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DESIGN CRITERIA FOR DEFLECTION CAPACITY OF CONVENTIONALLY REINF--ETC(U)

OCT 80 T TAKAYANAGI, A T DERECHO   N68305-79-C-0009

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Naval Construction Battalion Center
Port Hueneme, CA

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DESIGN CRITERIA FOR DEFLECTION CAPACITY OF CONVENTIONALLY REINFORCED CONCRETE SLABS, PHASE III - SUMMARY OF DESIGN CRITERIA AND DESIGN AND CONSTRUCTION DETAILS - DESIGN EXAMPLES

October 1980

An Investigation Conducted by
CONSTRUCTION TECHNOLOGY LABORATORIES
Structural Analytical Section
5420 Old Orchard Road
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A design procedure is given for one-way and two-way standard reinforced concrete slabs allowed to respond to incipient collapse under blast loads. When tensile membrane action can be developed, larger incipient collapse deflections can be used in design than are currently allowed in the design manual, NAVFAC P-397 ("Structures to Resist the Effects of Accidental Explosions" also TM 5-1300 and AFM 88-22). Design and construction requirements are specified to insure adequate tensile membrane resistance. A supplement is proposed for consideration as a supplement to ~~P-397~~ the design manual.

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DESIGN CRITERIA FOR DEFLECTION CAPACITY OF CONVENTIONALLY REINFORCED CONCRETE SLABS

Phase III - Summary of Design Criteria and Design and Construction Details - Design Examples

by

T. Takayanagi and A. T. Derecho*

1. INTRODUCTION

1.1 Objective and Scope

This report summarizes major results of the investigation conducted in the first two phases of the project, reported in Refs. 1 and 2. In addition, design examples, intended to illustrate the use of recommended procedures for estimating the incipient collapse deflection of conventionally reinforced concrete slabs, are presented. The material has been arranged in a format suitable for adoption as a supplement to NAVFAC P-397 (3).

The primary objective of the investigation is to develop design criteria, based on existing data, for the incipient collapse deflection of conventionally reinforced concrete one-way and two-way slabs under uniform load. The slabs may have either laterally and rotationally restrained or unrestrained edges. A related objective is to recommend design requirements and construction details necessary to develop tensile membrane capacity of reinforced concrete slabs under uniform load.

Major emphasis is placed on the deflection capacity associated with incipient collapse. Incipient collapse for conventionally reinforced concrete slabs is defined here as that state of a slab characterized by a drop in the load capacity following mobilization of tensile membrane action. The collapse condition is associated with tensile rupture of the flexural reinforcement. It is assumed that the slab is properly designed to preclude premature failure due to shear or inadequate anchorage at the supports.

An expression for estimating incipient collapse deflection of conventionally reinforced concrete slabs has been developed based on evaluation of available analytical and experimental data. The significant parameters, in terms of their effect on incipient collapse, were identified primarily on the basis of

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experimental data. Design criteria that account for the influence of the major design parameters are developed and presented in useful format. Design and construction requirements necessary to develop tensile membrane action and incipient collapse in slabs are also recommended.

Example problems are solved to illustrate the recommended design procedure for slabs loaded to incipient collapse. Design criteria and minimum design and construction requirements, as well as the design examples, are presented in a form that can serve as a supplement to NAVFAC P-397, "Structures to Resist the Effects of Accidental Explosions" (3).

1.2 Background

NAVFAC P-397 (3) is a government standard for designing structures subjected to accidental explosion. Although this standard is simple to apply, it does not take into account the influence of slab geometry, section properties, boundary conditions, material properties, and load distribution on incipient collapse deflection of reinforced concrete slabs. In view of this, a need was felt to re-examine the approach used in the manual in the light of informatin that has become available since its publication in 1969. Specifically, it was felt that the requirements presently found in NAVFAC P-397 may be too conservative for certain cases. For these particular cases, significant reductions in cost of protective structures may be achieved by the use of more realistic design criteria.

Many structural systems may provide sufficient lateral restraint to develop tensile membrane action in slabs. In fact, it is likely that boundary conditions of many slab systems allow individual slabs to behave and carry load as tensile membranes well before the rotational capacity of plastic hinges is exceeded. To utilize this stage of slab behavior, design criteria must consider the effects of lateral edge restraint and anchorage of reinforcing bars at slab boundaries. Also of interest are construction details required for reinforcing bars at slab boundaries.

The use of yield-line theory (4) for calculating the collapse load of reinforced concrete slabs is prescribed in NAVFAC P-397. Yield-line theory, which considers only flexural action in slabs, gives collapse load values that are theoretically upper bounds, i.e., "on the unsafe side". However, experimental investigations have shown that the actual maximum load, and in many cases, the collapse load, are usually higher than that indicated by yield-line theory. This enhancement in strength has been attributed to membrane action. Several analytical and experimental studies dealing with the subject have been reported in the literature (1). However, design criteria for reinforced concrete slabs near incipient collapse have not been presented.

A number of papers and reports describing experimental and analytical works on membrane action of slabs with and without lateral and rotational edge restraints have been reviewed and summarized in Ref. 1.

The review of experimental and analytical investigations of membrane action in slabs indicates that both compressive and tensile membrane actions, which occur at different deflection stages, can enhance slab load-carrying capacity. In particular, when sufficient lateral restraint exists at boundaries of a slab and a sufficient amount of reinforcement is provided, the ultimate load may be significantly higher than that given by yield-line theory. Approximate analytical techniques for predicting this enhanced capacity have been proposed. However, these are based on simplifying assumptions which do not allow examination of the effects of parameters other than the few included in the formulations.

Based on the literature review, design criteria for the incipient collapse deflection of conventionally reinforced slabs under static uniform loads have been developed and are presented in this report.

2. DESIGN CRITERIA

2.1 Review of State-of-the-Art

To provide a basis for the development of design criteria for slabs loaded to incipient collapse, a review of experimental and analytical work on slabs was carried out in Phase I of this investigation (1). Highlights of the review and evaluation are summarized below.

2.1.1 Yield-Line Theory. Yield-line theory is extensively used to calculate the collapse load of reinforced concrete slabs in NAVFAC P-397. Present knowledge on the design of reinforced concrete slabs using yield-line theory is based mainly on Johansen's work (4). The theory has proved effective in predicting the initial hinging load in reinforced concrete slabs with negligible membrane forces.

Yield-line theory is based on the premise that, under increasing load, a certain characteristic pattern of cracks (yield-lines) is formed in slabs that ultimately leads to failure. Along these yield-lines the plastic moment capacity of the slab cross section is assumed to have been reached, thereby transforming the slab into a mechanism. The pure moment capacity of slab sections in the direction of the reinforcement is used to calculate the load capacity of the slab in yield-line theory. The deformation of the slab takes place due to rotation of slab segments along yield-lines. The portion of the slab between yield-lines is assumed to remain rigid. In addition, all elastic deformations are neglected.

Solutions obtained from yield-line theory give an upper bound to the flexural capacity of slabs. Collapse loads calculated using yield-line theory were then considered essentially as unsafe solutions since the true collapse load was thought to be less than or equal to that calculated from the theory. However, the theory considers only the flexural action in slabs and does not take membrane action into account.

The ultimate load predicted by yield-line theory is sometimes referred to as the Johanson Load and serves as a useful index in examining the load-versus-deflection relationships of slabs.

2.1.2 Membrane Action in Reinforced Concrete Slabs. As mentioned earlier, yield-line theory is based on the pure moment capacity of the slab cross section and does not take into account in-plane (membrane) forces. Presence of in-plane forces results in an increase in the ultimate load to a magnitude often exceeding that predicted by the theory.

In laterally restrained slabs two types of membrane action can occur. Compressive membrane action, the so-called arching effect, occurs at the early stages of deflection. This is followed by tensile membrane action at more advanced stages of loading.

For under-reinforced slabs, a substantial shift occurs in the neutral axis position in the post-cracking range. This creates a tendency for the slab edge to move outwards as slab deflection increases. If the outer edges are restrained against movement, compressive forces are induced in the slab, as shown in Fig. 2-1.

Arching action occurs because the compressive force at the center of slab acts above the slab mid-depth, while along the edges it acts below the slab mid-depth. Due to arching action, the load-carrying capacity of a restrained slab is increased substantially above that predicted by yield-line theory.

As the deflection increases further and the load carried by the slab decreases, membrane action in the central region of the slab shifts from compressive to tensile. Thereafter, the slab carries load by the reinforcement net acting as a plastic tensile membrane, with cracking penetrating the slab thickness, as shown in Fig. 2-2. The ultimate tensile membrane capacity is reached when the reinforcement starts to rupture.

2.1.3 Load-versus-Deflection Relationship. The load versus-deflection relationship of uniformly loaded reinforced concrete slabs is significantly influenced by the boundary conditions along the slab edges. This is shown in Fig. 2-3.

The solid curve in the figure shows that a two-way, laterally unrestrained slab deflects elastically and then elastoplastically as the load is increased from A to B. Near load stage B, a yield-line pattern develops and the slab deflects at a faster rate. Beyond this stage, the slab is continuously stretched at the center, with cracks penetrating through the entire slab thickness. The center portion of the slab acts essentially as a tensile membrane. Tensile forces at the center require existence of a compressive ring capable of resisting radial tension. Compression is often taken by edge beams, but if beams are absent, the slab will naturally develop a compressive ring as shown in Fig. 2-4.

In the tensile membrane regime, depth of the compressive stress block in the yield-line near the corners is greatly increased. Tensile cracks at the center may go completely through the slab. The increase in deflection gives rise to an increasing elongation that ultimately leads to rupture of the reinforcement.

When the slab edges are restrained against lateral movement, slab capacity is enhanced in the early stages of loading due to arching (compressive membrane) action, as shown by the dashed curve in Fig. 2-3. This restraint reaches a maximum at Point D,

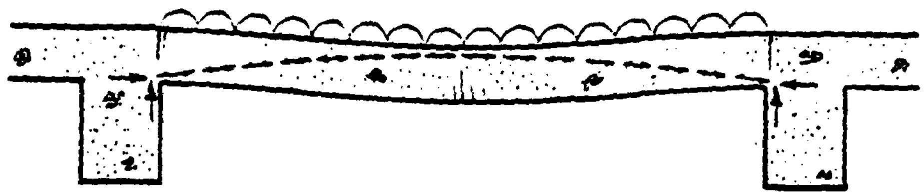

Fig. 2-1 Arching Action in Restrained Reinforced Concrete Slabs

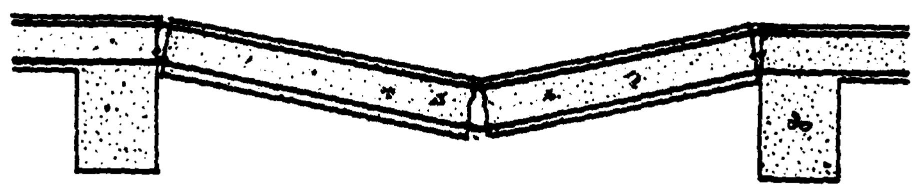

Fig. 2-2 Plastic Tensile Membrane in Restrained Reinforced Concrete Slabs

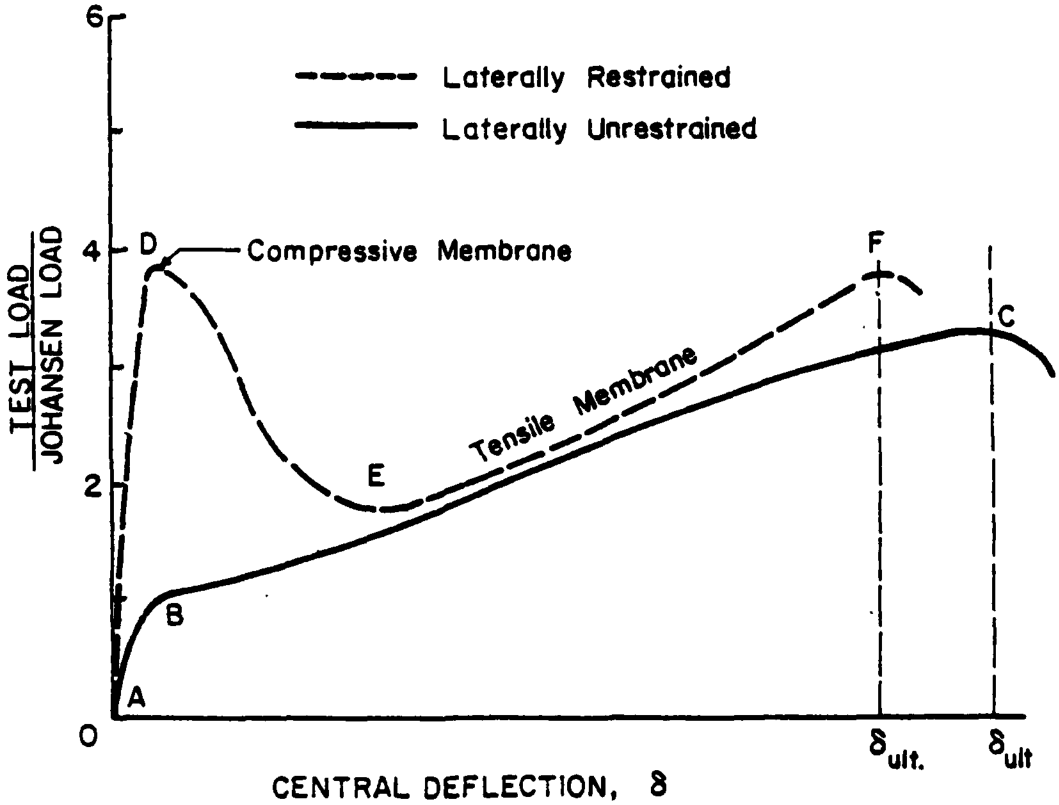

Fig. 2-3 Load-versus-Deflection Relationship for Two-Way Reinforced Concrete Slabs

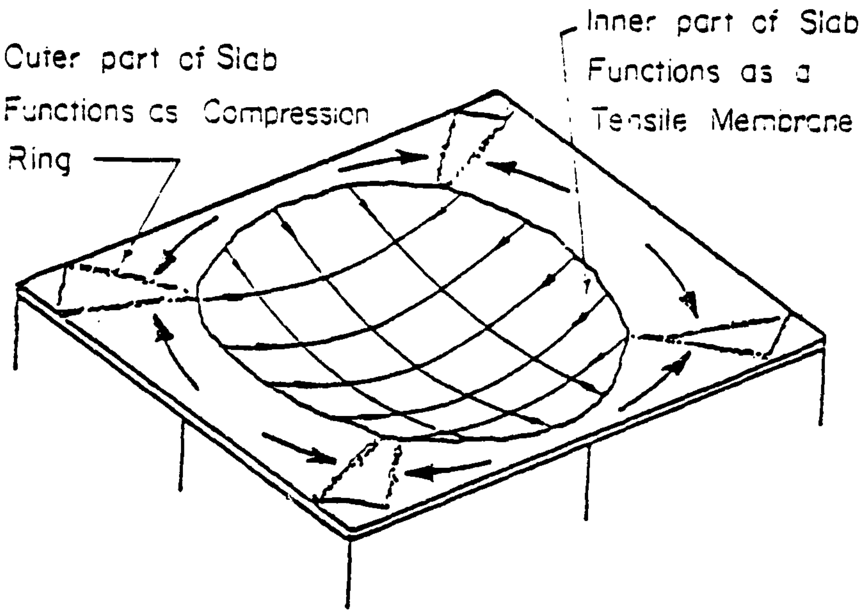

Fig. 2-4 Behavior of Laterally Unrestrained Slab
Near Incipient Collapse Deflection

when crushing of the concrete in compression occurs. Immediately beyond D, the load carried by the slab decreases rapidly. This is sometimes referred to as the "snap through" phase.

After the concrete crushes and Point E is approached, membrane action in the central region of the slab changes from compressive to tensile. Beyond E, the slab carries load by the reinforcement acting as a plastic tensile membrane, with cracks penetrating the slab thickness. The slab continues to carry greater load with increasing deflection until the reinforcement ruptures at F. Point F (or Point C for laterally unrestrained slabs) in Fig. 2-3 corresponds to the condition of incipient collapse which is of major interest in this investigation.

In both unrestrained and restrained slabs, rupture of the reinforcement precipitates collapse. However, failure of bond between reinforcement and concrete in boundary support elements causes pull-out of the reinforcement from the anchorage zones and can trigger premature collapse. Therefore, effective end anchorage of the reinforcement is vital to develop the tensile membrane load capacity and incipient collapse deflection of slabs.

2.1.4 Review of Experimental Work A review of available literature on behavior of reinforced concrete slabs was conducted in Phase I of this investigation. The results have been summarized in Ref. 1. The types of slabs considered in the report were classified as follows:

a. Laterally restrained two-way slabs

b. Laterally unrestrained two-way slabs
(referred to as simply-supported two-way slabs in Phase I and II reports)

c. One-way slabs (laterally restrained or unrestrained)

The laterally restrained slabs considered are generally also rotationally restrained along their edges. Similarly, the laterally unrestrained slabs are also rotationally unrestrained at the boundaries.

a. Laterally Restrained Two-Way Slabs. The report presents data and results of 107 restrained two-way slab specimens tested by various investigators. The test data indicate that both compressive and tensile membrane actions, though occurring at different deflection stages, enhance slab load-carrying capacity. When sufficient lateral restraint exists at boundaries of a slab, the slab capacity is increased to several times that predicted by Johansen's yield-line theory. The ratio of slab deflection at the peak load in the compressive membrane regime, corresponding to

Point D in Fig. 2-3, to the slab thickness varies between 0.11 and 0.97. Major parameters affecting this ratio are degree of edge restraint, span-depth ratio, and reinforcement ratio.

Near the end of the snap-through range, corresponding to Point E in Fig. 2-3, the large stretch in the central region of the slab surface causes cracks there to penetrate the entire thickness of the slab. At this stage, load in the central region is carried mainly by the reinforcing bars acting as a tensile membrane.

Beyond Point E in Fig. 2-3, the boundary restraints begin to resist inward movement of the slab edges. Initially, the outer regions of the slab act with the edge restraint as part of the compressive ring supporting the tensile membrane in the inner region of the slab. With further deflection beyond Point E, tensile membrane action gradually spreads throughout the slab. Subsequently, the load carried by the yielding reinforcement increases until the reinforcement starts to fracture at Point F. Point F represents the condition of incipient failure for restrained slabs.

Knowledge of the region DE is important since the load may drop suddenly as soon as Point D is reached. In this respect, tensile membrane action is useful in preventing a catastrophic failure. If the load corresponding to Point D is maintained, this will require that a resistance equal to or greater than that corresponding to Point D be developed in tensile membrane action. Test data show that for heavily reinforced slabs the collapse load at Point F in Fig. 2-3 can significantly exceed the peak load at Point D. The ultimate load associated with tensile membrane action tends to increase with increasing reinforcement ratio.

Most of the tests on restrained slab specimens were not carried into the tensile membrane range. Instead, the tests were terminated once a slab showed a decrease in its load-carrying capacity. This termination would usually occur just after the state represented by Point D in Fig. 2-3 is reached. The limited number of tests on slabs loaded into the tensile membrane range indicate that the ultimate deflection just before rupture of the reinforcement lies between 0.10 and 0.15 of the slab span.

b. <u>Laterally Unrestrained Two-Way Slabs</u>. Test results of 65 unrestrained two-way slab specimens show that slab capacity is always greater than that predicted by Johansen's yield-line theory. This is because the geometry of deformation permits development of some membrane forces in the slab. This occurs in uniformly loaded two-way slabs at relatively large deflections, when the slab regions near the edges tend to move inwards but are restrained from doing so by the adjacent outer regions. The result is an outer ring of compression resisting tensile membrane forces in the inner region of the slab. A representative load-versus-deflection curve for a laterally unrestrained two-way slab is shown as a solid curve in Fig. 2-3.

A plot of the ratio of ultimate deflection/slab thickness based on test results for laterally unrestrained two-way slabs shows considerable scatter. One reason behind such scatter is that not all specimens were loaded to incipient collapse. Some tests were terminated earlier either due to the loading system being inoperable at large deflections or disinterest in slab behavior in the region where tensile membrane action predominates.

c. One-Way Slabs. Test results on 44 one-way slab specimens with restrained and unrestrained edges were reviewed in the Phase I report (1). A significant number of tests were carried out using two equal loads at the middle-third points.

None of the tests on *restrained slab* strips was carried into the tensile membrane range.

2.1.5 Review of Analytical Work. In order to evaluate the feasibility of using computer methods in analyzing membrane action in slabs with different material and geometric properties, the literature on nonlinear analysis of slabs was reviewed in Phase I of this investigation (1).

Analysis of reinforced concrete systems considering cracking as well as material and geometric nonlinearities involves complex problems in finite element modeling. Although there have been some notable breakthroughs in the use of the finite element method for nonlinear problems, even the speed and storage capacities of today's large digital computers are sometimes insufficient to provide solutions at reasonable costs because of the nature of such nonlinear problems.

A survey of available computer programs showed that no available code is capable of determining the response of slabs, considering cracking as well as material and geometric non-linearities, up to the tensile membrane range. Also, no program that considers all the relevant parameters has been developed for calculating the incipient collapse deflection of reinforced concrete slabs.

Most finite element methods are based on small displacement theory. This may represent a serious shortcoming when applying the method to the analysis of slab behavior in the tensile membrane range where large displacements are expected. Another difficulty in using a finite element model arises due to presence of an unstable region, represented by the descending branch DE in the load-versus-deflection relationship as shown in Fig. 2-3. In this region, the load decreases with increasing deflections, creating a negative stiffness problem. No finite element study is available which attempts to predict the behavior in the unstable region.

2.2 Development of Design Criteria

2.2.1 Introduction. The primary objective of this investigation is to develop design criteria for the incipient collapse deflection of conventionally reinforced concrete slabs under uniform load.

The literature review conducted in Phase I (1) showed that a considerable amount of experimental and analytical work has been done on the behavior of reinforced concrete slabs. Very few of these studies, however, have considered the response of slabs beyond the compressive membrane action range. A good number of tests were carried out primarily to show that compressive membrane action in slabs can result in loads far in excess of the load indicated by yield-line analysis, i.e., the Johansen load. It was noted that the load-versus-deflection relationship of uniformly loaded reinforced concrete slabs is significantly influenced by the restraint conditions along the edges, as shown in Fig. 2-3.

Very little information is available on slabs loaded into the tensile membrane range. Most of the tests were terminated once the specimen showed a decrease in load-carrying capacity. This would be just after the stage represented by Point D in Fig. 2-3. The principal interest in this investigation is the behavior of slabs at deflection levels equal to or near that corresponding to incipient collapse.

The literature review indicated the lack of a satisfactory tool i.e., computer program, to analyze the response of slabs over the entire range of loading. Because of this, reliance had to be placed on data from a limited number of tests to examine the influence of certain parameters on incipient collapse deflection capacity.

2.2.2 Parameters Affecting Slab Behavior. One of the main objectives in examining test data is to identify the most important parameters affecting the deflection capacity at incipient collapse. Park (5), Keenan (6), and Black (7) suggested that the short span of a slab is the only parameter affecting incipient collapse deflection capacity. Herzog (8), and Hawkins and

Mitchell (9) hypothesized that, in addition to slab span, the breaking strain of the reinforcement affects deflection capacity. In addition to short span and reinforcement breaking strain, the following parameters have been thought to influence the deflection capacity of slabs: span-depth ratio, aspect ratio, size of slab and boundary conditions. An examination of the correlation between selected parameters and incipient collapse deflection using available experimental data is given below.

As mentioned earlier, most slab tests were not carried to deflection levels equal to or near the incipient collapse deflection capacity. Therefore, only slabs loaded to incipient collapse are examined in terms of the effects of selected parameters on incipient collapse deflection.

a. Short Span of Slab. The effect of short span length on the slab deflection capacity is significant, as shown in Fig. 2-5. The incipient collapse deflection, δ_{ult}, increases almost linearly with an increase in the short span of slabs, L_y.

   Park (5) and Keenan (6) determined empirically the safe maximum value of central deflection for restrained slabs in tensile membrane action, δ_{ult}, to be:

$$\delta_{ult} = 0.1 L_y \tag{2-1}$$

   where

$$L_y = \text{short span of slab}$$

   Later, Black (7) claimed Eq. 2-1 to be too conservative for an estimate of the deflection capacity. Black suggested that the deflection capacity, δ_{ult}, be calculated using the expression

$$\delta_{ult} = 0.15 L_y \tag{2-2}$$

   A comparison of these equations with available test data indicates that Eq. 2-1 yields a lower bound whereas Eq. 2-2 yields an upper bound on the test data, as shown in Fig. 2-5.

b. Boundary Conditions. A comparison between restrained and unrestrained two-way slab test results shows that the deflection capacity-span ratio of unrestrained

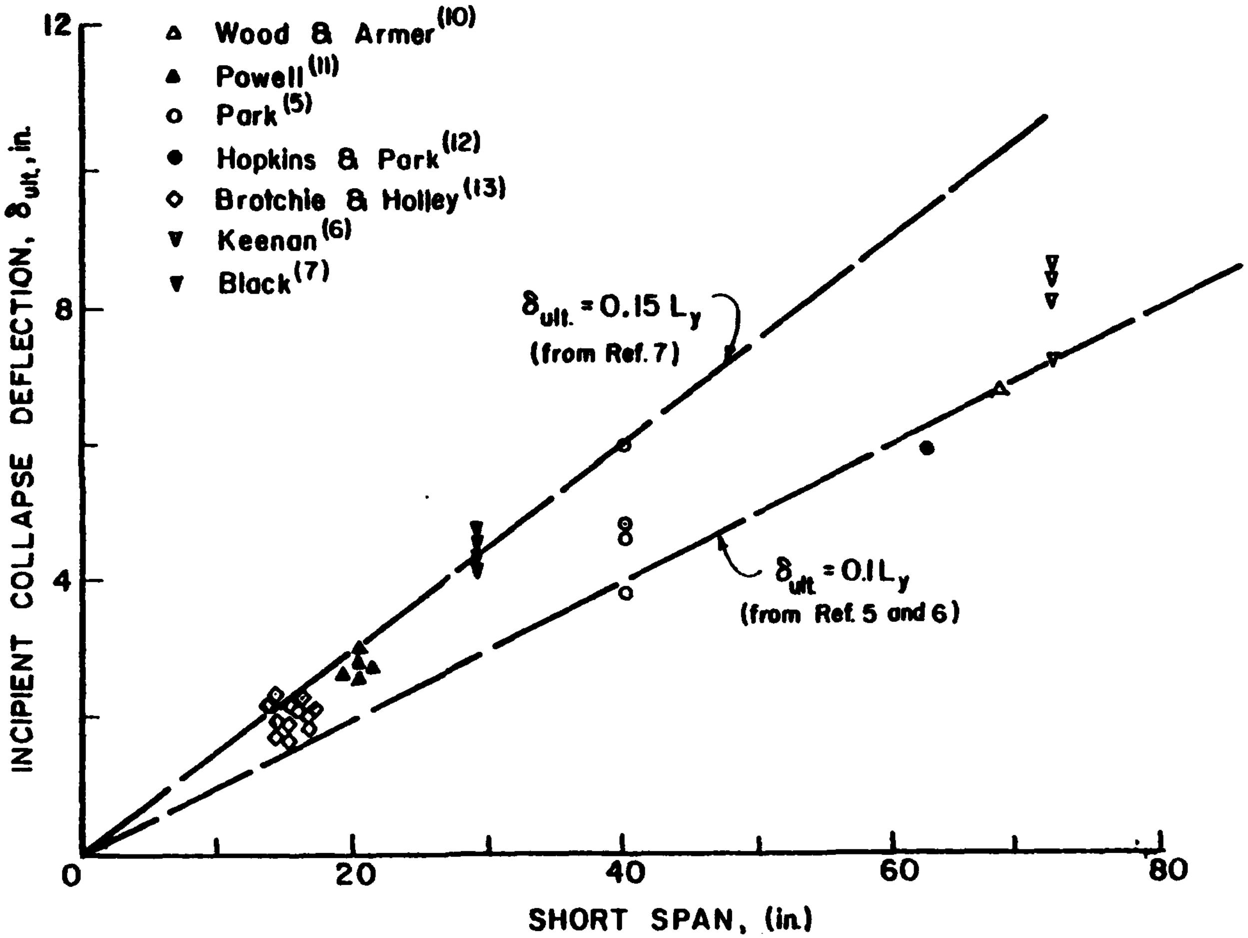

Fig. 2-5 Effect of Short Span Length on Incipient Collapse Deflection Capacity of Restrained Two-Way Slabs

slabs is slightly greater than that of restrained slabs. The average δ_{ult}-span ratio for unrestrained slabs is 0.18 compared to a value of 0.14 for restrained slabs. This is indicated in Fig. 2-6.

c. Span-Depth Ratio. A plot of incipient collapse deflection, δ_{ult}, versus short span-depth ratio for restrained two-way slabs shows a wide scatter as shown in Fig. 2-7.

The scatter implies that short span-depth ratio has no notable effect on δ_{ult}. The observation lends support to the hypothesis that a slab acts essentially as a cable net in the tensile membrane action range.

d. Combined Short Span - Reinforcement Breaking Strain. The number of tests for which data on short span of slab and the breaking strain of reinforcement are available is quite limited. Only a few investigators have reported the breaking strain of the reinforcement used in their respective test specimens. The geometric and material properties of those specimens for which the ultimate strain of the reinforcement at rupture was reported are shown in Table 2-1, along with relevant test results.

Two approaches have been proposed to relate δ_{ult} with the combined effect of short span and breaking strain of reinforcement. Herzog (8) hypothesized that the incipient collapse deflection capacity, δ_{ult}, depends mainly on the short span of slab and the reinforcement strain at rupture. Assuming the slab to take the shape of a parabolic cable, and allowing for the irregular strain distribution in the slab reinforcement, Herzog proposed the following expression for the midspan deflection at incipient collapse:

$$\delta_{ult} = L_y \sqrt{\frac{3\ \varepsilon_u}{32}} = 0.31\ L_y \sqrt{\varepsilon_u} \qquad (2\text{-}3)$$

where L_y = short span of slab

ε_u = rupture strain of reinforcement

A comparison of Eq. 2-3 with the selected test data, as shown in Fig. 2-8, indicates that the equation gives a reasonable estimate of the incipient collapse deflection capacity of restrained slabs. For large specimens, Eq. 2-3 appears to be slightly on the unsafe side.

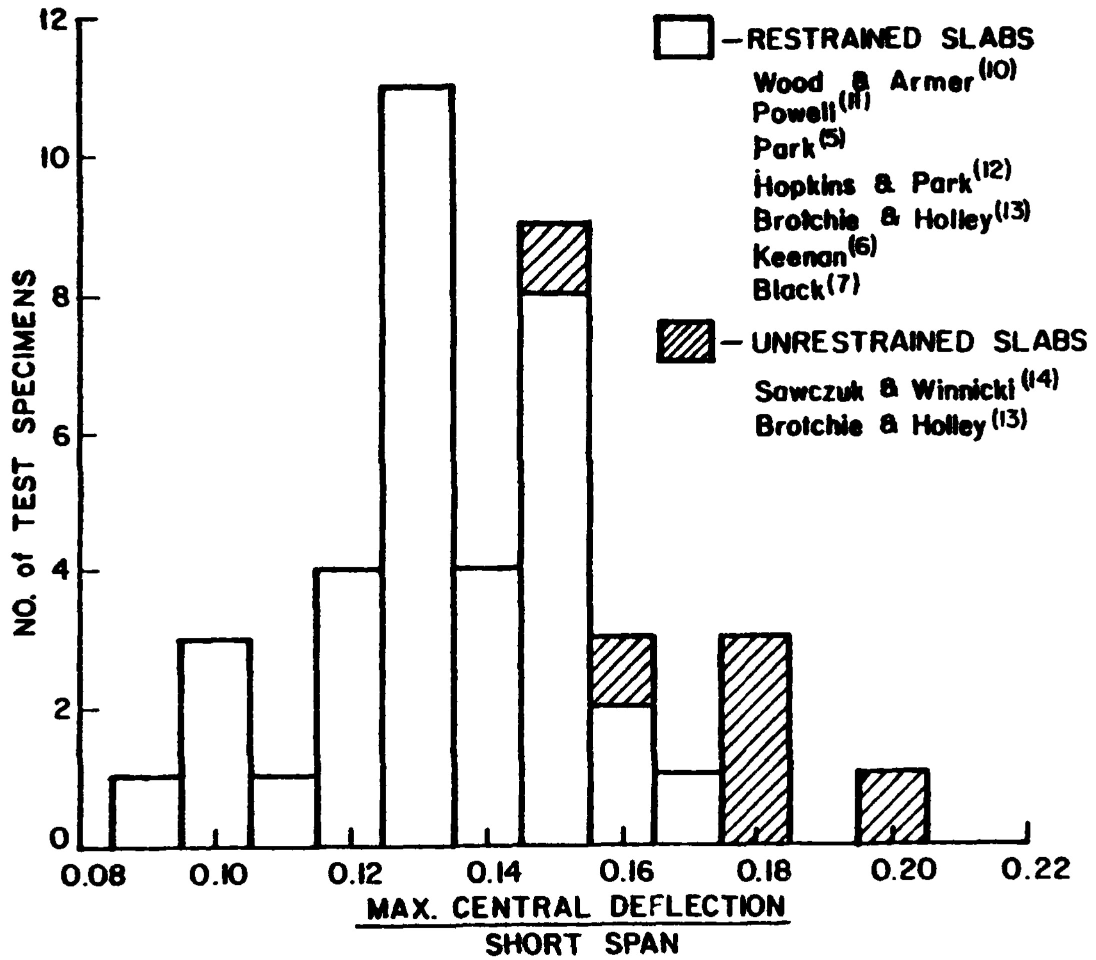

Fig. 2-6 Effect of Lateral Edge Restraint on Incipient Collapse Deflection Capacity of Two-Way Slabs

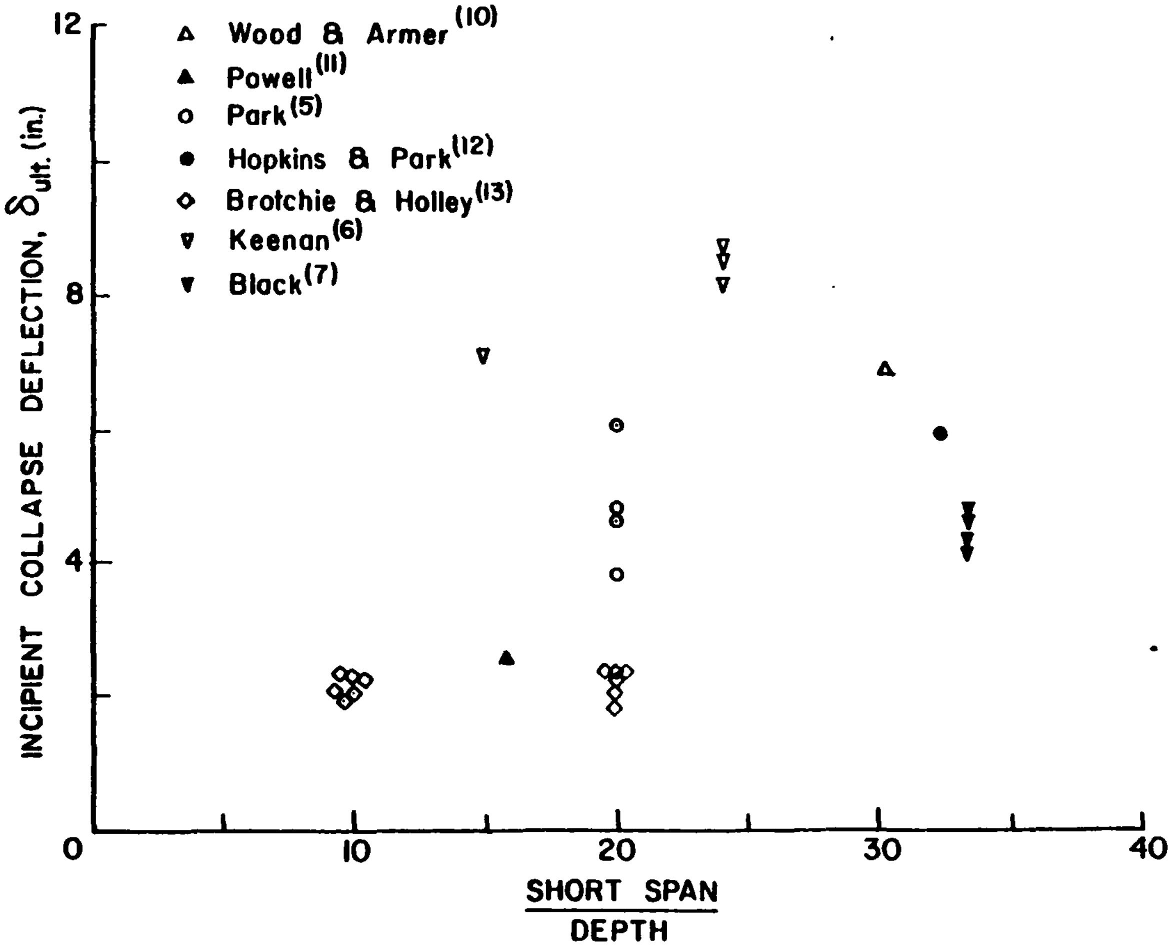

Fig. 2-7 Effect of Span-Depth Ratio on Incipient Collapse Deflection Capacity of Restrained Two-Way Slabs

Table 2-1 Properties and Test Results of Two-Way Restrained Slabs

Investigators	Mark	Geometric and Material Properties									Test Results			Remarks
		Clear Slab Dimension $L_y \times L_x \times h$ in.	L_y/L_x	L_x/h	Steel Reinforcement, Short Span Top	Steel Reinforcement, Short Span Bot.	Steel Reinforcement, Long Span Top	Steel Reinforcement, Long Span Bot.	f_y ksi	ε_u	δ_t in.	δ_t/L_y	Maximum Edge Rotation degrees	
Park[5]	A1	40x60x2	1.5	20	0.38	0.19	0.41	0.20	45.5	0.11	3.0	0.10	11	Rectangular; orthotropic; edges fully restrained
	A2				0.84	0.42	0.43	0.21			4.0	0.12	14	
	A3				1.44	0.72	0.45	0.22			6.0	0.15	17	
	A4				2.42	1.21	0.47	0.23			4.6	0.12	13	
Keenan[6]	3S1	72x72x3	1.0	24	0.82	0.82	0.82	0.82	49.6	0.19	8.4	0.12	13	Fully restrained
	3S3				0.82	0.82	0.82	0.82	49.6	0.19	8.5	0.12	13	
	3S4				0.82	0.82	0.82	0.82	49.6	0.19	8.0	0.11	13	
	4.75S1	72x72x4.75	1.0	15.2	0.89	0.89	0.89	0.89	47.4	0.20	7.0	0.10	11	
Black[7]	IS1	29x29x0.89	1.0	33	0	0.07	0	0.07	43.6	0.12	4.47	0.15	17	Fully restrained
	IS2				0	0.07	0	0.07	43.6		4.1	0.14	16	
	IS3				0	0.07	0	0.87	43.6		4.64	0.16	18	
	IS4				0	1.17	0	1.17	43.6		4.1	0.14	16	
Wood and Armer[10]	FS12	60x60x2½	1.0	30	0	0.25	0	0.25	33.0		6.0	0.10	11	Square; singly-reinforced; restrained against expansion
Powell[11]	S46	36x20.6x1.29	1.75	16	0.25	0.25	0.25	0.25			2.60 min.	0.13	14	Rectangular; doubly-reinforced; retrained against expansion
	S47				0.25	0.25	0.25	0.25						
	S50				0.45	0.45	0.45	0.45						
	S54				0.71	0.71	0.71	0.71						

Table 2-1 Properties and Test Results of Two-Way Restrained Slabs (Cont.)

Investigators	Mark	Geometric and Material Properties: Clear Slab Dimension $L_y \times L_x \times h$ in.	L_y/L_x	L_x/h	Steel Reinforcement, Short Span Top	Short Span Bot.	Long Span Top	Long Span Bot.	f_y ksi	ε_u	Test Results: δ_t in.	δ_t/L_y	Maximum Edge Rotation degrees	Remarks
	S55				0.71	0.71	0.71	0.71						
	S58				0.97	0.97	0.97	0.97						
	S59				0.97	0.97	0.97	0.97						
	S62				1.53	1.53	1.53	1.53						
	S63				1.53	1.53	1.53	1.53						
Hopkins and Park [12]		62.5x6.5x 1-15/16	1.0	32	0.15	0.15	0.15	0.15	52.0		5.9	0.09	11	Interior panel in a 9-panel floor system
Brotchie and Holley [13]	33	15x15x0.75	1.0	20	0	0.5	0	0.5	55		1.9	0.13	14	Restrained against lateral elongation
	34					1.0		1.0			2.1	0.14	16	
	35					2.0		2.0			2.15	0.14	16	
	36					3.0		3.0			2.2	0.15	17	
	46					1.0		1.0			2.2	0.15	17	Fully restrained
	48					2.0		2.0			2.5	0.17	18	
	28	15x15x1.5		10		0.5		0.5	55		2.0	0.13	15	Restrained against lateral elongation
	29					1.0		1.0			2.31	0.15	17	
	31					2.0		2.0			2.25	0.15	17	
	30					3.0		3.0			2.15	0.15	16	
	47					1.0		1.0			2.4	0.16	18	
	49					2.0		2.0			2.2	0.15	16	

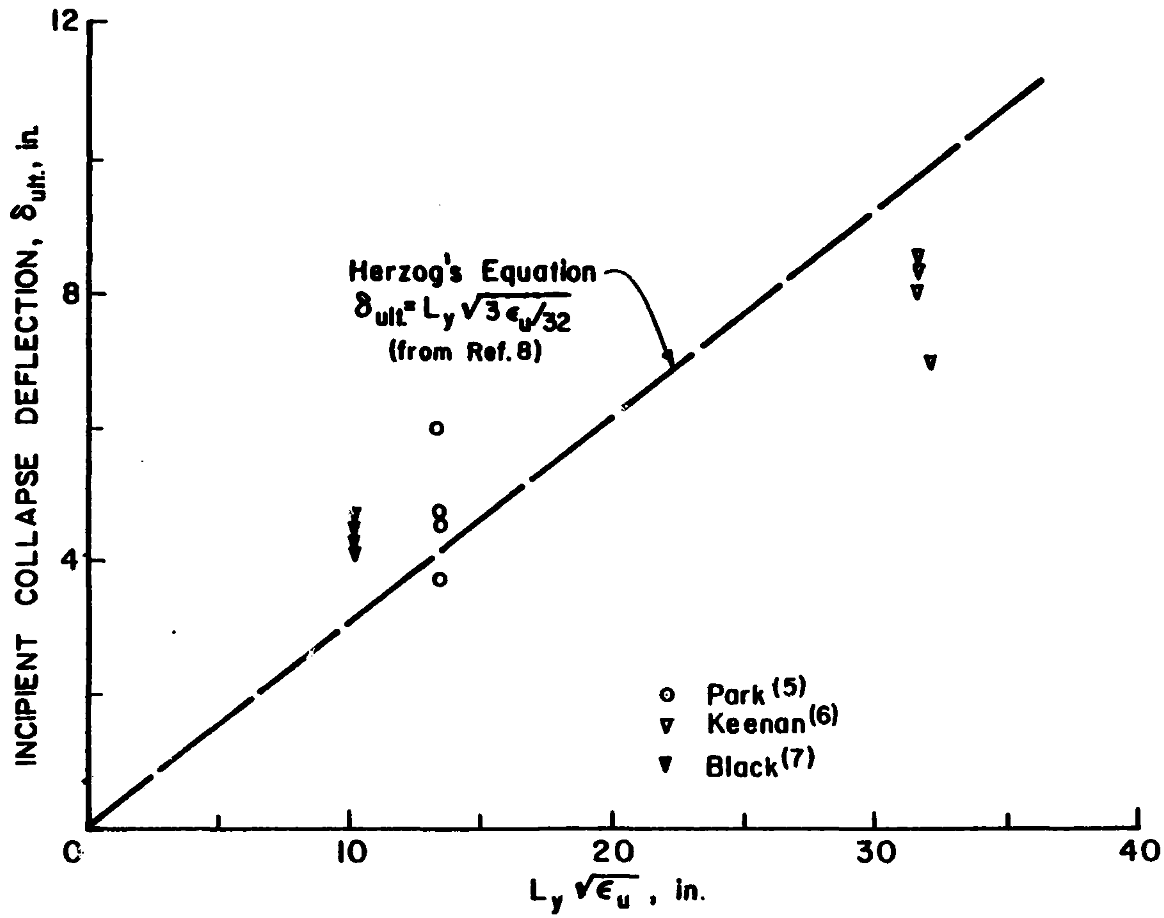

Fig. 2-8 Combined Short Span-Steel Breaking Strain Effect on Incipient Collapse Deflection Capacity, as Proposed by Herzog

Hawkins and Mitchell (9) assumed that the tensile membrane takes the shape of a circular arc and developed the following expression for δ_{ult}:

$$\delta_{ult} = \frac{1.5\ L_y\ \varepsilon_u}{\sin\ \sqrt{6\varepsilon_u}} \tag{2-4}$$

Figure 2-9 shows that Eq. 2-4 grossly overestimates the incipient collapse deflection of the test slabs considered.

Both Figures 2-8 and 2-9 indicate strong correlation between the selected two parameters, namely, short span of slab, L_y, and breaking strain of reinforcement, ε_u, and the incipient collapse deflection, δ_{ult}.

2.2.3 Proposed Expression (Design Criterion) for Incipient Collapse Deflection. Comparison of analytically predicted deflections with available experimental data was used as the principal basis for assessing the reliability of any proposed expression for incipient collapse deflection. From such a comparison, it was determined that both short-span of slab and breaking strain of reinforcement are the most significant parameters affecting the incipient collapse deflection, δ_{ult}.

Proposed analytical expressions for predicting incipient collapse deflection of two-way slabs are approximate and based on the assumption of pure membrane action. The assumption of pure membrane action implies a uniform strain distribution along the length of the slab reinforcement. This results in a predicted collapse deflection corresponding to rupture of reinforcement that is considerably greater than that observed in tests.

It is obvious that because of cracking, the strain distribution in the flexural reinforcement of a slab, even in the tensile membrane range, is nonuniform. The magnitude of the strains at various points along the reinforcement may also be affected by previous flexural response history.

From a correlation of the basic expression for pure membrane action and available experimental data, an expression which provides a reasonable lower bound to the test data is proposed. Probability concepts associated with test results on ultimate deflection and steel rupture strain are introduced in an evaluation of the proposed equation. The proposed expression for the incipient collapse deflection of conventionally reinforced concrete slabs under uniform load is given by:

$$\delta_{ult} = k\ L_y\ \sqrt{\varepsilon_u} \tag{2-5}$$

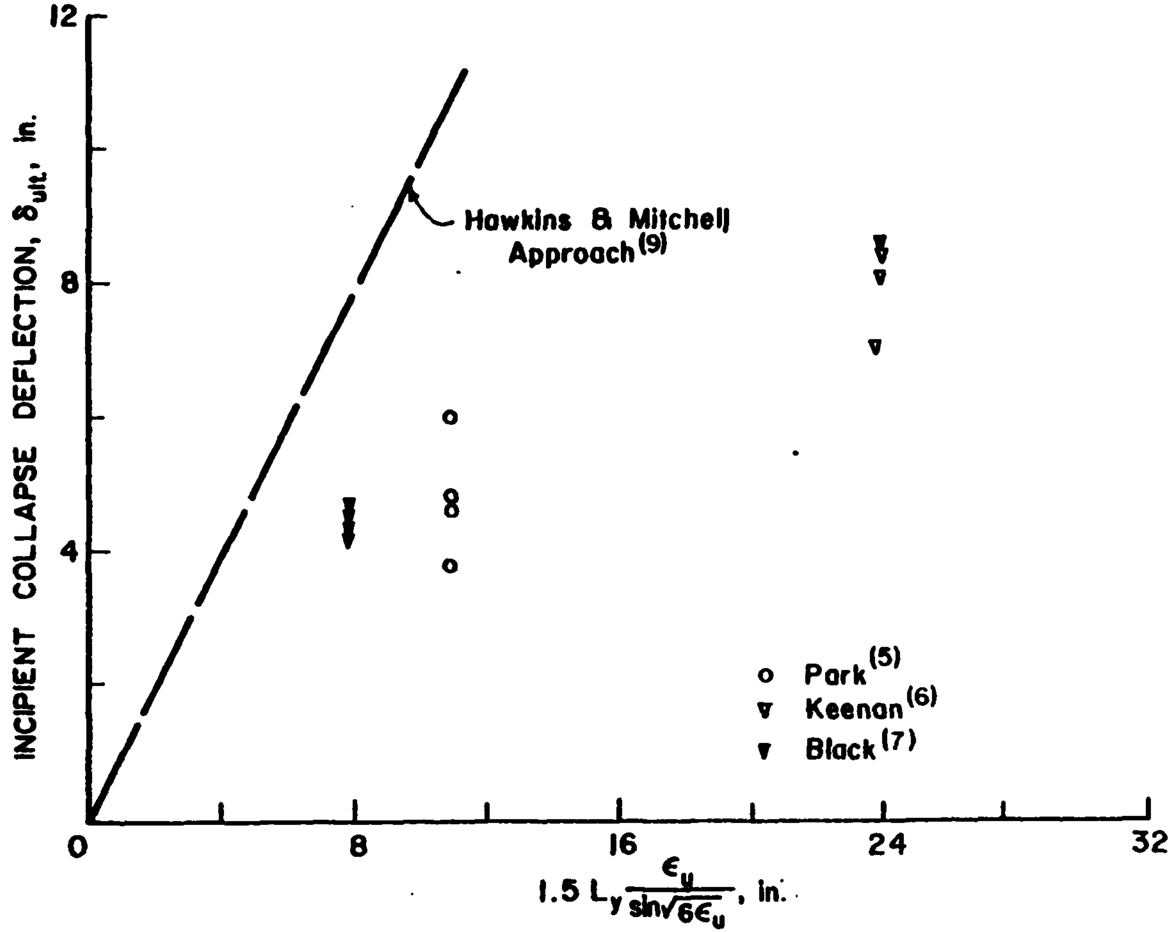

Fig. 2-9 Combined Short Span-Steel Breaking Strain Effect on Incipient Collapse Deflection Capacity, as Proposed by Hawkins and Mitchell

where L_y = short span of slab

ε_u = breaking strain of flexural reinforcement

k = a factor to account mainly for the nonuniform distribution of strain along the length of reinforcing bars

The basic form of Eq. 2-5 was derived on the assumption that a representative slab strip takes the form of a parabolic cable in the tensile membrane range. It has essentially the same forms as Eq. 2-3 proposed by Herzog (8), except that instead of the constant 0.31 in Herzog's equation a variable factor k is used. The factor k is introduced to account for the disparity between the deflection associated with pure cable action and observed slab behavior. A major difference between the two is the non-uniform strain distribution along the length of the reinforcement associated with cracking in the slab. The use of a variable factor in Eq. 2-5 allows the determination of an appropriate value of k for the different categories of slabs based on a consistent probability that the predicted value will be exceeded.

Two parameters are needed to estimate slab incipient collapse deflection, δ_{ult}. These are slab short span, L_y, and breaking strain of flexural reinforcement, ε_u. Figure 2-10 shows the proposed equation, with k assigned a value of 0.25, compared with available experimental data on restrained two-way slabs. The plotted test data represent only those tests in which incipient collapse was observed and the ultimate strain of the steel reinforcement at rupture was reported. The geometric and material properties of these test specimens are shown in Table 2-1 along with relevant test results.

Although Eq. 2-5 is based on the results of tests on small-scale specimens, it is assumed that the expression is directly applicable to full-scale slabs. Since the primary mechanism of resistance in the tensile membrane range is relatively simple and the variables considered in Eq. 2-5 are directly related to this simple mechanism, there is good reason to believe that the application of Eq. 2-5 to full-scale slabs is justifiable.

There are several points that should be noted in relation to the proposed Eq. 2-5. First, the test data on which the equation is based include both square and rectangular slabs. The short span of the slabs range from a low value of 29 in. to a high value of 72 in., more than twice the shortest span as noted in Table 2-1. Finally, the test results are from the work of not just one investigator but of three different investigators. In spite of the scarcity of data, the fact that the proposed relationship (i.e., design criterion) represents a reasonable lower bound on

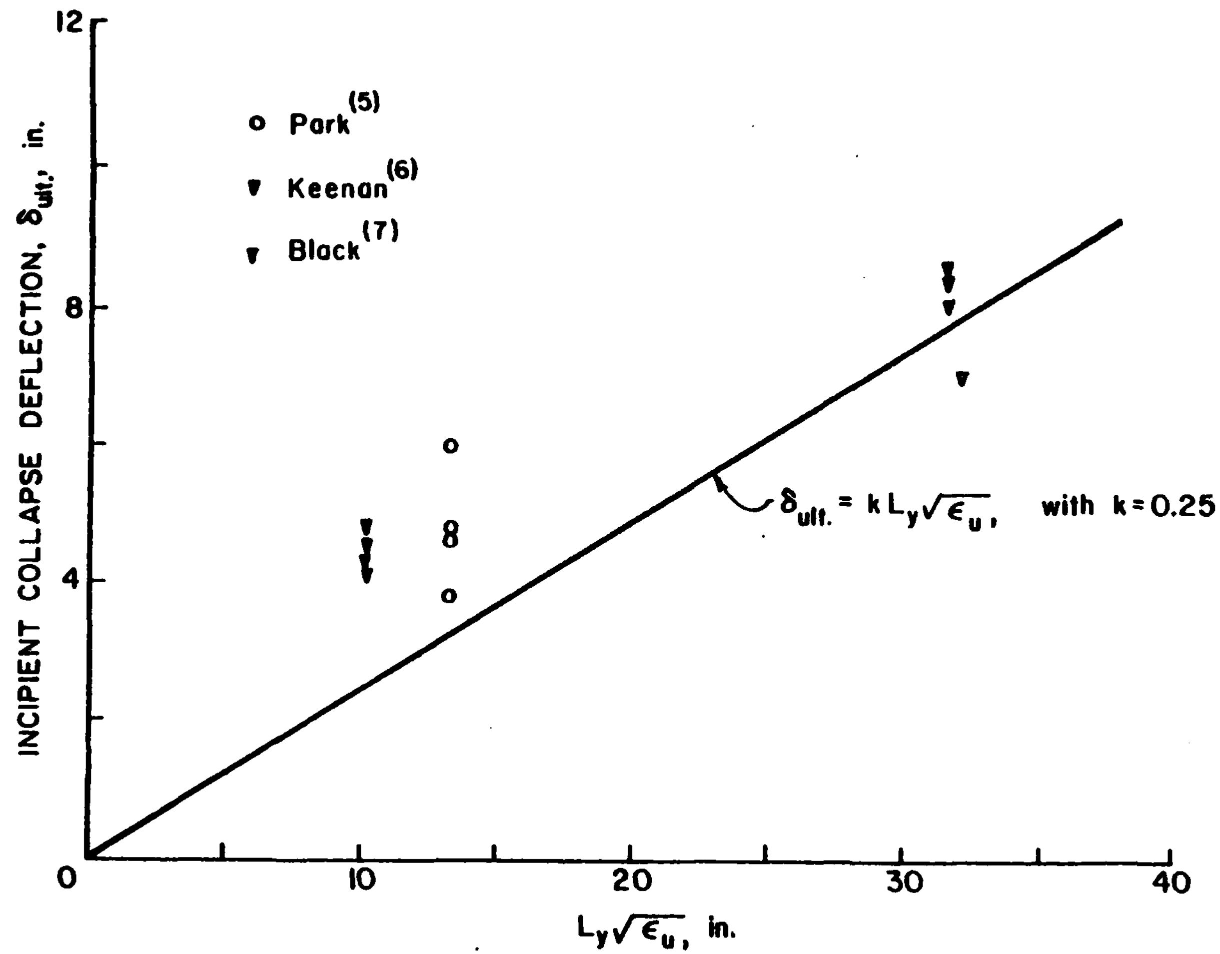

Fig. 2-10 Incipient Collapse Deflection of Restrained Two-Way Slabs as a Function of Short Span, L_y, and Reinforcement Rupture Strain, ε_u

data covering a wide range of conditions provides some assurance of its reliability.

Although available experimental data is meager, the prediction of incipient collapse deflection using Eq. 2-5 may be treated within a probabilistic framework if the appropriate probability density functions can be assumed as adequately defined. The assumed density functions can be refined as more data become available. In the following, a probabilistic approach is used in determining the value of the factor k corresponding to specified probabilities that the incipient collapse deflection exceeds the value predicted by Eq. 2-5.

Probability density functions of the factor k are derived as a function of two random variables. These are the ratio of ultimate deflection to short span, δ_{ult}/L_y, and steel rupture strain, ε_u. These two random variables have their own probability density functions. However, these two variables, δ_{ult}/L_y and ε_u, are not independent of each other. Uncertainty in δ_{ult}/L_y includes the effect of variations in ε_u.

The probability density function of the ratio δ_{ult}/L_y can be obtained from selected test results as listed in Tables 2-1 and 2-2. Because of the scarcity of data on slabs tested to incipient collapse, a more accurate definition of the probability density function for δ_{ult}/L_y will have to await further tests. There are, however, sufficient data on steel rupture strain, in addition to values related to the slab tests, to allow a much better definition of the associated probability density function. For this purpose, the results of coupon tests of reinforcing bars from various sources can be used.

For a random variable, Y, specified as a function of two other statistically independent variables X_1 and X_2, i.e.,

$$Y = g\ (X_1,\ X_2) \tag{2-6}$$

The approximate mean or expectation, E(Y), and variance, Var(Y), of Y are given by (15):

$$E(Y) \simeq g(\mu_{x_1}, \mu_{x_2}) \tag{2-7}$$

$$\mathrm{Var}(Y) \simeq \sum_{i=1}^{2} C_i^2\ \mathrm{Var}\ (X_i) \tag{2-8}$$

Where the C_i are values of the partial derivatives $\partial g/\partial X_i$, evaluated at μ_{x_1}, and μ_{x_2}.

Table 2-2 Properties and Test Results of Two-Way Unrestrained Slabs

Investigators	Mark	Geometric and Material Properties						Test Results			Remarks
		Clear Slab Dimensions $L_y \times L_x \times h$ in.	L_y/L_x	L_x/h	Steel Reinforcement, Short Span	Steel Reinforcement, Long Span	f_y ksi	δ_t in.	δ_t/L_y	Maximum Edge Rotation degrees	
Sawczuk and Winnicki (14)	I1	43x63x1.18	1.45	37.0	0.91	0.91	38.3	7.9	0.18	20	
	I2	39x79x1.18	2.00	33.0	0.91	0.91		7.9	0.20	22	
	II1	43x63x1.18	1.45	37.0	0.45	0.45		7.9	0.18	20	
Brotchie and Holley (13)	8	15x15x0.75	1.00	20.0	1.00	1.00	60.0	2.2	0.15	16	Square Isotropic
	9			20.0	3.00	3.00		2.4	0.16	18	
	12			10.0	1.00	1.00		2.6	0.17	19	

By taking k and ε_u in Eq. 2-5 as the statistically independent variables X_1 and X_2, respectively, in Eq. 2-6, and δ_{ult}/L_y as the dependent random variable Y, the coefficient of variation (i.e., standard deviation, σ/mean, μ) of k, that is, X_1 in Eqs. 2-7 and 2-8, can be expressed in terms of the coefficients of variation of Y and X_2 as follows:

$$\Omega_{x_1} = \left(\Omega_Y^2 - \tfrac{1}{4}\,\Omega_{x_2}^2\right)^{\frac{1}{2}} \tag{2-9}$$

Assuming all three variables to be normally distributed, the parameters E(k) and Var(k) defining the approximate probability distribution of k can be obtained by using Eqs. 2-7 and 2-9.

Figure 2-11 shows a histogram of steel rupture strain, ε_u, for 92 coupon tests listed in Ref. 16. Number 6, 7 and 8 bars were used in the tests. All specimens in this series were tested as full sections of the as-rolled bars and elongations were measured on an 8-in. gauge length. Also shown is the corresponding continuous distribution of ε_u based on an assumed normal distribution.

Figure 2-12 shows the histogram of the ratio δ_{ult}/L_y for the thirty-five restrained slab tests listed in Table 2-1, together with the corresponding continuous normal distribution. Figure 2-13 shows the histogram of the ratio δ_{ult}/L_y and the corresponding continuous distribution for the six unrestrained slab tests listed in Table 2-2.

Values of the mean, μ, and standard deviation, σ, for δ_{ult}/L_y, ε_u, and k for restrained and unrestrained slabs are summarized in Table 2-3. The μ and σ for the factor k were calculated from the μ and σ for δ_{ult}/L_y and ε_u using Eqs. 2-7 and 2-9. Also listed are the corresponding values of k for 90%, 95%, and 99% probabilities of exceedance based on assumed normal distributions. Values of μ, σ, and k are shown for restrained and unrestrained slabs.

Since data for only six unrestrained slab specimens are available, the coefficient of variation of k, calculated on the basis of these data, cannot be too reliable. Therefore, the same value of the coefficient of variation of k for restrained slabs is assumed for unrestrained slabs. The paucity of data on δ_{ult} for unrestrained slabs used to define the probability distribution for k would justify a more conservative approach to the choice of a design probability of exceedance for unrestrained slabs than for restrained slabs. In view of this, it is recommended that values of k corresponding to a 99% probability of exceedance be selected for the purpose of calculating incipient collapse deflection of unrestrained slabs. The corresponding value recommended for restrained slabs is 90%. These values are listed in Table 2-3. Ultimate deflection, i.e., incipient

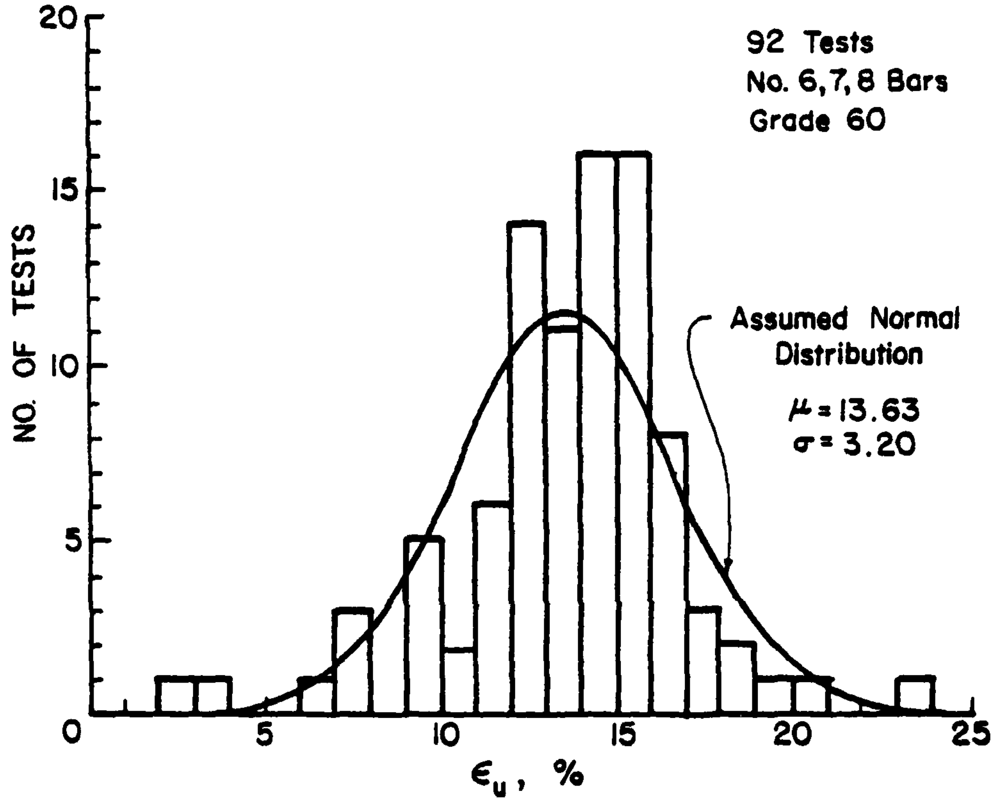

Fig. 2-11 Distribution of Steel Rupture Strain, ε_u, from Coupon Test Data (from Ref. 16)

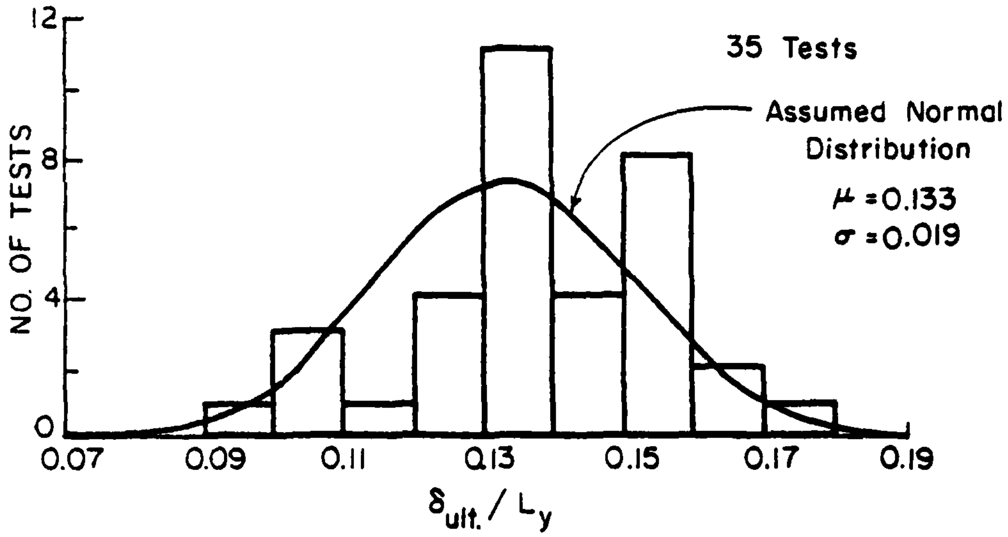

Fig. 2-12 Distribution of δ_{ult}/L_y for Restrained Slab Test Data

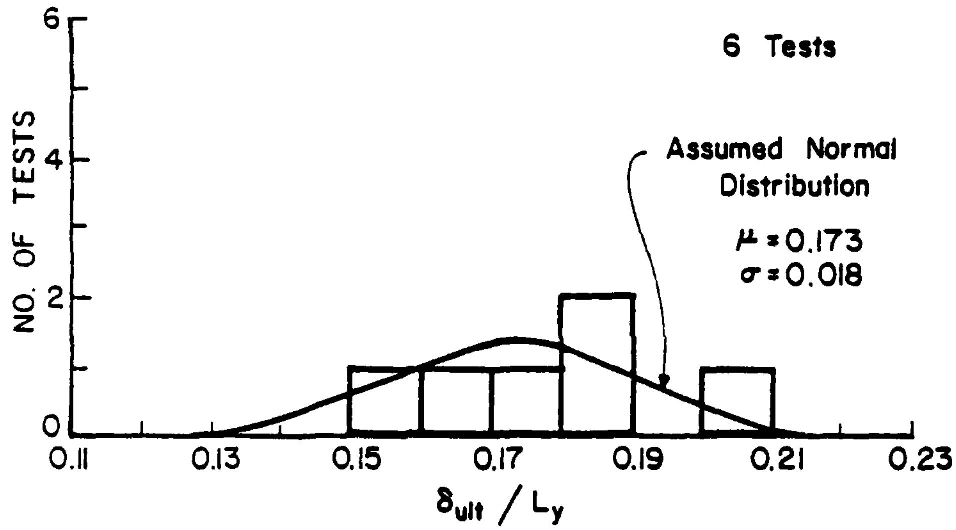

Fig. 2-13 Distribution of δ_{ult}/L_y for Unrestrained Slab Test Data

Table 2-3 Values of k for Selected Probabilities of Exceedance

Support Conditions	ε_u		δ_{ult}/L_y		Ω_k	k		Values of k Corresponding to Probabilities of Exceedance*		
	μ	σ	μ	σ		μ	σ	90%	95%	99%
Restrained 35 Tests	0.136	0.032	0.133	0.019	0.081	0.361	0.029	0.324	0.313	0.293
Unrestrained 6 Tests			0.173	0.018	0.081**	0.469	0.038	0.420	0.407	0.380

*Probability that the calculated value of $\delta_{ult} = k\ L_y \sqrt{\varepsilon_u}$ will be exceeded.

**The same value as for restrained slabs is assigned because of scarcity of data.

collapse deflection of a slab, is calculated using Eq. 2-5 with the appropriate value of k depending on slab support conditions.

The values of k determined here are based on two-way slab tests. As stated in the Phase I report, very little information is available on one-way restrained strips tested to incipient collapse under uniformly distributed load. A significant number of tests were carried out using two equal loads at the middle-third points. However, none of these tests was carried to incipient collapse. In the absence of specific test data, it is believed that the use of k values derived for two-way slabs for the case of one-way slabs is reasonable. It is pointed out that the single-cable model used in deriving Eq. 2-5 represents a closer approximation of the one-way slab strip.

Equation 2-5 cannot be applied to one-way unrestrained slabs since tensile membrane action cannot develop in this type of slab. The ultimate deflection of a unrestrained one-way slab is determined by the flexural rotation capacity of the hinging region.

For slabs restrained along two adjacent sides only, Eq. 2-5 cannot be applied, since no tensile membrane can form in either direction. In the case of a rectangular slab restrained along three sides, the slab should be treated as a one-way slab in the direction perpendicular to the two restrained sides, since tensile membrane action can form only in this direction. It is suggested that when Eq. 2-5 is not applicable the current NAVFAC P-397 criteria be used.

Often, a design has to be prepared without information being available on the steel rupture strain. To allow for this likelihood, a simplified form of Eq. 2-5, involving only the short span, L_y, as the independent variable may be used. This is given by Eq. 2-10 below. A new factor, k', is introduced in place of k in Eq. 2-5. The value of the new factor reflects, to some degree, the effect of ε_u without making it an explicit variable.

$$\delta_{ult} = k' L_y \qquad (2\text{-}10)$$

The factor k' can be thought of as a product of k and $\sqrt{\varepsilon_u}$. In determining values of k', it is recommended that values of k associated with Eq. 2-5 and values of ε_u corresponding to a probability of exceedance (relative to test values) of 90% be used. Equation 2-10 is essentially the equivalent of Eq. 2-5 with the value of ε_u assumed fixed. Recommended values of the factor k' are shown in Table 2-4. Also shown in the table are the corresponding values of k for use with Eq. 2-5. Only the most commonly encountered support conditions for rectangular slabs are shown.

Table 2-4 Recommended k values for Use in Eq. 2-10* Corresponding to Selected Slab Support Conditions

Support Conditions	Values of k**	Values of k' (Design Values)	Corresponding Support Rotation Angle, in Degrees
Two-Way Slabs			
a) Laterally Restrained	0.32	0.10	11.3
b) Laterally Unrestrained	0.38	0.12	13.5
One-Way Slabs			
a) Laterally Restrained	0.32	0.10	11.3
b) Laterally Unrestrained	Not Applicable***	-	-
Two-Way Slabs with Three Sides Laterally Restrained	Treated as One-Way Restrained Slabs, k = 0.32	0.10	11.3
Two-Way Slabs with Two Adjacent Sides Laterally Restrained	Not Applicable***	-	-

*$\delta_{ult} = k' L_y$ **$\delta_{ult} = k L_y \sqrt{\varepsilon_u}$ ***Use Existing P-397 Criteria

3. DESIGN AND CONSTRUCTION REQUIREMENTS

3.1 General Design Considerations

3.1.1 Introduction. The major parameters affecting incipient collapse deflection were identified and the appropriate relationship presented in the preceding chapter. In this chapter, design and construction requirements necessary to develop tensile membrane behavior at incipient collapse are discussed.

To develop effective tensile membrane action, design requirements beyond those associated with development of a flexural yield-line mechanism in a slab should be considered. In tensile membrane action, a slab acts essentially as a cable with cracks penetrating the entire slab thickness. The design of such slabs must carefully consider reinforcement detail to assure development of its full tensile strength.

3.1.2 Importance of Details in Structures. Details in structures have always commanded the attention of engineers concerned with unusual loads, whether the loads be unusual in terms of magnitude or character or both. Proper detailing allows a structure to survive loading not normally accounted for in design or expected only infrequently. The major objective in detailing for satisfactory performance under unusual loading is to provide adequate ductility and energy dissipation capacity.

Detailing requirements covering structures subjected to normal live loads as well as dynamic loads are described in most model building codes (17-21). The basic detailing requirements designed to provide ductility may be stated as follows:

1. All required minimum reinforcement should be continuous through joints.

2. All principal reinforcement in critical regions, particularly at joints and tension splices, should be enclosed by confinement reinforcement.

The first of these provisions stipulates continuity throughout the structure and allows for redistribution of loads. The second requirement provides for the formation of flexural "hinges" and plastic deformation. This particular requirement would normally apply to beam-column frames. Except where the special effort is justifiable, the use of confinement reinforcement (e.g., lacing, stirrups, etc.) in slabs is not common. Requirements contained in Appendix A of the ACI Building Code (17), intended for earthquake-resistant structures, embody both of the above detailing considerations.

In detailing for tensile membrane action, the basic requirements listed above must be supplemented to provide for special features associated with the tensile membrane mechanism of resistance. In the tensile membrane range, ductile performance in slabs is measured not so much in terms of the flexural rotational capacity of hinging regions near the supports as by the deflection capacity of the slab as a whole. As such, the performance of the slab becomes less dependent on such considerations as the shear strength of the critical flexural regions of the slab and is influenced more by the ductility of the reinforcement itself in tension as well as the integrity of its anchorage.

3.1.3 Development of Tensile Membrane Action. The basic requirements for development of tensile membrane action in slabs are twofold. First, the slab must possess tensile strength and extensibility beyond the snap-through range, or Point E in Fig. 2-3. Second, either the slab itself or the surrounding structure must be able to provide the necessary vertical and horizontal restraints.

Extensibility of the reinforcement depends on its breaking strain. This value may be determined from coupon tests of the reinforcing bars used. Alternatively, a reasonably conservative value may be assumed. The reinforcement must be carefully detailed so that it is effectively continuous. The results of a few tests by Taylor, Maher and Hayes (22) indicate that stopped-off and bent-up bars tend to reduce the tensile membrane capacity of slabs. Until more information is available on the effects of these design features on slab performance in the tensile membrane action range, it is recommended that stopped-off and bent-up bars be avoided.

Because cracks penetrate the entire slab thickness in the tensile membrane range, little reliance can be placed on lap splices located in the region where these cracks occur. In view of this, lap splices should be avoided in reinforcement intended to contribute to tensile membrane action. Near the periphery of the slab, when some circumferential compression may be present in the tensile membrane range, reinforcement may perhaps be spliced, provided the splices are staggered.

Concerning the necessary restraint along slab edges, Wood (23) remarked that no allowance for membrane action should be made if the supporting beams are involved in the mechanism of collapse and hence provide no restraint for development of tensile membrane action.

Anchorage lengths for bars within supports must be capable of developing the full tensile capacity of the reinforcement. Thus, anchorage of the tensile reinforcement in support regions or members providing adequate horizontal and vertical restraints is the primary requirement for the development of incipient collapse deflection as defined here. If anchorage is inadequate to develop the full tensile capacity of the bars, incomplete tensile membrane action will result.

Because slabs are relatively thin and generally do not have web reinforcement, the degree of confinement of main flexural steel bars is not as good as in deep members with web reinforcement. The need to ensure proper anchorage of slab bars is one of the reasons why beams of adequate section should be provided along discontinuous edges of slabs.

One of the simplest reinforcement arrangements is to have bars parallel to the edges but concentrated in the middle strip. Tests (22) indicate that this arrangement strengthens the tensile membrane action at the center and increases the circumferential compression in the outer regions. The reduction of reinforcement in the outer regions is not detrimental since the yield moment is raised considerably because of compression in the region.

3.1.4 End Anchorage Problems for Tensile Membrane Action. No matter how well a slab is reinforced in its clear span, collapse can occur if adequate anchorage of the reinforcement is not provided. Therefore, effectiveness of end anchorage of the reinforcement is vital to develop the tensile membrane load capacity and incipient collapse deflection of slabs.

Experimental data on incipient collapse deflection, as reported in Ref. (1), were obtained by testing small scale specimens. In these specimens, the reinforcement was either welded to stiff boundary elements or anchored in such a way as to eliminate the possibility of bar pull-out. The anchorage details used in the test specimens are not normally used in practice. Because of this, the tests on slab specimens did not provide useful information on bar anchorage details for incipient collapse.

To develop recommendations on reinforcement anchorage for slabs intended to deform well into the tensile membrane action range, a review of literature on development length for both straight and bent bar anchorages was carried out in Phase II of this investigation (2). In that report, particular importance is given to evaluation of these tests and the resulting anchorage requirements suggested by ACI Committee 408 (24).

a. Development Length of Deformed Bars in Tension. Results of a large number of tests dealing with development length, splices and hooks for reinforcing bars in tension have been reported in the literature. Development length is that length of reinforcing bar necessary to transfer the force corresponding to a specified stress level from the bar to the surrounding concrete. Parameters that may affect anchorage strength are: concrete strength, yield stress of reinforcing bars, bar diameter, position of bars, embedment length, concrete cover, bar spacing, transverse reinforcement, confinement, hook geometry, and aggregate properties, i.e., lightweight versus normal weight concrete. Confinement may take the form of a compression stress field, stirrups, hoops, ties, spirals or lacings.

Most tests on development length of reinforcing bars have been performed on either simple-span beams or cantilever overhangs. Geometrical and material properties of test specimens from selected papers (25-28) are summarized in Table 3-1.

In assessing applicability of available data on development length to slabs used in common practice, the ranges of selected parameters in the development length tests were compared with expected ranges of the same parameters in slabs (29). The particular parameters considered are concrete cover, bar spacing, and depth of concrete below bar. The comparison is shown in Table 3-1. It will be noted in Table 3-1 that the cover thickness considered in the tests represent the upper range of values normally expected in slabs whereas the rest of the parameters are representative of practical slab sizes. These test results were used as basis for the ACI 318-77 recommendations (17) and ACI Committee 408 recommendations (24).

ACI 318-77 recommendations are design provisions governing reinforced concrete that are the most commonly referred to in practice. In ACI 318-77, the minimum basic development length required for No. 11 or smaller bars is expressed as a function of bar area, bar diameter, concrete strength, and yield stress of reinforcing bars, as follows:

$$\ell_d = 0.04\ A_b f_y \sqrt{f'_c} \geq 0.0004\ d_b f_y \quad \text{(in.)} \qquad (3\text{-}1)$$

where:

A_b = cross sectional area of bar, in^2

d_b = bar diameter, in.

f'_c = concrete compressive strength, psi

f_y = yield stress of reinforcing bar, psi

Table 3-1 Geometrical and Material Properties of Development Length Test Specimens

Investigators	Bar Size d_b in.	No. of Specimens n	Concrete Compression Strength f'_c psi	Steel Yield Strength f_y ksi	Beam Width b in.	Beam Depth D in.	Concrete* Cover C_c/d_b	Effective** Depth d/d_b	Bar*** Spacing C_s/d_b	Embedment Length L/d_b
Chamberlin [27]	#4 0.500	88	3680 ~ 5870	50.0	6.0	6.0	2.0 ~ 4.5	7.0 ~ 9.5	1.0 ~ 5.0	6.0 ~ 21.3
	#6 0.750	12	4470	50.0	9.0	9.0	1.3	10.2	2.0 ~ 5.0	21.3
Mathey and Watstein [28]	#4 0.500	9	3675 ~ 4625	114.7	8.0	18.0	3.5	32.0	16.0	14.0 ~ 34.0
	#8 1.000	9	3495 ~ 4485	97.0	8.0	18.0	1.5	16.0	8.0	7.0 ~ 34.0
Ferguson and Thompson [25]	#3 0.375	8	2380 ~ 3850	82.0	6.0 ~ 10.0	4.3 ~ 5.2	1.8 ~ 4.5	6.7 ~ 11.5	16.0 ~ 26.7	20.0 ~ 40.0
	#7 0.875	60	2380 ~ 5670	87.5	12.0 ~ 18.0	6.0 ~ 11.2	0.9 ~ 3.1	3.4 ~ 10.2	13.7 ~ 20.6	18.0 ~ 48.0
	#11 1.410	8	3020 ~ 3620	86.0	18.0	9.7 ~ 11.0	1.0 ~ 1.9	4.5 ~ 6.4	12.8	32.0 ~ 48.0
Ferguson and Thompson [26]	#11 1.410	32	3000	86.0	18.0 ~ 24.0	13.0 ~ 18.0	1.1 ~ 3.5	8.2 ~ 10.9	3.8 ~ 17.0	24.0 ~ 48.0
Typical Slab [29]	#3 0.375 ~ #9 1.128					4.0 ~ 10.0	0.6 ~ 2.0	6.0 ~ 16.0	9.0 ~ 36.0	

*Thickness of concrete cover measured from extreme tension fiber to outermost surface of bar divided by bar diameter
**Distance from extreme compression fiber to centroid of tensile bar divided by bar diameter
***Center to center bar spacing or twice the distance measured from lateral concrete surface to centroid of outermost bar divided by bar diameter

The effect of position of the reinforcing bar in a section, concrete aggregate properties and confinement are considered by means of specified factors to be applied to the basic development length. The expression for basic development length, such as given by Eq. 3-1, assumes a uniform distribution of bond stress along the development length and a maximum tensile stress to be developed equal to 1.25 the yield stress of the bar.

Based on more recent work by Jirsa and associates, and other investigators (30-34), recommendations on development length, splices, and standard hooks for deformed bars in tension were proposed by ACI Committee 408. A major improvement in the ACI Committee 408 report over the ACI 318-77 expressions for development length is the inclusion of the effects of new parameters that recent tests and a reevaluation of previous tests have shown to be significant (35). Thus, the ACI Committee 408 equations take into account the effects of concrete cover, bar spacing and transverse reinforcement on the anchorage strength of bars in tension. These parameters do not appear in the current ACI Code equations.

In the recommendations, basic development length ℓ_{db} for Grade 60 deformed bars in tension is given as:

$$\ell_{db} = \frac{5500\ A_b}{\Phi k \sqrt{f'_c}} \quad \text{(in.)} \qquad (3\text{-}2)$$

where:

K = the smaller of (a) $C_c + K_{tr}$ or (b) $C_s + K_{tr}$

C_c = thickness of concrete cover measured from extreme tension fiber to cente r of bar, in.

C_s = the smaller of the cover to the center of bar measured along the line through the layer of bars, or half the center-to-center distance of bars in the layer as illustrated in Fig. 3-1.

K_{tr} = an index of the transverse reinforcement provided along the anchored bar, $A_{tr}\ f_{yt}/1500s$, in.

A_{tr} = area of transverse reinforcement crossing plane of splitting (a) parallel to the layer of bars for C_c or (b) total area of transverse reinforcement divided by n for C_s, in.^{2}

n = number of bars in layer

f_{yt} = specified yield stress of transverse reinforcement, psi

s = maximum spacing of transverse reinforcement within ℓ_d, center-to-center, in.

Φ = strength reduction factor for development length and splices = 0.8

The required development length, ℓ_d, for any particular application is computed as the product of the basic development length and applicable modification factors. Modification factors to account for effects of yield stress of reinforcement other than Grade 60, top horizontal reinforcement and lightweight aggregate are given in the ACI Committee 408 report. These effects are essentially the same as those appearing in ACI 318-77, except that the values of the factors are different.

In contrast to the unconservativeness found in the present ACI Code, the equations proposed by ACI Committee 408 provide reasonably conservative estimates of development length for large concrete covers and spacings, and for cases where transverse reinforcement length is present.

b. Bent Bar Anchorage Along Exterior Slab Edges. The development length discussed in the preceding section refers mainly to straight segments of embedded bars, whether at member ends or as they occur in lap splices.

A problem associated with anchorage of slab reinforcement in support regions, besides concrete cover and bar spacing, is the limited space available for anchorage at discontinuous edges. In interior spans of slabs, reinforcement can usually be continued through to adjacent slabs. However, along exterior edges, there may be difficulty in providing space for adequate anchorage of reinforcing bars. This is especially true when small supporting beams are provided. At discontinuous edges, the bent bar anchorage appears to be the most practical alternative if space for the required straight embedment length is not available.

Current ACI 318-77 design provisions for hooked bars in tension are a combination of special equations for hooked bars and standard development length provisions

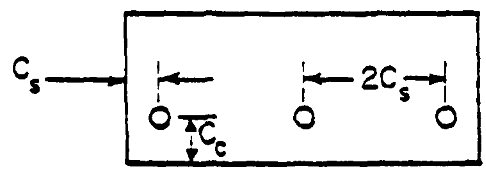

Fig. 3-1 Definition of Cover Parameters Related to Eq. 3-2

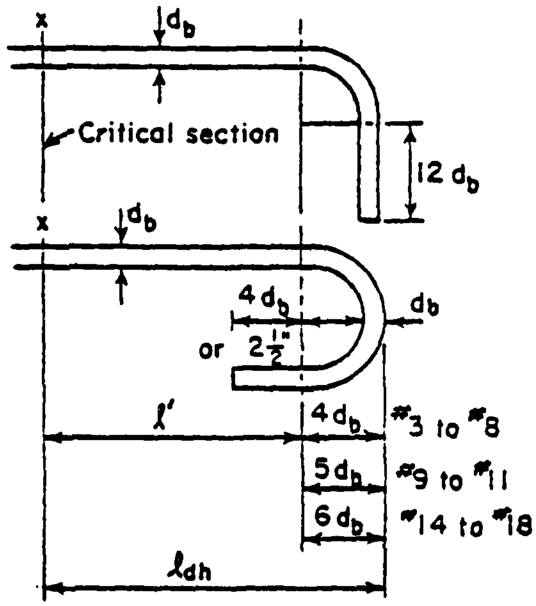

Fig. 3-2 Embedded Length of Standard Hooked Bars

as expressed in Eq. 3-1. In these provisions, the required straight bar development length between the standard hook and the critical section, ℓ', is derived by subtracting the equivalent development length for the hooked portion from the total standard development length as follows:

$$\ell' = 0.04\ A_b\ f_y/\sqrt{f'_c} - 0.04\ A_b\ f_h/\sqrt{f'_c} \quad \text{(in.)} \qquad (3\text{-}3)$$

where

f_h = tensile stress developed by the hook $= \xi\sqrt{f'_c}$

ξ = a function of bar diameter, yield stress of bars and casting position, which can be selected from a table.

Application of Eq. 3-3 is not straightforward because the adjustments in standard hook stress, f_h to account for top bars, lightweight concrete, etc., are not clearly defined.

A simple relationship between embedded length of a hooked bar and anchorage strength has been proposed by Pinc, Watkins, and Jirsa (34) based on results of their tests and previous tests (33). In the relationship, the hook and the straight lead embedment are considered as a unit, as illustrated in Fig. 3-2. Strength of a hooked bar anchorage is treated separately from that for straight bars. The proposed relationship has been incorporated into the recommendations of ACI Committee 408 (24). In the recommendations, basic development length for Grade 60 hooked reinforcement, ℓ_{hb} is given by:

$$\ell_{hb} = \frac{960\ d_b}{\Phi\sqrt{f'_c}} \quad \text{(in.)} \qquad (3\text{-}4)$$

Effects of yield stress of reinforcement other than Grade 60, confinement, and lightweight aggregate are accounted for by multiplying Eq. 3-4 by specified factors.

The study by Jirsa, et al (34) indicates that failure of a hooked bar is governed primarily by splitting of the cover parallel to the plane of the hook rather than by pulling out. Also, splitting originates at the inside of the hook where the local stress concentrations are very high. For this reason, Eq. 3-4 is a function of bar diameter, d_b, which governs the magnitude of compressive stresses on the inside of the hook. Only standard ACI hooked bars, as indicated in Fig. 3-2, were considered in tests reported in Ref. 34.

Tests by Minor and Jirsa (36) indicate that there is little difference in strength between straight and bent bar anchorages. They further concluded that in terms of reducing slip and maintaining a stiffness of the anchorage comparable to that of a straight bar, 90° hooks are preferable to 180° hooks and the radius of bend should be as large as practicable.

c. Strength Reduction Factor, Φ. In ACI 318-77 no strength reduction factor is specified in the calculation of development length. A strength reduction factor may be thought of as indirectly considered in Eq. 3-1 for basic development length because of the assumption that the reinforcement develops a stress equal to 1.25 f_y. This is in addition to the $\Phi = 0.90$ used in all flexural calculations.

Equation 3-2 for development length and Eq. 3-4 for hooked bar anchorage in the ACI Committee 408 recommendations are based on developing a stress equal to f_y in the bar. However, the Committee recommends a strength reduction factor $\Phi = 0.80$ for use with Eqs. 3-2 and 3-4. It should be noted that if Φ is viewed as a factor designed to account for variabilities in material properties and structural dimensions, then the anchorage indicated by these equations is intended to develop a maximum stress in the bar equal to f_y only.

Since, in the tensile membrane action range and prior to rupture of the reinforcement, the critical bars are subjected to the ultimate stress, i.e., to stresses generally equal to or greater than 1.25 f_y, there is reason to consider an increase in the anchorage requirements indicated by Eqs. 3-2 and 3-4.

Also, it is not clear whether Eqs. 3-2 and 3-4 based on the series of tests listed in Table 3-1 can be properly applied to cases where the concrete cover is less than those used in the tests and particularly to slabs loaded to incipient collapse. This is another reason for applying a multiplier to these expressions.

Until additional information becomes available, it is recommended that the values given by Eq. 3-2 for development length and Eq. 3-4 for hooked bar anchorage be increased by a factor of 1.2 for slabs designed for incipient collapse conditions.

In accordance with the above recommendation, Eqs. 3-2 and 3-4 are modified to read as follows:

For straight bar embedment

$$\ell_{db} = \frac{1.2 \times 5500A_b}{\Phi K \sqrt{f'_c}} \quad \text{(in.)} \qquad (3\text{-}5)$$

For hooked bar embedment

$$\ell_{hb} = \frac{1.2 \times 960d_b}{\Phi \sqrt{f'_c}} \quad \text{(in.)} \qquad (3\text{-}6)$$

3.2 Recommended Design and Construction Requirements

Based on the investigation conducted in this project, the following design requirements are recommended to ensure development of incipient collapse in slabs caused by tensile rupture of flexural reinforcement:

1. The immediate support system for the slab must be adequate to allow development of tensile membrane action. The surrounding structure or support beams should be capable of providing the necessary vertical, and where indicated, horizontal restraints.

   A minimum requirement is that support beams not be involved in the yield-line "collapse mechanism" of the slab. Also, design must ensure that shear failure does not occur at column supports.

   Beams of sufficient cross section and stiffness to develop membrane action of slabs should be provided along all discontinuous edges of a slab. Beams not only provide horizontal and vertical supports for the plastic membrane but also provide a means of anchoring the slab reinforcement.

2. Principal flexural reinforcement should be continuous throughout the spans of slabs. No cut-off or splicing of the reinforcement intended to contribute to tensile membrane action should be allowed within the span. In the tensile membrane action range, cracks in the central region of a slab penetrate the entire thickness and transfer of stress between reinforcing bars and concrete may be destroyed. In the outer, peripheral, region of a slab where circumferential compression occurs, splices may be used provided these are staggered.

The use of double (i.e., top and bottom) reinforcement is desirable from the point of view of confinement of the concrete between the two layers.

3. The amount of slab reinforcement relative to gross section should be sufficient to ensure development of the required tensile membrane strength.

4. Positive moment reinforcement should be extended and anchored in the supports by the same amount as required for negative moment reinforcement since both types of reinforcement are subjected to tension in the tensile membrane action stage.

5. Adequate anchorage of the main flexural reinforcement in boundary support elements must be provided. Such anchorage should be sufficient to develop the full tensile capacity of the reinforcement.

   Use of the expressions for development length for straight bar and hooked bar anchorages suggested by ACI Committee 408 (24), modified by a multiplicative factor of 1.2, is recommended. The current ACI 318-77 (17) requirements can be unconservative for cases where the concrete cover is small, which is typical in slabs.

6. In case of unrestrained two-way slabs, primary flexural reinforcement should be securely hooked around longitudinal bars. Adequate concrete cover should be provided for the compression ring portion.

   The use of diagonally arranged reinforcement may be considered as a means of providing improved bar anchorage for slabs that are square or nearly square in plan.

Figure 3-3 illustrated some of these design and construction requirements as they apply to an edge panel of a restrained two-way slab system. Figure 3-4 shows the corresponding requirements for unrestrained two-way slabs.

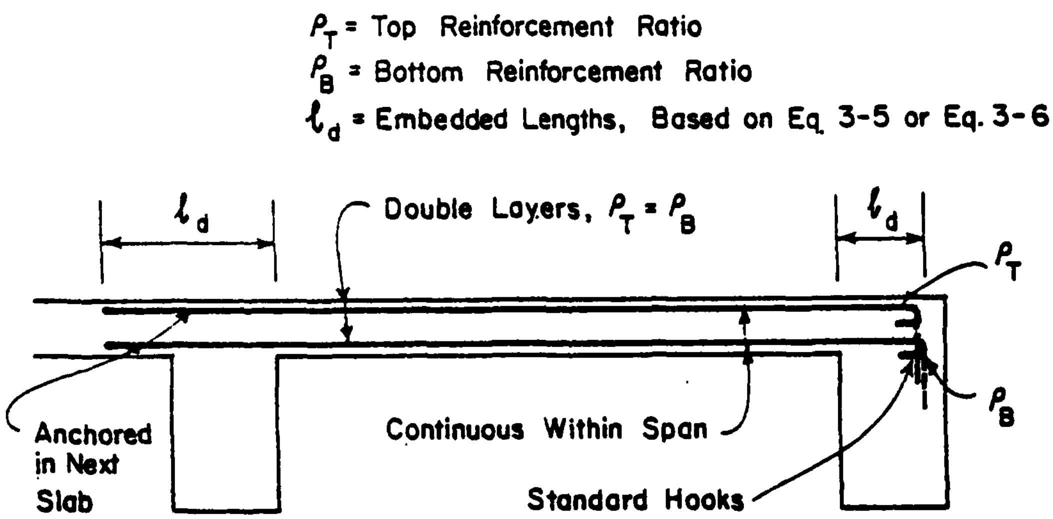

Fig. 3-3 Recommended Construction Details for Restrained Slabs

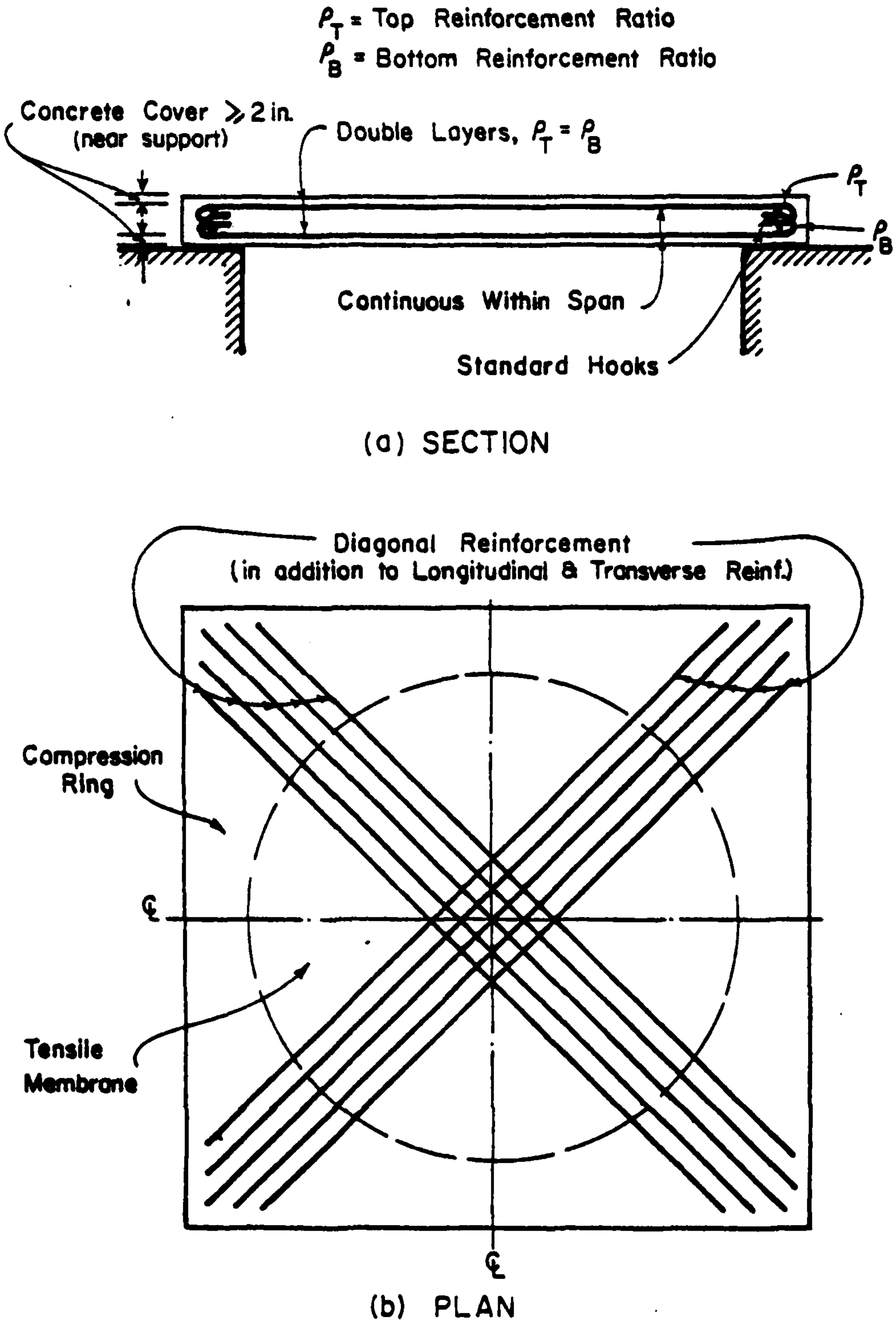

Fig. 3-4 Recommended Construction Details for Unrestrained Slabs

4. DESIGN PROCEDURE

4.1 General

The design criteria developed for incipient collapse deflection of conventionally reinforced concrete slabs are based on tests of statically loaded small-scale slab specimens. Investigators whose works were reviewed in Ref. 1 did not use special construction details, except those used in anchoring flexural reinforcement, to obtain relatively large deflections at incipient collapse. No splicing of reinforcement was used since the slab specimens were of relatively small size. Most specimens were singly reinforced and had no confinement reinforcement.

Where a slab is specifically designed to resist blast loading, additional design requirements may have to be considered. Thus, the need to prevent large gaps in the slab due to loss of concrete fragments under blast loading will require effective confinement of concrete. The degree of confinement required will vary with the intensity of the blast loading (i.e., the donor system) as well as the nature or sensitivity of the protected system (i.e., the receiver system).

4.2 Design Ranges

It is of primary interest to identify ranges where the design criteria for incipient collapse deflection of conventionally reinforced concrete slabs can be applied in terms of pressure ranges, protection categories, and structural behavior, etc., as described in NAVFAC P-397.

The ranges, in terms of different parameters, are summarized here:

4.2.1. Pressure Design Ranges. The response of a protective structure to blast pressure is classified according to the design pressure intensity range as: (a) high (b) intermediate and (c) low. These design ranges are related to the location of the protective structure with respect to the explosion.

a. High-Pressure Design Range. In this range the duration of the applied loads is short in comparison to the response time (i.e. time to reach maximum displacement) of the structural element. Structures subjected to blast effects in the high-pressure range are designed primarily for the impulse rather than for the peak pressures used with longer-duration blast pressures.

b. Intermediate-Pressure Design Range. Because the duration of the load in this pressure range is of the order of magnitude of the response time of the structure, structural elements designed for the intermediate range respond to the combined effects of both the pressure and impulse associated with the blast output.

c. Low-Pressure Design Range. Duration of blast loads acting on structures designed for this range are long in comparison to the response time of the structure. For this case, the structure is designed primarily for the peak pressure.

Design criteria developed in this investigation relate mainly to "incipient collapse deflection". Emphasis is on deflection capacity rather than strength of slabs. As such, the criteria are intended to be applied to the design of structures subjected to loading in the high-pressure range, i.e., where the load duration is short compared to the response time or natural period of vibration of the structure. For this case, the response of the structure is governed mainly by the impulse, represented by the area under the load-versus-time curve. This is approximately equal to the area under the resistance-versus-deflection curve of the structure.

Because loading in the high-pressure range can be very intense and nonuniform, provision must be made when conventionally reinforced slabs (i.e., without lacing) are used to minimize spalling and scabbing. The use of welded wire fabric to contain the concrete between the layers of flexural steel may be considered here.

4.2.2 Protection Categories. For the purpose of analysis, NAVFAC P-397 recognizes four categories of structures according to the degree of protection afforded by the facility, as follows:

a. Category 1. Protect personnel from fragments, blast pressures and excessive structural motions. This category limits the response of the structural element to the initial portion of the ductile mode. The maximum permissible support rotations for this category are assumed to be equal to, or less than, 5 degrees.

b. Category 2. Protect equipment and supplies from the same hazards as listed for Category 1. The degree of damage sustained by structures designed in this category can be greater than that of structures designed to protect personnel. Larger deflections are allowed for the design of protective structures in this category than for structures designed for Category 1.

c. Category 3. Prevent communication of detonation by fragments and high-blast pressures. The explosion must be confined to a donor cell. Controlled failure of the structural elements is allowed, thereby permitting limited post-failure fragment formation. Structures providing protection called for under this category can be designed for both ductile and brittle types of behavior.

d. Category 4. This category is similar to Category 3 except that limited communication of detonation is permitted. However, mass detonation should be prevented.

The first two protection categories apply to protective structures classified as shelters. The last two protection categories pertain to the design of barriers.

Shelters are generally enclosed and are designed to respond within the initial portion of the ductile mode with limited brittle mode behavior. Elements composing a shelter are, in most cases, restrained from displacing in the direction normal to the plane of the elements along all edges. On the other hand, barriers are generally open and are designed for a combination of ductile and brittle behavior. Consequently, barrier elements may have free edges.

It should be noted that the proposed design criteria were developed on the basis of test results on uniformly loaded slabs, either laterally restrained or unrestrained at all edges. Therefore, the design criteria for incipient collapse deflection can be considered more applicable to shelters than to barriers. Considering the large magnitude of deflections at incipient collapse, the most appropriate application of the proposed design criteria for incipient collapse deflection would be in design of structures providing protection described under Category 2.

4.2.3 Modes of Structural Behavior. The response of a structural element can be described in terms of two modes of structural behavior (3). These are (a) the ductile mode, in which the element attains large inelastic deflections without complete collapse, and (b) the brittle mode, in which partial failure or total collapse of the element occurs. The selected behavior of an element for a particular design is governed by (1) the magnitude and duration of the blast output, (2) the occurrence of primary fragments, and (3) the function of the protective structure.

The principal effects of an explosion are dynamic in nature and cause a resisting element to deflect until such time that: (1) the strain energy of the element is developed sufficiently to balance the kinetic energy produced by the applied load and the element comes to rest, or (2) fragmentation of the concrete occurs resulting in either partial or total collapse of the element.

a. Ductile Mode Behavior. Ductility of flexural reinforcement and integrity of concrete between two layers of flexural reinforcement are vital for successful ductile behavior of a slab under blast pressure. To insure ductile behavior, the slab should be reinforced symmetrically i.e., the same amount of compression and tension reinforcement should be provided.

Laced reinforcement is required for structures located inside the immediate-high blast intensity area (high-pressure design range). Because blast loads associated with the immediate high-intensity area are non-uniform, high-pressure concentrations are developed. The concentrations can produce local failure of an element. With the use of lacing, the local high shears produced by the conentrated load tends to be redistributed over the broader area.

Where lacing is not used, other means, e.g., welded wire fabric, should be used to redistribute the load and prevent local punching failure as well as minimize spalling and scabbing under high-pressure loading.

Structures loaded in the intermediate- and low-pressure ranges can be designed without lacing since deflection required to absorb the loading is considerably smaller than the deflection that can be realized with laced concrete elements. Also, in these ranges the distribution of the applied loads is fairly uniform.

Results of high-explosive structural response tests indicate that unlaced concrete elements begin to lose their structural integrity after support rotations in the order of 2 degrees are reached.

b. Brittle Mode of Behavior. Brittle behavior of reinforced concrete is characterized by two types of concrete failure; (1) spalling of concrete cover over the flexural reinforcement, and (2) post-failure fragments formed by collapse of the structure. The size of failed sections of laced elements is fixed by the regions of maximum stress in the element where hinges form, whereas the portion between the hinges usually remains intact. On the other hand, fragments in unlaced elements are usually in the form of concrete rubble.

Tests have shown that support rotations of laced elements can reach or exceed 12 degrees before failure occurs.

Ductile mode of behavior is required for slabs to reach the incipient collapse deflection stage. However, brittle mode of behavior may be observed in unlaced slabs in the process of loading toward the incipient collapse deflection. The concrete cover may be fragmented after large deflections are experienced. Where incipient collapse deflection is expected to develop in conventionally reinforced concrete slabs, debris hazard from direct spalling is unavoidable. Some means of containing potential loose fragments of concrete will have to be provided.

4.2.4 Types of Cross Section Geometry. Three types of slab cross sections may be distinguished for slabs that undergo blast loads. These depend primarily on the intensity of the loads.

a. Type I is the case where the concrete cover over the reinforcement on both surfaces of the element remains intact. This type of cross section is observed in the slabs designed for intermediate- and/or low-pressure ranges with support rotations less than 2 degrees.

b. Type II is characterized by crushing of the concrete cover over the compression reinforcement without any additional disengagement of the concrete cover. The slabs deflect within a range where support rotations are greater than 2 degrees and less than 5 degrees. Type II cross sections are encountered in slabs at the intermediate- and low-pressure design ranges.

c. Type III is the case where the concrete cover on both surfaces of the element is disengaged. The ultimate strength of slabs with Type III is no less than that of slabs with Type II. However, the overall capacity to resist the blast output is reduced due to the spalling of the concrete covers on both sides caused by either the direct transmission of the high pressures at the high-pressure range or the large deflections at the intermediate- and low-pressure ranges.

These three types of cross sections are illustrated in Fig. 4-1. All three types can be utilized for calculating the resisting moment. Futhermore, the strength of slabs can be calculated on the basis of "yield-line theory" in which the resisting moment (the moment capacity of slab cross section) is assumed along yield-lines.

Tensile membrane action developes at deflections beyond that where the flexural resistance mechanism associated with yield-line theory operates. As incipient collapse deflection is approached, cracks will have penetrated through the entire slab thickness. Therefore, none of the slab cross section types mentioned above typifies the condition of a slab in the tensile membrane range.

4.2.5 Deflection Ranges. The degree of protection required defines the maximum permissible deflection, in terms

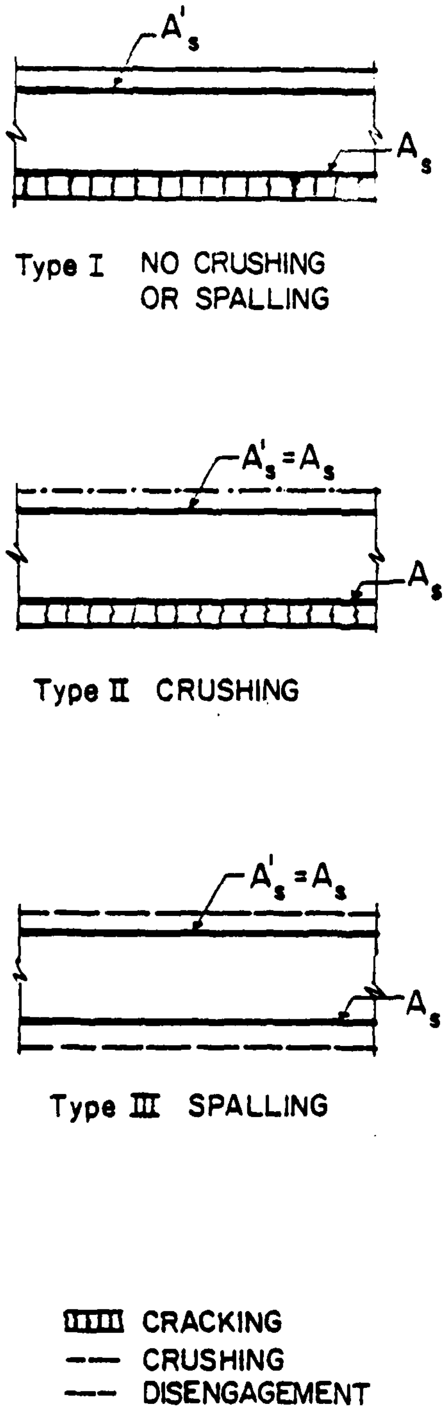

Fig. 4-1 Types of Slab Cross Section Geometry

of the support rotation angle. NAVFAC P-397 recognizes three distinct deflection ranges:

a. Limited deflection zone where maximum support rotation is less than 5 degrees.

b. Large deflection zone where maximum support rotation is larger than 5 degrees but less than the incipient failure rotation of 12 degrees.

c. Total destruction zone where maximum support rotation exceeds the incipient failure rotation (12 degrees).

It should be noted that spalled fragments are generally found in all three deflection zones except in the initial portion (less than 2 degrees) of the limited deflection zone. Post-failure fragments are present only in the total destruction zone.

Design for limited deflections covers all three of the specified design ranges, high-pressure, intermediate-pressure, and low-pressure. On the other hand, large deflection and total failure zones are used only for the high-pressure design range.

It is apparent from the nature of incipient collapse and the magnitude of maximum support rotation that the design criteria for incipient collapse deflection is applicable only to the large deflection and total destruction zones.

In summary, application of the design criteria for incipient collapse deflection of conventionally reinforced concrete slabs developed in this investigation would appear to be most appropriate for the following set of conditions and categories recognized in NAVFAC P-397:

a. Pressure design range: high-pressure range, with design primarily for impulse.

b. Protection category: Category 2, i.e., structures designed to function as shelters for equipment and supplies rather than personnel.

c. Mode of structural behavior: ductile mode, characterized by large inelastic deformations.

d. Cross section geometry: Type III with tensile cracks penetrating through thickness of slab.

e. Deflection range: large deflection to total destruction zone, with support rotations in excess of 5 degrees.

In the absence of lacing, some provision must be made to minimize debris hazards and the likelihood of sizable concrete fragments disengaging from the flexural reinforcement net under high-intensity blast loading. This may take the form of layers of welded wire fabric fastened to the flexural reinforcement.

4.3 Use of Proposed Design Criteria

Dynamic analysis is generally required for design of slabs subjected to impulse loading. Where the deformation of a structure is of primary interest, analysis based on work done and energy considerations become convenient. In the energy method, the work done by the externally applied load must be equal to the sum of the kinetic energy and the strain energy,

$$WD = KE + SE \qquad (4\text{-}1)$$

where

WD = work done by externally applied load

KE = kinetic energy of structure

SE = strain energy of structure

For practical problems, a structure is replaced by a simple concentrated mass-spring-load system to evaluate the quantities that appear in Eq. 4-1. This equivalent single-degree-of-freedom (SDF) system is a convenient approximation used in the analysis of practical slab design problems.

Solution of the governing equations of motion gives the maximum deflection of the equivalent system for a given load-versus-time relationship and specified structural properties of the system. The maximum deflection can then be compared with prescribed deflection limits, such as incipient collapse deflection, to evaluate adequacy of the assumed structural parameters.

Structural elements must be capable of developing a resistance sufficient to limit the dynamic motions to a range within the deflection capacity of the structure. The elements should be capable of sustaining the associated forces over a specified period of time, depending on the nature of applied load.

When the duration of the load is short compared to the natural period of vibration of the system, which is the case of interest here, the work done by the external load depends primarily on the area under the load- or pressure-time curve and is independent of the shape of that curve and the properties of the dynamic system. With the element at its maximum deflection, the condition of equilibrium requires that its impluse capacity be equal numerically to the impulse of the applied blast load. The impulse capacity is given by the area under the resistance-time curve corresponding to maximum deflection. In the impulse method the ultimate deflection is expressed in terms of the angle of rotation at the support or the deflection at the center of slab rather than the ductility ratio.

Thus, when a structural element responds to the impulse, the maximum response, corresponding to the area under the resistance-time curve, depends upon the area under the pressure-time curve. Typical pressure-time and resistance-time curves are shown in Fig. 4-2.

Parameters used in Fig. 4-2 are:

t_m = time at which maximum deflection occurs, ms

t_o = duration of positive phase of blast pressure, ms

t_y = time to reach yield, ms

r_u = ultimate unit resistance, psi

P_o = peak pressure, psi

i_b = unit blast impluse, psi-ms

The resistance-time curve shown in Fig. 4-2 is based on a resistance-deflection curve proposed for laterally restrained slabs which exhibit a tensile membrane stage. For two-way slabs with sufficient tensile membrane resistance, the reduction in resistance assumed in NAVFAC P-297 in the post-ultimate range, between X_1 and X_u in Fig. 4-3, does not occur. Provided that a slab has a calculated resistance at least equal to r_u at a deflection X_1*, it is considered permissible to calculate the work done on the basis of a uniform value of r_u.

Parameters used in Fig. 4-3 are:

X_1 = partial failure deflection, in.

X_u = incipient collapse (ultimate) deflection, in.

Additional resistances due to compressive and tensile membrane actions beyond the "yield-line" resistance are ignored in the proposed resistance-deflection relationship.

For a slab with a resistance-deflection function characterized by a uniform reistance, r_u, the maximum response deflection, X_m, under an implusive load is given by (3)

*or X_u, where X_u calculated from Eq. 2-10 is less than X_1 obtained from Table 5-9 of Ref. 3.

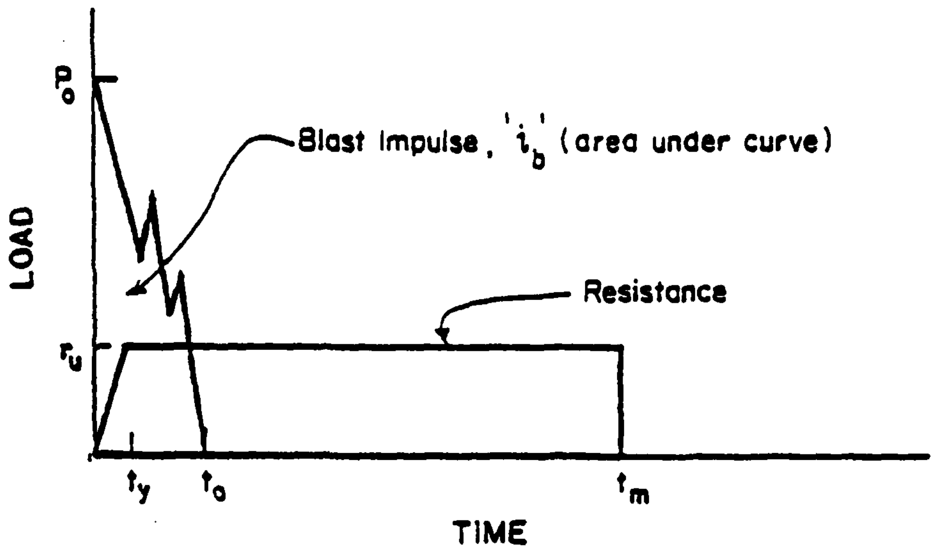

Fig. 4-2 Pressure-Time and Resistance-Time Curves for Slabs Subjected to Impulse Loading

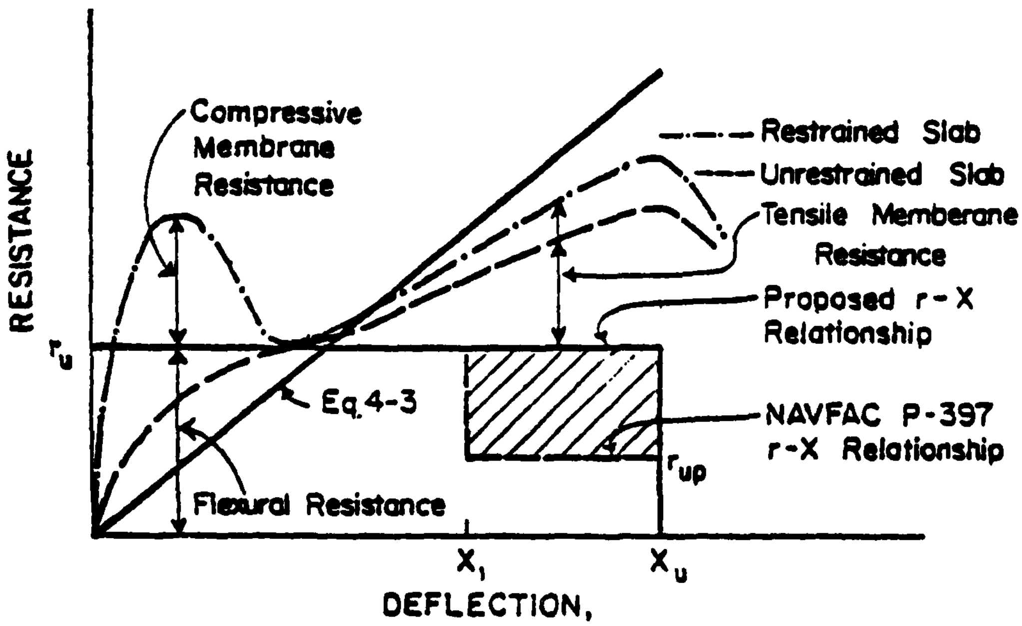

Fig. 4-3 Resistance-Deflection Curves for Two-Way Slabs Loaded to Incipient Collapse ($\theta_m > 5°$)

$$\frac{i_b^2}{2\,m_u} = r_u X_m \tag{4-2}$$

where:

m_u = effective unit mass in the ultimate range, psi-ms^2/in.

Equation 4-2 is applicable to either two-way or one-way slabs provided the associated resistance-deflection function can be defined by an essentially uniform resistance, as shown in Fig. 4-3.

In deriving Eq. 4-2, it is assumed that the unit blast impulse, i_b, is applied instantaneously at $t = 0$, and that the time to reach yield, t_y, is also close to zero.

The left side of this equation is the initial kinetic energy resulting from the applied blast impulse and the right side represents the potential energy of the element.

It should be noted that Eq. 4-2 is valid only for a design involving large deflections, i. e., with support rotations greater than 5 degrees. Also, use of the equation is limited to cases where the time to reach yield, t_y, and the duration of the impulse, t_o, are short in comparison to the response time, t_m.

The expressions for the resistance, r_u, deflections, X_1 and X_u, and effective mass, m_u are listed in Tables 5-5 and 5-6, Tables 5-8 and 5-9, and Table 6-1 respectively in NAVFAC P-397 (3).

In design, the maximum response deflection, X_m, calculated from Eq. 4-2 is compared to the ultimate deflection, X_u. If the maximum response deflection, X_m, is larger than the ultimate deflection, X_u, the asssumed member properties should be revised.

In NAVFAC P-397, the partial failure deflection, X_1, and the ultimate deflection, X_u, are based on the development of a maximum support rotation of 12 degrees prior to failure. The maximum value of 12 degrees for support rotation is specified irrespective of the geometrical or material properties of the slab. In the case of slabs designed for the particular set of conditions for which the design criteria developed in this investigation are applicable, it is recommended that the incipient collapse deflection, δ_{ult}, be used in place of the ultimate deflection X_u, in Fig. 4.3.

It should be noted that the use of δ_{ult} in place of X_u cannot be justified unless the tensile membrane resistance, r_t, at the partial failure deflection, X_1, or at the ultimate deflection, X_u, where X_u is less than X_1, is comparable to or preferably higher than the assumed uniform resistance, r_u. This is apparent in Fig. 4-3.

Tensile membrane resistance, r_t, at a deflection, X, can be approximately evaluated by using the following expressions suggested by Park (5), based on concept of plastic tensile membrane:

For two-way slabs

$$r_t = \alpha X \left[\frac{T_y}{L_y^2} \frac{\pi^3}{4 \sum\limits_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh\left(\frac{n \pi L_x}{2L_y} \sqrt{\frac{T_y}{T_x}}\right)} \right)} \right] \qquad (4\text{-}3)$$

For one-way slabs

$$r_t = X \left[\frac{8T_y}{L_y^2} \right] \qquad (4\text{-}4)$$

where:

α = modification factor based on the factor k proposed by Keenan (6); the ratio of the factor of k = 20 proposed by Keenan for a square slab to the factor of k = 13.5 based on the original equation by Park, equal to 1.5

L_x = long span of slab, in.

L_y = short span of slab, in.

T_x = yield force, per unit width, of the reinforcement in long span direction, lbs/in.

T_y = yield force, per unit width, of the reinforcement in short span direction, lbs/in.

It is recommended that the tensile membrane resistance, r_t, obtained from Eq. 4-3 or Eq. 4-4 at X_1, or at X_u where X_u is less than X_1, be equal to or larger than the assumed uniform ultimate resistance, r_u. For some cases X_u becomes less than X_1. The reason for this is that X_1 is determined from Tables 5-8 and 5-9 of NAVFAC P-397, where X_1 is based on the development of a specified value of rotation, namely, 12 degrees, whereas X_u is independently determined from Eq. 2-5 in this report.

4.4 Steps in Design

A procedure for applying the proposed design criteria for incipient collapse deflection to the design of slabs subjected to an impulsive load is presented in this section. A design for large deflections is assumed. The procedure consists of several steps. Each step is described below in the order to be followed in the calculation.

Step 1 Establish design parameters, such as impulse load applied, geometry of a slab, support conditions, material properties, dynamic increase factors and so forth.

Step 2 Assume reinforcement ratios in both directions of slab.

Step 3 Select an appropriate value of the factor k' from Table 2-4, according to support conditions of the slab.

Step 4 Calculate the incipient collapse deflection, δ_{ult}, from Eq. 2-10. The calculated value of δ_{ult} should be used as the ultimate deflection, X_u, instead of the values listed in Tables 5-8 and 5-9 of NAVFAC P-397.

Step 5 Calculate the partial failure deflection, X_1, for two-way slabs from Table 5-9 of NAVFAC P-397.

Step 6 Ensure that assumed reinforcement ratios provide enough tensile membrane strength. To do this, calculate the tensile membrane resistance, r_t, corresponding to the partial failure deflection, X_1, or that corresponding to the ultimate deflection, X_u, where X_u is less than X_1, from Eq. 4-3 for two-way slabs or from Eq. 4-4 for one-way slabs.

Calculate the ultimate unit resistance, r_u, of the slab from equations listed in Tables 5-5 and 5-6 of NAVFAC P-397. If r_t is less than r_u, repeat Step 2 through Step 6 until the condition that r_t is larger than r_u is satisfied.

Step 7 Evaluate the effective unit mass of the slab in the ultimate resistance range from Fig. 6-5 (for two-way slabs) or Table 6-1 (for one-way slabs) of NAVFAC P-397.

Step 8 Given a blast impluse, i_b, calculate the maximum response deflection, X_m, by solving Eq. 4-2 for X_m. Parameters to be used in this equation have been calculated in the foregoing steps, such as the ultimate deflection, X_u, ($= \delta_{ult}$), in Step 4, the ultimate unit resistance, r_u, in Step 6, and the effective unit mass in the ultimate range, m_u, in Step 7. If the calculated value of X_m is larger than the specified ultimate deflection, X_u, repeat Step 2 through Step 8 until X_m becomes less than X_u.

After completing Step 8, the slab can be considered to be safely designed for the incipient collapse condition when subjected to the specified impulsive load. In addition, the slab is assured of having sufficient reinforcement to exhibit ductile performance in the post-ultimate range, that is, in the tensile membrane action range.

4.5 Illustrative Examples

Designs of conventionally reinforced concrete slabs subjected to impulsive loads are worked out to demonstrate the application of the proposed design criteria. The design process follows the procedure described in the preceding section.

Problem 1:

Design a restrained two-way slab subjected to an impulsive load for incipient collapse conditions. Relevant data are given below. The slab is illustrated in Fig. 4-4.

Step 1 Design parameters:

a. Short span length $L_y = 180$ in.

b. Long span length $L_x = 240$ in.

c. Slab thickness $T_c = 12$ in.

d. Slab is fixed at all four edges.

e. Impulse load $i_b = 900$ psi-ms.

f. Material properties,

Reinforcement $f_y = 60{,}000$ psi
$f_u^y = 90{,}000$ psi

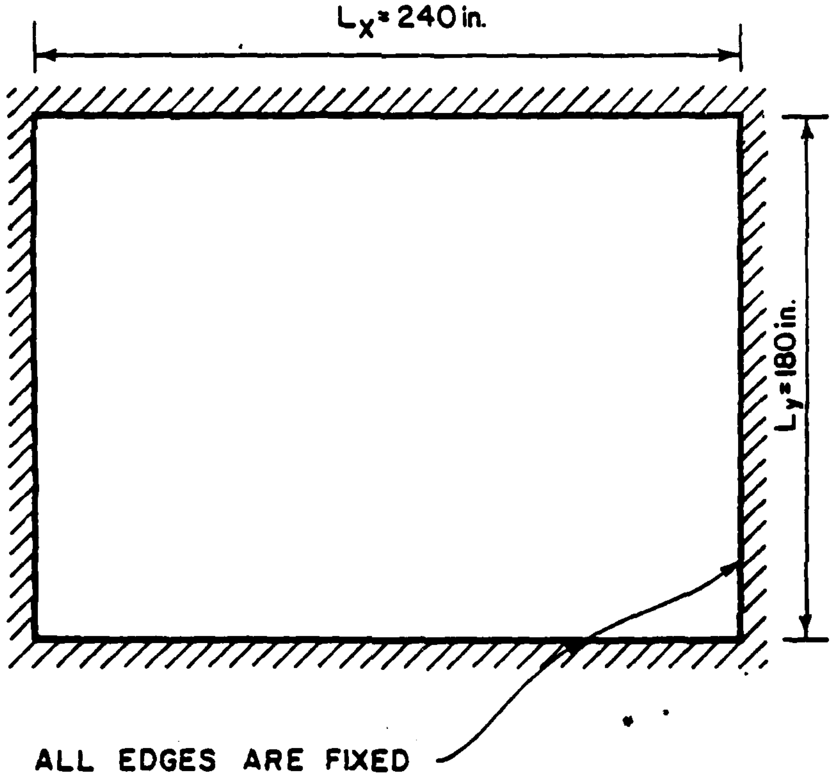

Fig. 4-4 Plan View of the Slab for Problem 1

Concrete f'_c = 4,000 psi

g. Dynamic increase factor for reinforcement, DIF = 1.20 (from Table 5-3 of NAVFAC P-397)

h. Static design stress for reinforcement, (Paragraph 5-6 of NAVFAC P-397)

$$f_s = \frac{f_y + f_u}{2} = \frac{60{,}000 + 90{,}000}{2}$$

$$= 75{,}000 \text{ psi}$$

i. Dynamic design stress for reinforcement,

$$f_{ds} = (DIF)\ f_s = 1.20 \times 75{,}000$$

$$= 90{,}000 \text{ psi}$$

j. Equal amount of reinforcement is assumed for top and bottom layers.

<u>Step 2</u> Assume reinforcement ratios.

Reinforcement ratio in short span on each face,

$$P_y = 0.005$$

Reinforcement ratio in long span on each face,

$$P_x = 0.003$$

Noted that an optimum value of P_y/P_x can be approximately evaluated from either Fig. 6-17 or Fig. 6-18 of NAVFAC P-397.

<u>Step 3</u> Select an appropriate value of the factor k' from Table 2-4.

For a restrained two-way slab, k' = 0.10

<u>Step 4</u> Calculate the incipient collapse deflection, δ_{ult}, from Eq. 2-10. This value of δ_{ult} is used in place of the ultimate deflection, X_u, in NAVFAC P-397.

$$X_u = \delta_{ult} = k'\ L_y$$

$$= 0.10 \times 180 = 18.0 \text{ in.}$$

Step 5 Calculate the partial failure deflection, X_1, from Table 5-9 of NAVFAC P-397.

From Fig. 5-11 (NAVFAC P-397),

$$x = 0.35L_x = 0.35 \times 240 = 84 \text{ in.}$$

$$X_1 = x \tan 12^\circ = 84 \tan 12^\circ$$

$$= 17.9 \text{ in.} < X_u = 18.0 \text{ in.}$$

Step 6 Check approximate tensile membrane strength provided by reinforcement ratio assumed in Step 2.

a. Calculate the tensile membrane resistance corresponding to $X = X_1$ from Eq. 4-3.

$$r_t = \alpha X \left[\frac{T_y}{L_y^2} \frac{\pi^3}{4 \sum\limits_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh\left(\frac{n \pi L_x}{2L_y} \sqrt{\frac{T_y}{T_x}} \right)} \right)} \right]$$

$$= 1.5 \times 17.9 \left[\frac{8{,}640}{180^2} \frac{\pi^3}{4 \sum\limits_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh\left(\frac{n \pi\, 240}{2 \times 180} \sqrt{\frac{8{,}640}{5{,}184}} \right)} \right)} \right]$$

$$= 26.85 \times 0.2667 \times 31.01/(4 \times 0.8377) = 66.3 \text{ psi}$$

where;

T_x and T_y = yield forces, per unit width, of the reinforcement.

Short span direction,

$$T_y = 2\, T_c\, P_y\, f_y\, (\text{DIF})$$

$$= 2 \times 12 \times 0.005 \times 60{,}000 \times 1.2$$

$$= 8{,}640 \text{ lbs/in.}$$

Long span direction,

$$T_x = 2\, T_c\, P_x\, f_y\, (\text{DIF})$$

$= 2 \times 12 \times 0.003 \times 60{,}000 \times 1.2$

$= 5{,}184$ lbs/in.

b. Calculate the ultimate unit resistance, r_u, from Table 5-6 of NAVFAC P-397.

Distance between the centroids of top and bottom reinforcement layers,

$$d_c = 7.5 \text{ in.}$$

Ultimate moment in short span direction,

$$M_y = \frac{A_s\, f_{ds}\, d_c}{b} \quad \text{(Eq. 5-7 of NAVFAC P-397)}$$

$$= 0.005 \times 12 \times 1 \times 90{,}000 \times 7.5$$

$$= 40{,}500 \text{ in-lbs/in.}$$

Ultimate moment in long span direction,

$$M_x = 0.003 \times 12 \times 1 \times 90{,}000 \times 7.5$$

$$= 24{,}300 \text{ in-lbs/in.}$$

Determine yield line location from Fig. 5-11 of NAVFAC P-397.

$$\frac{L_x}{L_y}\left(\frac{M_y}{M_x}\right)^{1/2} = \frac{240}{180}\left(\frac{40{,}500}{24{,}300}\right)^{1/2} = 1.72$$

From Fig. 5-11 (NAVFAC P-397),

$$x = 0.35L_x = 0.35 \times 240 = 84.0 \text{ in.}$$

Considering

$$x = 84.0 \text{ in.} < \frac{L_x}{2} = \frac{240}{2} = 120.0 \text{ in.}$$

the ultimate unit resistance, r_u, is obtained using the following expression from Table 5-6 (NAVFAC P-397).

$$r_u = \frac{5x2xM_x}{x^2} = \frac{10x24,300}{84^2} = 34.4 \text{ psi}$$

c. Compare r_t with r_u.

$$r_t = 66.3 \text{ psi} > r_u = 34.4 \text{ psi}$$

The condition stated in Step 6 is satisfied.

Step 7. Evaluate the effective unit mass of the slab for the ultimate range from Fig. 6-5 of NAVFAC P-397.

a. Calculate the unit mass of the slab.

$$m = d_c w/g = d_c \left(\frac{150}{12^3}\right) x \frac{10^6}{386}$$

$$= 225d_c = 225 \times 7.5$$

$$= 1,688 \text{ psi-ms}^2\text{/in.}$$

where

w = weight density of concrete, lbs/in.^{3}

g = gravity acceleration, in./ms^2

b. Calculate the effective unit mass for the ultimate range using a load-mass factor from Fig. 6-5 (NAVFAC P-397).

$$\frac{x}{L_x} = \frac{84}{240} = 0.35$$

From Fig. 6-5 (NAVFAC P-397), the load-mass factor is

$$K_{LM} = 0.56$$

The effective unit mass for the ultimate range is calculated as follows:

$$m_u = K_{LM}\, m = 0.56 \times 1,688$$

$$= 945 \text{ psi-ms}^2\text{/in.}$$

Step 8 Calculate the maximum response deflection, X_m, by solving Eq. 4-2 for X_m and compare X_m with X_u.

$$X_m = \frac{i_b^2}{2m_u r_u} = \frac{900^2}{2 \times 945 \times 34.4} = 12.5 \text{ in.}$$

Compare X_m with X_u.

$$X_m = 12.5 \text{ in.} < X_u = 18.0 \text{ in.}$$

The condition stated in Step 8 is satisfied.

Problem 2:

Design a unrestrained two-way slab subjected to an impulsive load for incipient collapse conditions. Relevant data are given below. The slab is illustrated in Fig. 4-5.

Step 1 Design parameters:

a. Short span length L_y = 180 in.

b. Long span length L_x = 240 in.

c. Slab thickness T_c = 12 in.

d. Slab is simply-supported and laterally unrestrained at all four edges.

e. Impulse load i_b = 800 psi-ms.

f. Material properties,

Reinforcement f_y = 60,000 psi
f_u = 90,000 psi

Concrete f'_c = 4,000 psi

g. Dynamic increase factor for reinforcement, DIF = 1.20 (from Table 5-3 of NAVFAC P-397)

h. Static design stress for reinforcement, (Paragraph 5-6 of NAVFAC P-397)

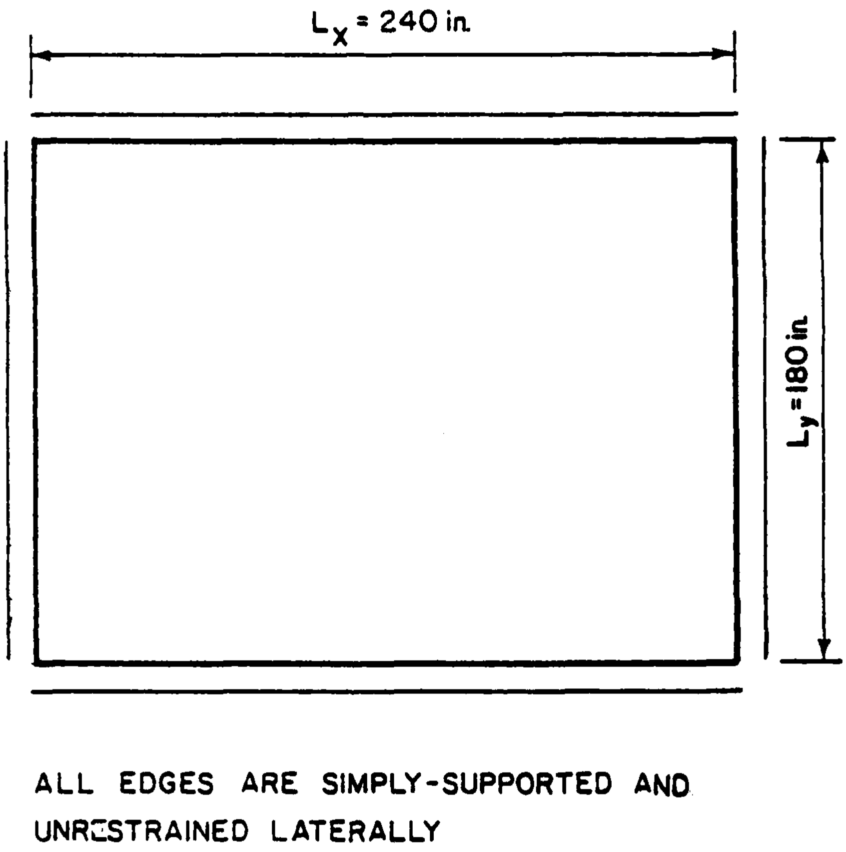

Fig. 4-5 Plan View of the Slab for Problem 2

$$f_s = \frac{f_y + f_u}{2} = \frac{60{,}000 + 90{,}000}{2}$$

$$= 75{,}000 \text{ psi}$$

i. Dynamic design stress for reinforcement,

$$f_{ds} = (\text{DIF})\ f_s = 1.20 \times 75{,}000$$

$$= 90{,}000 \text{ psi}$$

j. Equal amount of reinforcement is assumed for top and bottom layers (double layered slab).

<u>Step 2</u> Assume reinforcement ratios.

Reinforcement ratio in short span on each face,

$$p_y = 0.005$$

Reinforcement ratio in long span on each face,

$$p_x = 0.003$$

<u>Step 3</u> Select an appropriate value of the factor k' from Table 2-4.

For a unrestrained two-way slab, k' = 0.12

<u>Step 4</u> Calculate the incipient collapse deflection, δ_{ult}, from Eq. 2-10. This value of δ_{ult} is used in place of the ultimate deflection, X_u, in NAVFAC P-397.

$$X_u = \delta_{ult} = k'\ L_y$$

$$= 0.12 \times 180 = 21.6 \text{ in.}$$

<u>Step 5</u> Calculate the partial failure deflection, X_1, from Table 5-9 of NAVFAC P-397.

From Fig. 5-11 (NAVFAC P-397),

$$x = 0.35L_x = 0.35 \times 240 = 84 \text{ in.}$$

$$X_1 = x \tan 12^\circ = 84 \tan 12^\circ$$

$$= 17.9 \text{ in.} < X_u = 21.6 \text{ in.}$$

Step 6 Check approximate tensile membrane strength provided by reinforcement ratio assumed in Step 2.

a. Calculate the tensile membrane resistance corresponding to $X = X_1$ from Eq. 4-3.

$$r_t = \alpha X \left[\frac{T_y}{L_y^2} \frac{\pi^3}{4 \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh\left(\frac{n \pi L_x}{2L_y} \sqrt{\frac{T_y}{T_x}} \right)} \right)} \right]$$

$$= 1.5 \times 17.9 \left[\frac{8,640}{180^2} \frac{\pi^3}{4 \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh\left(\frac{n \pi 240}{2 \times 180} \sqrt{\frac{8,640}{5,184}} \right)} \right)} \right]$$

$$= 26.85 \times 0.2667 \times 31.01/(4 \times 0.8377) = 66.3 \text{ psi}$$

where;

T_x and T_y = yield forces, per unit width, of the reinforcement.

Short span direction,

$$T_y = 2\ T_c\ P_y\ f_y\ (DIF)$$
$$= 2 \times 12 \times 0.005 \times 60,000 \times 1.2$$
$$= 8,640 \text{ lbs/in.}$$

Long span direction,

$$T_x = 2\ T_c\ P_x\ f_y\ (DIF)$$
$$= 2 \times 12 \times 0.003 \times 60,000 \times 1.2$$
$$= 5,184 \text{ lbs/in.}$$

b. Calculate the ultimate unit resistance, r_u, from Table 5-6 of NAVFAC P-397.

Distance between the centroids of top and bottom reinforcement layers,

d_c = 7.5 in.

Ultimate moment in short span direction,

$$M_y = \frac{A_s \, f_{ds} \, d_c}{b} \quad \text{(Eq. 5-7 of NAVFAC P-397)}$$

$$= 0.005 \times 12 \times 1 \times 90{,}000 \times 7.5$$

$$= 40{,}500 \text{ in-lbs/in.}$$

Ultimate moment in long span direction,

$$M_x = 0.003 \times 12 \times 1 \times 90{,}000 \times 7.5$$

$$= 24{,}300 \text{ in-lbs/in.}$$

Determine yield line location from Fig. 5-11 of NAVFAC P-397.

$$\frac{L_x}{L_y}\left(\frac{M_y}{M_x}\right)^{\frac{1}{2}} = \frac{240}{180}\left(\frac{40{,}500}{24{,}300}\right)^{\frac{1}{2}} = 1.72$$

From Fig. 5-11 (NAVFAC P-397),

$$x = 0.35L_x = 0.35 \times 240 = 84.0 \text{ in.}$$

Considering

$$x = 84.0 \text{ in.} < \frac{L_x}{2} = \frac{240}{2} = 120.0 \text{ in.}$$

the ultimate unit resistance, r_u, is obtained using the following expression from Table 5-6 (NAVFAC P-397).

$$r_u = \frac{5M_x}{x^2} = \frac{5\text{x}24{,}300}{84^2} = 17.2 \text{ psi}$$

c. Compare r_t with r_u.

$$r_t = 66.3 \text{ psi} > r_u = 17.2 \text{ psi}$$

The condition stated in Step 6 is satisfied.

Step 7. Evaluate the effective unit mass of the slab for the ultimate range from Fig. 6-5 of NAVFAC P-397.

a. Calculate the unit mass of the slab.

$$m = d_c w/g = d_c \left(\frac{150}{12^3}\right) \times \frac{10^6}{386}$$

$$= 225 d_c = 225 \times 7.5$$

$$= 1{,}688 \text{ psi-ms}^2/\text{in.}$$

where

w = weight density of concrete, lbs/in.^{3}

g = gravity acceleration, in./ms^2

b. Calculate the effective unit mass for the ultimate range using a load-mass factor from Fig. 6-5 (NAVFAC P-397).

$$\frac{x}{L_x} = \frac{84}{240} = 0.35$$

From Fig. 6-5 (NAVFAC P-397), the load-mass factor is

$$K_{LM} = 0.56$$

The effective unit mass for the ultimate range is calculated as follows:

$$m_u = K_{LM}\, m = 0.56 \times 1{,}688$$

$$= 945 \text{ psi-ms}^2/\text{in.}$$

Step 8 Calculate the maximum response deflection, X_m, by solving Eq. 4-2 for X_m and compare X_m with X_u.

$$X_m = \frac{i_b^{\,2}}{2 m_u\, r_u} = \frac{800^2}{2 \times 945 \times 17.2} = 19.7 \text{ in.}$$

Compare X_m with X_u.

$$X_m = 19.7 \text{ in.} < X_u = 21.6 \text{ in.}$$

The condition stated in Step 8 is satisfied.

5. SUMMARY

This report summarizes results of the investigation conducted in the first two phases of the project.

Design criteria based on incipient collapse deflection of conventionally reinforced concrete one-way and two-way slabs under uniform static load are developed. Existing test data are used as the basis for these criteria. An expression for deflection capacity associated with incipient collapse is proposed in Chapter 2. The expression was derived on the assumption that a representative slab strip takes the form of a parabolic cable in the tensile membrane range. A factor, k, is introduced to account for the difference between deflection associated with pure cable action and observed slab behavior. Appropriate values of k for different slab support conditions are determined based on a consistent probability that the predicted value will be exceeded. For design purposes, a new factor, k', which reflects the combined effect of k and ε_u, is recommended for use. Proposed values of k' are presented in Table 2-4.

Design requirements and construction details necessary to develop tensile membrane capacity of a slab are recommended in Chapter 3. Basic requirements for development of tensile membrane action include sufficient reinforcement to provide the required tensile strength of slabs, continuity of reinforcement through support regions, and adequate vertical and horizontal restraints. Of particular importance is the need for adequate anchorage in support regions. Use of expressions for development length for straight bar and hooked bar anchorages suggested by ACI Committee 408 (24), multiplied by a factor of 1.2, is recommended.

A design procedure for slabs loaded to incipient collapse is recommended in Chapter 4. Design ranges where the design criteria for incipient collapse deflection can be most appropriately applied are identified. It is noted that the criteria appear to be most appropriate for the design of slabs subjected to impulsive loading. A procedure for design of slabs subjected to impulsive loading is summarized in Section 4.4 in a step-by-step fashion. Design examples are given to illustrate the use of the proposed criteria in practical problems.

ACKNOWLEDGMENT

This work was undertaken as part of the activities of the Structural Analytical Section, Engineering Development Division, Construction Technology Laboratories of Portland Cement Association.

The draft of this report was reviewed and valuable suggestions were provided by Dr. W. G. Corley, Divisional Director, Engineering Development Division.

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APPENDIX

PROPOSED SUPPLEMENT TO NAVFAC P-397

APPENDIX - PROPOSED SUPPLEMENT TO NAVFAC P-397
Design Criteria for Deflection Capacity of Conventionally Reinforced Concrete Slabs

Background

The existing NAVFAC P-397 does not take into account the influence of slab geometry, section properties, boundary conditions, and material properties on incipient collapse deflection of reinforced concrete slabs. Since the requirements on deflection capacity presently found in NAVFAC P-397 may be too conservative for certain cases, there is a need to update design criteria for the incipient collapse deflection of conventionally reinforced concrete slabs.

The use of yield-line theory for calculating the collapse load of reinforced concrete slabs is prescribed in NAVFAC P-397. Yield-line theory is based on the pure moment capacity of the slab cross section and does not take into account in-plane forces. The presence of in-plane forces results in an increase in the ultimate resistance as well as the deflection capacity. Prior to incipient collapse the slabs carry load as a tensile membrane. Incipient collapse for conventionally reinforced concrete slabs is defined here as that state of a slab characterized by a drop in the load capacity following mobilization of tensile membrane action. The collapse condition is associated with tensile rupture of the flexural reinforcement.

Formation of tensile membrane at the center portion of slabs requires existence of a compressive ring capable of resisting radial tension. Compression is often taken by edge beams for restrained slabs, but if no lateral restraint is provided (as for unrestrained slabs), a slab will develop a compressive ring along its periphery.

In this supplement, design criteria for the incipient collapse deflection of conventionally reinforced concrete one-way and two-way slabs with laterally and rotationally restrained and unrestrained edges are presented based on the mobilization of tensile membrane action. A uniformly distributed applied load is assumed. Also, reinforcement details necessary to develop the tensile membrane capacity of slabs are recommended. These criteria may be considered as an alternative method for estimating slab deflection at incipient collapse.

Failure Deflection Criteria

Based on a review of experimental and analytical work on slabs, major parameters affecting the deflection capacity of slabs were found to be the short span of slab and reinforcement rupture strain, and edge restraint conditions.

An expression for predicting incipient collapse deflection of slabs is proposed, based on the assumption that a representative slab strip takes the form of a parabolic cable in the tensile membrane range. Probability concepts associated with test results on ultimate deflection and reinforcement rupture strain are introduced in an evaluation of the proposed equation. The proposed expression for the incipient collapse deflection of conventionally reinforced concrete slabs under uniform load is given by

$$\delta_{ult} = k\, L_y \sqrt{\varepsilon_u} \qquad \text{(A1)}$$

where

L_y = short span of slab

ε_u = rupture strain of flexural reinforcement

k = a factor to mainly account for the distribution of strain in the reinforcing bars.

The factor k is introduced to account for the disparity between the deflection associated with pure cable action and observed behavior. A major difference lies in the nonuniform distribution of strains along the length of the reinforcement in a cracked slab.

Often, a design has to be prepared without information being available on the reinforcement rupture strain. To allow for this likelihood, a simplified form of Eq. A1, involving only the short span, L_y, as the independent variable may be used. This is given by Eq. A2 below. A new factor, k', is introduced in place of k in Eq. A1. The value of the new factor reflects, to some degree, the effect of ε_u without making it an explicit variable. The factor k' can be thought of as a product of k and $\sqrt{\varepsilon_u}$.

$$\delta_{ult} = k'\, L_y \qquad \text{(A2)}$$

Equation A2, instead of Eq. A1, is recommended for use for design purposes.

Values of the factors k' and k for different boundary conditions have been determined by treating available experimental data within a probabilistic framework. Recommended values of the factor k' for use with Eq. A2 are listed in Table A1 along with values of k for use with Eq. A1. Only the most commonly encountered support conditions for rectangular slabs are shown.

Neither Eq. A1 nor Eq. A2 can be applied to one-way unrestrained slabs since no tensile membrane action develops in this type of

Table A1 Recommended k' values for Use in Eq. A2* Corresponding to Selected Slab Support Conditions

Support Conditions	Values of k**	Values of k' (Design Values)	Corresponding Support Rotation Angle, in Degrees
Two-Way Slabs			
a) Laterally Restrained	0.32	0.10	11.3
b) Laterally Unrestrained	0.38	0.12	13.5
One-Way Slabs			
a) Laterally Restrained	0.32	0.10	11.3
b) Laterally Unrestrained	Not Applicable***	-	-
Two-Way Slabs with Three Sides Laterally Restrained	Treated as One-Way Restrained Slabs, $k = 0.32$	0.10	11.3
Two-Way Slabs with Two Adjacent Sides Laterally Restrained	Not Applicable***	-	-

*$\delta_{ult} = k' L_y$ **$\delta_{ult} = k L_y \sqrt{\varepsilon_u}$ ***Use Existing P-397 Criteria

slab. Also, for slabs restrained along two adjacent sides only, Eqs. A1 and A2 are not applicable as no tensile membrane can form in either direction. It is suggested that when Eq. A1 and A2 are not applicable the current NAVFAC P-397 criteria be used. In the case of a rectangular slab restrained along three sides only, the slab should be treated as a one-way restrained slab in the direction perpendicular to the restrained edges since tensile membrane action can form only in this direction.

Design and Construction Requirements

To develop effective tensile membrane action, design requirements beyond those associated with development of a flexural yield-line mechanism in a slab should be considered. In view of this, the following design requirements are recommended to ensure development of incipient collapse in slabs caused by tensile rupture of flexural reinforcement:

a. The immediate support system for the slab must be adequate to allow development of tensile membrane action. The surrounding structure or support beams should be capable of providing the necessary vertical, and where indicated, horizontal restraints.

   A minimum requirement is that support beams not be involved in the yield-line "collapse mechanism" of the slab. Also, design must ensure that shear failure does not occur at column supports.

b. Principal flexural reinforcement should be continuous throughout the spans of slabs. No cut-off or splicing of the reinforcement intended to contribute to tensile membrane action should be allowed within the span. In the tensile membrane action range, cracks in the central region of a slab penetrate the entire thickness and transfer of stress between reinforcing bars and concrete may be destroyed. In the outer, peripheral, region of a slab where circumferential compression occurs, splices may be used provided these are staggered.

   The use of double (i.e., top and bottom) reinforcement is desirable from the point of view of confinement of the concrete between the two layers.

c. The amount of slab reinforcement relative to gross section should be sufficient to ensure development of the required tensile membrane strength.

d. Positive moment reinforcement should be extended and anchored in the supports by the same amount as required for negative moment reinforcement since both types of reinforcement are subjected to tension in the tensile membrane action stage.

e. Adequate anchorage of the main flexural reinforcement in boundary support elements must be provided. Such anchorage should be sufficient to develop the full tensile strength of the reinforcement.

   Use of the expressions for development length for straight bar and hooked bar anchorages suggested by ACI Committee 408*, modified by a multiplicative factor of 1.2, is recommended.

f. In case of unrestrained slabs, primary flexural reinforcement should be securely hooked around longitudinal bars. Adequate concrete cover should be provided for the compression ring portion.

   The use of diagonally arranged reinforcement may be considered as a means of providing improved bar anchorage for slabs that are square or nearly square in plan.

Design Ranges

It is of primary interest to identify ranges where the design criteria for incipient collapse deflection of conventionally reinforced concrete slabs can be applied in terms of pressure ranges, protection categories, and structural behavior, etc., as described in NAVFAC P-397.

Based on an evaluation of the conditions associated with each design consideration, it is believed that the design criteria for incipient collapse deflection presented here would be most appropriate for the following set of conditions and categories recognized in NAVFAC P-397:

a. Pressure design range: high-pressure range, with design primarily for impulse.

b. Protection category: Category 2, i.e., structures designed to function as shelters for equipment and supplies rather than personnel.

c. Mode of structural behavior: ductile mode, characterized by large inelastic deformations.

d. Cross section geometry: Type III with tensile cracks penetrating through thickness of slab.

e. Deflection range: large deflection to total destruction zone, with support rotations in excess of 5 degrees.

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*Reference No. 16 in Reference List

Design Procedure

Dynamic analysis is generally required for design of slabs subjected to impulse loading. Where the deformation of a structure is of primary interest, analysis based on work done and energy considerations become convenient.

Solution of the governing equations of motion gives the maximum deflection of the equivalent system for a given load-versus-time relationship and specified structural properties of the system. The maximum deflection can then be compared with prescribed deflection limits, such as incipient collapse deflection, to evaluate adequacy of the assumed structural parameters.

Structural elements must be capable of developing a resistance sufficient to limit the dynamic motions to a range within the deflection capacity of the structure. Such elements should be capable of sustaining the associated forces over a specified period of time, depending on the nature of applied load.

When the duration of the load is short compared to the natural period of vibration of the system, which is the case of interest here, the work done by the external load depends primarily on the area under the load- or pressure-time curve. It is independent of the shape of that curve and the properties of the dynamic system. With the element at its maximum deflection, the condition of equilibrium requires that its impluse capacity be equal numerically to the impulse of the applied blast load. The impulse capacity is given by the area under the resistance-time curve corresponding to maximum deflection. In the impulse method, the ultimate deflection is expressed in terms of the angle of rotation at the support or the deflection at the center of slab rather than the ductility ratio.

Thus, when a structural element responds to an impulse, the maximum response, corresponding to the area under the resistance-time curve, depends upon the area under the pressure-time curve. Typical pressure-time and resistance-time curves are shown in Fig. A1.

Parameters used in Fig. A1 are:

t_m = time at which maximum deflection occurs, ms

t_o = duration of positive phase of blast pressure, ms

t_y = time to reach yield, ms

r_u = ultimate unit resistance, psi

P_o = peak pressure, psi

i_b = unit blast impluse, psi-ms

The resistance-time curve shown in Fig. A1 is based on a resistance-deflection curve proposed for laterally restrained slabs which exhibit a tensile membrane stage. For two-way slabs with sufficient tensile membrane resistance, the reduction in resistance assumed in NAVFAC P-297 in the post-ultimate range, between X_1 and X_u in Fig. A2, does not occur. Provided that a slab has a calculated resistance at least equal to r_u at a deflection X_1*, the work done may be calculated on the basis of a uniform value of r_u.

Parameters used in Fig. A2 are:

X_1 = partial failure deflection, in.

X_u = incipient collapse (ultimate) deflection, in.

Additional resistances due to compressive and tensile membrane actions beyond the "yield-line" resistance (Johansen load) are ignored in the proposed resistance-deflection relationship.

For a slab with a resistance-deflection function characterized by a uniform resistance, r_u, the maximum response deflection, X_m, under an implusive load is given by

$$\frac{i_b^{\,2}}{2\,m_u} = r_u X_m \qquad \text{A3(a)}$$

from which

$$X_m = \frac{i_b^{\,2}}{2 m_u r_u} \qquad \text{A3(b)}$$

where

m_u = effective unit mass in the ultimate range, psi-ms^2/in.

Equation A3 is applicable to either two-way or one-way slabs provided the associated resistance-deflection function can be defined by an essentially uniform resistance, as shown in Fig. A2.

In deriving Eq. A3, it is assumed that the unit blast impulse, i_b, is applied instantaneously at $t = 0$, and that the time to reach yield, t_y, is also close to zero.

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*or X_u, where X_u calculated from Eq. A2 is less than X_1 obtained from Table 5-9 of NAVFAC P-397.

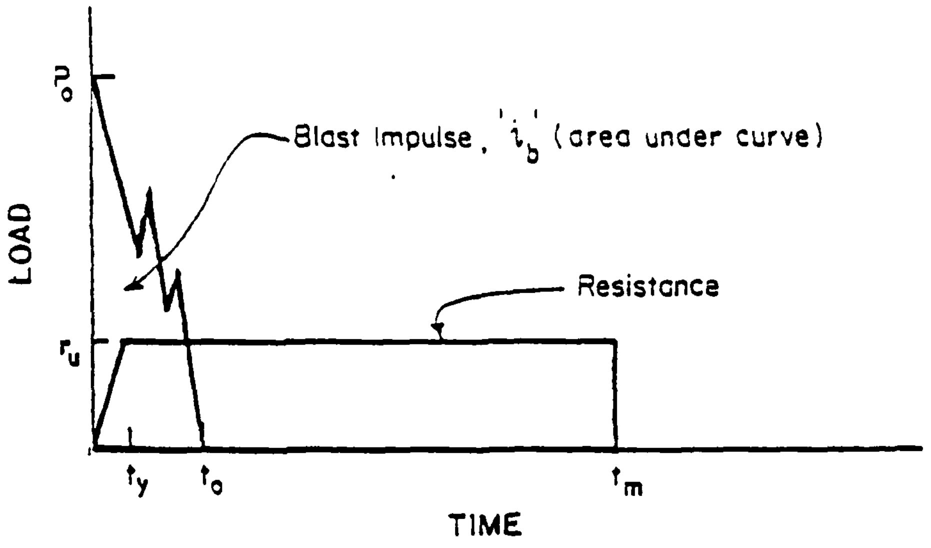

Fig. A1 Pressure-Time and Resistance-Time Curves for Slabs Subjected to Impulse Loading

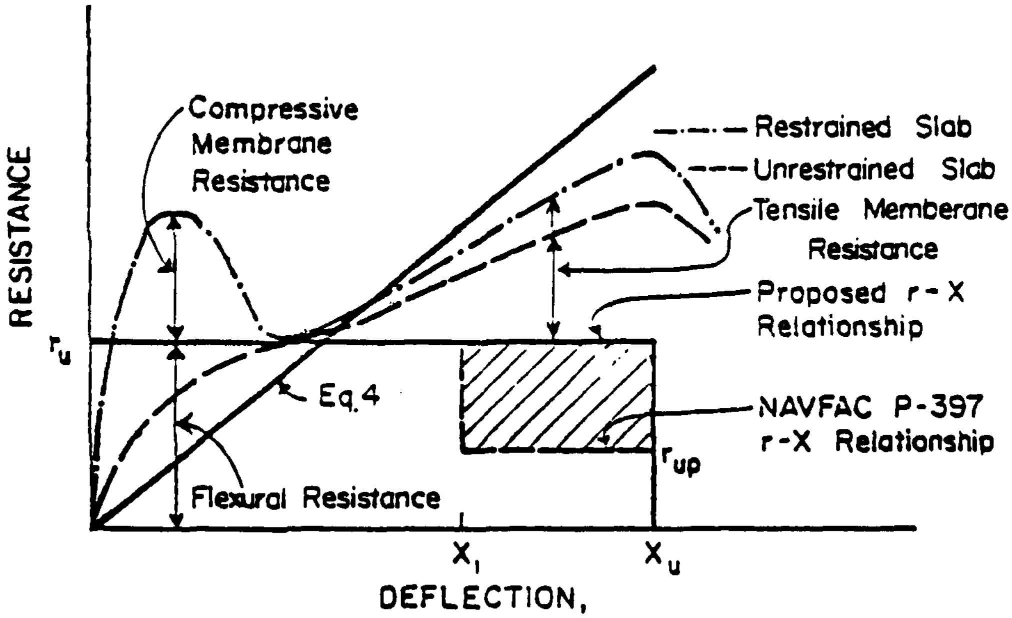

Fig. A2 Resistance-Deflection Curves for Two-Way Slabs Loaded to Incipient Collapse ($\theta_m > 5°$)

The left side of Eq. A3(a) is the initial kinetic energy resulting from the applied blast impulse and the right side represents the potential energy of the element.

It should be noted that Eq. A3 is valid only for a design involving large deflections, i.e., with support rotations greater than 5 degrees. Also, use of the equation is limited to cases where the time to reach yield, t_y, and the duration of the impulse, t_o, are short in comparison to the response time, t_m.

The expressions for the resistance, r_u, deflections, X_1 and X_u, and effective mass, m_u are listed in Tables 5-5 and 5-6, Tables 5-8 and 5-9, and Table 6-1, respectively, of NAVFAC P-397.

In design, the maximum response deflection, X_m, calculated from Eq. A3 is compared to the ultimate deflection, X_u. If the maximum response deflection, X_m, is larger than the ultimate deflection, X_u, the asssumed member properties should be revised.

In NAVFAC P-397, the partial failure deflection, X_1, and the ultimate deflection, X_u, are based on the development of a maximum support rotation of 12 degrees prior to failure. The maximum value of 12 degrees for support rotation is specified irrespective of the geometrical or material properties of the slab. In the case of slabs designed for the particular set of conditions for which the design criteria developed in this investigation are applicable, it is recommended that the incipient collapse deflection, δ_{ult}, be used in place of the ultimate deflection X_u, in Fig. A2.

It should be noted that the use of δ_{ult} in place of X_u cannot be justified unless the tensile membrane resistance, r_t, at the partial failure deflection, X_1, or at the ultimate deflection, X_u, where X_u is less than X_1, is comparable to or preferably higher than the assumed uniform resistance, r_u. This is apparent in Fig. A2.

Tensile membrane resistance, r_t, at a deflection, X, can be approximately evaluated by using the following expressions suggested by Park, based on concept of plastic tensile membrane:

For two-way slabs

$$r_t = \alpha X \left[\frac{T_y}{L_y^2} \frac{\pi^3}{4 \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh\left(\frac{n\pi L_x}{2L_y} \sqrt{\frac{T_y}{T_x}} \right)} \right)} \right] \tag{A4}$$

For one-way slabs

$$r_t = X \left[\frac{8T_y}{L_y^2} \right] \tag{A5}$$

where:

α = modification factor based on the factor k proposed by Keenan; the ratio of the factor of k = 20 proposed by Keenan for a square slab to the factor of k = 13.5 based on the original equation by Park, equal to 1.5

L_x = long span of slab, in.

L_y = short span of slab, in.

T_x = yield force, per unit width, of the reinforcement in long span direction, lbs/in.

T_y = yield force, per unit width, of the reinforcement in short span direction, lbs/in.

It is recommended that the tensile membrane resistance, r_t, obtained from Eq. A4 or Eq. A5 at X_1, or at X_u where X_u is less than X_1, be equal to or larger than the assumed uniform ultimate resistance, r_u. For some cases X_u becomes less than X_1. The reason for this is that X_1 is determined from Tables 5-8 and 5-9 of NAVFAC P-397, where X_1 is based on the development of a specified value of rotation, namely, 12 degrees, whereas X_u is independently determined from Eq. A2 in this supplement.

The design procedure consists of several steps. These steps are described briefly below in the order to be followed in a typical calculation.

Step 1 Establish design parameters, such as applied impulse load, geometry of a slab, support conditions, material properties, dynamic increase factors and so forth.

Step 2 Assume reinforcement ratios in both directions of slab.

Step 3 Select an appropriate value of the factor k' from Table A1, according to support conditions of the slab.

Step 4 Calculate the incipient collapse deflection, δ_{ult}, from Eq. A2. The calculated value of δ_{ult} should be used as the ultimate deflection, X_u, instead of the values listed in Tables 5-8 and 5-9 of NAVFAC P-397.

Step 5 Calculate the partial failure deflection, X_1, for two-way slabs from Table 5-9 of NAVFAC P-397.

Step 6 Ensure that assumed reinforcement ratios provide enough tensile membrane strength. To do this, calculate the tensile membrane resistance, r_t, corresponding to the partial failure deflection, X_1, or that corresponding to the ultimate deflection, X_u, where X_u is less than X_1, from Eq. A4 for two-way slabs or from Eq. A5 for one-way slabs.

Calculate the ultimate unit resistance, r_u, of the slab from equations listed in Tables 5-5 and 5-6 of NAVFAC P-397. If r_t is less than r_u, repeat Step 2 through Step 6 until the condition that r_t is larger than r_u is satisfied.

Step 7 Evaluate the effective unit mass of the slab in the ultimate resistance range from Fig. 6-5 (for two-way slabs) or Table 6-1 (for one-way slabs) of NAVFAC P-397.

Step 8 Given a blast impluse, i_b, calculate the maximum response deflection, X_m, from Eq. A3(b). Parameters to be used in this equation have been calculated in the foregoing steps, such as the ultimate deflection, X_u, ($= \delta_{ult}$), in Step 4, the ultimate unit resistance, r_u, in Step 6, and the effective unit mass in the ultimate range, m_u, in Step 7. If the calculated value of X_m is larger than the specified ultimate deflection, X_u, repeat Step 2 through Step 8 until X_m becomes less than X_u.

After completing Step 8, the slab can be considered to be safely designed for the incipient collapse condition when subjected to the specified impulsive load. In addition, the slab is assured of having sufficient reinforcement to exhibit ductile performance in the post-ultimate range, that is, in the tensile membrane action range.

Illustrative Example

The design of a restrained conventionally reinforced concrete slab subjected to impulse loading is worked out to demonstrate the application of the proposed design criteria. The design process follows the procedure described in the preceding section.

Problem:

Design a restrained two-way slab subjected to an impulsive load for incipient collapse conditions. Relevant data are given below. The slab is illustrated in Fig. A3.

Step 1 Design parameters:

a. Short span length L_y = 180 in.

b. Long span length L_x = 240 in.

c. Slab thickness T_c = 12 in.

d. Slab is fixed along all four edges.

e. Impulse load i_b = 900 psi-ms.

f. Material properties,

Reinforcement f_y = 60,000 psi
f_u = 90,000 psi

Concrete f'_c = 4,000 psi

g. Dynamic increase factor for reinforcement, DIF = 1.20 (from Table 5-3 of NAVFAC P-397)

h. Static design stress for reinforcement, (Paragraph 5-6 of NAVFAC P-397)

$$f_s = \frac{f_y + f_u}{2} = \frac{60,000 + 90,000}{2}$$

$$= 75,000 \text{ psi}$$

i. Dynamic design stress for reinforcement,

$$f_{ds} = (\text{DIF})\ f_s = 1.20 \times 75,000$$

$$= 90,000 \text{ psi}$$

j. Equal amount of reinforcement is assumed for top and bottom layers.

Step 2 Assume reinforcement ratios.

Reinforcement ratio in short span on each face,

$$p_y = 0.005$$

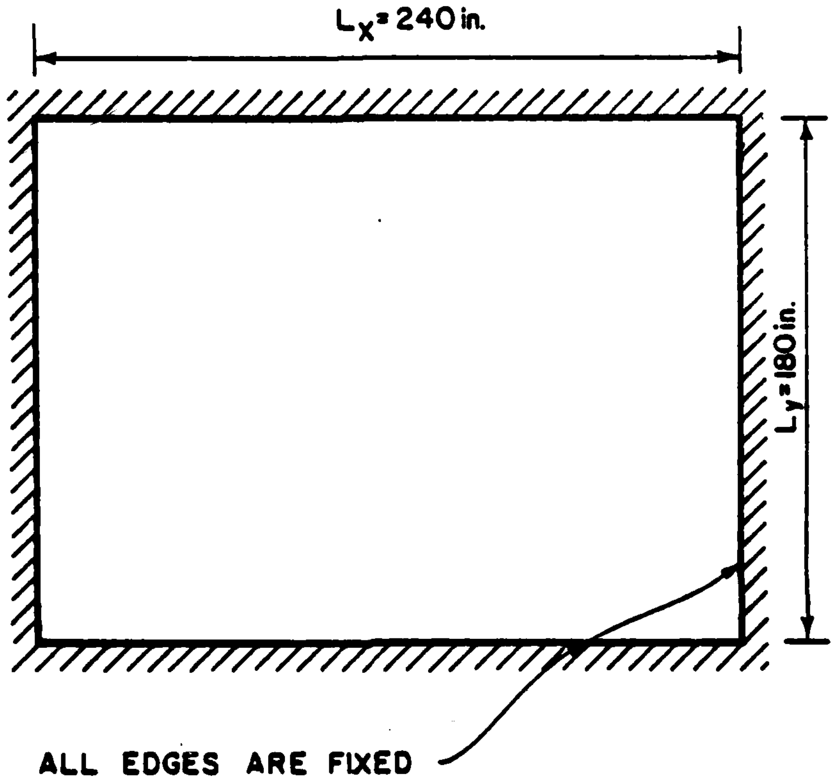

Fig. A3 Plan View of the Slab for Example Problem

Reinforcement ratio in long span on each face,

$$P_x = 0.003.$$

An approximate optimum value of P_y/P_x can be evaluated from either Fig. 6-17 or Fig. 6-18 of NAVFAC P-397.

Step 3 Select an appropriate value of the factor k' from Table A1.

For a restrained two-way slab, k' = 0.10.

Step 4 Calculate the incipient collapse deflection, δ_{ult}, from Eq. A2. This value of δ_{ult} is used in place of the ultimate deflection, X_u, in NAVFAC P-397.

$$X_u = \delta_{ult} = k' \, L_y$$

$$= 0.10 \times 180 = 18.0 \text{ in.}$$

Step 5 Calculate the partial failure deflection, X_1, from Table 5-9 of NAVFAC P-397.

From Fig. 5-11 (NAVFAC P-397),

$$x = 0.35 L_x = 0.35 \times 240 = 84 \text{ in.}$$

$$X_1 = x \tan 12^\circ = 84 \tan 12^\circ$$

$$= 17.9 \text{ in.} < X_u = 18.0 \text{ in.}$$

Step 6 Check approximate tensile membrane strength provided by reinforcement ratio assumed in Step 2.

a. Calculate the tensile membrane resistance corresponding to $X = X_1$ from Eq. A4.

$$r_t = \alpha X \left[\frac{T_y}{L_y^2} \frac{\pi^3}{4 \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh\left(\frac{n \pi L_x}{2 L_y} \sqrt{\frac{T_y}{T_x}} \right)} \right)} \right]$$

$$= 1.5 \times 17.9 \left[\frac{8,640}{180^2} \frac{\pi^3}{4 \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh\left(\frac{n \pi \, 240}{2 \times 180} \sqrt{\frac{8,640}{5,184}} \right)} \right)} \right]$$

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$= 26.85 \times 0.2667 \times 31.01/(4 \times 0.8377) = 66.3$ psi

where

T_x and T_y = yield forces, per unit width, of the reinforcement.

Short span direction,

$$T_y = 2\ T_c\ P_y\ f_y\ (DIF)$$

$$= 2 \times 12 \times 0.005 \times 60{,}000 \times 1.2$$

$$= 8{,}640 \text{ lbs/in.}$$

Long span direction,

$$T_x = 2\ T_c\ P_x\ f_y\ (DIF)$$

$$= 2 \times 12 \times 0.003 \times 60{,}000 \times 1.2$$

$$= 5{,}184 \text{ lbs/in.}$$

b. Calculate the ultimate unit resistance, r_u, from Table 5-6 of NAVFAC P-397.

Distance between the centroids of top and bottom reinforcement layers,

$$d_c = 7.5 \text{ in.}$$

Ultimate moment in short span direction,

$$M_y = \frac{A_s\ f_{ds}\ d_c}{b} \quad \text{(Eq. 5-7 of NAVFAC P-397)}$$

$$= 0.005\text{x}12\text{x}1\text{x}90{,}000\text{x}7.5$$

$$= 40{,}500 \text{ in-lbs/in.}$$

Ultimate moment in long span direction,

$$M_x = 0.003\text{x}12\text{x}1\text{x}90{,}000\text{x}7.5$$

$$= 24{,}300 \text{ in-lbs/in.}$$

Determine yield line location from Fig. 5-11 of NAVFAC P-397.

$$\frac{L_x}{L_y}\left(\frac{M_y}{M_x}\right)^{\frac{1}{2}} = \frac{240}{180}\left(\frac{40,500}{24,300}\right)^{\frac{1}{2}} = 1.72$$

From Fig. 5-11 (NAVFAC P-397),

$$x = 0.35L_x = 0.35 \times 240 = 84.0 \text{ in.}$$

Considering

$$x = 84.0 \text{ in.} < \frac{L_x}{2} = \frac{240}{2} = 120.0 \text{ in.}$$

the ultimate unit resistance, r_u, is obtained using the following expression from Table 5-6 (NAVFAC P-397).

$$r_u = \frac{5x2xM_x}{x^2} = \frac{10x24,300}{84^2} = 34.4 \text{ psi}$$

c. Compare r_t with r_u.

$$r_t = 66.3 \text{ psi} > r_u = 34.4 \text{ psi}$$

The condition stated in Step 6 is satisfied.

Step 7. Evaluate the effective unit mass of the slab for the ultimate range from Fig. 6-5 of NAVFAC P-397.

a. Calculate the unit mass of the slab.

$$m = d_c w/g = d_c\left(\frac{150}{12^3}\right) \times \frac{10^6}{386}$$

$$= 225d_c = 225 \times 7.5$$

$$= 1,688 \text{ psi-ms}^2/\text{in.}$$

where

w = weight density of concrete, lbs/in.^3

g = gravity acceleration, in./ms^2

b. Calculate the effective unit mass for the ultimate range using a load-mass factor from Fig. 6-5 (NAVFAC P-397).

$$\frac{x}{L_x} = \frac{84}{240} = 0.35$$

From Fig. 6-5 (NAVFAC P-397), the load-mass factor is

$$K_{LM} = 0.56$$

The effective unit mass for the ultimate range is calculated as follows:

$$m_u = K_{LM}\, m = 0.56 \times 1{,}688$$
$$= 945 \text{ psi-ms}^2\text{/in.}$$

<u>Step 8</u> Calculate the maximum response deflection, X_m, by solving Eq. A3(b) for X_m and compare X_m with X_u.

$$X_m = \frac{i_b^{\,2}}{2m_u\, r_u} = \frac{900^2}{2 \times 945 \times 34.4} = 12.5 \text{ in.}$$

Compare X_m with X_u.

$$X_m = 12.5 \text{ in.} < X_u = 18.0 \text{ in.}$$

The condition stated in Step 8 is satisfied.

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