	1	0.203	
Due to Duration of Excursion Above Zero	0.006	1	0.006	
Due to Duration of Excursion Above ε_f	0.073	1	0.073	
Due to Band Width	0.057	1	0.057	
Due to Average Amplitude Above ε_f	0.005	1	0.005	
Due to Regression	5.642	8	0.705	4.796
Residuals	0.147	1	0.147	
Total	5.789	9		

F-ratio is smaller than the table value 239 with 8 and 1 degrees of freedom at 95% significance level. So regression is not effective and the model is not accepted.

Table 7. Results of 10 Tests, First Order Model of 8 Variables
Life predicting equation:

$$y = 5.90 - 0.226x_1 - 0.687x_2 - 0.0519x_3 + 0.261x_4 - 0.378x_5 - 0.382x_6 - 0.285x_7 - 0.509x_8$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
1	1363.43	7.218	7.31	1495.18	-0.09	2.61	12.01	13.58	164591.20
2	938.83	6.845	6.71	820.57	0.13	2.28	9.48	9.78	13092.56
3	165.08	5.106	5.05	156.02	0.05	0.22	9.88	1.25	19494.74
4	391.97	5.971	6.20	492.75	-0.23	2.39	10.01	10.89	22287.91
5	156.20	5.051	5.09	162.39	-0.04	0.26	9.92	1.30	20298.05
6	160.83	5.080	4.99	146.94	0.10	0.29	9.69	1.33	16174.96
7	1011.42	6.919	6.71	820.57	0.21	2.65	10.78	14.07	47854.48
8	259.08	5.560	5.64	281.46	-0.08	0.94	10.34	2.56	30983.80
9	347.50	5.851	5.89	361.41	-0.04	1.06	10.72	2.89	45174.14
10	370.33	5.914	5.92	372.41	-0.006	1.09	10.75	2.98	46549.89

Table 8. Analysis of Variance of 18 Tests
First Order Model of 3 Variables
Life predicting equation:

$$\hat{y} = 5.9008 - 0.1117x_1 - 0.7291x_2 - 0.0124x_3$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.1212	1	0.1212	
Due to Variance	8.4720	1	8.4720	
Due to Zero Upcrossings	0.0070	1	0.0070	
Due to Regression	8.6002	3	2.8667	33.86
Residual	1.1841	14	0.0846	
Total	9.7843	17		

F-ratio is greater than the table value 3.34 with 3 and 14 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 9. Results of 18 Tests, First Order Model of 3 Variables
Life predicting equation:

$$\hat{y} = 5.9008 - 0.1117x_1 - 0.729x_2 - 0.013x_3$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
1	1363.43	7.218	6.863	956.23	0.354	6.571	7.155	714.28	1280.13
2	938.83	6.845	6.711	821.39	0.134	6.387	7.035	594.13	1135.58
3	165.08	5.106	5.281	196.57	-0.174	5.006	5.555	149.37	256.67
4	391.97	5.971	6.512	673.17	-0.541	6.227	6.797	506.09	895.41
5	156.20	5.051	5.016	150.81	0.035	4.649	5.383	104.50	217.63
6	160.83	5.080	4.994	147.53	0.087	4.692	5.296	109.02	199.63
7	1011.42	6.919	6.637	762.80	0.282	6.334	6.939	563.72	1032.20
8	259.08	5.557	5.267	193.83	0.290	4.956	5.578	142.02	264.55
9	347.50	5.851	5.894	362.85	-0.043	5.742	6.046	311.59	422.54
10	370.33	5.914	5.922	373.16	-0.008	5.772	6.072	321.13	433.61
11	346.00	5.846	5.879	357.45	-0.033	5.725	6.033	306.30	417.15
12	371.00	5.916	5.868	353.54	0.049	5.711	6.024	302.30	413.47
13	467.83	6.148	5.817	335.96	0.332	5.392	6.242	219.71	513.74
14	407.33	6.010	6.091	441.86	-0.082	5.752	6.430	314.85	620.12
15	98.67	4.592	4.524	92.20	0.068	4.191	4.856	66.12	128.57
16	1327.33	7.191	7.215	1399.67	-0.024	6.880	7.526	972.99	1854.96
17	189.08	5.242	5.883	358.88	-0.641	5.520	6.245	249.76	515.69
18	273.67	5.612	5.696	297.67	-0.084	5.469	5.923	237.13	373.69

Table 10. Analysis of Variance of 18 Tests
First Order Model of 8 Variables
Life predicting equation:

$$\hat{y} = 5.823 + 0.039x_1 - 0.904x_2 - 0.082x_3 + 0.127x_4 - 0.231x_5 + 0.100x_6 - 0.112x_7 + 0.015x_8$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.1212	1	0.1212	
Due to Variance	8.4720	1	8.4720	
Due to Zero Upcrossings	0.0070	1	0.0070	
Due to ε_f Level Upcrossings	0.1440	1	0.1440	
Due to Duration of Excursion Above Zero	0.0692	1	0.0692	
Duration of Excursion Above ε_f Level	0.2291	1	0.2291	
Due to Band Width	0.1988	1	0.1988	
Due to Average Amplitude Above ε_f Level	0.0036	1	0.0036	
Due to Regression	9.2449	8	1.1556	19.29
Residual	0.5394	9	0.0599	
Total	9.7843	17		

F-ratio is greater than the table value 3.23 with 8 and 9 degrees of freedom at 95% significance level. So regression is effective and the model is accepted.

Table 11. **Results of 18 Tests, First Order Model of 8 Variables**
Life predicting equation:

$$\hat{y} = 5.823 + 0.039x_1 - 0.904x_2 - 0.082x_3 - 0.127x_4 - 0.231x_5 + 0.100x_6 - 0.112x_7 + 0.015x_8$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
1	1363.43	7.218	7.142	1241.41	0.076	6.845	7.515	938.96	1835.79
2	938.83	6.845	6.589	727.05	0.256	6.193	6.985	489.39	1080.14
3	165.08	5.106	5.413	224.30	-0.307	5.028	5.797	152.70	329.49
4	391.97	5.971	6.431	620.79	-0.460	6.137	6.431	462.76	621.08
5	156.20	5.051	5.095	163.20	-0.044	4.575	5.815	97.05	335.22
6	160.83	5.080	4.887	132.56	0.193	4.500	5.274	90.03	195.16
7	1011.42	6.919	6.787	886.25	0.132	6.445	7.129	629.82	1247.08
8	259.08	5.557	5.441	230.67	0.116	5.124	5.758	160.06	316.61
9	347.50	5.851	5.953	384.91	-0.084	5.609	6.297	272.92	542.85
10	370.33	5.914	5.722	305.52	0.192	5.238	6.207	188.28	495.75
11	346.00	5.846	5.958	386.84	-0.112	5.759	6.157	317.01	472.04
12	371.00	5.916	5.909	368.34	0.007	5.699	6.119	298.46	454.58
13	467.83	6.148	6.059	427.95	0.089	5.577	6.541	264.33	692.84
14	407.33	6.010	6.096	444.08	-0.086	5.655	6.537	285.69	690.27
15	98.67	4.592	4.575	97.03	0.017	4.213	4.937	67.56	139.34
16	1327.33	7.191	7.203	1343.45	-0.012	6.870	7.536	963.42	1873.41
17	189.08	5.242	5.328	206.03	-0.085	4.785	5.871	119.72	354.56
18	273.67	5.612	5.501	244.94	0.111	5.060	5.942	157.58	380.73

Table 12. Analysis of Variance of 24 Tests
First Order Model of 3 Variables
Life predicting equation:

$$\hat{y} = 5.855 - 0.121x_1 - 0.760x_2 - 0.006x_3$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.1172	1	0.1172	
Due to Variance	11.5157	1	11.5157	
Due to Zero Upcrossings	0.0021	1	0.0021	
Due to Regression	11.6350	3	3.8783	
Residuals	1.4781	20	0.0739	52.48
Total	13.1131	23		

F-ratio is greater than the table value 3.10 with 3 and 20 degrees of freedom at 95% significance level. So regression is effective and the model is accepted.

Table 13. Results of 24 Tests, First Order Equation of 3 Variables
Life predicting equation:

$$\hat{y} = 5.855 - 0.121x_1 - 0.760x_2 - 0.006x_3$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
1	1363.43	7.218	6.854	947.66	0.364	6.612	7.096	743.99	1207.10
2	938.83	6.845	6.714	823.86	0.131	6.449	6.983	629.49	1078.15
3	165.08	5.106	5.220	184.93	-0.113	5.019	5.420	151.37	225.94
4	391.97	5.971	6.482	653.28	-0.510	6.054	6.909	425.97	1001.88
5	156.20	5.051	4.942	140.05	0.109	4.194	5.240	66.27	188.73
6	160.83	5.080	4.908	135.37	0.173	4.672	5.144	106.94	171.35
7	1011.42	6.919	6.598	733.63	0.321	6.346	6.850	569.98	944.26
8	259.08	5.557	5.192	179.83	0.365	4.968	5.415	143.85	224.80
9	347.50	5.851	5.847	346.19	0.004	5.724	5.970	306.10	391.50
10	370.33	5.914	5.877	356.74	0.038	5.754	6.000	315.43	403.46
11	346.00	5.846	5.833	341.38	0.013	5.708	5.958	301.22	386.90
12	371.00	5.916	5.821	337.31	0.095	5.694	5.948	297.01	383.08
13	467.83	6.148	5.729	307.66	0.419	5.385	6.073	218.07	434.06
14	407.33	6.010	6.061	428.80	-0.051	5.815	6.307	335.24	548.48
15	98.67	4.592	4.421	83.18	0.171	4.171	4.671	64.76	106.84
16	1327.33	7.191	7.222	1369.22	-0.032	6.949	7.495	1041.83	1799.50
17	189.08	5.242	5.810	333.62	-.568	5.509	6.110	247.06	450.51
18	273.67	5.612	5.651	284.58	-0.039	5.459	5.843	234.88	344.78
19	430.00	6.064	5.970	391.51	0.093	5.732	6.208	308.65	496.61
20	157.17	5.057	5.217	184.38	-.160	5.021	5.413	151.55	224.32
21	127.00	4.844	4.992	147.23	-.148	4.773	5.211	118.27	183.28
22	137.42	4.923	5.239	188.48	-.316	5.012	5.466	150.15	236.60
23	484.42	6.183	6.445	629.55	-.263	6.226	6.664	505.71	783.70
24	136.75	4.918	5.014	150.51	-0.095	4.785	5.243	119.65	189.32

Table 14. Analysis of Variance of 24 Tests
First Order Model of 8 Variables
Life predicting equation:

$$\hat{y} = 5.826 - 0.037x_1 - 0.919x_2 - 0.027x_3 + 0.078x_4 - 0.139x_5 + 0.053x_6 - 0.083x_7 + 0.044x_8$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.1172	1	0.1172	
Due to Variance	11.5157	1	11.5157	
Due to Zero Upcrossings	0.0021	1	0.0021	
Due to ε_f Level Upcrossings	0.0707	1	0.0707	
Due to Duration of Excursion Above Zero	0.0554	1	0.0554	
Due to Duration of Excursion Above ε_f Level	0.2026	1	0.2026	
Due to Band Width	0.1556	1	0.1556	
Due to Average Amplitude Above ε_f Level	0.0351	1	0.0351	
Due to Regression	12.1545	8	1.5193	23.78
Residuals	0.9585	15	0.0639	
Total	13.1131	23		

F-ratio is greater than the table value 2.64 with 8 and 15 degrees of freedom at 95% significance level. So regression is effective and the model is accepted.

Table 15. Results of 24 Tests, First Order Model of 8 Variables. Life predicting equation:

$$\hat{y} = 5.826 - 0.037x_1 - 0.919x_2 - 0.027x_3 + 0.078x_4 - 0.139x_5 + 0.053x_6 - 0.083x_7 + 0.044x_8$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
1	1363.43	7.218	7.045	1147.11	0.173	6.721	7.369	829.72	1585.91
2	938.83	6.845	6.687	801.91	0.158	6.338	7.036	565.39	1137.68
3	165.08	5.106	5.316	203.57	-0.209	4.962	5.670	142.92	289.96
4	391.97	5.971	6.438	625.16	-0.467	6.192	6.683	489.28	798.76
5	156.20	5.051	5.065	158.38	-0.014	4.568	5.561	96.40	260.22
6	160.83	5.080	4.847	127.36	0.234	4.555	5.139	95.11	170.54
7	1011.42	6.919	6.698	810.78	0.221	6.453	6.943	634.56	1035.94
8	259.08	5.557	5.281	196.57	0.276	5.036	5.526	153.84	251.15
9	347.50	5.851	5.950	383.75	-0.099	5.637	6.263	280.55	524.92
10	370.23	5.914	5.738	310.44	0.176	5.338	6.183	208.02	484.63
11	346.00	5.846	5.873	353.31	-0.027	5.696	6.050	297.71	424.06
12	371.00	5.916	5.829	340.02	0.088	5.639	6.018	281.28	411.03
13	467.83	6.148	5.923	373.53	0.225	5.482	6.364	240.30	580.63
14	407.33	6.010	6.071	433.11	-0.061	5.741	6.401	311.28	602.63
15	98.67	4.592	4.424	83.43	0.168	4.123	4.724	61.78	112.67
16	1327.33	7.191	7.227	1376.09	-0.036	6.921	7.534	1012.46	1870.32
17	189.08	5.242	5.308	154.16	-0.066	4.783	5.832	119.55	341.12
18	273.67	5.612	5.434	229.06	0.178	5.023	5.845	151.82	345.60
19	430.00	6.064	6.020	411.58	0.044	5.728	6.312	307.37	551.12
20	152.17	5.057	5.082	161.10	0.025	4.839	5.324	126.35	205.39
21	127.00	4.844	4.919	136.87	0.074	4.644	5.194	103.97	180.17
22	137.42	4.923	5.367	214.22	-0.444	5.096	5.638	163.43	280.80
23	484.42	6.183	6.371	584.64	-0.188	6.130	6.612	459.53	743.82
24	136.75	4.918	5.147	171.91	-0.229	4.883	5.411	131.99	223.91

Table 16. Comparison of Regression Sum of Squares for 18 and 24 Tests

Source	Sum of Squares	
	18 Tests	24 Tests
Due to Variance	8.4720	11.5157
Due to Mean	0.1212	0.1172
Due to Duration of Excursion Above ε_f Level	0.2291	0.2026
Due to Band Width	0.1988	0.1556
Due to Regression		
Variance Alone	8.4720	11.5157
All 4 Significant Variables	9.0211	11.9911
Total Sum of Squares	9.7843	13.1131
	Regression Sum of Squares Percent of the Total	
Variance Alone	86.6	87.8
All 4 Significant Variables	92.2	91.4
	Percent Reduction in	
Residual Sum of Squares	5.6	3.6

Table 17. Comparison of Percent Deviations of Predicted Lives and Residual Sum of Squares for Six Models

Test No.	Actual Life T	Percent Deviations of Predicted Lives					
		Eq(1)	Eq(2)	Eq(3)	Eq(4)	Eq(5)	Eq(6)
1	1363.43	17.2	-9.7	29.9	8.9	32.7	15.9
2	938.83	13.6	12.6	12.6	22.6	12.2	14.6
3	165.08	-34.7	5.5	-19.1	-35.9	-12.0	-23.1
4	391.97	-62.1	-25.7	-71.9	-58.4	-59.0	-59.6
5	156.20	17.9	-4.0	3.4	-4.5	10.3	-1.4
6	160.83	14.1	8.6	8.2	17.6	15.8	20.8
7	1011.42	17.8	18.9	24.6	12.4	27.5	19.8
8	259.08	3.7	-8.6	25.2	11.0	30.6	24.1
9	347.50	-8.5	-4.0	-4.4	-10.8	0.4	-10.4
10	370.33	-5.5	-0.6	-0.8	17.5	3.7	16.2
11	346.00			-3.3	-11.8	1.3	-2.7
12	371.00			4.7	0.7	9.1	8.4
13	467.83			28.2	8.5	34.2	20.2
14	407.33			-8.3	-9.0	-5.3	-6.3
15	98.67			6.6	5.7	15.7	15.5
16	1327.33			-5.5	-1.2	-3.2	-3.7
17	189.08			-89.8	-9.0	-76.5	-6.8
18	273.67			-8.8	10.5	-4.0	16.3
19	430.00					8.6	4.3
20	157.17					-17.3	-2.4
21	127.00					-16.0	-7.8
22	137.42					-37.2	-55.9
23	484.42					-30.0	-20.7
24	136.75					-13.4	-25.7

	Average Deviations					
Negative side	27.7	8.8	23.5	17.6	24.9	17.4
Positive side	14.1	11.4	15.9	11.5	15.5	16.0
	Residuals					
Percent residual sum of squares of the total	8.5	2.4	12.1	5.5	11.3	7.3

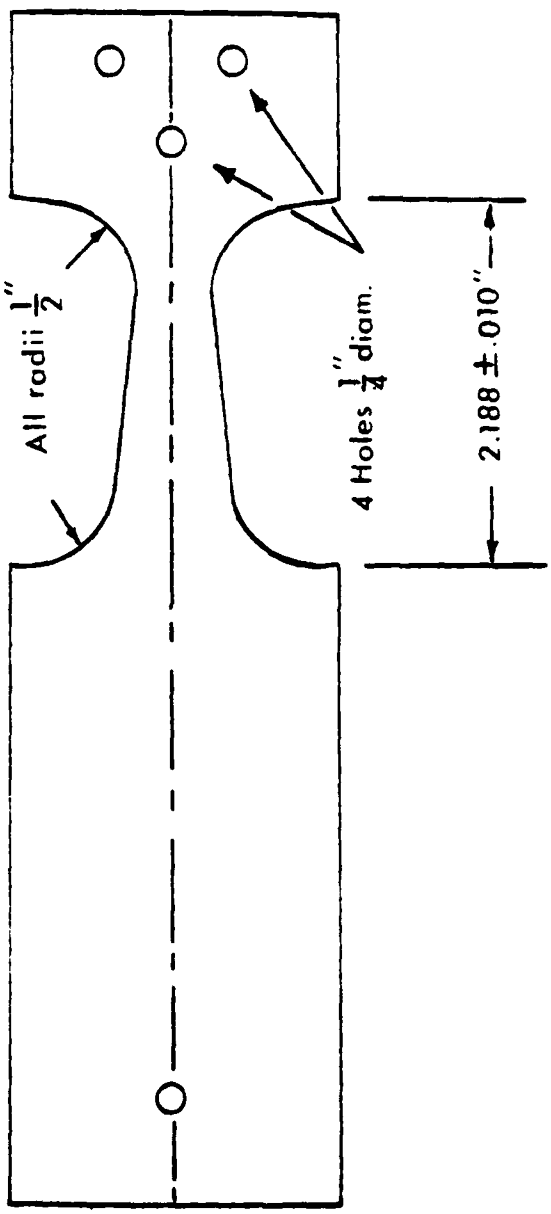

Fig. 1 Random Fatigue Test Specimen

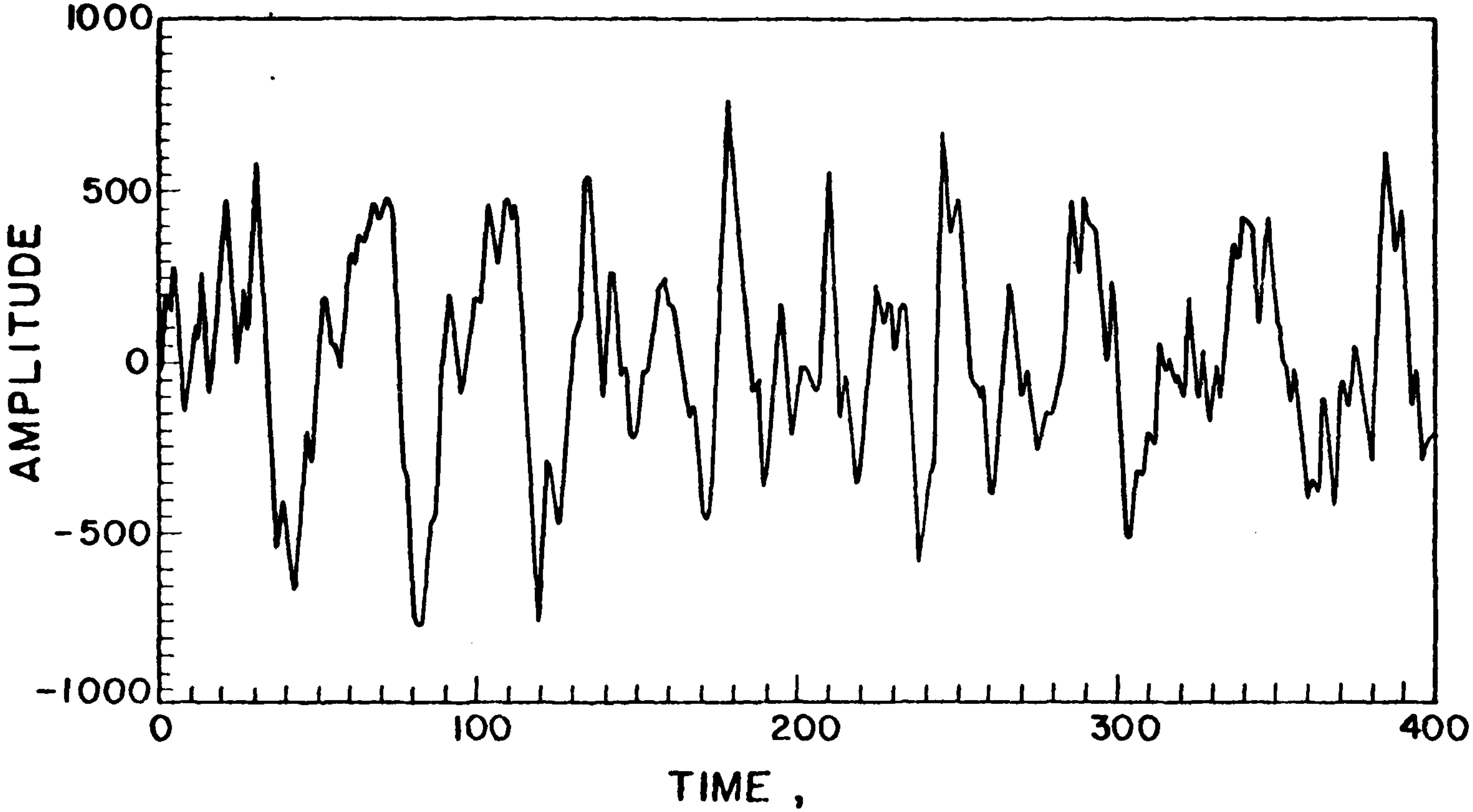

Fig. 2 Typical Response Signal of a Test Specimen

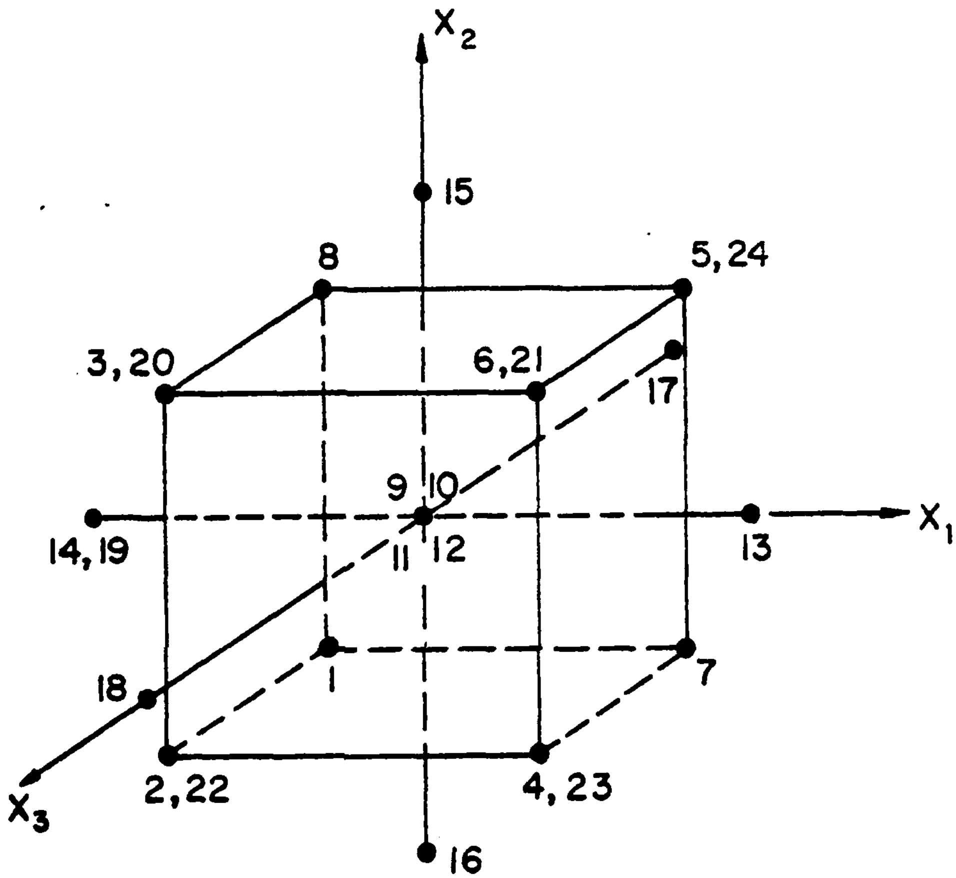

Fig. 3 Central Composite Design with Four Center Points and Six Replications

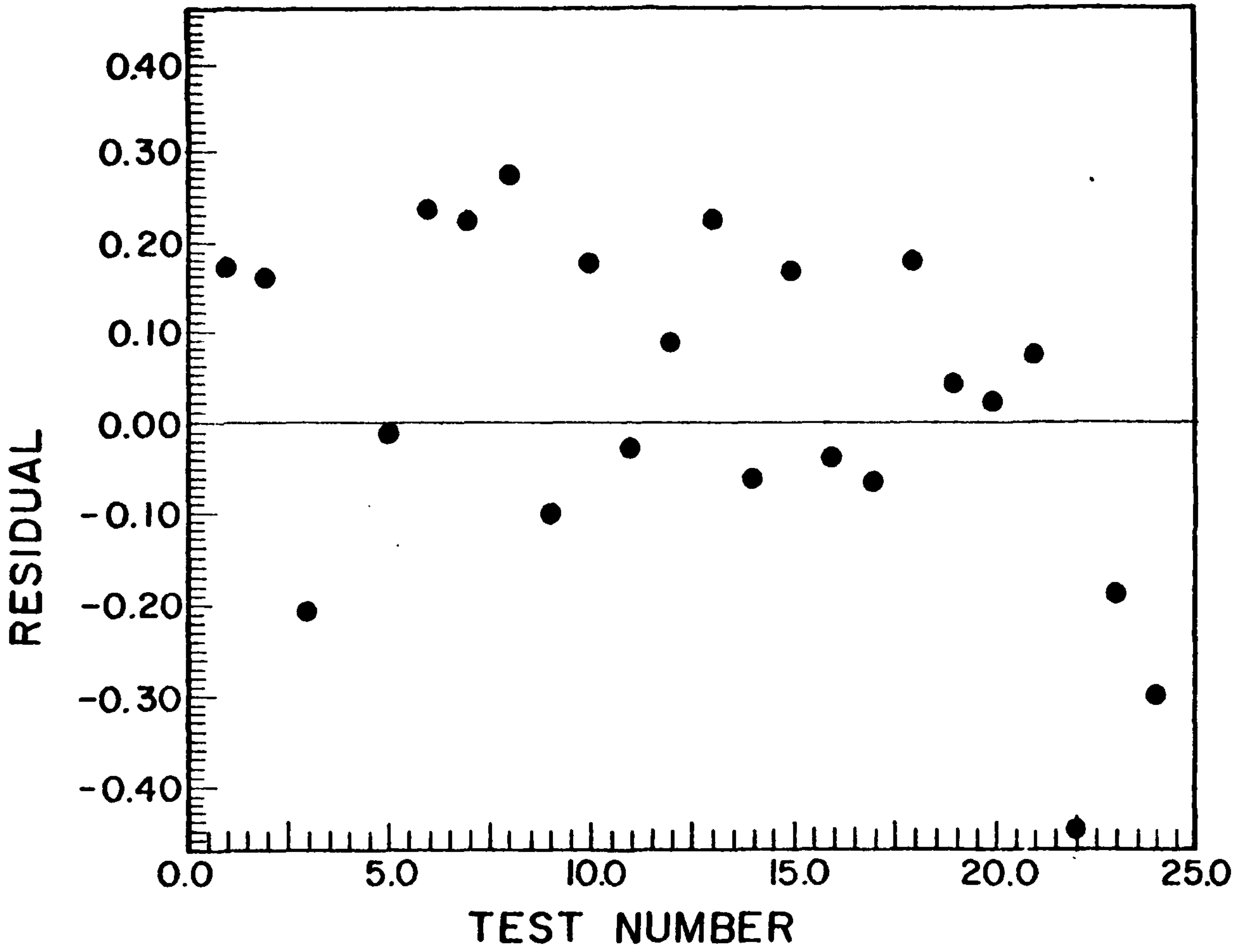

Fig. 4 Distribution of Residuals for the Best First Order Model

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An experiment program, based on 8 probabilistic parameters and 3 experiment designs, was conducted for 24 specimens for the fatigue life of materials in elastic range under random vibrations. The magnitude levels of all probabilistic parameters for the response of every specimen were computed from its spectral moments and were coded. By regressing the log of fatigue life on coded probabilistic parameters the first order models to predict fatigue life were obtained.

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