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New York University
251 Mercer Street
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Final Technical Report

Contract No. N00014-76-C-0010

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# "Propagation, Scattering, and Reflection of Waves in Continuous and Random Inhomogeneous Media, and Nonlinear Boundary Value Problems"

Edward L. Reiss
Principal Investigator

## 1. Publications.

The following papers were published that describe the research work performed under this contract.

| | | |
|---|---|---|
| ONR-H66 | G. A. Kriegsmann<br>E. W. Larsen | On the Parabolic Approximation to the Reduced Wave Equation<br>Pub: SIAM J. Appl. Math., 34 (1978), pp. 200-204. |
| ONR-H67 | J. Tavantzis<br>E. L. Reiss<br>B. J. Matkowsky | On the Smooth Transition to Convection<br>Pub: SIAM J. Appl. Math., 34 (1978), pp. 322-337. |
| ONR-H68 | B. J. Matkowsky<br>E. L. Reiss | Singular Perturbations of Bifurcations<br>Pub: SIAM J. Appl. Math., 33 (1977), pp. 230-255. |
| ONR-H69 | J. B. Keller | The Ray Theory of Ship Waves and the Class of Streamlined Ships<br>Acc: J. Fluid Mech., in press. |
| ONR-H70 | E. L. Reiss | Cutoff Wavenumbers and Modes of Hexagonal Waveguides<br>Pub: SIAM J. Appl. Math., 35 (1978), pp. 508-514. |
| ONR-H71 | G. A. Kriegsmann<br>E. L. Reiss | New Magnetohydrodynamic Equilibria by Secondary Bifurcation<br>Pub: Phys. Fluids, 21 (1978), pp. 258-264. |
| ONR-H72 | T. J. Mahar<br>B. E. Willner | Eigenvalue Problems and Continuous Dependence<br>Sub: Comm. Pure Appl. Math. |
| ONR-H73 | J. B. Keller | Rays, Waves and Asymptotics<br>Pub: Bull. Am. Math. Soc., 84 (1978), pp. 727-750. |
| ONR-H74 | R. C. Morgan<br>S. N. Karp | Multi-Mode Cylindrical Surface Wave Excitation<br>Pub: Inter. J. Eng. Sci., 16 (1978), pp. 155-171. |

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|---|---|---|
| ONR-H75 | D. U. Anyanwu<br>J. B. Keller | Asymptotic Solution of Higher Order Differential Equations with Several Turning Points, and Application to Wave Propagation in Slowly Varying Waveguides<br>Pub: Comm. Pure Appl. Math., 31 (1978), pp. 107-121. |
| ONR-H76 | J. B. Keller | Modification of Bubble Oscillations by Nearby Objects<br>Pub: J. Acoust. Soc. Am., 64 (1978), p. 937. |
| ONR-H77 | E. L. Reiss<br>J. Tavantzis | A Boundary Value Problem of Thermal Convection<br>Acc: J. Diff. Eqns. |
| ONR-H78 | T. J. Mahar<br>B. Willner | The Existence of Differential Equations Whose Solutions Have Certain Properties<br>Sub: Comm. Pure Appl. Math. |
| ONR-H79 | D. S. Ahluwalia<br>E. L. Reiss | Sound Propagation in a Temporally Varying Ocean<br>Sub: J. Acoust. Soc. Am. |
| ONR-H80 | J. B. Keller | Elastic Waveguides<br>Pub: Modern Problems in Elastic Wave Propagation, J. Miklowitz and J. D. Achenbach, eds., Wiley, New York, 1978, pp. 401-415. |
| ONR-H81 | J. B. Keller<br>J. Geer | Slender Streams<br>Acc: J. Fluid Mech., in press. |

## 2. Research Program.

The research work on this project centered about two major subjects. The first is the development of asymptotic, perturbation, and numerical methods for sound wave propagation in the oceans, atmosphere, and other waveguides. The second area is the development of similar methods for solving nonlinear problems. Particular emphasis was given to the nonlinear problems that arise in, bifurcation theory, the nonlinear buckling of elastic plates and shells, and hydrodynamic stability.

## 3. Wave Propagation.

ONR-H73 contains the 1977 Gibbs Lecture by J. B. Keller to the American Mathematical Society. It is an expository paper that summarizes some of the asymptotic methods for wave propagation that have been developed at NYU. These asymptotic methods have been generalized in ONR-H69 to obtain a new method for studying the waves that are produced by ship motions.

A new numerical method for solving waveguide problems is presented in ONR-H70. The method was applied to obtain the first 21 normal modes and corresponding cutoff frequencies for the E-modes of waveguides with regular hexagonal cross-sections.

In ONR-H75, the method of Lynn and Keller for determining asymptotic expansions of solutions of differential equations with turning points, was extended to systems of equations of higher order. Applications of the method are given to study the wave propagation in slowly varying waveguides.

In ONR-H66, a new asymptotic method was presented for studying high frequency propagation. It combines the features of the parabolic equation method, and the methods of geometrical optics.

One of the most important problems concerning sound wave propagation is the effect of variability in the sound velocity upon propagation of sound waves in the atmosphere and the ocean. In addition to spatial variations in the sound velocity, there are temporal fluctuations. These temporal variations of the sound velocity are caused by internal and surface tides, diurnal heating and cooling, and other atmosphereic and oceanic phenomena. We have analyzed the sound propagation from a periodic point source in an ocean with a temporally, slowly varying sound velocity. The sound velocity is taken in the form of a product of a function of depth and a slowly varying function of time. The small parameter $\varepsilon$ in this problem, is the ratio of the frequency of the tides, heating, etc., to the frequency of the sound source. In ONR-H79, asymptotic expansions of the solution have been obtained by a combination of the methods of, normal modes, two times, and asymptotic evaluation of integrals. A preliminary study of the resulting asymptotic expansions suggests that the pressure amplitudes change suddenly at a sequence of times from large to small, and conversely. These "fades" are accompanied by phase shifts.

In ONR-H74, we study the solution of the 3-dimensional reduced wave equation above an infinite plane 2-dimensional surface wave supporting structure. The structure is simulated by a multiple impedance boundary condition that allows for the excitation of two different cylindrical surface wave modes. We investigate the problem of a simple source which is then generalized to multi-pole sources. For the cases considered, we are able to give an explicit solution that exhibits the character of the surface wave field. Furthermore, using a virtual structure notion, we are able to identify the power travelling within the impedance structure associated with a given external surface wave mode. Thus, a definition of the total surface wave power is applied in a manner that modal power flow separability is maintained. The far field pattern is observed to vanish in certain directions, in particular along the impedance plane. Furthermore, the far field power is calculated in terms of a non-elementary definite integral which is then estimated at high and low frequencies so that a partial verification of our calculations can be made by observing that power is conserved to these orders.

## 4. Perturbations of Bifurcations Caused by Inhomogeneities.

Bifurcation occurs in mathematical models of "perfect" systems. In reality (experiments), bifurcation never occurs. Small imperfections or impurities in the system distort the bifurcation. For some systems, such as occur in the buckling of elastic rods and plates, the imperfections may have only a minor effect, and the results of the bifurcation model give good approximations to the actual phenomena. In other systems, such as occur in the buckling of elastic shells, the imperfections may dominate the response and destroy the predictions of the bifurcation models.

In ONR-H68, an asymptotic theory is presented to analyze perturbations of bifurcations of the solutions of nonlinear problems. The perturbations may result from imperfections, impurities, or other inhomogeneities in the corresponding physical problem. It is shown that for a wide class of problems the perturbations are singular. The method of matched asymptotic expansions is used to obtain asymptotic expansions of the solutions. Global representations of the solutions of the perturbed problem are obtained when the bifurcation solutions are known globally. This procedure also gives a quantitative method for analyzing singularities of nonlinear mappings and their unfoldings. Applications are given to a simple elasticity problem, and to nonlinear boundary value problems.

In ONR-H67, we have applied this method to study the smooth transition to thermal convection. In the classical mathematical description of the convection of a viscous fluid heated from below, there is a sharp transition from conduction to convection as the Rayleigh number exceeds a critical value. The convection state bifurcates from the conduction state at the critical Rayleigh number. This is in contrast to experiments, where the transition is smooth as the Rayleigh number is increased. We employ our theory of singular perturbations of bifurcations to show that small imperfections, such as thermal noise in the applied temperatures, are sufficient to explain the smooth transition. Finally, we show that when the imperfections satisfy a special condition, the transition remains sharp, but it is slightly perturbed. Since the imperfections are arbitrary, smooth transitions are most likely to occur in experiments.

We have slightly generalized the method of ONR-H68 to study the motion of a viscous fluid through a channel with driven walls. In this way, new results have been obtained concerning the transition to turbulence for two-dimensional channel flows. This work constituted the Ph.D. thesis of Gary Strumolo. A paper describing these results is in preparation.

By using various analytical techniques of nonlinear functional analysis and partial differential equations, we have shown in ONR-H77 that the thermal convection problem considered in ONR-H67 has a solution. Furthermore, we have rigorously justified most of the formal analysis in ONR-H67. The feature of this analysis is that the results are proved for the convection problem in a rectangle. The corners provided the main difficulties in the analysis.

In ONR-H71, we have applied the previously developed method of cascading bifurcations to obtain new states of magnetohydrodynamic (MHD) equilibrium. We use the Grad-Shaffranov theory of MHD equilibria. The secondary bifurcation states are the new equilibria. To lowest order the secondary states are a linear combination of the eigenfunctions corresponding to the simple bifurcation points that coalesce to form the multiple bifurcation point. As the weights in this linear combination are varied, the equipotential lines are distorted. Various interesting pinched configurations are thus obtained. The results of this paper suggest that there are many more possible MHD equilibria than had been believed previously. The results may also help to explain some numerical studies of MHD equilibria.

We have recently applied this method to study secondary transitions in thermal convection. The points of secondary transition are the secondary bifurcation points. In addition, the secondary bifurcation states have been obtained. These states have been observed experimentally for many years. This seems to be the first time they have been obtained mathematically. A paper describing these results is in preparation.