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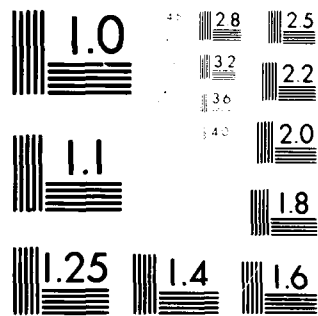   JUL 79   T C HUANG, V K NAGPAL   N00014-76-C-0825
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EXPERIMENTAL RANDOM FATIGUE
IN ELASTIC RANGE - MODELS
OF SIGNIFICANT VARIABLES

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EXPERIMENTAL RANDOM FATIGUE IN ELASTIC RANGE-MODELS OF

SIGNIFICANT VARIABLES

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Abstract

In the previous study of first order models with 8 variables to predict fatigue life under random vibrations, 4 variables showed significant effects. In this report both first and second order models of these 4 significant variables based on 10, 18, and 24 tests are developed. The tables of analysis of variance, and of predicted lives together with their residuals and 95% confidence intervals are constructed for each of the first and second order models. A second order model of 4 significant variables consisting of 14 terms, based on 24 tests, has been found to be the statistically best one. The deviations of the lives predicted by this model range from -26.6% to 20.4% with an average of 5.5% on the negative side and 7.0% on the positive side. These results are in contrast with those which are obtained by the principle of linear damage accumulation based on cycle counting and involve several hundred percent error.

INTRODUCTION

In the previous report [1] it was noted that the principle of linear damage accumulation based on equivalent cycle counting gives unreliable estimates of fatigue life under random vibrations [2-11]. In this group of selected references [2-11], the principle of linear damage accumulation was applied and the fatigue life was found to be overestimated by a factor ranging from -5.0 to 15.0 in [2-9]. In [10,11] it was reported that the fatigue life was overestimated but the factor of overestimation was not given.

A new history-dependent stochastic model of cumulative damage is being developed by Bogdanoff [12,13,14] by taking a comprehensive view of entire failure process to improve the predictive accuracy. Another phenomenological approach entirely different from that of all the cumulative damage approaches mentioned above was undertaken in the previous study [1] to develop first order models or life predicting equations. In this approach a test program based on 8 probabilistic parameters and experiment design was conducted in 3 designs. The first order models are developed for each design and the four variables which showed significant effects on the fatigue life are considered in the present report. Both the first order and second order models of the significant variables are developed for the same three designs as in [1]. From a comparison of all the models considered the statistically best one is singled out.

Considering the models of significant variables is essential in our search for the statistically best model. The other models, which consist of terms of any combination of insignificant variables are inferior because the number of degrees of freedom of regression

increases with a negligible increase in the regression sum of squares. Simultaneously, the confidence intervals are widened depending upon the number of degrees of freedom added to regression. Therefore such models need not be considered.

I. EXPERIMENTS, PARAMETERS, DESIGNS AND MODELS

The mean, variance, duration of excursion above ϵ_f level, and the band width were found to be 4 significant variables on the basis of the analysis in the previous report [1] of the first order models of all 8 variables. In the present report these 4 variables are considered for both first and second order models. In addition, a model with all second order terms of significant variables and first order terms of all 8 variables is also constructed. All these models are expected to improve the prediction of fatigue life under random vibrations over the first order models in [1].

In this report 3 designs which were used in [1] will be considered. For the first design, a full factorial design consisting of 10 tests, only first order model of significant variables could be constructed because a minimum of 15 tests are required for a second order model based on 4 variables. For the second design, a central composite design of 18 tests including 4 center points, first order and second order models using only the significant variables are developed. For the third design, a central composit design of 24 tests including 4 center points and 6 replications, three models are developed. The first two are the first and second order models involving the four significant variables only as was done for the second design. The third model involves first order terms of all 8 variables and all second order terms of significant variables. A minimum of 19 tests are

required to develop this model. For this reason, the third model could not be developed for the first and second designs.

For each of these three designs the tables of analysis of variance, and of the predicted lives together with residuals and 95% confidence intervals are constructed. The F-ratio was computed for each model to check if the model is acceptable. The confidence intervals are computed from the standard deviations of the predicted lives and the t values from the t-table corresponding to the degrees of freedom of the residuals of the model. The distribution of residuals was studied for any trend or pattern present. In case of any trend or pattern present the model was considered inadequate.

Finally a comparison of all models is made.

II. FULL FACTORIAL DESIGN WITH TWO CENTER POINTS

1. First Order Model of 4 Significant Variables

A first order model is obtained using the first ten tests of the experiment by regressing log of fatigue life, y, on the coded variables under consideration. These 10 tests form a full factorial design with 2 center points. The first order life predicting equation is obtained as

$$\hat{y} = 5.87 - 0.094x_1 - 0.782x_2 - 0.0510x_6 - 0.100x_7 \quad (1)$$

The analysis of variance is given in Table 1. The F-ratio for this equation is 16.91 which is greater than the corresponding F value of 5.19 from F-table with 4 and 5 degrees of freedom at 95% significance level. So the regression is effective and the model is acceptable. The residual sum of squares is 0.3983 in comparison to a total of 5.788, a 6.9%. The other 93.1% of the total is due to regression.

The sum of squares due to duration of excursion above ϵ_f level is fairly low in comparison to sum of squares due to other 3 variables. This variable will be considered in the remaining two designs based on high number of tests in order to further investigate its individual effects in the first order models, and its individual and interaction effects in the second order models.

Table 2 gives the lives predicted by Eq. (1) together with their residuals and 95% confidence intervals. All the actual lives fall within the predicted confidence intervals. These confidence intervals are wide because the number of observations is relatively small. On the basis of the 10 tests, it will not be very meaningful to draw any conclusion on the basis of the distribution of residuals.

2. Second Order Model of 4 Significant Variables

For the first 10 tests only the first order model could be developed for this design because the number of tests are not sufficient to develop the second order model.

III. CENTRAL COMPOSITE DESIGN WITH FOUR CENTER POINTS

This design consists of 18 tests. Two life predicting equations, a first order and a second order, are obtained for this design which are given below.

1. First order model of 4 Significant Variables

The first order life predicting equation is obtained as

$$\hat{y} = 5.84 - 0.039x_1 - 0.708x_2 - 0.080x_6 - 0.108x_7 \quad (2)$$

The analysis of variance of this equation is given in Table 3. The F-ratio is computed to be 33.86 with 4 and 13 degrees of freedom. The corresponding F value from the F-table is 3.18 at 95% significance level. The comparison of the two F values shows that the regression

is effective and the model is acceptable. The residual sum of squares
is 0.8568 in comparison to a total of 9.7843, a 8.8%. The other 91.2%
of the total is due to regression. The sum of squares contributed due
to the duration of excursion above ε_f level, in this case also, is
comparatively low, which implies that this variable may not have a
significant effect on the fatigue life. In order to verify this fact,
this variable will be considered for the analysis of 24 tests which
is described in the third design.

The predicted lives together with their residuals and 95 percent
confidence intervals are given in Table 4. The confidence intervals
cover all the actual lives of the tests except for the test no. 4.
The lower limit of the confidence interval of test 4 is slightly above
its actual life. That the residuals appear to be randomly distributed
indicates that this model is adequate.

2. Second Order Model of 4 Significant Variables

The life predicting equation involving all first and second order
terms of significant variables is obtained as

$$\hat{y} = 6.24 - 0.255x_1 - 2.15x_2 + 0.0001x_5 - 0.377x_7 - 0.890x_1^2$$
$$+ 0.471x_2^2 - 0.0225x_5^2 - 0.260x_7^2 + 1.25x_1x_2 + 0.254x_1x_5$$
$$+ 0.128x_1x_7 - 1.39x_2x_5 + 1.28x_2x_7 - 0.270x_5x_7 \qquad (3)$$

The analysis of variance of this equation is given in Table 5. The
F-ratio was computed to be 59.02 with 14 and 3 degrees of freedom. The
corresponding F value from the F-table is 8.71 at 95% significance
level. The comparison of two F values indicates that the regression
is effective and the model is acceptable. The residual sum of squares
is 0.0353 in comparison to a total of 9.7843, a 1.4%. The other 99.6%

of the total is due to regression. Observing the sum of squares due to each individual term, it is obvious that for the first order terms the mean, variance and band width are significant. For the second order terms, the product of mean and variance, and the product of mean and duration of excursion above ε_f level are significant. The individual effect of the duration of excursion above ε_f level is found to be insignificant.

The predicted lives together with their residuals and 95% confidence intervals are given in Table 6. The confidence intervals here cover the actual fatigue lives of all tests including number 4. The reason is that the confidence intervals are fairly wide because of low degrees of freedom associated with t. The residuals are fairly random and are very small in magnitude. It can be considered a very good model.

IV. CENTRAL COMPOSITE DESIGN WITH FOUR CENTER POINTS AND SIX REPLICATIONS

This design consists of 24 tests for which three models have been developed. The first two are the first and second order models involving significant variables only. The third one involves first order terms of all 8 variables and second order terms of significant variables. These models are described below.

1. First Order Model of 4 Significant Variables

The first order life predicting equation of 4 significant variables is obtained as

$$\hat{y} = 5.82 - 0.056x_1 - 0.750x_2 - 0.0595x_6 - 0.0780x_7 \qquad (4)$$

The analysis of variance of this equation is given in Table 7. The F-ratio is computed to be 46.5 with 4 and 19 degrees of freedom which is greater than the corresponding F value from F-table of 2.90 at 95%

significance level. This comparison of two F values indicates that the regression is effective and the model is acceptable. The residual sum of squares is 1.2165 in comparison to a total of 13.1131, a 10.3%. The other 90.7% of the total is due to regression. The sum of squares due to the duration of excursion above ε_f level is very low in comparison to the other three variables.

The predicted lives together with their residuals and 95% confidence interval of this equation are given in Table 8. The actual fatigue lives for test numbers 4, 13 and 17 fall out of the confidence interval. The confidence intervals are fairly narrow. The residuals appear to be randomly distributed.

2. Second Order Model of 4 Significant Variables

The second order life predicting equation using all first and second order terms of significant variables is obtained as

$$\hat{y} = 6.23 + 0.185x_1 - 1.88x_2 + 0.002x_6 - 0.310x_7 - 0.773x_1^2$$
$$+ 0.373x_2^2 - 0.0420x_6^2 - 0.248x_7^2 + 0.982x_1x_2 + 0.236x_1x_6$$
$$+ 0.178x_1x_7 - 1.10x_2x_6 + 1.16x_2x_7 - 0.267x_6x_7 \qquad (5)$$

The analysis of variance of the equation above is given in Table 9. The F-ratio of 41.17 is obtained with 14 and 9 degrees of freedom. The corresponding F value from the F-table of 2.90 at 95% significance level is less than the F-ratio which implies that the regression is effective and the model is acceptable. The residual sum of squares is 0.2018 in comparison to a total of 13.1131, a 1.5%. The other 98.5% of the total is due to regression. Even though the duration of excursion above ε_f level and its square are insignificant, its interactions with mean and band width are not insignificant.

The predicted life together with their residuals and 95% confidence intervals are given in Table 10. All confidence intervals cover the actual fatigue lives of all tests except for the test number 22 which is 0.7% below the lower level. The residuals are small and appear to be randomly distributed. The plot of residuals is given in Fig. 1.

3. Second Order Model Involving First Order Terms of All 8 Variables
and Second Order Terms of 4 Significant Variables

The life predicting equation of this model is obtained as

$$\hat{y} = 6.09 + 0.182x_1 - 1.81x_2 + 0.0866x_3 + 0.0711x_4 + 0.172x_5$$
$$- 0.130x_6 - 0.370x_7 - 0.0487x_8 - 0.572x_1^2 + 0.408x_2^2$$
$$- 0.0510x_6^2 - 0.144x_7^2 + 0.969x_1x_2 + 0.194x_1x_6 + 0.161x_1x_7$$
$$- 1.15x_2x_6 + 0.936x_2x_7 - 0.226x_6x_7 \tag{6}$$

The analysis of variance of the above equation is given in Table 11. The F-ratio of 21.34 with 18 and 5 degrees of freedom is obtained for the above equation. The corresponding F value from the F-table at 95% significance level is 9.61 which is smaller than the F-ratio. Therefore, the regression is effective and the model is acceptable. The residual sum of squares is 0.1685 in comparison to a total of 13.1311, a 1.4%. The other 98.6% of the total is due to regression.

The predicted lives together with their residuals and 95% confidence intervals are given in Table 12. The residuals for all tests are very small. The low degrees of freedom associated with t makes the confidence intervals fairly wide. The actual lives of all tests fall within the predicted confidence intervals. The residuals appear to be randomly distributed.

On the basis of the analyses of variances of various models it is worth remarking that the duration of excursion above ε_f level has an insignificant effect as an individual variable but its interactions with the other variables is not insignificant. It may also be observed that lower limit of confidence interval gives the safe estimate of fatigue life.

V. DISCUSSIONS AND CONCLUSIONS

On the basis of the first order models obtained in [1], the best predicted lives and the lowest residual sum of squares are obtained for the model with all 8 variables. It is also found that there are only 4 variables which have significant effects on the fatigue life. These 4 variables are mean, variance, duration of excursion above ε_f level, and the band width. In the present report, both first and second order models involving significant variables have been investigated.

In a comparative study of all 6 models investigated in this report and the best model of [1] as shown in Table 13, the second order model represented by equation (6) gives the best predicted lives and lowest residual sum of squares. From the analysis of this model it is observed that there are several terms which contribute a very small sum of squares to the regression. In the meantime they add one degree of freedom each to the regression. This leaves a less number of degrees of freedom associated with t which makes the 95% confidence intervals fairly wide. The residuals are very small because the number of variables regressed in this equation is relatively large considering the number of tests available.

The second order model represented by equation (5) has 4 terms less than those of the model represented by equation (6) but the change in the residual sum of squares is nominal as shown in Table 13. The predicted lives of equation (5) are also very close to actual lives. However, the confidence intervals are on an average 36.2% narrower than those of equation (6) as shown in Table 14.

Based on the overall consideration of the accuracy of the life prediction, residual sum of squares and the width of the confidence interval it may be concluded that the model of equation (5) is statistically better than the model of equation (6).

VI. <u>SUMMARY</u>

(1) On the basis of analysis of first order models presented in [1], 4 variables showed significant effects on the fatigue life under random vibrations. These variables have been considered for further developing first and second order models in the present study.

(2) The first and second order models of significant variables have been developed separately for each of the three designs. In addition, one model involving first order terms of all 8 variables and second order terms of 4 significant variables is also obtained.

(3) For each model the analysis of variance and predicted lives together with residuals and 95 percent confidence intervals are obtained.

(4) Among the first order models in the present and the previous report [1], the best first order model is found to be the one which consists of all 8 variables based on 24 tests as it should be.

(5) Among all the second order models investigated in the present report and the best first order model, a second order model of significant variables is found to be the statistically best one.

ACKNOWLEDGMENTS

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Table 1.   Analysis of Variance of 10 Tests
            First Order Model of 4 Significant Variables
            Life predicting equation:

$$\hat{y} = 5.87 - 0.094x_1 - 0.782x_2 - 0.051x_6 - 0.100x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.3337	1	0.3337	
Due to Variance	4.8717	1	4.8717	
Due to Duration of Excursion Above ε_f Level	0.0206	1	0.0206	
Due to Band Width	0.1645	1	0.1645	
Due to Regression	5.3905	4	1.3476	
Residuals	0.3983	5	0.0797	16.91
Total	5.7888	9		

F-ratio is greater than the table value of 5.19 with 4 and 5 degrees
of freedom at 95% significance level. So the regression is effective
and the model is accepted.

Table 2.   Results of 10 Tests, First Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 5.87 - 0.094x_1 - 0.782x_2 - 0.051x_6 - 0.100x_7$$

| Test No. | Actual Life | | Predicted Life | | Residuals | 95% Confidence Interval | | | |
	T	y	$\hat{y}$	$\hat{T}$	$y - \hat{y}$	$\hat{y}$ Lower	$\hat{y}$ Upper	$\hat{T}$ Lower	$\hat{T}$ Upper
1	1363.43	7.218	7.180	1312.91	0.038	6.640	7.720	765.16	2252.76
2	938.83	6.845	6.718	827.16	0.127	6.152	7.284	469.83	1456.25
3	165.08	5.106	5.402	221.85	-0.296	4.960	5.844	142.56	345.23
4	391.97	5.971	6.339	566.23	-0.367	5.858	6.820	350.10	915.78
5	156.20	5.051	5.171	176.09	-0.120	4.634	5.708	102.89	301.37
6	160.83	5.080	4.796	121.03	0.284	4.254	5.338	70.35	208.20
7	1011.42	6.919	6.736	842.19	0.183	6.206	7.266	495.90	1430.28
8	259.08	5.557	5.434	229.06	0.123	4.992	5.876	147.20	356.46
9	347.50	5.851	5.920	372.41	-0.069	5.617	6.223	274.96	504.40
10	370.33	5.914	5.817	335.96	0.097	5.148	6.539	172.18	691.91

Table 3.   Analysis of Variance of 18 Tests
           First Order Model of 4 Significant Variables
           Life predicting equation:

$$\hat{y} = 5.84 - 0.039x_1 - 0.708x_2 - 0.080x_6 - 0.108x_7$$

	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.1212	1	0.1212	
Due to Variance	8.4720	1	8.4720	
Due to Duration of Excursion Above ϵ_f Level	0.0398	1	0.0398	
Due to Band Width	0.2945	1	0.2945	
Due to Regression	8.9276	4	2.2319	
Residuals	0.8568	13	0.0659	33.86
Total	9.7843	17		

F-ratio is greater than the table value of 3.18 with 4 and 13 degrees
of freedom at 95% significance level. So the regression is effective
and the model is accepted.

Table 4.   Results of 18 Tests, First Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 5.84 - 0.039x_1 - 0.708x_2 - 0.080x_6 - 0.108x_7$$

Test No.	Actual Life T	Actual Life y	Predicted Life $\hat{y}$	Predicted Life $\hat{T}$	Residuals $y-\hat{y}$	95% Confidence Interval $\hat{y}$ Lower	$\hat{y}$ Upper	$\hat{T}$ Lower	$\hat{T}$ Upper
1	1363.43	7.218	7.120	1236.45	0.097	6.760	7.480	863.83	1769.69
2	938.83	6.845	6.642	766.63	0.202	6.355	6.930	575.20	1021.76
3	165.08	5.106	5.212	183.46	0.175	4.834	5.590	125.71	267.76
4	391.97	5.971	6.320	555.57	-0.349	6.000	6.640	403.56	764.85
5	156.20	5.051	5.191	179.65	-0.140	4.841	5.541	126.61	254.91
6	160.83	5.080	4.896	133.75	0.180	4.615	5.177	101.01	177.12
7	1011.42	6.919	6.752	855.77	0.167	6.439	7.065	625.66	1170.52
8	259.08	5.557	5.458	234.63	0.100	5.143	5.773	171.17	321.62
9	347.50	5.851	5.937	378.80	-0.087	5.790	6.084	327.05	438.73
10	370.33	5.914	5.755	315.77	0.159	5.461	6.049	235.39	423.59
11	346.00	5.846	5.892	362.13	-0.046	5.756	6.028	316.06	414.92
12	371.00	5.916	5.918	371.67	-0.001	5.778	6.058	322.98	427.69
13	467.83	6.148	5.869	353.89	0.279	5.530	6.208	252.11	496.77
14	407.33	6.010	5.978	394.65	0.031	5.665	6.291	288.53	539.80
15	98.67	4.592	4.528	92.57	.064	4.221	4.835	68.12	125.80
16	1327.33	7.191	7.211	1354.24	-0.020	6.924	7.498	1016.09	1804.94
17	189.08	5.242	5.910	368.70	-0.668	5.668	6.152	289.47	469.62
18	273.67	5.612	5.480	239.85	0.132	5.186	5.774	178.79	321.74

Table 5.   Analysis of Variance of 18 Tests
            Second Order Model of 4 Significant Variable
            Life predicting equation:

$$\hat{y} = 6.24 - 0.255x_1 - 2.15x_2 + 0.0001x_6 - 0.337x_7$$
$$- 0.890x_1^2 + 0.471x_2^2 - 0.0225x_6^2 - 0.260x_7^2$$
$$+ 1.25x_1x_2 + 0.254x_1x_6 + 0.128x_1x_7 - 1.39x_2x_6$$
$$+ 1.28x_2x_7 - 0.270x_6x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio
Due to Mean	0.1212	1	0.1212	
Due to Variance	8.4720	1	8.4720	
Due to Duration of Excursion Above ϵ_f Level	0.0398	1	0.0398	
Due to Band Width	0.2945	1	0.2945	
Due to Mean Square	0.0653	1	0.0653	
Due to Variance Square	0.0251	1	0.0251	
Due to Duration of Excursion Above ϵ_f Level Square	0.0666	1	0.0666	
Due to Band Width Square	0.0115	1	0.0115	
Due to Mean * Variance	0.1948	1	0.1948	
Due to Mean * Duration of Excursion Above ϵ_f Level	0.4194	1	0.4194	
Due to Mean * Band Width	0.0006	1	0.0006	
Due to Variance * Band Width	0.0030	1	0.0030	
Due to Variance * Duration of Excursion	0.0070	1	0.0070	
Due to Duration of Excursion Above ϵ_f Level * Band Width	0.0281	1	0.0281	
Due to Regression	9.7490	14	0.6964	
Residuals	0.0353	3	0.0118	59.02
Total	9.7843			

F-ratio is greater than the table value 8.71 with 14 and 3 degrees of
freedom at 95% significance level. So the regression is effective and
the model is accepted.

Table 6. Results of 18 Tests, Second Order Model of 4 Significant Variables

Life predicting equation:

$$y = 6.24 - 0.255x_1 - 2.15x_2 + 0.0001x_6 - 0.337x_7 - 0.190x_1^2 + 0.471x_2^2$$
$$- 0.0225x_6^2 - 0.260x_7^2 + 1.25x_1x_2 + 0.254x_1x_6 + 0.128x_1x_7 - 1.39x_2x_6$$
$$+ 1.28x_2x_7 - 0.270x_6x_7$$

Test No.	Actual Life T	Actual Life y	Predicted Life $\hat{T}$	Predicted Life $\hat{y}$	Residuals $y-\hat{y}$	95% Confidence Interval $\hat{y}$ Lower	$\hat{y}$ Upper	$\hat{T}$ Lower	$\hat{T}$ Upper
1	1363.43	7.218	1398.28	7.243	-0.025	6.903	7.583	994.78	1965.45
2	938.83	6.845	980.44	6.888	-0.044	6.554	7.222	701.97	1369.37
3	165.08	5.106	162.55	5.091	0.016	4.747	5.435	115.28	229.21
4	391.97	5.971	429.23	6.062	-0.091	5.763	6.361	318.27	578.89
5	156.20	5.051	156.02	5.050	0.001	4.703	5.397	110.29	220.70
6	160.83	5.080	161.09	5.082	-0.002	4.738	5.426	114.24	227.16
7	1011.42	6.919	1036.91	6.944	-0.025	6.600	7.288	735.35	1462.14
8	259.08	5.557	251.14	5.526	0.031	5.186	5.866	178.67	353.00
9	347.50	5.851	352.83	5.866	-0.015	5.675	6.057	291.52	427.06
10	370.23	5.914	353.19	5.867	0.047	5.533	6.201	252.87	493.30
11	346.00	5.846	369.08	5.911	-0.064	5.714	6.108	302.99	449.57
12	371.00	5.916	369.81	5.913	0.004	5.709	6.167	301.57	453.34
13	467.83	6.148	444.52	6.097	0.051	5.767	6.428	319.28	618.89
14	407.33	6.010	409.53	6.015	-0.005	5.671	6.359	290.42	577.47
15	98.67	4.592	102.92	4.634	-0.042	4.300	4.968	73.69	143.75
16	1327.33	7.191	1211.96	7.100	0.091	6.801	7.399	898.65	1634.52
17	189.08	5.242	186.05	5.226	0.016	4.882	5.570	131.94	262.34
18	273.67	5.612	258.79	5.556	0.056	5.228	5.884	186.47	359.15

Table 7.   Analysis of Variance of 24 Tests
            First Order Model of 4 Significant Variables
            Life predicting equation:

$$\hat{y} = 5.82 - 0.0656x_1 - 0.750x_2 - 0.0595x_6 - 0.0780x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio
Due to Mean	0.1172	1	0.1172	
Due to Variance	11.5157	1	11.5157	
Due to Duration of Excursion Above ε_f	0.0305	1	0.0305	
Due to Band Width	0.2331	1	0.2331	
Due to Regression	11.8966	4	2.9742	
Residuals	1.2165	19	0.0640	46.57
Total	13.1131			

*F-ratio is greater than the table value 2.90 with 4 and 19 degrees
of freedom at 95% significance level. So the regression is effective
and the model is accepted.*

Table 8. Results of 24 Tests, First Order Model of 4 Significant Variables Life predicting equation:

$$\hat{y} = 5.82 - 0.0656x_1 - 0.750x_2 - 0.0595x_6 - 0.0780x_7$$

Test No.	Actual Life T	Actual Life y	Predicted Life $\hat{y}$	Predicted Life $\hat{T}$	Residuals $y-\hat{y}$	95% Confidence Interval $\hat{y}$ Lower	95% Confidence Interval $\hat{y}$ Upper	95% Confidence Interval $\hat{T}$ Lower	95% Confidence Interval $\hat{T}$ Upper
1	1363.43	7.218	7.057	1160.95	0.161	6.751	7.363	855.27	1575.89
2	938.83	6.845	6.674	791.56	0.171	6.431	6.917	620.93	1009.07
3	165.08	5.106	5.163	174.69	-0.056	4.820	5.506	123.93	246.23
4	391.47	5.971	6.359	577.67	-0.388	6.108	6.610	449.37	742.60
5	156.20	5.051	5.063	158.06	-0.011	4.768	5.358	117.67	212.32
6	160.83	5.080	4.838	126.22	0.242	4.543	5.133	93.96	169.54
7	1011.42	6.919	6.705	816.48	0.215	6.437	6.973	624.59	1067.32
8	259.08	5.557	5.332	206.85	0.225	5.093	5.571	162.94	262.59
9	347.50	5.851	5.887	360.32	-0.036	5.761	6.013	317.80	408.54
10	370.33	5.914	5.760	317.35	0.154	5.500	6.019	244.81	411.39
11	346.00	5.846	5.850	347.23	-0.003	5.737	5.963	310.13	388.78
12	371.00	5.916	5.864	352.13	0.052	5.745	6.035	312.53	417.92
13	467.83	6.148	5.791	327.34	0.357	5.502	6.080	245.22	436.96
14	407.33	6.010	5.984	397.02	0.026	5.743	6.225	312.09	505.06
15	98.67	4.592	4.418	82.93	0.173	4.175	4.661	65.05	105.72
16	1327.33	7.191	7.239	1392.70	-0.048	6.984	7.494	1078.85	1797.85
17	189.08	5.242	5.849	346.89	-0.607	5.633	6.065	279.62	430.34
18	273.67	5.612	5.497	243.96	0.115	5.273	5.721	195.01	305.19
19	430.00	6.064	5.929	375.78	0.135	5.703	6.155	299.75	471.09
20	157.17	5.057	5.104	164.68	-0.046	4.870	5.338	130.27	208.18
21	127.00	4.844	4.871	130.45	-0.027	4.632	5.110	102.76	165.60
22	137.42	4.923	5.359	212.51	-0.436	5.120	5.597	167.40	269.78
23	484.42	6.183	6.320	555.57	-0.138	6.077	6.563	435.81	708.24
24	136.75	4.918	5.147	171.91	-0.228	4.904	5.390	134.86	219.16

Table 9.   Analysis of Variance of 24 Tests
            Second Order Model of 4 Significant Variables
            Life predicting equation:

$$\hat{y} = 6.23 + 0.185x_1 - 1.88x_2 + 0.0002x_6 - 0.310x_7$$
$$- 0.773x_1^2 + 0.373x_2^2 - 0.042x_6^2 - 0.248x_7^2 + 0.982x_1x_2$$
$$+ 0.236x_1x_6 + 0.178x_1x_7 - 1.10x_2x_6 - 1.116x_2x_7$$
$$- 0.267x_6x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio
Due to Mean	0.1172	1	0.1172	
Due to Variance	11.5157	1	11.5157	
Due to Duration of Excursion Above ε_f Level	0.0305	1	0.0305	
Due to Band Width	0.2331	1	0.2331	
Due to Mean Square	0.0633	1	0.0633	
Due to Variance Square	0.0240	1	0.0240	
Due to Duration of Excursion Above ε_f Level Square	0.0004	1	0.0004	
Due to Band Width Square	0.0025	1	0.0025	
Due to Mean * Variance	0.1252	1	0.1252	
Due to Mean * Duration of Excursion Above ε_f Level	0.5617	1	0.5617	
Due to Mean * Band Width	0.0013	1	0.0013	
Due to Variance * Duration of Excursion Above ε_f Level	0.0200	1	0.0200	
Due to Variance * Band Width	0.1084	1	0.1084	
Due to Band Width * Duration of Excursion Above ε_f	0.1078	1	0.1078	
Due to Regression	12.9113	14	0.9222	
Residuals	0.2018	9	0.0224	41.17
Total	13.1131	23		

F-ratio is greater than the table value 2.90 with 14 and 9 degrees of freedom at 95% significance level. So the regression is effecitve and the model is accepted.

Table 10. Results of 24 Tests, Second Order Model of 4 Significant Variables. Life predicting equation:

$$y = 6.23 + 0.185x_1 - 1.88x_2 + 0.0002x_6 - 0.310x_7 - 0.773x_1^2 + 0.373x_2^2 - 0.0420x_6^2$$
$$- 0.248x_7^2 + 0.982x_1x_2 + 0.238x_1x_6 + 0.178x_1x_7 - 1.10x_2x_6 + 1.16x_2x_7 - 0.267x_6x_7$$

Test No.	Actual Life T	Actual Life y	Predicting Life $\hat{y}$	Predicting Life $\hat{T}$	Residuals $y-\hat{y}$	95% Confidence Interval $\hat{y}$ Lower	95% Confidence Interval $\hat{y}$ Upper	95% Confidence Interval $\hat{T}$ Lower	95% Confidence Interval $\hat{T}$ Upper
1	1363.43	7.218	7.238	1391.31	-0.021	6.910	7.566	1002.26	1931.38
2	938.83	6.845	6.883	975.55	-0.039	6.591	7.175	728.66	1306.09
3	165.08	5.106	5.105	164.84	0.001	4.845	5.365	127.09	213.82
4	391.97	5.971	5.979	395.05	-0.008	5.741	6.216	311.53	500.95
5	156.20	5.051	5.034	153.55	0.017	4.713	5.355	111.36	211.71
6	160.83	5.080	5.093	162.88	-0.013	4.871	5.315	130.50	203.30
7	1011.42	6.919	6.954	1047.33	-0.035	6.732	7.284	839.09	1457.17
8	259.08	5.557	5.328	206.03	0.229	5.156	5.500	173.48	244.67
9	347.50	5.851	5.868	353.54	-0.017	5.696	6.032	297.70	416.51
10	370.23	5.914	5.863	351.78	0.051	5.605	6.121	271.82	455.26
11	346.00	5.846	5.910	368.71	-0.064	5.724	6.095	306.28	443.85
12	371.00	5.916	5.907	367.60	0.009	5.722	6.092	305.37	442.52
13	467.83	6.148	6.090	441.42	0.058	5.780	6.400	323.79	601.78
14	407.33	6.010	6.167	476.75	-0.158	5.927	6.407	375.11	605.93
15	98.67	4.592	4.621	101.60	-0.029	4.336	4.906	76.40	135.10
16	1327.33	7.191	7.106	1219.26	0.085	6.821	7.391	916.89	1621.35
17	189.08	5.242	5.245	189.62	-0.002	4.917	5.573	136.59	263.22
18	273.67	5.612	5.525	250.89	0.087	5.525	5.794	250.88	328.83
19	430.00	6.064	5.885	359.60	0.179	5.632	6.138	288.07	463.28
20	157.17	5.057	5.051	156.18	0.007	4.741	5.361	114.56	212.92
21	127.00	4.844	4.862	129.28	-0.018	4.620	5.104	101.49	164.68
22	137.42	4.923	5.159	173.99	-0.236	4.931	5.747	138.45	313.39
23	484.42	6.183	6.262	524.27	-0.079	6.038	6.486	419.08	655.85
24	136.75	4.918	4.924	137.55	-0.006	4.600	5.247	99.54	190.08

Table 11.   Analysis of Variance of 24 Tests
Second Order Model of All 8 Variables
Life predicting equation:

$$\hat{y} = 6.09 + 0.182x_1 - 1.81x_2 + 0.087x_3 + 0.071x_4$$
$$+ 0.172x_5 - 0.130x_6 - 0.370x_7 - 0.049x_8$$
$$- 0.572x_1^2 + 0.408x_2^2 - 0.051x_6^2 - 0.144x_7^2$$
$$+ 0.969x_1x_2 + 0.194x_1x_6 + 0.161x_1x_7 - 1.15x_2x_6$$
$$+ 0.936x_2x_7 - 0.226x_6x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio
Due to Mean	0.1172	1	0.1172	
Due to Variance	11.5157	1	11.5157	
Due to Zero Upcrossings	0.0021	1	0.0021	
Due to ε_f Level Upcrossings	0.0708	1	0.0708	
Due to Duration of Excursion Above Zero	0.0554	1	0.0554	
Due to Duration of Excursion Above ε_f	0.0041	1	0.0041	
Due to Band Width	0.6993	1	0.6993	
Due to Average Amplitude Above ε_f	0.0015	1	0.0015	
Due to Mean Square	0.0009	1	0.0009	
Due to Variance Square	0.0417	1	0.0417	
Due to Duration of Excursion Above ε_f Square	0.0286	1	0.0286	
Due to Band Width Square	0.0427	1	0.0427	
Due to Mean * Variance	0.0047	1	0.0047	
Due to Mean * Duration of Excursion Above ε_f	0.0927	1	0.0927	
Due to Mean * Band Width	0.0223	1	0.0223	
Due to Variance * Duration of Excursion Above ε_f	0.1095	1	0.1095	
Due to Variance * Band Width	0.0873	1	0.0873	
Due to Band Width * Duration of Excursion Above ε_f	0.0480	1	0.0480	
Due to Regression	12.9446	18	0.7191	
Residuals	0.1685	5	0.0337	21.34
Total	13.1311	23		

F-ratio is greater than the table value 9.61 with 18 and 5 degrees of
freedom at 95% significance level. So the regression is effective and
the model is accepted.

Table 12. Results of 24 Tests, Second Order Model of All 8 Variables. Life predicting equation:

$$\hat{y} = 6.09 + 0.182x_1 - 1.81x_1^2 + 0.087x_3 + 0.071x_4 + 0.172x_5 - 0.130x_6 - 0.370x_7 - 0.049x_8$$
$$- 0.572x_1^2 + 0.408x_2^2 - 0.057x_6^2 - 0.144x_7^2 + 0.969x_1x_2 + 0.194x_1x_6 + 0.161x_1x_7$$
$$- 1.15x_2x_6 + 0.936x_2x_7 - 0.266x_6x_7$$

Test No.	Actual Life T	Actual Life y	Predicted Life $\hat{T}$	Predicted Life $\hat{y}$	Residuals $y-\hat{y}$	95% Confidence Interval $\hat{y}$ Lower	$\hat{y}$ Upper	$\hat{T}$ Lower	$\hat{T}$ Upper
1	1363.43	7.218	1381.60	7.231	-0.013	6.768	7.694	869.76	2194.65
2	938.83	6.845	993.27	6.901	-0.056	6.450	7.352	632.99	1558.60
3	165.08	5.106	156.49	5.053	0.053	4.610	5.496	100.48	243.71
4	391.97	5.971	405.45	6.005	-0.034	5.658	6.352	286.48	573.84
5	156.20	5.051	154.78	5.042	0.009	4.581	5.503	97.65	245.33
6	160.83	5.080	176.09	5.171	-0.090	4.801	5.541	121.63	254.93
7	1011.42	6.919	1055.75	6.962	-0.043	6.506	7.417	669.43	1664.99
8	259.08	5.570	218.55	5.387	0.171	4.999	5.775	148.32	322.02
9	347.50	5.851	329.64	5.798	0.053	5.380	6.216	217.06	500.61
10	370.23	5.914	363.94	5.897	0.018	5.436	6.358	229.61	576.87
11	346.00	5.846	360.68	5.888	-0.042	5.588	6.188	267.33	486.64
12	371.00	5.916	372.04	5.959	-0.043	5.634	6.284	279.86	535.76
13	467.83	6.148	429.66	6.063	0.085	5.612	6.514	273.82	674.21
14	407.33	6.010	453.05	6.116	-0.106	5.723	6.509	305.93	670.92
15	98.67	4.592	98.49	4.590	0.002	4.185	4.995	65.68	147.71
16	1327.33	7.191	1213.18	7.101	0.090	6.701	7.501	813.05	1810.22
17	189.08	5.242	189.62	5.245	-0.003	4.784	5.706	119.63	300.55
18	273.67	5.612	242.50	5.491	0.121	5.030	5.914	152.99	370.13
19	430.00	6.064	375.78	5.929	0.135	5.564	6.294	260.87	541.30
20	157.17	5.057	162.88	5.093	-0.036	4.647	5.539	104.32	254.30
21	127.00	4.844	124.96	4.828	0.016	4.500	5.206	85.67	182.28
22	137.42	4.923	111.74	5.146	-0.223	4.766	5.526	117.44	215.15
23	484.42	6.183	518.53	6.251	-0.069	5.876	6.626	356.37	754.48
24	136.75	4.918	136.18	4.914	0.004	4.453	5.375	85.92	215.86

Table 13   Comparison of Percent Deviations of Predicted Lives
and Residual Sum of Squares for Seven Models

Test No.	Actual Life T	Percent Deviations of Predicted Lives						
		Eq(1)	Eq(2)	Eq(3)	Eq(4)	Eq(5)	Eq(6)	Eq(6) of [1]
1	1363.43	3.7	9.3	-2.6	14.8	-2.1	-1.3	15.9
2	938.83	11.9	18.3	-4.4	15.7	-3.9	5.8	14.6
3	165.08	-25.6	-11.1	1.5	-5.8	0.2	5.2	-23.1
4	391.97	-44.5	-41.7	-9.5	-47.4	-0.8	3.4	-59.7
5	156.20	-12.7	-15.0	0.1	-1.2	1.7	0.9	-1.4
6	160.83	24.7	16.8	-0.1	21.5	-1.3	9.5	20.8
7	1011.42	16.7	15.4	-2.5	19.3	-3.6	-4.4	19.8
8	259.08	11.6	9.4	3.1	20.2	20.4	15.6	24.1
9	347.50	-7.2	-9.0	-1.5	-3.7	-1.7	5.1	-10.4
10	370.23	9.3	14.7	4.6	14.3	5.0	1.7	16.2
11	346.00		-4.7	-6.7	-0.4	-6.6	-4.2	-2.7
12	371.00		-0.2	0.3	5.1	0.9	-0.3	8.4
13	467.82		24.4	5.0	30.0	0.9	8.2	20.2
14	407.33		3.1	-0.5	2.5	-17.0	-11.2	6.3
15	98.67		6.2	-4.3	15.9	-3.0	0.2	15.5
16	1327.33		-2.0	8.7	-4.9	8.1	8.6	3.7
17	189.08		-95.0	1.6	-83.5	0.3	-0.3	6.8
18	273.67		12.4	5.4	10.9	8.3	11.4	16.3
19	430.00				12.6	16.4	12.6	4.3
20	152.17				-4.8	-2.6	-0.4	-2.4
21	127.00				-2.7	-1.8	1.6	-7.8
22	137.42				-54.6	-26.6	-24.9	-55.9
23	484.42				-14.7	8.2	-7.0	-20.7
24	136.75				-25.7	-0.6	0.4	-25.7

	Average Deviations						
Negative side	22.5	22.3	3.6	20.8	5.3	6.0	17.4
Positive side	13.0	13.0	3.4	15.2	6.8	6.0	16.0

	Residuals						
Percent residual sum of squares of the total	17.0	8.8	0.4	9.3	1.5	1.3	7.3

Table 14 Comparison of Widths of the Confidence Intervals for Two Models

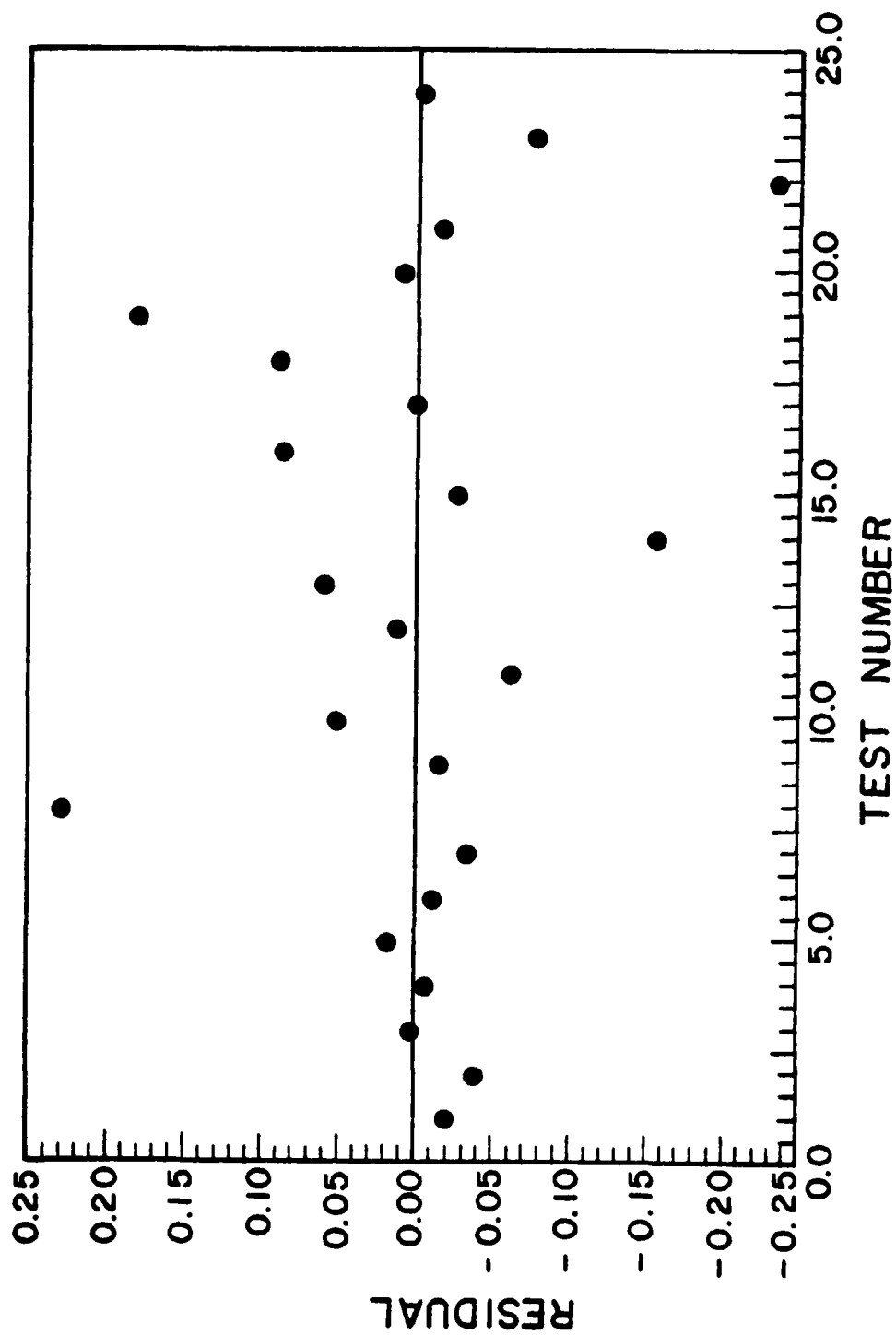| Test No.	Actual Life T	Equation (5)			Equation (6)			Percent Ratio $\frac{(U-L)_5}{(U-L)_6} \times 100$
		Predicted Life T	Lower Limit L	Upper Limit U	Predicted Life T	Lower Limit L	Upper Limit U	
1	1363.43	1391.31	1002.26	1931.38	1381.60	869.78	2164.65	71.7
2	938.83	975.55	728.66	1306.09	993.27	632.99	1558.60	62.4
3	165.08	164.83	127.09	213.82	156.49	100.48	243.71	60.6
4	391.97	395.05	311.53	500.95	405.45	286.48	573.84	65.9
5	156.20	153.55	111.36	211.71	154.78	97.65	245.33	68.0
6	160.83	162.88	130.50	203.30	176.09	121.63	254.93	54.6
7	1011.42	1047.33	839.09	1457.17	1055.75	669.43	1664.99	62.1
8	259.08	206.03	173.48	244.67	218.55	148.32	322.02	41.0
9	347.50	353.54	297.70	416.51	329.64	217.06	500.61	42.0
10	370.23	351.78	271.82	455.26	363.94	229.61	576.87	52.8
11	346.00	368.71	306.28	443.85	360.68	263.33	486.64	61.6
12	371.00	367.70	305.37	442.52	372.04	279.86	535.76	53.6
13	467.83	441.42	323.79	601.78	429.66	273.82	674.21	69.4
14	407.33	476.75	375.11	605.93	453.05	305.93	670.92	63.2
15	98.67	101.60	76.40	135.10	98.49	65.68	147.71	71.6
16	1327.33	1219.26	916.89	1621.35	1213.18	813.05	1810.22	70.6
17	189.08	189.62	136.59	263.22	189.62	119.63	300.55	70.1
18	273.67	250.89	250.88	328.83	242.50	152.99	370.13	35.9
19	430.00	359.60	288.07	463.28	375.78	260.87	541.30	62.5
20	157.17	156.18	114.56	212.92	162.88	104.32	254.30	65.6
21	127.00	129.28	101.49	164.68	124.96	85.67	182.28	65.4
22	137.42	173.99	138.45	313.39	171.74	117.44	251.14	130.8
23	484.42	524.27	419.08	655.85	518.53	356.37	754.48	59.5
24	136.75	137.55	99.54	190.08	136.18	85.92	215.86	69.6

Average: 63.8%

Fig. 1   Distribution of Residuals for the Best Second Order Model, Eq. (5)

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In the previous study of first order models with 8 response variables to predict fatigue life in elastic range under random vibrations, 4 variables showed significant effects. In this part of the research both first and second order models of these 4 significant variables based on 10, 18, and 24 tests are developed. The tables of analysis of variance, and of predicted lives together with their residuals and 95% confidence intervals are constructed for each of the first and second order models.

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