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DECISION FLEXIBILITY IN A LEARNING ENVIRONMENT

Final Report

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By: Miley W. Merkhofer
William M. Saade

Prepared for:

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A problem example is analyzed with the object of identifying the characteristics of a decision environment that influence the choice of decision flexibility. The results support a growing concern that current methods of analysis tend to produce plans that are insufficiently flexible. A sensitivity analysis clarifies how the desired level of flexibility depends on uncertainty, information, the cost of retaining flexibility, and the cost of decision error. Procedures are suggested for modifying current methods of analysis to account more accurately for the value of flexibility.

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SUMMARY

The research described in this report was undertaken in response to an increasing need for methods of analysis that will adequately measure the value of flexibility in decision making. Decision flexibility is defined as the ability to modify a decision in accordance with changing information. This capability is more essential than ever in our rapidly evolving world, in which technology has resulted in an "information explosion."

Although the value of flexibility has long been recognized by decision makers, decision analysis theory has had little to say on the subject. More importantly, there is a growing concern that because of inherent limitations, current methods of analysis tend to produce plans with insufficient flexibility. Therefore, current planning methods should be expanded to enable decision analysts and decision makers to develop decision strategies with adequate flexibility.

The first step toward developing methods that accurately represent the value of flexibility is to understand how the value of flexibility depends on characteristics of the decision environment. To improve the understanding of the value of flexibility, a problem example concerning the choice of decision flexibility has been designed and analyzed. The results of that analysis, and the conclusions drawn from it are contained in this report. The principal result is elucidation of just how the desired level of flexibility varies with the cost of retaining it, uncertainty as reflected in the decision maker's prior state of knowledge, information flow, and the cost of decision error.

There are three sections to the report. Section 1 explains the failure of current planning methods to accurately represent the value of flexibility. Section 2 describes the example presented herein and its analysis. Section 3 draws conclusions and suggests directions for further research.

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The problem analyzed in this report was designed by M. W. Merkhofer, William Saade, and Burke Robinson. Problem analysis and solution were performed by William Saade.

1 NEED FOR METHODS THAT ACCURATELY REPRESENT THE VALUE OF FLEXIBILITY IN DECISION ANALYSIS

1.0 Introduction

Flexibility is commonly defined as the capability to respond to a changing environment. A rod is flexible if it bends to compensate for forces applied to it. Similarly, a decision strategy is flexible if it can be adjusted to compensate for changes in the decision maker's state of information regarding the decision environment.

1.1 Distinction Between Flexible Strategies, Compromises, and Hedges

A simple example devised from Howard's "Party Problem"[1] can be used to illustrate the distinction between a compromise, a hedge, and a flexible strategy. According to the example, a host must decide whether to hold a party outdoors or indoors, given uncertainty about the weather. If the sun shines, the guests would prefer to be outdoors and, in any case, holding the party indoors would be somewhat inconvenient. If it rains, however, being indoors is clearly preferable. A friend suggests three additional alternatives: a compromise, a hedge, and a flexible strategy. The compromise is to hold the party on the porch. Although not as good as outdoors, if it is sunny, or as indoors, if it rains, holding the party on the porch is acceptable in either case. The hedge is to set up the bar and hors d'oeuvres outdoors and serve dinner indoors. One element of the hedge pays off regardless of the uncertain outcome: if the sun shines, the guests will be disappointed only because they have to eat dinner inside the house; if it rains, at least dinner will be a success. The flexible strategy is to plan an outdoor party, but prepare for an indoor one as well. If it starts to rain, the entire party can easily be moved inside.

Flexible strategies, compromises, and hedges, are useful in situations of uncertainty. The key difference is that hedges and compromises are options that make the worst outcome a little bit better at the expense of making the best outcome a little bit worse. A flexible strategy, on the other hand, justifies the cost of keeping options open by the expectation that additional information will permit the decision to be made more effectively later.

1.2 Increasing Importance of Flexibility in Decision-Making

At least three trends in society are contributing to an increasing awareness by managers of the importance of retaining flexibility in their decisions. First, technology provides the decision-maker with more information than ever before. Decision-makers should, therefore, retain more decision flexibility to effectively use this information. Second, because of an accelerating rate of technological and social change, plans or policies instituted by decision-makers are more likely than in the past to become rapidly outdated. Decisions with long-term impacts should therefore be adaptive if frequent shifts in strategy are to be avoided. Third, greater awareness of public issues has led to an increasing demand for excellence. Concerned parties, as well as the decision maker's superiors, are less likely to be forgiving of mistakes. To keep the confidence of others, the decision-maker should avoid rigidity in his thinking; he should retain the capacity to modify a plan or course of action in response to a changing environment.

1.3 Current Decision Analysis Methods Lead to Decisions with Insufficient Flexibility

Current methods of planning that use decision analysis often lead to decisions with inadequate flexibility, because the decision models seriously underestimate the uncertainties that may affect the decision. The logic of this reasoning will become clear as we review what has been learned about the concept of flexibility from earlier research.

1.3.1 The Value of Decision Flexibility Depends on the Anticipated Resolution of Uncertainty

That flexibility is motivated by uncertainty has long been recognized. Nearly 20 years ago economist W. J. Baumal[2] suggested that, "the existence of uncertainty will lead to the (increased) use of equipment whose scale of operation is flexible." This observation was refined by Hart[3], who argued that the desire to retain flexibility should be related to the rate of information accumulation over time.

Merkhofer[4] applied concepts of decision analysis to quantify the relationship between flexibility and information. The number of alternatives available for a decision was chosen as a measure of the decision's flexibility--and the value of information calculation used in decision analysis was expanded to provide a means for placing an economic value on flexibility. Although the research investigated flexibility in the context of a very simple decision situation, it showed clearly that the value of retaining decision flexibility is directly related to the amount of information expected by the decision maker prior to his commitment of resources. The more information expected, the greater the value of retaining flexibility will be. The fact that the flexibility is valued according to the anticipated resolution of uncertainty means that it will be undervalued if uncertainty is underestimated.

1.3.2 Uncertainty is Typically Underestimated in Decision Analysis

In a decision analysis, uncertainties are identified and represented in an explicit model. The model frequently takes the form of a decision tree. Solution of the decision tree yields a strategy--an initial action plus a decision rule for subsequent decisions--that takes acount of the effectiveness given the occurrence of various uncertain events as well as the likelihood of those events. If all future uncertainties that may affect the strategy are anticipated and explicitly included in the analysis, the solution to the decision tree should yield the strategy with the desired level of flexibility. In practice, however, significant uncertainties are typically omitted from the analysis.

There are two reasons decision analyses usually fail to account for all significant uncertainties faced by the decision maker. First, anticipating all the uncertainties that may affect a decision is difficult, if not impossible. In military planning, for example, defense systems may be operational for ten, twenty, or more years following implementation. The military environment is so diverse and dynamic that the planner cannot be expected to foresee all the events that may affect his decision over such a long period. Events that frequently take the planner by surprise include changes in political constraints, unexpected developments in technology, and shifts in the level of military appropriations. Such events are often difficult to predict; for instance, it is not surprising that military planners a decade ago failed to include the possibility of a costly all-volunteer army in their computation of manpower costs.

Recognizing the uncertainties that may influence a decision is only part of the problem. The second reason that decision analyses fail to represent all relevant uncertainties is that limited resources make simplification necessary. Even if the analyst is aware of all uncertainties, time and budget constraints make it impossible to delineate all possible contigencies. Less important uncertainties and detailed subsequent decisions are not explicitly represented in the decision tree. Consequently, the model developed for the analysis substantially underestimates the true uncertainty in the decision environment. Moreover, the value of a flexible decision strategy (initial decisions that leave many options open) is underestimated, and the analysis often yields an overly constrained and potentially high-risk course of action. Because only a small fraction of the actual uncertainty in future events can be encompassed in any given decision model, the use of that model for decision making yields an insufficiently flexible strategy.

1.4 Need for Methods That Accurately Represent the Value of Decision Flexibility

Military planners have already commented on the need for procedures and methodology that rectify the tendency of current planning methods to lead to inflexible decisions. For example, in his keynote address to the 37th Military Operations Research Symposium, Dr. Seth Bonder stated: "I believe we... must redirect our focus from the classic cost-effectiveness philosophy and criteria to a related but philisophically different criterion--that of maximizing system and force versatility." Rather than throw out the present methods for analysis, procedures should be developed to improve those methods so as to represent the value of decision flexibility accurately.

1.4.1 Purpose of the Research

The first step toward the development of methods that accurately represent the value of flexibility is to understand how that value depends on characteristics of the decision environment. The following section of this report, Section II, describes a decision problem that was designed and analyzed to explore the importance of flexibility in a learning environment. The purpose of the example is to indicate how characteristics of the decision situation, such as the decision maker's prior state of knowledge and anticipated information affect the decision of retaining flexibility. Section III discusses some approaches to accomplishing the ultimate objective of this research: the development of techniques that reduce the tendency of traditional decision analysis methods to produce inflexible decision strategies.

2 DECISION PROBLEM EXAMPLE REQUIRING A CHOICE OF FLEXIBILITY

This section describes a simple decision problem example designed to explore the flexibility desired by a decision maker as a function of various problem parameters. We first describe the example, which has been discussed in slightly different form by Howard[5], and point out the special features that make it relevant to many real world problems. The later subsections describe its solution and interpret the results.

2.1 A Thumbtack-Tossing Problem

Suppose that an eccentric millionaire with a taste for gambling offers you the chance to participate in a thumbtack-tossing game. He produces a common, ordinary thumbtack and points out that if the tack were to be dropped onto a large flat surface it would come to rest in one of two landing positions, which are labeled "heads" and "tails" as shown in Figure 1. The object of the game is to guess the fraction

FIGURE 1 THUMBTACK POSITIONS

of times, Φ, that the tack will come to rest in a "heads" position. To encourage you to play, he offers a large sum of money, but points out that there are some additional rules:

1. To obtain the "true" fraction of heads, Φ, the tack will be sent to a reputable research laboratory where it will be tossed a great number of times (say, ten million).

2. You are allowed to toss the thumbtack m times before making your guess.

3. Your guess, g, is constrained to lie between two values, a and b, which you select before you begin tossing the tack.

4. Your payoff for playing the game is

$$K_1 - K_2 (b-a) - K_3 (g-\Phi)^2$$

In other words, you get an amount of money for playing, K_1, but you give up an amount equal to K_2 dollars times the range you allow for your guess, plus an amount equal to K_3 times the square of the difference between the value you guess and the true value.

The objective of the analysis is to determine the values of a and b and the guess, g, that maximize your expected gain when playing the game. Intuitively, how should your selected range of flexibility, a to b, change if the number of trial flips, m, were increased? If the cost parameter K_3 were decreased, how should that affect your choice? Should the flexibility desired by someone familiar with thumbtacks be very different from that of someone with little prior knowledge? These questions are answered in Section 2.3.

Before discussing the results, Section 2.2 points out the characteristics of the thumbtack tossing example that make it relevant to real-world decisions involving flexibility. The next section also describes some additional assumptions upon which the solution is based.

2.2 Characteristics Relevant to the Value of Decision Flexibility

The thumbtack- ,sing example stated above contains, in simplified form, many factors that are present in real-world decisions involving a choice of decision flexibility. These are briefly discussed in subsections below.

2.2.1 Retaining Flexibility is Often Costly

Keeping options open usually costs something. In the thumbtack-tossing example the primary decision to be made is your guess, g, as to the fraction of times the thumbtack will come up heads. Flexibility will be defined as the range of allowed guesses, b-a. The cost of flexibility, $K_2(b-a)$, is assumed to be linearly related to the magnitude of the range. The parameter K_2 can be varied to investigate the effect of the cost of flexibility on the decision.

Many decision problems relating to system development have a structure very similar to the preceding example. In the development of a new generation of missiles, for example, desired propulsion system characteristics may depend on many factors that will not be known precisely until payload, guidance system, and airframe have been selected and tested. The risk of having to compromise performance because of an inadequate propulsion system can be decreased by developing several alternative propulsion system designs simultaneously. Although conducting research and development in several competing technologies is an effective way to maincain flexibility, costs increase approximately in proportion to the number of alternative designs that are developed.

2.2.2 Information Improves over Time

The motivation for delaying the commitment to a specific alternative is the hope that better information will be available later. The tendency for information to improve over time is represented in the example by the m trial tosses of the thumbtack that the decision maker is allowed to see before making a guess. The sensitivity of the decision to the amount of information can be checked by varying the number of trial tosses m. In a more realistic decision problem, accruing information could consist of intelligence concerning enemy actions, the results of experiments or testing, or natural events such as weather.

2.2.3 The Inability to Select the Best Alternative Entails a Cost

In any decision problem there will be some opportunity cost associated with selecting the wrong alternative. The cost of error can take many functional forms. In the thumbtack-tossing example the cost of guessing wrong is assumed to be proportional to the square of the error, $K_3(\Phi-g)^2$. Because of its simplicity, a quadratic cost function such as this is frequently used to approximate more complicated relationships. The parameter K_3 can be varied to gauge the sensitivity of the decision to the cost of decision error.

2.2.4 The Decision-Maker Brings to Each Decision a Prior State of Information

The desirability of retaining flexibility is clearly related to the decision-maker's state of information. If he is confident that additional data will not change his decision, then flexibility has little value.

In the thumbtack-tossing example the decision-maker's prior state of information concerning the fraction of heads is assumed to be represented by a probability distribution. Several techniques have been developed for encoding such a distribution. To be specific, we assume that it is well approximated by a beta distribution, which represents a family of functions characterized by two parameters, m_o and r_o. By varying m_o and r_o, we can obtain a large number of different shapes for the probability density function. Some of these are shown in Figure 2. It is useful to note that the mean and variance of the beta distribution are

$$\text{mean} = \frac{r_o}{m_o}, \quad \text{variance} = \frac{r_o}{m_o}\left(1 - \frac{r_o}{m_o}\right)\frac{1}{m_o + 1}$$

Roughly speaking, as m_o increases uncertainty decreases; this is shown in Figure 2 by the fact that the distributions become narrower for larger values of m_o. Thus, by varying the parameters m_o and r_o, one can investigate the sensitivity of the decision to variations in the decision-maker's prior state of information.

2.3 Solution to the Thumbtack Example

As shown in the appendix, the strategy that maximizes the expected value of the thumbtack game involves a guess regarding the fraction

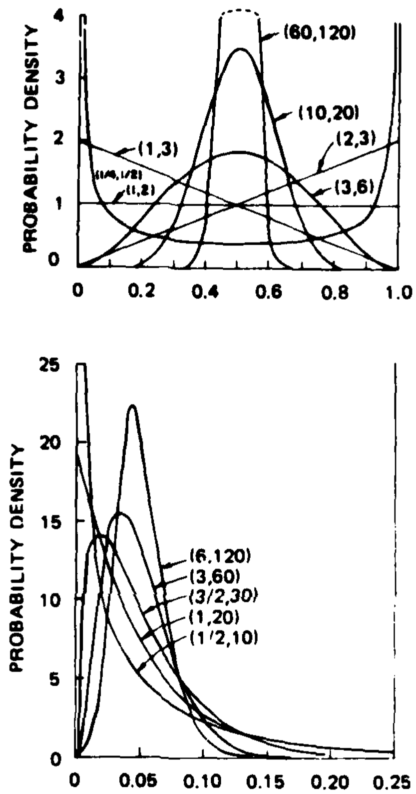

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FIGURE 2 BETA DISTRIBUTIONS FOR VARIOUS PARAMETERS (r_o, m_o)

of heads that depends on the posterior mean of the beta distribution. The posterior mean is the expected value of the distribution describing the fraction of heads, updated by the number of heads obtained in the m trial tosses. If the posterior mean lies between a and b, the best guess for the fraction of heads is the posterior mean. If the posterior mean lies below a, the best guess is a; if the posterior mean lies above b, the best guess is b.

Before seeing the results of m flips of the thumbtack, the decision-maker must select his degree of flexibility (a and b). The best choices for a and b depend on the number of trial flips (m), the cost of flexibility relative to the cost of decision error (K_2/K_3), and the prior distribution for the fraction of heads, as described by the parameters m_o and r_o. Figures 3 and 4 show how optimal flexibility varies with m and the ratio $K = K_2/K_3$ for typical prior distributions. Figure 3 assumes that the decision-maker's uncertainty is characterized by the symmetric prior distribution for the fraction of heads shown in Figure 2 with $m_o=6$, $r_o=3$. Figure 4 assumes the asymmetric prior distribution shown in Figure 2 with $m_o=3$, $r_o=1$. A number of interesting characteristics can be observed.

1. The optimal range of flexibility (b-a) increases in proportion to the number of trial tosses (m). Thus, the desired degree of flexibility increases as the amount of expected information increases. This result is consistent with the earlier research cited in Section 1.3.1.

2. If no information is expected (m=0), there is no incentive to retain flexibility (a=b). Thus, flexibility has value only if the decision-maker expects uncertainty to be resolved before he has to make his decision.

3. Optimal flexibility diminishes as the ratio of the cost of flexibility to the cost of decision error ($K = K_2/K_3$) increases.

4. Even when the cost of flexibility is zero (K=0), the range of flexibility is not necessarily maximized (by setting a=0, b=1). In fact, when the cost of flexibility is zero, optimal

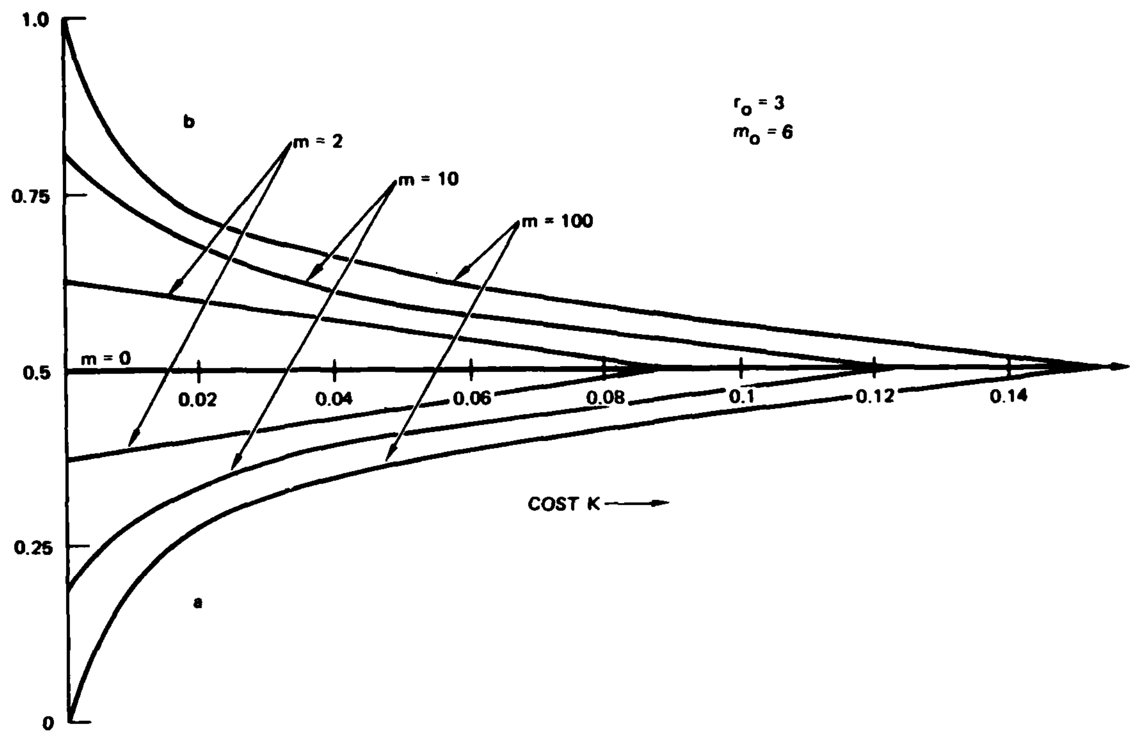

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FIGURE 3 OPTIMAL RANGE OF FLEXIBILITY VERSUS COST OF FLEXIBILITY FOR A SYMMETRICAL PRIOR PROBABILITY DISTRIBUTION

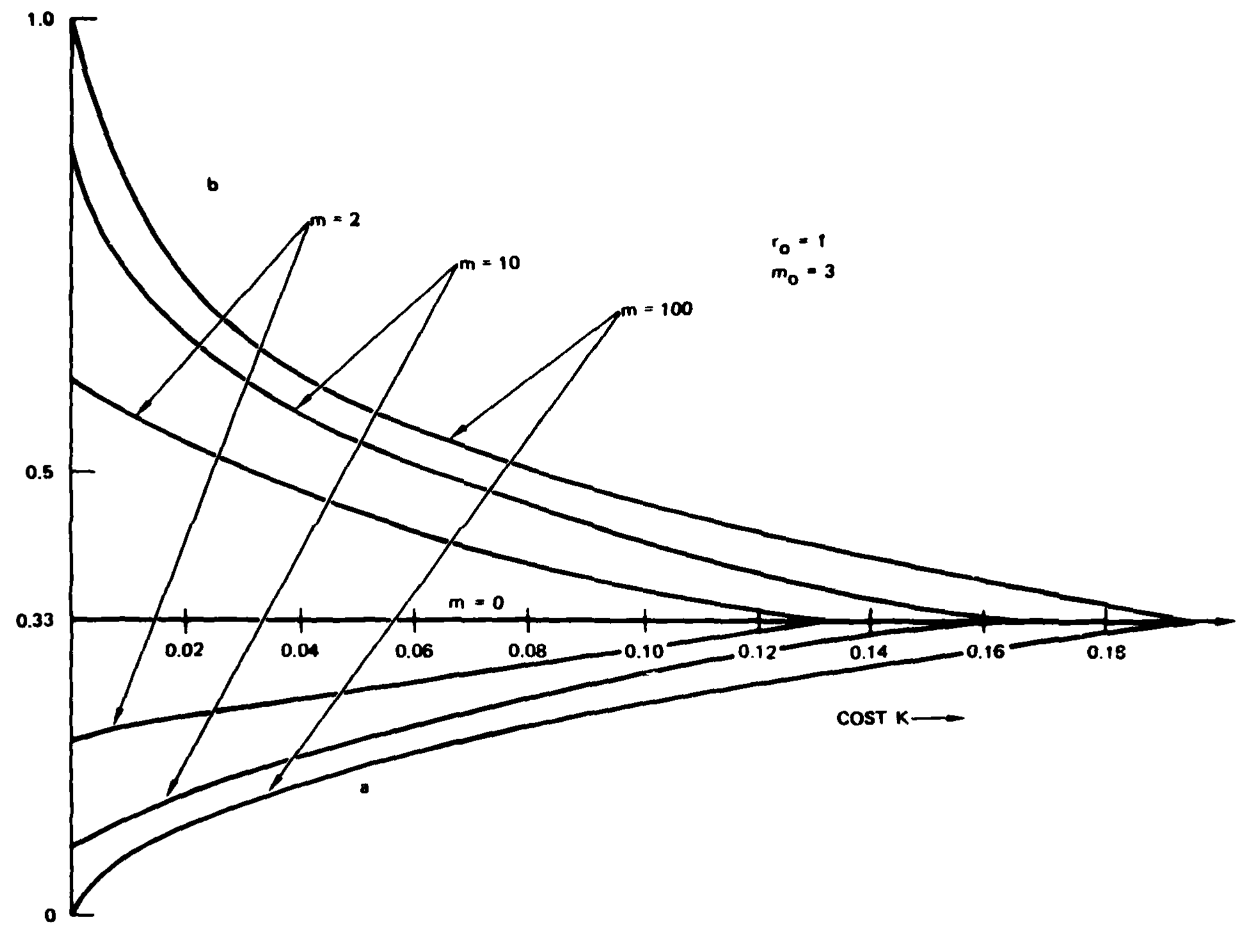

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FIGURE 4 OPTIMAL RANGE OF FLEXIBILITY VERSUS COST OF FLEXIBILITY FOR AN ASYMMETRICAL PRIOR PROBABILITY DISTRIBUTION a and b correspond to the minimum and maximum conceivable guesses (a is the value that should be guessed if the trial tosses produced no heads, while b is the value that should be guessed if every trial toss were a head). It makes no sense to retain flexibility on options that will never be selected.

5. If the cost of flexibility is high enough in relation to the cost of decision error, the optimal strategy is to retain no flexibility and to immediately guess the prior mean (a=b=g).

6. Comparison of Figures 3 and 4 shows the similarity between the case of a symmetrical prior distribution and that of its assymmetrical counterpart. In general, if the decision maker has greater prior information (m_o increases), the desired range of flexibility is narrowed. (This will be illustrated more clearly in Figure 9.)

2.3.1 Value per Unit of Additional Flexibility

In the thumbtack-tossing example, as in many decision situations, selecting the appropriate amount of decision flexibility requires a trade-off between the cost of flexibility and the cost of decision error. Increasing flexibility costs money, but also increases the likelihood that a good alternative can be selected later. Figure 5 illustrates the trade-off. The figure shows the two cost terms in the payoff plotted as a function of the upper bound b on the flexibility range. As noted earlier, the cost of flexibility is assumed to increase linearly with range. The expected loss resulting from decision error is also illustrated. As shown in the figure, the optimal choice for b is the value that minimizes the sum of the two cost components.

Greater insight into the value of flexibility can be obtained by plotting the slope of the game's expected value as a function of b. The slope indicates the value per unit of additional flexibility.

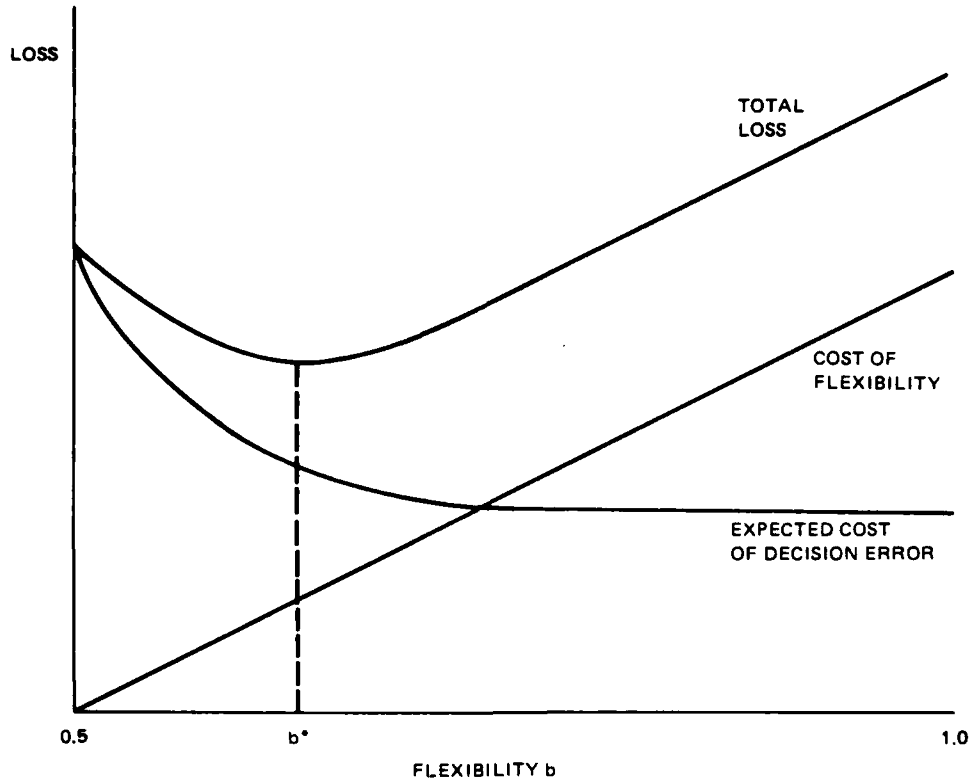

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FIGURE 5 COMPONENTS OF EXPECTED LOSS VERSUS DEGREE OF FLEXIBILITY CHOSEN (SYMMETRICAL PRIOR PROBABILITY DISTRIBUTION)

Figure 6 shows how the value per unit of additional flexibility

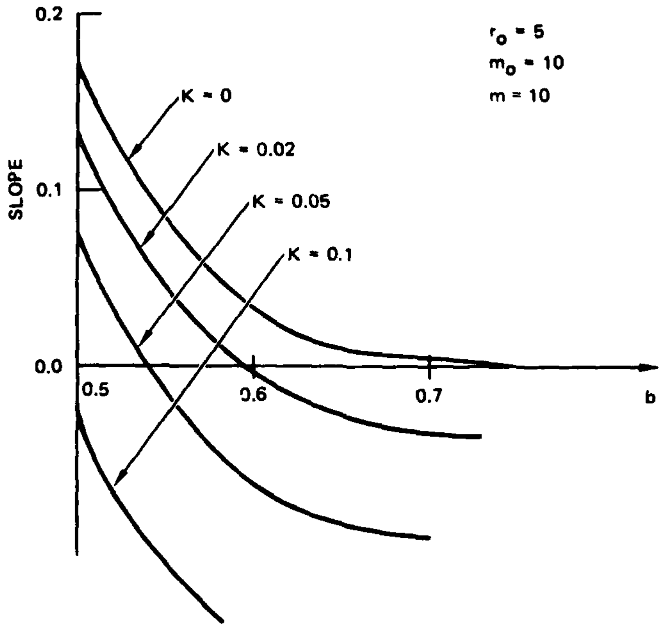

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FIGURE 6 VALUE PER UNIT OF ADDITIONAL FLEXIBILITY FOR DIFFERENT COSTS OF FLEXIBILITY

declines with increasing flexibility. For fixed flexibility b, this value also declines with the cost of flexibility (K). Since the optimal value for b corresponds to the point at which the plot crosses the x axis, it is again apparent that the desired amount of flexibility decreases with its cost. Figure 7 shows how the value per unit of additional flexibility increases with increasing m, while Figure 8 shows how the value per unit of additional flexibility declines with increasing m_o. From these plots it can be concluded that the value of flexibility rises with an increase in the amount of expected information, but goes down as the degree of prior knowledge increases.

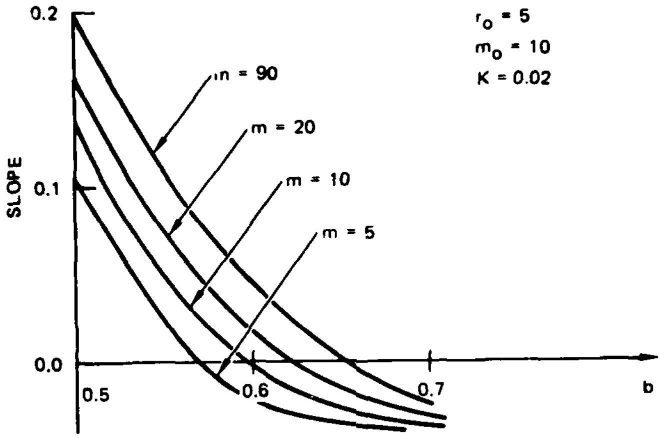

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FIGURE 7 VALUE PER UNIT OF ADDITIONAL FLEXIBILITY WHEN DIFFERING AMOUNTS OF INFORMATION ARE EXPECTED

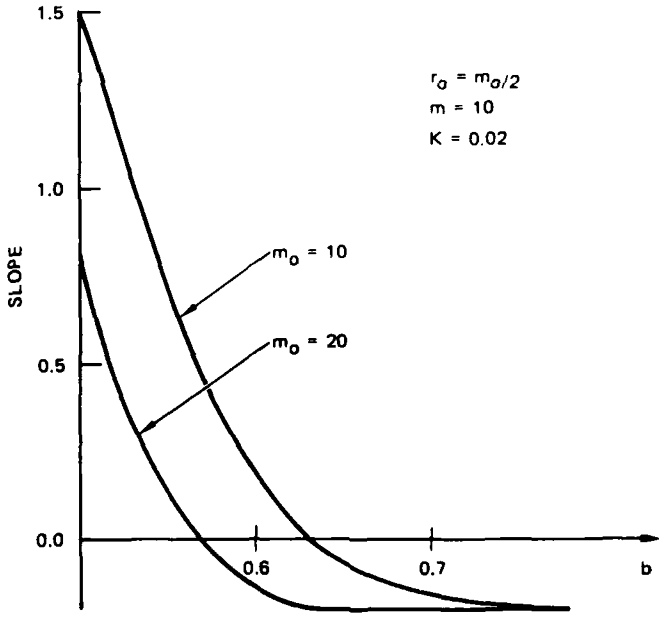

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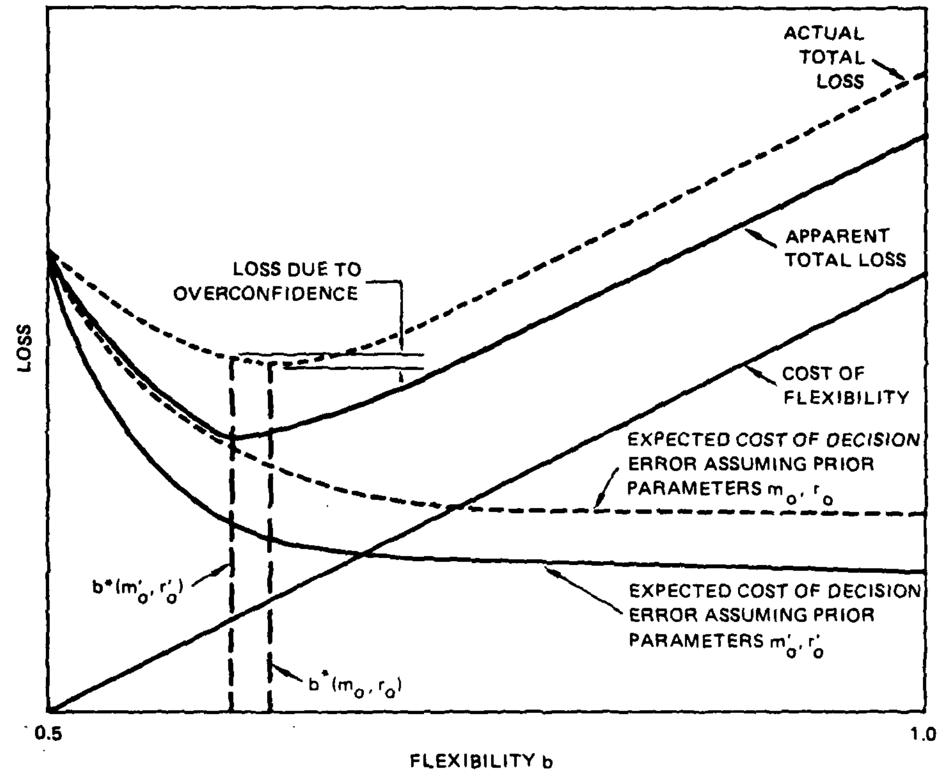
FIGURE 8 VALUE PER UNIT OF ADDITIONAL FLEXIBILITY UNDER DIFFERING AMOUNTS OF PRIOR INFORMATION

2.4 Overconfidence and the Neglect of Revealed State Variables Lead to Insufficient Flexibility

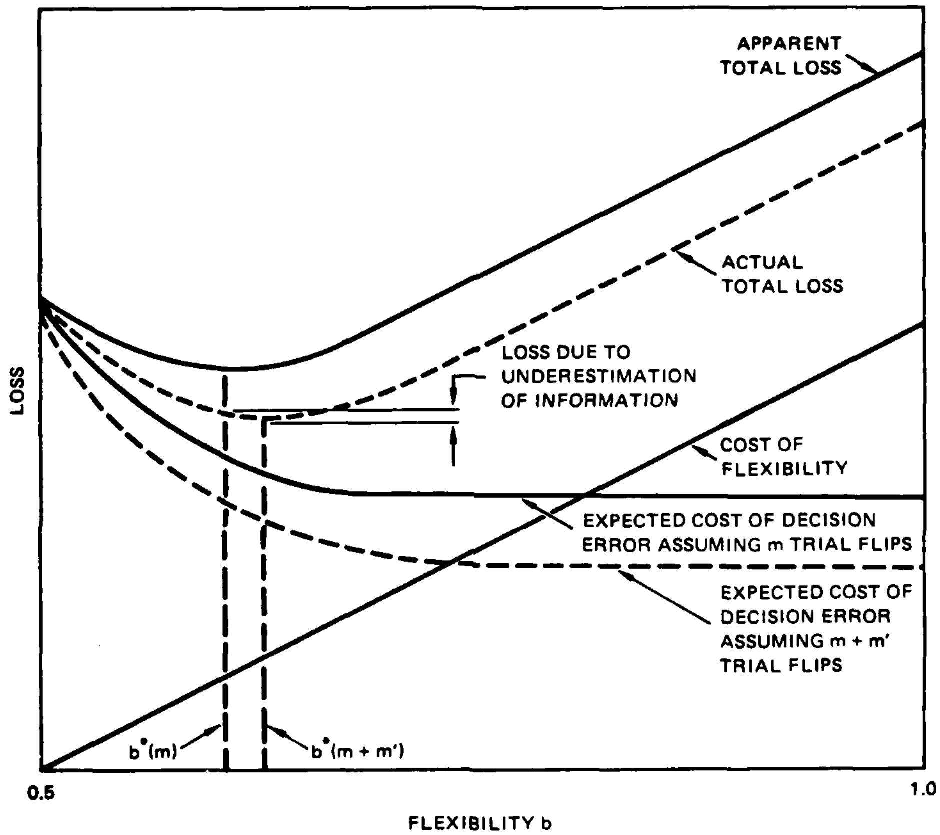
The thumbtack-tossing example can be used to demonstrate the tendency to select decision strategies that are insufficiently flexible. Figure 9 shows the effect of assuming a too narrow prior distribution. The lower curve is the expected loss from flexibility cost and decision error that would be computed if an overly narrow prior distribution, with parameters m_o' and r_o', is assumed rather than a "correct" prior distribution, with parameters m_o and r_o. As illustrated in the figure, the result is the selection of a lower degree of flexibility $b^*(m_o', r_o')$ than that which would be produced if the proper prior distribution were used, $t^*(m_o, r_o)$. Because distributions assessed from individuals are often too narrow (see reference [6]) there is a tendency to select decision strategies with insufficient flexibility.

Figure 10 shows the effect of ignoring information. The top curve is the expected loss that would be computed if m trial thumbtack tosses are assumed. If, in fact, m+m' tosses occur, the decision maker has more information than he anticipated. The upper curve is the expected cost that should have been computed. Again, the result is the selection of a degree of flexibility $b^*(m)$ that is less than the desired level $b^*(m+m')$. Overlooking or ignoring information thus results in decision strategies with insufficient flexibility.



SOURCE: SRI International

FIGURE 9 LOSS RESULTING FROM AN OVERLY NARROW PRIOR DISTRIBUTION WITH PARAMETERS m'_o, r'_o, (OVERCONFIDENCE)



SOURCE: SRI International

FIGURE 10 LOSS RESULTING FROM FAILING TO ANTICIPATE m' TRIAL THUMBTACK FLIPS (UNDERESTIMATION OF INFORMATION)

3 CONCLUSION AND AREAS FOR FURTHER RESEARCH

An example of the selection of decision flexibility has been defined and analyzed. Although highly simplified, the example does imbody several important characteristics relevant to the choice of decision flexibility in more realistic situations. A sensitivity analysis was conducted to investigate the importance of the cost of retaining flexibility, the accumulation of information over time, the cost of being unable to select the best alternative, and the prior information possessed by the decision maker.

3.1 Conclusions

Analysis of the example supports the following assertions:

1. Decision flexibility is most valuable when receipt of important information is anticipated prior to the final commitment of resources. Decision flexibility has no value if it is impossible for the decision-maker to receive information that could change the decision.

2. The greater the decision-maker's initial uncertainty, the greater is the value of retaining flexibility. As his uncertainty is reduced, so does the value of flexibility.

3. The optimal level of decision flexibility will increase if the cost of flexibility goes down or the cost of decision error rises.

4. Overconfidence, or the observed tendency of subjective probability distributions to be too narrow, and the failure to model completely the uncertainty in a decision analysis both contribute to the selection of decision strategies with insufficient flexibility.

3.2 Techniques for Reducing the Tendency to Produce Inflexible Decision Strategies

The awareness that conventional methods of analysis promote decision strategies with insufficient flexibility suggests a need for additional research. The goal of this research should be the development of procedures for generating and realistically evaluating flexible decision strategies. Although not formally part of this effort, the results derived from analysis of the example confirm the feasibility of developing more reliable evaluation techniques.

Some indication as to whether an analysis has incorrectly favored an inflexible strategy can be obtained by investigating the difference between the expected value of the inflexible strategy and the expected value of the flexible strategy. As in the example, the flexible strategy is defined as an initial decision that culminates in a larger number of choices for subsequent decisions. If the difference in expected values is a small percentage of the value of the decision (i.e., the inflexible decision is not much better than the flexible one), it is probably reasonable to assume that the value of flexibility not represented in the analysis is sufficient to motivate the selection of the flexible decision. On the other hand, if the difference in expected values is considerable, the additional value of flexibility not represented in the analysis is probably not sufficient to motivate selection of the flexible decision.

If the difference between the expected value of the alternative with the highest expected value and that of a more flexible decision is of moderate magnitude, it is useful to explore the sensitivity of that difference to changes in model assumptions. Suppose the difference in expected values between an inflexible and a flexible strategy is Δv.* The concern is that if the decision model were expanded to include

*If the risk attitude of the decision-maker has been represented in the analysis through the encoding of a utility function, the comparison to be made is between the expected utilities and certain equivalents of the strategies rather than their expected values. See reference [7] for a discussion of utility theory.

additional uncertainties, Δv would become smaller and perhaps change signs; that is, a more accurate representation of uncertainty might indicate that the flexible strategy is the one with the highest expected value.

The sensitivity of Δv to overconfidence can be explored by increasing the spread of distributions of critical state variables (by some suitable transformation) both individually and jointly, and then observing the effect on Δv. If Δv rapidly decreases with the spread of critical distributions, a more careful representation of uncertainty may be called for.

Sensitivity to the omission of state variables is more difficult to check, but there may be ways to obtain an approximate indication as to whether the omission might be leading to a strategy with insufficient flexibility. One approach is to construct dummy state variables to represent the occurrence of events not yet represented in the analysis. The dummy events could be defined in terms of their influence on decision outcomes. For example, the path through the tree which contributes the most to the expected value of the inflexible strategy could be identified and the question posed: "Is it possible that some event might cause the outcomes associated with this path to be significantly worse than previously estimated?"* Checklists derived from previous, similar decisions could be used to help in the identification of omitted events and their probabilities. If the inclusion of "surprise" events defined in this way significantly reduces Δv, the analyst has an indication that further modeling may be necessary to accurately represent the value of flexible decision strategies.

More research is necessary to develop methods to ensure the accurate valuing of flexibility in the analysis of decisions. Procedures are needed for generating flexible decision strategies and for comparing the value of such strategies with the value of strategies with less

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*Research on decision structuring currently being conducted by SRI International is investigating the feasibility of identifying surprise events through this approach. See Merkhofer et al, [8].

flexibility. It is hoped that the research that has been described here helps to motivate additional work and suggests some fruitful avenues for exploration.

Appendix

MATHEMATICAL SOLUTION TO THE THUMBTACK-TOSSING EXAMPLE

Following the notation in the main text, let g denote the value guessed for the long-run fraction of heads for tossed thumbtacks. The decision maker is paid an amount K_1 for playing the game. The guess g is constrained to lie between values a and b. Depending on his selected values for a and b, he deducts an amount $K_2(b-a)$ from his pay. After selecting a and b, but before making his guess, the decision maker is allowed to see m sample tosses of the thumbtack.

To clarify the solution techniques, inferential notation will be used. If x is a random variable, $\{x|\zeta\}$ denotes the probability density function of x given the state of knowledge ζ. ε denotes the state of knowledge available at the beginning of the problem. The expectation of x based on ζ is denoted $\langle x|\zeta\rangle$.

For given values of a, b, m, and r, the decision maker must choose his guess, g, to maximize

$$\langle v|m,a,b,r,g,\varepsilon\rangle = K_1-K_2(b-a) - K_3\langle(\Phi-g)^2|m,a,b,r,g,\varepsilon\rangle \quad ,$$

which implies an optimal guessing strategy

$$g^* = \begin{cases} a & \text{if } \langle\Phi|m,r,\varepsilon\rangle < a \\ \langle\Phi|m,r,\varepsilon\rangle & \text{if } a \le \langle\Phi|m,r,\varepsilon\rangle \le b \\ b & \text{if } b < \langle\Phi|m,r,\varepsilon\rangle \quad . \end{cases}$$

Since r may take any value from 0 to m, a and b must be chosen to maximize

$$\langle v|m,a,b,\varepsilon\rangle = \sum_{r=o}^{m} \langle v|m,a,b,r,g^*,\varepsilon\rangle\{r|m,\varepsilon\} \quad ,$$

where

$$\langle v|m,a,b,r,g^*,\varepsilon\rangle = K_1-K_2(b-a)-K_3 \begin{cases} \langle(\Phi-a)^2|r,m,\varepsilon\rangle \text{ if } \langle\Phi|r,m,\varepsilon\rangle<a \\ \langle(\Phi-\langle\Phi|r,m,\varepsilon\rangle)^2|r,m,\varepsilon\rangle \\ \qquad \text{if } a\le\langle\Phi|r,m,\varepsilon\rangle\le b \\ \langle(\Phi-b)^2|r,m,\varepsilon\rangle \text{ if } b<\langle\Phi|m,r,\varepsilon\rangle \end{cases} .$$

Since

$$\langle\Phi|r,m,\varepsilon\rangle = \frac{r+r_o}{m+m_o} ,$$

for $r<a(m+m_o)-r_o = r_1$ and $r<b(m+m_o)-r_o = r_2$, we have, respectively, $\langle\Phi|r,m,\varepsilon\rangle<a$ and $\langle\Phi|r,m,\varepsilon\rangle>b$. Thus

$$\begin{aligned}\langle v|m,a,b,\varepsilon\rangle = K_1-K_2(b-a)-K_3 \Bigg\{ & \sum_{r=0}^{r_1^-} \langle(\Phi-a)^2|r,m,\varepsilon\rangle\{r|m,\varepsilon\} \\ & + \sum_{r=r_1}^{r_2} \langle(\Phi-\langle\Phi|r,m,\varepsilon\rangle)^2|r,m\ \varepsilon\rangle\{r|m,\varepsilon\} \\ & + \sum_{r=r_2^+}^{m} \langle(\Phi-b)^2|r,m,\varepsilon\rangle\{r|m,\varepsilon\}\Bigg\} .\end{aligned} \tag{1}$$

Letting

$$\Theta\ (x) = \begin{cases} 0 & x<0 \\ \frac{1}{2} & x=0 \\ 1 & x>0 \end{cases}$$

be the unit step,

$$\begin{aligned}\langle v|m,a,b,\varepsilon\rangle = K_1-K_2(b-a)-K_3 \Bigg\{ \sum_{r=0}^{m} \Big[& \langle(\Phi-a)^2|r,m,\varepsilon\rangle\Theta(r_1-r) \\ & + \langle(\Phi-\langle\Phi|r,m,\varepsilon\rangle)^2|r,m,\varepsilon\rangle\Theta(r-r_1)\Theta(r_2-r) \\ & + \langle(\Phi-b)^2|r,m,\varepsilon\rangle\Theta(r-r_2)\Big]\{r|m,\varepsilon\}\Bigg\} .\end{aligned}$$

Taking the derivitive with respect to a and setting the result equal to zero,

$$\frac{\partial\langle v|m,a,b,\varepsilon\rangle}{\partial a} = K_2-K_3\left\{\sum_{r=0}^{m}\Big[\langle -2(\Phi-a)|r,m,\varepsilon\rangle\Theta(r_1-r)\right.$$

$$+\langle(\Phi-a)^2|r,m,\varepsilon\rangle\delta(r_1-r)(m+m_o)$$

$$\left.-\langle(\Phi-\langle\Phi|r,m,\varepsilon\rangle)^2|r,m,\varepsilon\rangle\delta(r-r_1)\Theta(r_2-r)(m+m_o)\Big]\{r|m,\varepsilon\}\right\} \quad .$$

Since $\langle\Phi|r_1,m,\varepsilon\rangle = a$, the second and third terms cancel.

$$\frac{\partial\langle v|m,a,b,\varepsilon\rangle}{\partial a} = K_2-2K_3\left[\sum_{r=0}^{r_1}[a-\langle\Phi|r,m,\varepsilon\rangle]\{r|m,\varepsilon\}\right]$$

$$= K_2-2K_3\left[\sum_{r=0}^{r_1}\left[a-\frac{r+r_o}{m+m_o}\right]\{r|m,\varepsilon\}\right] = 0,$$

or, in terms of r_1, only

$$\sum_{r=0}^{r_1}(r_1-r)\ \{r|m,\varepsilon\} = \frac{K_2}{2K_3}(m+m_o) \quad . \tag{2}$$

The derivative is continuous, since the left-hand side of this equation is continuous in r_1. A similar equation results when we set the derivative with respect to b equal to zero:

$$\sum_{r=r_2}^{m}(r-r_2)\{r|m,\varepsilon\} = \frac{K_2}{2K_3}(m+m_o) \tag{3}$$

For the specific case where K_2=0, an explicit solution may be obtained: r_1=0, r_2=m, which implies

$$a^* = \frac{r_o}{m+m_o} \qquad b^* = \frac{r_o+m}{m+m_o} \quad .$$

In the general case, a* and b* may be found using Newton's method to solve equations (2) and (3). The results are plotted in Figures 3 and 4. The expected value of the game, given the optimal decision strategy, is then obtained by substituting the solution into equation (1). The expression for $\{r|m,\varepsilon\}$ given by Howard [5] is

$$\{r|m,\varepsilon\} = \frac{m!}{r!(m-r)!} \quad \frac{\Gamma(m_o)}{\Gamma(r_o)\Gamma(m_o-r_o)} \quad \frac{\Gamma(r+r_o)\Gamma(m+m_o-r-r_o)}{\Gamma(m+m_o)}$$

For the special case where $K_2=0$ the numerical results obtained correspond to the results derived by Howard [5, eq. 55]

$$\langle v|m,a,b,K_3=0,\varepsilon\rangle = K_1-K_3\,\frac{r_o(m_o-r_o)}{m_o(m_o+1)(m_o+m)} \quad .$$

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