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GENERAL RESULTS, TIME SERIES IN M DIMENSIONS.(U)
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GENERAL RESULTS, TIME SERIES IN M DIMENSIONS.

Leo A. Aroian and Josef Schmee
~~Union College and University~~
Institute of Administration and Management
Schenectady, New York 12308

ABSTRACT

General results in the theory of time series in m dimensions are obtained, thus providing a broad view applicable to the various models. The inter-relationships among the various types of moving average models, MA, and autoregressive models, AR, and the general ARMA autoregressive moving average models are stressed.

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INTRODUCTION

General results in the theory of time series in m dimensions are determined. These are: definitions of moving average, MA, models; autoregressive, AR, models; and autoregressive moving average, ARMA models; stationarity and invertibility; the characteristic function; the autocovariance and autocorrelation function; the power spectrum; properties of MA, AR, and ARMA models; the partial autocorrelation function; estimation; and a brief look at the multivariate problem. Simulation and forecasting are considered by Aroian and Taneja (1).

Important assumptions are outlined below: The characteristic of an event is $z_{x,t}$, $x = (x_1,x_2,\ldots,x_m),t$, $-\infty<x<\infty$, $x-\ell = (x_1-\ell_1,x_2-\ell_2,\ldots,x_m-\ell_m)$. Weak stationarity is assumed in time and space as a minimum assumption:

$$\mu_z = E(z_{x,t}) = 0,\ \sigma_z^2 = E(z_{x,t}-\mu_z)^2$$

$$Ea_{x,t} = 0,\ \sigma_a^2 > 0,\ \rho_{\ell,k} = \{E(z_{x,t}z_{x-\ell,t-k})\}/\sigma_z^2,$$

$$\ell = (\ell_1,\ell_2,\ldots,\ell_m).$$

All second order moments exist. Note x may be any coordinate system; if x is dropped, the time series is the usual one at a point; if t is dropped, then the series is purely spatial. Note ℓ may be plus or minus but k is plus except in forecasting.

MODELS

The MA model is defined:

$$z_{x,t} = \sum_{n=-\infty}^{\infty}\sum_{k=1}^{\infty} \psi_{n,k}a_{x-n,t-k}+a_{x,t}\ , \qquad (2.1)$$

$$n = (n_1,n_2,\ldots,n_m),\ \sum_{n=-\infty}^{\infty} = \sum_{n_m=-\infty}^{\infty}\cdots\sum_{n_1=-\infty}^{\infty}\ ,$$

$a_{x,t}$ is an i.i.d. variable with mean zero, $\sigma_a^2 > 0$, $Ea_{x,t}z_{x-\ell,t-k} = 0$, unless $\ell = k = 0$. Usually

$$-p\le n\le q,\ 1\le k\le r,\ z_{x,t} = \sum_{n=-p}^{q}\sum_{k=1}^{r} \psi_{n,k}a_{x+n,t-k}+a_{x,t}, \qquad (2.2)$$

an MA model of temporal order r, spatial order p_i+q_i in each spatial variable x_i, $1\le i\le m$. As an example $m = r = 1$:

CORRECTIONS

GENERAL RESULTS: TIME SERIES IN M DIMENSIONS

1. P2 ℓ11 $-\infty < t < \infty$ not $0 < t < \infty$

2. P3 equation (3.7) replace by:

$$a_{x,t} = \sum_{d=0}^{\infty} \left(- \sum_{n=-q}^{p} \sum_{k=1}^{r} \Psi_{n,k} F_x^n B_t^k\right)^d$$

3. P3 equation (3.11), replace by:

$$z_{x,t} = \Phi^{-1}(B_x, B_t) a_{x,t} = \sum_{d=0}^{\infty} \left(\sum \sum \phi_{n,k} F_x^n B_t^k\right)^d a_{x,t}$$

4. P4 equation (3.14) in denominator:

$\Psi^2_{n,k}$ <u>not</u> $\Psi_{n,k}$

$$z_{x,t} = -\theta_1 a_{x,t-1} - \theta_2 a_{x-1,t-1} + a_{x,t} \ , \tag{2.3}$$

replacing the ψ's by θ's.

The AR model is defined:

$$z_{x,t} = \sum_{n=-\infty}^{\infty} \sum_{k=1}^{\infty} \phi_{n,k} z_{x+n,t-k} + a_{x,t}, \tag{2.4}$$

temporal order r if $1 \le k \le r$, spatial order $p_i + q_i$, $-q \le n \le p$, in each variable, x_i, $1 \le i \le m$. An example is

$$z_{x,t} = \phi_1 z_{x,t-1} + \phi_2 z_{x-1,t-1} + a_{x,t} \ . \tag{2.5}$$

The ARMA model is defined:

$$z_{x,t} = \sum_{n=-p}^{q} \sum_{k=1}^{r} \phi_{n,k} z_{x+n,t-k} - \sum_{n=-u}^{v} \sum_{k=1}^{s} \theta_{n,k} a_{x+n,t-k} + a_{x,t} \tag{2.6}$$

of order r+s in the temporal domain, q+p, and u+v in each spatial variable.

The general case would be $-\infty<n<\infty$, $0<t<\infty$. This model is denoted by ARMA (r,s;p,q;u,v). An example is:

$$z_{x,t} = \phi_1 z_{x,t-1} + \phi_2 z_{x-1,t-1} - \theta_1 a_{x,t-1} - \theta_2 a_{x-1,t-1} + a_{x,t} \tag{2.7}$$

All the preceding are univariate cases.

Practical examples for m=1 are the flow characteristics of a river or a manufacturing process; for m=2, the characteristics of a process in the plane such as storms, social or economic processes, geological and geographical processes in time including earthquakes; and for m=3 processes in space such as weather processes, sunspots, communications, satellite tracking, and oil exploration.

PROPERTIES OF MA, AR, AND ARMA MODELS

An MA model is stationary, and invertible dependent on $\psi_{n,k}$, and its representation as an infinite AR model. Define

$$B_t z_{x,t} = z_{x,t-1}, F_t = B_t^{-1}, \ B_{x_i} z_{x,t} = z_{x-\delta_i,t} \ , \tag{3.1}$$

$$F_{x_i} = B_{x_i}^{-1}, \ \delta_i = (\delta_{i1}, \delta_{i2}, \ldots, \delta_{im}), \delta_{ij} = \begin{pmatrix} 1 & i=j \\ 0 & i \neq j \end{pmatrix}.$$

Rewrite (2.2) in terms of (3.1).

$$z_{x,t} = (1 + \sum_{n=-p}^{q} \sum_{k=1}^{r} \psi_{n,k} F_x^n B_t^k) a_{x,t}, \tag{3.2}$$

where $F_x^n = (F_{x_1}^{n_1}, F_{x_2}^{n_2}, \ldots, F_{x_m}^{n_m})$.

The characteristic function is

$$\Psi(B_x, B_t) = 1 + \sum_{n=-p}^{q} \sum_{k=1}^{r} \psi_{n,k} F_x^n B_t^k, \tag{3.3}$$

and $a_{x,t} = \Psi^{-1}(B_x, B_t) z_{x,t}$. (3.4)

For invertibility (3.3) must converge on $S_i \ \underset{i=-m}{\overset{m}{X}}$

$$\text{where } S_0 = \{B_t : |B_t| < 1\}, \ S_{-i} = \{B_{x_i} : |B_x| < 1\},$$
$$S_i = \{F_{x_i} : |F_{x_i}| > 1\}, \ 1 \le i \le m. \tag{3.5}$$

Define $\gamma_{\ell,k} = E(z_{x,t} z_{x-\ell,t-k})$, the autocovariance between $z_{x,t}$ and $z_{x-\ell,t-k}$.

Theorem 3.1. The autocovariance function of an MA process may be found by multiplying (2.2) by $z_{x-\ell,t-k}$, where $\ell = (\ell_1, \ldots, \ell_m)$, and taking expectations; or better, the autocovariance function

$$\Gamma(B_x,B_t) = \sigma_a^2 \Psi(B_x,B_t)\Psi(F_x,F_t), \tag{3.6}$$

and $\gamma_{\ell,k}$ is the coefficient of both $B_x^{\ell}B_t^{k}$ and $B_x^{-\ell}B_t^{-k}$, with $\gamma_{00} = \sigma_a^2$ being the coefficient of $B_x^0B_t^0 = 1$. Theorem 3.1 may be used to find $\rho_{\ell_1,\ell_2,\ldots,\ell_m,k}$ for MA, AR, and ARMA models. The autocorrelation function is not symmetric in m dimensions; it is symmetric to the origin x = 0, t = 0. Thus $\rho_{\ell 0} = \rho_{-\ell 0}, \rho_{0k} = \rho_{0-k}, \rho_{\ell k} = \rho_{-\ell -k}$, but $\rho_{\ell m} \neq$ to $\rho_{-\ell m}$, $\rho_{0k} \neq \rho_{k0}$. For m=1, $\rho_{\ell k}$ will have the same values in the first and third quadrants, and the second and fourth quadrants. For m=2, four sets of equal ρ's occur. In m variable similar results hold, as reported in Perry's ongoing Ph.D. thesis. An important cut-off property for the MA models is given by:

Theorem 3.2. The autocorrelation function for a finite MA model is finite. Use theorem 3.1 for the proof. This cutoff property is an important way of determining where a process is MA, AR, or ARMA.

Theorem 3.3. If the conditions for invertibility are satisfied, every finite MA process in m dimensions may be expressed as an infinite AR model. From (3.4)

$$a_{x,t} = \left(-\sum_{n=-q}^{p}\sum_{k=1}^{r}\psi_{n,k}F_x^nB_t^k\right)^d z_{x,t} \quad 0\le d\le\infty , \tag{3.7}$$

and condition (3.5) must be satisfied.

Theorem 3.4. The power spectrum. Let $B_t = \exp{-2\pi i f}$, $B_{x_j} = \exp{-2\pi i g_j}$, in the autocovariance function, theorem 3.1, the power spectrum of an MA process is:

$$\begin{aligned} p(f,g) &= 2\sigma_a^2\Psi(\exp{-2\pi i f}, \exp{-2\pi i g}) \ast \\ &\qquad \Psi(\exp 2\pi i f, \exp 2\pi i g) \\ &= 2\sigma_a^2|\Psi(\exp{-2\pi i f}, \exp{-2\pi i g})|^2 \end{aligned} \tag{3.8}$$

$0\le|f|\le 1/2,\ 0\le|g_j|\le 1/2,\ 1\le j<m.$

Next AR processes are considered.

Theorem 3.5. The autocorrelation function of an AR process is found by multiplying (2.4) by $z_{x-\ell,t-k}$ and take expectations. For $\ell = k = 0$

$$\sigma_z^2 = \sigma_a^2\left\{1-\sum_{n=-p}^{q}\sum_{k=1}^{r}\phi_{n,k}\rho_{n,k}\right\}^{-1}. \tag{3.9}$$

Note $\sigma_z^2 > 0$, and

$$\rho_{x,t} = \sum_{n=-p}^{q}\sum_{k=1}^{r}\phi_{n,k}\rho_{x+n,t-k} . \tag{3.10}$$

Replace x,t by m,n. Note $\rho_{\ell k}$ satisfies the same form as (2.4) for all $\{\ell,k\}$, except $\ell = k = 0$. The autocorrelation (cross correlation) function may be plotted in m+1 dimensions and is infinite in extent. As ℓ or k or both approach $\pm\infty$, the corresponding ρ's approach zero, provided the ϕ's are such that the AR process is stationary. The difference system needs some $\rho_{\ell k}$ to start the process.

Theorem 3.6. If conditions for stationarity (3.5) are satisfied, every finite AR process may be represented by an infinite MA process.

Proof: From (2.4)

$$z_{x,t} = \phi^{-1}(B_x,B_t)a_{x,t} = (\Sigma\Sigma\phi_{n,k}B_x^nB_t)^d\, a_{x,t} \tag{3.11}$$

$$0 \le d \le \infty$$

Theorem 3.7. For invertibility of an MA model $|B_x|\le 1, |B_t|\le 1$ in (3.3) set equal to zero restricting $\theta_{n,k}(\psi_{n,k})$. For AR restrictions on $\phi_{n,k}$ the same method is used in the corresponding characteristic equation of the AR model

$$\phi(B_x, B_t) = (1 - \sum_{n=-q}^{p} \sum_{k=1}^{\infty} \phi_{n,k} B_x^n B_t^k). \tag{3.12}$$

This theorem may also be stated for the MA(AR) models, or in fact for the ARMA model. The roots of the characteristic equation must lie outside the unit circle in each variable B_i when all other B's are set to one. This condition holds as stationarity in m dimensions, m+1 variables, requires stationarity in every direction in the m+1 variables.

Theorem 3.8. The power spectrum for an AR process is:

$$p(f,g) = 2\sigma_a^2 |\phi(\exp-2\pi if, \exp-2\pi ig)|^{-2}$$
$$0 \le |f| \le 1/2,\ 0 \le |g_j| \le 1/2,\ 1 \le j \le m . \tag{3.13}$$

Note the similarity to (3.8).

Theorem 3.9. Given an AR process to determine $\rho_{\ell,m}$, it is necessary to first find the corresponding MA expansion of the AR process. Then

$$\rho_{\ell,m} = \frac{C_{\ell,m}}{C_{00}} = \frac{\sum_{n=-\infty}^{\infty} \sum_{k=0}^{\infty} \psi_{n,k}\psi_{n+\ell,k+m}}{\sum_{n=-\infty}^{\infty} \sum_{k=0}^{\infty} \psi_{n,k}^2} \tag{3.14}$$

where $C_{\ell,m}$ is the covariance $E(z_{x,t}z_{x-\ell,t-m})$. This is very important since the recurrence relationship of $\rho_{\ell,m}$ given by (3.10) does not provide a way of finding all the $\rho_{\ell,m}$. An AR model may be simulated given $\{\phi_{n,k}\}$. If this is done a set of estimated $\rho_{m,n}$ are found. The true values are found from (3.14).

THE PARTIAL AUTOCORRELATION FUNCTION

One of the most important questions that must be faced is the choice of m. Usually spatial considerations make clear the value of m. Otherwise choose that m which minimizes σ_z^2, but theoretical considerations should have the greater weight. If m is given, how is r the order determined? For the MA model use the cutoff property of the autocorrelation function. For the AR model the cutoff property is provided by the partial autocorrelation function. For m=0, it is known that the last coefficient ϕ_r in an AR model with terms $\phi_1,\ldots,\phi_r$ is a partial coefficient of correlation and more importantly ϕ_{r+i}, $i > r$ are all zero. AR models in m dimensions have a similar property. For r=1 in m dimensions m+1 "last" coefficients ϕ will be partial coefficients of correlation, non-zero, but all other ϕ's would be exactly zero. In samples of n, n large these other ϕ's will tend to be small instead of being exactly zero as in the theoretical model.

The definition for the partial coefficient of correlation for AR models is $\phi_{ij} \ne 0$ for $i=1,2,\ldots,m$, $j=1,2,\ldots,r$, $-1 \le \phi_{ir} \le 1$ and $\phi_{ij} = 0$ for $i > m$, $j > r$. For any AR models for ϕ_{ij}, $i > m$, $i > r$, one may prove $\phi_{ij} = 0$.

YULE-WALKER EQUATIONS, AR MODELS

The Yule-Walker equations for AR models in m dimensions will be given. Suppose

$$z_{x,t} = \phi_1 z_{x,t-1} + \phi_2 z_{x-1,t-1} + a_{x,t}. \tag{5.1}$$

Multiply (5.1) by the coefficients of ϕ_1 and ϕ_2, take expected values and obtain the Yule-Walker equations

$$\rho_{01} = \phi_1 + \phi_2\rho_{10}$$
$$\rho_{11} = \phi_1\rho_{10} + \phi_2 , \tag{5.2}$$

which solved for ϕ_1 and ϕ_2

$$\phi_1 = (\rho_{01} - \rho_{11}\rho_{10})/(1-\rho_{10}^2)$$

$$\phi_2 = (\rho_{11}-\rho_{01}\rho_{10})/(1-\rho_{10}^2). \tag{5.3}$$

This is the usual least squares used in regression. Hence the Yule-Walker equations in m dimensions for any order r may be found by the same method.

Thus the Yule-Walker equations will be of the form $\rho = P_\rho\phi$, ρ and ϕ are column vectors and P_ρ is the matrix of correlations. By the usual least squares theory

$$V(\hat{\phi}) \simeq n^{-1}(1-\rho^1\phi)P_2^{-1}, \tag{5.4}$$

and if we substitute the sample values r for ρ then

$$V(\hat{\phi}) = n^{-1}(1-r^1\hat{\phi})R_2^{-1},$$

$$r^1 = (r_{01}, r_{11}),\ R_2 = \begin{pmatrix} 1 & r_{10} \\ r_{10} & 1 \end{pmatrix}$$

$$\phi^1 = (\phi_1, \phi_2),\ R_2^{-1} = (1-r_{10}^2)^{-1}\begin{pmatrix} 1 & -r_{10} \\ -r_{10} & 1 \end{pmatrix}, \tag{5.5}$$

$$V(\hat{\phi}_1, \hat{\phi}_2) = n^{-1}(1-r_{01}\hat{\phi}_1-r_{11}\hat{\phi}_2)R_2^{-1}$$

$$\sigma^2_{\hat{\phi}_1} = \sigma^2_{\hat{\phi}_2} = n^{-1}(1-r_{10}^2)^{-1}(1-\hat{\phi}_1^2-\hat{\phi}_2^2-2\hat{\phi}_1\hat{\phi}_2 r_{10}),$$

and $\hat{\rho}_{\hat{\phi}_1, \hat{\phi}_2} = -r_{10}$. This method is general and if $a_{x,t}$ are distributed normally the estimates $\hat{\phi}$ are asympototicaly unbiased, consistent and approximately the maximum likelihood estimates; Perry and Aroian (2), Aroian and Taneja (1) have extended this method to the MA and ARMA models.

ARMA MODELS

Denote the characteristic equation of the MA model by $\Theta(B_x, B_t)$ and that of the AR model by $\Phi(B_x, B_t)$, then

$$\Phi(B_x, B_t)z_{x,t} = \Theta(B_x, B_t)a_{x,t} \tag{6.1}$$

$B_x = (B_{x_1}, B_{x_2}, \ldots, B_{x_m})$, an m dimensional ARMA model of order $(r,s;p,q;u,v)$.

Now (6.1) may be written as an infinite MA model:

$$z_{x,t} = \Theta(B_x, B_t)\Phi^{-1}(B_x, B_t)a_{x,t}, \tag{6.2}$$

or as an infinite AR model

$$a_{x,t} = \Phi(B_x, B_t)\Theta^{-1}(B_x, B_t)z_{x,t}. \tag{6.3}$$

Both results are important, particularly (6.2), since it is useful in finding $\rho_{m,n}$, and in forecasting $z_{x+\ell_1, t+\ell_2}$, $\ell_1, \ell_2 > 0$. The restrictions on ϕ_i and θ_i for invertibility and stationarity of (6.1) are exactly those of the MA and AR model jointly as given in theorem 3.7. An example is given for m=1, $r_1 = r_2 = 2$ in Aroian and Taneja (1). The autocovariance functions is a combination of those of the MA and AR models. The partial autocorrelation is similar to that of the MA model.

The power spectrum is:

$$p(f,g) = 2\sigma_a^2 \frac{|\Theta(\exp-2\pi if, \exp-2\pi ig)|^2}{|\Phi(\exp-2\pi if, \exp-2\pi ig)|^2} \tag{6.4}$$

$$0 \le |f| \le 1/2,\ 0 \le |g_{x_j}| \le 1/2,\ 1 \le x_j \le m.$$

The ARMA model may differ markedly from either the MA or AR model.

MULTIVARIATE AR MODELS

A brief description of the definition of an AR multivariate model is given:

$$z^p_{x,t} = \phi(p,r) z^p_{x-i,t-j} + a^p_{x,t} \quad , \tag{7.1}$$

where x is the vector $x = (x_1, x_2, \ldots, x_m)$, $\phi(p,r)$ is a matrix of ϕ_{ij}'s of p rows by rp columns, $z^p_{x,t}$ a column vector of p components, $z^p_{x-i,t-j}$, a column vector of rp components, and $a^p_{x,t}$ a column vector of p components, a multivariate AR (p,m,r) model. The properties of the model, with an example p=2, m=r=1, are given by Aroian (3).

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General results in the theory of time series in m dimensions are obtained, thus providing a broad view applicable to the various models. The interrelationships among the various types of moving average models, MA, and autoregressive models, AR, and the general ARMA autoregressive moving average models are stressed.

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