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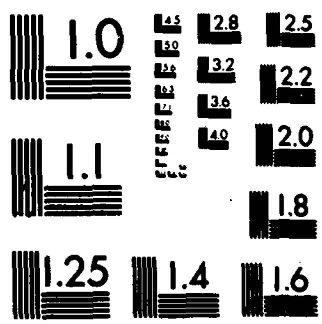

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PSR Report 1028

IMPROVED NOISE SUPPRESSION FOR MULTIBAND IMAGERY

P. J. Ready
D. A. Anderson
B. J. Gaitley

August 1980

Final Technical Report
Contract F49620-78-C-0059

Sponsored by
Air Force Office of Scientific Research
Bolling Air Force Base
Washington, D.C. 20332

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Developed and tests an approach to improvement of noise degraded multiband imagery based on a class dependent Karhunen-Loeve (K-L) transformation and transform domain spatial filtering. The technique is attractive because of itstendency to preserve the high spatial frequency information (e.g., edges in thepicture as compared with conventional minimum mean square error linear filtering of the original bands. Multiband images (especially color and multispectral characteristically exhibit significant class structure, and within a particular class show high spectral correlations. The filtering

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A technique developed during this research exploits both the class structure and the intraclass correlation to achieve noise suppression.

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CONTENTS

FINAL TECHNICAL REPORT

IMPROVED NOISE SUPPRESSION FOR MULTIBAND IMAGERY

1.0 INTRODUCTION

This research has developed and tested an approach to improvement of noise degraded multiband imagery based on a class dependent Karhunen-Loeve (K-L) transformation and transform domain spatial filtering. The technique is attractive because of its tendency to preserve the high spatial frequency information (e.g., edges) in the picture as compared to conventional minimum mean square error linear filtering of the original bands. By multi-band we mean a set of images of the same scene as recorded in different optical wavelengths, at different times, or a combination of both. For example, an ordinary color photograph is composed of three bands--red, green, and blue. Multispectral imagery is another example for which the same scene is imaged in several bands ranging from the visible to the infrared. Color video (e.g., television) is an example of the combination of spectral and temporal bands.

Multi-band images (especially color and multispectral) characteristically exhibit significant class structure, and within a particular class shows high spectral correlations. The filtering technique developed during this research exploits both the class structure and the intra class correlation to achive noise suppression.

2.0 DEFINITIONS

The multi-band image is viewed as a vector random process $\underline{P}(x, y)$ where

$$\underline{P}(x, y) \triangleq \begin{bmatrix} P_1(x, y) \\ P_2(x, y) \\ \vdots \\ P_N(x, y) \end{bmatrix} \qquad (1)$$

and $P_i(x, y)$ is the amplitude of the ith image having spatial variables x and y. We define the N × N spectral covariance matrix as

$$\underset{\sim}{C}(x, y) \triangleq E\{\underline{P}(x, y)\ \underline{P}^t(x, y)\} \tag{2}$$

where E is the expectation, t indicates transpose, and $E\{\underline{P}(x, y)\} = \underline{0}$ is assumed. We also assume spatial stationarity so that

$$\underset{\sim}{C}(x, y) = C\ . \tag{3}$$

The general stationary covariance matrix for the process is

$$\underset{\sim}{K}(\tau_x, \tau_y) \triangleq E\{\underline{P}(x, y)\ \underline{P}^t(x + \tau_x, y + \tau_y)\}\ , \tag{4}$$

which includes the spatial covariance. The (i, j) the element of $K(\tau_x, \tau_y)$ is

$$k_{i,j}(\tau_x, \tau_y) = E\{P_i(x, y)\ P_j(x + \tau_x, y + \tau_y)\}\ . \tag{5}$$

To a good approximation,

$$k_{i,j}(\tau_x, \tau_y) = c_{i,j}\ R(\tau_x, \tau_y) \tag{6}$$

where $c_{i,j}$ is the (i, j)th element of $\underset{\sim}{C}$ and

$$R(\tau_x, \tau_y) \triangleq E\{P_i(x, y)\ P_i(x + \tau_x, y + \tau_y)\}/\sigma^2 \quad i = 1, 2, \ldots, N \tag{7}$$

is the normalized scaler autocovariance function for all elements of $\underline{P}(x, y)$. That is, each band is assumed to have the same spatial autocovariance function $R(\tau_x, \tau_y)$ and variance σ^2. This is verified experimentally as shown in Figure 1 where we show the (azimuthally averaged) autocovariance function for the red, green, and blue bands

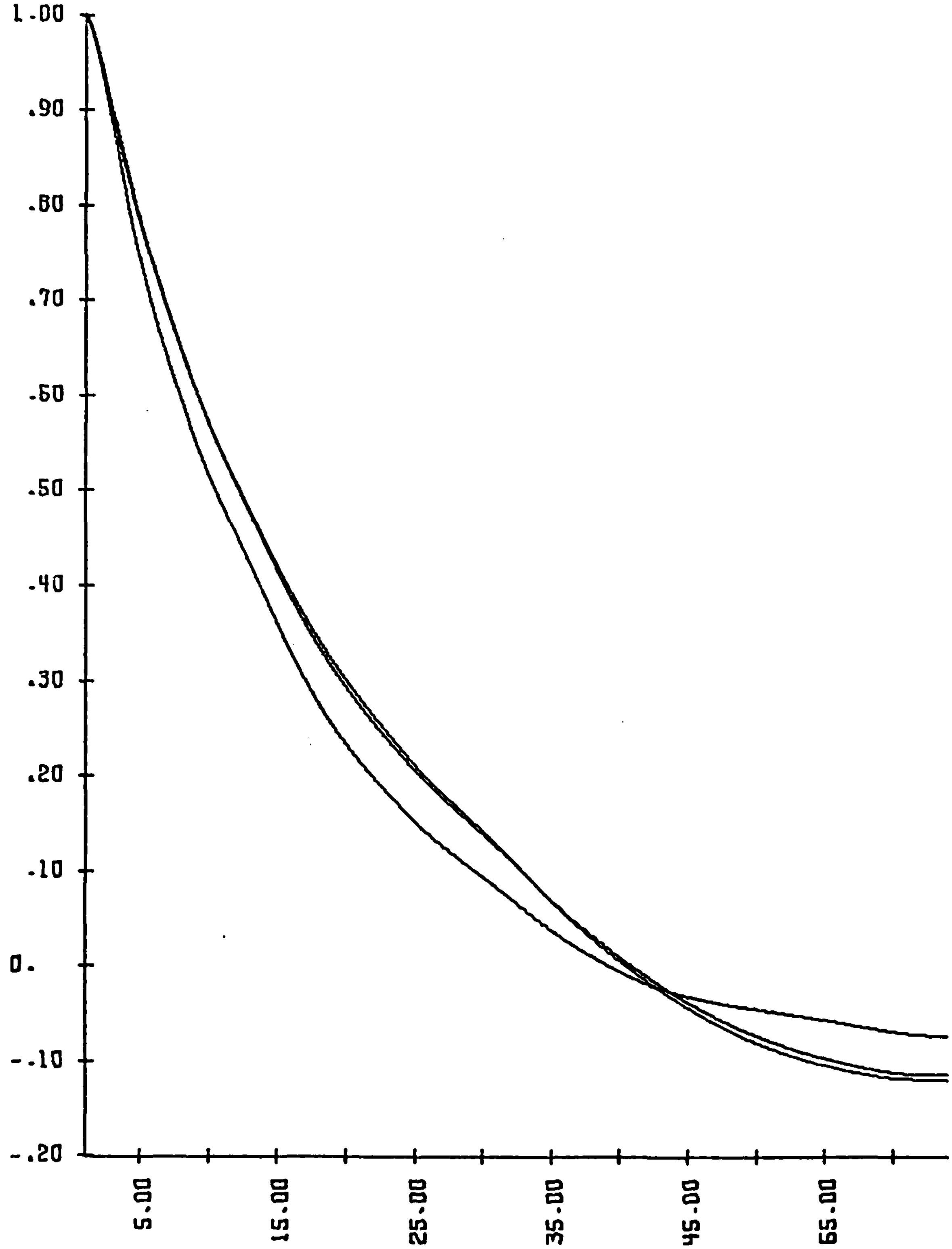

Fig. 1--Azimuthal Average of Red, Green, and Blue Autocorrelation Functions.

of a "typical" test image.

From (4) and (6),

$$\underset{\sim}{K}(\tau_x, \tau_y) = \underset{\sim}{C}\, R(\tau_x, \tau_y) \ . \tag{8}$$

With the above definitions and assumptions we next derive the general covariance matrix for the principle component vector $\underline{\alpha}(x, y)$,

$$\underline{\alpha}(x, y) \triangleq \underset{\sim}{T}\, \underline{P}(x, y) \tag{9}$$

where $\underset{\sim}{T}$ is the N × N matrix of eigenvectors of $\underset{\sim}{C}$. Thus,

$$\begin{aligned}\underset{\sim}{K}_{\alpha}(\tau_x, \tau_y) &\triangleq E\{\underline{\alpha}(x, y)\, \underline{\alpha}^t(x + \tau_x, y + \tau_y)\} \\ &= E\{\underset{\sim}{T}\, \underline{P}(x, y)\, \underline{P}^t(x + \tau_x, y + \tau_y)\underset{\sim}{T}^t\} \\ &= \underset{\sim}{T}\, \underset{\sim}{K}(\tau_x, \tau_y)\underset{\sim}{T}^t \ . \end{aligned} \tag{10}$$

But $\underset{\sim}{K}(\tau_x, \tau_y) = \underset{\sim}{C}\, R(\tau_x, \tau_y)$. Thus (10) becomes

$$\begin{aligned}\underset{\sim}{K}_{\alpha}(\tau_x, \tau_y) &= (\underset{\sim}{T}\, \underset{\sim}{C}\, T^t)\, R(\tau_x, \tau_y) \\ &= R(\tau_x, \tau_y) \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_N \end{bmatrix} \\ &= R(\tau_x, \tau_y)\underset{\sim}{\lambda} \end{aligned} \tag{11}$$

since $\underset{\sim}{C}$ is diagonalized by the orthogonal similarity transformation $\underset{\sim}{T}\,\underset{\sim}{C}\,\underset{\sim}{T}^t$. The λ_i, i = 1, 2, ..., N are the eigenvalues of $\underset{\sim}{C}$ ordered such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$.

Equation (11) is a key result. It states that the principal component images are orthogonal, that their variances are destributed as the λ_i, and that their individual spatial autocovariance functions are

$$R_{i_\alpha}(\tau_x, \tau_y) \triangleq E\{\alpha_i(x, y)\ \alpha_i(x + \tau_x, y + \tau_y)\}$$

$$= \lambda_i\ R(\tau_x, \tau_y) \qquad i = 1, 2, \ldots, N. \tag{12}$$

That is, each $R_{i_\alpha}(\tau_x, \tau_y)$ is a scaled (by λ_i) replica of the original spatial autocovariance function. The impact of this result is developed next.

3.0 DERIVATION OF THE MATRIX WIENER-HOPF EQUATION

To determine the filter generating the minimum variance linear estimate of a vector picture process received in the presence of additive white noise the following problem formulation is needed. Suppose $\underline{P}(\underline{x})$ is the n-element vector picture process, with $\underline{x} = (x, y)$ the generic point on the picture surface, and $\underline{n}(\underline{x})$ is the additive white observation noise. Then, the problem is to find a matrix spatial impulse response $\underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})$ such that

$$\min_{\text{all } \underset{\sim}{H}(\underline{x}, \underline{y})} E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})[\underline{P}(\underline{y}) + \underline{n}(y)]d\underline{y}\|^2\}$$

$$= E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})[\underline{P}(\underline{y}) + \underline{n}(\underline{y})]d\underline{y}\|^2\} \tag{13}$$

for all $\underline{x}$ in the 2-dimensional area A defined by the picture. Here "all $\underset{\sim}{H}(\underline{x}, \underline{y})$" means all square-integrable $\underset{\sim}{H}(\underline{x}, \underline{y})$, i.e., all matrices $\underset{\sim}{H}(\underline{x}, \underline{y})=[\{h_{ij}(\underline{x}, \underline{y})\}^n_{i,j=1}]$ such that

$$\int_A\int_A \| \underset{\sim}{H}(\underline{x}, \underline{y})\|^2 d\underline{x}d\underline{y} \triangleq \int_A\int_A \sum_{i,j=1}^{n} h^2_{ij}(\underline{s}, \underline{y})\, d\underline{x}d\underline{y}$$

$$< +\infty$$

For convenience, $\underline{P}(\underline{x}) + \underline{n}(\underline{x})$, the received picture process, will be denoted by $\underline{r}(x)$:

$$\underline{r}(\underline{x}) \triangleq \underline{P}(\underline{x}) + \underline{n}(\underline{x}) .$$

Here

$$E[\underline{r}(\underline{x})\underline{r}^t(y)] = E[\underline{P}(\underline{x})P^t(\underline{y})]$$

$$+ E[\underline{n}(\underline{x})\underline{n}^t(\underline{y})] = E[\underline{P}(\underline{x})\underline{P}^t(\underline{y})]$$

$$+ \frac{1}{2}N_0\delta(\underline{x}-\underline{y})\underset{\sim}{I} \tag{14}$$

$\{\underset{\sim}{I} = \{\delta_{ij}\}$ and for any $\underline{v}$, $\underline{v}^t$ is the transpose of $\underline{v}\}$.

Now it will be verified that if $\underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})$ satisfies a matrix analogue of the scalar Wiener-Hopf integral equation $H^{(0)}(\underline{x}, \underline{y})$ does indeed provide the minimization of mean square error described by (1). It will be assumed that

$$E\{[\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}]\underline{r}^t(\underline{z})\} = 0, \text{ all } \underline{z} \text{ in } A \tag{15}$$

Note that (15) is equivalent to

$$R_{\underline{P},\underline{r}}(\underline{x}, \underline{z}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})R_{\underline{r},\underline{r}}(\underline{y}, \underline{z})d\underline{y} = 0 \tag{16}$$

where

$$R_{\underline{P},\underline{r}}(\underline{x}, \underline{z}) \triangleq E[\underline{P}(\underline{x})\underline{r}^t(\underline{z})]$$

and

$$R_{\underline{r},\underline{r}}(\underline{y}. \underline{z}) \triangleq E[\underline{r}(y)\underline{r}^t(z)].$$

Thus, (15) is, in the form (16), a matrix Wiener-Hopf equation.

Write the arbitrary square integrable $\underset{\sim}{H}(\underline{x}, \underline{y})$ as a perturbation of $H^{(0)}(\underline{x}, \underline{y})$:

$$\underset{\sim}{H}(\underline{x}, \underline{y}) = \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y}) = \underline{\Delta}(\underline{x}, \underline{y}) .$$

Then, the following sequence of equalities holds, where the operator "tr" is defined by the equation

$$\mathrm{tr}(\{m_{ij}\}) \triangleq \sum_{i=1}^{n} m_{ii}, \text{ all matrices } \{m_{ij}\}$$

$$E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\}$$

$$= E\{\mathrm{tr}[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\underline{P}(x) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\}$$

$$= E\{\mathrm{tr}[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y} - \int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})$$

$$\times (\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y} - \int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\}$$

$$= E\{\mathrm{tr}[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t$$

$$- (\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t$$

$$+ E\{tr[-(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t$$

$$+ (\int_A \underline{\Delta}(\underline{x}. \underline{y})\underline{r}(\underline{y})d\underline{y})(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\} \quad (17)$$

Since the "tr" operator is linear and is invariant under matrix transposition, (17) becomes

$$E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\}$$

$$= E\{tr[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\underline{P}(\underline{x} - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\}$$

$$- 2E\{tr[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\}$$

$$+ E\{tr[(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\} \quad (18)$$

But, first,

$$E\{tr[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\}$$

$$= E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\} \quad (19)$$

Moreover,

$$E\{tr[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\}$$

$$= E\{tr[\int_A [(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})\underline{r}^t(\underline{z})]\underline{\Delta}^t(\underline{x}, \underline{z})d\underline{z}]\}$$

$$= tr\{\int_A E[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})\underline{r}^t(\underline{z})]\underline{\Delta}^t(\underline{x}, \underline{z})d\underline{z}\} \quad (20)$$

Finally, using (14),

$$E\{tr[(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})^t]\}$$

$$= E\{tr[(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})(\int_A \underline{r}^t(\underline{y})\underline{\Delta}^t(\underline{x}, \underline{y})d\underline{y})]\}$$

$$= E\{tr[\int_A \int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{r}(\underline{y})r^t(\underline{z})\underline{\Delta}^t(\underline{x}, \underline{z})d\underline{y}d\underline{z}]\}$$

$$= tr[\int_A \int_A \underline{\Delta}(\underline{x}, \underline{y})E[\underline{r}(\underline{y})\underline{r}^t(\underline{z})]\underline{\Delta}^t(\underline{x}, \underline{z})d\underline{y}d\underline{z}]$$

$$= tr[\int_A \int_A \underline{\Delta}(\underline{x}, \underline{y})E[\underline{P}(\underline{y})\underline{P}^t(\underline{z})]\Delta^t(\underline{x}, \underline{z})d\underline{y}d\underline{z}]$$

$$+ \operatorname{tr}[\int_A \int_A \underline{\Delta}(\underline{x}, \underline{y})(\frac{N_0}{2} \underset{\sim}{I})\underline{\Delta}^t(\underline{x}, \underline{z})d\underline{y}d\underline{z}$$

$$= E\{\operatorname{tr}[(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{P}(\underline{y})d\underline{y})(\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{P}(\underline{y})d\underline{y})^t]\}$$

$$+ \operatorname{tr}[(\text{Area}(a))\int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{\Delta}^t(\underline{x}, \underline{y})d\underline{y}]$$

$$= E\{\| \int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{P}(\underline{y})d\underline{y}\|^2\} + \text{Area}(A)\int_A \| \underline{\Delta}(\underline{x}, \underline{y})\| \, d\underline{y} \qquad (21)$$

where

$$\| \underline{\Delta}(\underline{x}, \underline{y})\|^2 = \sum_{i,j=1}^{3} \Delta_{ij}^2(\underline{x}, \underline{y}) \; .$$

Hence, combining (18), (19), (20) and (21), there results

$$E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\} = E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\}$$

$$+ \operatorname{tr}\{\int_A E[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})d\underline{y})\underline{r}^t(\underline{z})]\underline{\Delta}^t(\underline{x}, \underline{z})d\underline{z}\}$$

$$+ E\{\| \int_A \underline{\Delta}(\underline{x}, \underline{y})\underline{P}(\underline{y})d\underline{y}\|^2\} + \text{Area}(A)\int_A \| \underline{\Delta}(\underline{x}, \underline{y})\|^2 d\underline{y} \qquad (22)$$

Then, invoking the Wiener-Hopf equation (15), there results

$$E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\} = E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\}$$

$$+ E\{\| \int_A \underset{\sim}{\Delta}(\underline{x}, \underline{y})\underline{P}(\underline{y})d\underline{y}\|^2\} + \text{Area}(A)\int_A \| \underset{\sim}{\Delta}(\underline{x}, \underline{y})\|^2 d\underline{y} \qquad (23)$$

Thus, whenever for some i_0 and j_0

$$\int_A \Delta^2_{i_0 j_0}(\underline{x}, \underline{y})d\underline{y} > 0,$$

it follows from (22) that

$$E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\} > E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\}$$

But, when for all i and j

$$\int_A \Delta^2_{ij}(\underline{x}, \underline{y})d\underline{y} = 0,$$

i.e., when

$$\int_A \| \underset{\sim}{\Delta}(\underline{x}, \underline{y})\|^2 = 0,$$

$$E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\| \} = E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})\|^2\}$$

by (22).

Hence, not only does $\underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})$ minimize $E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\}$

but any minimizing $\underset{\sim}{H}(\underline{x}, \underline{y})$ has the property that

$$\int_A\int_A \| \underset{\sim}{H}(\underline{x}, \underline{y}) - H^{(0)}(\underline{x}, \underline{y})\|^2 d\underline{x}d\underline{y}$$

$$= \int_A\int_A \| \underline{\Delta}(\underline{x}, \underline{y})\|^2 d\underline{x}d\underline{y}$$

$$= 0,$$

i.e., $\underset{\sim}{H}(\underline{x}, \underline{y})$ differs from $H^{(0)}(\underline{x}, \underline{y})$ by a null function.

Thus a solution of the Wiener-Hopf equation (15) (or (16)) not only minimizes the mean square error but it is unique in that respect to within a null function. This means, in addition, that if the Wiener-Hopf equation has one square-integrable solution it is unique to within a null function.

To see that the existence of a solution of the Wiener-Hopf equation is a necessary, as well as sufficient, condition for the existence of a minimizing choice of $\underset{\sim}{H}(\underline{x}, \underline{y})$ one must simply analyze equation (22). This equation is true independent of whether or not $\underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})$ is a solution of the Wiener-Hopf equation. But if $\underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})$ is not a solution of the Wiener-Hopf then set

$$\Delta(\underline{x}, \underline{z}) = -\epsilon \underset{\sim}{M}(\underline{x}, \underline{z}) \tag{24}$$

where ϵ is an arbitrary positive number and

$$\underset{\sim}{M}(\underline{x}, \underline{z}) \triangleq E[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})d\underline{y})\underline{r}^t(\underline{z})] \tag{25}$$

In this case,

$$\mathrm{tr}\{\int_A E[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})d\underline{y})\underline{r}^t(\underline{z})]\ \underline{\Delta}^t(\underline{x}, \underline{y})d\underline{z}$$

$$= -\varepsilon\,\mathrm{tr}\{\int_A \underset{\sim}{M}(\underline{x}, \underline{z})\underset{\sim}{M}^t(\underline{x}, \underline{z})d\underline{z}\}$$

$$= -\varepsilon \int_A \mathrm{tr}\{\underset{\sim}{M}(\underline{x}, \underline{z})\underset{\sim}{M}^t(\underline{x}, \underline{z})\}d\underline{z}$$

$$= -\varepsilon \int_A \| \underset{\sim}{M}(\underline{x}, \underline{z}) \|^2 d\underline{z} \qquad (26)$$

As a result of (26), (22) becomes

$$E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\} = E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\}$$

$$- \varepsilon\int_A \| \underset{\sim}{M}(\underline{x}, \underline{z}) \|^2 d\underline{z} + E\{\| \int_A (-\varepsilon)\underset{\sim}{M}(\underline{x}, \underline{y})\underline{P}(\underline{y})d\underline{y}\|^2\}$$

$$+ \mathrm{Area}(A)\int_A \| (-\varepsilon)\underset{\sim}{M}(\underline{x}, \underline{y}) \|^2 d\underline{y}$$

$$= E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\} - \varepsilon\int_A \| \underset{\sim}{M}(\underline{x}, \underline{z}) \|^2 d\underline{z}$$

$$+ \varepsilon^2\{E\{\| \int_A \underset{\sim}{M}(\underline{x}, \underline{y})\underline{P}(\underline{y})d\underline{y}\|^2\} + \text{Area}(A)\int_A \| \underset{\sim}{M}(\underline{x}, \underline{y})\|^2 d\underline{y}\} \quad (27)$$

Assuming that $\underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})$ is not a solution of the Wiener-Hopf equation and remembering the definition of $\underset{\sim}{M}(\underline{x}, \underline{y})$, (25) makes it clear that

$$- \varepsilon \int_A \| \underset{\sim}{M}(\underline{x}, \underline{z})\|^2 d\underline{z} < 0$$

Thus, for ε sufficiently small (27) shows that

$$E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\} < E\{\| \underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y}\|^2\}$$

where, by (24) and (25),

$$H(\underline{x}, \underline{z}) = \underset{\sim}{H}^{(0)}(\underline{x}, \underline{z}) + \underset{\sim}{\Delta}(\underline{x}, \underline{z})$$

$$= H^{(0)}(\underline{x}, \underline{z}) - \varepsilon E[(\underline{P}(\underline{x}) - \int_A \underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})\underline{r}(\underline{y})d\underline{y})\underline{r}^t(\underline{z})]$$

Hence, it has been shown that if $\underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})$ is not a solution to the Wiener-Hopf equation $\underset{\sim}{H}^{(0)}(\underline{x}, \underline{y})$ does not minimize the mean-square error.

4.0 SOLUTION UNDER THE ASSUMPTION OF STATIONARITY AND NON-CAUSALITY

If we allow the matrix filter to be non-causal and further assume spatial stationarity, we find the result,

$$\underset{\sim}{H}(\omega_x, \omega_y) = \underset{\sim}{T}^t \underset{\sim}{G}(\omega_x, \omega_y)\underset{\sim}{T} \tag{28}$$

where

$$\underset{\sim}{G}(\omega_x, \omega_y) = \begin{bmatrix} \frac{\lambda_1 S(\omega_x, \omega_y)}{\lambda_1 S(\omega_x, \omega_y) + \sigma_n^2} & & \\ & \ddots & \\ & & \frac{\lambda_N S(\omega_x, \omega_y)}{\lambda_N S(\omega_x, \omega_y) + \sigma_N^2} \end{bmatrix} \tag{29}$$

and $S(\omega_x, \omega_y)$ is the spatial power spectral density of the original bands, and σ_N^2 is the noise variance. This matrix filter consists of N^2 filters given by

$$H_{i,j}(\omega_x, \omega_y) = \sum_{k=1}^{N} e_{k,i}\, e_{k,j} \left(\frac{\lambda_n\, S(\omega_x, \omega_y)}{\lambda_n\, S(\omega_x, \omega_y) + \sigma_n^2} \right) \tag{30}$$

where $e_{k,i}$ is the k,i-th element of $\underset{\sim}{T}$ (i-th element of the k-th eigenvector of $\underset{\sim}{C}$).

Implementation of the N^2 filters requires

$$O_H = N^2 N_S^2 (1 + 4 \log_2 N_S)$$

operations (multiplications and additions), where N_S is the number of samples per line in each of the original (square) images. If we implement the filters using the $\underset{\sim}{G}$ matrix preceded and proceeded by the K-L transform as indicated in (28), then the number of operations is

$$O_{K-L} = N N_S^2 [2N + (1 + 4 \log_2 N_S)] \ .$$

Clearly,

$$\frac{O_{K-L}}{O_H} < 1 \text{ for } N \geq 2,\ N_S \geq 2 \ .$$

In fact,

$$\lim_{N_S \to \infty} \frac{O_{K-L}}{O_H} = \frac{1}{N}$$

and the filter implementation in (28) results in an approximate 1/N reduction in the number of operations for typical pictures ($N_S \geq 256$). For example, with $N_S = 256$ and $N = 3$ (color data), $O_G/O_H = 1/2.54$ (7.68×10^6 ops versus 19.5×10^6 ops).

5.0 CLASS-DEPENDENT K-L TRANSFORMATION

The presence of linear class structure within multiband data suggests the use of K-L transforms matched to the covariance matrices of each class. In so doing we achieve a piece wise linear approximation to a non-linear transformation of the multiband data, and approximate the "intrinsic dimensionality" of the multiband data. The result is a further compression of data variance into a smaller number of principal components compared to the global K-L based on a single covariance matrix computed over the entire picture.

Our approach to classification is based on a simple unsupervised clustering algorithm in which an initial (random) guess of both the number of classes and their mean vectors are iterated until the mean vectors remain constant and the inter class distances (Bhattacharyya distance) satisfy a threshold. Upon convergence of the algorithm we have the number of class and their covariance matrices. This information is used to compute the required set of K-L transforms, and to implement a conventional liklehood-ratio hypothesis test for each data vector comprising the multiband image.

Once the data is classified we then perform the class dependent K-L transformation. The MSE-optimum spatial filters are applied to the principal component images, followed by inverse transformation.

The effectiveness of the above procedure has been verified using a color test image (3 bands: red, green, and blue) of an F-16 aircraft obtained from the USC Image Processing Institute.

In Figure 2 we show histograms (estimates of the probability density function) of a) the original data, b) global principal components and c) the class dependent principal components. Note the significant reduction in variance in c).

For this particular date set the clustering algorithm generated five classes. These classes are depected in Figure 3. The uniqueness of each class is somewhat evident in Figures (4) and (5) where we show a two dimensional projection of the three dimensional joint density function for the original bands. Note the varying directions of maximum variance for each class, and therefore, the appropriateness of separate K-L transform for each class.

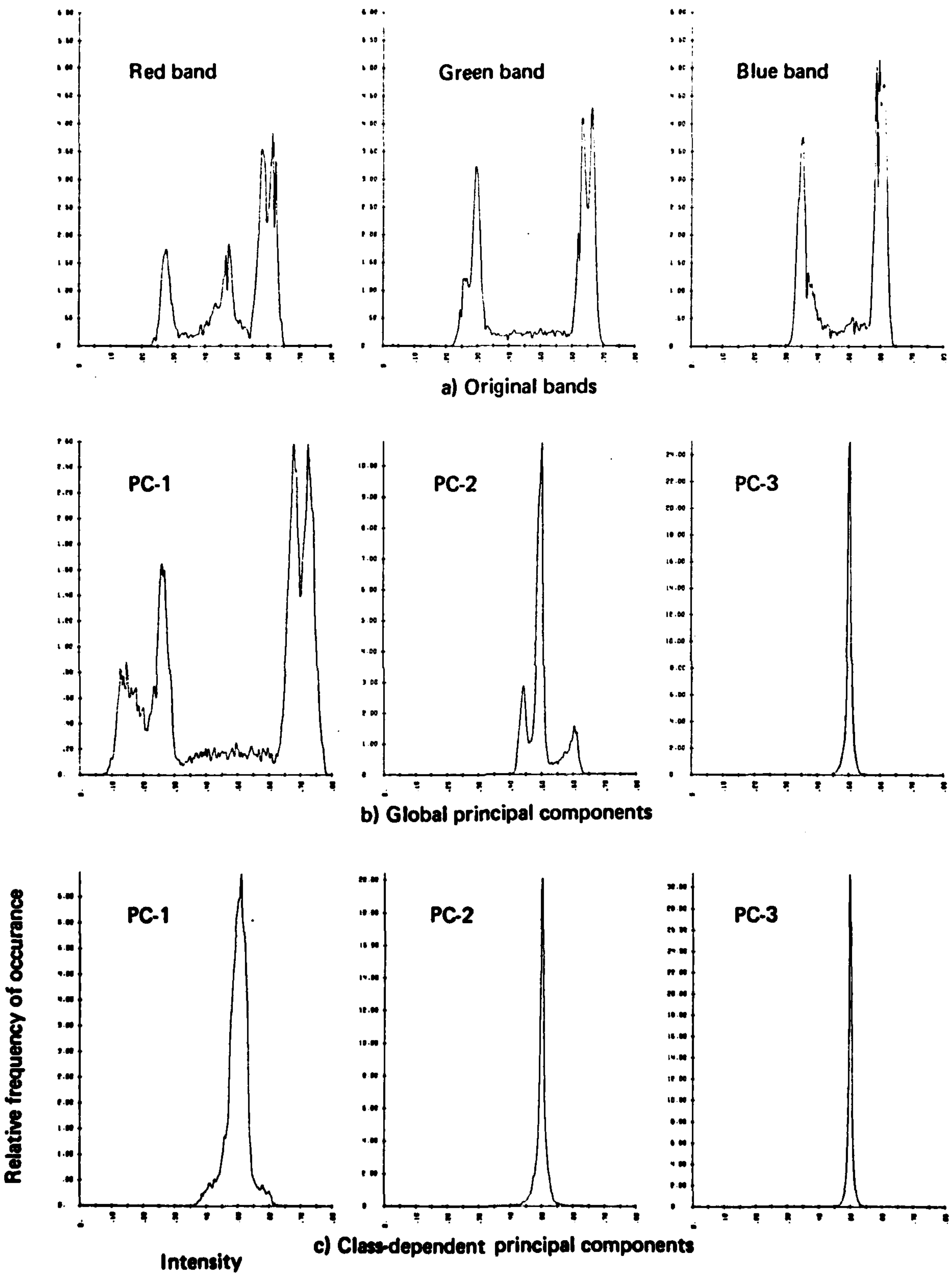

Fig. 2--Comparison of density functions

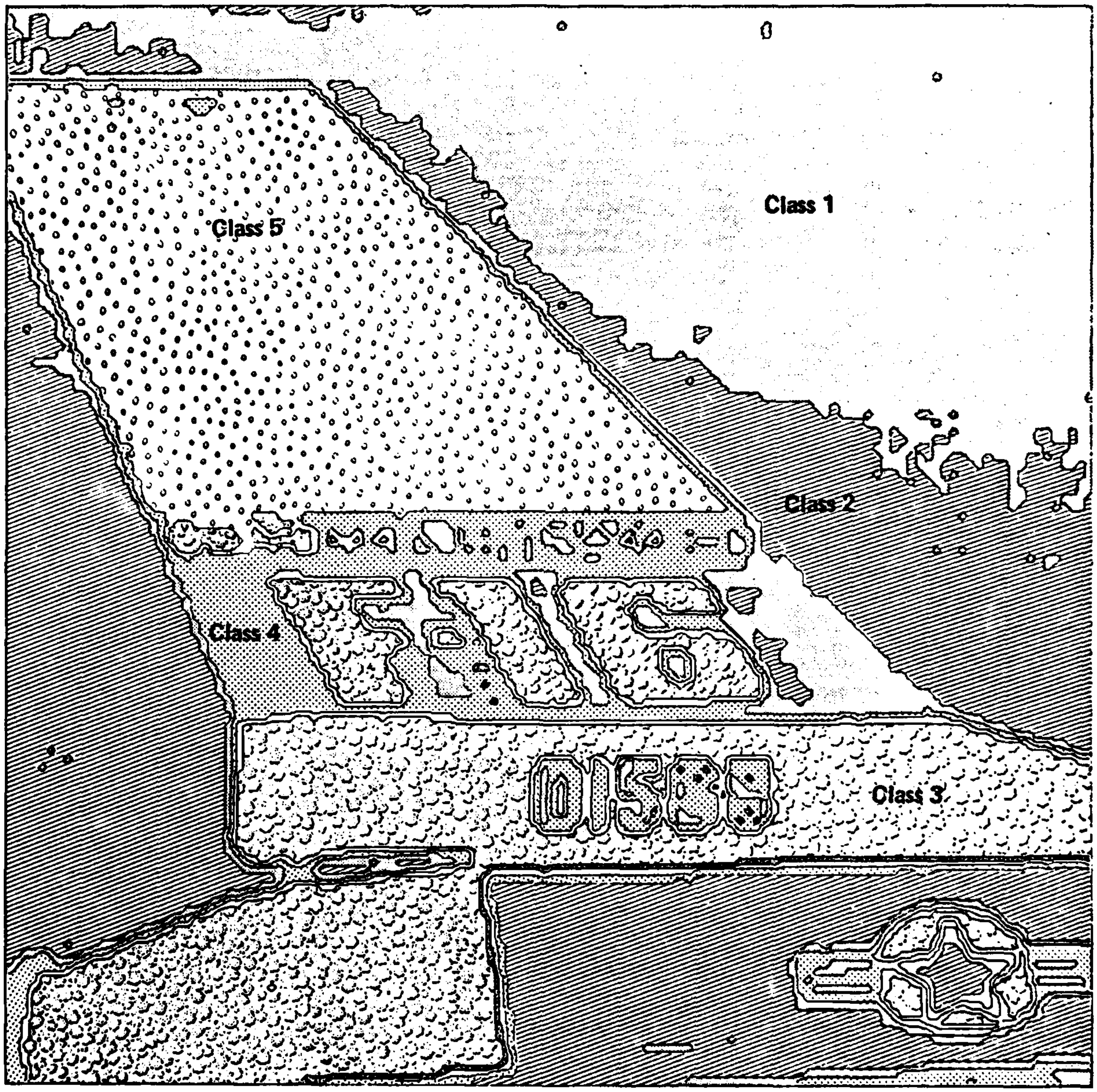

Fig. 3--Map of estimated classes

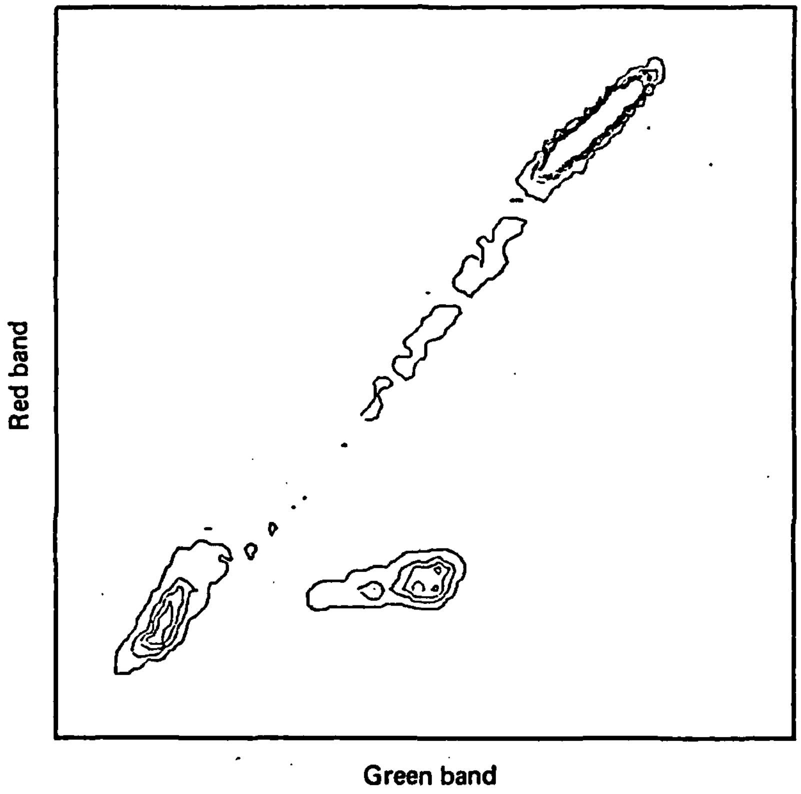

Fig. 4--Contour plot of red-green joint density

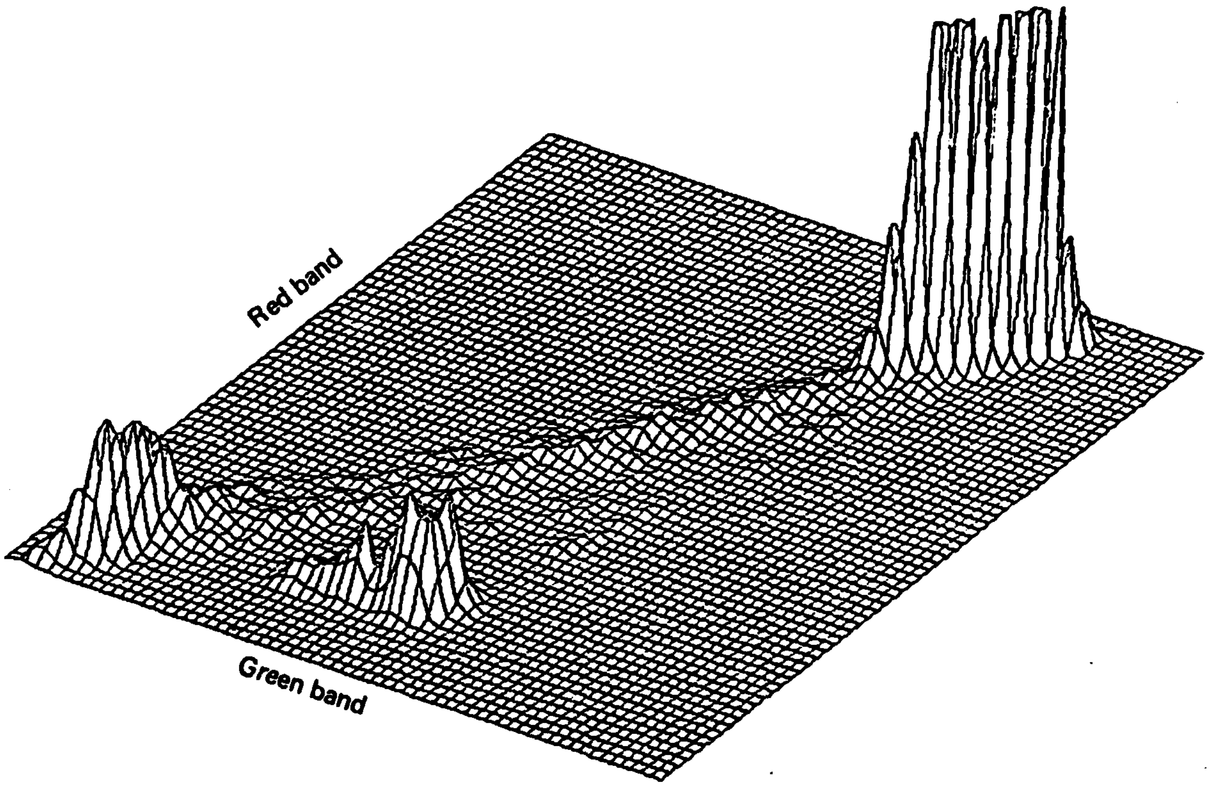

Fig. 5--Three dimensional view of Fig. (4)