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EXPERIMENTAL RANDOM FATIGUE
IN ELASTIC-PLASTIC RANGE -
FIRST ORDER MODELS

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ABSTRACT

An experiment based on 11 probabilistic parameters and experiment design is conducted for fatigue of materials in an elastic-plastic range under random vibrations to develop regression models which will predict the fatigue life. The experiment consisted of a total of 24 tests and is conducted in 3 designs. The response of each test specimen is digitized, recorded, and is fitted with time series models. The time-series model parameters are then used to compute the spectrum and spectral moments from which the magnitude levels of probabilistic parameters are computed and coded. The first-order regression models are developed for each design by regressing the log of fatigue life on the corresponding coded parameters. The tables of the analysis of variance, and of the predicted lives together with residuals and 95% confidence intervals are constructed for each model. From the analysis of variance the F-ratio is computed to check whether the model is acceptable. The first-order model involving all 11 parameters based on 24 tests is considered to be the statistically best one. The methodology of this report is found to be an accurate and reliable approach and is suitable for industrial applications. This method contrasts with the prevailing method which is based on linear damage accumulation and cycle counting, involves a several hundred-percent error as a rule, and is hardly applicable to industrial problems.

INTRODUCTION

The principle of linear damage accumulation based on cycle counting has been consistently found to give unreliable and inaccurate estimates of fatigue life under random vibrations [1-10]. This group of representative references has been discussed in [11].

Recently a new history-dependent, stochastic model of cumulative damage is being developed by Bogdanoff [12-14] by taking a comprehensive view of the entire failure process to improve the predictive accuracy. Another approach, entirely different from all above mentioned approaches, has been developed in [11,15] to predict the fatigue life of materials under random vibrations in the elastic range. The same approach is used now for elastic-plastic range. An experiment program of 24 tests based on 11 probabilistic parameters and experiment design is conducted in 3 designs. For each design the regression models are developed to predict fatigue life under random vibrations. Again, this approach has been found to give very reliable and accurate estimates.

The purpose of this research is to develop a novel methodology for random fatigue based on the probabilistic parameters and experiment design. Therefore, the choice of the specimen materials, types of loadings (axial, shear or bending), and range of parameters is irrelevant and they are arbitrarily chosen.

I. EXPERIMENTS, PARAMETERS, DESIGNS AND MODELS

1. Experiment

The experiment program was conducted with 24 specimens in 3 designs. These specimens were numbered from 25-48 to distinguish them from specimens numbered from 1-24 used in the elastic case [11].

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The bending specimen of aluminum alloy 6061-T6 and the experimental set-up described in [11,16,17] were also used for this experiment.

2. Parameters

A typical random response signal in elastic-plastic range which is characterized by 11 probabilistic parameters is given in Fig. 1. They are mean, variance, zero upcrossings, ϵ_f level upcrossings, ϵ_y level upcrossings, duration of excursion above zero, duration of excursion above ϵ_f level, duration of excursion above ϵ_y level, band width, average amplitude above ϵ_f level, and average amplitude above ϵ_y level. In these parameters ϵ_f represents the strain level corresponding to a material life of 10^7 cycles in the deterministic fatigue tests, and ϵ_y represents the yield point strain of the material. A mathematical analysis of these 11 probabilistic parameters is given in [18]. Hereafter, these probabilistic parameters will be simply referred to as variables as well.

3. Designs

The first design of the experiment consisted of 10 tests which formed a full factorial design with 2 center points. The second design, a central composite design with 4 center points, was formed by adding 8 more tests to the first design. The third design was formed by adding to the second design random replications of six tests from the second design.

a. Range The first design was based on the three different magnitude levels of the 3 variables, namely, mean, variance and zero upcrossings. Two more levels from each of the 3 variables were added in the second design so that one was lower and the other one higher

than the 3 previous levels. In the third design, 6 tests were replicated without introducing new levels of the variables. Thus, the second and the third designs had 5 levels from each of the above 3 variables. The lowest and the highest of the 5 levels represent the lower and upper bounds of the range of variables. The upper bounds of the mean and variance are limited such that the maximum capability of the shaker table displacement is not exceeded. Whenever the displacement exceeds the maximum capability of the shaker, the overtravel limit is reached and the shaker is shut off automatically. Otherwise, the range of the variables can be arbitrarily chosen. The ranges of mean, variance and zero upcrossings based on 5 levels are 241-2657 micro inches/inch, $2.42 \times 10^6 - 4.36 \times 10^6$ (micro inches/inch)², and 9.12 - 15.32, respectively.

b. Code The above 3 variables which were controlled in the experiment were coded. The remaining 8 variables which could not be controlled in the experiment were computed from the random response of each test specimen as described in [11]. The variables and the actual levels, and the coded variables and coded levels are shown in Table 1. Tables 2 and 3 show the actual levels of variables and coded levels of coded variables, respectively, of all 11 variables for the 24 tests.

4. Models

The regression models, also referred to as life predicting equations, were obtained for each of the 3 designs as was done in [11]. For the first design only one life predicting equation of 3 variables was obtained. For each of the second and third designs two life predicting equations were obtained, one involves only 3 variables and the other

one involves all 11 variables. These equations were obtained by regressing the log of actual life, y , on the corresponding coded variables involved in the model. The models predicted the log of life, \hat{y} . The predicted fatigue life, \hat{T} , is computed by taking the antilog of \hat{y} .

The table of analysis of variance, and the table of predicted lives together with residuals and 95% confidence intervals, are constructed for each model. From the analysis of variance, the F-ratio for each model was computed to determine whether the model is acceptable. The confidence intervals were computed using the standard deviations of \hat{y} and the t values from the t -table with degrees of freedom equal to that of residuals. The distribution of residuals was studied for any pattern or trend present. If the pattern or trend existed, the model was considered to have missed some significant variables. The analysis of 3 designs is given below.

II. FULL FACTORIAL DESIGN WITH TWO CENTER POINTS

The first ten tests from 25-34 of the 24 tests formed a full factorial design with 2 center points. One life predicting equation of three variables is obtained by regressing log of actual life, y , on coded levels of mean, variance and zero upcrossings. These three variables are represented by symbols x_1 - x_3 . The life predicting equation of all 11 variables could not be obtained.

1. Three Variables

The life predicting equation is obtained as

$$\hat{y} = 3.52 + 0.0538x_1 - 0.341x_2 - 0.0514x_3 \quad (1)$$

The analysis of variance of equation (1) is given in Table 4. The F-ratio of 4.328 with 3 and 6 degrees of freedom is found. The

corresponding F value from the F-table is 4.76 which is greater than the F-ratio of the equation. This implies that the regression is not effective and the model is not acceptable. The residual sum of squares is 0.4862 in comparison to a total of 2.0269, a 32.6%. The other 68.4% of the total sum of squares is due to regression.

This model does not qualify the F-test and may not be acceptable. Nevertheless the predicted lives together with their residuals and 95% confidence intervals are computed as given in Table 5 in order to see how the results of an unacceptable model appear. The confidence intervals are fairly wide because the t value with six degrees of freedom is high. The actual life of the test number 30 only is slightly below the lower limit of the predicted interval.

2. All 11 Variables

The life predicting equation based on all 11 variables could not be obtained because the number of tests in this design is smaller than the variables involved.

III. CENTRAL COMPOSITE DESIGN WITH FOUR CENTER POINTS

The full factorial design is augmented by 8 more tests to form a central composite design with 4 center points resulting in a total of 18 tests. Two life predicting equations are obtained for this design. The first one consists of the same 3 variables, while the second one consists of all 11 variables.

1. Three Variables

The life predicting equation is obtained as

$$\hat{y} = 3.55 + 0.0242x_1 - 0.327x_2 - 0.0597x_3 \quad (2)$$

The analysis of variance of equation (2) is given in Table 6. The F-ratio of 14.80 with 3 of 14 degrees of freedom is obtained. The corresponding F value from the F-table at 95% significance level is 3.34. The table value of F is smaller than the F-ratio which implies that the regression is effective and the model is acceptable. The residual sum of squares is 0.4862 in comparison to a total of 2.0269, a 24.0%. The other 76.0% of the total sum of squares is due to regression. In this case the sum of squares due zero upcrossings is higher than the sum of squares, due to mean.

The predicted lives together with residuals and 95% confidence interval, are given in Table 7. The confidence intervals are fairly narrow. The actual lives of test numbers 30, 33 and 34 are slightly off from the corresponding predicted interval. The residuals also are relatively large in magnitude. The investigation of this model suggests that more variables which have significant effects on the fatigue life should be introduced.

2. All 11 Variables

The life predicting equation is obtained as

$$\begin{aligned} \hat{y} = & 3.39 - 0.0143x_1 - 0.463x_2 - 0.144x_3 + 0.020x_4 \\ & - 0.314x_5 - 0.727x_6 + 0.637x_7 + 0.112x_8 - 0.0103x_9 \\ & + 0.101x_{10} - 0.0803x_{11} \end{aligned} \quad (3)$$

The analysis of variance of equation (3) is given in Table 8. The F-ratio of 6.036 with 11 and 6 degrees of freedom is obtained for this model. The corresponding F value from the F-table is 2.92 at 95% significance level. The F-ratio is greater than the table value of F

which means that the regression is effective and that the model is acceptable. The sum of squares due to residuals is 0.1682 in comparison to a total of 2.0269, a 8.3%. The other 91.7% of the total sum of squares is due to regression. The sum of squares due to 6 of the 11 variables is considered insignificant. These six variables are mean, ϵ_y level upcrossings, duration of excursion above zero and ϵ_y levels, band width and average amplitude above ϵ_f level.

The predicted lives together with residuals and 95% confidence intervals are given in Table 9. The confidence intervals are relatively wide because t value with 6 degrees of freedom associated with the residuals is high. The actual lives of all tests fall within the predicted confidence intervals. The residuals are small in magnitudes.

IV. CENTRAL COMPOSITE DESIGN WITH FOUR CENTER POINTS AND SIX REPLICATIONS

Six more tests are added to the central composite design by randomly replicating six of the 18 tests. Adding these six tests resulted in a total of 24 tests in the design. Two life predicting equations are obtained for this design also, one using 3 variables and another one using all 11 variables. The equations and their analyses are as follows.

1. Three Variables

The life predicting equation is obtained as

$$\hat{y} = 3.56 + 0.0195x_1 - 0.325x_2 - 0.0518x_3 \quad (4)$$

The analysis of variance of equation (4) is given in Table 10. The F-ratio for the equation above is 24.23 with 3 and 20 degrees of freedom which is greater than the critical F-value of 3.10 from the F-table at 95% significance level. This implies that the regression is effective and the model is acceptable. The residual

sum of squares is 0.6192 in comparison to a total sum of squares of 3.1801, a 20.5%. The other 79.5% of the total is due to regression.

The predicted lives together with residuals and 95% confidence intervals are given in Table 11. The sum of squares due to zero upcrossings in this case is lower than that due to mean. The confidence intervals are comparatively narrow but the actual lives of test nos. 27, 29, 30, 33, 34, 35, 44 and 45 fall out of the corresponding predicted confidence intervals. The residuals are relatively large which implies that some variables which may have significant effects on the fatigue life are missing from the model. The life predicting equation using more variables is discussed below.

2. All 11 Variables

The life predicting equation is obtained as

$$\begin{aligned} \hat{y} = & 3.50 - 0.0105x_1 - 0.398x_2 - 0.0748x_3 + 0.0459x_4 \\ & - 0.111x_5 - 0.317x_6 + 0.290x_7 + 0.0091x_8 - 0.0383x_9 \\ & + 0.0739x_{10} + 0.0231x_{11} \end{aligned} \quad (5)$$

The analysis of variance based on 24 tests of equation (5) is given in Table 12. The F-ratio for the above equation is computed to be 8.812 with 11 and 12 degrees of freedom which is larger than the corresponding F value of 2.73 from the F-table at 95% significance level. The comparison of two F values indicates that the regression is effective and the model is acceptable. The residual sum of squares is 0.3498 in comparison to a total sum of squares of 13.1801, a 11.0%. The other 89% of the total is due to regression, which is significantly high. The six of the 11 variables are considered to be insignificant because they contribute a very low sum of squares to regression. These

6 variables are ϵ_f level upcrossings, duration of excursion above zero level, duration of excursion above ϵ_y level, band width, and average amplitudes above ϵ_f and ϵ_y levels.

The predicted lives together with residuals and 95% confidence intervals are given in Table 13. The confidence intervals are fairly narrow. Actual lives of all the 24 tests fall within the predicted confidence intervals. The residuals appear to be relatively small and their plot is given in Fig. 2. The plot shows that the residuals are concentrated more on the positive side with fairly small magnitudes.

V. DISCUSSIONS AND CONCLUSIONS

On the basis of the analysis of all models of equations from (1) to (6) it is observed that there are 5 variables which have significant effects on the fatigue life. These 5 significant variables are mean, variance, zero upcrossings, ϵ_y level upcrossings, and the duration of excursion above ϵ_f level. The remaining 6 variables are considered to be less significant.

Among the 5 significant variables the variance has the most significant effect. This is obvious because the variance is a measure of square of the amplitude of the strain signal. Consequently a large variance of the signal means large amplitude of the strain. The magnitudes of effects of the remaining four significant variables are low as compared to that of the variance but are not negligible. Variance, ϵ_y level upcrossings, and duration of excursion above ϵ_f level show significant effects in all 3 analyses of 10, 18 and 24 tests. The effect of zero upcrossings is found to be pronounced in the analysis of 10 tests and the effect of mean, in the analysis of 24 tests. Nevertheless

both mean and zero upcrossings have been included in the list of 5 significant variables.

A comparison of all the first order models investigated in this report with respect to their percent deviations of the predicted lives from the actual ones, and residual sum of squares as percent of the total is given in Table 14. This table shows that the models of all 11 variables for both 18 and 24 tests give a lower percent deviations and a lower percent residual sum of squares in comparison to the model of 3 variables for the same design. Based on 24 tests, the model of equation (5) of all 11 variables is considered the best first order model to predict fatigue life under random vibrations in the elastic-plastic range. The percent deviations of the best model range from -26.0% to 23.2% with an average deviations of 13.4% on the negative side and 8.4% on the positive side. These deviations are negligible as compared to several hundred percent deviations obtained in linear damage accumulation theory.

The lives predicted by the best first order model are expected to be the mean lives of the material for the given levels of variables. The 95% confidence intervals give the range which is expected to include the actual fatigue life for 95% of the tests. The actual life of almost all the tests is greater than the lower level of confidence interval so this level can be considered a conservative estimate of fatigue life.

A few first order and second order models of significant variables will be given in [19].

VI. SUMMARY

(1) An experiment of 24 tests based on 11 variables and experiment design has been performed to predict fatigue life under random vibrations.

(2) The experiment was conducted in 3 designs. For the first design one first order life predicting equation of 3 variables has been developed. For each of the other two designs two first order life predicting equations of 3 and all 11 variables, respectively, have been developed.

(3) The tables of analysis of variance, and of the predicted lives together with residuals and 95% confidence intervals are constructed for each equation.

(4) From the analysis of variance the F-ratio is computed to check whether the regression is effective and the model is acceptable.

(5) Five out of 11 variables have been found to have significant effects on the fatigue life. These 5 variables are mean, variance, zero upcrossings, ϵ_y level upcrossings and duration of excursion above ϵ_f level.

(6) The first order life predicting equation of all 11 variables and 24 tests was found to be the statistically best one.

(7) The percent deviations of the predicted lives obtained by the best equation are within a range from -26.0% to 23.2% with an average of 13.4% on the negative side and 8.4% on the positive side. These results are in contrast with those of linear damage accumulation principle.

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REFERENCES

1. Hillberry, B. M., "Fatigue Life of 2024-T3 Aluminum Alloy Under Narrow-and Broad-Band Random Loading," Effects of Environment and Complex Load History on Fatigue Life, ASTM, STP462, American Society for Testing Materials, 1970, pp. 167-183.
2. Head, A. K., and Hooke, F. H., "Random Noise Fatigue Testing," International Conference on Fatigue of Metals, Institute of Mechanical Engineers, 1956, pp. 301-303.
3. Cleaverson, S. A. and Steiner, Roy, "Fatigue Life Under Random Loading for Several Power Spectral Shapes," Technical Report, NASA R-266, Sept. 1967.
4. Lowcock, M. T. and Williams, T. R. G., "Effects of Random Loading on Fatigue Life of Aluminum Alloy L-73," Department of Aeronautics and Astronautics, July 1962, TR-10943, Unclass Code 3.
5. Brown, G. W., and Ikegemi, R., "The Fatigue Life of Aluminum Alloys Subjected to Random Loading," Experimental Mechanics, Aug. 1970, pp. 321-327.
6. Dowling, N. E., "Fatigue Failure Predictions for Complicated Stress Strain Histories," Journal of Materials, JMLSA, Vol. 7, No. 1, March 1972, pp. 71-87.

7. Schutz, W., "Fatigue Life Predictions of Aircraft Structures-Past, Present and Future," Engineering Fracture Mechanics, Vol. 6, Pergamon Press, 1974, pp. 745-773.
8. Buch, Alfred, "The Damage Sum in Fatigue of Structure Components," Engineering Fracture Mechanics, Vol. 10, Pergamon Press, 1978, pp. 233-247.
9. Smith, Jr., P. W., and Malme, C. I., "Fatigue Test of a Resonant Structure with Random Excitation," Journal of Acoustical Society of America, Vol. 35, No. 1, Jan. 1963, pp. 43-46.
10. Swanson, S. R., "An Introduction of the Fatigue of Aluminum Alloy Due to Random Loading," UTIA Report No. 84, Feb. 1963.
11. Huang, T. C. and Nagpal, Vinod K., "Experimental Random Fatigue in Elastic Range-First Order Models," ONR Technical Report No. UW/RF-3, University of Wisconsin-Madison, 1979.
12. Bogdanoff, J. L., "A New Cumulative Damage Model-Part 1," Journal of Applied Mechanics, Vol. 45, June 1978, pp. 246-250.
13. Bogdanoff, J. L., and Kreiger, W., "A New Cumulative Damage Model-Part 2," Journal of Applied Mechanics, Vol. 45, June 1978, pp. 251-257.
14. Bogdanoff, J. L., "A New Cumulative Damage Model-Part 3," Journal of Applied Mechanics, Vol. 45, Dec. 1978, pp. 733-739.
15. Huang, T. C., and Nagpal, Vinod K., "Experimental Random Fatigue in Elastic Range-Models of Significant Variables, ONR Technical Report No. UW/RF-4, University of Wisconsin-Madison, 1979.

16. Huang, T. C., Hubbard, Ralph B., and Lanz, R. W., "Fatigue Failure of Materials Under Narrow Band Random Vibration-Part 1, Proceedings of the Twenty-first International Astronautical Congress- North Holland, 1971, pp. 593-609.
17. Huang, T. C., Hubbard, Ralph B., and Lanz, R. W., "Fatigue Failure of Materials Under Narrow Band Random Vibrations-Part 2," Astronautical Research, 1971, pp. 331-342.
18. Huang, T. C., Shen, K. S. and Nagpal, Vinod K., "Probabilistic Parameters in Random Fatigue," ONR-Technical Report No. UW/RF-1, University of Wisconsin-Madison, Oct. 1977.
19. Huang, T. C., and Nagpal, Vinod K., "Experimental Random Fatigue in Elastic-Plastic Range - Models of Significant Variables," Forthcoming ONR-Technical Report No. UW/RF-6, University of Wisconsin-Madison, 1979.

Table 1 Actual Levels, Coded Levels and Coded Variables

| No. | Variables | Actual Level | | | Coded Level | | | Coded Variables |
|-----|---|---------------------|---------------------|--------------------|-------------|--------|------|-----------------|
| | | Low | Center | High | Low | Center | High | |
| 1 | Mean | 400.00 | 800.00 | 1600.00 | -1 | 0 | +1 | x_1 |
| 2 | Variance | 27.40×10^5 | 32.80×10^5 | 38.5×10^5 | -1 | 0 | +1 | x_2 |
| 3 | Zero Upcrossings | 10.50 | 11.91 | 13.50 | -1 | 0 | +1 | x_3 |
| 4 | ϵ_f Level Upcrossings | 10.50 | 11.91 | 13.50 | -1 | 0 | +1 | x_4 |
| 5 | ϵ_y Level Upcrossings | 0.25 | 0.6124 | 1.50 | -1 | 0 | +1 | x_5 |
| 6 | Duration of Excursion Above Zero Level | 0.600 | 0.693 | 0.800 | -1 | 0 | +1 | x_6 |
| 7 | Duration of Excursion Above ϵ_f Level | 0.300 | 0.387 | 0.50 | -1 | 0 | +1 | x_7 |
| 8 | Duration of Excursion Above ϵ_y Level | 0.003 | 0.0085 | 0.01 | -1 | 0 | +1 | x_8 |
| 9 | Band Widths | 0.96 | 0.97 | 0.98 | -1 | 0 | +1 | x_9 |
| 10 | Average Amplitude Above ϵ_f Level | 3000.0 | 3646.0 | 4000.0 | -1 | 0 | +1 | x_{10} |
| 11 | Average Amplitude Above ϵ_y Level | 5500.0 | 5979.1 | 6500.0 | -1 | 0 | +1 | x_{11} |

Table 2 Test Numbers and Actual Levels of 11 Variables

| Test No. | Actual Levels of Variables | | | | | | | | | | |
|----------|----------------------------|---------|-------|-------|--------|------|------|--------|--------|---------|---------|
| | 1 | 2* | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 25 | 1601.15 | 27.7441 | 10.50 | 10.44 | 0.0000 | 0.82 | 0.55 | 0.0000 | 0.9636 | 3777.61 | 0.00 |
| 26 | 410.82 | 27.2941 | 13.46 | 11.05 | 0.0000 | 0.82 | 0.56 | 0.0000 | 0.9823 | 2926.32 | 0.00 |
| 27 | 1597.20 | 38.1361 | 15.32 | 15.26 | 0.6739 | 0.81 | 0.52 | 0.0100 | 0.9778 | 3802.80 | 6999.11 |
| 28 | 1605.17 | 27.3237 | 12.60 | 12.54 | 0.0000 | 0.83 | 0.56 | 0.0000 | 0.9852 | 3870.84 | 0.00 |
| 29 | 410.62 | 38.4906 | 11.36 | 10.24 | 0.0000 | 0.57 | 0.29 | 0.0000 | 0.9867 | 3389.56 | 0.00 |
| 30 | 403.38 | 27.4555 | 10.75 | 8.77 | 0.0000 | 0.61 | 0.27 | 0.0000 | 0.9649 | 2722.23 | 0.00 |
| 31 | 397.36 | 38.6158 | 10.95 | 9.35 | 0.1045 | 0.59 | 0.32 | 0.0035 | 0.9478 | 2989.73 | 6505.24 |
| 32 | 1605.49 | 38.0926 | 10.46 | 10.41 | 0.5933 | 0.80 | 0.55 | 0.0100 | 0.9608 | 3898.88 | 6642.57 |
| 33 | 802.89 | 32.3718 | 13.30 | 12.28 | 0.0615 | 0.67 | 0.38 | 0.0015 | 0.9658 | 2900.65 | 6416.13 |
| 34 | 805.73 | 32.7229 | 10.62 | 10.10 | 0.0000 | 0.67 | 0.38 | 0.0000 | 0.9775 | 3386.69 | 0.00 |
| 35 | 795.21 | 33.0729 | 11.11 | 10.51 | 0.0000 | 0.68 | 0.40 | 0.0000 | 0.9788 | 3308.33 | 0.00 |
| 36 | 799.08 | 32.4496 | 12.29 | 11.52 | 0.0000 | 0.67 | 0.39 | 0.0000 | 0.9728 | 3140.04 | 0.00 |
| 37 | 251.38 | 32.6025 | 11.23 | 9.36 | 0.0000 | 0.58 | 0.28 | 0.0000 | 0.9724 | 3044.00 | 0.00 |
| 38 | 791.33 | 43.5084 | 11.36 | 10.90 | 0.4979 | 0.65 | 0.42 | 0.0042 | 0.9764 | 3723.71 | 6835.44 |
| 39 | 801.54 | 32.3287 | 13.79 | 13.01 | 0.2607 | 0.65 | 0.37 | 0.0034 | 0.9855 | 3216.36 | 6565.29 |
| 40 | 2645.87 | 32.6288 | 11.89 | 9.67 | 2.5200 | 0.93 | 0.74 | 0.0300 | 0.9753 | 5374.21 | 6847.03 |
| 41 | 800.52 | 31.8401 | 9.12 | 8.55 | 0.1324 | 0.68 | 0.38 | 0.0008 | 0.9523 | 3129.60 | 6525.04 |
| 42 | 794.69 | 23.7001 | 11.82 | 10.85 | 0.0000 | 0.69 | 0.36 | 0.0000 | 0.9704 | 2849.34 | 0.00 |
| 43 | 1596.36 | 39.0830 | 11.89 | 11.86 | 1.4923 | 0.80 | 0.50 | 0.0040 | 0.9859 | 4300.32 | 6869.77 |
| 44 | 1601.56 | 39.3295 | 11.39 | 11.34 | 1.0621 | 0.79 | 0.53 | 0.0100 | 0.9434 | 4129.94 | 6771.88 |
| 45 | 399.12 | 27.3123 | 10.52 | 8.67 | 0.0000 | 0.61 | 0.28 | 0.0000 | 0.9614 | 2783.97 | 0.00 |
| 46 | 805.23 | 31.1355 | 12.22 | 11.65 | 0.0000 | 0.67 | 0.39 | 0.0000 | 0.9896 | 3434.78 | 0.00 |
| 47 | 793.90 | 43.6298 | 13.41 | 12.49 | 0.3548 | 0.66 | 0.41 | 0.0005 | 0.9686 | 3310.01 | 6614.79 |
| 48 | 803.13 | 24.6076 | 11.18 | 10.47 | 0.0000 | 0.70 | 0.37 | 0.0000 | 0.9763 | 3097.03 | 0.00 |

* All numbers in the column for Variable 2 to be multiplied by 10^5 .

Table 3 Test Numbers and Coded Levels of Coded Variables

| Test No. | Coded Levels of Variables | | | | | | | | | | |
|----------|---------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|
| | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | x_7 | x_8 | x_9 | x_{10} | x_{11} |
| 25 | 1.0020 | -0.9266 | -1.0000 | -1.0456 | 0.0000 | 1.1717 | 1.3732 | 0.0000 | -0.7679 | 0.6023 | 0.0000 |
| 26 | -0.9644 | -1.0227 | 0.9764 | -0.5937 | -1.6443 | 1.1717 | 1.4437 | -0.7439 | 1.2273 | -1.1729 | 1.0096 |
| 27 | 0.9981 | 0.9442 | 2.0065 | 1.9752 | 0.2058 | 1.0864 | 1.1536 | 1.0000 | 0.7820 | 0.6485 | -2.1435 |
| 28 | 1.0023 | -1.0164 | -0.4509 | 0.4429 | 0.0000 | 1.2559 | 1.4437 | 0.0000 | 1.5133 | 0.7718 | 0.0000 |
| 29 | -0.9644 | 0.9986 | -0.3735 | 1.1995 | 0.0000 | -1.3566 | -1.1327 | 0.0000 | 1.6609 | -0.1512 | 0.0000 |
| 30 | -0.9879 | -0.9881 | -0.8127 | -2.4328 | 0.0000 | -0.8850 | -1.4125 | 0.0000 | -0.5062 | -1.6761 | 0.0000 |
| 31 | -1.0108 | 1.0176 | -0.6660 | -1.9231 | 0.0000 | -1.1168 | -0.7473 | 0.0000 | -2.2406 | -1.0255 | 0.0000 |
| 32 | 1.0023 | 0.9787 | -1.0304 | -1.0685 | 0.0793 | 1.0000 | 1.3732 | 1.0000 | -0.9192 | 0.8220 | 1.2587 |
| 33 | 0.0036 | -0.0195 | 0.8812 | 0.2462 | -2.1701 | -0.2328 | -0.0745 | -2.1514 | -0.4157 | -1.2357 | 0.8443 |
| 34 | 0.0090 | 0.0439 | -0.9096 | -1.3091 | 0.0000 | -0.2328 | 0.0745 | 0.0000 | 0.7522 | -0.1572 | 0.0000 |
| 35 | -0.0090 | 0.1065 | -0.5506 | -0.0000 | 0.0000 | -0.1298 | 0.1263 | 0.0000 | 0.8811 | 0.6102 | 0.0000 |
| 36 | -0.0018 | -0.0054 | 0.2527 | 0.2622 | -0.0000 | 0.2328 | 0.0272 | 0.0000 | 0.2847 | 0.5030 | 0.0000 |
| 37 | -1.6723 | 0.0223 | -0.3529 | -1.9146 | 0.0000 | -1.2357 | -1.2701 | 0.0000 | 0.2448 | -0.8988 | 0.0000 |
| 38 | -0.0163 | 1.7191 | -0.3735 | -0.7025 | -0.0947 | -0.4435 | 0.3174 | -0.4411 | 0.6430 | 0.5024 | 1.6016 |
| 39 | 0.0018 | -0.0273 | 1.1691 | 0.7058 | -0.7369 | -0.4435 | -0.1789 | -0.7921 | 1.5428 | -0.5159 | 1.1191 |
| 40 | 1.7252 | 0.0270 | -0.0106 | -0.7025 | 1.5149 | 2.0468 | 2.5349 | 2.8250 | 0.5337 | 4.1062 | 1.6227 |
| 41 | 0.0000 | -0.1169 | -2.1213 | -2.6350 | -1.4094 | -0.1298 | -0.0745 | -3.1957 | -1.7811 | 0.4962 | 1.0459 |
| 42 | -0.0108 | -1.8530 | -0.0576 | -0.7390 | 0.0000 | -0.0283 | -0.2862 | 0.0000 | 0.0451 | 0.3036 | 0.0000 |
| 43 | 0.9964 | 1.0883 | -0.0106 | -0.0307 | 0.9949 | 1.0000 | 1.0000 | -0.5221 | 1.5822 | 1.5031 | 1.6610 |
| 44 | 1.0009 | 1.1253 | -0.3525 | -0.3875 | 0.6573 | 0.9126 | 0.8550 | 1.0000 | -2.6919 | 1.2207 | 1.4906 |
| 45 | -1.0036 | -1.0188 | -0.9849 | -2.5241 | 0.0000 | -0.8850 | -1.2701 | 0.0000 | -0.8586 | 0.2559 | 0.0000 |
| 46 | 0.0090 | -0.2485 | 0.2072 | -0.1729 | 0.0000 | -0.2328 | 0.0272 | 0.0000 | 1.8475 | 0.6872 | 0.0000 |
| 47 | -0.0127 | 1.7355 | 0.9468 | 0.3817 | -0.4310 | -0.3374 | 0.2230 | -3.9764 | -0.1349 | 0.6113 | 1.2096 |
| 48 | 0.0054 | -1.6321 | -0.5006 | -1.0228 | 0.0000 | 0.0717 | -0.1789 | 0.0000 | 0.6331 | 0.4747 | 0.0000 |

Table 4. Analysis of Variance of 10 Tests
 First Order Model of 3 Variables
 Life predicting equation:

$$\hat{y} = 3.52 + 0.0538x_1 - 0.341x_2 - 0.0514x_3$$

| Source | Sum of Squares | Degrees of Freedom | Mean Square | F-Ratio |
|-------------------------|----------------|--------------------|-------------|---------|
| Due to Mean | 0.0100 | 1 | 0.0100 | |
| Due to Variance | 0.8439 | 1 | 0.8439 | |
| Due to Zero Upcrossings | 0.0264 | 1 | 0.0264 | |
| Due to Regression | 0.8857 | 3 | 0.2952 | |
| Residuals | 0.4094 | 6 | 0.0682 | 4.328 |
| Total | 1.2951 | 9 | | |

F-ratio is smaller than the table value of 4.76 with 3 and 6 degrees of freedom at 95% significance level. So regression is not effective and the model is rejected.

Table 5. Results of 10 Tests, First Order Model of 3 Significant Variables
Life predicting equation:

$$\hat{y} = 3.52 + 0.0538x_1 - 0.341x_2 - 0.0516x_3$$

| Test No. | Actual Life | | Predicted Life | | Residuals $y - \hat{y}$ | 95% Confidence Interval | | | |
|----------|-------------|-------|----------------|-----------|----------------------------|-------------------------|-------|-----------|-------|
| | T | y | \hat{y} | \hat{f} | | \hat{y} | | \hat{f} | |
| | | | | | | Lower | Upper | Lower | Upper |
| 25 | 49.58 | 3.904 | 3.940 | 51.42 | -0.036 | 3.502 | 4.378 | 33.18 | 79.68 |
| 26 | 50.75 | 3.927 | 3.766 | 43.21 | 0.161 | 3.323 | 4.208 | 27.75 | 67.28 |
| 27 | 18.41 | 2.913 | 3.147 | 23.27 | -0.234 | 2.628 | 3.666 | 13.85 | 39.09 |
| 28 | 52.50 | 3.957 | 3.896 | 49.21 | 0.061 | 3.500 | 4.302 | 32.79 | 73.86 |
| 29 | 22.33 | 3.106 | 3.249 | 25.76 | -0.143 | 2.909 | 3.589 | 18.34 | 36.20 |
| 30 | 30.67 | 3.423 | 3.844 | 46.72 | -0.421 | 3.440 | 4.248 | 31.19 | 83.77 |
| 31 | 24.00 | 3.178 | 3.151 | 23.36 | 0.027 | 2.674 | 3.628 | 14.49 | 37.64 |
| 32 | 30.00 | 3.401 | 3.291 | 26.87 | 0.110 | 2.899 | 3.683 | 18.16 | 39.75 |
| 33 | 40.92 | 3.712 | 3.480 | 32.46 | 0.232 | 3.198 | 3.761 | 24.50 | 43.01 |
| 34 | 44.42 | 3.794 | 3.551 | 34.85 | 0.243 | 3.282 | 3.820 | 26.62 | 45.62 |

Table 6. Analysis of Variance of 18 Tests
 First Order Model of 3 Variables
 Life predicting equation:

$$\hat{y} = 3.55 + 0.0242x_1 - 0.327x_2 - 0.0597x_3$$

| Source | Sum of Squares | Degrees of Freedom | Mean Square | F-Ratio |
|-------------------------|----------------|--------------------|-------------|---------|
| Due to Mean | 0.0108 | 1 | 0.0108 | |
| Due to Variance | 1.4728 | 1 | 1.4728 | |
| Due to Zero Upcrossings | 0.0572 | 1 | 0.0572 | |
| Due to Regression | 1.5407 | 3 | 0.5136 | |
| Residuals | 0.4862 | 14 | 0.0347 | 14.80 |
| Total | 2.0269 | 17 | | |

F-ratio is greater than the F value of 3.34 from the table with 3 and 14 degrees of freedom at 95% significance level. So regression is effective and the model is accepted.

Table 7. Results of 18 Tests, First Order Model of 3 Variables
Life predicting equation:

$$\hat{y} = 3.55 + 0.0242x_1 - 0.327x_2 - 0.0597x_3$$

| Test No. | Actual Life | | Predicted Life | | Residuals $y - \hat{y}$ | 95% Confidence Interval | | | |
|----------|-------------|-------|----------------|----------------|----------------------------|-------------------------|-------|-------|-------|
| | T | y | \hat{y} | $\hat{\sigma}$ | | Lower | Upper | Lower | Upper |
| 25 | 49.58 | 3.904 | 3.934 | 51.11 | -0.030 | 3.732 | 4.136 | 41.79 | 62.53 |
| 26 | 50.75 | 3.927 | 3.800 | 44.70 | 0.127 | 3.581 | 4.109 | 35.93 | 55.62 |
| 27 | 18.41 | 2.913 | 3.142 | 23.15 | -0.229 | 2.873 | 3.411 | 17.69 | 30.29 |
| 28 | 52.50 | 3.957 | 3.876 | 48.23 | 0.081 | 3.699 | 4.053 | 40.41 | 57.57 |
| 29 | 22.33 | 3.106 | 3.265 | 26.18 | -0.159 | 3.088 | 3.432 | 21.93 | 30.92 |
| 30 | 30.67 | 3.423 | 3.894 | 49.11 | -0.471 | 3.704 | 4.084 | 40.61 | 59.39 |
| 31 | 24.00 | 3.178 | 3.229 | 25.25 | -0.051 | 3.021 | 3.437 | 20.52 | 31.08 |
| 32 | 30.00 | 3.401 | 3.312 | 27.44 | 0.089 | 3.120 | 3.504 | 22.64 | 33.26 |
| 33 | 40.92 | 3.712 | 3.500 | 33.16 | 0.212 | 3.358 | 3.642 | 28.73 | 38.17 |
| 34 | 44.42 | 3.794 | 3.587 | 36.13 | 0.207 | 3.462 | 3.712 | 31.89 | 40.92 |
| 35 | 38.17 | 3.642 | 3.545 | 34.64 | 0.097 | 3.440 | 3.650 | 31.19 | 38.47 |
| 36 | 36.92 | 3.609 | 3.533 | 34.23 | 0.076 | 3.428 | 3.638 | 30.82 | 38.01 |
| 37 | 33.08 | 3.498 | 3.520 | 33.78 | -0.022 | 3.297 | 3.743 | 27.04 | 42.22 |
| 38 | 23.33 | 3.150 | 3.006 | 20.21 | 0.144 | 2.792 | 3.220 | 16.31 | 25.03 |
| 39 | 30.83 | 3.429 | 3.486 | 32.66 | -0.057 | 3.320 | 3.652 | 27.66 | 38.55 |
| 40 | 32.17 | 3.471 | 3.580 | 35.87 | -0.109 | 3.375 | 3.786 | 29.21 | 44.05 |
| 41 | 40.42 | 3.693 | 3.711 | 40.89 | -0.018 | 3.488 | 3.934 | 32.72 | 51.10 |
| 42 | 71.33 | 4.267 | 4.156 | 63.82 | 0.111 | 3.935 | 4.377 | 51.18 | 79.57 |

Table 8. Analysis of Variance of 18 Tests
First Order Model of 11 Variables

Life predicting equation:

$$\hat{y} = 3.39 - 0.0143x_1 - 0.463x_2 - 0.144x_3 - 0.0200x_4 \\ - 0.314x_5 - 0.727x_6 + 0.637x_7 + 0.112x_8 + 0.0103x_9 \\ + 0.101x_{10} - 0.0803x_{11}$$

| Source | Sum of Squares | Degrees of Freedom | Mean Squares | F-Ratio |
|--|----------------|--------------------|--------------|---------|
| Due to Mean | 0.0108 | 1 | 0.0108 | |
| Due to Variance | 1.4728 | 1 | 1.4728 | |
| Due to Zero Upcrossings | 0.0572 | 1 | 0.0572 | |
| Due to Level Upcrossings of ϵ_f Level | 0.0009 | 1 | 0.0009 | |
| of ϵ_y Level | 0.0794 | 1 | 0.0794 | |
| Due to Duration of Excur- sion Above Zero Level | 0.0349 | 1 | 0.0349 | |
| Due to Duration of Excur- sion Above ϵ_f Level | 0.1650 | 1 | 0.1650 | |
| Above ϵ_y Level | 0.0015 | 1 | 0.0015 | |
| Due to Band Width | 0.0002 | 1 | 0.0002 | |
| Due to Average Amplitude Above ϵ_f Level | 0.0097 | 1 | 0.0097 | |
| Above ϵ_y Level | 0.0262 | 1 | 0.0262 | |
| Due to Regression | 1.8587 | 11 | 0.1690 | |
| Residuals | 0.1682 | 6 | 0.0280 | 6.036 |
| Total | 2.0269 | 17 | | |

F-ratio is greater than the table value of 2.92 with 11 and 6 degrees of freedom at 95% significance level. So regression is effective, and the model is accepted.

Table 9. Results of 18 Tests, First Order Model of all 11 Variables
Life predicting equation:

$$\hat{y} = 3.39 - 0.0143x_1 - 0.463x_2 - 0.144x_3 + 0.200x_4 - 0.314x_5 - 0.727x_6 + 0.637x_7 + 0.112x_8 + 0.0103x_9 - 0.101x_{10} - 0.083x_{11}$$

| Test No. | Actual Life | | Predicted Life | | Residuals $y - \hat{y}$ | 95% Confidence Interval | | | |
|----------|-------------|-------|----------------|-----------|----------------------------|-------------------------|-------|-------|-------|
| | T | y | \hat{y} | \hat{T} | | Lower | Upper | Lower | Upper |
| 25 | 49.58 | 3.904 | 4.000 | 54.65 | -0.097 | 3.676 | 4.326 | 39.47 | 75.67 |
| 26 | 50.75 | 3.927 | 4.037 | 56.66 | -0.110 | 3.665 | 4.409 | 39.06 | 82.18 |
| 27 | 18.41 | 2.913 | 2.925 | 18.63 | -0.012 | 2.524 | 3.326 | 12.47 | 27.84 |
| 28 | 52.50 | 3.957 | 3.887 | 48.76 | 0.070 | 3.591 | 4.183 | 36.27 | 65.57 |
| 29 | 22.33 | 3.106 | 3.254 | 25.89 | -0.148 | 2.902 | 3.606 | 18.20 | 36.83 |
| 30 | 30.67 | 3.423 | 3.497 | 33.02 | -0.074 | 3.115 | 3.789 | 22.54 | 48.36 |
| 31 | 24.00 | 3.178 | 3.234 | 25.38 | -0.056 | 2.865 | 3.603 | 17.54 | 36.73 |
| 32 | 30.00 | 3.401 | 3.254 | 25.89 | 0.147 | 2.919 | 3.589 | 18.52 | 36.21 |
| 33 | 40.92 | 3.712 | 3.640 | 38.09 | 0.072 | 3.297 | 3.983 | 27.04 | 53.66 |
| 34 | 44.42 | 3.794 | 3.617 | 37.23 | 0.177 | 3.355 | 3.789 | 28.65 | 48.37 |
| 35 | 38.17 | 3.642 | 3.662 | 38.94 | -0.020 | 3.439 | 3.885 | 31.16 | 48.65 |
| 36 | 36.92 | 3.609 | 3.588 | 36.16 | 0.021 | 3.390 | 3.786 | 29.66 | 44.09 |
| 37 | 33.08 | 3.498 | 3.490 | 32.79 | 0.008 | 3.089 | 3.891 | 21.95 | 48.97 |
| 38 | 23.33 | 8.150 | 3.064 | 21.41 | 0.086 | 2.741 | 3.387 | 15.50 | 29.58 |
| 39 | 30.83 | 3.429 | 3.465 | 31.98 | -0.036 | 3.169 | 3.761 | 23.78 | 42.99 |
| 40 | 32.17 | 3.471 | 3.579 | 35.84 | -0.108 | 3.200 | 3.958 | 24.53 | 52.37 |
| 41 | 40.42 | 3.693 | 3.772 | 43.47 | -0.079 | 3.383 | 4.161 | 29.46 | 64.14 |
| 42 | 71.33 | 4.267 | 4.109 | 60.89 | 0.158 | 3.822 | 4.395 | 45.73 | 81.07 |

Table 10. Analysis of Variance of 24 Tests
 First Order Model of 3 Variables
 Life predicting equation:

$$\hat{y} = 3.56 + 0.0195x_1 - 0.325x_2 - 0.0518x_3$$

| Source | Sum of Squares | Degrees of Freedom | Mean Squares | F-Ratio |
|-------------------------|----------------|--------------------|--------------|---------|
| Due to Mean | 0.1223 | 1 | 0.1223 | |
| Due to Variance | 2.3905 | 1 | 2.3905 | |
| Due to Zero Upcrossings | 0.0482 | 1 | 0.0482 | |
| Due to Regression | 2.5609 | 3 | 0.8536 | |
| Residuals | 0.6192 | 20 | 0.0310 | 27.54 |
| Total | 3.1801 | 23 | | |

F-ratio is greater than the table value of 3.10 with 3 and 20 degrees of freedom at 95% significance level. So regression is effective, and the model is accepted.

Table 11. Results of 24 Tests, First Order Model of 3 Variables
Life predicting equation:

$$\hat{y} = 3.56 + 0.0195x_1 - 0.0325x_2 - 0.0518x_3$$

| Test No. | Actual Life | | Predicted Life | | Residuals | | 95 % Confidence Interval | | | |
|----------|-------------|-------|----------------|-----------|---------------|-------|--------------------------|-------|-------|-----------|
| | T | y | \hat{y} | \hat{t} | $y - \hat{y}$ | Lower | Upper | Lower | Upper | \hat{t} |
| 25 | 49.58 | 3.904 | 3.932 | 51.01 | -0.028 | 3.767 | 4.097 | 43.26 | 60.15 | |
| 26 | 50.75 | 3.927 | 3.823 | 45.74 | 0.104 | 3.644 | 4.002 | 38.23 | 54.73 | |
| 27 | 18.41 | 2.913 | 3.167 | 23.74 | -0.254 | 2.958 | 3.376 | 19.27 | 29.24 | |
| 28 | 52.50 | 3.957 | 3.886 | 48.72 | 0.071 | 3.727 | 4.045 | 41.57 | 57.08 | |
| 29 | 22.33 | 3.106 | 3.272 | 26.36 | -0.166 | 3.147 | 3.397 | 23.26 | 29.88 | |
| 30 | 30.67 | 3.423 | 3.904 | 49.60 | -0.481 | 3.760 | 4.048 | 42.95 | 57.28 | |
| 31 | 24.00 | 3.178 | 3.243 | 25.61 | -0.065 | 3.072 | 3.414 | 21.58 | 30.39 | |
| 32 | 30.00 | 3.401 | 3.314 | 27.49 | 0.087 | 3.162 | 3.466 | 23.61 | 32.02 | |
| 33 | 40.92 | 3.712 | 3.520 | 33.78 | 0.192 | 3.403 | 3.637 | 30.06 | 37.97 | |
| 34 | 44.42 | 3.794 | 3.592 | 36.31 | 0.202 | 3.492 | 3.692 | 32.85 | 40.13 | |
| 35 | 38.17 | 3.642 | 3.553 | 34.92 | 0.089 | 3.469 | 3.636 | 32.12 | 37.96 | |
| 36 | 36.92 | 3.609 | 3.548 | 34.74 | 0.061 | 3.465 | 3.631 | 31.96 | 37.77 | |
| 37 | 33.08 | 3.498 | 3.538 | 34.40 | -0.040 | 3.352 | 3.724 | 28.57 | 41.42 | |
| 38 | 23.33 | 3.150 | 3.019 | 20.41 | 0.131 | 2.860 | 3.177 | 17.47 | 23.99 | |
| 39 | 30.83 | 3.429 | 3.508 | 33.38 | -0.079 | 3.370 | 3.646 | 29.09 | 38.31 | |
| 40 | 32.17 | 3.471 | 3.585 | 36.05 | -0.114 | 3.416 | 3.754 | 30.45 | 42.69 | |
| 41 | 40.42 | 3.693 | 3.707 | 40.73 | -0.014 | 3.521 | 3.893 | 33.83 | 49.04 | |
| 42 | 71.33 | 4.267 | 4.165 | 64.39 | 0.102 | 4.000 | 4.330 | 54.61 | 75.93 | |
| 43 | 19.75 | 2.983 | 3.225 | 25.15 | -0.242 | 3.100 | 3.350 | 22.19 | 28.51 | |
| 44 | 29.92 | 3.398 | 3.231 | 25.30 | 0.167 | 3.100 | 3.344 | 22.19 | 28.34 | |
| 45 | 48.17 | 3.875 | 3.922 | 50.50 | -0.047 | 3.772 | 4.072 | 43.46 | 58.68 | |
| 46 | 44.50 | 3.795 | 3.630 | 37.71 | 0.165 | 3.544 | 3.716 | 34.62 | 41.08 | |
| 47 | 21.00 | 3.046 | 2.945 | 19.01 | 0.101 | 2.770 | 3.120 | 15.96 | 22.65 | |
| 48 | 64.92 | 4.173 | 4.116 | 61.31 | 0.057 | 3.966 | 4.266 | 52.76 | 71.25 | |

Table 12. Analysis of Variance of 24 Tests
 First Order Model of 11 Variables
 Life predicting equation:

$$\hat{y} = 3.50 - 0.0105x_1 - 0.398x_2 - 0.0748x_3 + 0.0459x_4 \\
 - 0.111x_5 - 0.317x_6 + 0.290x_7 + 0.0091x_8 - 0.0383x_9 \\
 + 0.0739x_{10} + 0.0231x_{11}$$

| Source | Sum of Squares | Degrees of Freedom | Mean Squares | F-Ratio |
|---|----------------|--------------------|--------------|---------|
| Due to Mean | 0.1223 | 1 | 0.1223 | |
| Due to Variance | 2.3905 | 1 | 2.3905 | |
| Due to Zero Upcrossings | 0.0482 | 1 | 0.0482 | |
| Due to Level Upcrossings | | | | |
| of ϵ_f Level | 0.0006 | 1 | 0.0006 | |
| of ϵ_y Level | 0.0838 | 1 | 0.0838 | |
| Due to Duration of Excursion Above Zero Level | 0.0180 | 1 | 0.0180 | |
| Due to Duration of Excursion Above ϵ_f Level | 0.1010 | 1 | 0.1010 | |
| Above ϵ_y Level | 0.0006 | 1 | 0.0006 | |
| Due to Band Width | 0.0238 | 1 | 0.0238 | |
| Due to Average Amplitude | | | | |
| Above ϵ_f Level | 0.0362 | 1 | 0.0362 | |
| Above ϵ_y Level | 0.0054 | 1 | 0.0054 | |
| Due to Regression | 2.8302 | 11 | 0.2573 | |
| Residuals | 0.3498 | 12 | 0.0292 | 8.812 |
| Total | 3.1801 | 23 | | |

F-ratio is greater than the table value of 2.73 with 11 and 12 degrees of freedom at 95% significance level. So regression is effective, and the model is accepted.

Table 13. Results of 24 Tests, First Order Model of all 11 Variables
Life predicting equation:

$$\hat{y} = 3.50 - 0.0105x_1 - 0.398x_2 - 0.0748x_3 + 0.0459x_4 - 0.111x_5 - 0.317x_6 + 0.297x_7 + 0.0091x_8 - 0.0383x_9 + 0.0734x_{10} + 0.0231x_{11}$$

| Test No. | Actual Life | | Predicted Life | | Residuals $y - \hat{y}$ | 95% Confidence Interval | | | |
|----------|-------------|-------|----------------|-----------|----------------------------|-------------------------|-------|-------|-------|
| | T | y | \hat{y} | \hat{t} | | Lower | Upper | Lower | Upper |
| 25 | 49.58 | 3.904 | 3.990 | 54.05 | -0.086 | 3.717 | 4.262 | 41.17 | 70.98 |
| 26 | 50.75 | 3.927 | 3.934 | 51.11 | -0.007 | 3.620 | 4.248 | 37.35 | 69.95 |
| 27 | 18.41 | 2.913 | 3.004 | 20.17 | -0.091 | 2.651 | 3.357 | 14.17 | 28.70 |
| 28 | 52.50 | 3.957 | 3.904 | 49.60 | 0.053 | 3.693 | 4.115 | 40.15 | 61.25 |
| 29 | 22.33 | 3.106 | 3.207 | 24.70 | -0.101 | 2.908 | 3.506 | 18.33 | 33.30 |
| 30 | 30.67 | 3.423 | 3.624 | 37.49 | -0.201 | 3.347 | 3.901 | 28.42 | 49.44 |
| 31 | 24.00 | 3.178 | 3.235 | 25.41 | -0.057 | 2.939 | 3.531 | 18.89 | 34.17 |
| 32 | 30.00 | 3.401 | 3.339 | 28.19 | 0.062 | 3.119 | 3.559 | 22.62 | 35.13 |
| 33 | 40.92 | 3.712 | 3.675 | 39.45 | 0.037 | 3.396 | 3.954 | 29.85 | 52.14 |
| 34 | 44.42 | 3.794 | 3.530 | 34.12 | 0.264 | 3.362 | 3.698 | 28.85 | 40.36 |
| 35 | 38.17 | 3.642 | 3.592 | 36.31 | 0.050 | 3.439 | 3.745 | 31.17 | 42.29 |
| 36 | 36.92 | 3.609 | 3.584 | 36.02 | 0.025 | 3.451 | 3.717 | 31.53 | 41.14 |
| 37 | 33.08 | 3.498 | 3.575 | 35.69 | -0.077 | 3.200 | 3.930 | 25.02 | 50.92 |
| 38 | 23.33 | 3.150 | 3.105 | 22.31 | 0.045 | 2.878 | 3.332 | 17.78 | 27.98 |
| 39 | 30.83 | 3.429 | 3.548 | 34.74 | -0.119 | 3.324 | 3.772 | 27.76 | 43.48 |
| 40 | 32.17 | 3.471 | 3.702 | 40.53 | -0.231 | 3.390 | 4.010 | 29.68 | 55.35 |
| 41 | 40.42 | 3.693 | 3.864 | 47.66 | -0.171 | 3.550 | 4.178 | 34.82 | 65.22 |
| 42 | 71.33 | 4.267 | 4.160 | 64.07 | 0.107 | 3.946 | 4.374 | 51.75 | 79.32 |
| 43 | 19.75 | 2.983 | 3.007 | 20.23 | -0.024 | 2.793 | 3.312 | 16.34 | 27.44 |
| 44 | 29.92 | 3.398 | 3.178 | 23.99 | 0.220 | 2.886 | 3.470 | 17.92 | 32.14 |
| 45 | 48.17 | 3.875 | 3.843 | 46.67 | 0.032 | 3.551 | 4.096 | 34.85 | 60.10 |
| 46 | 44.50 | 3.795 | 3.642 | 38.17 | 0.153 | 3.476 | 3.808 | 32.34 | 45.04 |
| 47 | 21.00 | 3.046 | 3.022 | 20.53 | 0.024 | 2.697 | 3.347 | 14.84 | 28.41 |
| 48 | 64.92 | 4.173 | 4.081 | 59.20 | 0.092 | 3.915 | 4.247 | 50.17 | 69.87 |

Table 14 . Comparison of Percent Deviations of Predicted Lives
and Residual Sum of Squares for Five Models

| Test No. | Actual Life T | Percent Deviations of Predicted Lives | | | | |
|-------------|------------------|---------------------------------------|-------|-------|-------|-------|
| | | Eq(1) | Eq(2) | Eq(3) | Eq(4) | Eq(5) |
| 25 | 49.58 | -3.7 | -3.1 | -10.2 | -2.9 | -9.0 |
| 26 | 50.75 | 14.9 | 11.9 | -11.6 | 9.9 | -0.7 |
| 27 | 18.41 | -26.4 | -25.7 | -2.0 | -28.9 | -9.6 |
| 28 | 52.50 | 6.3 | 8.1 | 7.1 | 7.2 | 5.5 |
| 29 | 22.33 | -15.4 | -17.2 | -15.9 | -18.0 | -10.6 |
| 30 | 30.67 | -52.3 | -60.1 | -7.7 | -61.7 | -22.2 |
| 31 | 24.00 | 2.7 | -5.2 | -5.7 | -6.7 | -5.8 |
| 32 | 30.00 | 10.4 | 8.5 | 13.7 | 8.4 | 6.0 |
| 33 | 40.92 | 2.1 | 19.0 | 6.9 | 17.4 | 3.6 |
| 34 | 44.42 | 21.5 | 18.5 | 16.2 | -22.3 | 23.2 |
| 35 | 38.17 | | 9.2 | -2.0 | 8.5 | 5.3 |
| 36 | 36.92 | | 7.3 | 2.1 | 5.9 | 2.4 |
| 37 | 33.08 | | -2.1 | 0.9 | -4.0 | -7.9 |
| 38 | 23.33 | | 13.3 | 8.2 | 12.5 | 4.4 |
| 39 | 30.83 | | -5.9 | -3.7 | -8.3 | -12.7 |
| 40 | 32.17 | | -11.5 | -11.4 | -12.1 | -26.0 |
| 41 | 40.42 | | -1.1 | -7.5 | -0.8 | -17.9 |
| 42 | 71.33 | | 10.5 | 14.6 | 9.7 | 10.2 |
| 43 | 19.75 | | | | -27.3 | -2.4 |
| 44 | 29.92 | | | | 15.4 | 19.8 |
| 45 | 48.17 | | | | -4.8 | 3.1 |
| 46 | 44.50 | | | | 15.3 | 14.2 |
| 47 | 21.00 | | | | 9.5 | 2.2 |
| 48 | 64.92 | | | | 5.6 | 8.8 |

| | Average Deviations | | | | |
|--|--------------------|------|-----|------|------|
| Negative side | 24.5 | 14.7 | 7.8 | 16.5 | 13.4 |
| Positive side | 9.7 | 11.8 | 8.7 | 10.4 | 8.4 |
| | Residuals | | | | |
| Percent residual sum of squares of the total | 32.6 | 24.0 | 8.3 | 20.5 | 11.0 |

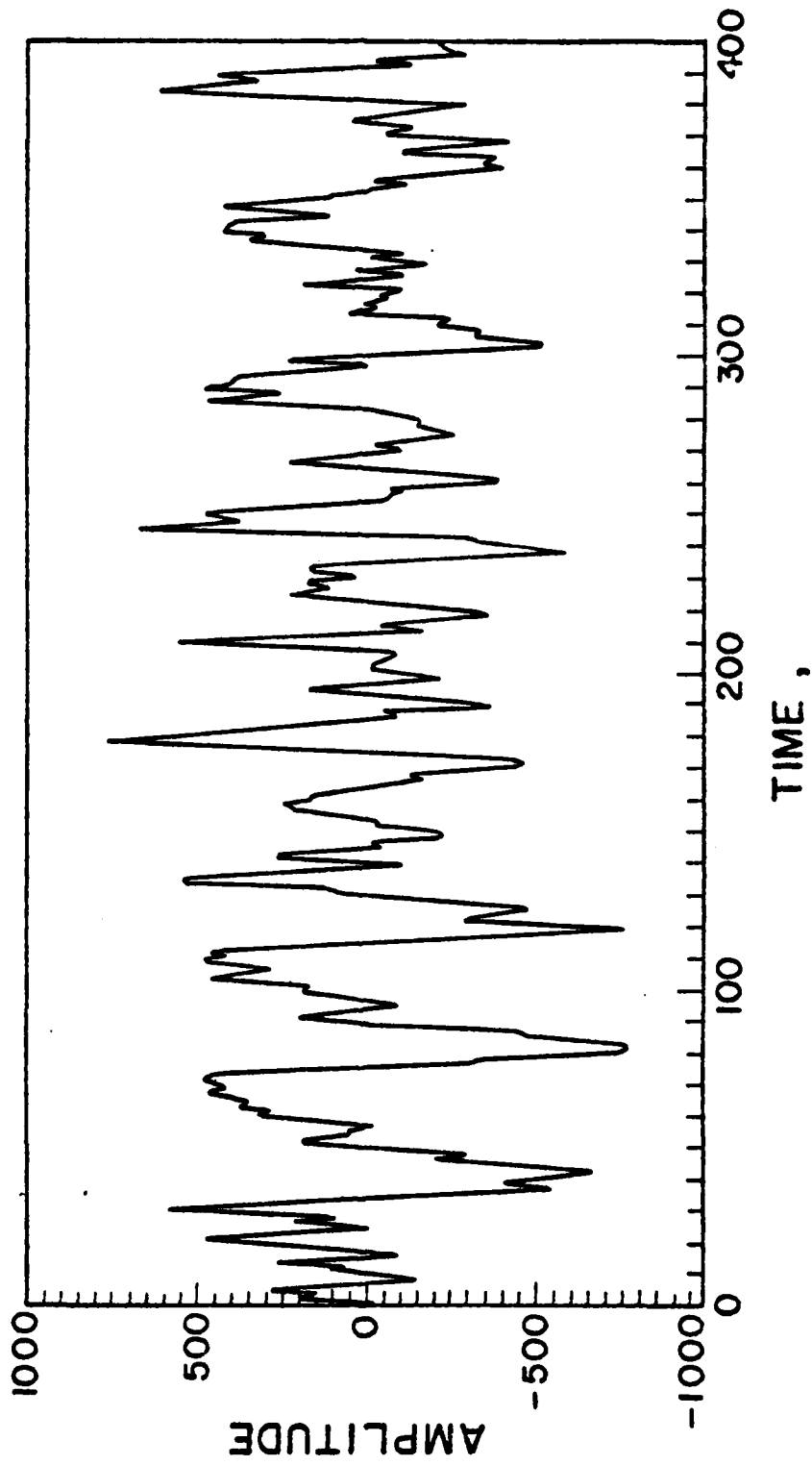


Fig. 1 Typical Response Signal of a Test Specimen

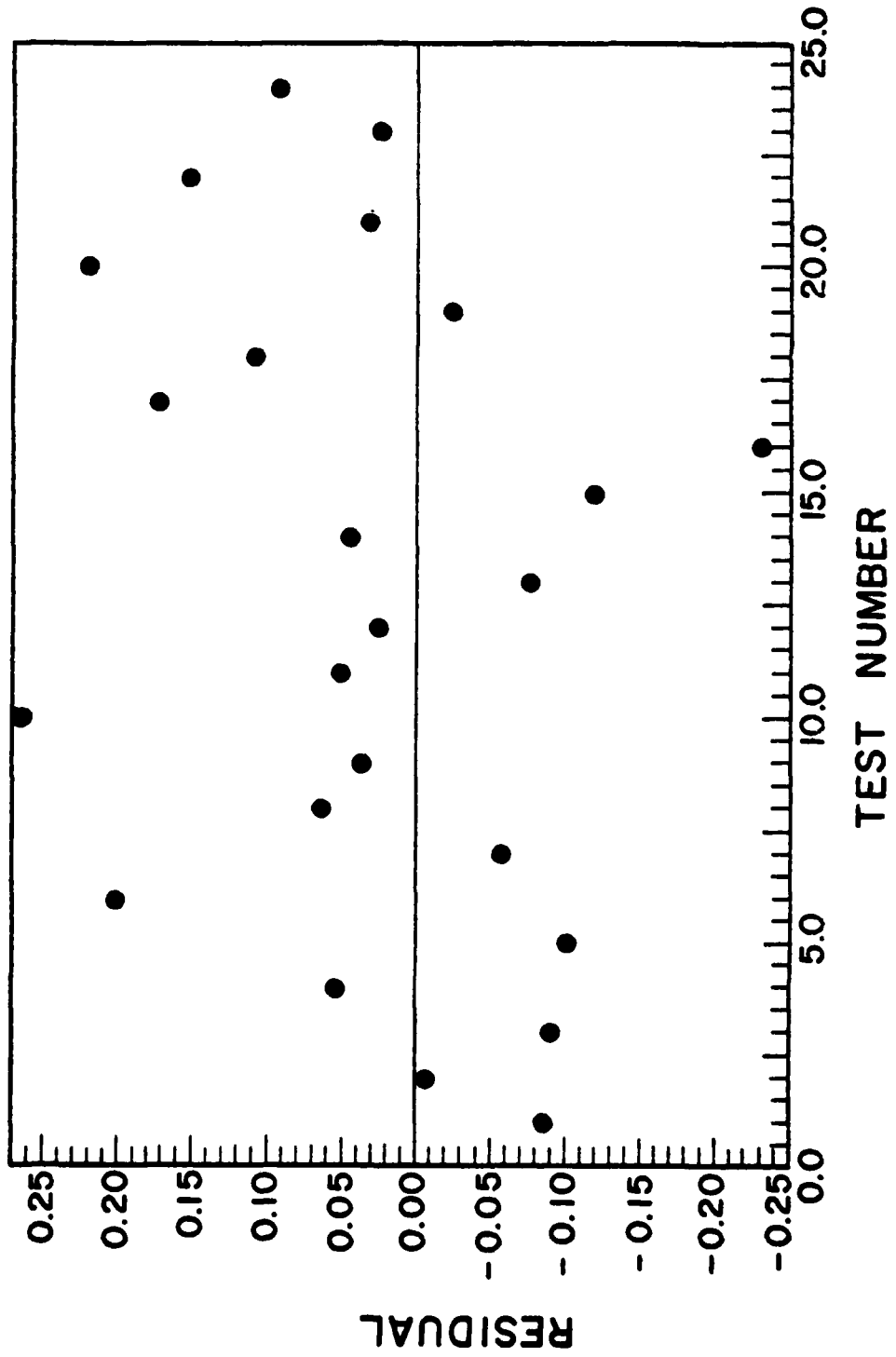


Fig. 2 Distribution of Residuals for the Best First Order Model, Eq. (5)

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An experiment program, based on 11 probabilistic parameters and 3 experiment designs was conducted for 24 specimens for fatigue of materials in an elastic-plastic range under random vibrations. The magnitude levels of all probabilistic parameters are completed from the spectral moments and coded. By regressing the log of fatigue life on the corresponding coded parameters, the first-order regression models are developed. A | | | |

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