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SOME SIMPLE EXAMPLES OF TIME SERIES IN M DIMENSIONS: AN INTRODU--ETC(U)
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SOME SIMPLE EXAMPLES OF TIME SERIES IN M DIMENSIONS: AN INTRODUCTION

Leo A. Aroian
Union College and University
Institute of Administration
and Management
Schenectady, N.Y. 12308

Vidya Taneja
Western Illinois University
Professor of Mathematics
Macomb, Illinois 61455

ABSTRACT

Some simple theoretical examples of time series in *m dimensions*, their main properties, estimation, and forecasting are given. These are generalizations of the usual time series to events in both time and space. How these models may be simulated is explained. One practical example of flooding on the Mohawk River is given. Included are new results in forecasting; the regions for permissible values of the autocorrelation and cross correlations; and methods for obtaining these correlations.

1. INTRODUCTION

The purpose of this paper is to give a short introduction to the main ideas of time series in *m dimensions*. These simple examples are concerned with results in only one dimension. Full generality to m dimensions is given in the papers of Aroian and his collaborators. The usual time series is in zero dimensions. The same methods apply to purely spatial series if time is not a consideration; spatial series are explored in a forthcoming paper of Aroian and Gebizlioglu (1). Every event occurs in time and space. An event has many characteristics; only the univariate case is treated here. For the multivariate case, see Aroian (2). As examples, if $m = 1$, note the characteristics of a river, and as a generalization any process involving a flow. If $m = 2$, examples are physical processes as a storm, rainfall over a region, the changing physical or social processes over time in a restricted region; for $m = 3$, examples are the positions of satellites, sunspots on the sun, geological processes and earthquakes, storms in space, or characteristics over an (ξ_1, ξ_2, ξ_3, t) coordinate system. Social, industrial, scientific, geophysical as well as other processes may be explained by these models.

2. ARMA MODEL

The autoregressive moving average model, $m = 1$, is defined for $m = 1$, $r_1 = r_2 = 2$ by:

$$z_{x,t} = \phi_1 z_{x,t-1} + \phi_2 z_{x-1,t-1} - \theta_1 a_{x,t-1} - \theta_2 a_{x-1,t-1} + a_{x,t} \tag{2.1}$$

The characteristic $z_{x,t}$ in which one is interested is given as a linear combination of two past values of z at $(x,t-1)$ and $(x-1,t-1)$, $r_2 = 2$; and two past values of a variable $a_{x,t-1}$, $a_{x-1,t-1}$, random shocks, with an error $a_{x,t}$, $r_1 = 2$. We assume $z_{x,t}$ is weakly stationary, $E(z_{x,t}) = 0$ (with no loss generality), $E(z^2_{x,t}) = \sigma^2_z < \infty$, $E(z_{x,t_1} z_{y,t_2}) = \sigma^2_z \rho_{x-y,t_1-t_2}$, $-\infty < x < \infty$, $-\infty < t < \infty$; $a_{x,t}$ is an independent random variable with mean zero, variance $E(z_{x,t_1} z_{x,t_2})$ $E(a_{x,t} a_{x-\ell,t-k}) = 0$, unless $\ell = k = 0$, when it is σ^2_a; and $a_{x,t}$ is independent of $z_{x-\ell,t-k}$ unless $\ell = k = 0$.

If $\phi_1 = \phi_2 = 0$, then

$$z_{x,t} = a_{x,t} - \theta_1 a_{x,t-1} - \theta_2 a_{x-1,t-1} \tag{2.2}$$

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SOME SIMPLE EXAMPLES OF TIME SERIES IN M DIMENSIONS: AN INTRODUCTION

by

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P. 1, $\ell 4\uparrow$	$Ea^2_{x,t} = \sigma^2_a$
P. 2, $\ell 2\downarrow$	(2.3) $z_{x,t} = \phi_1 z_{x,t-1} + \phi_2 z_{x-1,t-1} + a_{x,t}$
P. 2, $\ell 3\uparrow$	$\theta_2 = -\rho_{10}/\rho_{01}$
P. 3	(4.1) $z_{x,t} = a_{x,t} - \theta_1 a_{x,t-1} - \theta_2 a_{x-1,t-1}$
P. 3, $\ell 4\uparrow$	$r = s = 10$
P. 4, $\ell 22\downarrow$	Replace (3) by (2)
P. 4	(5.1) $\hat{\rho}\hat{\theta}_1, \hat{\theta}_2 = -r_{10}$.
P. 6	(6.7) $\phi_1 = (\rho_{01} - \rho_{11}\rho_{10})(1-\rho^2_{10})^{-1}$
P. 6, $\ell 3\uparrow$	(6.5) and (6.9) are satisfied.
P. 7	(6.10) $\hat{\rho}\hat{\phi}_1, \hat{\phi}_2 = -r_{10}$
P. 9	(7.6) $\psi_{ii} = \phi_2 \psi_{i-1,i-1}$
P. 10	(8.5) $\sigma^2_{e_{x,t}}(1,2)$
P. 10	(8.10) $\sigma^2_{e_{x,t}}(\ell_1, \ell_2)$
P. 11	(8.13) $\sigma^2_{e_{x,t}}(\ell_1, \ell_2)$
P. 12	Add (10) Campbell, D. & Schaeffer, D.J. (1979). Pollution in the Chicago Sanitary and Ship Canal. <u>Environmental Management</u>, 3, 283-288.

is a moving average MA model $m_1 = 1$, $r_1 = 2$; if $\theta_1 = \theta_2 = 0$

$$a_{x,t} = \phi_1 z_{x,t-1} + \phi_2 z_{x-1,t-1} + a_{x,t} \tag{2.3}$$

is an autoregressive AR model $m = 1$, $r_2 = 2$.

If the t variable is omitted and if in Equation (2.1) one replaces x and x-1 on the right hand side by x-1 and x-2 except in $a_{x,t}$ one gets a purely spatial model; while if the x variable is omitted, one gets the usual time series. A simpler ARMA model is obtained if one sets $\phi_1 = \theta_1 = 0$, or $\phi_2 = \theta_2 = 0$. The MA model, the AR model, and the ARMA model with their properties will be considered.

3. PROPERTIES OF THE MA MODEL

The autocorrelation function of the MA model, the values which θ_1 and θ_2 may have, and the corresponding AR model with an infinite set of coefficients will be given. From Voss et al. (3), the variance

$$\sigma_z^2 = \sigma_a^2(1+\theta_1^2+\theta_2^2) \tag{3.1}$$

and the correlations

$$\rho_{01}(1+\theta_1^2+\theta_2^2) = -\theta_1, \quad \rho_{10}(1+\theta_1^2+\theta_2^2) = \theta_1\theta_2,$$

$$\rho_{11}(1+\theta_1^2+\theta_2^2) = -\theta_2, \tag{3.2}$$

all other autocorrelations are zero. Note $\rho_{11} = \rho_{-1-1}, \rho_{01} = \rho_{0-1}, \rho_{10} = \rho_{-10}$.

This curtoff property of the autocorrelation function aids in the identification of an MA model $m = 1$, $r_1 = 2$. A corresponding cutoff property is true for any MA model, (m,r). Rewrite (2.2) as

$$z_{x,t} = (1-\theta_1 B_t - \theta_2 B_x B_t) a_{x,t} \tag{3.3}$$

where $B_t a_{x,t} = a_{x,t-1}$, $B_x a_{x,t} = a_{x-1,t}$, and $1-\theta_1 B_t-\theta_2 B_x B_t$ is called the generating function or characteristic polynomial of the MA model. Rewrite (3.3) as

$$a_{x,t} = (1-\theta_1 B_t-\theta_2 B_x B_t)^{-1} z_{x,t} = \left(\sum_{j=0}^{\infty} (\theta_1+\theta_2 B_x)^j B_t^j\right) z_{x,t}. \tag{3.4}$$

This is an infinite AR model representation of (2.2) provided $|\theta_1|+|\theta_2|<1$, since $|B_x|<1$, $|B_t|<1$ for convergence of (3.4). Every MA process is stationary but for a representation as an infinite AR model, the condition $|\theta_1|+|\theta_2|<1$ is necessary. Note the coefficients of $z_{x-\ell,t-k}$ approach zero as $\ell,k \to \infty$. These results may also be obtained by successive substitutions in (2.2), or by straightforward division. From (3.2) we infer

$$1+\theta_1^2+\theta_2^2 = -\theta_1/\rho_{01} = \theta_1\theta_2/\rho_{10} = -\theta_2/\rho_{11} \quad , \tag{3.5}$$

$$\theta_1 = -\rho_{10}/\rho_{11}, \quad \theta_2 = -\rho_{10}/\rho_{01}, \quad \theta_1 = (\theta_2\rho_{01})/\rho_{11}, \tag{3.6}$$

$$\theta_2 = \frac{-\rho_{11} \pm \rho_{11}[1-4(\rho_{01}^2+\rho_{11}^2)]^{1/2}}{2(\rho_{01}^2+\rho_{11}^2)} \tag{3.7}$$

Hence $1-4(\rho_{01}^2+\rho_{11}^2) > 0$, or $\rho_{01}^2+\rho_{11}^2 < 1/4$, (3.8)

a circle of radius 1/2. From $\theta_2 = -\rho_{10}/\theta_{01}$, we conclude $\rho_{10} = -\rho_{01}\theta_2$. From $|\theta_1| + |\theta_2|<1$, the added restrictions apply:

$$-1/\rho_{10} < 1/\rho_{11} + 1/\rho_{01} < 1/\rho_{10} \text{ if } 0 \le \rho_{10} < 1, \text{ or}$$

$$1/\rho_{10} < 1/\rho_{11} + 1/\rho_{01} < -1/\rho_{10} \text{ if } -1 < \rho_{10} \le 0 . \tag{3.9}$$

Thus the only values of $\{\rho_{01},\rho_{10},\rho_{11}\}$ possible are those in the intersection of (3.8) and (3.9).

In summary, given θ_1 and θ_2 then ρ_{01}, ρ_{10}, and ρ_{11} may be found from (3.2). If a representation of (2.2) as an infinite AR model (3.4) is desired, then $|\theta_1|+|\theta_2|<1$ and the coefficients of this AR model are given by (3.4). Additionally $\{\rho_{01},\rho_{10},\rho_{11}\}$ satisfy (3.6), (3.7), (3.8), and (3.9). Conversely, given $\rho_{01},\rho_{10},\rho_{11}$, which define an MA process (2.2), then if (3.4) is satisfied, all equations (3.5), (3.6), (3.7), and (3.8) are satisfied. Note ρ_{01} and ρ_{11} must satisfy (3.7) and consequently $\rho_{10} = -\theta_2\rho_{01}$. Hence, given $\{\rho_{01},\rho_{11}\}$, ρ_{10} is determined. If an arbitrary set $\{\rho_{01},\rho_{10},\rho_{11}\}$ are from some unknown time series in m dimensions the equations (3.5), (3.6), (3.7), (3.8), and $|\theta_1|+|\theta_2|<1$ will not be satisfied. For instance, let $\theta_1 = .2$, $\theta_2 = -.5$, $\sigma_z^2 = 1.29\sigma_a^2$, $\rho_{01} = -.1550$, $\rho_{10} = -.0775$, $\rho_{11} = .3876$. All equations (3.5-3.9) are satisfied. The corresponding AR model is $a_{x,t} = z_{x,t}+.2z_{x,t-1}-.5z_{x-1,t-1}+.04z_{x,t-2}-.2z_{x-1,t-2}+.25z_{x-2,t-2}+.008z_{x,t-3}-.06z_{x-1,t-3}+.15z_{x-2,t-3}-.125z_{x-3,t-3}+\ldots$etc. Suppose from a *simulation of this process* the sample values are $r_{01} = -.14$, $r_{10} = -.10$, $r_{11} = .40$, to determine the estimated values of the constants $\hat\theta_1,\hat\theta_2$ three equations must be satisfied, an impossibility. Instead iterations are used in order to satisfy the (3.5) as closely as possible. The result is $\hat\theta_1=.22$, $\hat\theta_2 = -.55$, $1+\theta_1^2+\theta_2^2 = 1.3509$, and resulting r's from (3.5) are $r_{01} = -.163$, $r_{10} = -.090$, $r_{11} = .407$.

We rewrite

$$\begin{aligned}
&a_{x,t} = \sum_0^\infty \sum_0^\infty \pi_{i,j} z_{x-i,t-j},\ \pi_{00} = 1,\ \pi_{01} = \theta_1,\ \psi_{11} = \theta_2,\\
&\psi_{02} = \theta_1^2,\ \psi_{12} = 2\theta_1\theta_2,\ \psi_{22} = \theta_2^2,\ \pi_{03} = \theta_1^3,\ \pi_{13} = 3\theta_1^2\theta_2,\\
&\pi_{23} = 3\theta_1\theta_2^2,\ \pi_{33} = \theta_3^2,\ \pi_{0j} = \theta_1\pi_{0j-1},\ \pi_{ij} = \theta_1\psi_{i,j-1} +\\
&\qquad \theta_2\psi_{i-1,j-1},\ \pi_{jj} = \theta_2\pi_{j-1,j-1} .
\end{aligned} \tag{3.10}$$

4. SIMULATION OF THE MA MODEL

Given $\{\theta_1,\theta_2\}$, $|\theta_1|+|\theta_2|<1$, simulate

$$z_{x,t} = a_{x,t}-\theta_1 a_{x,t}-\theta_2 a_{x-1,t-1} \tag{4.1}$$

by first generating a set of random numbers $(a_{x+i,t+j})$, $i=0,1,2,\ldots, r+4, j=0, 1,2,\ldots,s+4$, from a distribution $\mu=0,\sigma=1$. This gives $(r+5)(s+5)$ values of $a_{x+i,t+j}$ random numbers. Then from these values of a's calculate $(r+5)(s+5)$ values of $(z_{x+i,t+j})$ with the chosen theoretical set $\{\theta_1,\theta_2\}$ using (4.1). All a's not generated as above are identically zero. Delete the first two rows, first two columns, the last three rows and the last three columns of the Z matrix. This produces a reduced Z matrix of rs value. Note the number five is chosen for convenience, so an inner Z matrix may represent the process properly. In the example $\theta_1=.2$, $\theta_2=-.5$, and the $a_{x,t}$'s are normally distributed with $\mu=0$, $\sigma_a=1$, $r=s=10$, $z_{x,t}$ is given as the first number in each cell of Table 1. The $a_{x,t}$, the second number in each cell of Table 1 is not the 10X10 inner part of 15X15 original Z but is determined from the 10X10 Z matrix by using

$$a_{x,t} = z_{x,t} + \theta_1 a_{x,t-1}+\theta_2 a_{x-1,t-1}, \tag{4.2}$$

with $\theta_1=.2$, $\theta_2=-.5$ where now $a_{x,t-1}$ and $a_{x-1,t-1}$ are taken as zero for initial starting values. Thus in the first row $a_{x,t}=z_{x,t}$. In the first column all unknown a's for $x<1$ are zero. In this way a set of a's are built up slightlv different from the inner 10X10 $a_{x,t}$'s obtainable from the 15X15 original Z matrix. This is done since in a sample of $z_{x,t}$ the $a_{x,t}$ must be determined in this way (although in practice the $\{\theta_1,\theta_2\}$ would be unknown and would be a set $\{\hat{\theta}_1,\hat{\theta}_2\}$ estimated from the sample $z_{x,t}$'s). In the second row all needed a's are known as soon as the first a is determined, and subsequently for each row, since the needed a's in any cell are found by using the $z_{x,t}$ in that cell and the $a_{x,t-1}$ in the cell directly above and the $a_{x-1,t-1}$ in the cell one to the left and above using (4.2). This set of a's is used in finding all further $\hat{Z}$'s for all subsequent sets $\{\hat{\theta}_1,\hat{\theta}_2\}$. From the 10X10 Z matrix determine $r_{m,n}$, $\bar{z}$ (the sample mean), $\hat{\sigma}_z^2$ (the sample variance of z, $\hat{r}_{01}$, r_{10}, r_{11}, $\hat{\theta}_1,\hat{\theta}_2$, using (3.5-3.7) iteratively as in 3. New estimates $\hat{z}_{x,t}$ may be found using any desired $(\hat{\theta}_1,\hat{\theta}_2)$ particularly a set satisfying (3.5-3.7) and having a minimum prediction error variance $\hat{\sigma}_\varepsilon^2 = \Sigma\Sigma(z_{x,t}-\hat{z}_{x,t})^2 /(n-1)$. This is essentially a least squares procedure. The minimum variance procedure is to vary a set (θ_1,θ_2) in the plane such that $\hat{\sigma}_\varepsilon^2$ is a minimum. *The theoretical value of* σ_ε^2 *is*

$$\sigma_\varepsilon^2 = \sigma_a^2\ \{(\theta_1-\hat{\theta}_1)^2+(\theta_2-\hat{\theta}_2)^2\}$$

where θ_1 and θ_2 are population values. This method is remarkably effective, working well for autoregressive models also (3). Note $\hat{z}(x,t)_1$ are values from the minimum prediction error variance set $\hat{\theta}_1=.208,\hat{\theta}_2=-.493$ almost identical to the theoretical set $\{.2,-.5\}$!; while $\hat{z}(x,t)_2$ are from $\{.333-.418\}$ the least squares set most nearly satisfying (3.5)-(3.7), which leads to a theoretical $\{\rho_{01},\rho_{10},\rho_{11}\}$ set of $\{-.259,-.108,.325\}$ compared with an actual $\{r_{10},r_{01},r_{11}\}$ of $\{-.255,-.133,.299\}$ from $z_{x,t}$. The values of $r_{m,n}$ in Table 2 are those from $z_{x,t}$(the notation of z(x,t) in Table 1 is used for convenience.) Further from (5.1) in the next section given $\{.2,-.5\}$, the 95% confidence interval for θ_1 is (.036,.364) and for θ_2 is (-.664,-.336) and $\{.333,-.418\}$ cuts these limits.

5. ESTIMATION

The variances and covariances of $\{\hat{\theta}_1,\hat{\theta}_2\}$ are approximated by use of the sample estimates r_{01}, r_{10}, r_{11} in a sample of n of $\rho_{01},\rho_{10},\rho_{11}$ and the relationship of the variance-covariance matrix of MA models which are the same as those of AR models. From the results of Perry and Aroian (4),

$$\hat{\sigma}_{\hat{\theta}_1}^2 = \hat{\sigma}_{\hat{\theta}_2}^2 = n^{-1}(1-r_{10}^2)^{-1}(1-\theta_1^2-\theta_2^2-2\theta_1\theta_2 r_{10}),\ \text{and}$$

$$\hat{\rho}_{\hat{\theta}_1,\hat{\theta}_2} = r_{10}. \qquad (5.1)$$

These correspond to the least squares solution of the corresponding AR model and if the $a_{x,t}$ are assumed to be distributed normally, these are close approximations to the maximum likelihood estimates of $\{\theta_1\theta_2\}$. (See Box and Jenkins, (5) p. 283, for similar case, m=0.) The starting values of $a_{x,t}$ as noted in the simulation would need to be considered for the maximum likelihood solution. The method leading to (5.1) is general and applies to the m dimensional MA model. The minimum variance estimates $\{\hat{\theta}_1,\hat{\theta}_2\}$ and the resulting estimates of the correlation between $\hat{\theta}_1$ *and* $\hat{\theta}_2$ *may be found as noted in* the next section for any set of data.

TABLE 1

Values of $Z(x,t)$, $a(x,t)$, $\hat{Z}(x,t)_1$, $\hat{Z}(x,t)_2$

t	x	1	2	3	4	5	6	7	8	9	10
1	$z(x,t)$	-1.478	0.815	-0.301	0.337	-1.277	0.527	0.210	-0.625	0.867	-1.668
	$a(x,t)$	-1.478	0.815	-0.301	0.337	-1.277	0.527	0.210	-0.625	0.867	-1.668
	$\hat{z}(x,t)_1$	-1.478	0.815	-0.301	0.337	-1.277	0.527	0.210	-0.625	0.867	-1.668
	$\hat{z}(x,t)_2$	-1.478	0.815	-0.301	0.337	-1.277	0.527	0.210	-0.625	0.867	-1.668
2	$z(x,t$	-1.110	-1.410	1.541	0.126	-0.681	0.245	0.132	1.506	-1.602	-0.741
	$a(x,t)$	-1.406	-0.508	1.073	0.343	-1.105	0.989	-0.090	1.276	-1.116	-1.508
	$\hat{z}(x,t)_1$	-1.098	-1.406	1.538	0.125	-0.673	0.250	0.127	1.510	-1.605	-0.734
	$\hat{z}(x,t)_2$	-0.913	-1.397	1.514	0.106	-0.539	0.280	0.061	1.572	-1.666	-0.590
3	$z(x,t)$	-0.105	-1.199	-0.701	-0.649	1.620	0.425	0.148	-0.124	0.595	-0.060
	$a(x,t)$	-0.386	-0.598	-0.232	-1.116	1.228	1.175	-0.364	0.176	-0.266	0.196
	$\hat{z}(x,t)_1$	-0.094	-1.185	-0.706	-0.659	1.626	0.425	0.142	-0.134	0.595	-0.040
	$\hat{z}(x,t)_2$	0.082	-1.016	-0.802	-0.783	1.739	0.384	0.079	-0.286	0.639	0.232
4	$z(x,t)$	0.450	-0.484	-1.375	-0.278	-1.252	0.540	-0.343	0.767	-2.043	-0.903
	$a(x,t)$	0.372	-0.411	-1.123	-0.385	-0.448	0.161	-1.003	0.984	-2.184	-0.731
	$\hat{z}(x,t)_1$	0.453	-0.477	-1.369	-0.267	-1.254	0.522	-0.348	0.768	-2.042	-0.903
	$\hat{z}(x,t)_2$	0.501	-0.373	-1.295	-0.110	-1.324	0.283	-0.391	0.773	-2.022	-0.907
5	$z(x,t)$	-2.642	0.460	-0.264	-1.163	-0.555	2.482	1.121	-1.062	0.911	-1.187
	$a(x,t)$	-2.568	0.191	-0.284	-0.679	-0.452	2.739	0.840	-0.364	-0.018	-0.241
	$\hat{z}(x,t)_1$	-2.645	0.461	-0.252	-1.152	-0.549	2.484	1.128	-1.063	0.922	-1.166
	$\hat{z}(x,t)_2$	-2.692	0.484	-0.081	-1.020	-0.464	2.497	1.241	-1.111	1.121	-0.911
6	$z(x,t)$	1.355	0.040	-0.401	-0.857	2.023	-1.367	-0.648	0.602	-0.817	-1.329
	$a(x,t)$	0.842	1.362	-0.553	-0.851	2.272	-0.593	-1.850	0.109	-0.639	-1.368
	$\hat{z}(x,t)_1$	1.376	0.056	-0.400	-0.850	2.031	-1.386	-0.674	0.599	-0.814	-1.327
	$\hat{z}(x,t)_2$	1.696	0.225	-0.379	-0.743	2.139	-1.694	-0.984	0.581	-0.785	-1.295
7	$z(x,t)$	0.554	-0.191	1.936	0.196	-0.843	1.234	-0.742	0.557	0.775	0.542
	$a(x,t)$	0.722	-0.340	1.144	0.303	0.037	-0.021	-0.815	1.503	0.593	0.588
	$\hat{z}(x,t)_1$	0.547	-0.208	1.931	0.207	-0.855	1.223	-0.723	0.569	0.779	0.557
	$\hat{z}(x,t)_2$	0.442	-0.441	1.898	0.355	-1.075	1.127	-0.447	0.694	0.851	0.776
8	$z(x,t)$	0.012	2.509	-1.481	0.012	0.692	-0.702	0.124	-2.361	0.689	1.718
	$a(x,t)$	0.157	2.080	-1.082	-0.499	0.548	-0.725	-0.029	-1.653	0.056	1.539
	$\hat{z}(x,t)_1$	0.006	2.507	-1.488	0.002	0.690	-0.702	0.131	-2.367	0.674	1.709
	$\hat{z}(x,t)_2$	-0.084	2.495	-1.605	-0.122	0.662	-0.702	0.234	-2.494	0.487	1.591
9	$z(x,t)$	0.298	-1.145	2.832	0.688	-0.602	-1.427	-1.601	-0.279	-1.305	-0.124
	$a(x,t)$	0.329	-0.808	1.576	1.129	-0.243	-1.846	-1.244	-0.596	-0.467	0.156
	$\hat{z}(x,t)_1$	0.297	-1.163	2.826	0.700	-0.603	-1.425	-1.596	-0.266	-1.294	-0.137
	$\hat{z}(x,t)_2$	0.277	-1.434	2.805	0.843	-0.634	-1.376	-1.538	-0.057	-1.177	-0.333
10	$z(x,t)$	-1.572	1.603	-0.668	1.921	1.776	1.193	-1.140	-0.279	1.135	-1.252
	$a(x,t)$	-1.506	1.277	0.051	1.359	1.162	0.945	-0.466	0.224	1.339	-0.988
	$\hat{z}(x,t)_1$	-1.575	1.607	-0.675	1.901	1.770	1.209	-1.117	-0.266	1.143	-1.250
	$\hat{z}(x,t)_2$	-1.616	1.683	-0.811	1.642	1.716	1.458	-0.823	-0.098	1.246	-1.234

MEAN OF $z(x,t) = -0.0757732$ VAR. OF $z(x,t) = 1.29559$
MEAN OF $a(x,t) = -0.0620963$ VAR. OF $a(x,t) = 1.03742$
MEAN OF $\hat{z}(x,t)_1 = -0.0745114$ VAR. OF $\hat{z}(x,t)_1 = 1.29484$

The prediction error variance of $\hat{z}(x,t)_1 = .00009444634$
For $z(x,t)_1$, $\hat{\theta}_1 = .208$, $\hat{\theta}_2 = -.493$

MEAN OF $\hat{z}(x,t)_2 = -0.0575717$ VAR. OF $\hat{z}(x,t)_2 = 1.32942$

The prediction error variance of $\hat{z}(x,t)_2 = .0245026$
For $z(x,t)_2$, $\hat{\theta}_1 = .333$, $\hat{\theta}_2 = -.418$

Where $\hat{z}(x,t)_1$ is the prediction with minimum error without regard to (3-5) to (3-8)
$z(x,t)_2$ is the prediction with minimum error with grgard to (3-5) to (3-8).

TABLE 2

ESTIMATED CORRELATION COEFFICIENTS

$r_{m,n}$ from $z_{x,t}$

m \ n	0	1	2	3	4	5
-5	-.02440	-.08929	.01751	.09018	-.07126	.01399
-4	-.00683	-.01955	-.15060	.12190	-.06967	.06822
-3	.00744	-.21941	-.09758	-.16610	.14620	.06586
-2	-.04944	-.02406	.09758	.02137	.03338	.00839
-1	-.13263	.15886	-.05186	.07963	-.08132	.03068
0	1.0000	-.25518	.10146	.01626	-.05956	.02659
1	-.13263	.29921	.03112	-.13549	.03900	-.12985
2	-.04944	.05475	.02902	.05822	-.10431	.04176
3	.00744	-.00829	-.04685	.12587	.01059	.00946
4	-.00683	-.08132	.06358	-.00833	.07500	-.04081
5	-.02440	.00370	-.06670	-.05665	.05308	.01927

6. PROPERTIES OF THE AR MODEL

The properties of the AR model

$$z_{x,t} = \phi_1 z_{x,t-1} + \phi_x z_{x-1,t-1} + a_{x,t} \tag{6.1}$$

are taken from Taneja and Aroian (6) and Perry and Aroian (4). These are: the characteristic function,

$$\phi(B_x, B_t) = 1-(\phi_1+\phi_2 B_x)B_t; \tag{6.2}$$

the autocorrelation function,

$$\rho_{m,n} = \phi_1 \rho_{m,n-1} + \phi_2 \rho_{m-1,n-1}, \quad m = n \neq 0; \tag{6.3}$$

the variance,

$$\sigma_z^2 = \sigma_a^2 (1-\phi_1\rho_{01}-\phi_2\rho_{11})^{-1}; \tag{6.4}$$

and the stationarity condition

$$|\phi_1| + |\phi_2| < 1; \tag{6.5}$$

the Yule-Walker equations,

$$\rho_{01} = \phi_1 + \phi_2\rho_{10} \qquad \rho_{11} = \phi_1\rho_{10} + \phi_2 \tag{6.6}$$

with the solution:

$$\phi_1 = (\rho_{01}-\rho_{10}\rho_{10})\ (1-\rho_{10}^2)^{-1}$$
$$\phi_2 = (\rho_{11}-\rho_{01}\rho_{10})\ (1-\rho_{10}^2)^{-1} \tag{6.7}$$

Since $\sigma_2^2 > 0$, then $\{\rho_{01}, \rho_{10}, \rho_{11}\}$ are restricted by

$$\rho_{10}^2 + \rho_{01}^2 + \rho_{11}^2 - 2\rho_{01}\rho_{10}\rho_{11} - 1 < 0 \quad . \tag{6.8}$$

If (6.8) is set equal to zero, then for each value of ρ_{10} (6.8) represents an ellipse. The permissible values lie within the rectangle described in the ellipses. The rectangle is bounded by

$$-1+\rho_{10} < \rho_{01} - \rho_{11} < 1-\rho_{10}, \text{ and}$$

$$-1-\rho_{10} < \rho_{01} + \rho_{11} < 1+\rho_{10} \quad . \tag{6.9}$$

While an AR model is invertible, it is not stationary unless condition (6.9) is satisfied. Note that given a permissible set of $\{\rho_{10}, \rho_{11}, \rho_{01}\}$ then ϕ_1 and ϕ_2 are determined by (6.7). However, if (ϕ_1, ϕ_2) are given, then $\{\rho_{10}, \rho_{11}, \rho_{01}\}$

are unique and may be determined from the corresponding AM model of the AR model. An example, the depths of the Mohawk River, discussed briefly in the Perry and Aroian paper (4) is being expanded from three points on the river to ten points. The results will be used to predict floods along the Mohawk. Let $\phi_1 = .2$, $\phi_2 = -.6$, $\rho_{10} = -.20871$, $\rho_{01} = .32523$, $\rho_{11} = -.64174$. The simulation of this set of $\{\rho_{10}, \rho_{01}, \rho_{11}\}$ is accomplished by using the ϕ's and any set of starting values with the a's generated by any desired distribution. The values of $\rho_{m,n}$ for this example are given in the table for (m,n) in the first and second quadrants; the results in the third quadrant are the same as those in the first quadrant, while the results in the fourth quadrant are the same as those in the second quadrant. Thus $\rho_{m,n} = \rho_{-m,-n}$, $\rho_{m,-n} = \rho_{-m,n}$, $\rho_{m,0} = \rho_{-m,0}$, $\rho_{0,-n} = \rho_{0,n}$, but $\rho_{m,n} \neq \rho_{-m,n}$. This follows from the definition of ρ_{x-y,t_1-t_2}, given in section 2. A table of the correlation $\rho_{m,n}$ for $\phi_1 = .2$, $\phi_2 = -.6$ is presented.

TABLE 3

CORRELATIONS $\rho_{m,n}$ IN AR MODEL

$\phi_1 = .2$, $\phi_2 = -.6$

m \ n	0	1	2	3	4	5	6	7	8
8								.01	.03
7							-.01	-.04	-.06
6					.01	.02	.07	.09	.07
5				-.01	-.04	-.11	-.12	-.09	-.05
4		.01	.02	.06	.17	.17	.11	.06	.03
3	-.01	-.03	-.09	-.26	-.22	-.12	-.06	-.02	-.01
2	.04	.13	.41	.28	.13	.05	.02	.01	
1	-.21	-.64	-.32	-.13	-.05	-.05	-.02	-.01	
0	1.00	.33	.11	.03	.01				
-1	-.21	-.07	-.02	-.01					
-2	.04	.01							
-3	-.01								

The numbers in the blank spaces are zero.

Every AR model has a cutoff property given by the partial autocorrelation function, ϕ_i in this case; $\phi_i \neq 0$ for $i = 1,2$; $\phi_i = 0$, $i \neq 1,2$, where ϕ_i is determined by the Yule-Walker equations corresponding to AR models, $r > 2$. For an $m = 1$ model is of order one if $\phi_3, \phi_4, \ldots, \phi_r$, are zero as determined by the partial coefficient of correlation, Taneja and Aroian (6).

The approximate estimated variances and covariances of (ϕ_1, ϕ_2) given by

$$\hat{\sigma}^2_{\hat{\phi}_1} = \hat{\sigma}^2_{\hat{\phi}_2} = n^{-1}(1-r_{10}^2)^{-1}(1-\phi_1^2-\phi_2^2-2\phi_1\phi_2\, r_{10}) \text{ and}$$
$$\hat{\rho}_{\hat{\phi}_1,\hat{\phi}_2} = r_{10} \qquad (6.10)$$

are approximate maximum likelihood results, if the $a_{x,t}$'s are distributed normally, are asymptotically unbiased, consistent, and minimum variance. Let

$$R_2 = \begin{pmatrix} 1 & r_{10} \\ r_{10} & 1 \end{pmatrix}, \quad r^1 = (r_{01}, r_{11}), \quad \hat{\phi}^1 = (\hat{\phi}_1, \hat{\phi}_2)$$

$$\hat{V}(\hat{\phi}_1,\hat{\phi}_2) = n^{-1}(1-r^1\hat{\phi})\ R_2^{-1} = n^{-1}(1-r_{01}\hat{\phi}_1-r_{11}\hat{\phi}_2)\ R_2^{-1}\ ,$$

where $\hat{V}(\hat{\phi}_1,\hat{\phi}_2)$ is the variance-covariance matrix of $(\hat{\phi}_1,\hat{\phi}_2)$. The minimum variances of $(\hat{\phi}_1,\hat{\phi}_2)$ may be found numerically by varying $(\hat{\phi}_1,\hat{\phi}_2)$ until the variance σ_a^2 is minimized for the particular sample.

The AR model may be given as an infinite AM model

$$z_{x,t} = \sum_{i=0}^{\infty}\sum_{i=0}^{\infty}\psi_{ij}a_{x-i,t-j},$$

$$\psi_{00}=1,\ \psi_{01}=\phi_1,\ \psi_{11}=\phi_2,\ \psi_{02}=\phi_1^2,\ \psi_{12}=2\phi_1\phi_2,\ \ \psi_{22}=\phi_2^2, \tag{6.11}$$

$$\psi_{0j}=\phi_1\psi_{0j-1},\ \psi_{ij}=\phi_1\psi_{i,j-1}+\phi_2\psi_{i-1,j-1},\ \psi_{jj}=\phi_2\pi_{j-1,j-1}\ ,$$

like (4.1) replacing π_{ij} by ψ_{ij}, and θ_i by ϕ_i.

For $\phi_1=.2$, $\phi_2=-.6$, $\psi_{00}=1$, $\psi_{01}=.2$, $\psi_{11}=-.6$, $\psi_{02}=.04$,

$\psi_{12}=-.24$, $\psi_{22}=.36$, $\psi_{03}=.008$, $\psi_{13}=-.072$, $\psi_{23}=.216$,

$\psi_{33}=.216$, etc.

An AR model simulation is given in Perry and Aroian (4).

7. ARMA MODELS

The results of Oprian et al. (7) are used freely in addition to some new results mainly in the determination of values of $\{\phi_1,\phi_2,\theta_1,\theta_2\}$ as related to the values of $\{\rho_{01},\rho_{10},\rho_{11},\rho_{-1,1}\}$. The results are:

$$\sigma_z^2 = \sigma_a^2\{1-\theta_1(\phi_1-\theta_1)-\theta_2(\phi_2-\theta_2)\}(1-\phi_1\rho_{01}-\phi_2\rho_{10})^{-1}\ ,$$

$$\rho_{01} = \phi_1+\phi_2\rho_{10}-\theta_1\sigma_a^2/\sigma_z^2;\ \theta_1\sigma_a^2/\sigma_z^2 = \phi_1+\phi_2\rho_{10}-\rho_{01} \tag{7.1}$$

$$\rho_{11} = \phi_1\rho_{10}+\phi_2-\theta_2\sigma_a^2/\sigma_z^2;\ \theta_2\sigma_a^2/\sigma_z^2 = \phi_1\rho_{10}+\phi_2-\rho_{11}$$

$$\rho_{10} = \phi_1\rho_{1-1}+\phi_2\rho_{01}-\theta_2(\phi_1-\theta_1)\sigma_a^2/\sigma_z^2\ .$$

The second and third equations may be solved for $\theta_i\sigma_a^2/\sigma_z^2$ and these results substituted in the first and fourth equations to obtain $\{\phi_1,\phi_2,\theta_1,\theta_2\}$. If $\phi_1=\phi_2=0$, or $\theta_1=\theta_2=0$, the corresponding MA or AR models are obtained. For $m,n \geq 2$,

$$\rho_{m,n} = \phi_1\rho_{m,n-1}+\phi_2\rho_{m-1,n-1}\ . \tag{7.2}$$

The ARMA model may be represented as an infinite moving MA model

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\psi_{ij}a_{x-i,t-j},\ \text{or} \tag{7.3}$$

an infinite AR model

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\pi_{ij}z_{x-i,t-j}, \tag{7.4}$$

restrictions already noted $|\phi_1|+|\phi_2|<1$ and $|\theta_1|+|\theta_2|<1$, naturally restricting the values of $\{\rho_{01},\rho_{10},\rho_{11}\}$ also. The infinite MA model is given by

$$z_{x,t} = (1-\theta_1 B_t - \theta_2 B_x B_t)(1-\phi_1 B_t - \phi_2 B_x B_t)^{-1} a_{x,t} =$$
$$(1-\theta_1 B_t - \theta_2 B_x B_t)\left(\sum_{i=0}^{\infty} (\phi_1 + \phi_2 B_x)^i B_t^i\right) a_{x,t}, \tag{7.5}$$

with $\psi_{00}=1,\ \psi_{01}=\phi_1-\theta_1,\ \psi_{11}=\phi_2-\theta_2,\ \psi_{02}=\phi_1,$

$$\psi_{12}=-\phi_1\theta_2+2\phi_1\phi_2-\phi_2\theta_1,\ldots,\ \psi_{0j}=\phi_1\psi_{0,j-1},$$
$$\psi_{ij}=\phi_1\psi_{i,j-1}+\phi_2\psi_{i-1,j-1},\ \psi_{ii}=\phi_2\psi_{i-1,\dot{8}-1}. \tag{7.6}$$

Similarly the ARMA model as an AR model:

$$\pi_{00}=1,\ \pi_{01}=\theta_1-\phi_1,\ \pi_{11}=\theta_2-\phi_2,\ \pi_{02}=\theta_1(\theta_1-\phi_1)$$
$$\pi_{12}=-\theta_1\phi_2+2\theta_1\theta_2-\theta_2\phi_1,\ \pi_{22}=\theta_2(\theta_2-\phi_2), \tag{7.7}$$
$$\pi_{0j}=\theta_1\pi_{0,j-1},\ \pi_{ij}=\theta_1\pi_{i,j-1}+\theta_2\pi_{i-1,j-1},$$
$$\pi_{ii}=\theta_2\pi_{i-1,i-1}\,.$$

Note (3.10) and (6.11) are special cases of (7.5) and (7.6). For

$$\phi_1=.2,\ \phi_2=-.6,\ \theta_1=.2,\ \theta_2=-.5,$$
$$\psi_{00}=1,\ \psi_{11}=-.1,\ \psi_{12}=-.02,\ \psi_{22}=-.6,\ \psi_{13}=-.004, \tag{7.8}$$
$$\psi_{23}=.024,\ \psi_{33}=-.036,\ \text{etc.}$$

The autocorrelation function must be found by using (7.5) and

$$\rho_{m,n}=(\Sigma\Sigma\psi_{ij}\psi_{i-m,j-n})\,(\Sigma\Sigma\psi_{ij}^2)^{-1},\ m=n\neq 0 \tag{7.9}$$

and

$$\sigma_z^2=\sigma_a^2\Sigma\Sigma\psi_{ij}^2\ . \tag{7.10}$$

Thus $\sigma_z^2/\sigma_a^2 = 1.01741$, obtained from (7.9) and (7.10), must be used in (7.1) to solve for $\{\rho_{01},\rho_{10},\rho_{11},\rho_{1-1}\} = \{.058,-.091,-.127,-.213\}$ as compared to the MA model $\{-.155,-.078,.389,0\}$, and the AR model $\{.33,-.21,-.64,-.07\}$. In fact, had $\phi_1=.2$, $\phi_2=-.6$, $\theta_1=.2$, $\theta_2=-.6$, been chosen, then the ARMA model would have been reduced to $(1-.2B_t+.6B_xB_t)z_{x,t}=(1-.2B_t+.6B_xB_t)a_{x,t}$, or $z_{x,t}=a_{x,t}$! Conversely, had a sample set of s_z^2/s_a^2, $\{r_{01},r_{10},r_{11},r_{1-1}\}$ been given, estimates $(\hat{\phi}_1,\hat{\phi}_2,\hat{\theta}_1,\hat{\theta}_2)$ could be found from (7.1) as already given by substituting the sample estimates for the population values. These results with the exception of (7.1) and the methods of solution given there are due to Robert Perry in his ongoing Ph.D. thesis.

How may the estimates of $(\phi_1,\phi_2,\theta_1,\theta_2)$ in a sample of n be found? One method is to choose the set $(\hat{\phi}_1,\hat{\phi}_2,\hat{\theta}_1,\hat{\theta}_2)$ which minimizes s_ε^2, the sample estimates of σ_ε^2 as done in 4 by varying the four constants numerically. The other method is to find the corresponding AR model through π_{22} and then apply the method given in section 5.

8. FORECASTING

Now that the models have been defined, their properties given, it is necessary to show how these results may be used for forecasting. It should be realized that in forecasting it is not necessary to assume the future repeats the past. All that is needed is to assume that the errors of the past will be repeated in the future, although the events themselves will not necessarily be formulas must not be used blindly, but must be checked carefully.

In the MA case (2.1):

$$z_{x+\ell_1,t+\ell_2} = a_{x+\ell_1,t+\ell_2} - \theta_1 a_{x+\ell_1,t+\ell_2-1} - \theta_2 a_{x+\ell_1-1,t+\ell_2-1} , \tag{8.11}$$

$$e_{x+\ell_1,t+\ell_2} = a_{x+\ell_1,t+\ell_2} - \theta_1 a_{x+\ell_1,t+\ell_2-1} - \theta_2 a_{x+\ell_1,t+\ell_2-1} \tag{8.12}$$

$$\hat{z}_{x,t}(\ell_1,\ell_2) = 0$$

$$\sigma_z^2(\ell_1,\ell_2) = \sigma_a^2(1+\theta_1^2+\theta_2^2) , \tag{8.13}$$

for $\ell_1 \geq 2$ or $\ell_2 \geq 2$. The other cases are

$$\hat{z}_{x,t}(0,1) = -\theta_1 a_{x,t} - \theta_2 a_{x-1,t},\ \hat{z}_{x,t}(1,0) = -\theta_2 a_{x,t-1}$$

$$e_{x,t}(0,1) = a_{x,t+1},\ e_{x,t}(1,0) = a_{x+1,t} - \theta_1 a_{x+1,t-1} \tag{8.14}$$

$$\hat{z}_{x,t}(1,1) = -\theta_2 a_{x,t},\ e_{x,t}(1,1) = a_{x+1,t+1} - \theta_1 a_{x+1,t}$$

and $\sigma_e^2(0,1) = \sigma_a^2$, $\sigma_e^2(1,0) = \sigma_a^2(1+\theta_1^2)$, $\sigma_e^2(1,1) = \sigma_a^2(1+\theta_1^2)$.

The more general ARMA model is changed to infinite MA model given in (7.5), and terms as far as $\psi_{10,10}$ carried for accuracy. The method is exactly the same. The models may be updated as new results are obtained.

Forecasts and forecast errors are correlated. In any MA model (and by extension any AR model or ARMA model) the forecast errors are.

$$e_{x,t}(0,1) = a_{x,t+1},\ e_{x,t}(1,0) = a_{x+1,t} - \theta_1 a_{x+1,t-1} ,$$

and

$$e_{x,t}(1,1) = a_{x+1,t+1} - \theta_1 a_{x+1,t} . \tag{8.15}$$

Hence

$$\rho\{e_{x,t}(0,1),\ e_{x,t}(1,0)\} = \rho\{(e_{x,t}(0,1),\ e_{x,t}(1,1)\} = 0,$$

$$\rho\{e_{x,t}(1,0),\ e_{x,t}(1,1)\} = -\theta_1(1+\theta_1^2)^{-1} . \tag{8.16}$$

Correlations among other forecast errors may be found. Forecasts are also correlated. From (8.14)

$$\rho\{\hat{z}_{x,t}(0.1),\hat{z}_{x,t}(1,0)\} = \rho\{\hat{z}_{x,t}(1,0),\hat{z}_{x,t}(1,1)\} = 0,$$

but

$$\rho\{\hat{z}_{x,t}(0,1),\hat{z}_{x,t}(1,1)\} = \theta_1(\theta_1^2+\theta_2^2)^{-1/2} . \tag{8.17}$$

The autocorrelation function for forecasts and forecast errors may be found in a similar manner.

It has been assumed in the foregoing that all constants θ_i are exact where in fact estimates of θ_i are used, including the sample mean $\bar{z}_{x,t}$. A discussion of this matter is given in Box and Jenkins (5), pages 267-268. For the mean, an additional source of variation, the σ_a^2/n is added to the formula for σ_z^2 given in (8.10) and is important if n is fairly small, $n \leq 100$, depending on the model. For the power spectra of the discussed models, see the papers of Voss et al. (3), Taneja and Aroian (6), and Oprian et al. (7).

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Some simple theoretical examples of time series in m dimensions, their main properties, estimation, and forecasting are given. These are generalizations of the usual time series to events in both time and space. How these models may be simulated is explained. One practical example of flooding on the Mohawk River is given. Included are new results in forecasting; the regions for permissible values of the autocorrelation and cross correlations; and methods for obtaining these correlations.

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