

✓

⑥ LEVEL II

AD A092017



STUDENTS FACULTY STUDY R
ESEARCH DEVELOPMENT FUT
URE CAREER CREATIVITY CC
MMUNITY LEADERSHIP TECH
NOLOGY FRONTIER DESIGN
ENGINEERING APPREHENSIVE
GEORGE WASHINGTON UNIV

DTIC
ELECTE
S NOV 24 1980 D
B

DDC FILE COPY

DISTRIBUTION STATEMENT A
Approved for public release
Distribution Unlimited

SCHOOL OF ENGINEERING
AND APPLIED SCIENCE



⑥ LEVEL II

11) Apr 12

by

R. H. / Lang

Department of Electrical Engineering and Computer Science
The George Washington University
Washington, D.C. 20052

15) 1977-78-77-2-1171

Supplement to Final Report.

April 1980

DTIC
ELECTE

NOV 24 1980

Approved for public release; distribution unlimited

Prepared for:

U.S. Army Engineer Topographic Laboratories
Fort Belvoir, Virginia 22060
Contract DAAK 70-77-C-0142

DESTROY THIS REPORT WHEN NO LONGER NEEDED.
DO NOT RETURN IT TO THE ORIGINATOR.

THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL
DEPARTMENT OF THE ARMY POSITION UNLESS SO DESIGNATED BY OTHER
AUTHORIZED DOCUMENTS.

THE CITATION IN THIS REPORT OF TRADE NAMES OF COMMERCIALY
AVAILABLE PRODUCTS DOES NOT CONSTITUTE OFFICIAL ENDORSEMENT
OR APPROVAL OF THE USE OF SUCH PRODUCTS.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ETL-0215	2. GOVT ACCESSION NO. AD-A092 047	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DISCRETE SCATTERING APPROACH TO VEGETATION MODELING		5. TYPE OF REPORT & PERIOD COVERED Contract Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Roger H. Lang		8. CONTRACT OR GRANT NUMBER(s) DAAK70-77-C-0142
9. PERFORMING ORGANIZATION NAME AND ADDRESS George Washington University Washington, D.C.		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 4A161102B52CA3
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Engineer Topographic Laboratories Fort Belvoir, Virginia 22060		12. REPORT DATE April 1980
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Electromagnetic Wave Scattering, Random Media, Vegetation		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report studies microwave backscattering from a forest canopy which is modeled by a collection of dielectric discs with random orientation and position. The report begins by analyzing the mean field in a tenuous distribution of discrete scatterers. The correlation of the field is found by employing the distorted Born approximation. The above is then specialized to a half space of discrete scatterers with azimuthal symmetry. Horizontal,		

vertical and cross polarized backscattering coefficients for the half space are found. A comparison with experiment is made for the special case of lossy dielectric discs.

M

FOREWORD

This work was performed under U.S. Army Engineer Topographic Laboratories contract DAA70-77-6-0142. The support and encouragement of Mr. R. Hevenor of ETL is gratefully acknowledged. We would also like to thank Mr. Selim Seker of the George Washington University for performing the numerical calculations reported herein.

Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

TABLE OF CONTENTS

I. Introduction	<u>Page</u> 1
II. Problem Formulation	4
III. Single Scatterer-Transition Operator	7
IV. Coherent Field	11
V. Correlation	18
VI. Backscattering Coefficients for a Half Space of Dipoles	20
VII. Discussion and Numerical Evaluation	31
VIII. Conclusion and Recommendations	43
IX. References	44
Appendix A - Relationship Between the Transition Operator and the Scattering Amplitude	46
Appendix B - Polarizability Statistics	47
Appendix C - Relationship Between the Backscattering Coefficients and the Transverse Spectral Density	49

List of Figures

Figure 1	Distribution of particles within volume V	5
Figure 2	Incidence wave on half space of uniformly distributed particles	21
Figure 3	Principal axes of scatterer	23
Figure 4	Backscattering coefficients versus angle of incident for $\Delta\theta_{//} = 10^\circ$	34
Figure 5	Backscattering coefficients versus angle incident for $\Delta\theta_{//} = 30^\circ$	35
Figure 6	Backscattering coefficients versus angle incident for $\Delta\theta_{//} = 60^\circ$	36
Figure 7	Backscattering coefficients versus angle of incident for $\Delta\theta_{//} = \Delta\theta_{\perp} = 90^\circ$	37
Figure 8	Backscattering coefficients versus angle of incident for $\Delta\theta_{\perp} = 60^\circ$	38
Figure 9	Backscattering coefficients versus angle of incident for $\Delta\theta_{\perp} = 10^\circ$	39
Figure 10	Comparison of experimental and theoretical results for $\Delta\theta_{\perp} = 60^\circ$	40

INTRODUCTION

This report studies microwave backscattering from a forest canopy. The canopy is modeled by lossy dielectric discs having random orientation. Explicit expressions are derived for the horizontal, vertical and cross polarized backscattering coefficients. The results are then compared with experiment.

This work was motivated by the need to relate radar return to the characteristics of the scattering objects. The situation is particularly complex in the case of vegetated terrain which consists of an ensemble of many highly irregular objects placed in a more or less random fashion. Using the scattered return from a vegetated terrain one would like to obtain such information as height, density, average leaf and branch orientation so that the vegetation can be classified. This type of information is important for military analysis of the terrain. In addition, if the terrain has been determined adequately from an electromagnetic point of view, the backscatter information can be used to predict radar returns at other angles and frequencies not measured. This would be of great aid to the development of manageable data bases for radar simulations.

The above applications have served as a motivation for the development of electromagnetic models for the vegetated terrain. These models have been constructed by replacing the vegetated region with a random medium whose statistical characteristics are related to the physical quantities of the medium. The models can be divided into two categories:

continuous and discrete. In the continuous case, the vegetation is modeled by assuming that its permittivity $\epsilon(\underline{x})$ is a random process whose moments are known. The average back-scattering cross section is then calculated from a knowledge of the statistics of $\epsilon(\underline{x})$. Usually it is only the mean and correlation of the permittivity that are required. The analysis of the problem proceeds by first calculating the mean wave in the random medium [Keller, 1962] and [Tatarskii and Gertsenshtein, 1963]. The mean wave allows the characterization of the vegetation by an equivalent dielectric constant. The scattered field is then calculated by using single scattering theory in this equivalent medium - a technique employed by Rosenbaum and Bowles [1974]. Particular application of the method to vegetated medium has been made by Lang [1974], Hevenor [1976], Fung and Fung [1977], Fung and Ulaby [1978], Fung [1979], and Zuniga, et al, [1979]. The theory is scalar in origin and thus does not provide cross polarization information. In addition, a correlation function for medium fluctuations must be assumed. As yet the detailed structure of this correlation function has not been related to medium properties.

In this report, we have adopted the alternative approach - modeling by discrete random media techniques. Here, the individual objects - such as leaves - are characterized by their scattering cross sections or dipole moments. Each object is then given a random placement and orientation. The analysis proceeds in a similar manner to the analysis of the continuous case. First the mean wave in the medium is found by using techniques developed by Foldy [1945], Lax [1951],

Twersky [1962], and Keller [1964]. Then an equivalent dielectric tensor is found and the backscattering is computed by using single scattering in the equivalent medium. In applications for vegetation modeling, the method has been used previously by Lang and Sokolakis [1979] for spherical scatterers. We now extend the method to arbitrary shaped scatterers. Now we find that the equivalent medium is anisotropic in nature. This leads to zeroth order depolarization of the incident wave. In addition, the backscattering coefficients are explicitly related to scatterer volume and dielectric constant.

Previous work using discrete scatterers to model vegetation has been done by Du and Peake [1969], but they employed single scattering (Born approximation) without introducing an equivalent medium. Thus they did not take into account the decay of the incident wave in the vegetation. This limits their theory to much lower frequencies and thin layers of vegetation.

PROBLEM FORMULATION

Consider the problem of scattering of time harmonic electromagnetic waves from N discrete scatterers located in a volume V as is shown in Figure 1. The particles are all identical and each has volume V_p , relative dielectric constant ϵ_r and free space permeability μ_0 . It is assumed that the background medium is free space having permittivity ϵ_0 and permeability μ_0 .

The position of the i^{th} particle is specified by the vector $\underline{X}_i$ extending from an origin O to the center of that particle. The particle's center is located by the center of the smallest circumscribed sphere in which the particle can be placed. Although the particles are identical they have a rotation with respect to a fixed direction. The rotation for the i^{th} particle is specified by $\underline{\Omega}_i = (\theta_i, \phi_i)$ where θ_i and ϕ_i are polar and azimuth angles respectively with $0 \leq \theta_i \leq \pi$ and $0 \leq \phi_i \leq 2\pi$.

The electric field obeys the vector wave equation

$$\nabla \times \nabla \times \underline{E} - k_0^2 \epsilon_r(\underline{x}) \underline{E} = i\omega \mu_0 \underline{J} \quad (1)$$

where a time dependence $e^{-i\omega t}$ has been assumed. In (1) $k_0 = \omega \sqrt{\epsilon_0 \mu_0}$ is the free space wavenumber and $\underline{J}$ is the current density of the source. The relative dielectric constant $\epsilon_r(\underline{x})$ can be expressed in terms of individual particles by employing translations and rotations of the particle located at the origin. Let us assume that a particle located at the origin is characterized by the function $U(\underline{x})$ where

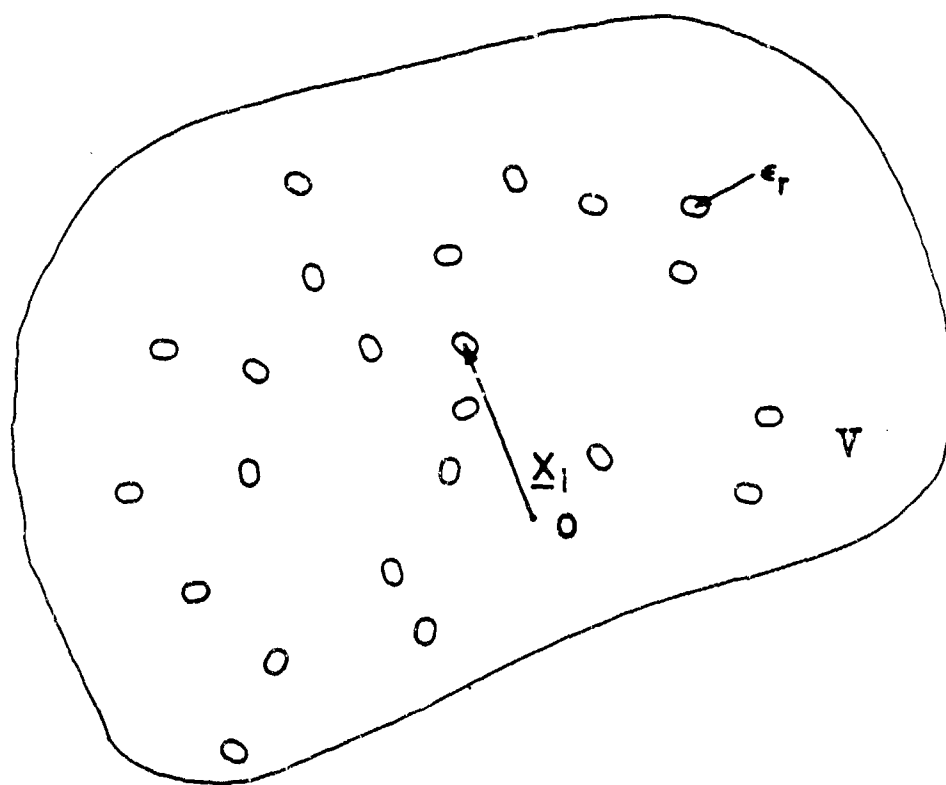


Figure 1 Distribution of particles within volume V

$$U(\underline{x}) = \begin{cases} 1 & , \quad \underline{x} \in V_p \\ 0 & , \quad \underline{x} \notin V_p \end{cases} \quad (2)$$

Using (2) we express $\epsilon_r(\underline{x})$ as

$$\epsilon_r(\underline{x}) = 1 + \Delta \sum_{i=1}^N U(\underline{x} - \underline{x}_i, \underline{\Omega}_i) \quad , \quad \Delta = \epsilon_r^{-1} \quad (3)$$

where

$$U(\underline{x}, \underline{\Omega}) = U(\underline{R}(\underline{\Omega}) \cdot \underline{x}) \quad (4)$$

Here $U(\underline{x}, \underline{\Omega})$ is the function $U(\underline{x})$ rotated by $\underline{\Omega}$ and $\underline{R}(\underline{\Omega})$ is a rotation dyadic.

We will find it convenient to express (1) and (3) in a more abstract notation. We have

$$(\underline{L} - \sum_{i=1}^N \underline{V}_i) \cdot \underline{E} = \underline{g} \quad (5)$$

where

$$\underline{L} = \nabla \times \nabla \times \underline{I} - k_0^2 \underline{I} \quad (6)$$

$$\underline{V}_i = k_0^2 \Delta U(\underline{x} - \underline{x}_i, \underline{\Omega}_i) \underline{I} \quad , \quad \underline{g} = i\omega\mu_0 \underline{J} \quad (7)$$

Here $\underline{I}$ is the unit dyadic and $\underline{g}$ can be viewed as a normalized source term. At times it will be convenient to write

$$\underline{E}(\underline{x}) = \underline{E}_0(\underline{x}) + \underline{E}_s(\underline{x}) \quad (8)$$

where $\underline{E}_0(\underline{x})$ is the solution to (5) when no scatterers are present, i.e.,

$$\underline{L} \cdot \underline{E}_0(\underline{x}) = \underline{g} \quad (9)$$

and $\underline{E}_s(\underline{x})$ is the scattered field from the particles.

SINGLE SCATTERER - TRANSITION OPERATOR

Before considering the N particle scattering problem, we will consider scattering from one particle located at the origin. Putting $N=1$ in (5) with $\underline{X}_1=0$ and $\underline{\Omega}_1=\underline{\Omega}$, we have

$$(\underline{L} - \underline{V}) \cdot \underline{e} = \underline{g} \quad , \quad \underline{V} = \Delta k_0^2 U(\underline{x}, \underline{\Omega}) \quad (10)$$

where

$$\underline{e} = \underline{e}_0 + \underline{e}_s \quad , \quad \underline{L} \cdot \underline{e}_0 = \underline{g} \quad (11)$$

and $\underline{e}_s$ is outgoing as $|\underline{x}| \rightarrow \infty$. We have used the small $\underline{e}$ notation for the field here to remind us that there is only one scatterer present.

If we use (11) in (10), we obtain

$$\underline{L} \cdot \underline{e}_s = \underline{V} \cdot \underline{e} \quad (12)$$

From (12) we see that the term on the left, $\underline{V} \cdot \underline{e}$, can be viewed as the source of the scattered field. We write

$$\underline{g}_{eq} = \underline{V} \cdot \underline{e} \quad (13)$$

where $\underline{g}_{eq}$ is an equivalent source term. Since $\underline{V}=0$ when $\underline{x} \notin V_p$, the sources, $\underline{g}_{eq}$, exist inside the particle boundaries.

It is more natural to think of the equivalent sources as being caused by the incident field $\underline{e}_0$. Because Maxwell's equations are linear, we can write

$$\underline{g}_{eq} = \underline{T} \cdot \underline{e}_0 \quad (14)$$

where the dyadic operator $\underline{T}$ is known as the transition operator in the scattering literature [Lax, 1951]. Now using (13) and (14) in (12) and multiplying through by $\underline{L}^{-1}$, we have

$$\underline{e}_s = \underline{L}^{-1} \cdot \underline{T} \cdot \underline{e}_0 \quad (15)$$

Thus the knowledge of $\underline{T}$ completely characterizes the scattering properties of the particle. The operator $\underline{T}$ is related to the dyadic scattering amplitude of the particle and for dipole scatterers $\underline{T}$ can be determined from the polarizability of the particle. Thus $\underline{T}$ is directly connected with quantities of physical interest.

The transition operator is a linear bounded operator and, as a result, can be expressed in integral form:

$$\underline{g}_{eq}(\underline{x}) = \underline{T} \cdot \underline{e}_0 = \int d\underline{x}' \underline{t}(\underline{x}, \underline{x}') \cdot \underline{e}_0(\underline{x}') \quad (16)$$

where the limits for the integral extend over all space. One can show that $\underline{t}$ is 0 when $\underline{x}$ and $\underline{x}'$ are outside the particle [Frisch, 1968], i.e.,

$$\underline{t}(\underline{x}, \underline{x}') = 0, \quad \underline{x} \notin V_p \quad \text{or} \quad \underline{x}' \notin V_p \quad (17)$$

The property follows directly from the fact that the equivalent sources for the scattered field are located within the particle boundaries.

We will now represent $\underline{t}$ in terms of plane waves. The representation of $\underline{t}$ can be directly related to the dyadic scattering amplitude. We proceed by representing $\underline{e}_0(\underline{x})$ by its Fourier transform, putting this in (16) and taking the Fourier transform of (16). We obtain

$$\underline{\tilde{g}}_{eq}(\underline{k}) = \int d\underline{k}' \underline{\tilde{t}}(\underline{k}, \underline{k}') \cdot \underline{\tilde{e}}_0(\underline{k}') \quad (18)$$

where

$$\underline{\tilde{t}}(\underline{k}, \underline{k}') = \frac{1}{(2\pi)^3} \int d\underline{x} d\underline{x}' \underline{t}(\underline{x}, \underline{x}') e^{-i(\underline{k} \cdot \underline{x} - \underline{k}' \cdot \underline{x}')} \quad (19)$$

In (18) we have used the notation that $\underline{\tilde{h}}$ is the Fourier

transform of $\underline{h}$. More specifically

$$\tilde{\underline{h}}(\underline{k}) = \int d\underline{x} \underline{h}(\underline{x}) e^{-i\underline{k} \cdot \underline{x}} \quad (20)$$

Inverting (19) the transition kernel $\underline{t}$ can be expressed in terms of its plane wave representation $\tilde{\underline{t}}$:

$$\underline{t}(\underline{x}, \underline{x}') = \frac{1}{(2\pi)^3} \int d\underline{k} d\underline{k}' \tilde{\underline{t}}(\underline{k}, \underline{k}') e^{i(\underline{k} \cdot \underline{x} - \underline{k}' \cdot \underline{x}')} \quad (21)$$

The dyadic scattering amplitude will now be defined.

Consider a plane wave incident upon a scatterer located at the origin. An arbitrary plane wave can be decomposed into two mutually orthogonal linearly polarized plane waves. The polarization directions are taken as $\underline{\alpha}^\circ$ and $\underline{\beta}^\circ$ where $\underline{\alpha}^\circ$ and $\underline{\beta}^\circ$ are orthogonal unit vectors with $\underline{\alpha}^\circ$ and $\underline{\beta}^\circ$ being perpendicular to the direction of propagation. The two incident waves are

$$\underline{e}_0(\underline{x}, \underline{i}; q) = \underline{\alpha}^\circ e^{-k_0 \underline{i} \cdot \underline{x}} \quad , \quad q \in \{\alpha, \beta\} \quad (22)$$

where $\underline{i}$ is a unit vector in the direction of incidence. It is more convenient to consider both polarizations simultaneously so we introduce the dyadic incident wave [Twersky, 1967]

$$\underline{e}_0(\underline{x}, \underline{i}) = \underline{e}_0(\underline{x}, \underline{i}; \alpha) \underline{\alpha}_0 + \underline{e}_0(\underline{x}, \underline{i}; \beta) \underline{\beta}_0 \quad (23)$$

$$= (\underline{\alpha}_0 \underline{\alpha}_0 + \underline{\beta}_0 \underline{\beta}_0) e^{ik_0 \underline{i} \cdot \underline{x}} \quad (24)$$

$$= (\underline{I} - \underline{i} \underline{i}) e^{ik_0 \underline{i} \cdot \underline{x}} \quad (25)$$

The dyadic scattered field from the particle is given by

$$\underline{e}_s(\underline{x}, \underline{i}) = \underline{e}_s(\underline{x}, \underline{i}; \alpha) \underline{\alpha}_0 + \underline{e}_s(\underline{x}, \underline{i}; \beta) \underline{\beta}_0 \quad (26)$$

where $\underline{e}_s(\underline{x}, \underline{i}; q)$ is the scattered field due to polarization q . The dyadic scattering amplitude, $\underline{f}$, is defined in terms of the asymptotic expression for $\underline{e}_s$ in the radiation zone. We have

$$\underline{\underline{g}}_s(\underline{x}, \underline{i}) \sim \underline{\underline{f}}(\underline{0}, \underline{i}) \frac{e^{ik_0|\underline{x}|}}{|\underline{x}|}, \quad |\underline{x}| \rightarrow \infty \quad (27)$$

where $\underline{0}$ is a unit vector in the $\underline{x}$ direction, $\underline{0} = \underline{x}/|\underline{x}|$.

The relationship between $\underline{\underline{f}}$ and $\underline{\underline{t}}$ can be found by employing (15) for large $|\underline{x}|$ (Appendix A). The result is

$$\underline{\underline{f}}(\underline{0}, \underline{i}) = 2\pi^2 (\underline{\underline{I}} - \underline{0} \underline{0}) \cdot \underline{\underline{t}}(k_0 \underline{0}, k_0 \underline{i}) \cdot (\underline{\underline{I}} - \underline{i} \underline{i}) \quad (28)$$

From this relation, we see

$$\underline{0} \cdot \underline{\underline{f}} = 0, \quad \underline{\underline{f}} \cdot \underline{i} = 0 \quad (29)$$

Thus $\underline{\underline{f}}$ is a four component tensor - all combinations of two incident polarizations and two scattered polarizations. We also note that $\underline{\underline{f}}$ does not completely determine $\underline{\underline{t}}$ but only partially specifies it. In particular, a knowledge of $\underline{\underline{f}}$ for a free space wave number k_0 only determines $\underline{\underline{t}}(\underline{k}, \underline{k}')$ at $|\underline{k}| = |\underline{k}'| = k_0$; also only four of the nine components of $\underline{\underline{t}}$ in the polarization directions are determined.

Before concluding this section, transition operators for particles not located at the origin will be needed. As before, the equivalent sources $\underline{g}_{eq}^{(i)}$ for a particle located at $\underline{x}_i$ can be related to the incident field. It follows that

$$\underline{g}_{eq}^{(i)}(\underline{x}) = \underline{\underline{T}}_i \cdot \underline{e}_0 = \int \underline{\underline{t}}_i(\underline{x}, \underline{x}') \cdot \underline{e}_0(\underline{x}') d\underline{x}' \quad (30)$$

By shifting the sources and the incident field to the origin $\underline{\underline{t}}_i$ can be related to $\underline{\underline{t}}$. One finds

$$\underline{\underline{t}}_i(\underline{x}, \underline{x}') = \underline{\underline{t}}(\underline{x} - \underline{x}_i, \underline{x}' - \underline{x}_i) \quad (31)$$

Note that throughout the discussion the dependence of $\underline{\underline{t}}$ on rotations has been suppressed for convenience of notation.

COHERENT FIELD

In this section we will develop an approximate equation for the coherent field by employing the Foldy approximation [Foldy, 1945]. The equation is in terms of the transition operator and thus, when the scattering amplitude is known, the equation is completely specified. After the equation has been derived it is pointed out that outside V the coherent fields obeys Maxwells' equations with free space permittivity and permeability. Inside V , the coherent fields obey Maxwells equations with free space permeability and a macroscopic permittivity that is inhomogeneous, anisotropic and spatially dispersive.

Before discussing the coherent field, the statistics that govern the particles position and rotation must be specified. It will be assumed that the position vectors $\underline{x}_i$, $i=1\dots N$ and rotation vectors $\underline{\Omega}_i$, $i=1\dots N$ are random variables that are specified by a $5N$ dimensional distribution function. In addition, it is assumed that interchanging particles leaves the distribution function unaffected. From this general distribution function we can obtain the probability density function for the i^{th} particle. It is

$$p_{\underline{x}_i \underline{\Omega}_i}(\underline{x}, \underline{\omega}) = p_{\underline{x} \underline{\Omega}}(\underline{x}, \underline{\omega}) \quad i=1\dots N \quad (32)$$

where $\underline{\omega}=(\theta, \phi)$. In (32) we have explicitly noted the fact that the particles are identically distributed by omitting the index i on the left hand side of (32). We will assume that the particles location and rotation are independent, thus

$$p_{\underline{x} \underline{\Omega}}(\underline{x}, \underline{\omega}) = p_{\underline{x}}(\underline{x}) p_{\underline{\Omega}}(\underline{\omega}) \quad (33)$$

with the usual property:

$$\int_V p_{\underline{X}}(\underline{x}) d\underline{x} = 1 \quad \int_{4\pi} p_{\underline{\Omega}}(\underline{\omega}) d\underline{\omega} = 1 \quad (34)$$

The particle density is defined by

$$\rho(\underline{x}) = N p_{\underline{X}}(\underline{x}) \quad (35)$$

so that

$$\int_V \rho(\underline{x}) d\underline{x} = N. \quad (36)$$

In addition to the one particle density - when treating the correlation of the field - the two particle density will be required. We have

$$p_{\underline{X}_i \underline{X}_j \underline{\Omega}_i \underline{\Omega}_j}(\underline{x}, \hat{\underline{x}}, \underline{\omega}, \hat{\underline{\omega}}) = p_{\underline{X} \underline{X} \underline{\Omega} \underline{\Omega}}(\underline{x}, \hat{\underline{x}}, \underline{\omega}, \hat{\underline{\omega}}) = p_{\underline{X} \underline{\Omega}}(\underline{x}, \underline{\omega}) p_{\hat{\underline{X}} \hat{\underline{\Omega}}}(\hat{\underline{x}}, \hat{\underline{\omega}}) \quad (37)$$

$$i, j = 1, \dots, N$$

In (37) we have assumed that the i^{th} and j^{th} particles are independent. The independence assumption is valid when the particles are sparsely distributed; the case we intend to treat.

We will now develop the approximate equation for the coherent field. We start by noting that the total field $\underline{E}$ can be thought of as a sum of the incident field $\underline{E}_0$ plus a sum of the fields scattered from each particle, $\underline{E}_s^{(i)}$. We have

$$\underline{E} = \underline{E}_0 + \sum_{i=1}^N \underline{E}_s^{(i)} \quad (38)$$

The total field incident on the i^{th} particle is called the effective field and is denoted by $\underline{E}^{(i)}$. Thus $\underline{T}_i \cdot \underline{E}^{(i)}$ represents the equivalent sources generated by the incident field in the i^{th} particle and

$$\underline{E}_s^{(i)} = \underline{L}^{-1} \cdot \underline{T}_i \cdot \underline{E}^{(i)} \quad (39)$$

Using (39) in (38), we have

$$\underline{E} = \underline{E}_0 + \sum_{i=1}^N \underline{L}^{-1} \cdot \underline{T}_i \cdot \underline{E}^{(i)} \quad (40)$$

This is the equation that we wished to obtain.

Now we average this equation. The result is

$$\langle \underline{E} \rangle = \underline{E}_0 + \sum_{i=1}^N \underline{L}^{-1} \cdot \langle \underline{T}_i \cdot \underline{E}^{(i)} \rangle \quad (41)$$

To obtain an approximate equation for the mean we follow [Foldy, 1945] and assume

$$\underline{E}^{(i)} \approx \langle \underline{E} \rangle \quad (42)$$

This means that the random quantity $\underline{E}^{(i)}$ is to first order equal to a deterministic quantity, i.e., to first order it is a ergodic quantity. Using (42) in (41) and noting that $\langle \underline{T}_i \cdot \underline{E}^{(i)} \rangle \approx \langle \underline{T}_i \rangle \cdot \langle \underline{E} \rangle = \langle \underline{T}_i \rangle \cdot \langle \underline{E} \rangle$ we have the approximate equation for the mean field

$$\langle \underline{E} \rangle = \underline{E}_0 + \sum_{i=1}^N \underline{L}^{-1} \cdot \langle \underline{T}_i \rangle \cdot \langle \underline{E} \rangle \quad (43)$$

Denoting explicitly the dependence of $\underline{T}_i$ upon $\underline{X}_i$ and $\underline{\Omega}_i$, averaging and then using (33), we have

$$\begin{aligned} \langle \underline{T}_i \rangle &= \langle \underline{T}(\underline{X}_i, \underline{\Omega}_i) \rangle = \int_V d\underline{s} \int_{4\pi} d\underline{\omega} p_{\underline{X}\underline{\Omega}}(\underline{s}, \underline{\omega}) \underline{T}(\underline{s}, \underline{\omega}) \\ &= \int_V d\underline{s} p_{\underline{X}}(\underline{s}) \bar{\underline{T}}(\underline{s}) \end{aligned} \quad (44)$$

where

$$\bar{\underline{T}}(\underline{s}) = \int_{4\pi} d\underline{\omega} p_{\underline{\Omega}}(\underline{\omega}) \underline{T}(\underline{s}, \underline{\omega}) \quad (45)$$

In (45) the bar over $\underline{T}$ has been used to indicate an average of

angular varies only. By putting (44) in (43), by noting that the scattered terms are identical and by introducing $\rho(\underline{s})$ via (35), we obtain

$$\langle \underline{E} \rangle = \underline{E}_0 + \int_V d\underline{s} \rho(\underline{s}) \underline{L}^{-1} \cdot \underline{T}(\underline{s}) \cdot \langle \underline{E} \rangle \quad (46)$$

Multiplying from the left by $\underline{L}$ and using (9), we get

$$\underline{\mathcal{L}} \cdot \langle \underline{E} \rangle = \underline{g} \quad (47)$$

where

$$\underline{\mathcal{L}} = \underline{L} - \int_V d\underline{s} \rho(\underline{s}) \underline{T}(\underline{s}) \quad (48)$$

This is the equation for the coherent field.

The arguments that have led to the approximate equation (47) have been largely heuristic. The essential approximation is contained in (42) where the effective field is assumed approximately equal to the average field. Although we will not discuss the conditions under which (42) is valid, it will be shown elsewhere that the approximation is valid when the fraction of volume occupied by the particles is small compared to the total volume, i.e., $NV_p/V \ll 1$. We shall refer to a distribution of scatterers satisfying this condition as a sparse distribution.

Before proceeding we will write the equation for the mean in more concrete form. Using (30) and (31) in (47) and (48), we obtain

$$\underline{L} \cdot \langle \underline{E}(\underline{x}) \rangle - \int_V d\underline{s} \int d\underline{x}' \rho(\underline{s}) \underline{T}(\underline{x}-\underline{s}, \underline{x}'-\underline{s}) \cdot \langle \underline{E}(\underline{x}') \rangle = \underline{g} \quad (49)$$

where

$$\underline{T}(\underline{x}, \underline{x}') = \int_{4\pi} d\omega p_{\underline{\Omega}}(\omega) \underline{t}(\underline{x}, \underline{x}'; \omega) \quad (50)$$

Here the kernel $\underline{t}(\underline{x}, \underline{x}'; \underline{\omega})$ is the same as given in (31), however we have explicitly shown its dependence on the angular coordinate $\underline{\omega}$.

We can now use (49) to obtain a macroscopic form of Maxwell's equations. First averaging the Faraday's law equation; we have

$$\nabla \times \langle \underline{E}(\underline{x}) \rangle = i\omega\mu_0 \langle \underline{H}(\underline{x}) \rangle . \quad (51)$$

Then by using (51) in (49), we obtain the macroscopic Ampere's law equation.

$$\nabla \times \langle \underline{H}(\underline{x}) \rangle = \underline{J} - i\omega \langle \underline{D} \rangle , \quad \langle \underline{D} \rangle = \epsilon_0 \underline{\epsilon} \cdot \langle \underline{E} \rangle \quad (52)$$

when $\underline{\epsilon}$ is a macroscopic permittivity operator which describes the average behavior of the medium and $\underline{J} = \underline{g} / (i\omega\mu_0)$. It is

$$\underline{\epsilon} = \underline{I} + \frac{1}{k_0^2} \int_V d\underline{s} \int d\underline{x}' \rho(\underline{s}) \underline{t}(\underline{x} - \underline{s}, \underline{x}' - \underline{s}) . \quad (53)$$

This expression simplifies to $\underline{I}$ (free space) when $\underline{x} \notin V$. To see this we note that when $\underline{x} \notin V$, we have $\underline{x} - \underline{s} \notin V_p$ since $\underline{s} \in V$. Now using (17) we have $\underline{t} = 0$. When $\underline{x} \in V$ (47) does not simplify in general. It describes an anisotropic, inhomogeneous, spatially dispersion medium.

Let us examine how (53) reduces to some more familiar expressions in some special situations. We will assume that V is infinite through the remainder of this section. First we will consider the case when the density is constant, i.e., $\rho(\underline{s}) = \rho$. In this case the permittivity is translationally invariant or homogeneous. To see this we substitute (21) into (53) and we perform the integrations over $\underline{s}$ and $\underline{k}'$. We obtain

$$\underline{\underline{\epsilon}} = \underline{\underline{I}} + \frac{\rho}{k_0^2} \int d\underline{x}' \int d\underline{k} e^{i\underline{k} \cdot (\underline{x} - \underline{x}')} \underline{\underline{\tilde{\epsilon}}}(\underline{k}, \underline{k}) \quad (54)$$

$$\text{when } \underline{\underline{\tilde{\epsilon}}}(\underline{k}, \underline{k}) = \int_{4\pi} d\omega P_{\underline{\Omega}}(\omega) \underline{\underline{\tilde{t}}}(\underline{k}, \underline{k}'; \omega) \quad (55)$$

Since the integrand is a function of $\underline{x} - \underline{x}'$ the permittivity is translationally invariant however it is still anisotropic and spatially dispersive.

Another special case of interest is when $\underline{\underline{t}}$ is scalar, i.e. $\underline{\underline{t}}(\underline{x}, \underline{x}') = \underline{\underline{t}}(\underline{x}, \underline{x}') \underline{\underline{I}}$. This occurs when the scatterers are spherical. Then the permittivity is isotropic but inhomogeneous and spatially dispersive.

The last special case to be treated is when the wavelength is large compared to the size of a scatterer. Here the particle can be treated as an electric dipole. Its equivalent source distribution is given by

$$\underline{\underline{g}}_{eq} = i\omega\mu_0 \underline{\underline{J}}_{eq} = i\omega\mu_0 (-i\omega \underline{\underline{p}} \delta(\underline{x})) \quad (56)$$

where $\underline{\underline{p}}$ is the electric dipole moment of the scatterer and $\delta(\underline{x})$ is the Dirac delta function. The dipole moment is related to the incident field $\underline{\underline{e}}_0$ by the polarizability tensor $\underline{\underline{\alpha}}$ [Jones, 1964]

$$\underline{\underline{p}} = \epsilon_0 \underline{\underline{\alpha}} \cdot \underline{\underline{e}}_0 \quad (57)$$

Using (57) in (56) and comparing it with (14), we find

$$\underline{\underline{T}} = k_0^2 \underline{\underline{\alpha}} \delta(\underline{x}) \quad (58)$$

or

$$\underline{\underline{t}}(\underline{x}, \underline{x}') = k_0^2 \underline{\underline{\alpha}} \delta(\underline{x}) \delta(\underline{x}') \quad (59)$$

Now putting (59) in (19), we obtain

$$\underline{\underline{\tilde{t}}}(\underline{k}, \underline{k}') = k_0^2 \underline{\underline{\alpha}} / (2\pi)^3 \quad (60)$$

Thus we see in the dipole limit $\tilde{\underline{\epsilon}}$ is independent of $\underline{k}$ and $\underline{k}'$.

Since we have an expression for $\underline{\epsilon}$ in the low frequency or dipole limit, the special form of $\underline{\epsilon}$ can be easily obtained. Using (59) in (54), we find

$$\underline{\epsilon} = \underline{I} + \rho(\underline{x})\overline{\underline{\alpha}} \quad (61)$$

Thus in the low frequency case, the permittivity is no longer spatially dispersive however it is still anisotropic and inhomogeneous.

CORRELATION

In this section we will calculate the correlation of the electric field. Rather than following procedures used to find the coherent wave, the distorted Born approximation will be employed. This is a single scattering approximation where the scatterers are assumed to be embedded in the equivalent medium which has been found in the previous section. The method is useful when the fractional volume is small ($NV_p/V \ll 1$) and the albedo of a single particle is small. The latter condition implies that the energy absorbed by a particle must be much larger than the energy scattered by it.

We start by considering a volume V of equivalent medium surrounded by free space. There are N particles embedded in V as shown in Figure 1. The scattered field due to the i^{th} particle can be calculated by modifying (39). We assume that the incident field on the particle is the mean field $\langle \underline{E} \rangle$ and that the free space operator $\underline{L}$ is replaced by the equivalent medium operator $\underline{\mathcal{L}}$ as given in (48). We have

$$\underline{E}_s = \sum_{i=1}^N \underline{E}_s^{(i)} = \sum_{i=1}^N \underline{\mathcal{L}}^{-1} \cdot \underline{T}_i \cdot \langle \underline{E} \rangle \quad (62)$$

Before proceeding, we point out that our main interest in finding the correlation of the field is to use it to calculate the backscattering cross section. Since this cross section is related to the correlation of the field fluctuations, we now define

$$\underline{E}_f = \underline{E}_s - \langle \underline{E}_s \rangle, \quad \langle \underline{E}_f \rangle = 0 \quad (63)$$

Now computing the correlation of the fluctuating field, we obtain

$$\langle \underline{E}_f(\underline{x}) \underline{E}_f^*(\hat{\underline{x}}) \rangle = \langle \underline{E}_s(\underline{x}) \underline{E}_s^*(\hat{\underline{x}}) \rangle - \langle \underline{E}_s(\underline{x}) \rangle \langle \underline{E}_s^*(\hat{\underline{x}}) \rangle \quad (64)$$

where $\underline{z}^*$ is the conjugate of $\underline{z}$. Putting (62) in (64) and noting that a portion of $\langle \underline{E}_s \underline{E}_s^* \rangle$ cancels with $\langle \underline{E}_s \rangle \langle \underline{E}_s^* \rangle$ if we use the fact that $N \gg 1$. We find

$$\langle \underline{E}_f(\underline{x}) \underline{E}_f^*(\hat{\underline{x}}) \rangle = \int_{4\pi} d\omega \, p_\omega(\omega) \langle \underline{E}_f(\underline{x}) \underline{E}_f^*(\hat{\underline{x}}) \rangle_\omega \quad (65)$$

where

$$\langle \underline{E}_f(\underline{x}) \underline{E}_f^*(\hat{\underline{x}}) \rangle_\omega = \int_V d\underline{s} \, \rho(\underline{s}) \, \underline{\xi}(\underline{x}, \underline{s}) \underline{\xi}^*(\hat{\underline{x}}, \underline{s}) \quad (66)$$

with

$$\underline{\xi}(\underline{x}, \underline{s}) = \underline{\mathcal{L}}^{-1} \cdot \underline{T}(\underline{s}) \cdot \langle \underline{E} \rangle \quad (67)$$

Here we have separated the average into rotation and coordinate space averages, thus introducing the conditional expectation, $\langle \underline{E}_f \underline{E}_f^* \rangle_\omega$, with respect to given ω .

To write (67) more explicitly, we introduce the dyadic Green's function $\underline{G}(\underline{x}, \underline{x}')$ for the operator $\underline{\mathcal{L}}$. It satisfies

$$\underline{\mathcal{L}} \cdot \underline{G}(\underline{x}, \underline{x}') = \underline{I} \delta(\underline{x} - \underline{x}') \quad (68)$$

+ $\underline{G}$ - outgoing as $|\underline{x}| \rightarrow \infty$

where $\underline{\mathcal{L}}$ is given in (48). Now (67) becomes

$$\underline{\xi}(\underline{x}, \underline{s}) = \int d\underline{x}' \underline{G}(\underline{x}, \underline{x}') \cdot \int d\underline{x}'' \underline{t}(\underline{x}' - \underline{s}, \underline{x}'' - \underline{s}) \cdot \langle \underline{E}(\underline{x}'') \rangle \quad (69)$$

The expression simplifies greatly in the low frequency limit.

Assuming that $\underline{t}$ is given by (59) and using this in (69) gives

$$\underline{\xi}(\underline{x}, \underline{s}) = k_0^2 \underline{G}(\underline{x}, \underline{s}) \cdot \underline{\alpha} \cdot \langle \underline{E}(\underline{s}) \rangle \quad (70)$$

BACKSCATTERING COEFFICIENTS FOR A HALF SPACE OF DIPOLES

To illustrate the application of the methods developed in the previous sections, we will calculate the backscattering coefficients from a half space of scatterers that are small compared to wavelength. We will also assume that the density of scatterers ρ is constant. The physical configuration is shown in Figure 2. There, we have shown the direction of the incident wave and the polarization vectors $\underline{h}^0$ and $\underline{v}^0$ representing horizontal and vertical polarizations respectively.

To compute the scattered field using the distorted Born approximation, we must first calculate the mean field in the half space containing the particles. In the low frequency approximation the mean wave is computed by replacing the particles with an equivalent medium having relative permittivity tensor $\underline{\underline{\epsilon}} = \underline{\underline{I}} + \rho \underline{\underline{\alpha}}$ and free space permeability μ_0 . The usual continuity conditions associated with macroscopic Maxwell's equations are assumed to hold at the interface $z=0$.

Before proceeding we would like to emphasize that the scatterers are sparsely distributed or that the fractional volume they occupy is small ($NV_p/V \ll 1$) - a condition necessary for the validity of the mean equation. This restriction is reflected in the equivalent permittivity tensor. Small fractional volume requires that $|\rho \bar{\alpha}_{ij}| \ll 1$ where $\bar{\alpha}_{ij}$ are the components of $\underline{\underline{\alpha}}$. We can exhibit the dependence of this condition on the fractional volume $\delta \equiv NV_p/V = \rho V_p$ explicitly by introducing a normalized polarizability tensor $\underline{\underline{a}}$ as follows:

$$\underline{\underline{a}} = \underline{\underline{\alpha}}/V_p \quad (71)$$

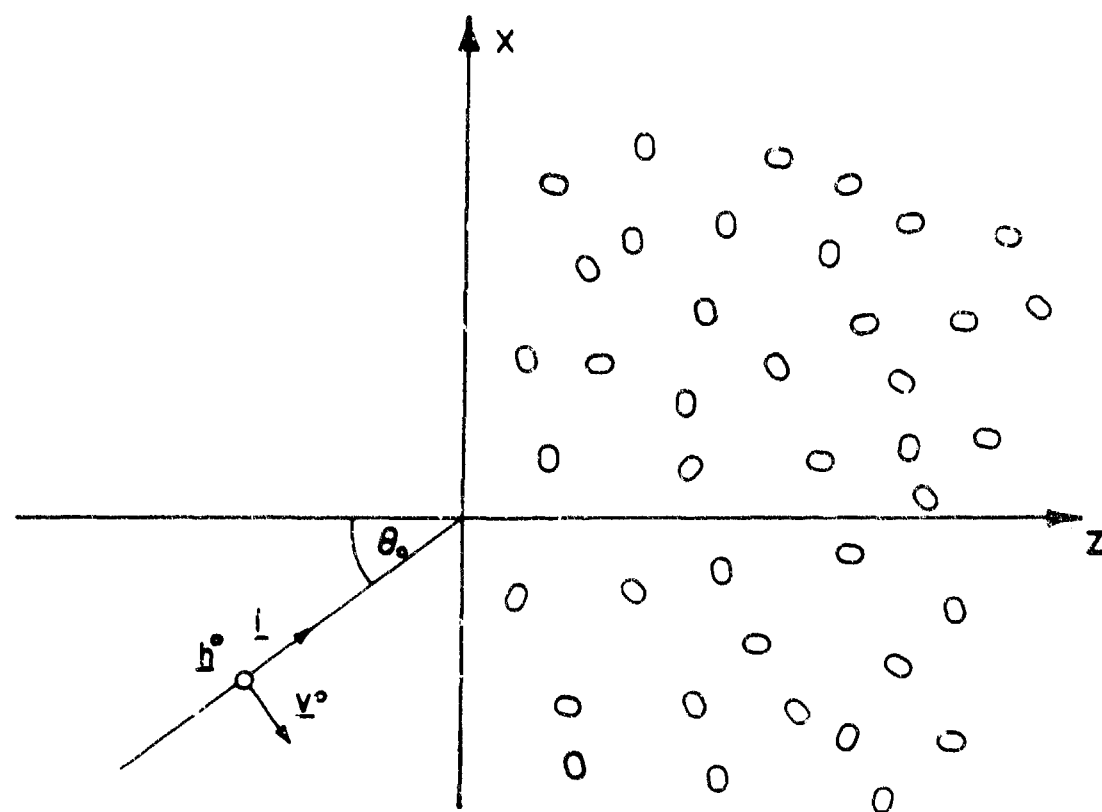


Figure 2 Incidence wave on half space of uniformly distributed particles

where we can show that the components of $\underline{a}$ remain bounded as $V_p \rightarrow 0$. Now the permittivity can be written as

$$\underline{\epsilon} = \underline{I} + \delta \underline{\bar{a}} \quad (72)$$

and thus we have a small parameter for ordering purposes.

Although we are able to carry out the calculation of the mean wave for an arbitrary average polarizability tensor $\underline{\bar{a}}$, it is convenient to partially specify the angular probability density, $p_{\Omega}(\underline{\omega})$ in order to make $\underline{\bar{a}}$ diagonal. First we choose a spherical coordinate system of mutually orthogonal unit vectors $\underline{r}_0$, $\underline{\theta}_0$ and $\underline{\phi}_0$. The position of these vectors is completely determined by the spherical angles θ and ϕ as shown in Figure 3. Now we align the principal axes of the scatterer along these unit vectors. Then we write

$$\underline{a} = a_r \underline{r}_0 \underline{r}_0^\circ + a_{\theta} \underline{\theta}_0 \underline{\theta}_0^\circ + a_{\phi} \underline{\phi}_0 \underline{\phi}_0^\circ \quad (73)$$

By using the usual transformation between spherical and cartesian coordinates, (73) becomes

$$\underline{a} = \sum_{i=1}^3 \sum_{j=1}^3 a_{x_i x_j}(\theta, \phi) \underline{x}_i^\circ \underline{x}_j^\circ \quad (74)$$

where $x_1=x$, $x_2=y$ and $x_3=z$ and $\underline{x}_i^\circ, \underline{y}^\circ$ and $\underline{z}^\circ$ are cartesian unit vectors. The relationship between the $a_{x_i x_j}$ and $a_r, a_{\theta}, a_{\phi}$ are given in Appendix

Now assuming that the random variables θ_i and ϕ_i are independent, we write

$$p_{\Omega}(\underline{\omega}) = P_{\theta}(\theta) p_{\phi}(\phi) \quad (75)$$

Averaging (74), we have

$$\underline{\bar{a}} = \sum_{i=1}^3 \sum_{j=1}^3 \bar{a}_{x_i x_j} \underline{x}_i^\circ \underline{x}_j^\circ \quad (76)$$

where

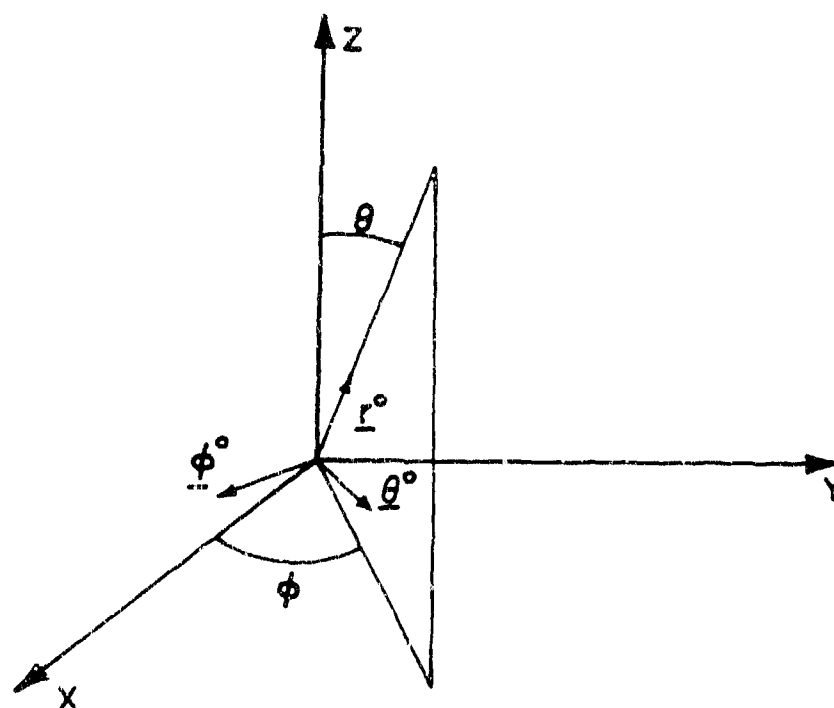


Figure 3 Principal axes of scatterer

$$\bar{a}_{x_i x_i} = \int_0^\pi d\theta \int_0^{2\pi} d\phi p_\theta(\theta) p_\phi(\phi) a_{x_i x_j}(\theta, \phi) \quad (77)$$

We will now assume that the particles are distributed uniformly in the ϕ variable, i.e., $p_\phi(\phi) = 1/2\pi$. We find

$$\bar{\underline{a}} = \bar{a}_{xx} \underline{x}^\circ \underline{x}^\circ + \bar{a}_{yy} \underline{y}^\circ \underline{y}^\circ + \bar{a}_{zz} \underline{z}^\circ \underline{z}^\circ \quad (78)$$

where

$$\bar{a}_{xx} = \bar{a}_{yy} = \frac{1}{2} [a_r \overline{\sin^2 \theta} + a_\theta \overline{\cos^2 \theta} + a_\phi] \quad (79)$$

$$\bar{a}_{zz} = a_r \overline{\cos^2 \theta} + a_\theta \overline{\sin^2 \theta} \quad (80)$$

Thus all off diagonal terms averaged to zero and two of the on diagonal terms are equal. Using this in (72), we see that the equivalent dielectric is uniaxial.

The mean wave in the equivalent medium will now be found for the case of particles uniformly distributed in the azimuth coordinate ϕ . The incident wave is given

$$\underline{E}_0(\underline{x}, q) = \underline{q}^\circ e^{i \underline{k} \cdot \underline{x}} \quad q \in \{h, v\} \quad (81)$$

where

$$\underline{k} = \underline{k}_{t_0} + k_{z_0} \underline{z}^\circ, \quad (82)$$

with

$$\underline{k}_{t_0} = k_0 \sin \theta_0 \underline{x}^\circ, \quad k_{z_0} = k_0 \cos \theta_0 \quad (83)$$

and the polarization vectors are

$$\underline{h}^\circ = \underline{y}^\circ, \quad \underline{v}^\circ = -\cos \theta_0 \underline{x}^\circ + \sin \theta_0 \underline{z}^\circ \quad (84)$$

The average electric field in the equivalent half space satisfies

$$[\nabla \times (\nabla \times \underline{I}) - k_0^2 (\underline{I} + \delta \bar{\underline{a}})] \cdot \langle \underline{E}(\underline{x}) \rangle = 0, \quad z < 0 \quad (85)$$

Let us assume a plane wave solution of the form

$$\langle \underline{E}(\underline{x}) \rangle = \underline{A} e^{i\kappa \cdot \underline{x}} \quad (86)$$

where $\underline{\kappa} = \underline{\kappa}_t + \kappa_z \underline{z}^0$. In order to match fields at the interface, the transverse phase velocity of the incident and transmitted waves must be the same, thus $\underline{\kappa}_t = \underline{\kappa}_t = k_0 \sin \theta_0 \underline{x}^0$. Putting (86) in (85), we have

$$\underline{\kappa} \cdot \underline{x} (\underline{\kappa} \cdot \underline{x} \underline{A}) + k_0^2 \underline{A} + \delta k_0^2 \underline{\bar{a}} \cdot \underline{A} = 0 \quad (87)$$

Representing $\underline{A}$ in cartesian components (87) can be written as

$$\begin{bmatrix} \kappa_z^2 - k_0^2 \beta_{xx} & 0 & k_0 \sin \theta_0 \kappa_z \\ 0 & \kappa_z^2 - k_0^2 \beta_{yy} & 0 \\ k_0 \sin \theta_0 \kappa_z & 0 & -k_0^2 \beta_{zz} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = 0 \quad (88)$$

where

$$\beta_{xx} = 1 + \delta \bar{a}_{xx}, \quad \beta_{yy} = \cos^2 \theta_0 + \delta \bar{a}_{yy}, \quad \beta_{zz} = \cos^2 \theta_0 + \delta \bar{a}_{zz} \quad (89)$$

Since (88) is a homogeneous system, the determinant of coefficients must be zero for a solution to exist. This condition determines the allowable values of κ_z . We find

$$\kappa_z = \pm \kappa_z^{(h)} = \pm k_0 \sqrt{\beta_{yy}} \quad (90)$$

$$\kappa_z = \pm \kappa_z^{(v)} = \pm k_0 \left[\frac{\beta_{xx} \beta_{zz}}{\sin^2 \theta_0 + \beta_{zz}} \right]^{1/2} \quad (91)$$

where the superscripts h and v have been used to designate the propagation constants associated with horizontal and vertical polarizations. For an incident wave that is not grazing, i.e. $\theta_0 \neq \pi/2$, expressions (90) and (91) can be simplified using the small δ parameter. We have

$$\kappa_z^{(h)} = k_0 \left(\cos \theta_0 + \frac{\delta \bar{a}_{yy}}{2 \cos \theta_0} \right) + O(\delta^2) \quad (92)$$

$$\kappa_z^{(v)} = k_0 \left[\cos \theta_0 + \frac{\delta}{2} \left(\frac{\bar{a}_{zz} + \cos^2 \theta_0 (\bar{a}_{xx} - \bar{a}_{zz})}{\cos \theta_0} \right) \right] + O(\delta^2) \quad (93)$$

We cannot calculate these propagation constants to higher accuracies than $O(\delta)$ since the original mean equation has only been found to this accuracy. We note that since we are considering lossy particles, the $\kappa_z^{(q)}$ are complex and thus the mean wave will decay away from the interface.

Next we calculate the amplitude coefficients for the mean wave of both polarizations. We have

$$\langle \underline{E}(\underline{x}, h) \rangle = \begin{cases} (e^{ik_z z} + \Gamma_h e^{-ik_z z}) e^{i \underline{k}_{t_0} \cdot \underline{x}} \underline{y}^0, & z < 0 \\ A_y e^{i \kappa_z^{(h)} \cdot \underline{x}} \underline{y}^0, & z > 0 \end{cases} \quad (94)$$

and

$$\langle \underline{E}(\underline{x}, v) \rangle = \begin{cases} (e^{ik_z z} + \Gamma_v e^{-ik_z z}) e^{i \underline{k}_{t_0} \cdot \underline{x}} \underline{v}^0, & z < 0 \\ (A_x \underline{x}^0 + A_z \underline{z}^0) e^{i \kappa_z^{(v)} \cdot \underline{x}}, & z > 0 \end{cases} \quad (95)$$

where we have introduced a reflected wave in the free space medium at the specular angle. Now by using the fact that the tangential $\langle \underline{E} \rangle$ and $\langle \underline{H} \rangle$ must be continuous at the interface, the unknown reflection and transmission coefficients can be calculated. Since the major effect of the equivalent medium is to produce exponential decay, we next expand the coefficient for small δ and we keep only zero order terms. We find that $\Gamma_q = O(\delta)$ and thus it can be neglected. The transmitted

mean fields are:

$$\langle \underline{E}(\underline{x}, q) \rangle = \underline{q}^0 e^{i(\kappa_z^{(q)} z + \underline{k}_t \cdot \underline{x})} + 0(\delta), \quad z > 0 \quad (96)$$

$q \in \{h, v\}$

Proceeding with our developemnt, we now relate the transverse Fourier transform of the correlation to the backscattering coefficients. This is done within the context of the distorted Born approximation developed in the previous section. We start by taking the transverse Fourier transform with respect to $\underline{x}$ and $\hat{\underline{x}}$ of (65), (66) and (70). We have

$$\langle \tilde{\underline{E}}_f(\underline{k}_t, z) \tilde{\underline{E}}_f^*(\hat{\underline{k}}_t, \hat{z}) \rangle = \int_{4\pi} d\omega \, p_{\underline{\Omega}}(\omega) \langle \tilde{\underline{E}}_f(\underline{k}_t, z) \tilde{\underline{E}}_f^*(\hat{\underline{k}}_t, \hat{z}) \rangle_{\omega} \quad (97)$$

$$\langle \tilde{\underline{E}}_f(\underline{k}_t, z) \tilde{\underline{E}}_f^*(\hat{\underline{k}}_t, \hat{z}) \rangle_{\omega} = \rho \int_V d\underline{s} \, \tilde{\underline{\xi}}(\underline{k}_t, z, \underline{s}) \cdot \tilde{\underline{\xi}}^*(\hat{\underline{k}}_t, \hat{z}, \underline{s}) \quad (98)$$

where

$$\tilde{\underline{\xi}}(\underline{k}_t, z, \underline{s}) = k_0^2 \tilde{\underline{G}}(\underline{k}_t, z, \underline{s}) \cdot \underline{q} \cdot \langle \underline{E}(\underline{s}) \rangle \quad (99)$$

Since we will only require $\underline{G}$ when $\underline{x}$ and $\underline{s}$ are in the equivalent medium and since the reflection at the interface is small we can replace $\underline{G}$ by the dyadic Green's function for an infinite equivalent medium, i.e.,

$$\underline{G}(\underline{x}, \underline{s}) = \underline{G}^{(\infty)}(\underline{x} - \underline{s}) + 0(\delta) \quad (100)$$

We have written the infinite space Green's function in terms of $\underline{x} - \underline{s}$ since it is translationally invariant. We then have

$$\tilde{\underline{G}}^{(\infty)}(\underline{k}_t, z, \underline{s}) = \tilde{\underline{G}}^{(\infty)}(\underline{k}_t, z - \underline{s}) e^{-i \underline{k}_t \cdot \underline{s}} \quad (101)$$

Now by putting (96), (99), (100) and (101) in (98), by integrating over $\underline{s}_t$ and by setting $z = \hat{z} = 0$, we have

$$\langle \tilde{\underline{E}}_f(\underline{k}_t, 0) \tilde{\underline{E}}_f^*(\hat{\underline{k}}_t, 0) \rangle_{\omega} = \underline{S}(\underline{k}_t, q | \omega) \delta(\underline{k}_t - \hat{\underline{k}}_t) \quad (102)$$

where

$$\underline{\underline{S}}(\underline{k}_t, q | \underline{\omega}) = (2\pi)^2 \delta k_0^4 V_P \int_0^\infty ds \langle \tilde{\underline{G}}^{(\infty)}(\underline{k}_t, -s) \cdot \underline{\underline{a}} \cdot \underline{q}^\circ \rangle (\tilde{\underline{G}}^{(\infty)})^*(\underline{k}_t, -s) \cdot \underline{\underline{a}}^* \cdot \underline{q}^\circ \rangle \cdot e^{-2\text{Im}\kappa_z^{(q)} s} \quad (103)$$

Here $\underline{\underline{S}}(\underline{k}_t, q | \underline{\omega})$ is the transverse dyadic spectral density at the interface assuming $\underline{\omega}$ is fixed. The normalized polarizability has been introduced by using (71).

By using the results of Appendix C and by noting that $\overline{\underline{\underline{S}}(\underline{k}_t, q | \underline{\omega})}$ we obtain the backscattering coefficients

$$\sigma_{pq}^\circ = \frac{k_0^2 \cos^2 \theta_0}{4\pi^3} \underline{\underline{p}}^\circ \cdot \overline{\underline{\underline{S}}(-\underline{k}_{t_0}, q | \underline{\omega})} \cdot \underline{\underline{p}}^\circ, \quad p, q \in \{h, v\} \quad (104)$$

To evaluate the integral of $\underline{\underline{g}}$ in (103), we will need the transformed Green's function. To obtain it, we first write the governing equation for $\underline{\underline{G}}^{(\infty)}$. It is

$$[\nabla \times (\nabla \times \underline{\underline{I}}) - k_0^2 (\underline{\underline{I}} + \delta \underline{\underline{a}})] \cdot \underline{\underline{G}}^{(\infty)}(\underline{x}) = \underline{\underline{I}} \delta(\underline{x}), \quad (105)$$

$$\underline{\underline{G}}^{(\infty)}(\underline{x}) - \text{outgoing as } |\underline{x}| \rightarrow \infty$$

Using

$$\underline{\underline{G}}^{(\infty)}(\underline{x}) = \frac{1}{(2\pi)^3} \int d\underline{\kappa} \underline{\underline{g}}(\underline{\kappa}) e^{i\underline{\kappa} \cdot \underline{x}} \quad (106)$$

in (105), we find

$$[(\underline{\kappa} \times \underline{\kappa} \times \underline{\underline{I}}) + k_0^2 (\underline{\underline{I}} + \delta \underline{\underline{a}})] \cdot \underline{\underline{g}}(\underline{\kappa}) = -\underline{\underline{I}} \quad (107)$$

Then

$$\tilde{\underline{\underline{G}}}^{(\infty)}(\underline{k}_t, z) = \frac{1}{2\pi} \int d\kappa_z \underline{\underline{g}}(\underline{\kappa}) e^{i\underline{\kappa} \cdot \underline{x}} \quad (108)$$

To simplify the remaining computation for $\tilde{\underline{\underline{G}}}^{(\infty)}$ we note from (104) and (103) that $\tilde{\underline{\underline{G}}}^{(\infty)}$ will only be required for $\underline{k}_t = -\underline{k}_{t_0} = -k_0 \sin \theta_0 \underline{x}^\circ$.

Inverting (107) and performing the integral in (107) by the method of residues, we obtain

$$\begin{aligned} \tilde{G}^{(\infty)}(-\underline{k}_{t_0}, z) = & \left\{ \beta_{zz} \underline{x}^0 \underline{x}^0 + \frac{\sin \theta_0 \kappa_z^{(v)} \sigma(z)}{k_0} (\underline{x}^0 \underline{z}^0 + \underline{z}^0 \underline{x}^0) \right. \\ & \left. - \frac{(\kappa_z^{(v)})^2 - k_0^2 \beta_{xx}}{k_0^2} \underline{z}^0 \underline{z}^0 \right\} \frac{e^{i \kappa_z^{(v)} |z|}}{2i \kappa_z^{(v)}} + \frac{e^{i \kappa_z^{(h)} |z|}}{2i \kappa_z^{(h)}} \underline{y}^0 \underline{y}^0 \quad (109) \end{aligned}$$

where

$$\sigma(z) = \begin{cases} 1, & z > 0 \\ -1, & z < 0 \end{cases} \quad (110)$$

and the β 's are defined in (89). Approximating the coefficients to zeroth order in δ , we have the simplified expression

$$\tilde{G}^{(\infty)}(-\underline{k}_{t_0}, z) = \underline{v}^0 \underline{v}^0 \frac{e^{i \kappa_z^{(v)} |z|}}{2i k_0 \cos \theta_0} + \underline{h}^0 \underline{h}^0 \frac{e^{i \kappa_z^{(h)} |z|}}{2i k_0 \cos \theta_0} \quad (111)$$

If we use this in (103), perform the integration and use the result in (104), we have our final form for the backscattering coefficients. It is

$$\sigma_{pq}^0 = \frac{\delta k_0^4 \underline{v}_p \overline{|a_{pq}|^2}}{8\pi (\text{Im} \kappa_z^{(p)} + \text{Im} \kappa_z^{(q)})} \quad (112)$$

where

$$\overline{|a_{pq}|^2} = \int_{4\pi} d\omega |\underline{p}^0 \cdot \underline{a} \cdot \underline{q}^0|^2 \quad (113)$$

and $\kappa_z^{(s)}$, $s \in \{h, v\}$ are given in (92) and (93). The dependence of $\overline{|a_{pq}|^2}$ on angle of incidence is worked out explicitly in Appendix B for scatterers that are distributed uniformly in the ϕ coordinate.

The final result given in (112) can be expressed in terms of scattering cross sections of individual particles. Using (28), (60) and (71), we have

$$f_{pq} = p^\circ \cdot \underline{f} \cdot q^\circ = 2\pi^2 S_{pq} = k_0^2 V_p a_{pq} / 4\pi \quad (114)$$

Now by recalling that the backscattering cross section from a particle $\sigma_{pq}^{(b)} = 4\pi |f_{pq}|^2$, (112) becomes

$$\sigma_{pq}^\circ = \frac{\rho \sigma_{pq}^{(b)}}{2\text{Im}\kappa_z^{(p)} + 2\text{Im}\kappa_z^{(q)}} \quad , \quad p, q \in \{h, v\} \quad (115)$$

Following Attema and Ulaby [1978] we can give a one dimensional interpretation of (115). If we rewrite (115) as

$$\sigma_{pq}^\circ = \int_{-\infty}^0 dz \rho \sigma_{pq}^{(b)} e^{-2\text{Im}\kappa_z^{(q)} |z|} e^{-2\text{Im}\kappa_z^{(p)} |z|} \quad (116)$$

we can view the scattering as being decomposed into scattering from slabs of width dz . An intensity of $\exp(-2\text{Im}\kappa_z^{(q)} |z|)$ is incident on the slab located at z . The incident intensity is backscattered with reflectivity factor $\rho \sigma_{pq}^{(b)}$. The backscattered wave then decays as $\exp(-2\text{Im}\kappa_z^{(p)} |z|)$ until it reaches the interface.

DISCUSSION AND NUMERICAL EVALUATION

In this section, we will first discuss several general properties of σ_{pq}° that are independent of the particular scatterer chosen. Following this discussion, we use our method to model a forest canopy by a collection of lossy dielectric discs. The theoretical curves computed from this model are then compared with some experimental data.

Because of the simple dependence of σ_{pq}° on the medium properties and incidence angle certain general observations can be made that are independent of the particular nature of the scatterer. First, we note that σ_{pq}° as given by (115) is independent of the density of scatterers ρ . This follows directly from (92) and (93) where we see that the $\text{Im} \kappa_z^{(s)}$, $s \in \{h, v\}$ are directly proportional to ρ . Thus the linear ρ dependence in the numerator of (115) is cancelled out by the denominator. Second, we note that $\sigma_{hh}^{\circ} = \cos \theta_0$. This is the same result as predicted by the scalar theory. Finally, we note that $\sigma_{hh}^{\circ} = \sigma_{vv}^{\circ}$ at normal incidence ($\theta = 0$). This is an expected result. Since the scatterers are uniformly distributed in ϕ , the two polarizations see the same medium at normal incidence.

We now proceed to model a forest canopy by a collection of leaves. The leaves are in turn assumed to be lossy dielectric discs as mentioned previously. The discs have radius a and thickness h . Typical dimensions are radii of one to several centimeters and thicknesses of tenths of a millimeter. The electrical properties of discs can be characterized by their normalized polarizability tensor $\underline{\underline{a}}$ when the wavelength is large compared to the disc. From Jones [1964] and

Van de Hulst [1957] the polarizability of a disc along its principal axes is given by

$$a_r = \frac{\alpha_r}{V_p} = \frac{\Delta}{1+\Delta} \quad , \quad a_\theta = a_\phi = \Delta \quad , \quad \Delta = \epsilon_r - 1 \quad (117)$$

when r, θ, ϕ are defined in Figure 3.

Because of the large volume of water present in vegetation, we can usually assume $|\epsilon_r| \gg 1$ in the microwave region. Using this assumption in (117), we find that $|a_\theta| = |a_\phi| \gg |a_r|$. This inequality can now be used to simplify the scattering cross section of (115). We find that $\sigma_{pq}^0 \approx |\epsilon_r|^2 V_p / \text{Im} \epsilon_r$. Thus it follows that the magnitude of the backscattering cross sections are directly related to the volume and complex dielectric constant of the discs in a simple manner. Therefore, as leaves grow and as their moisture content changes these effects should be observable by measuring σ_{pq}^0 at different periods of the growing season.

Before computing the backscattering cross sections as a function of incidence angle, we will require the relative dielectric constant of the leaves and the angular distribution of leaves. First the relative dielectric constant is considered. Our calculation follows that of Fung and Ulaby [1978] who in turn have based his results upon de Loor [1968] and Carlson [1967]. They model the leaves as a mixture of water and solid materials. For illustrative purposes we have chosen 50% water and 50% solid for our calculations. By using (3) and (4) of Fung and Ulaby [1978] at a frequency of 1.1 GHz we find that $\epsilon_r = 30.8 + j1.8$. Our choice of frequency has been motivated by experimental results that appear in the literature. We have chosen to

compare our results with those of Bush, et al [1976] who has measured σ_{pq}^0 , $p, q \in \{h, v\}$ from forests for frequencies 1-18 GHz. Because of the dipole approximation made in our model, only the lowest frequency (1.1GHz) Ulaby measured was used for comparison purposes.

The angular distribution of leaves will now be considered. Field measurements of leaf orientations have been made by Smith [1973] and others. It has been found that the leaves are distributed uniformly in the ϕ coordinate (Figure 2). The distribution of leaves with respect to θ is more vegetation type dependent. Several are given by Smith [1977]. Since no measurements of this type exist for the Ulaby data, we have assumed that ϕ is uniformly distributed. For θ we have considered the following two density functions:

$$p_{\theta}(\theta) = \begin{cases} \frac{1}{\Delta\theta_{//}} & , \quad 0 \leq \theta \leq \Delta\theta_{//} \\ 0 & , \quad \Delta\theta_{//} \leq \theta \leq \pi \end{cases} \quad (118)$$

or

$$p_{\theta}(\theta) = \begin{cases} \frac{1}{2\Delta\theta_{\perp}} & , \quad \pi - \Delta\theta_{\perp} \leq \theta \leq \pi + \Delta\theta_{\perp} \\ 0 & , \quad \text{elsewhere} \end{cases} \quad (119)$$

In (118) when $\Delta\theta_{//}$ is small, the leaves are approximately parallel to the interface ($z=0$); when $\Delta\theta_{//} = \pi/2$, they are uniformly distributed in θ . In (119) when $\Delta\theta_{\perp}$ is small, the leaves are perpendicular to the interface; when $\Delta\theta_{\perp}$ is $\pi/2$ they are uniformly distributed in θ .

The numerical calculations are presented in Figures 4-9. In these figures the backscattering coefficient is plotted as a function of the angle of incidence θ_0 . In Figures 4-7 we

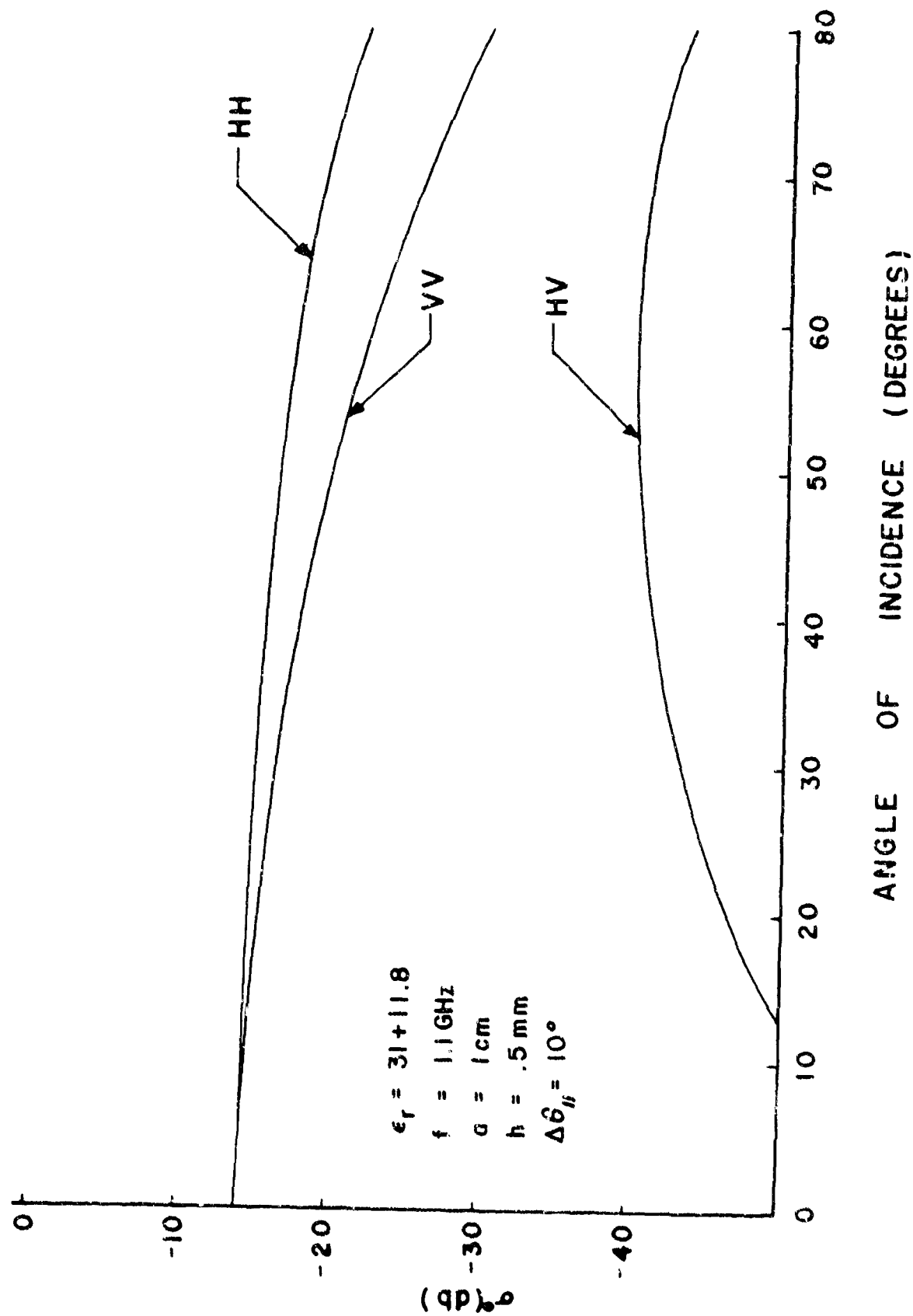


Figure 4 Backscattering coefficients versus angle of incident for $\Delta\theta_{//} = 10^\circ$

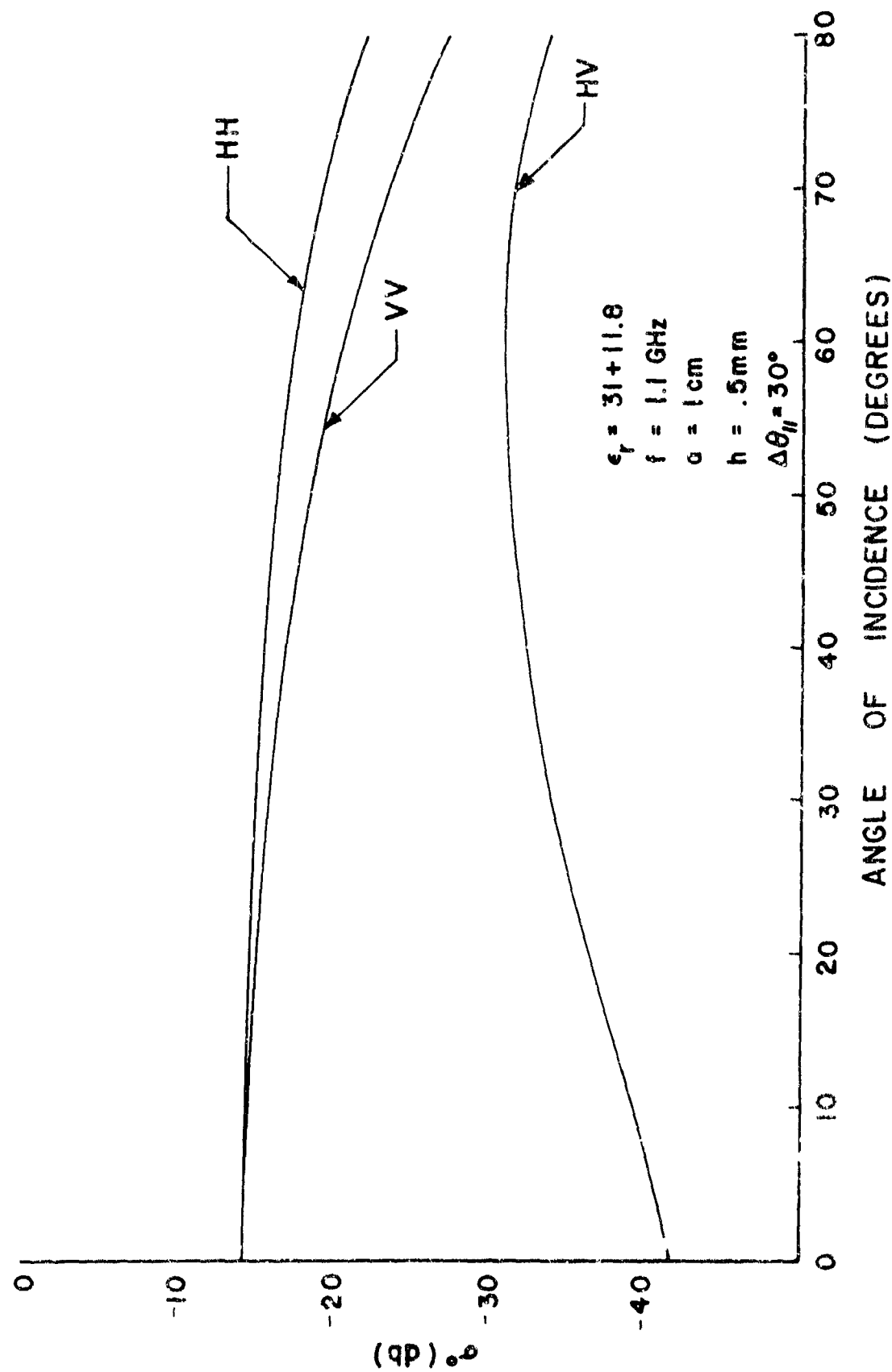


Figure 5 Backscattering coefficients versus angle of incident for $\Delta\theta_{//} = 10^\circ$

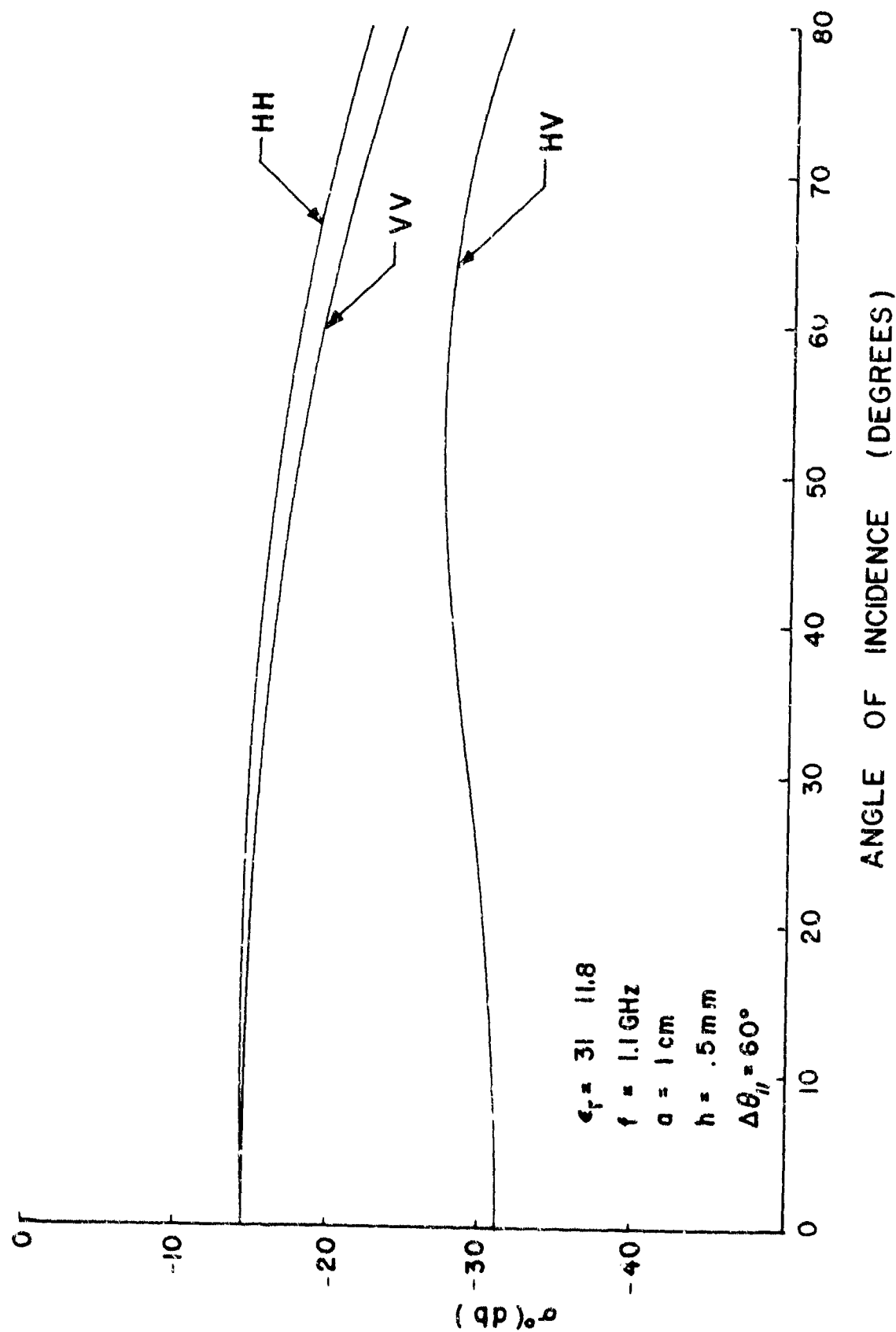


Figure 6 Backscattering coefficients versus angle of incident for $\Delta\theta_{//} = 30^\circ$

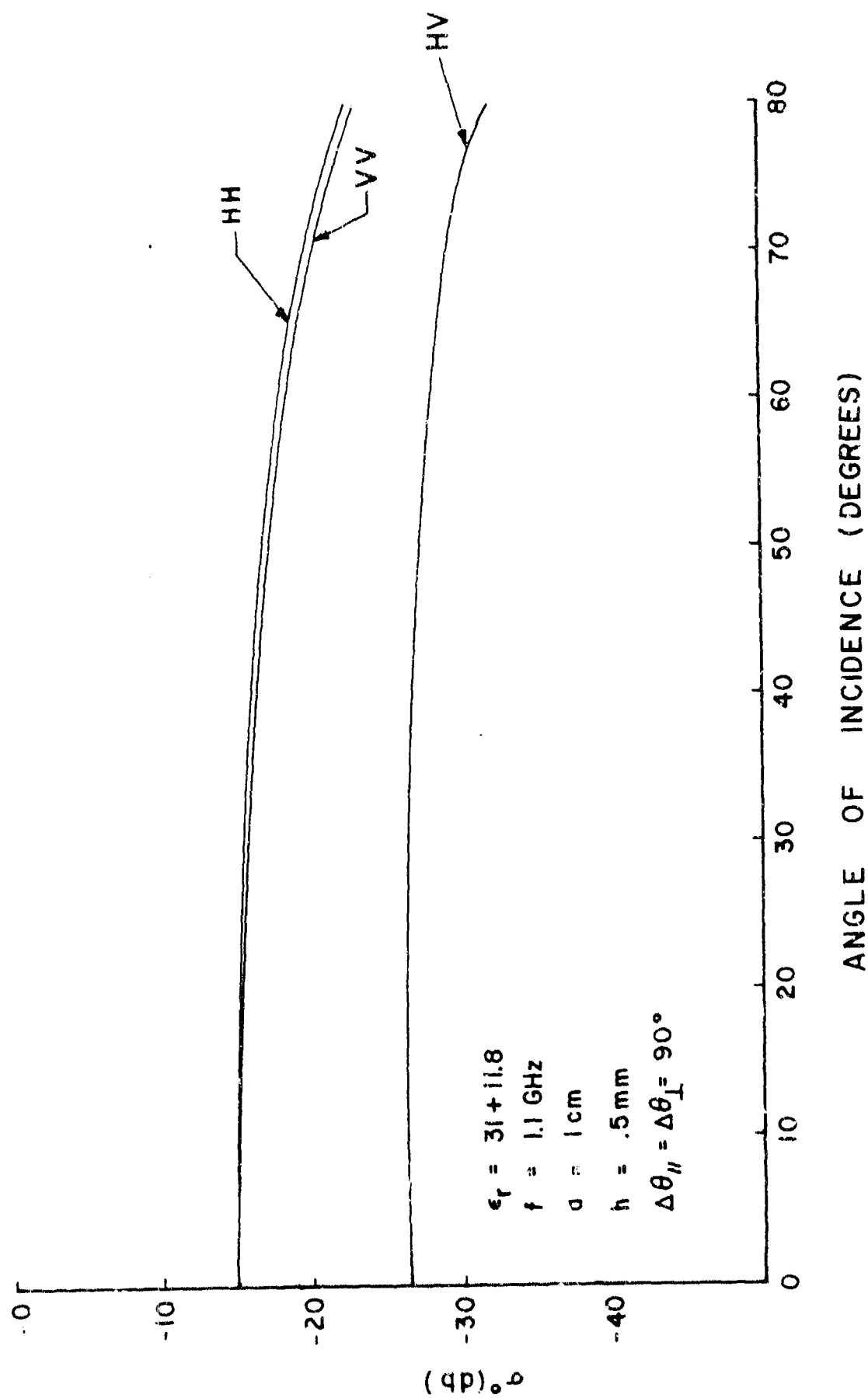


Figure 7 Backscattering coefficients versus angle of incident for $\Delta\theta_{II} = \Delta\theta_I = 90^\circ$

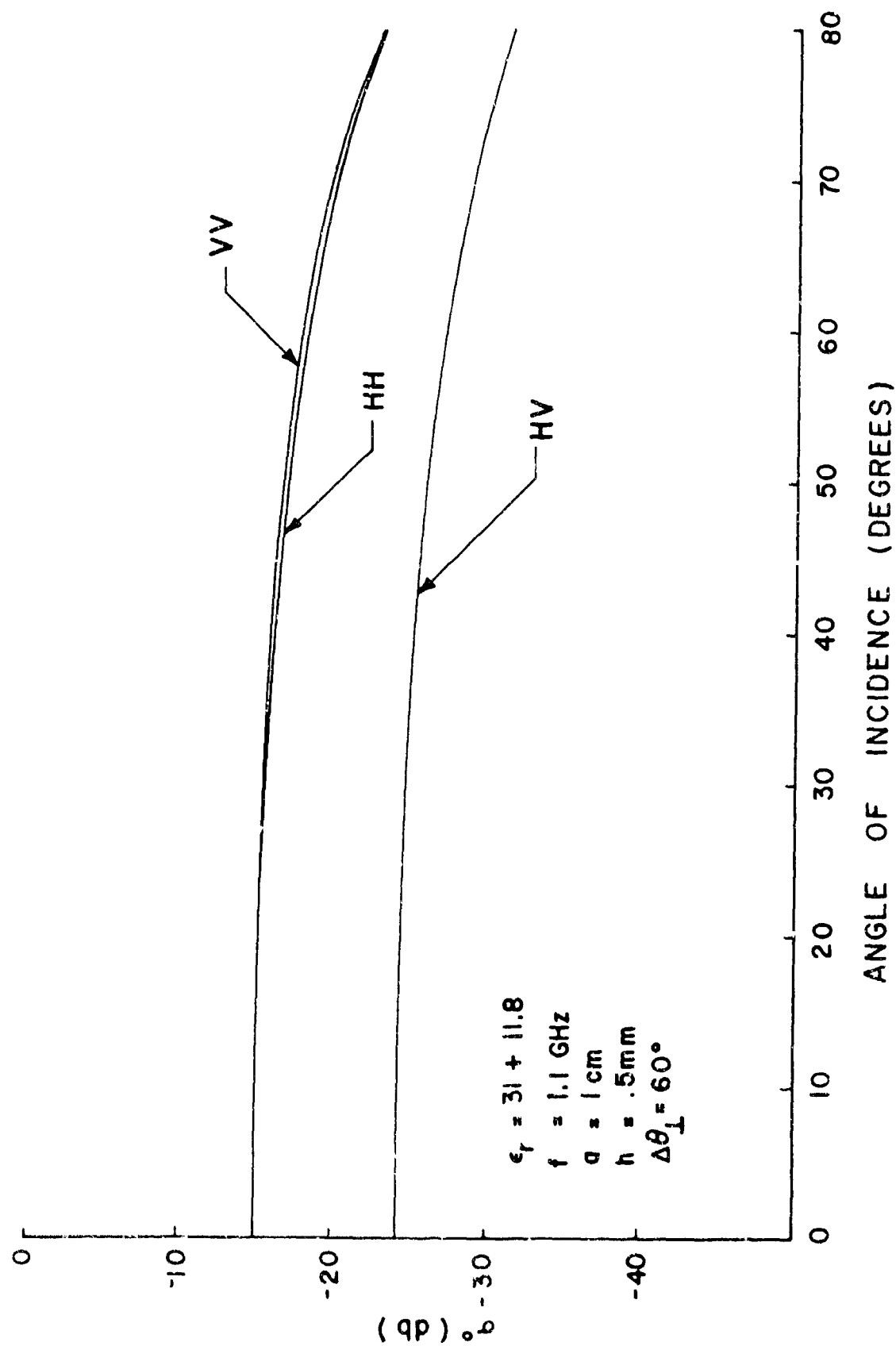


Figure 8 Backscattering coefficients versus angle in incident for $\Delta\theta_I = 60^\circ$

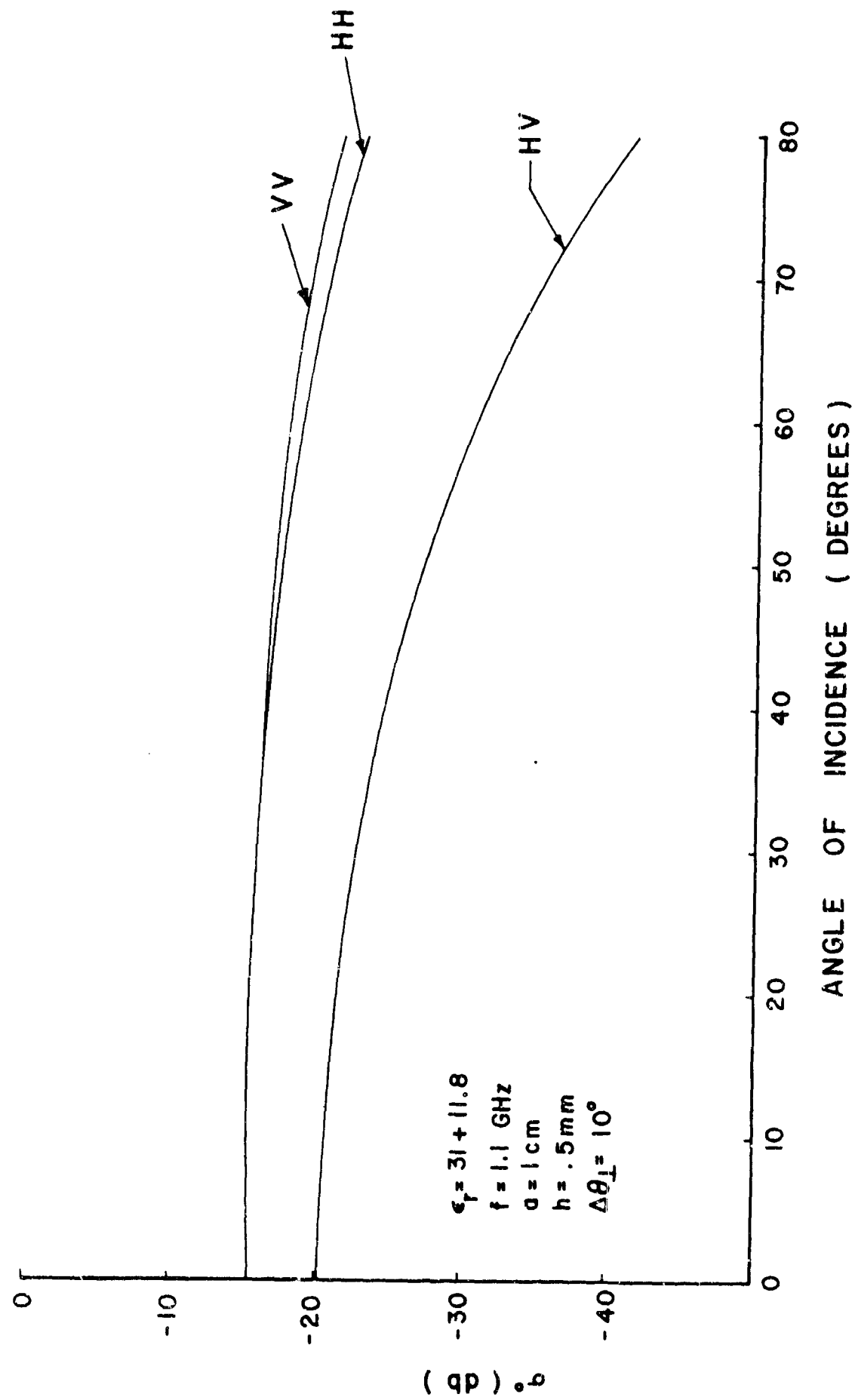


Figure 9 Backscattering coefficients versus angle of incident for $\Delta\theta_I = 10^\circ$

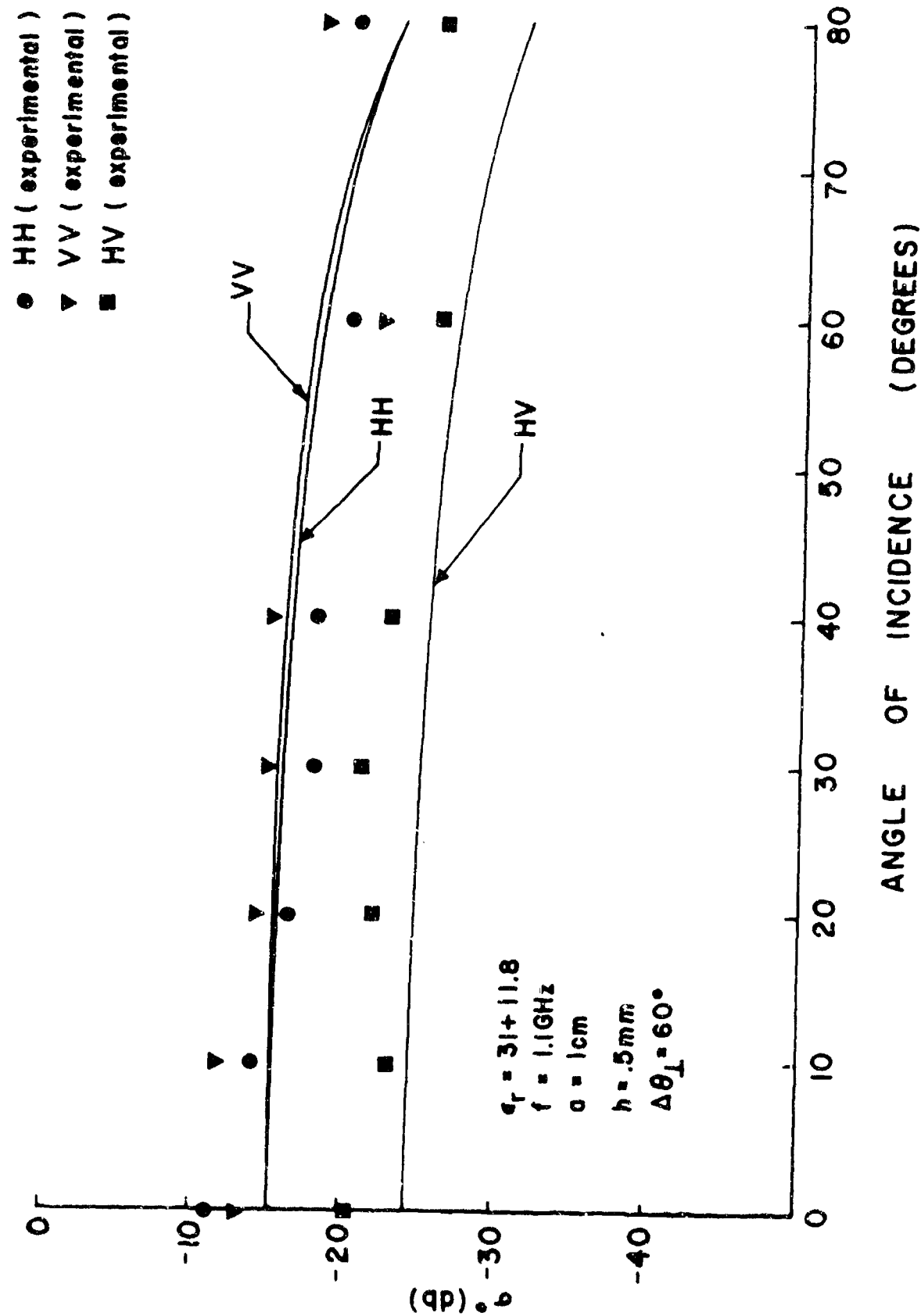


Figure 10 Comparison of experimental and theoretical results for $\Delta\theta_L = 60^\circ$

have used the angular distribution given in (118) while in Figures 7-9 we have used (119). Figure 7 corresponds to a uniform distribution over all θ and thus, for this case, (118) and (119) give the same results. One should note that since $|\epsilon_r| \gg 1$, a change in f , V_p or ϵ_r just shifts the level of the curves but it does not change their shape. Their shape only depends on $p_\theta(\theta)$.

The following trends are observed in Figures 4-7: First, σ_{hh}° is always greater than σ_{vv}° . Their difference increases as $\Delta\theta_{//}$ becomes smaller. Second, the cross polarized back-scatter becomes smaller as $\Delta\theta_{//}$ becomes smaller. In Figure 7-9, we observe that: First, σ_{vv}° becomes greater than σ_{hh}° as $\Delta\theta_\perp$ is decreased; second, the curve for σ_{hv}° tends to flatten out as $\Delta\theta_\perp$ is increased; and third, the difference $\sigma_{hh}^\circ - \sigma_{hv}^\circ$ at $\theta_0 = 0$ becomes smaller as $\Delta\theta_\perp$ increases.

A comparison of our theory with the experimental results of Bush, et.al [1976] is made in Figure 10. There, we have plotted our Figure 8 along with his data for Kansas deciduous trees measured in the springtime at a frequency of 1.1 GHz. Figure 8 was chosen since it most clearly appears to follow the trends of the data i.e., flat cross polarization and $\sigma_{vv}^\circ > \sigma_{hh}^\circ$.

Although our theory follows the trends of the data, it is clear from the results that additions to our model should be made. The fact that $\sigma_{vv}^\circ > \sigma_{hh}^\circ$ is most likely due to the vertically oriented tree branches other than due to the leaves that tend to be parallel to the interface. In addition, an examination of the numerical results shows that the skin depth

for the mean wave is large. Thus at a frequency of 1 GHz the underlying ground should be taken into account.

CONCLUSIONS AND RECOMMENDATIONS

We have developed and analyzed a discrete scattering model for vegetation. The model replaces the vegetation by simple scattering objects such as discs, spheres or rods so that leaves, branches and trunks can be modeled. The objects are given random placement and orientation in order that the complicated features of the individual scattering objects are averaged out and simple expressions are developed for the backscattering coefficients. The model directly relates the backscattering coefficients to easily measured medium parameters such as leaf size, density and moisture content. The theory as presented also takes into account depolarization effects that agree with experimental data.

At present only the leaves in the forest have been included in this model and no effects of the ground have been taken into account. We recommend: first, that the effect of the ground be included. If this is done agricultural crops can be modeled. Second, the numerical calculations indicated the branches in forests will make an important contribution of microwave frequencies. They can be modeled by dielectric cylinders and averaged over position.

Finally we feel that the inverse problems related to the above approach should be investigated. Questions such as: Can the probability density function of leaf inclination angles be determined from backscatter data? Once the distribution of inclination angles is known, forest or crop identification will be much easier.

REFERENCES

- Attema, E., and F. Ulaby, (1978), "Vegetation Modeled as a Water Cloud," Radio Sci., 13(2), 357-364.
- Bush, T., F. Ulaby, T. Metzler, and H. Stiles, (1976), "Seasonal Variations of the Microwave Scattering Properties of the Deciduous Trees as Measured in the 1-18GHz Spectral Range," Tech. Rep., 177-60, Remote Sensing Lab., University of Kansas, Lawrence, Kansas.
- Carlson, N. L., (1967), "Dielectric Constant of Vegetation at 8.5GHz," Tech. Rep. 1903-5, Electro-Science Lab., Ohio State University, Columbus, Ohio.
- Collin, R. and F. Zucker (1969), Antenna Theory Part I, 62-69, McGraw Hill, New York.
- de Loor, G. P. (1968), "Dielectric Properties of Heterogeneous Mixtures Containing Water," J. Microwave Power 3, 67-73.
- Du, L., and W. H. Peake (1969), "Rayleigh Scattering From Leaves," Proc. IEEE, 57(6), 1227-1229.
- Foldy, L. (1945), "The Multiple Scattering of Waves," Phys. Rev., 67(3), 107-119.
- Frisch, U. (1968), "Wave Propagation in a Random Medium," in Probabilistic Methods in Applied Math. Vol. I, edited by A.T. Bharucha-Reid, 75-198, Academic Press, New York.
- Fung, A. and H. Fung (1977), "Application of First-Order Renormalization Method to Scattering From a Vegetated-like Half-space," IEEE Trans. Geosci. Electron., 15(4), 189-195.
- Fung, A. and F. Ulaby (1978), "A Scatter Model for Leafy Vegetation," IEEE Trans. Geosci. Electron., 16(4), 281-286.
- Fung, A. (1979), "Scattering From a Vegetation Layer," IEEE Trans. Geosci. Electron., 17(1), 1-6.
- Hevenor, R. (1977), "Backscattering of Radar Waves by Vegetated Terrain," Tech. Rep. ETL-0105, U.S. Army Engineer Topographical Laboratories, Fort Belvoir, VA.
- Jones, D. S. (1964), The Theory of Electromagnetism, 807 pp., Pergamon Press, New York.
- Keller, J. B. (1962), "Wave Propagation in Random Media," in Proc. Sympos. Appl. Math., Vol. 13, pp. 145-170, Amer. Math. Soc., Providence, RI.
- Keller, J. B. (1964), "Stochastic Equations and Wave Propagation in Random Media," Proc. Symp. Appl. Math., 16 145-170, Amer. Math. Soc., Providence, RI.

16. Keller, J. B. (1964), "Stochastic Equations and Wave Propagation in Random Media," Proc. Symp. Appl. Math., 16 145-170, Amer. Math. Soc., Providence, RI.
17. Keller, J. B. and F. C. Karal, Jr. (1966), "Effective Dielectric Constant, Permeability, and Conductivity of a Random Medium and the Velocity and Attenuation Coefficient of Coherent Waves," J. Math. Phys., 7(4), 661-670.
18. Lang, R. H., and S. Sokolakis (1979), "Radar Backscattering From a Vegetated Terrain; A Discrete Scattering Approach," Tech. Rept. ETL-0159, U.S. Army Engineer Topographical Laboratories, Fort Belvoir, VA.
19. Lax, M. (1951), "Multiple Scattering of Waves," Rev. Mod. Phys., 23(4), 287-310.
20. Lax, M. (1952), "Multiple Scattering of Waves. II. The Effective Field in Dense Systems," Phys. Rev., 85(4), 621-629.
21. Rosenbaum, S. and W. Bowles (1974), "Clutter Return From Vegetated Areas," IEEE Trans. Antennas Propaga., AP-22(2), 227-236.
22. Smith, J., R. Oliver and J. Berry (1977), "A Comparison of Two Photographic Techniques for Estimating Foliage Angle Distribution," Aust. J. Bot., 25, 545-53.
23. Tartarskii, V. I., and M. E. Gertsenshtein (1963), "Propagation of Waves in a Medium With Strong Fluctuation of the Refractive Index," Soviet Physics JETP, 17(2), 458-463.
24. Tsang, L. and J. A. Kong (1978), "Radiative Transfer Theory for Active Remote Sensing of Half-Space Random Media," Radio Sci., 13(5), 763-773.
25. Twersky, V. (1962), "On Scattering of Waves by Random Distributions. I. Free Space Scatter Formalism," J. Math. Phys., 3(4), 700-715.
26. Twersky, V. (1967), "Multiple Scattering of Electromagnetic Waves by Arbitrary Configurations," J. Math. Phys., 8(3), 589-610.
27. Twersky, V. (1978), "Coherent, Electromagnetic Waves in Pair-Correlated Random Distribution of Aligned Scatterers," J. Math. Phys., 19(1), 215-230.
28. Van de Hulst, H. C. (1957), Light Scattering by Small Particles, 470 pp, Wiley, New York.
29. Zuniga, M., T. Habashy, and J. Kong (1979), "Active Remote Sensing of Layered Random Media," IEEE Trans. Geosci. Electron., 17(4), 276-302.

APPENDIX A

Relationship Between the Transition Operator and the Scattering Amplitude

To find the relationship between $\underline{\underline{f}}$ and $\underline{\underline{\tilde{t}}}$, we start with (15). Using (23) and (26), we can write (15) in terms of dyadic incident and scattered wave

$$\underline{\underline{e}}_s = \underline{\underline{L}}^{-1} \cdot \underline{\underline{T}} \cdot \underline{\underline{e}}_0 \quad (1A)$$

Next we use (16) in (1A) along with the free space dyadic Green's function $\underline{\underline{\Gamma}}$ for $\underline{\underline{L}}^{-1}$. We have

$$\underline{\underline{e}}_s(\underline{x}, \underline{i}) = \int d\underline{x}' \underline{\underline{\Gamma}}(\underline{x}, \underline{x}') \cdot \int d\underline{x}'' \underline{\underline{t}}(\underline{x}', \underline{x}'') \cdot \underline{\underline{e}}_0(\underline{x}'', \underline{i}) \quad (2A)$$

Here the free space dyadic Green's function is given by

$$\underline{\underline{\Gamma}}(\underline{x}, \underline{x}') = (\underline{\underline{I}} + \frac{\nabla \nabla}{k_0^2}) \frac{e^{ik_0 |\underline{x} - \underline{x}'|}}{4\pi |\underline{x} - \underline{x}'|} \quad (3A)$$

To obtain $\underline{\underline{e}}_s$ in the radiation zone, the far field expression for $\underline{\underline{\Gamma}}$ will be required. It is [Twersky, 1967]

$$\underline{\underline{\Gamma}}(\underline{x}, \underline{x}') \sim (\underline{\underline{I}} - \underline{\underline{00}}) e^{-ik_0 \underline{0} \cdot \underline{x}'} \frac{e^{ik_0 |\underline{x}|}}{4\pi |\underline{x}|}, \quad |\underline{x}| \rightarrow \infty \quad (4A)$$

Now putting (4A) and (25) in (2A)

$$\begin{aligned} \underline{\underline{e}}_s(\underline{x}, \underline{i}) \sim (\underline{\underline{I}} - \underline{\underline{00}}) \frac{1}{4\pi} \int d\underline{x}' d\underline{x}'' \underline{\underline{t}}(\underline{x}', \underline{x}'') e^{ik_0 (\underline{i} \cdot \underline{x}'' - \underline{0} \cdot \underline{x}')} \\ \cdot (\underline{\underline{I}} - \underline{\underline{ii}}) \frac{e^{ik_0 |\underline{x}|}}{|\underline{x}|} \end{aligned} \quad (5A)$$

Finally employing (19) in (5A), and comparing with (27) we obtain the required result

$$\underline{\underline{f}}(\underline{0}, \underline{i}) = 2\pi^2 (\underline{\underline{I}} - \underline{\underline{00}}) \cdot \underline{\underline{\tilde{t}}}(\underline{k}_0 \underline{0}, \underline{k}_0 \underline{i}) \cdot (\underline{\underline{I}} - \underline{\underline{ii}}) \quad (6A)$$

APPENDIX B - Polarizability Statistics

In this appendix calculate the mean square polarization statistics used in (112). The calculation will be performed with the assumption that the scatterers are uniformly distributed in ϕ .

First we derive the components of the polarizability tensor in cartesian coordinates in terms of the principal axis components. The unit vectors $\underline{r}^\circ$, $\underline{\theta}^\circ$ and $\underline{\phi}^\circ$ are related to x° , y° and z° as follows:

$$\begin{aligned}\underline{r}^\circ &= \sin\theta\cos\phi\underline{x}^\circ + \sin\theta\sin\phi\underline{y}^\circ + \cos\theta\underline{z}^\circ \\ \underline{\theta}^\circ &= \cos\theta\cos\phi\underline{x}^\circ + \cos\theta\sin\phi\underline{y}^\circ - \sin\theta\underline{z}^\circ \quad (1B) \\ \underline{\phi}^\circ &= -\sin\phi\underline{x}^\circ + \cos\phi\underline{y}^\circ\end{aligned}$$

Using this in (73), the cartesian components of (74) are:

$$\begin{aligned}a_{xx} &= (a_r\sin^2\theta + a_\theta\cos^2\theta)\cos^2\phi + a_\phi\sin^2\phi \\ a_{yy} &= (a_r\sin^2\theta + a_\theta\cos^2\theta)\sin^2\phi + a_\phi\cos^2\phi \\ a_{zz} &= a_r\cos^2\theta + a_\theta\sin^2\theta \\ a_{xy} &= [(a_r\sin^2\theta + a_\theta\cos^2\theta) - a_\phi]\cos\phi\sin\phi \\ a_{xz} &= (a_r - a_\theta)\sin\theta\cos\theta\cos\phi \\ a_{yz} &= (a_r - a_\theta)\sin\theta\cos\theta\sin\phi\end{aligned} \quad (2B)$$

The other components are gotten from the fact that $\underline{a}$ is a symmetric dyadic.

Now we will obtain the components of $\underline{a}$ in the polarization directions in terms of the cartesian components. We have using (84)

$$a_{hh} = \underline{h}^0 \cdot \underline{a} \cdot \underline{h}^0 = a_{yy}$$

$$a_{hv} = \underline{h}^0 \cdot \underline{a} \cdot \underline{v}^0 = -\cos\theta_0 a_{yx} + \sin\theta_0 a_{yz}$$

(3B)

$$a_{vh} = \underline{v}^0 \cdot \underline{a} \cdot \underline{h}^0 = a_{hv}$$

$$a_{vv} = \underline{v}^0 \cdot \underline{a} \cdot \underline{v}^0 = \cos^2\theta_0 a_{xx} - 2\cos\theta_0 \sin\theta_0 a_{xz} + \sin^2\theta_0 a_{zz}$$

Following this, the mean square polarizabilities are computed. When we assume $a_\theta = a_\phi = a_t$ and we use the approximation that $a_t \gg a_r$, the mean square polarizabilities become

$$\overline{|a_{hh}|^2} = |a_t|^2 [3 \sin^4\theta/8 + \cos^2\theta]$$

$$\overline{|a_{hv}|^2} = \overline{|a_{vh}|^2} = |a_t|^2 [\sin^4\theta \cos^2\theta_0/8 + \sin^2\theta \cos^2\theta \sin^2\theta_0/2]$$

$$\begin{aligned} \overline{|a_{vv}|^2} = |a_t|^2 \left\{ [3 \sin^4\theta/8 + \cos^4\theta] \cos^4\theta_0 + \sin^4\theta \sin^4\theta_0 \right. \\ \left. + [2\sin^2\theta \cos^2\theta - \sin^4\theta + 2\sin^2\theta] \sin^2\theta_0 \cos^2\theta_0 \right\} \end{aligned}$$

APPENDIX C - Relationship Between the Backscattering Coefficients and the Transverse Spectral Density

We start by considering the fluctuating portion of the scattered field, $\underline{E}_f$, as defined in (6.3) in the region $z \geq 0$. This field can be viewed as arising from sources on the interface. So that far field quantities can be found, we initially consider that portion of $\underline{E}_f$ that arises from sources contained within a finite region A on the interface. The radiated field from the region A will be denoted by $\underline{E}_{fA}(\underline{x}, q)$.

The field $\underline{E}_{fA}(\underline{x}, q)$ can be related to the interface fluctuations by employing a plane wave expansion in the region $z \leq 0$. We have

$$\underline{E}_{fA}(\underline{x}, q) = \frac{1}{(2\pi)^2} \int d\underline{k}_t \int_A d\underline{x}_t \underline{E}_f(0, \underline{x}_t, q) e^{-ik_z z + i\underline{k}_t \cdot (\underline{x}_t - \underline{x}_t')}, \quad z \leq 0 \quad (1C)$$

$$k_z = \sqrt{k_0^2 - |\underline{k}_t|^2}, \quad \text{Im} k_z \geq 0$$

where $\underline{E}_f(0, \underline{x}_t, q)$ is the fluctuating field on the interface due to an incident wave of polarization q. The $\underline{k}_t$ integral in (1C) can be asymptotically evaluated for large $|\underline{x}|$ [Collin and Zucker, 1969]. We find

$$\underline{E}_{fA}(\underline{x}, q) \sim \frac{+ik_0 \cos \theta_0}{2\pi |\underline{x}|} e^{+ik_0 |\underline{x}|} \int_A d\underline{x}_t \underline{E}_f(0, \underline{x}_t, q) e^{i\underline{k}_{t0} \cdot \underline{x}_t'} \quad (2C)$$

where $\underline{x}$ has been specialized to the backscatter direction θ_0 and $\underline{k}_{t0}$ is given by (83).

The backscattering coefficients are now defined. They are

$$\sigma_{pq}^0 = \lim_{\Lambda \rightarrow \infty} \lim_{|\underline{x}| \rightarrow \infty} \frac{4\pi |\underline{x}|^2 I_s(p, q)}{A I_1(q)} \quad q, p \in \{h, v\} \quad (3C)$$

where $I_i(q)$ is the incident intensity per unit area with polarization q , $I_s(p,q)$ is the average intensity at the observation point with polarization p due to an incident wave with polarization q . In view of (81), $I_i(q)=1$. We also have

$$I_s(p,q) = \langle |E_{fA}(\underline{x},q) \cdot \underline{p}^0|^2 \rangle \quad (4C)$$

We can complete the development by first representing the field on the interface by its Fourier transform, $\tilde{E}_f(\underline{k}_t, q)$:

$$E_f(0, \underline{x}_t, q) = \frac{1}{(2\pi)^2} \int \tilde{E}_f(\underline{k}_t, q) e^{i \underline{k}_t \cdot \underline{x}_t} d\underline{k}_t \quad (5C)$$

Now by using (2C), (4C) and (5C) in (3C), we have

$$\begin{aligned} \sigma_{pq}^0 = \lim_{A \rightarrow \infty} \lim_{|\underline{x}| \rightarrow \infty} \frac{k_0^2 \cos^2 \theta_0}{2^4 \pi^5 A} \int d\underline{k}_t' d\underline{k}_t'' \int_{A \times A} d\underline{x}_t' d\underline{x}_t'' & \langle \underline{p}^0 \cdot \tilde{E}_f(\underline{k}_t', q) \\ \tilde{E}_f^*(\underline{k}_t'', q) \cdot \underline{p}^0 \rangle & e^{i [\underline{k}_t' \cdot \underline{x}_t' - \underline{k}_t'' \cdot \underline{x}_t'' + \underline{k}_{t0} \cdot (\underline{x}_t'' - \underline{x}_t')] } \end{aligned} \quad (6C)$$

Next we introduce the transverse dyadic spectral density $\underline{\underline{S}}(\underline{k}_t, q)$ which is given by

$$\langle \tilde{E}_f(\underline{k}_t', q) \tilde{E}_f^*(\underline{k}_t'', q) \rangle = \underline{\underline{S}}(\underline{k}_t, q) \delta(\underline{k}_t' - \underline{k}_t'') \quad (7C)$$

Note that this definition requires that the process $\tilde{E}_f(\underline{k}_t, q)$ be homogeneous or stationary in $\underline{k}_t$.

By using (7C) in (6C) and carrying out the integrations, we obtain

$$\sigma_{pq}^0 = \frac{k_0^2 \cos^2 \theta_0}{4\pi^3} S_p(-\underline{k}_{t0}, q) \quad (8C)$$

where

$$S_p(\underline{k}_t, q) = \underline{p}^0 \cdot \underline{\underline{S}}(\underline{k}_t, q) \cdot \underline{p}^0 \quad (9C)$$

THE GEORGE WASHINGTON UNIVERSITY

BENEATH THIS PLAQUE
IS BURIED

A VAULT FOR THE FUTURE
IN THE YEAR 2036

THE STORY OF ENGINEERING IN THIS YEAR OF THE PLACING OF THE VAULT AND
ENGINEERING HOPES FOR THE TOMORROWS AS WRITTEN IN THE RECORDS OF THE
FOLLOWING GOVERNMENTAL AND PROFESSIONAL ENGINEERING ORGANIZATIONS AND
THOSE OF THIS GEORGE WASHINGTON UNIVERSITY.

BOARD OF COMMISSIONERS DISTRICT OF COLUMBIA
UNITED STATES ATOMIC ENERGY COMMISSION
DEPARTMENT OF THE ARMY UNITED STATES OF AMERICA
DEPARTMENT OF THE NAVY UNITED STATES OF AMERICA
DEPARTMENT OF THE AIR FORCE UNITED STATES OF AMERICA
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
NATIONAL BUREAU OF STANDARDS U.S. DEPARTMENT OF COMMERCE
AMERICAN SOCIETY OF CIVIL ENGINEERS
AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
THE SOCIETY OF AMERICAN MILITARY ENGINEERS
AMERICAN INSTITUTE OF MINING & METALLURGICAL ENGINEERS
DISTRICT OF COLUMBIA SOCIETY OF PROFESSIONAL ENGINEERS INC.
THE INSTITUTE OF RADIO ENGINEERS INC.
THE CHEMICAL ENGINEERS CLUB OF WASHINGTON
WASHINGTON SOCIETY OF ENGINEERS
FAULKNER KINGSBURY & STENHOUSE ARCHITECTS
CHARLES R. TOMPKINS COMPANY-BUILDERS
SOCIETY OF WOMEN ENGINEERS
NATIONAL ACADEMY OF SCIENCES NATIONAL RESEARCH COUNCIL

THE PURPOSE OF THIS VAULT IS INSPIRED BY AND IS DEDICATED TO
CHARLES HOOK TOMPKINS, DOCTOR OF ENGINEERING
BECAUSE OF HIS ENGINEERING CONTRIBUTIONS TO THIS UNIVERSITY, TO HIS
COMMUNITY, TO HIS NATION AND TO OTHER NATIONS.

BY THE GEORGE WASHINGTON UNIVERSITY

ROBERT V. FLEMING

CHAIRMAN OF THE BOARD OF TRUSTEES

CLOYD H. MARVIN

PRESIDENT

JUNE TWENTY-NINTH
1955

To cope with the expanding technology, our society must be assured of a continuing supply of rigorously trained and educated engineers. The School of Engineering and Applied Science is completely committed to this objective.