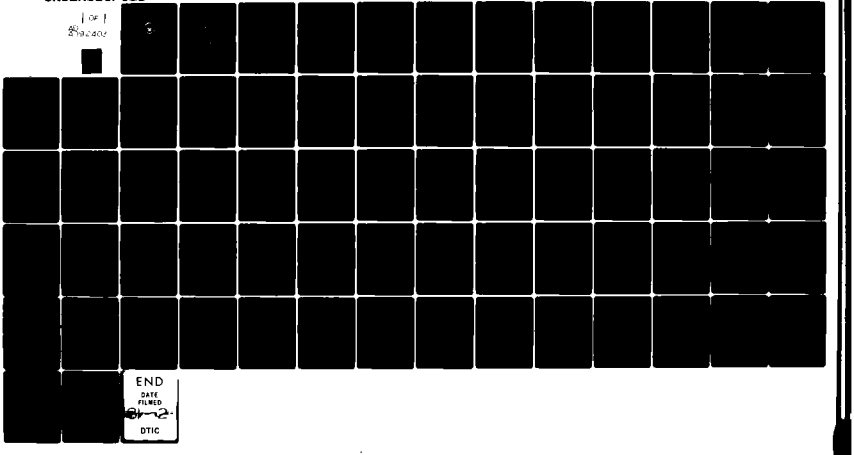
AD-A092 403 NAVAL POSTGRADUATE SCHOOL MONTEREY CA F/G 5/9
AN ATTEMPT TO MODEL THE USN UNRESTRICTED LINE OFFICER PROMOTION--ETC(U)
JUN 80 R W WEBER

UNCLASSIFIED NL



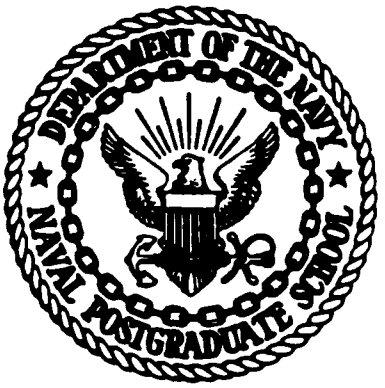
END
DATE FILMED
8-82
DTIC

LEVEL II

(2)

# NAVAL POSTGRADUATE SCHOOL
## Monterey, California








# THESIS

AN ATTEMPT TO MODEL THE
USN UNRESTRICTED LINE OFFICER PROCESS
USING AN ADAPTED "LENGTH OF SERVICE"
MARKOV MODEL

by

Richard W. Weber

June, 1980

Thesis Advisor: Prof. P.R. Milch

Approved for public release; distribution unlimited

# DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|---|
| 1. REPORT NUMBER | 2. GOVT ACCESSION NO. AD-A092403 (9) | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) (6) An Attempt to Model the USN Unrestricted Line Officer Promotion Process using an adapted "Length of Service" Markov Model. | 5. TYPE OF REPORT & PERIOD COVERED MASTER'S THESIS, June 1980 |
| | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s) WEBER, Richard Winsloe / Nelson (10) | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940 | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940 (11) | 12. REPORT DATE Jun[illegible]80 |
| | 13. NUMBER OF PAGES 68 |
| 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Naval Postgraduate School Monterey, CA 93940 (12) 74 | 15. SECURITY CLASS. (of this report) Unclassified |
| | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release, Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Approved for Public Release, Distribution Unlimited

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Promotion
Manpower Modelling

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A Markov Model is developed for the U.S. Navy Unrestricted Line Officer promotion process. The Transition Matrix is derived from historical promotion and attrition data.

The model is based on seniority, this being the fundamental measurable qualification for promotion. The model is tested using data on Lieutenant Commander movement in the period 1973-79.

DD FORM 1 JAN 73 1473 (Page 1) EDITION OF 1 NOV 65 IS OBSOLETE S/N 0102-014-6601

251450

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Stocks predicted in the test are compared with actual known stocks. The comparison shows the model to be a poor predictor of future stocks because of instability in the promotion process during the period in which data was collected. Large variations in cohort sizes have induced nonstationary promotion rates in the process. The Markov model based on seniority is impractical. Any alternative approach needs to account for, or be independent of, irregularities in cohort size.

Approved for Public Release; Distribution Unlimited

# An Attempt to Model the USN Unrestricted Line Officer Promotion Process using an adapted "Length of Service" Markov Model

by

Richard W. Weber
Captain, Royal Australian Engineers
B.E., University of New South Wales, 1971

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN MANAGEMENT

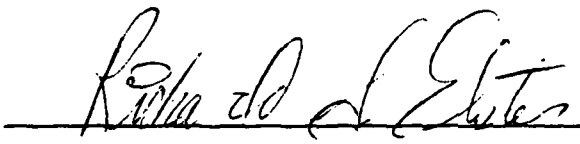
from the

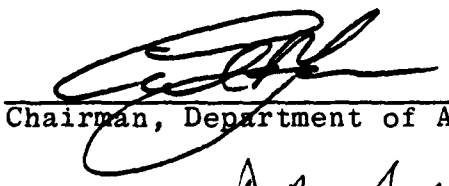
NAVAL POSTGRADUATE SCHOOL

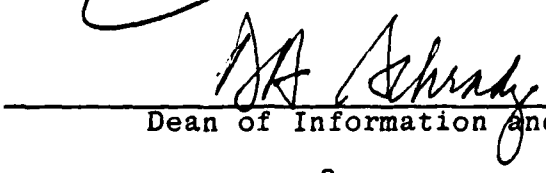
June, 1980

Author 

Approved by: 
Thesis Advisor


Second Reader


Chairman, Department of Administrative Sciences


Dean of Information and Policy Sciences

ABSTRACT

A Markov model is developed for the U.S. Navy Unrestricted Line Officer promotion process. The Transition Matrix is derived from historical promotion and attrition data.

The model is based on seniority, this being the fundamental measurable qualification for promotion. The model is tested using data on Lieutenant Commander movement in the period 1973-79.

Stocks predicted in the test are compared with actual known stocks. The comparison shows the model to be a poor predictor of future stocks because of instability in the promotion process during the period in which data was collected. Large variations in cohort sizes have induced nonstationary promotion rates in the process. The Markov model based on seniority is impractical. Any alternative approach needs to account for, or be independent of, irregularities in cohort size.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

LIST OF TABLES

# I. INTRODUCTION

The promotion process for officers of the U.S. Navy includes three major elements; namely eligibility, selection and promotion. Each of these elements is controlled by various laws, regulations and administrative procedures. The officer component is generally a strict hierarchical organisation, with a pyramid type structure, in which all who enter cannot reach the top. According to Ref. 5, each officer does however, have the same opportunity for promotion as his contemporaries.

This opportunity for promotion is a function of three important variables. These are the prescribed number, the promotion flow point, and the promotion opportunity (otherwise called selection rate, selection percentage or line fraction). These three variables are explained in detail in Chapter II.

At least once per year, the Secretary of the Navy prescribes the number of officers who may serve in each grade, and vacancies occur whenever the actual number of officers serving in a grade falls below the prescribed number. The number of officers to be promoted each year is determined from known and expected vacancies for that year.

Selection boards, composed of senior officers of experience, maturity and varied backgrounds, are convened by the Secretary

of the Navy in each fiscal year. The selection board considers all officers eligible for promotion to a given grade. The board evaluates each officer's record, by considering his professional background, the responsibilities he has carried, and his marks on fitness reports. The officers selected by the board for promotion, are those considered "best fitted" for promotion. The majority of selectees come from a promotion zone, while a relatively small number may come from other eligible officers above and below the promotion zone.

The promotion process appears to have well defined operating rules, but it is, nevertheless, a complex process with some factors being difficult to incorporate in a model.

This thesis attempts to develop a predictive model of the promotion process for U.S. Navy unrestricted line officers, based on seniority, which is a fundamental factor in the process. This apparently logical approach is unsuccessful, because of unstable behaviour in the promotion process during the period in which the historical data was collected.

## II. THE USN URL OFFICER PROMOTION PROCESS

### A. A BRIEF DESCRIPTION OF THE URL (UNRESTRICTED LINE)

The officer component of the USN is, in general, a strict hierarchical organisation. Members of the organisation, except for a few specialist professions, enter the organisation as Ensigns or Lieutenants (Junior Grade), and movement from these grades and subsequent grades takes place through promotion and attrition.

#### 1. Numerical Classifactions

These grades are numerically classified, and those classifications used in this document are as follows:

a. Lieutenant 0-3

b. Lieutenant Commander 0-4

c. Commander 0-5

d. Captain 0-6

#### 2. Designators

Each officer in the organisation has a designator which relates to his profession or specialty within the Navy. This thesis is concerned with Unrestricted Line (URL) officers including:

a. surface warfare officers (designator 1110)

b. submariners ( " 1120)

c. pilots ( " 1310)

d. naval flight officer ( " 1320)

B. PROMOTION, SELECTION, AND ELIGIBILITY

Promotion is based on length of service in grade which is referred to as seniority. Seniority is measured from an officer's Date of Rank (DOR). The DOR is normally the first day of the month in which promotion occurs. The minimum seniority for eligibility for promotion (denoted by e) is indicated in the following table:

TABLE I PROMOTION ELIGIBILITY CRITERIA

| Grade after Promotion | Minimum Seniority (e) for Promotion eligibility in years |
|---|---|
| Lieutenant | 2 |
| Lieutenant Commander | 4 |
| Commander | 3 |
| Captain | 3 |

Officers who are to be promoted in a given year are selected by a selection board which is convened at the beginning of that year. Officers considered by the selection board are those who will be eligible for promotion (in the terms stated above) in the year following the meeting of the selection board, excluding those who have been considered twice before and not selected by selection boards.

The actual promotion date of a selected officer occurs when a vacancy exists or when the officer becomes eligible,

whichever is later. Therefore, actual promotion of selected officers occurs in order of seniority, and should occur in the year following the selection board if predicted vacancies were accurate. The most junior selectee will normally be promoted one year after he is selected.

It is important to differentiate between selection and promotion.

This document is concerned with both selection and promotion. Promotion is a continuous process which occurs throughout the year and directly affects the structure and costs of the organisation. The selection process takes a relatively short time each year and lasts for as long as the selection board is convened. During this short period each year, an officer is classified as "In Zone", "Below Zone", or "Above Zone" (see explanations below). The selection process has essentially no direct effect on the structure of the organisation. It merely triggers the promotion process.

C. VACANCIES AND PRESCRIBED NUMBERS

There is a prescribed number of officers in each grade that may be authorized at any time. The prescribed number in each grade is calculated as a percentage of the total enlisted personnel in the Navy. The percentages for each grade are in accordance with resolution 5503 of the House of Representatives of the United States.

The number of vacancies in a given grade which will be created during the year following the meeting of the selection board is estimated from historical data on promotion and attrition. The immediate aim of the promotion process is to fill vacancies and thereby maintain the prescribed number in each grade. Minor variations in the promotion process for different officer grades are mentioned below. These variations relate to circumstances which follow when an officer fails to be selected after twice being considered for promotion.

D. PROMOTION OPPORTUNITY AND THE PROMOTION ZONES

Prior to each selection board, the Secretary of the Navy declares a promotion opportunity as a percentage. The promotion opportunity applies to all promotions except to Flag ranks and to Lieutenant (Junior Grade) where promotion occurs after 24 months of satisfactory service as an Ensign.

The promotion opportunity allows the promotion zone to be determined from the relationship:

$$\text{Promotion opportunity} = \frac{\text{Total No. Officers to be Selected}}{\text{No. Officers in the Promotion Zone}}$$

According to results published by the Office of the Deputy Chief of Naval Operations (DCNO) Manpower, Personnel and Training (MPT), at the 1980 Commander selection board meeting, 757 URL Lieutenant Commanders were to be selected. The promotion opportunity was declared as 80%.

The Number of Officers in the Promotion Zone was therefore 757 ÷ .80 = 946.

The promotion zone then consisted of the first 946 Lieutenant Commanders on the seniority list who had not previously been considered for promotion.

There were however, 4900 Lieutenant Commanders eligible for selection. Those eligible personnel who were not part of the promotion zone were either in the "below zone" or the "above zone".

The above zone includes those who have been in the promotion zone once previously, but who were not selected. Failure to be selected from the above zone normally eliminates any further opportunity for promotion. Regular Lieutenants and Lieutenants (Junior Grade) who twice fail selection to the next higher grade are discharged. Lieutenant Commanders, Commanders and Captains who twice fail selection may continue to serve without further opportunity for promotion, but they must retire on completion of 20, 26 and 30 years of total commissioned service respectively.

The below zone includes those officers who are eligible for selection but whose position on the seniority list is not high enough to place them in the promotion zone. The junior "below zone" officer has a seniority of e-1 years at the time the selection board meets.

The junior "in zone" officer has a seniority of a years at the time the selection board meets. The value of a depends on the promotion opportunity. Note that $a \geq e-1$.

The senior "in zone" officer has a seniority of b years at the time the selection board meets. In a given year, the value of b is 1 year greater than the value of a for the previous year.

The senior "above zone" officer has a seniority of c years at the time the selection board meets. In a given year the value of c is 1 year greater than b for the previous year.

According to the results reported by the Office of the Deputy Chief of Naval Operations after the 1980 Commander selection boards, up to 5% of all selectees could have come from below the zone (i.e., a maximum of 0.05 x 757 = 38). In actual fact, the board selected 39 Lieutenant Commanders from below the zone. There was no limit on the number who could be selected from above zone. The selection and promotion policy relating to the "below zone" and "above zone" varies from year to year at the discretion of the Secretary of the Navy.

Promotion Opportunity, as explained below, is apparently a paradox, because the size of the promotion zone is inversely proportional to the magnitude of the promotion opportunity. This means that for a given number of vacancies and therefore

for a given number of selectees, an increase in promotion opportunity is equivalent to a reduction in the number of officers placed in the promotion zone. Therefore, an officer of relatively low seniority is less likely to be put in the promotion zone and as a result his actual promotion opportunity diminishes.

## E. FLOW POINT

The flow point for a particular promotion is the number of years of commissioned service normally completed by promotees. Most documents on the promotion process use the flow point as a significant factor. However, flow point is not directly considered in this thesis because it is considered as merely a characteristic of the promotion process which is primarily based on seniority.

## III. MODELLING THE USN URL PROMOTION PROCESS

### A. SOME FUNDAMENTAL CONSIDERATIONS FOR THE MODEL

The promotion process is complicated by the existence of the 3 zones which classify officers eligible for selection. A model of the process needs to also account for the fact that certain aspects of promotion policy change. The most significant of these variable aspects is, of course, the promotion opportunity. Other variable aspects include the percentage of vacancies allowed to be filled from above the zone and below the zone.

It is clear that loss and promotion probabilities are not constant and the same for all officers. It is also clear that the process does not strictly follow the Markovian assumption, explained in Section 4.2 of Reference 1, that future events are independent of past events when conditioned on the present state. The process is more closely related to a renewal process where promotions are prompted by vacancies. Vacancies are however, grade specific only, and cannot be described in terms of seniority.

### B. MODELLING THE PROCESS USING ZONE-SPECIFIC STATES

Typical results from a selection board meeting are shown on Table 2 below. These results are taken from information

reported by the Office of the DCNO (MPT) after the 1980 Commander selection board meeting. The information relates to both unrestricted and restricted line officers.

TABLE II 1980 COMMANDER SELECTIONS

| | ELIGIBLE | SELECTED | % | % OF PROMOTED TOTAL |
|---|---|---|---|---|
| ABOVE ZONE | 1312 | 67 | 5.1 | 7.0 |
| IN ZONE | 1196 | 851 | 71.2 | 88.4 |
| BELOW ZONE | 2408* | 44 | 1.8 | 4.6 |
| | 4916* | 962 | | 100. |

* These totals were not included in the results reported by the DCNO (MPT). They have been derived from information contained in "Navy Times" issue of May 21, 1979.

The percentage of selectees who come from above the zone and below the zone is significantly smaller than the percentage who come from within the zone. In fact a model which ignores both "above-zone" and "below-zone" could be quite useful in view of the low proportion of selectees from outside the zone.

A model currently being used by the Navy [Ref. 8] ignores any selections from above the zone and assumes that "deep" selections are made only up to 1 year below the zone, i.e., the model assumes that the junior below zone or junior eligible has a seniority which is no less than 1 year behind the junior in zone.

A segment of a transition matrix which could be used in a Markov Model to describe the process for promotion through several grades is shown in Table III. For each grade, the states of the Markov Model are the promotion zone, the above and below zone, and the two states of ineligibility outside the zones.

The time at which this Markov Model would be observed, is at successive selection board meetings, (i.e., at one year intervals). In Table III, the row and column dimensions are not the same, but the matrix for all grades would have equal row and column dimensions, and could therefore, be termed a Markov matrix of transition rates.

The alphabetical characters included in Table III are positioned where actual non-zero transition probability values may occur. With the attrition column vector included, all row sums are 1.

A major problem in using the approach depicted in Table III is that the upper and lower limits of the "in zone" and the "above zone" are variable from one year to the next (i.e., the values of a, b and c are variable). This problem exists because of the variability of promotion opportunity as explained in Chapter II, and the variation in vacancies to be filled from one year to the next. At the time of each selection board meeting, the stocks in each state (in particular those in the promotion zone) are unknown

| SENIORITY | | | RANK I | | | | | | ATTRITION |
|---|---|---|---|---|---|---|---|---|---|
| | | | BELOW ELIGIBLE | BELOW ZONE | IN ZONE | ABOVE ZONE | NON ELIGIBLE | RANK I+1 | |
| 0 | RANK I | BELOW ELIGIBLE | a | b | c | | | | d |
| e-1 | | BELOW ZONE | | e | f | | | g | h |
| a | | IN ZONE | | | | i | | j | k |
| b | | ABOVE ZONE | | | | | l | m | n |
| c | | NON ELIGIBLE | | | | | o | | p |

TABLE III TABLE OF TRANSITION RATES BETWEEN SUCCESSIVE SELECTION BOARDS.

until the promotion opportunity and the vacancies become known. Although the stock vector at the time the selection board meets is known in terms of seniority, the stock vector in terms of the states shown in Table III (in particular in the promotion zone) is not known.

This means that a predictive model based on the zones cannot be built unless we assume constant promotion opportunity and constant vacancies and a beginning stock vector which is representative of several typical years in terms of the content of the various states of the model.

## C. A MODEL USING SENIORITY SPECIFIC RATES

It seems then, that a suitable approach which incorporates the variability of promotion zone limits would be one adapted from a "Length of Service" model. Table IV shows a segment of the transition matrix which could be used in a Markov Model to describe the process for promotion through several grades. The matrix accounts for officers according to seniority within each grade.

The alphabetical characters in Table IV are included where non-zero probability figures are possible.

Adaption of such a "Length of Service" model for the USN Officer promotion process is particularly appropriate because of the natural association between promotion probability and length of service in grade.

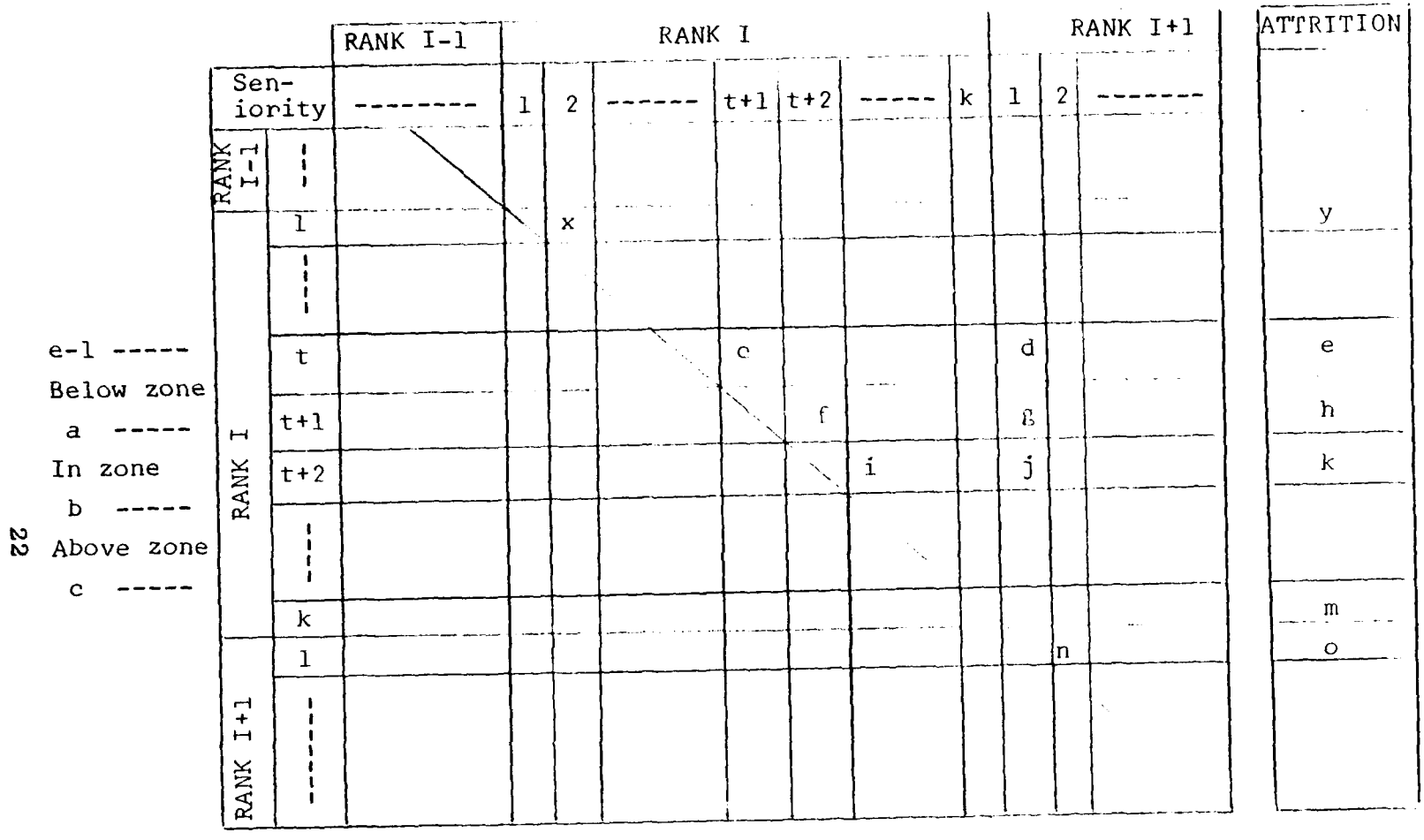



TABLE IV EXTRACT FROM TRANSITION MATRIX BASED ON SENIORITY AT SUCCESSIVE TRANSITIONS.

With a model based on seniority, manpower planners may superimpose the upper and lower extremities of the promotion zone on the seniority scale for each grade. Different values of a and b could be applied in using the model as an analytical tool, and in testing different values of promotion opportunity which may be set for each selection board meeting. Similarly, the model could be used in determining suitable values of promotion opportunity for selection boards to produce the required number of selectees.

With the attrition column vector included, row sums in Table IV are all 1, and each row has 3 possible entries. For example, an officer in grade i with seniority t (in years) at the time a selection board meets, will, 1 year later, be either:

(1) in grade i with seniority t + 1; or

(2) in grade i + 1 with seniority 1 year; or

(3) out of the organisation.

D. ACCOUNTING INTERVAL AND TRANSITION INTERVAL

It may be preferable for manpower analysts to select different seniority accounting intervals, and transition intervals. Table IV depicts a situation where the accounting interval and the transition interval are the same. This is apparent because the entries for those who stay in the same grade during a transition are positioned 1 column to the right of the main diagonal. This means that during a

transition, they advance to the next interval of seniority. If the accounting interval is less than the transition interval, then entries for those who stay in the same grade during a transition will be positioned further to the right of the main diagonal. In fact, with a 12 month transition interval and a 3 month accounting interval the entry for those who stay in the same rank during a transition will be 4 columns to the right of the main diagonal.

Similarly officers who are promoted during the 12 month transition interval from 1 particular three month interval of seniority, have a possibility of moving into any of the first 4 seniority intervals in their new rank depending on when the promotion actually occurs.

In the model developed in the following two chapters, the accounting interval will be variable. This situation complicates the transition matrix in a way which is best shown by the numerical example which is developed in the following two chapters.

# IV. A SENIORITY SPECIFIC MARKOV MODEL FOR THE PROMOTION PROCESS FROM THE LIEUTENANT COMMANDER GRADE

## A. THE SOURCE OF HISTORICAL DATA

In the remaining chapters, an adapted length of service model is developed for the Lieutenant Commander grade.

Data suitable for developing and testing the model has been extracted from the Officer Master Files of the Naval Personnel Research and Development Center (NPRDC), San Diego, on all officers who have served between 1953 and December, 1979. Within each officer's record, his promotion and attrition history (if applicable) is recorded in terms of the actual date of promotion and/or attrition.

From this historical data on promotion and attrition, 6 years worth of statistics on Lieutenant Commanders have been compiled and presented in Tables V through X.

## B. A SUMMARY OF PROMOTION AND ATTRITION STATISTICS 1973-78

Each of Tables V through X is a summary of movements of Lieutenant Commanders in the 12 months commencing June 1 of each year.

The seniority scale for each table has a variable interval which has been selected to allow the promotion and attrition trends to be compared easily by visual inspection across the years.

TABLE V

JUNE 1973 LCDR TRANSITIONS AFTER 12 MONTHS

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | NO. STAYING | NO. AND FRACTION PROMOTED | | NO. ATTRIT'D |
|---|---|---|---|---|
| 48 | 3449 | 7 | 0.02 | 68 |
| 51 | 124 | 3 | 0.01 | 3 |
| 54 | 134 | 1 | 0.00 | 0 |
| 57 | 156 | 4 | 0.01 | 5 |
| 60 | 96 | 3 | 0.01 | 2 |
| 63 | 157 | 9 | 0.02 | 0 |
| 66 | 108 | 51 | 0.13 | 0 |
| 69 | 91 | 134 | 0.33 | 2 |
| 72 | 42 | 60 | 0.15 | 3 |
| 75 | 0 | 0 | 0.0 | 0 |
| 78 | 156 | 130 | 0.32 | 11 |
| 81 | 212 | 3 | 0.01 | 7 |
| 84 | 149 | 0 | 0.0 | 18 |
| 87 | 107 | 0 | 0.0 | 5 |
| 90 | 45 | 0 | 0.0 | 2 |
| 93 | 322 | 0 | 0.0 | 5 |
| 96 | 362 | 0 | 0.0 | 15 |
| 99 | 355 | 0 | 0.0 | 12 |
| 102 | 27 | 0 | 0.0 | 1 |
| 105 | 63 | 0 | 0.0 | 7 |
| 108 | 683 | 0 | 0.0 | 63 |
| 111 | 145 | 0 | 0.0 | 17 |
| 114 | 148 | 0 | 0.0 | 32 |
| 117 | 127 | 0 | 0.0 | 31 |
| 120 | 285 | 0 | 0.0 | 96 |
| >120 | 14587 | 0 | 0.0 | 1049 |
| | 22337 | 404 | | 1454 |

AVERAGE SENIORITY OF PROMOTEES 70. MONTHS

## TABLE VI

JUNE 1974 LCDR TRANSITIONS AFTER 12 MONTHS

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | NO. STAYING | NO. AND FRACTION PROMOTED | | NO. ATTRITED |
|---|---|---|---|---|
| 48 | 2215 | 0 | 0.0 | 45 |
| 51 | 0 | 0 | 0.0 | 0 |
| 54 | 240 | 9 | 0.03 | 4 |
| 57 | 272 | 4 | 0.01 | 3 |
| 60 | 1294 | 11 | 0.04 | 20 |
| 63 | 123 | 0 | 0.0 | 1 |
| 66 | 130 | 0 | 0.0 | 4 |
| 69 | 116 | 37 | 0.13 | 5 |
| 72 | 47 | 46 | 0.16 | 4 |
| 75 | 60 | 97 | 0.34 | 0 |
| 78 | 50 | 65 | 0.23 | 3 |
| 81 | 108 | 7 | 0.02 | 5 |
| 84 | 60 | 1 | 0.00 | 5 |
| 87 | 0 | 0 | 0.0 | 0 |
| 90 | 175 | 3 | 0.01 | 11 |
| 93 | 202 | 3 | 0.01 | 9 |
| 96 | 124 | 3 | 0.01 | 22 |
| 99 | 107 | 0 | 0.0 | 0 |
| 102 | 39 | 1 | 0.03 | 5 |
| 105 | 336 | 0 | 0.0 | 16 |
| 108 | 343 | 1 | 0.00 | 18 |
| 111 | 316 | 0 | 0.0 | 39 |
| 114 | 20 | 0 | 0.0 | 7 |
| 117 | 54 | 0 | 0.0 | 9 |
| 120 | 527 | 0 | 0.0 | 156 |
| >120 | 14028 | 0 | 0.0 | 1264 |
| | 21030 | 288 | | 1659 |

AVERAGE SENIORITY OF PROMOTEES 72. MONTHS

## TABLE VII

JUNE 1975 LOOR TRANSITIONS AFTER 12 MONTHS

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | NO. STAYING | NO. AND FRACTION PROMOTED | NO. ATTRITED |
|---|---|---|---|
| 48 | 2270 | 0 0.0 | 31 |
| 51 | 109 | 0 0.0 | 2 |
| 54 | 145 | 0 0.0 | 1 |
| 57 | 189 | 0 0.0 | 5 |
| 60 | 0 | 0 0.0 | 0 |
| 63 | 0 | 0 0.0 | 0 |
| 66 | 240 | 0 0.0 | 0 |
| 69 | 271 | 1 0.00 | 0 |
| 72 | 979 | 301 0.54 | 14 |
| 75 | 23 | 98 0.18 | 2 |
| 78 | 45 | 84 0.15 | 1 |
| 81 | 70 | 50 0.09 | 1 |
| 84 | 47 | 5 0.01 | 2 |
| 87 | 64 | 9 0.02 | 0 |
| 90 | 55 | 2 0.00 | 3 |
| 93 | 106 | 3 0.01 | 1 |
| 96 | 58 | 0 0.0 | 2 |
| 99 | 0 | 0 0.0 | 0 |
| 102 | 165 | 1 0.00 | 11 |
| 105 | 194 | 2 0.00 | 8 |
| 108 | 112 | 0 0.0 | 15 |
| 111 | 101 | 1 0.00 | 5 |
| 114 | 39 | 0 0.0 | 1 |
| 117 | 249 | 2 0.00 | 55 |
| 120 | 279 | 0 0.0 | 64 |
| >120 | 13938 | 0 0.0 | 1007 |
| | 19859 | 559 | 1232 |

AVERAGE SENIORITY OF PROMOTEES 74. MONTHS

## TABLE VIII

JUNE 1976 LCDR TRANSITIONS AFTER 12 MONTHS

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | NO. STAYING | NO. AND FRACTION PROMOTED | | NO. ATTRITED |
|---|---|---|---|---|
| 48 | 2585 | 0 | 0.0 | 46 |
| 51 | 7 | 0 | 0.0 | 0 |
| 54 | 17 | 0 | 0.0 | 0 |
| 57 | 15 | 0 | 0.0 | 0 |
| 60 | 224 | 2 | 0.00 | 2 |
| 63 | 108 | 0 | 0.0 | 1 |
| 66 | 143 | 0 | 0.0 | 2 |
| 69 | 181 | 0 | 0.0 | 8 |
| 72 | 0 | 0 | 0.0 | 0 |
| 75 | 0 | 0 | 0.0 | 0 |
| 78 | 227 | 8 | 0.01 | 5 |
| 81 | 251 | 19 | 0.13 | 2 |
| 84 | 428 | 550 | 0.54 | 11 |
| 87 | 25 | 0 | 0.0 | 1 |
| 90 | 56 | 1 | 0.00 | 1 |
| 93 | 75 | 0 | 0.0 | 5 |
| 96 | 48 | 0 | 0.0 | 0 |
| 99 | 64 | 0 | 0.0 | 1 |
| 102 | 52 | 0 | 0.0 | 3 |
| 105 | 102 | 2 | 0.00 | 3 |
| 108 | 51 | 0 | 0.0 | 7 |
| 111 | 0 | 0 | 0.0 | 0 |
| 114 | 150 | 3 | 0.01 | 13 |
| 117 | 161 | 0 | 0.0 | 34 |
| 120 | 60 | 0 | 0.0 | 52 |
| >120 | 13329 | 0 | 0.0 | 1278 |
| | 18429 | 585 | | 1475 |

AVERAGE SENIORITY OF PROMOTEES 82. MONTHS

## TABLE IX

JUNE 1977 LCDR TRANSITIONS AFTER 12 MONTHS

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | NO. STAYING | NO. AND FRACTION PROMOTED | | NO. ATTRITED |
|---|---|---|---|---|
| 48 | 2405 | 0 | 0.0 | 65 |
| 51 | 97 | 0 | 0.0 | 3 |
| 54 | 0 | 0 | 0.0 | 0 |
| 57 | 41 | 0 | 0.0 | 3 |
| 60 | 748 | 3 | 0.00 | 14 |
| 63 | 7 | 0 | 0.0 | 0 |
| 66 | 17 | 0 | 0.0 | 0 |
| 69 | 13 | 0 | 0.0 | 2 |
| 72 | 213 | 7 | 0.01 | 4 |
| 75 | 59 | 48 | 0.08 | 1 |
| 78 | 56 | 86 | 0.14 | 1 |
| 81 | 67 | 111 | 0.18 | 3 |
| 84 | 0 | 0 | 0.0 | 0 |
| 87 | 0 | 0 | 0.0 | 0 |
| 90 | 66 | 158 | 0.26 | 3 |
| 93 | 73 | 175 | 0.29 | 4 |
| 96 | 401 | 15 | 0.02 | 36 |
| 99 | 22 | 1 | 0.01 | 2 |
| 102 | 45 | 3 | 0.00 | 8 |
| 105 | 65 | 0 | 0.0 | 10 |
| 108 | 45 | 0 | 0.0 | 3 |
| 111 | 61 | 1 | 0.01 | 2 |
| 114 | 51 | 0 | 0.0 | 1 |
| 117 | 82 | 0 | 0.0 | 21 |
| 120 | 35 | 0 | 0.0 | 16 |
| >120 | 12472 | 1 | 0.00 | 1229 |
| | 17256 | 609 | | 1431 |

AVERAGE SENIORITY OF PROMOTEES 85. MONTHS

TABLE X

JUNE 1978 LCDR TRANSITIONS AFTER 12 MONTHS

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | NO. STAYING | NO. AND FRACTION PROMOTED | NO. ATTRITED |
|---|---|---|---|
| 48 | 2596 | 0 0.0 | 106 |
| 51 | 86 | 0 0.0 | 0 |
| 54 | 44 | 0 0.0 | 1 |
| 57 | 111 | 0 0.0 | 1 |
| 60 | 313 | 2 0.00 | 10 |
| 63 | 95 | 1 0.00 | 1 |
| 66 | 0 | 0 0.0 | 0 |
| 69 | 39 | 0 0.0 | 1 |
| 72 | 369 | 369 0.58 | 10 |
| 75 | 2 | 5 0.01 | 0 |
| 78 | 12 | 4 0.01 | 1 |
| 81 | 4 | 8 0.01 | 1 |
| 84 | 47 | 164 0.26 | 2 |
| 87 | 11 | 47 0.07 | 1 |
| 90 | 44 | 10 0.02 | 2 |
| 93 | 53 | 15 0.02 | 5 |
| 96 | 0 | 0 0.0 | 0 |
| 99 | 0 | 0 0.0 | 0 |
| 102 | 59 | 7 0.01 | 1 |
| 105 | 69 | 0 0.0 | 5 |
| 108 | 371 | 2 0.00 | 32 |
| 111 | 21 | 0 0.0 | 2 |
| 114 | 38 | 1 0.00 | 9 |
| 117 | 50 | 0 0.0 | 15 |
| 120 | 27 | 0 0.0 | 18 |
| >120 | 11295 | 2 0.00 | 1405 |
| | 15873 | 637 | 1636 |

AVERAGE SENIORITY OF PROMOTEES 76. MONTHS

The first row of entries applies to those Lieutenant Commanders who had between zero and 48 months of seniority at the beginning of the year. The accounting interval used from 48 to 120 months is consistently 3 months. All entries applying to Lieutenant Commanders with more than 120 months of seniority at the beginning of the year are grouped together.

By way of further explanation, the column labelled "No. Staying" includes those Lieutenant Commanders who stayed for the full year commencing June 1. The fractions shown in the column labelled "No. and Fraction Promoted" are the fractions of total promotees for each year, who come from the corresponding seniority interval.

The average seniority of promotees shown below each table is determined from the formula:

$$\text{Average seniority of promotees} = \frac{\sum_i [(\#\text{ promotees in } i^{th} \text{ seniority interval}) \times (\text{mean of } i^{th} \text{ seniority interval})]}{\text{Total \# promotees}}$$

In Table XI, data from the period 1973-78 are combined into a table which shows the probability distribution of staying in grade, being promoted or attriting, over the seniority intervals. In other words, this table contains the average seniority distribution of the movements of Lieutenant Commanders over the period 1973-78.

The formula used in constructing this table is given below for the promotion column only.

TABLE XI

AVERAGE PROBABILITIES TABLE

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | PROPORTION STAYING | PROPORTION PROMOTED | PROPORTION ATTRITED |
|---|---|---|---|
| 48 | 0.14 | 0.00 | 0.04 |
| 51 | 0.00 | 0.00 | 0.00 |
| 54 | 0.01 | 0.00 | 0.00 |
| 57 | 0.01 | 0.00 | 0.00 |
| 60 | 0.02 | 0.01 | 0.01 |
| 63 | 0.00 | 0.00 | 0.00 |
| 66 | 0.01 | 0.02 | 0.00 |
| 69 | 0.01 | 0.06 | 0.00 |
| 72 | 0.01 | 0.25 | 0.00 |
| 75 | 0.00 | 0.08 | 0.00 |
| 78 | 0.00 | 0.12 | 0.00 |
| 81 | 0.01 | 0.06 | 0.00 |
| 84 | 0.01 | 0.23 | 0.00 |
| 87 | 0.00 | 0.12 | 0.00 |
| 90 | 0.00 | 0.06 | 0.00 |
| 93 | 0.01 | 0.06 | 0.00 |
| 96 | 0.01 | 0.01 | 0.01 |
| 99 | 0.00 | 0.00 | 0.00 |
| 102 | 0.00 | 0.00 | 0.00 |
| 105 | 0.01 | 0.00 | 0.01 |
| 108 | 0.01 | 0.00 | 0.02 |
| 111 | 0.01 | 0.00 | 0.01 |
| 114 | 0.00 | 0.00 | 0.01 |
| 117 | 0.01 | 0.00 | 0.02 |
| 120 | 0.01 | 0.0 | 0.05 |
| >120 | 0.69 | 0.00 | 0.81 |

$P_i$ = Probability of promotion in the $i^{th}$ seniority interval = $[\sum_t x_i(t)]/[\sum_t \sum_i x_i(t)]$

where $x_i(t)$ = No. of promotees who were in the $i^{th}$ seniority interval at the beginning of year t.

The columns labelled "Proportion Staying" and "Proportion Attriting" are derived in a similar way. All column sums in Table XI should be 1 except for rounding errors. Examination of Table XI suggests that the promotion zone lies between 66 and 93 months of seniority. Any of the promotion probabilities within this range of seniority are too large to occur outside of the promotion zone and those outside of this range are too small to be included as part of the normal promotion process.

C. PROMOTION, ATTRITION AND STAYING RATES DERIVED

If the stocks used in a model are accounted for in terms of seniority, then it is necessary for the rates of promotion, attrition and staying in grade to relate to each seniority accounting interval. For each separate year, the rates are derived as follows:

Let

$n_i(t)$ = No. in seniority interval i at beginning of year t

$s_i(t)$ = No. in seniority interval i at beginning of year t who stay in grade during year t

$a_i(t)$ = No. in seniority interval i at beginning of year t who attrite during year t

and

$x_i(t)$ = No. of promotees during year t who were in seniority interval i at beginning of year t

Then $n_i(t) = s_i(t) + a_i(t) + x_i(t)$.

The rates of promotion during year t, for individuals who are in the $i^{th}$ seniority interval at the beginning of the year, may be computed as the ratio:

$$p_i(t) = \frac{x_i(t)}{n_i(t)}$$

The rates of staying in grade during year t, and the rates of attrition during year t, may be similarly computed as the ratios of $s_i(t)$ to $n_i(t)$ and $a_i(t)$ to $n_i(t)$ respectively.

If these ratios are reasonably stable over the years for which data is available, it is then acceptable to estimate from such data, a common (or average) rate of promotion during any year by the formula:

$$p_i = \frac{\sum_i x_i(t)}{\sum_i n_i(t)} \text{ for all seniority intervals i.}$$

The average rate of attritions from each seniority interval and the average rate of staying in grade during the year, may be estimated in a similar manner to that shown above for the average promotion rate. Such average rates are presented in Tables XII, XIII and XIV, where each table is derived from a different sequence of years.

The rates of promotion, attrition and staying in grade, may then be used in a Markov Model, where the inherent assumption is that individuals move independently and with

TABLE XII

AVERAGE SATISFIABLE FOR YEARS 73 74 75 76 77 78

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | SAT STAYING | SAT PROMOTED | SAT ATTRITED |
|---|---|---|---|
| 48 | 0.98 | 0.00 | 0.02 |
| 51 | 0.97 | 0.01 | 0.02 |
| 54 | 0.97 | 0.02 | 0.01 |
| 57 | 0.97 | 0.01 | 0.02 |
| 60 | 0.97 | 0.01 | 0.02 |
| 63 | 0.98 | 0.02 | 0.01 |
| 66 | 0.92 | 0.07 | 0.01 |
| 69 | 0.79 | 0.19 | 0.02 |
| 72 | 0.67 | 0.32 | 0.01 |
| 75 | 0.36 | 0.63 | 0.01 |
| 78 | 0.58 | 0.40 | 0.02 |
| 81 | 0.77 | 0.21 | 0.02 |
| 84 | 0.49 | 0.48 | 0.03 |
| 87 | 0.77 | 0.21 | 0.03 |
| 90 | 0.69 | 0.27 | 0.03 |
| 93 | 0.79 | 0.19 | 0.03 |
| 96 | 0.91 | 0.02 | 0.07 |
| 99 | 0.97 | 0.00 | 0.03 |
| 102 | 0.90 | 0.03 | 0.07 |
| 105 | 0.94 | 0.00 | 0.06 |
| 108 | 0.92 | 0.00 | 0.08 |
| 111 | 0.91 | 0.00 | 0.09 |
| 114 | 0.87 | 0.01 | 0.12 |
| 117 | 0.81 | 0.00 | 0.18 |
| 120 | 0.75 | 0.0 | 0.25 |
| >120 | 0.92 | 0.00 | 0.08 |

TABLE XIII

AVERAGE RATE TABLE FOR YEARS 75 76 77

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | RATE STAYING | RATE RECYCLED | RATE ATTRITED |
|---|---|---|---|
| 48 | 0.98 | 0.0 | 0.02 |
| 51 | 0.98 | 0.0 | 0.02 |
| 54 | 0.99 | 0.0 | 0.01 |
| 57 | 0.97 | 0.0 | 0.03 |
| 60 | 0.98 | 0.01 | 0.02 |
| 63 | 0.99 | 0.0 | 0.01 |
| 66 | 1.00 | 0.0 | 0.00 |
| 69 | 0.98 | 0.00 | 0.02 |
| 72 | 0.79 | 0.20 | 0.01 |
| 75 | 0.35 | 0.63 | 0.01 |
| 78 | 0.64 | 0.35 | 0.01 |
| 81 | 0.68 | 0.31 | 0.01 |
| 84 | 0.46 | 0.53 | 0.01 |
| 87 | 0.90 | 0.09 | 0.01 |
| 90 | 0.51 | 0.47 | 0.02 |
| 93 | 0.57 | 0.40 | 0.02 |
| 96 | 0.91 | 0.03 | 0.07 |
| 99 | 0.96 | 0.01 | 0.03 |
| 102 | 0.91 | 0.01 | 0.08 |
| 105 | 0.94 | 0.01 | 0.05 |
| 108 | 0.89 | 0.0 | 0.11 |
| 111 | 0.95 | 0.01 | 0.04 |
| 114 | 0.93 | 0.01 | 0.06 |
| 117 | 0.81 | 0.00 | 0.18 |
| 120 | 0.74 | 0.0 | 0.26 |
| >120 | 0.92 | 0.00 | 0.08 |

TABLE XIV

AVERAGE RATE TABLE FOR YEARS 73 75 76

| SENIORITY (MONTHS) AT BEGINNING OF YEAR | RATE STAYING | RATE PROMOTED | RATE ATTRITED |
|---|---|---|---|
| 48 | 0.98 | 0.00 | 0.02 |
| 51 | 0.97 | 0.01 | 0.02 |
| 54 | 0.99 | 0.00 | 0.00 |
| 57 | 0.96 | 0.01 | 0.03 |
| 60 | 0.97 | 0.02 | 0.01 |
| 63 | 0.97 | 0.03 | 0.00 |
| 66 | 0.90 | 0.09 | 0.00 |
| 69 | 0.79 | 0.20 | 0.01 |
| 72 | 0.73 | 0.26 | 0.01 |
| 75 | 0.19 | 0.80 | 0.02 |
| 78 | 0.64 | 0.33 | 0.03 |
| 81 | 0.87 | 0.12 | 0.02 |
| 84 | 0.52 | 0.46 | 0.03 |
| 87 | 0.93 | 0.04 | 0.03 |
| 90 | 0.95 | 0.02 | 0.04 |
| 93 | 0.97 | 0.01 | 0.02 |
| 96 | 0.96 | 0.0 | 0.04 |
| 99 | 0.97 | 0.0 | 0.03 |
| 102 | 0.94 | 0.0 | 0.06 |
| 105 | 0.94 | 0.01 | 0.05 |
| 108 | 0.91 | 0.0 | 0.09 |
| 111 | 0.91 | 0.0 | 0.08 |
| 114 | 0.87 | 0.01 | 0.12 |
| 117 | 0.82 | 0.00 | 0.18 |
| 120 | 0.75 | 0.0 | 0.25 |
| >120 | 0.93 | 0.0 | 0.07 |

identical probabilities which do not vary over time. If this assumption is sufficiently realistic for the promotion process, then a Markov Model of the promotion process, based on seniority could be built. Such a model could then serve as the basis for featuring the promotion zone in terms of the fixed months of seniority for each year.

The rates presented in Table XII should be particularly representative of the staying, promotion and attrition processes (and therefore particularly suitable for a Markov Model) because they are based on 6 years of data.

As discussed in Chapter 7 of Ref. 1, the transition rates (rates of promotion, attrition and staying in grade) are said to be stationary, in a stable process. In an unstable process, where the rates of promotion, attrition and staying in grade do not follow the assumptions of stationarity, transition rates derived from historical data will not reflect the actual movement of personnel in the system.

The variation in the total number of promotees shown in Tables V through X gives no clear indication as to instability in the promotion process. The variation in the average seniority of promotees gives some idea of the instability in the process. An even more meaningful indication as to the degree of instability in the promotion process, is obtained through a comparison of the promotion rates shown

in Tables XII, XIII and XIV. These Tables are based on different sequences of years, but with a stable process, there should be reasonable similarity between the rates for most accounting intervals.

The real test that derived transition rates are a reasonable representation of the real behaviour of the system, is to apply the rates in a Markov Model, and to compare the results predicted by the model with actual data for the same years.

## D. DEVELOPMENT OF A TRANSITION MATRIX FROM PROMOTION, ATTRITION, AND STAYING RATES

Table XV is an example of the Transition Matrix adapted from a preceeding Rate Table (Table XIII) for years 1975, 76, 77. In developing the Transition Matrix from the Rate Table, several empirical assumptions have been made.

1. In determining the element in the left upper corner (the probability of staying in the 0-48 month seniority group during a transition) the distribution of individuals has been assumed to be uniform through all 3 month accounting intervals between 0 and 48 months. The element in the upper left corner is therefore assumed to be 36/48 x 0.98 = 0.74, where 0.98 is the probability that individuals in the 0-48 month seniority interval will neither attrit nor be promoted during the year. Similarly, the second,

TABLE XV

TRANSITION MATRIX - BASED ON LIEUTENANT COMMANDER
MOVEMENT 1975 - 77

TRANSITION MATRIX

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (24) | (25) | (26) | (27) | (28) | (29) | (30) | ATTRITION |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ( 1) | .74 | .06 | .06 | .06 | .06 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .02 |
| ( 2) | .0 | .0 | .0 | .0 | .0 | .98 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .02 |
| ( 3) | .0 | .0 | .0 | .0 | .0 | .0 | .99 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .01 |
| ( 4) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .97 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .03 |
| ( 5) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .00 | .00 | .00 | .00 | .02 |
| ( 6) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .01 |
| ( 7) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .01 |
| ( 8) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .00 | .00 | .00 | .00 | .02 |
| ( 9) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .05 | .05 | .05 | .05 | .01 |
| (10) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .16 | .16 | .16 | .16 | .01 |
| (11) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .09 | .09 | .09 | .09 | .01 |
| (12) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .08 | .08 | .08 | .08 | .01 |
| (13) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .13 | .13 | .13 | .13 | .01 |
| (14) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .02 | .02 | .02 | .02 | .01 |
| (15) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .12 | .12 | .12 | .12 | .02 |
| (16) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .10 | .10 | .10 | .10 | .02 |
| (17) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .01 | .01 | .01 | .01 | .07 |
| (18) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .03 |
| (19) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .00 | .00 | .00 | .00 | .08 |
| (20) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .94 | .0 | .0 | .00 | .00 | .00 | .00 | .05 |
| (21) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .89 | .0 | .0 | .0 | .0 | .0 | .11 |
| (22) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .95 | .00 | .00 | .00 | .00 | .04 |
| (23) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .93 | .0 | .0 | .0 | .0 | .06 |
| (24) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .81 | .00 | .00 | .00 | .00 | .18 |
| (25) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .74 | .0 | .0 | .0 | .0 | .26 |
| (26) | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .92 | .0 | .0 | .0 | .0 | .08 |

third, fourth and fifth elements of the first row (the probabilities of individuals in seniority interval 0-48 months at the beginning of the year moving into the 4 respective 3 month accounting intervals between 48 and 60 months during the year) are therefore each 3/48 x 0.98 = 0.06.

2. The elements in columns 27, 28, 29 and 30 of any row of the Transition Matrix are the probabilities of being promoted into the 0-3 month, 3-6 month, 6-9 month and 9-12 month seniority intervals of Commander grade respectively.

   The assumption here is that the actual dates of promotion for all promotees are distributed uniformly throughout the year. For example, the element in column 28 of any row is the probability of an officer being promoted in the third quarter of the year, so that the officer will be in the 3-6 month seniority group of the next grade at the beginning of the next year. As a result of this assumption, for a given row, each of the four transition probabilities in columns 27, 28, 29 and 30 will be one quarter of the promotion rate for that row. For example, the promotion rate for the 69-72 month seniority interval in Table XIII is 0.20. Consequently, in the Transition Matrix (Table XV), the four promotion probabilities in row 9 are each 0.05.

The non-zero elements in columns 6 through 26, and the attrition rates in column 31 are taken directly from the rate table.

There is apparent inconsistency in accounting for the first 12 months of Commander grade in terms of 3 monthly intervals, whereas the first 48 months of Lieutenant Commander grade was accounted for in one group. This has been done to show how the analyst may adapt the length of service model to fit variations in the seniority accounting interval from one grade to another.

# V. RESULTS OF A MODEL VALIDATION TEST

## A. MODEL VALIDATION

A test to validate the model was conducted by "predicting" stocks for later years using the equation repeatedly:

$$n(t+1) = n(t)\ M + r(t)$$

where

n(t) is the row vector of stocks in each seniority interval at the beginning of year t;

M is the transition matrix of the type shown in Table XV, and

r(t) is the row vector of accessions in each seniority interval during year t.

The results of the validation test are presented in Tables XVI-A and XVI-B. The transition matrix was derived from data for years 1975-77 and Lieutenant Commander stocks in June, 1975 have been used as the beginning stock vector, n(1975). The tables include a comparison between stocks predicted by the model and actual known stocks.

In each case, the accession vector r(t) consisted of only one non-zero component, namely the first one, representing promotions from the Lieutenant grade to the 0-48 month accounting interval (of the Lieutenant Commander grade). Up to the year commencing June 78, the accessions are the actual value derived from the historical data. Beyond that year, the accessions used in the predicted stock vector, are the

average accessions from the years which were used to derive the table of rates.

For example, in Table XVI-B, the accessions applied from 1979 onwards are the average of 1975-77. The approach used here was to run the model through the same years from which the transition matrix was derived (1975-77) and then for 1978 (for which actual data is also known) through to 1982.

Table XVI-A shows a significant difference between predicted and actual numbers of promotees in all of the first four years except 1975. At first it appears that the discrepancies have occurred as a result of the assumption of uniform distribution of officers in the 0-48 month seniority group. This could be based on the fact that the major differences between the predicted and actual stock vectors clearly occur in the accounting intervals which contain individuals who were in the 0-48 month seniority group of the initial stock vector of 1975.

Closer examination shows, however, that this is not the significant source of discrepancy. Take, for example, the year 1976 when the predicted number of promotees exceeded the actual number by 199. The beginning stock vector used in the model for 1976 is the same as the stock vector predicted for the end of 1975. In this stock vector, those individuals who were in the 0-48 month seniority group at the beginning of 1975, are contained only in the seniority intervals between 48 and 60 months.

## TABLE XVI-A

COMPARISON OF ACTUAL STOCKS AND MODEL PREDICTIONS (INCL ACCESSIONS) AT END OF 12 MONTHS COMMENCING JUNE OF YEAR INDICATED.

| SENIORITY (MONTHS) | 75 PRED ICTED | 75 ACTUAL | 76 PRED ICTED | 76 ACTUAL | 77 PRED ICTED | 77 ACTUAL | 78 PRED ICTED | 78 ACTUAL |
|---|---|---|---|---|---|---|---|---|
| 48 | 2322 | 2631 | 2575 | 2470 | 2784 | 2702 | 3091 | 3153 |
| 51 | 141 | 7 | 142 | 1 | 157 | 86 | 170 | 34 |
| 54 | 141 | 17 | 142 | 0 | 157 | 45 | 170 | 132 |
| 57 | 141 | 15 | 142 | 43 | 157 | 112 | 170 | 100 |
| 60 | 141 | 228 | 142 | 765 | 157 | 325 | 170 | 225 |
| 63 | 138 | 139 | 137 | 7 | 138 | 97 | 153 | 86 |
| 66 | 145 | 146 | 140 | 17 | 141 | 0 | 156 | 44 |
| 69 | 137 | 189 | 136 | 15 | 137 | 40 | 152 | 111 |
| 72 | 0 | 0 | 138 | 224 | 138 | 748 | 153 | 313 |
| 75 | 0 | 0 | 137 | 18 | 135 | 7 | 136 | 95 |
| 78 | 238 | 240 | 144 | 143 | 139 | 17 | 140 | 1 |
| 81 | 265 | 272 | 182 | 181 | 132 | 13 | 133 | 39 |
| 84 | 1016 | 989 | 0 | 0 | 108 | 213 | 168 | 376 |
| 87 | 43 | 26 | 0 | 0 | 37 | 59 | 47 | 3 |
| 90 | 63 | 58 | 152 | 227 | 92 | 56 | 88 | 14 |
| 93 | 81 | 80 | 179 | 252 | 123 | 73 | 89 | 6 |
| 96 | 24 | 48 | 462 | 452 | 1 | 0 | 49 | 53 |
| 99 | 65 | 65 | 38 | 25 | 0 | 0 | 33 | 17 |
| 102 | 30 | 55 | 42 | 56 | 77 | 67 | 47 | 54 |
| 105 | 63 | 137 | 46 | 75 | 102 | 74 | 71 | 67 |
| 108 | 54 | 58 | 21 | 48 | 418 | 405 | 1 | 1 |
| 111 | 0 | 0 | 62 | 64 | 36 | 23 | 0 | 0 |
| 114 | 161 | 166 | 27 | 52 | 38 | 48 | 70 | 66 |
| 117 | 190 | 195 | 58 | 103 | 43 | 65 | 95 | 69 |
| 120 | 113 | 112 | 48 | 51 | 18 | 45 | 373 | 373 |
| 121 | 14371 | 14607 | 13591 | 13702 | 12653 | 12702 | 11742 | 11432 |
| ACCESSIONS | ( 630) | | ( 867) | | ( 890) | | (1044) | |
| STAYING | 19493 | 19859 | 17986 | 18429 | 17227 | 17256 | 16561 | 15873 |
| PROMOTED | 540 | 559 | 784 | 585 | 360 | 609 | 360 | 627 |
| ATTRITED | 1494 | 1232 | 1339 | 1475 | 1251 | 1431 | 1183 | 1636 |

## TABLE XVI-B

COMPARISON OF ACTUAL STOCKS AND MODEL PREDICTIONS (INCL ACCESSIONS) AT END OF 12 MONTHS COMMENCING JUNE OF YEAR INDICATED.

| SENIORITY (MONTHS) | 79 PRED | 79 ACTUAL | 80 PRED | 80 ACTUAL | 81 PRED | 81 ACTUAL | 82 PRED | 82 ACTUAL |
|---|---|---|---|---|---|---|---|---|
| 48 | 3068 | 0 | 3051 | 0 | 3039 | 0 | 3030 | 0 |
| 51 | 189 | 0 | 188 | 0 | 187 | 0 | 186 | |
| 54 | 189 | 0 | 188 | 0 | 187 | 0 | 186 | 0 |
| 57 | 189 | 0 | 188 | 0 | 187 | 0 | 186 | 0 |
| 60 | 189 | 0 | 188 | 0 | 187 | 0 | 186 | 0 |
| 63 | 186 | 0 | 184 | 0 | 183 | 0 | 182 | |
| 66 | 188 | 0 | 187 | 0 | 186 | 0 | 185 | 0 |
| 69 | 164 | 0 | 182 | 0 | 182 | 0 | 181 | 0 |
| 72 | 186 | 0 | 185 | 0 | 184 | 0 | 183 | 0 |
| 75 | 151 | 0 | 164 | 0 | 182 | 0 | 181 | 0 |
| 78 | 155 | 0 | 167 | 0 | 186 | 0 | 185 | 0 |
| 81 | 148 | 0 | 160 | 0 | 177 | 0 | 177 | 0 |
| 84 | 130 | 0 | 139 | 0 | 145 | 0 | 144 | 0 |
| 87 | 48 | 0 | 53 | 0 | 58 | 0 | 64 | 0 |
| 90 | 89 | 0 | 99 | 0 | 106 | 0 | 118 | 0 |
| 93 | 89 | 0 | 101 | 0 | 108 | 0 | 119 | 0 |
| 96 | 49 | 0 | 54 | 0 | 59 | 0 | 66 | 0 |
| 99 | 42 | 0 | 43 | 0 | 47 | 0 | 52 | 0 |
| 102 | 45 | 0 | 45 | 0 | 50 | 0 | 54 | 0 |
| 105 | 51 | 0 | 51 | 0 | 57 | 0 | 62 | 0 |
| 108 | 44 | 0 | 44 | 0 | 48 | 0 | 53 | |
| 111 | 31 | 0 | 40 | 0 | 41 | 0 | 44 | 0 |
| 114 | 42 | 0 | 41 | 0 | 40 | 0 | 45 | 0 |
| 117 | 65 | 0 | 47 | 0 | 47 | 0 | 53 | 0 |
| 120 | 0 | 0 | 39 | 0 | 39 | 0 | 42 | 0 |
| 121 | 11295 | 0 | 10415 | 0 | 9710 | 0 | 9064 | 0 |
| ACCESSIONS | ( 795) | | ( 795) | | ( 795) | | ( 795) | |
| STAYING | 16067 | 15663 | 15437 | 0 | 14827 | 0 | 14233 | 0 |
| PROMOTED | 348 | 0 | 389 | 0 | 412 | 0 | 452 | 0 |
| ATTRITED | 1174 | 1254 | 1034 | 0 | 978 | 0 | 924 | 0 |

However, Table XIII (as well as Table VIII) shows that within this 48-60 month seniority group, a negligible number of promotees (1% of the 57-60 month seniority group) are promoted in the model. It is therefore obvious that the discrepancy in 1976 has not been affected by the above-mentioned, assumption because 1% of that seniority group accounts for only one individual, while the discrepancy is 199.

B. PROBLEMS AND CAUSES

The major cause of the discrepancy between actual and predicted values can be found in the historical data tabulated in Tables V - X. The following summary data has been extracted from Tables VII and VIII to illustrate the real source of trouble.

TABLE XVII EXTRACT FROM PROMOTION AND ATTRITION DATA 1975/76

| SENIORITY (MONTHS) | 1975 NO. STAYING | 1975 NO. PROMOTED | 1975 NO. ATTRITED | 1975 TOTAL | 1976 NO. STAYING | 1976 NO. PROMOTED | 1976 NO. ATTRITED | 1976 TOTAL |
|---|---|---|---|---|---|---|---|---|
| 48 | 443 | 0 | 8 | 451 | 263 | 2 | 2 | 267 |
| 60 | 1490 | 302 | 14 | 1806 | 432 | 0 | 11 | 443 |
| 72 | 185 | 237 | 6 | 428 | 906 | 577 | 18 | 1501 |
| 84 | | | | | | | | |

The numbers shown above are divided into 12 monthly accounting intervals for the ease of explanation. The entries in the Total column for 1975 would form a portion of the actual beginning stock vector for 1975 if a 12 monthly accounting interval were used. Similarly, the Total column for 1976 forms a portion of the actual beginning stock vector for 1976. The variation in numbers from one year to the next for a given seniority interval is obvious. The variation in the numbers for successive seniority intervals in the same year is also obvious.

This is an extreme example. A 12 month accounting interval is selected here because outputs from Naval Officer Training Institutions occur at least once every 12 months. According to Reference 4, the annual output from the U.S. Naval Academy is kept reasonably constant, as is the output from the NROTC program. The OCS (Officer Cadet School) output is, however, quite variable on certain occasions, to meet a particular need. This was apparently the case with the group who were Lieutenant Commanders in the 60-72 month seniority group (in particular the 69-72 month seniority group) in 1975.

Erratic variations in the seniority distribution such as that illustrated above, have a significant effect on promotion rates which may be used by manpower planners, because "bulges" such as that illustrated cannot be passed through the grade structure.

Consequently, the extreme variations have an effect on rate tables derived from the historical data. This implies that a Markovian model with constant transition rates estimated from historical data which involves cohorts of widely varying sizes passing through the promotion zone, will result in inaccurate predictions as shown in Table XVI-A.

The evidence suggests that the USN URL promotion process is dependent on several factors, the basic one being seniority, but another important factor being the size of accessions.

A Markov model with fixed transition rates assumes that the promotion process is not time dependent. The results displayed in Table XVI-A show that this assumption is invalid in an organisation where cohort sizes are significantly varied, and where, as a consequence, the promotion process is deliberately managed to meet the varying requirements of the next higher grade.

The difference between the actual and predicted numbers of attritions and those staying in grade, has been less significant than the difference between actual and predicted numbers of promotees. This is due to the fact that:

(1) The numbers staying in grade and attriting are larger than the numbers being promoted. In the notation introduced in Section C of Chapter III $s_i(t)$ and $a_i(t)$ are significantly larger when compared with $n_i(t)$, than $x_i(t)$ is.

(2) Both the attrition rate and the rate of staying in grade are relatively constant, because they are each driven by natural factors, whereas the promotion rates are deliberately managed from year to year by manpower managers.

# VI. CONCLUSIONS AND RECOMMENDATIONS

## A. UNSUITABLE ASPECTS OF THE MODEL

1. If the structure of the Lieutenant Commander group during the period 1973-79 is typical of the Unrestricted Line Officer group, then a Markov model based on seniority is not appropriate, because the assumptions that individuals move through the system independent of each other with identical probabilities which do not vary over time, are not sufficiently realistic.

2. Instability in the size of cohorts, forces manpower managers to use non-stationary promotion rates in the system, to avoid instability in cohort sizes in more senior grades. If a Markov model is to be used to manage the promotion process then non-stationary rates should also be used, but they are unsuitable for a model with small accounting intervals, and with a large number of states.

3. Small seniority accounting intervals, such as the 3 month interval used in this model, are necessary if such a model is to be used for manpower planning purposes. However, the statistics used in this study show that homogeneous subsystems are not clearly evident with an accounting interval of less than one year. An accounting interval in excess of one year would probably give unacceptable results in a length of service model with the type of cohort variation found in this thesis.

4. While a Markov model usually gives better results for fewer groups, the sizes of the groups, and their transition rates must follow a reasonably regular pattern for a Markov model to be useful. Such regularity is more likely to occur with large seniority groups, although in the present model, even that was not the case.

B. POSSIBLE ALTERNATIVE APPROACHES

1. The URL Officer Promotion Process may be more successfully incorporated in a vacancy model which is not directly dependent on the distribution of seniority within each grade, but is dependent on attrition and promotion rates, and therefore on the rate of creation of vacancies. The existence of an unstable cohort size distribution does, however, imply that vacancy distribution in the past, has also been unstable. Instability in cohort sizes can be logically traced back to instability in the number of accessions (primarily in the lowest grade). This means that there has also been instability in the number of vacancies, because accessions in any grade should equal vacancies. In summary, nonstationary behaviour in the rate of accessions will mean nonstationary behaviour in the creation of vacancies, and will therefore give problems in a vacancy model.

2. Any further research should concentrate on modelling the actual 'control' which is applied to the promotion process in dealing with variations in cohort sizes.

It seems that a successful promotion process model also needs to concentrate more closely on the selection process. For the selection process, the data used in this thesis is unsuitable. Instead, historical selection board data is required, and should be obtained directly from the office of DCNO (MPT).

# APPENDIX A

```
C          THIS PROGRAM EXTRACTS PROMOTION AND ATTRITION DATA
C          FROM MASTER TAPES- THEN WRITES THAT DATA ON TO
C          A NEW TAPE (NPS518).
C
C
C  PA AND PB ARE PROMOTION DATES REDUCED TO MONTHS WRT A DATUM.
C  PINT(I) IS TIME IN RANK I BEFORE PROMOTION (MONTHS)
C  PINTOT(I,K) IS TOTAL NO. WHO SPENT EXACTLY K MONTHS IN RANK I.
C  PY(I) IS YEAR OF PROMOTION TO RANK I.
C  PM(I) IS MONTH OF PROMOTION TO RANK I.
C  PPATK(I,K) IS PROB OF PROMOTION FROM RANK I AT K MONTHS SENIORITY
C
      INTEGER PY(7), PM(7), PA, PB, PINT(6), PINTOT(6,150),
     * ATT, DESIG
      DIMENSION PPATK(5,150), NRANK(5), NWASTE(150),
     *NSTAY(150), NPROM(150), NROW(150), PSTAY(150), PWASTE(150),
     *PPROM(150)
      DO 21 K=1,150
      I=4
      PINTOT(I,K) = 0
      NRANK(I) = 0
      NSTAY(K) = 0
      NPROM(K) = 0
      NWASTE(K) = 0

   21 CONTINUE
C
  999 FORMAT(':::::::::',I5)
      WRITE(6,999)I
      WRITE(6,999)K
    2 READ(8,100,END=1) GP,DESIG,LDY,LDM, ACBD, ((PY(I), PM(I)),I=2,7)
  100 FORMAT(63X,A1,19X,I4,21X,2I2,152X,I6,198X,6(I2,I2,2X),
     *232X,250X,250X,250X,150X)
      IF(DESIG.EQ.1110 .OR. DESIG.EQ.1120 .OR. DESIG.EQ.1310 .OR.
     *DESIG.EQ.1320 ) GO TO 3
      WRITE(6,999)DESIG
      GO TO 2
    3 IF( PY(4).EQ.72 ) GO TO 4
      GO TO 2
C
C
  101 FORMAT(1X,A1,1X,I4,1X,I2,'/',I2,1X,I6,6(1X,I2,'/',I2))
  104 FORMAT(A1,I4,I2,I2,I6,6(I2,I2))
    4 WRITE(2,104) GP,DESIG,LDY,LDM,ACBD,((PY(I),PM(I)),I=2,7)
      WRITE(6,101)GP,DESIG,LDY,LDM,ACBD,((PY(I),PM(I)),I=2,7)
C CALC ATTRIT TIME FOR THOSE NEVER PROMOTED FROM LCDR(4)
      IF( PY(5).EQ.0 .AND. LDY.GT.0 ) GO TO 41
```

```
      GO TO 5
   41 ATT = ((LDY - PY(4))* 12) + (LDM - PM(4))
      K=0
    6 K=K+1
      IF(ATT.GT.K) GO TO 6
      NWASTE(K) = NWASTE(K) + 1
    5 PA = (PY(4) * 12) + PM(4)
      PB = (PY(5) * 12) + PM(5)
      PINT(4) = PB - PA
      K = 0
    8 K = K + 1
      IF( PINT(4) .LE. K ) GO TO 7
      NSTAY(K) = NSTAY(K) + 1
      GO TO 8
    7 PINTOT(4,K) = PINTOT(4,K) + 1
C
C     WRITE(6,101) (PINT(I),I=2,5)
      GO TO 2
    1 CONTINUE
      WRITE (6,102)
  102 FORMAT (2X,' LCDR MONTHS',5X,'PROPORTION STAYING',
     *3X,'PROPORTION PROMOTED',3X,'PROPORTION WASTED')
  103 FORMAT(11X,I3,16X,F5.3,16X,F5.3,17X,F5.3)
      DO 12 K = 1,150

      NROW(K) = NSTAY(K) + NPROM(K) + NWASTE(K)
      A = NROW(K)
      B = NSTAY(K)
      C = NPROM(K)
      D = NWASTE(K)
      PSTAY(K) = B/A
      PPROM(K) = C/A
      PWASTE(K) = D/A
      WRITE(6,103) K,PSTAY(K), PPROM(K), PWASTE(K)
   12 CONTINUE
      ENDFILE 2
      STOP
      END
/*
//GO.FT08F001 DD DISP=(OLD,KEEP),UNIT=3400-4,LABEL=(1,BLP,,IN),
//  DSN=DUMMY,DCB=(RECFM=FB,LRECL=1636,BLKSIZE=6544,DEN=3),
//  VOL=SER=(OF1,OF2,OF3,AF1,AF2,AF3,AF4,AF5,AF6)
//GO.FT02F001 DD DISP=(NEW,KEEP),UNIT=3400-4,LABEL=(,SL),
//  DSN=PSTAT,DCB=(RECFM=FB,LRECL=50,BLKSIZE=4800),
//  VOL=SER=NPS518
//
/*
```

APPENDIX B

```
C          THIS PROGRAM EXTRACTS SPECIFIC PROMOTION AND ATTRITION
C          DATA FROM TAPE NPS518 IN ORDER TO PRODUCE STATISTICS
C          FOR LCDR TRANSITIONS DURING SELECTED YEARS.
C          THESE STATISTICS ARE THEN USED TO DEVELOP AND TEST
C          A MODEL.
C
C
C  PA AND PB ARE PROMOTION DATES REDUCED TO MONTHS WRT A DATUM.
C  PINT(I) IS TIME IN RANK I BEFORE PROMOTION (MONTHS)
C  PINTOT(I,K) IS TOTAL NO. WHO SPENT EXACTLY K MONTHS IN RANK I.
C  PY(I) IS YEAR OF PROMOTION TO RANK I.
C  PM(I) IS MONTH OF PROMOTION TO RANK I.
C  PPATK(I,K) IS PROB OF PROMOTION FROM RANK I AT K MONTHS SENIORITY
C
      INTEGER PY(7), PM(7), PA, PB, PINT(6), PINTOT(6,150),
     * ATT, DESIG, PT1, PT2, TPROM, TSTAY, TWASTE
      DIMENSION PPATK(5,150), NRANK(5), NWASTE(150),
     *NSTAY(150), NPROM(150), NROW(150), PSTAY(150), PWASTE(150),
     *PPROM(150)
      DO 21 K=1,150
      I=4
      PINTOT(I,K) = 0
      NRANK(I) = 0
      NSTAY(K) = 0
      NPROM(K) = 0
      NWASTE(K) = 0
      TSTAY = 0
      TPROM = 0
      TWASTE = 0

   21 CONTINUE
C
C          LL IS THE YEAR  SELECTED FOR EACH RUN.
      LL=79
C
      WRITE(6,560) LL
      WRITE(6,561)
      WRITE(6,562)
      WRITE(6,557)
  560 FORMAT(1H0,11X,'JUNE 19',I2,' LCDR TRANSITIONS AFTER 12 MONTHS')
  561 FORMAT(1H0,12X,'SENIORITY     NO.       NO.       NO.')
  562 FORMAT(13X,'(MONTHS)  STAYING  PROMOTED    WASTED')
  555 FORMAT    (20X,3(4X,I6),/,17X,I3)
  556 FORMAT(1H0,/,16X,'>120',3(4X,I6))
  557 FORMAT(1H0,'-------------------------------------------------',
     *'-------------------------')
  558 FORMAT(1H0,19X,3(4X,I6))
C
  104 FORMAT(A1,I4,I2,I2,I6,6(I2,I2))
C
```

```
    2 READ (2,104,END=1) GP,DESIG,LDY,LDM,ACBD,((PY(I),PM(I)),I=2,7)
      PA = (PY(4) * 12) + PM(4)
      PB = (PY(5) * 12) + PM(5)
      ATT = (LDY * 12 ) + LDM
      PT1 = (LL  * 12 ) + 6
      PT2 = (PT1 + 12 )
C  PROMOTED DURING YEAR
      IF( PA.LE.PT1 .AND. PB.GT.PT1 .AND. PB.LE.PT2 ) GO TO 100
C STAYS FOR THE YEAR
      IF( PA.LE.PT1 .AND. PB.GT.PT2 ) GO TO 101
C STAYS FOR THE YEAR BUT ATTRITS LATER BEFORE PROMOTION
      IF(  PA.LE.PT1 .AND. ATT.GT.PT2 ) GO TO 101
C ATTRITS DURING THE YEAR
      IF( PA.LE.PT1 .AND. ATT.GT.PT1 .AND. ATT.LE.PT2 ) GO TO 102
C STAYS FOR THE YEAR ---PROMOTION AND ATTRITION DATA IS BLANK
      IF( PA.LE.PT1 .AND. PB.EQ.0 .AND. ATT.EQ.0 ) GO TO 101
      GO TO 2
C
  100 PINT(4) = PB - PA
      K = 0
    8 K = K + 1
      IF ( K.EQ.121 ) GO TO 61
      IF ((PT1-PA).LE.K) GO TO 7
      GO TO 8
    7 PINTOT(4,K) = PINTOT(4,K) + 1
   61 NPROM(K) = NPROM(K) + 1
      TPROM = TPROM + 1
      GO TO 2
C
  101 K=0
   10 K=K+1
      IF ( K .EQ. 121 ) GO TO 9
      IF ( (PT1- PA ) .LE. K ) GO TO 9
      GO TO 10
    9 NSTAY(K) = NSTAY(K) + 1
      TSTAY = TSTAY + 1
      GO TO 2
  102 K=0
   11 K=K+1
      IF ( K .EQ. 121 ) GO TO 12
      IF ( (PT1 - PA ) .LE. K ) GO TO 12
      GO TO 11
   12 NWASTE(K) = NWASTE(K) + 1
      TWASTE = TWASTE + 1
      GO TO 2
    1 CONTINUE
C
```

```
      KK = 1
      NSTAY(KK) = 0
      NPROM(KK) = 0
      NWASTE(KK) = 0
      DO 51 K = 1,48
      NSTAY(KK) = NSTAY(KK) + NSTAY(K)
      NPROM(KK) = NPROM(KK) + NPROM(K)
      NWASTE(KK) = NWASTE(KK) + NWASTE(K)
   51 CONTINUE
      M = K - 3
      GO TO 53
   52 KK = KK + 1
      NSTAY(KK) = 0
      NPROM(KK) = 0
      NWASTE(KK) = 0
      M = M + 3
   54 K = K + 1
      NSTAY(KK) = NSTAY(KK) + NSTAY(K)
      NPROM(KK) = NPROM(KK) + NPROM(K)
      NWASTE(KK) = NWASTE(KK) + NWASTE(K)
      IF ( K.EQ.121 ) GO TO 56
      IF ( (K - M) .EQ. 3 ) GO TO 53
      GO TO 54
   53 WRITE(6,555)    NSTAY(KK), NPROM(KK), NWASTE(KK), K
      GO TO 52
   56 WRITE(6,556)    NSTAY(KK), NPROM(KK), NWASTE(KK)
      WRITE(6,557)
      WRITE(6,558) TSTAY, TPROM, TWASTE
      WRITE(6,557)
   55 STOP
      END
/*
//GO.FT02F001 DD DISP=(OLD,KEEP),UNIT=3400-4,LABEL=(,SL,,IN),
//  DSN=PSTAT,DCB=(RECFM=FB,LRECL=50,BLKSIZE=4800),
//  VOL=SER=NPS518
//
/*
```

APPENDIX C

```
C          THIS PROGRAM USES HISTORICAL DATA TO DERIVE A
C          TRANSITION MATRIX FOR USN LIEUTENANT COMMANDERS OF
C          VARIOUS SENIORITY. THE SUBROUTINE 'FORCAS' USES THE
C          TRANSITION MATRIX IN A MARKOV MODEL AND COMPARES
C          MODEL PREDICTIONS WITH ACTUAL DATA.
C
C
      DIMENSION K(30), NSTAY(30), NPROM(30), NWASTE(30), B(33,33)
      INTEGER TNS(26),TNP(26),TNW(26), TSTAY(28),TPROM(28),TWASTE(28),
     * T(100,26),C(40,33),HH(40,26), NACCES(100), TACCES, AVACES,LY(7),
     * MRATYR(28)
  560 FORMAT(1H1,/,/,/,/,/,11X,'JUNE 19',I2,' LCDR TRANSITIONS AFTER 12 M'
     *,'ONTHS')
  561 FORMAT(1H0,12X,'SENIORITY     NO.     NO. AND FRACTION     NO.')
  562 FORMAT(13X,'(MONTHS)    STAYING      PROMOTED      WASTED',/,
     *12X,'AT BEGINNING',/,14X,'OF YEAR')
  555 FORMAT    (20X,3(4X,I6),/,17X,I3)
  556 FORMAT(1H0,/,16X,'>120',2(4X,I6),F6.2,4X,I6)
  557 FORMAT(1H0,'-------------------------------------------------------',
     *'-------------------------------------------------------')
  558 FORMAT(1H0,19X,2(4X,I6),6X,4X,I6)
  601 FORMAT(4I5)
  600 FORMAT(1H0,20X,'AVERAGE SENIORITY OF PROMOTEES',F5.0,'  MONTHS')
  606 FORMAT (30X,3(7X,F6.2),/,23X,I4)
  607 FORMAT(
     *20X,'SENIORITY',3(4X,'    RATE  '),/,21X,'(MONTHS)',5X,'STAYING',
     *6X,'PROMOTED',6X,' WASTED',/,
     *18X,'AT BEGINNING',/,20X,'OF YEAR')
  608 FORMAT (1H1)
  610 FORMAT(1H0,22X,'>120',3X,3(7X,F6.2))
  617 FORMAT(1H1,/,/,/,/,/,30X,' ACCUMULATED PROBABILITIES TABLE   ',/,/,/,
     *20X,'SENIORITY',3(4X,'PROPORTION'),/,21X,'(MONTHS)',5X,'STAYING',
     *6X,'PROMOTED',6X,' WASTED',/,
     *18X,'AT BEGINNING',/,20X,'OF YEAR')
  629 FORMAT(1H1,/,/,/,/,/,30X,' ACCUMULATED RATE TABLE FOR',/,
     *40X,'YEARS',6I4,/,/)
  655 FORMAT(20X,2(4X,I6),F6.2,4X,I6,/,17X,I3)
  812 FORMAT (1H1,/,/,/,/,/,/,/,/,/,/,/,/,/,/,  40X,'TRANSITION MATRIX')
  813 FORMAT(18X,'(',I2,')',1X,8(1X,F3.2),'-------',3(1X,F3.2),
     *1X,4(1X,F3.2),2X,F3.2,/,15X,I3)
  814 FORMAT(24X,'(1) (2) (3) (4) (5) (6) (7) (8)        (24)(25)(26) ',
     *'(27)(28)(29)(30) WASTE')
      EPS=10E-05
      DO 701 II=1,26
          TNS(II)=0
          TNP(II)=0
          TNW(II)=0
          TSTAY(II)=0
          TPROM(II)=0
          TWASTE(II)=0
      DO 735 I=73,100
          T(I,II)=0
  735 CONTINUE
  701 CONTINUE
```

```
      DO 729 I=1,7
      LY(I) = 72 + I
  729 CONTINUE
C        IF LY(I) IS SET AT ZERO THEN YR I IS NOT USED IN DERIVING
C         THE RATE TABLE.
C       INYRS IS NO. OF YEARS USED IN DERIVING THE RATES.
      INYRS = 3
      LY(1) = 0
      LY(2) = 0
      LY(6) = 0
      LY(7) = 0
C
      MTSTAY=0
      MTPROM = 0
      MTWAST=0
      TACCES = 0
C
      DO 702 NN=1,7
      READ(5,601) LL
      A=0.
C
      DO 700 II=1,26
      READ(5,601) K(II),NSTAY(II),NPROM(II),NWASTE(II)
      A=A+NPROM(II)*(K(II)-1.5)
      IF( LL.EQ.79 ) GO TO 602
C
C       TN*( ) = TOTAL WHO STAY PROM WASTE IN EACH ACCOUNTING INTERVAL
C       FOR ALL YEARS.  USED FOR DERIVING THE TRANSITION MATRIX.
C
      IF(LY(NN).NE.LL) GO TO 602
      TNS(II)=TNS(II)+NSTAY(II)
      TNP(II)=TNP(II)+NPROM(II)
      TNW(II)=TNW(II)+NWASTE(II)
C
C       T(IYEAR,II) IS STOCK OF II SENIORITY OFFRS IN IYEAR.
C       WHICH LATER BECOMES INITIAL STOCK VECTOR....A( )....
C      OR VECTOR AGAINST WHICH MODEL RESULTANT STOCK VECTOR IS COMPARED
  602 T(LL,II) = NSTAY(II)+NPROM(II)+NWASTE(II)
C
  700 CONTINUE
      READ(5,601) TSTAY(NN), TPROM(NN), TWASTE(NN)
      IF( NN.EQ.1 ) GO TO 705
      NACCES(NN-1) = TSTAY(NN) + TPROM(NN) + TWASTE(NN) - TSTAY(NN-1)
            IF(LY(NN-1).NE.(LL-1)) GO TO 705
      TACCES = TACCES + NACCES(NN-1)
      AVACES = TACCES / INYRS
  705 CONTINUE
C
      IF( LL.EQ.79 ) GO TO 721
```

```
C          MT**** WILL BE TOTAL FOR ALL YEARS ANALYSED
C
      IF(LY(NN).NE.LL ) GO TO 603
      MTSTAY=MTSTAY+TSTAY(NN)
      MTPROM=MTPROM+TPROM(NN)
      MTWAST=MTWAST+TWASTE(NN)
      AVSENP=A/TPROM(NN)
C
      WRITE(6,560) LL
      WRITE(6,561)
      WRITE(6,562)
      WRITE(6,557)
C
  603 DO 704 KK=1,26
C
      X=NPROM(KK)
      Y=TPROM(NN)
      FTPROM=X/Y
C
C     IF(KK.EQ.26) GO TO 900
      WRITE(6,655)    NSTAY(KK), NPROM(KK),FTPROM, NWASTE(KK), K(KK)
      GO TO 901
  900 WRITE(6,556)    NSTAY(KK), NPROM(KK),FTPROM, NWASTE(KK)
  901 CONTINUE
  704 CONTINUE
      WRITE(6,557)
      WRITE(6,558) TSTAY, TPROM, TWASTE
      WRITE(6,557)
      WRITE(6,600)AVSENP
  702 CONTINUE
C
      WRITE(6,557)
C                    WRITE PROBABILITIES TABLE
C
  721 WRITE(6,617)
      WRITE(6,557)
      DO 753 II=1,26
         S=TNS(II)
         W=TNP(II)
         Z=TNW(II)
         SSS=MTSTAY
         PPP=MTPROM
         ZZZ=MTWAST
         PROPST=S/SSS
         PROPPR=W/PPP
         PROPWA=Z/ZZZ
      IF(II.EQ.26) GO TO 619
      WRITE(6,606) PROPST, PROPPR, PROPWA, K(II)
  753 CONTINUE
  619 WRITE(6,610) PROPST, PROPPR, PROPWA
      WRITE(6,557)
```

```
C                       WRITE RATE TABLE
C
      KRAT=0
      DO 723 I=1,7
            IF(LY(I).EQ.0) GO TO 723
            KRAT=KRAT+1
      MRATYR(KRAT)=LY(I)
  723 CONTINUE
      WRITE(6,629)(MRATYR(I),I=1,KRAT)
      WRITE(6,607)
      WRITE(6,557)
C
C   INITIALIZE B MATRIX ELEMENTS.
C
      DO 800 KA=1,33
      DO 801 KB=1,33
      B(KA,KB)=0.
  801 CONTINUE
  800 CONTINUE
      DO 703 II=1,26
          S=TNS(II)
          W=TNP(II)
          Z=TNW(II)
          Q=S+W+Z
          TPS=S/Q
          TPP=W/Q
          TPW=Z/Q
C
C SET UP B MATRIX
C
      IF (II.GT.1 ) GO TO 802
      B(II,II) = (36./48.) * TPS
      DO 804 KD=2,5
      B(II,KD) = ( 3./48.) * TPS
  804 CONTINUE
      GO TO 807
  802 IF (II.GT.22) GO TO 803
      B(II,II+4) = TPS
      GO TO 807
  803 B(II,26) = TPS
  807 CONTINUE
      DO 806 KF = 28,31
      B(II,KF) = TPP/4.
  806 CONTINUE
      B(II,33) = TPW
      IF(II.EQ.26) GO TO 609
      WRITE(6,606) TPS, TPP, TPW, K(II)
  703 CONTINUE
  609 WRITE(6,610) TPS, TPP, TPW
      WRITE(6,557)
      WRITE(6,812)
      WRITE(6,814)
      WRITE(6,557)
      DO 811 KG=1,26
      WRITE(6,813)KG, ( B(KG,M), M=1,8),(B(KG,M),M=24,26),
     *(B(KG,M),M=28,31), B(KG,33)
  811 CONTINUE
      WRITE(6,557)
```

```
C SELECT INITIAL STOCK VECTOR.
      DO 39 J=1,26
      HH(1,J)=T(75,J)
   39 CONTINUE
C
C
      CALL FORCAS (K,HH,B,C, T, AVACES, TSTAY, TPROM, TWASTE, NACCES)
C
      WRITE(6,608)
      STOP
C
C
      SUBROUTINE FORCAS (KR,A,B,C,T,AVACES,TSTAY,TPROM,TWASTE,NACCES)
C
      DIMENSION B(33,33), NACCES(100),KR(30)
      INTEGER  A(40,26),    C(40,33), T(100,26), AVACES,
     * TSTAY(28), TPROM(28), TWASTE(28), KYY(40), KYX(40), KPS(40),
     * KPP(40)
      EPS=10E-05
C
   30 FORMAT (10I8)
   35 FORMAT (1H0)
   38 FORMAT ('-------------------------------------------------------------
     1-------------------------------------------------------------')
  823 FORMAT (1X,33F3.2)
  824 FORMAT (1X,26I5)
```

```
      K=26
      M=33
C
C       NTIME1 IS START YEAR FOR MODEL TEST '73=1 '74=2 ETC
C
      NTIME1=3
      NTIME2=10
      DO 16 KY=NTIME1,NTIME2
      WRITE (6,824) (A(1,J),J=1,K)
      IF( KY.GT.0 ) GO TO 19
      WRITE (6,35)
      WRITE (6,38)
      DO 21 I=1,K
      WRITE (6,823) (B(I,J),J=1,M)
   21 CONTINUE
      WRITE (6,38)
C
   19 DO 11 J=1,M
      PP=0.
      DO 10 N=1,K
      X = A(1,N)
      IF(B(N,J).LT.EPS) GO TO 10
      IF ( X.LT.EPS ) GO TO 10
      PP = PP +  ( X * B(N,J) )
   10 CONTINUE
      C(KY,J)=PP
   11 CONTINUE
      KYY(KY) = 73 + KY
      KYX(KY)=73+KY-1
      KPS(KY)=0
      KPP(KY)=0
      DO 17 J=1,26
      KPS(KY)= KPS(KY) + C(KY,J)
   17 CONTINUE
      DO 18 J=28,31
      KPP(KY) = KPP(KY) + C(KY,J)
   18 CONTINUE
      WRITE (6,824)(C(KY,J),J=1,M)
C  ADD ACCESSIONS TO NEW STOCK VECTOR
      IF(KYY(KY).GT.79) NACCES(KY) = AVACES
      C(KY,1)  =C(KY,1) + NACCES(KY)
C
C               A(,,..) BECOMES NEW STOCK VECTOR.
C               (26 COMPONENT LCDR STOCKS BY SENIORITY)
      DO 14 J=1,K
      A(1,J) = C(KY,J)
   14 CONTINUE
      WRITE(6,829) KYX(KY), NACCES(KY)
   16 CONTINUE
C
```

```
      M= NTIME1 - 4
      DO 840 MM=1,10
      M= M + 4
      N = M + 3
      IF ( M.EQ.(NTIME2 + 1)) GO TO 841
      IF( N.GT.NTIME2 ) N = NTIME2
      WRITE(6,842) (KYX(J),J=M,N)
      WRITE(6,843)
      WRITE(6,38)
C
      DO 844 J=1,26
  844 WRITE(6,845) ((C(KY,J),T(KYY(KY),J)),KY=M,N),KR(J)
      WRITE(6,38)
      WRITE(6,849) (NACCES(KY),KY=M,N)
      WRITE(6,846) ((KPS(KY),TSTAY(KY)),KY=M,N)
      WRITE(6,847) ((KPP(KY),TPROM(KY)),KY=M,N)
      WRITE(6,848) ((C(KY,33),TWASTE(KY)),KY=M,N)
      WRITE (6,38)
      IF( N.EQ.NTIME2 ) GO TO 841
  840 CONTINUE
  841 CONTINUE
  842 FORMAT(1H1,/,/,/,23X,'RESULTS OF MODEL PREDICTIONS (INCLUDING '
     *'ACCESSIONS)',/,20X,'AT END OF 12 MONTHS COMMENCING JUNE OF',
     *' YEAR INDICATED.',/,/,/,30X,5(I2,12X))
  843 FORMAT(1HO,13X,'SENIORITY  ',4('PRED          '),/,14X,
     *'(MONTHS)  ',4(' ICTED ACTUAL '))
  845 FORMAT     (22X,8I7,/,16X,I3)
  846 FORMAT (1HO,15X,'STAYING ',10(I5,2X))
  847 FORMAT (1HO,15X,'PROMOTED',10(I5,2X))
  848 FORMAT (1HO,15X,'WASTED  ',10(I5,2X))
  849 FORMAT(1HO,15X,'ACCESSIONS ',4('(',I4,')',8X))
  829 FORMAT(1HO,30X,'ACCESSIONS DURING YEAR BEGINNING JUNE 19',I2,I6
      RETURN
      END
/*
//GO.SYSIN DD *
```

LIST OF REFERENCES AND SELECTED BIBLIOGRAPHY


1. Bartholomew, David J. and Forbes, Andrew F., Statistical Techniques for Manpower Planning, Wiley, 1979

2. Beckman, Martin J., "Rank in Organisations" Lecture Notes in Economics and Mathematical Systems No. 161

3. Navy Rights and Benefits - Officer Promotions All Hands, November, 1979. Section 12

4. Center for Naval Analysis CNS 1096, An Evaluation of the Navy's URL Officer Accessions Programs, by R.C. Kleinman, September, 1977

5. Naval Military Personnel Center, Unrestricted Line Officer Career Guidebook, NAVPERS 15197A, S/N 0500-LP-272-9000

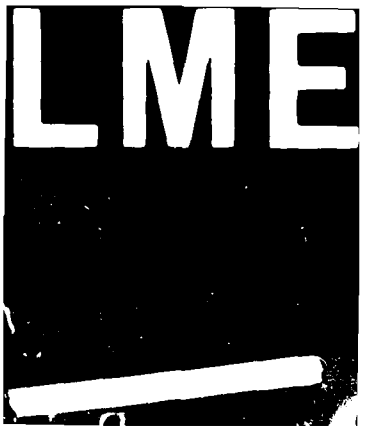
6. United States Code, Title Ten, Armed Forces, 1976 Edition. Government Printing Office, Washington, D.C.

7. Hansell, Ross C., Officer Promotion Opportunity within the Navy Unrestricted Line 1973-1979, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1979

8. Office of the Deputy Chief of Naval Operations (MPT) DP-130D2; A Model of Projected Officer Supply - POPI Washington, D.C., 1978

INITIAL DISTRIBUTION LIST

| | | No. Copies |
|---|---|---|
| 1. | Defense Technical Information Center<br>Cameron Station<br>Alexandria, VA 22314 | 2 |
| 2. | Library, Code 0142<br>Naval Postgraduate School<br>Monterey, CA 93940 | 2 |
| 3. | Library, Code 55<br>Naval Postgraduate School<br>Monterey, CA 93940 | 1 |
| 4. | Dr. Paul R. Milch, Code 55<br>Department of Operations Research<br>Naval Postgraduate School<br>Monterey, CA 93940 | 3 |
| 5. | Professor Richard Elster, Code 54<br>Department of Administrative Sciences<br>Naval Postgraduate School<br>Monterey, CA 93940 | 1 |
| 6. | Major R.W.Weber,<br>DPP, Russell Offices,<br>Canberra, 2600<br>ACT, AUSTRALIA. | 1 |


LME