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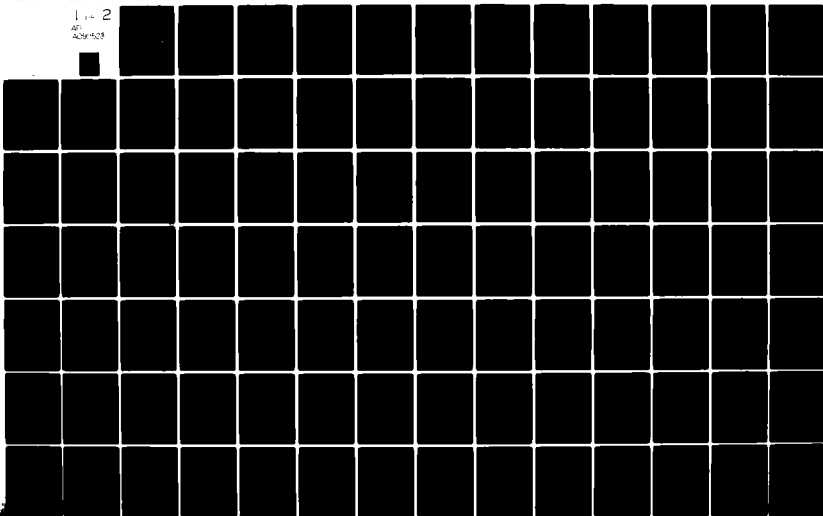
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EFFECTS OF CROSS-CORRELATION INTERFERENCE ON AN OPTICAL
CODE DIVISION MULTIPLE ACCESS COMMUNICATION SYSTEM

BY

GEORGE DENNIS PETERSON

B.S., North Carolina Agricultural and
Technical State University, 1966
M.S., Air Force Institute of Technology, 1971

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THESIS

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1980

Urbana, Illinois

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EFFECTS OF CROSS-CORRELATION INTERFERENCE ON AN OPTICAL
CODE DIVISION MULTIPLE ACCESS COMMUNICATION SYSTEM

George Dennis Peterson, Ph.D.

Department of Electrical Engineering

University of Illinois at Urbana-Champaign, 1980

The effects of cross-correlation interference and intensity fading on the error rate performance of an optical code division multiple-access communication system are investigated. The system model is a multiaccess satellite repeater in which user separation is accomplished by direct sequence modulation. The analysis is for direct detection, optical polarization modulation systems. Generalized expressions for the conditional probability of error for the uplink and downlink channels are derived in terms of the number of active users, signal strength and the normalized cross-correlation between user codes. Numerical results are obtained for systems of two users, for both length 31 and 127 Gold codes. Analysis of a random sampling of the length 31 codes has shown that performance is significantly dependent on the magnitude of the normalized second moment of the cross-correlation between user codes. Further analysis showed that the error probability decreases with increasing code length. The multiuser capabilities of the uplink and downlink channels were shown to differ significantly. It was shown that for K users at a given error rate in the uplink, K^2 users can be supported at the same error rate in the downlink. Fading effects were analyzed using the Nakagami m -distribution as a generalized statistical model of fluctuations in the received optical intensity. The

average probability of error was derived for both correlated and uncorrelated fading in terms of the number of interfering users, the Nakagami fading parameter and the average number of photons counted in a bit interval. The results are valid for negligible background noise and cross-correlation interference. Numerical results are given for both severe and moderate lognormal fading, using the Nakagami m-distribution to approximate the lognormal distribution. It was shown that the downlink performs consistently better than the uplink under all of the fading conditions evaluated. Under lognormal fading, with intensity variances comparable to measured data, it was shown that reasonable error rates for multiple users can be obtained by boosting the signal power in the uplink channel.

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CHAPTER 1

INTRODUCTION

1.1 Multiple Access Satellite Communications

Multiple access satellite systems enable a large number of ground stations to simultaneously transmit their respective information signals (voice, data, teletype, television, etc.) through a satellite repeater. An attractive feature of the satellite system is global accessibility. A single geostationary satellite is visible over approximately one-third of the earth's surface and is theoretically capable of providing global coverage to a large number of widely dispersed ground stations [Withers, 1977]. The information flow in a typical multiple access satellite communication network would be similar to that shown in Figure 1. To exploit the unique properties of global coverage and multiple accessibility inherent in satellite repeaters, the various communication links using them must be separated from each other. There are a variety of modulation techniques used to accomplish user separation. The most common are a basic or hybrid form of the following [Wittman, 1967].

- 1) Frequency division multiple access (FDMA): In FDMA systems, the bandwidth of the satellite repeater is divided into a number of nonoverlapping frequency bands which constitute access channels. Each user is given exclusive usage of an access channel at a predetermined frequency. Network synchronization is not required, but whenever more than one signal is present at the repeater, intermodulation noise develops. As the number of users increases

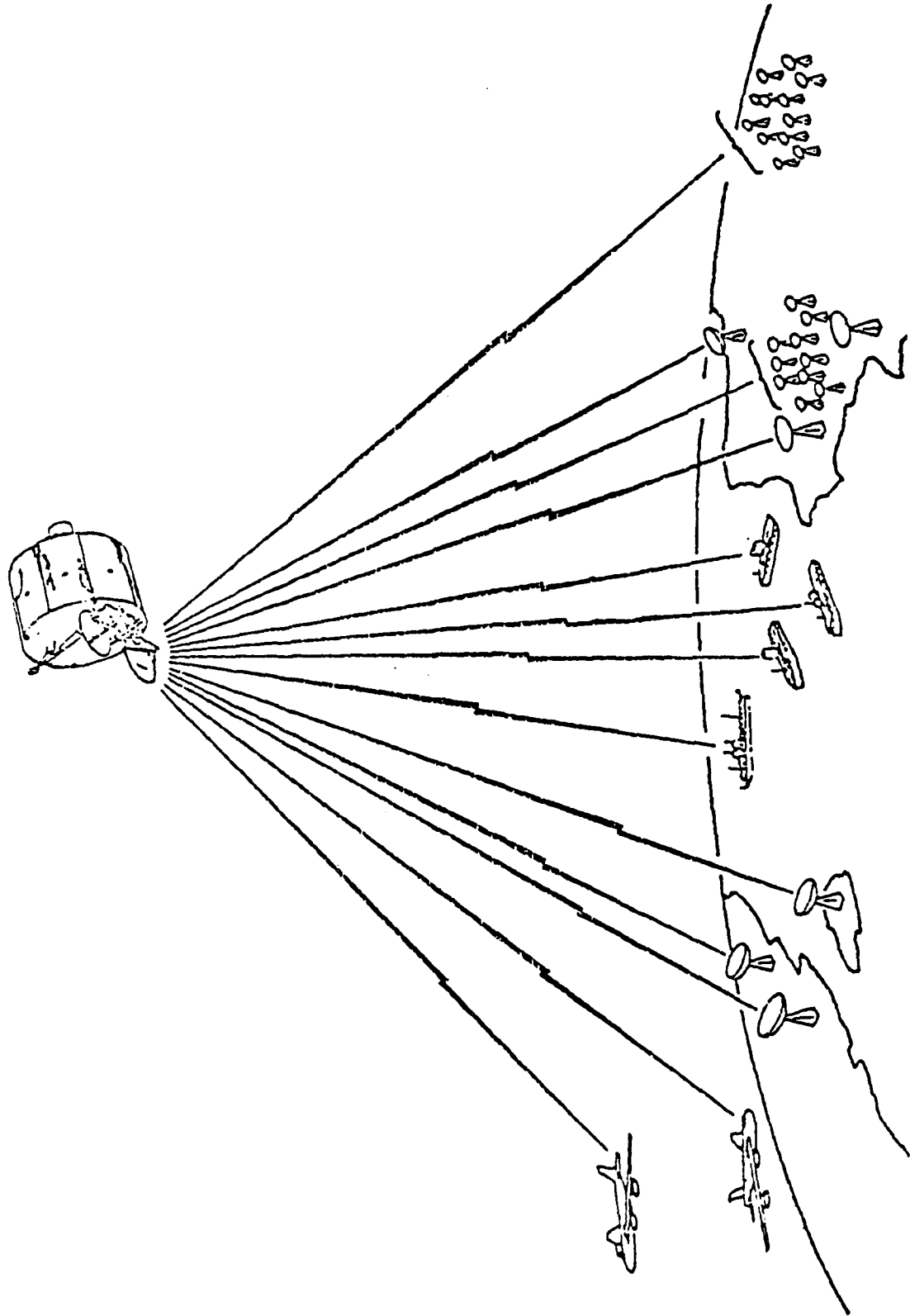


Figure 1. Network configuration of a satellite multiple access communication system.

this noise builds up and ultimately limits the system performance [Schwartz, J., 1966]. FDMA essentially allows each ground station continuous access to a predetermined portion of the available satellite bandwidth.

2) Time division multiple access (TDMA): In TDMA systems, an interval of time (usually called a frame) is divided into a number of discrete time slots (which are considered access channels). Each user is given exclusive usage of the satellite bandwidth, but transmission is restricted to one or more predetermined time slots within a frame. This technique avoids the problem of intermodulation noise since signals from different users are never present in the repeater at any one time. Consequently, TDMA systems require stringent network synchronization to avoid the problem of channel overlap. TDMA allows each user exclusive use of the satellite bandwidth but only for predetermined time slots within a frame [Puente, 1971].

3) Code division multiple access (CDMA): CDMA systems are designed so that multiple users can communicate simultaneously in the same frequency band. The multiple access capability is obtained by encoding the information signal of a particular ground station with a unique periodic code sequence. The most common methods of encoding the information signal are frequency hopping (FH), direct sequence (DS) modulation or hybrid combinations of the two [Dixon, 1975]. In FH systems, such as the Tactical Transmission System (TATS) [Drouilhet, 1969], each user imposes a code-selected frequency hopping pattern on his normal transmission so that cochannel interference occurs only when users are hopped to the same frequency at the same time. To minimize this interference, FH systems employ a large number of frequencies. The TATS modem, for example, has 2^{20}

discrete frequency choices. FH systems also require rapid response frequency synthesizers which introduce an extra measure of complexity to the system.

1.2 Direct Sequence/CDMA

Direct sequence modulation is one of the easiest schemes to implement [Schwartz, J., 1966], [Dixon, 1975]. In a simple implementation, the information signal is multiplied directly by a periodic code sequence and then modulated onto a multiple access carrier using one of the more conventional techniques such as AM or FM carrier modulation. A receiver extracts the desired information by correlation techniques. Each user transmits on the same carrier frequency, shares the same repeater bandwidth but modulates his information signal with a different code sequence. There is no requirement for precise timing or internal network control between the various users. However, interference is generated from the unavoidable cross-correlation between the desired user's code and the code-modulated information signals from all of the other active users.

Cross-correlation interference can be minimized by using specially constructed code sequences which are as nearly random as possible, i.e., the correlation between the encoding sequence and any other code sequence in the system is near zero. An example of such sequences is the family of Gold sequences [Gold, 1967]. If the cross-correlation interference can be held to acceptable levels, multiple users can communicate simultaneously at the same frequency. Note that CDMA does not allow each user exclusive use of the satellite repeater but it does provide each user continuous, uncontrolled access

and full use of the repeater bandwidth.

Both FH and DS modulating schemes are spread spectrum techniques, i.e., the transmitted signal occupies a bandwidth which is spread substantially wider than that of the information being transmitted. Therefore, space laser communication, operating at frequencies where the bandwidth allocations are substantially greater, is an attractive complement for the wideband signals generated in DS and FH systems. The technical feasibility of spaceborne laser communications is well-known for both heterodyne detection systems [McElroy, 1977] and direct detection systems [Ross, 1978]. In fact, the system cited in the reference [Ross, 1978] has progressed into system development with the scheduled launching of the first laser communication-equipped satellite in December 1981. The capability of the laser satellite multiaccess system can also be extended to include systems which provide service to geographically separated cells of several independent users. The multiple access receiver for such a system has been tested by McDonnell Douglas Astronautics Company [Olshan, 1979].

1.3 Outline of the Investigation

In this study, we analyze the effects of cross-correlation interference on the error performance of a direct detection optical CDMA communication system which uses direct sequence modulation. Generalized expressions for the uplink and downlink error probabilities are derived in terms of the number of simultaneous users and the normalized cross-correlation between user codes.

In Chapter 2, we define the multiple access system model, giving a general description of the optics for both the uplink and downlink channels. The specific implementation of the correlation process realized at the optical receivers is also outlined.

In Chapters 3 and 4, we provide the details necessary to develop the error probabilities for both channels. Approximations to the average probability of error are given which are valid for large signals and small values of the cross-correlation interference. Numerical results are given for various signal parameters and multiple users.

In Chapter 5, the analysis is extended to include the system's performance when the optical channel induces slow fading on the received intensity.

Finally in Chapter 6, we conclude with a summary of the pertinent results and recommendations for continued study.

CHAPTER 2

MULTIPLE ACCESS SYSTEM MODEL

In this investigation we have assumed the configuration of a multiaccess satellite repeater as an illustrative model. The links between the transmitter-to-satellite and the satellite-to-receiver are assumed to be direct detection, optical polarization modulation systems and are hereafter referred to as the uplink and the downlink channels, respectively. The basic transmitter and receiver optics are shown in Figure 2. Spatial acquisition, tracking and code synchronization are assumed to be handled by other subsystems and are not considered in this analysis.

2.1 Uplink Channel Model

In the uplink channel, we assume that we have K asynchronous users who transmit a coded data signal to a satellite receiver. As a necessary first step in minimizing interference, we assume that the users are sufficiently separated geographically so that the field-of-view of a particular user, when imaged onto the detecting surfaces of the satellite receiver, does not overlap that of another user. Hence, the incident optical fields do not overlap and we may consider the output of the satellite receiver as resulting from the additive intensities of each independent user. In both the data signal and the code sequence, a logical zero is represented by -1 , and a logical one is represented by $+1$. The data signal $b_i(t)$, from the i -th user, is a sequence of constant

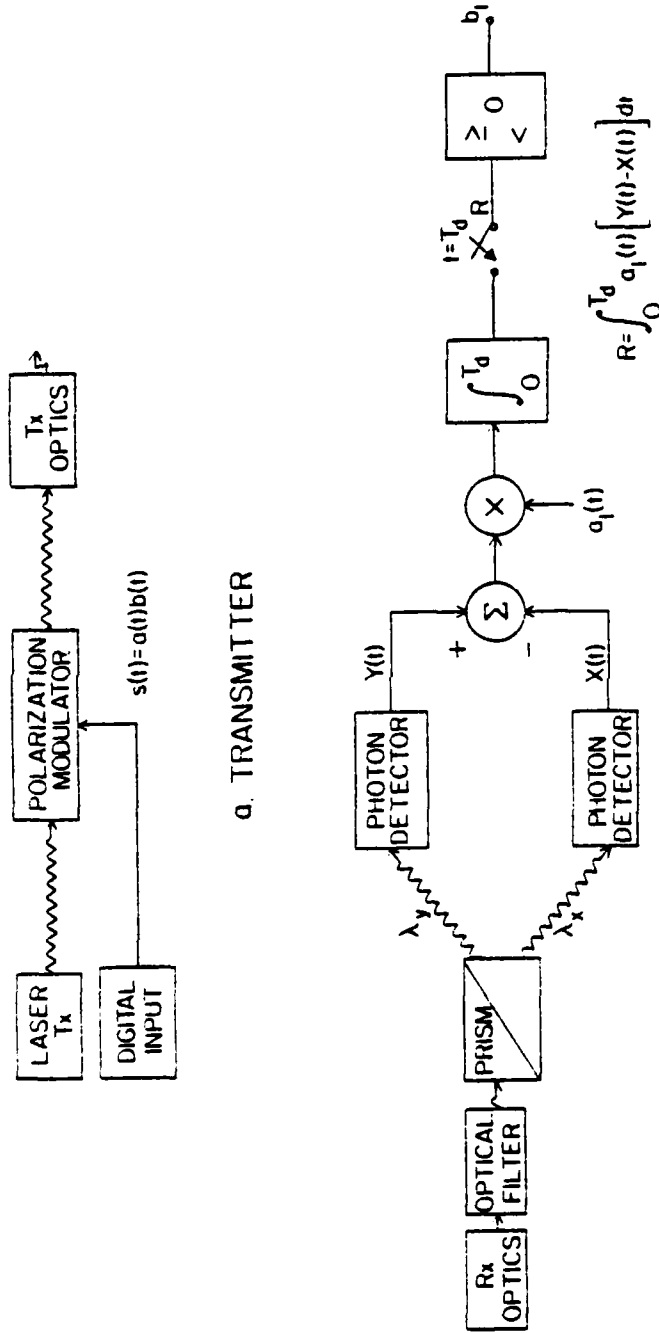


Figure 2. Transmit and receive optics. λ_y and λ_x are the emission rates of photodetector outputs $Y(t)$ and $X(t)$. The code sequence is $a(t)$ and the data sequence is $b(t)$.

amplitude, positive and negative pulses of duration T_d . The i -th user is assigned an access code $a_i(t)$, which consists of a periodic sequence of constant amplitude, positive and negative pulses of duration T_c . We refer to the time interval T_c as a "chip" so as to distinguish it from the "bit" which is usually reserved for the time interval of the data signal. The code consists of L chips and we require that the length of the data bit be given by $T_d = LT_c$. Generally, the code length is on the order of 10^2 to 10^5 chips.

The input $S_i(t)$ to the optical modulator of the i -th user is simply the data sequence $b_i(t)$ multiplied by the access code $a_i(t)$,

$$S_i(t) = a_i(t - \tau_i) b_i(t - \tau_i) \quad (2.1)$$

where τ_i is a random delay which accounts for the fact that the users are independent and not time-synchronous. The τ_i are uniformly distributed over $(0, T_d)$. The optical modulator, depicted in Figure 2a, switches the laser output between right- and left-circular polarization depending on the polarity of the input. The output intensity of each user is constant. Therefore, if the system has K active users, the signal at the receiver consists of K , equal intensity, circularly polarized laser beams.

2.2 Downlink Channel Model

The satellite repeater provides simultaneous user access by broadcasting a multiplexed output signal which is the asynchronous addition of all of the active users on the uplink. A particular downlink receiver extracts the desired signal from the satellite output by correlating the received signal with a synchronous copy of the appropriate multiple access code. The transmitted signal on the downlink differs considerably from that on the uplink in that the input $S(t)$ to the optical modulator of the satellite takes the form

$$S(t) = \sum_{i=1}^K a_i(t-\tau_i) \hat{b}_i(t-\tau_i) \quad (2.2)$$

where K is the number of simultaneous users and $\hat{b}_i(t)$ is an estimate of $b_i(t)$ which accounts for probable errors in signal processing in the satellite. $S(t)$ is the addition of the coded data signals from K independent users. As such, the magnitude and phase (polarity) of $S(t)$ depend on the number of simultaneous users. The satellite encodes this information by transmitting a laser beam which is both polarization and intensity modulated. The polarization, right- or left-circular, depends on the polarity of $S(t)$ and the intensity depends on the magnitude of $S(t)$.

2.3 Detection Process

The optical receivers on both the uplink and downlink channels perform the same detection process. The incident intensity is demodulated by spatially separating the polarization components so that light intensity of right-circular polarization strikes one photodetector and light intensity of left-circular polarization strikes another. This is shown schematically by the separation of the upper and lower photodetectors in Figure 2b. The photon counts from the upper and lower photodetectors are subtracted and the difference is then correlated with a synchronous copy of the appropriate multiple access code. This demodulation process closely parallels the PCM/Polarization Modulation technique described in the reference [Pratt, 1969], except that we require post-correlation with the multiple access code to extract the desired user. The receiver makes a decision based on whether the correlated difference is positive or negative. Other active users in the channel introduce left- and right-polarization components which interfere with the desired signal. This interference arises from the unavoidable cross-correlation between the code of the desired signal and the code-modulated information signals of the interferers. The correlation process generates interference terms of the form

$$\int_0^{T_d} a_1(t)a_i(t-\tau_i)b_i(t-\tau_i)dt \quad (2.3)$$

where a_1 is the access code of the desired signal, the product $a_i b_i$ is the code-modulated signal of the i -th interferer and T_d is the bit interval.

However, if the cross-correlation between the desired code and the code-modulated data sequences of the other users is uniformly small, this interference is negligible when averaged over the bit interval.

The optical receivers are subject to both thermal and shot noise. For low intensity, direct-detection receivers, the shot noise predominates and the thermal noise can be neglected. The shot noise is generated by the incident optical fields, background radiation and detector dark currents. The background radiation, which is generally unpolarized, is therefore equally distributed among the photodetectors. The average detector dark current is considered to be the same in both detectors. Therefore, without loss of generality, we consider the receivers to be shot noise-limited, with additive background noise which is the cumulative effect of background radiation and detector dark current.

For shot noise-limited detection, the photoelectron emissions from the detector surfaces are characterized by a Poisson process [Mandel, 1963]. If we let $I(t)$ be the incident optical intensity, then the probability of obtaining $n = 1, 2, \dots$, photons in an interval $(t, t+\tau)$ is given by

$$P(n) = \left[\eta/hf \int_t^{t+\tau} I(x) dx \right]^n (n!)^{-1} \exp \left[-\eta/hf \int_t^{t+\tau} I(x) dx \right] \quad (2.4)$$

where η is the quantum efficiency of the photodetectors, h is Planck's constant and f is the optical frequency.

If $I(t)$ is a known signal (deterministic), (2.4) is the well-known Poisson counting model. If $I(t)$ is unknown (random), (2.4) is conditionally Poisson and the counting probability is obtained, at least in principle, by taking the expectation over the statistics of $I(t)$. The result is normally not Poisson distributed.

CHAPTER 3

UPLINK CHANNEL ANALYSIS

3.1 Conditional Probability of Error

To determine the effects of interfering users on the detection error, we choose $b_1(t)$ as the desired data signal and the $K-1$ remaining data signals as the interfering signals. The receiver is synchronized to $a_1(t)$, the desired code. Since all time delays are relative to the desired code, we let $\tau_1 = 0$. The receiver is shown in Figure 2b. If we let $Y(t)$ be the output of the left-circular polarization detector and $X(t)$ be the output of the right-circular polarization detector, then the receiver output sampled at $t=T_d$ is

$$R = \int_0^{T_d} a_1(t)[Y(t)-X(t)]dt \quad (3.1)$$

where $Y(t)$ and $X(t)$ are independent Poisson processes.

During the bit interval, $a_1(t)$ takes on the values $+1$ and -1 . If we define

$$a^+ = \{ t : a_1(t) = +1, 0 \leq t \leq T_d \} \quad (3.2)$$

and

$$a^- = \{ t : a_1(t) = -1, 0 \leq t \leq T_d \} \quad (3.3)$$

we can separate (3.1) into integration over the positive and negative intervals of $a_1(t)$

$$R = R^+ - R^- \quad (3.4)$$

where

$$R^+ = \int_{a^+} Y(t)dt + \int_{a^-} X(t)dt \quad (3.5)$$

and

$$R^- = \int_{a^-} Y(t)dt + \int_{a^+} X(t)dt \quad (3.5)$$

The probability distribution of R , which is the difference of independent Poisson random variables, is completely defined if the mean values of R^+ and R^- are known [Pratt, 1969]. But since the mean and variance of Poisson random variables are equal, we have shown in Appendix A that the probability distribution of R can also be expressed as

$$P_R(R=n) = \exp[-\text{Var}(R)] \{ (\text{Var}(R) - E[R]) / (\text{Var}(R) + E[R]) \}^{-n/2} \\ \cdot I_{|n|} \{ \text{Var}(R)^2 - E[R]^2 \} \quad (3.7)$$

where $I_n(\cdot)$ is the modified Bessel function of order n and $n = 0, 1, 2, \dots$, is the number of photons counted.

To analyze the effects of interfering users on the error probability, we first derive the probability of error conditioned on the cross-correlation interference given in (2.3). If $b_1 = +1$ and $b_1 = -1$ are equally likely, the conditional probability of error is given by

$$P(E | \underline{C}) = \\ 1/2 \text{Pr}(R > 0 | \underline{C}, b_1 = -1) + 1/2 \text{Pr}(R < 0 | \underline{C}, b_1 = +1) \\ + 1/4 \text{Pr}(R = 0 | \underline{C}, b_1 = -1) \\ + 1/4 \text{Pr}(R = 0 | \underline{C}, b_1 = +1) \quad (3.8)$$

where

$$\underline{C} = \{ C_2, C_3, \dots, C_K \} \quad (3.9)$$

is the normalized cross-correlation vector for which the i -th element is given by

$$C_i = 1/T_d \int_0^{T_d} a_1(t) a_i(t-\tau_i) b_i(t-\tau_i) dt \quad (3.10)$$

The last two terms in (3.8) account for equality in the test, in which case a random choice is made with probability 1/2 of being correct. The average probability of error is then determined by averaging the conditional probability of error over the probability distribution of the cross-correlation C_i .

3.2 General Solution

The general solution to (3.8) is determined in a straightforward manner by obtaining the mean and variance of R in terms of the cross-correlation C_i . The mathematical details are worked out in Appendix B; the results are

$$E[R | Q] = b_1 U_1 + \sum_{i=2}^K U_i C_i$$

and

$$\text{Var}(R) = 2U_b + \sum_{i=1}^K U_i \quad (3.11)$$

where U_1 , U_i and U_b are the average number of photons collected in the bit interval due to the desired signal, interference, and background noise, respectively. By substituting (3.7) into (3.8), we get an expression that can be solved in terms of the modified Bessel functions and the Marcum's Q function [Marcum, 1960]. In section B.3 of Appendix B we have shown that the general result for K users is

$$P(E | \underline{C}) = 1/2[1 - Q(b,a) + Q(d,c)] \\ + 1/4 \exp[-(2U_b + \sum_{i=1}^K U_i)] [I_0(ab) - I_0(cd)] \quad (3.12)$$

where

$$a = [2U_b + \sum_{i=2}^K U_i (1 - C_i)]^{1/2} \\ b = [2U_b + 2U_1 + \sum_{i=2}^K U_i (1 + C_i)]^{1/2} \\ c = [2U_b + 2U_1 + \sum_{i=2}^K U_i (1 - C_i)]^{1/2} \\ d = [2U_b + \sum_{i=2}^K U_i (1 + C_i)]^{1/2} \quad (3.13)$$

and U_1, U_i and U_b are, as we previously defined, the average number of signal, interference and background noise photons collected in the bit interval T_d . The Marcum's Q-function is defined by the series expression

$$Q(a,b) = \exp[-(a^2 + b^2)/2] \sum_{n=0}^{\infty} (a/b)^n I_n(ab) \quad (3.14)$$

For no interference (single user), the probability of error reduces to

$$P(E) = 1/2 [1 - Q(b,a) + Q(a,b)] \quad (3.15)$$

where

$$a = [2U_b]^{1/2}$$

and

$$b = [2(U_b + U_1)]^{1/2} \quad (3.16)$$

This result, as we expected, is identical to that given in the reference [Pratt, 1969] for shot noise limited, PCM/Polarization modulation systems.

3.3 Special Case: No Cross-Correlation Interference

As we previously stated, interference is minimized by using codes which have low cross-correlation. The irreducible limit to cross-correlation interference can be evaluated by setting $C_1 = 0$ in the error expression. For the case of K independent users, the expression reduces to

$$P(E) = 1/2[1 - Q(b,a) + Q(a,b)] \quad (3.17)$$

where

$$a = [2U_b + \sum_{i=2}^K U_i]^{1/2}$$

and

$$b = [2U_b + 2U_1 + \sum_{i=2}^K U_i]^{1/2} \quad (3.18)$$

By comparing this result to (3.15) for the single user case, we note that even when we employ codes in which $C = 0$, we still encounter significant interference noise. In fact each interfering user introduces an equivalent background count of $U_i/2$.

3.4 Probability of Error: Large Signal Approximation

Generally, in the design process, we are concerned with obtaining a reasonable estimate of the minimum achievable error rate. We can derive this estimate by assuming $C = 0$ and large signal intensities ($U_1 \gg 1$). Under these conditions, the background noise can be neglected and we can use the asymptotic form of the Q-function [Schwartz, M., 1966] to approximate the error probability. For the case where

$$U_1 = U_2 = \dots = U_K = U$$

the error probability is approximately

$$P(E) \approx 1/2 \operatorname{erfc}\{\sqrt{2U} / (\sqrt{K+1} + \sqrt{K-1})\} \quad (3.19)$$

or for large K

$$P(E) \approx 1/2 \operatorname{erfc}(U/\sqrt{2KU}) \quad (3.20)$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function given by

$$\operatorname{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2) du \quad (3.21)$$

This approximation to the error probability is useful when making a tradeoff between the maximum number of active users and the required signal intensity for a specified error rate. Compare this result with phase-shift keying (PSK), a modulation technique similar to the polarization modulation employed here. The probability of error for synchronous detection of a PSK binary sequence in Gaussian noise [Schwartz, 1980] is

$$P(E) = 1/2 \operatorname{erfc}(A/\sqrt{2N}) \quad (3.22)$$

where A is the signal amplitude and N is the total noise power. By comparison, we see that, in the limit as the number of users and the signal intensities get very large, the shot noise effects tend toward a Gaussian process and accordingly, the probability of error takes the same form as that for PSK signalling in Gaussian noise, with the signal amplitude similar to U , the average number of signal photons and N , the total noise power similar to KU , the sum of the variances of the K independent users. Notice this equivalent signal-to-noise ratio, is inversely proportional to K , the number of users.

In Table 1, we give the generalized error expression for the uplink channel, with some special cases provided for comparison.

TABLE 1

SPECIAL CASES OF THE PROBABILITY OF ERROR EXPRESSION FOR THE UPLINK CHANNEL.

GENERALIZED ERROR EXPRESSION

$$P(E|C) = 1/2(1 - 0(b,a) + 0(c,a) + 1/4 \sum_{i=1}^K u_i(1+c_i) - I_0(c))$$

GENERAL

$$a = 12u_b + \sum_{i=2}^K u_i(1-c_i)^{1/2}$$

$$b = 12u_b + 2u_1 + \sum_{i=2}^K u_i(1+c_i)^{1/2}$$

$$c = 12u_b + 2u_1 + \sum_{i=2}^K u_i(1-c_i)^{1/2}$$

$$d = 12u_b + \sum_{i=2}^K u_i(1+c_i)^{1/2}$$

EQUAL POWER USERS

$$a = 12u_b + u \sum_{i=2}^K (1-c_i)^{1/2}$$

$$b = 12u_b + 2u + u \sum_{i=2}^K (1+c_i)^{1/2}$$

$$c = 12u_b + 2u + u \sum_{i=2}^K (1-c_i)^{1/2}$$

$$d = 12u_b + u \sum_{i=2}^K (1+c_i)^{1/2}$$

CROSS-CORRELATION, C=0

$$a = 12u_b + \sum_{i=2}^K u_i^{1/2}$$

$$b = 12u_b + 2u_1 + \sum_{i=2}^K u_i^{1/2}$$

$$c = b$$

$$d = a$$

LARGE SIGNAL APPROXIMATION

$$P(E) \approx 1/2 \operatorname{erfc}(\sqrt{a}) / (\sqrt{a} + \sqrt{a-1})$$

LARGE SIGNAL, LARGE USER APPROXIMATION

$$P(E) \approx 1/2 \operatorname{erfc}(0.6/\sqrt{a})$$

u_1 : desired signal count

u_b : interference count

u_i : background noise count

c_i : cross-correlation

3.5 Numerical Results

Gardner and Orr [Gardner, 1979] and Hanlon and Gardner [Hanlon, 1979] obtained good single-term approximations to the limiting error probability for fading, analog CDMA systems in terms of the normalized second moment of the cross-correlation. Since the shot noise behavior of optical detection is similar to the fading channel, we expected that the error performance of optical systems was also highly dependent on the magnitude of the normalized second moment. To test this conjecture, we tabulated the probability distribution for the cross-correlation values for a random selection of 10, length 31 Gold codes. We computed the error probabilities using (3.12) for the two code pairs having the smallest and largest second moment. This provided a best and worse case for comparison. The probability mass functions and density for the cross-correlation values are derived in the reference [Hanlon, R. C., Peterson, G. D., and Gardner, C. S., 1980]. The mass functions and density for the two code pairs having the smallest and largest second moment, respectively are given in Figure 3. However, in a practical multiaccess system a user will not have a priori knowledge of the other user codes. Therefore, an exact analysis for a particular pair of codes is of limited utility from a system's point of view. We would primarily be concerned with the average performance over the entire set of code sequences. Therefore, we also show the average error performance over all possible length 31 Gold code pair combinations. The results are shown in Figure 4. As expected, the best case is the code pair with the smallest second moment and the worse case is the code pair with the largest second moment. From this preliminary analysis, it appears that a useful technique for assigning user codes is to consider the magnitude of the second moment of the various

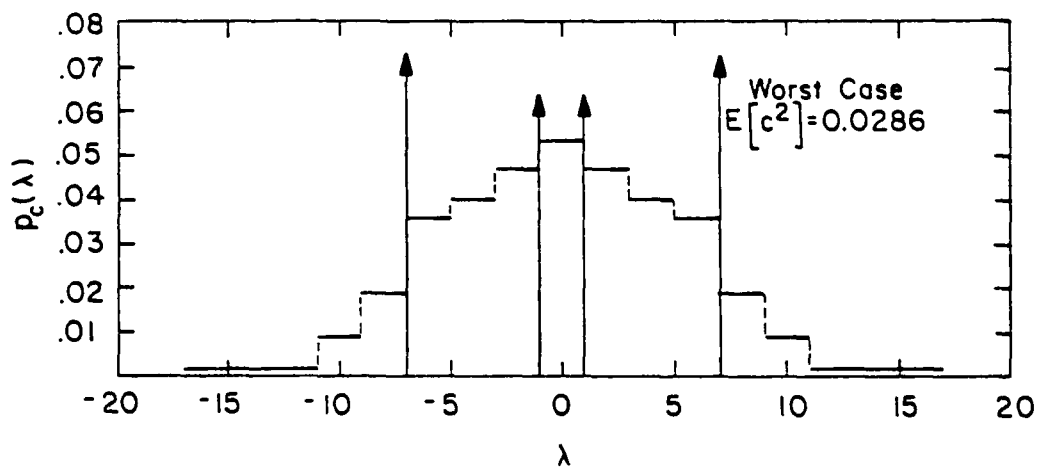
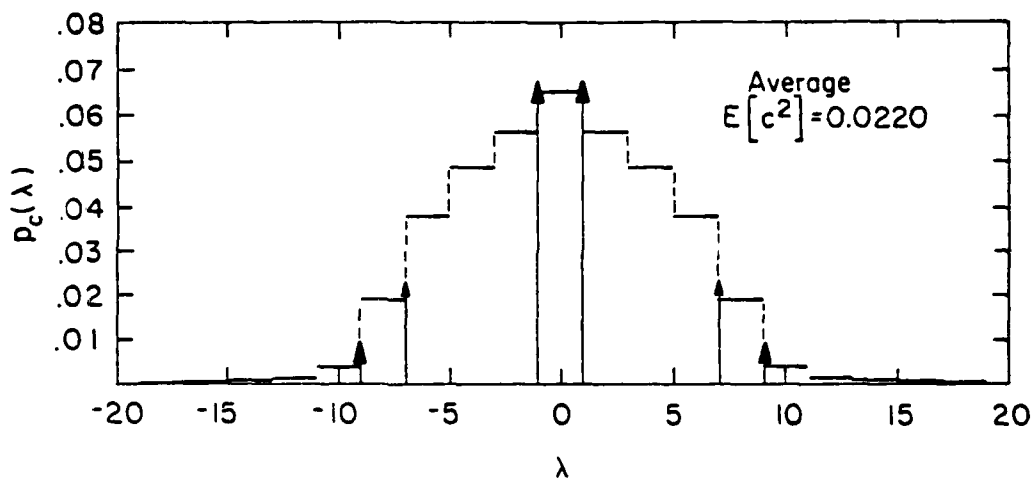
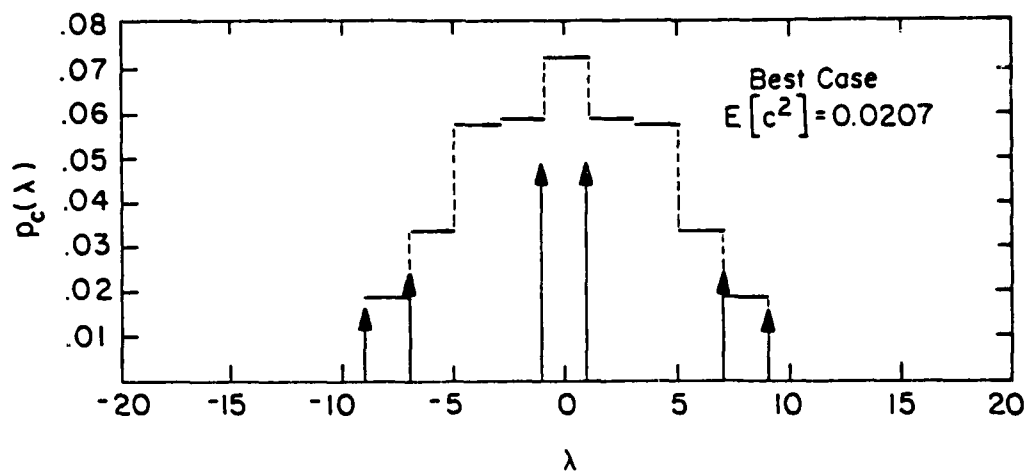


Figure 3. Probability mass functions and densities for cross-correlation values for the two best and worst cases, length 31 Gold code pairs.

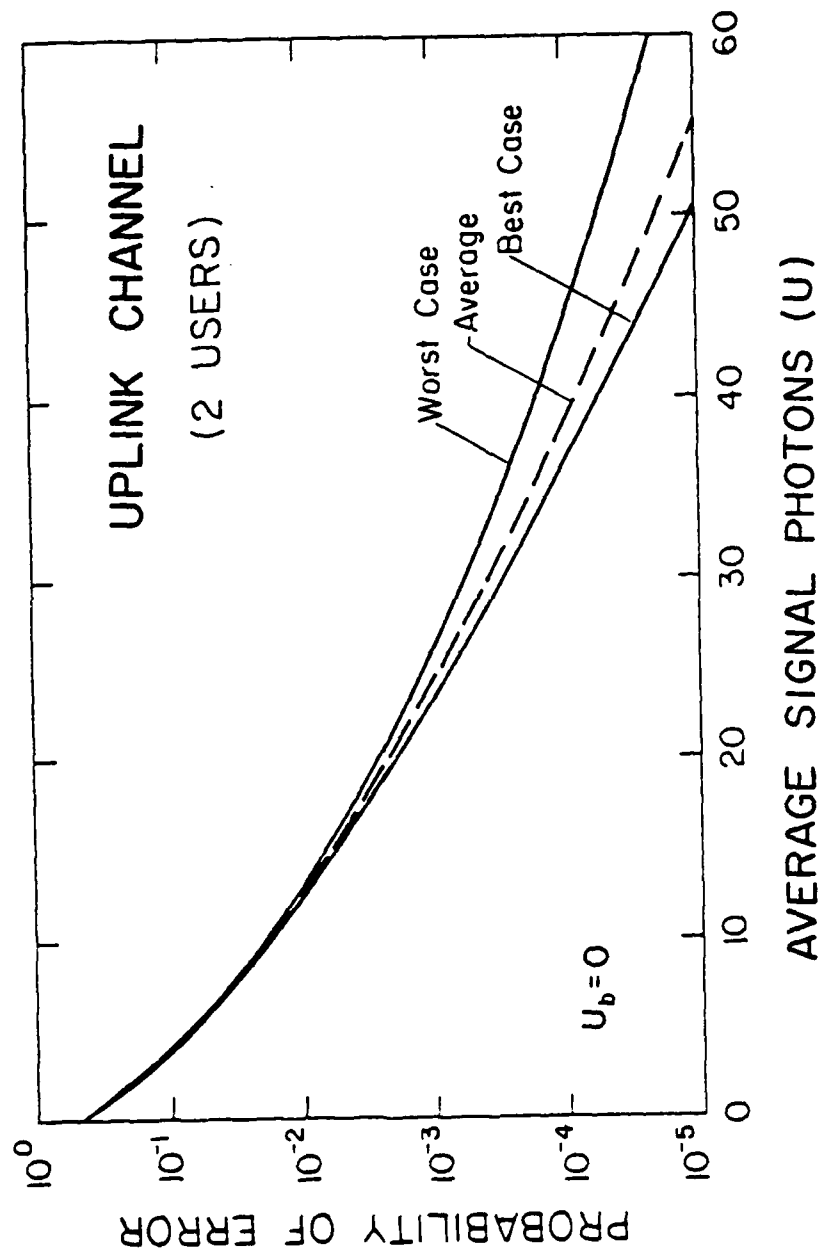


Figure 4. Probability of error vs. average signal photons for the uplink channel. Best and worse case plots are for length 31 Gold code pairs having smallest and largest normalized second moment, respectively. Average plot represents average performance over all length 31 Gold code pairs.

cross-correlations between users. Assign high duty cycle or high priority users, sequences which generate small second moments, and reserve the code sequences which generate high second moments to low duty cycle users.

The limiting error rate, as a function of the number of active users, was obtained by evaluating the error expression (3.17) for the ideal case in which the background noise and cross-correlation were set to zero. Results for systems with up to 10 users are given in Figure 5 for the uplink channel. This plot shows the degrading effects of increasing the number of users on the uplink channel. Each additional interferer, as we previously noted, has the effect of adding an equivalent photon count equal to $U/2$ to the interference noise, even though the cross-correlation between users is negligible. Therefore, as the number of users increases, much larger signal intensities are required to maintain satisfactory error rates. The curve also gives a reasonable estimate of the minimum achievable error rates for a given number of simultaneous users. It also gives a relative indication of how much the signal count has to be boosted to maintain the error rate goals as the system capacity increases. A significant point is that the error rate does not saturate at some fixed value. It appears that if the users are capable of continually boosting their transmitted power, the error rate can be continually lowered. This is an advantage the uplink has over the downlink because the satellite-to-user downlink is power-limited to the maximum output of the satellite transmitter and can not continually boost its output power to achieve lower error rates.

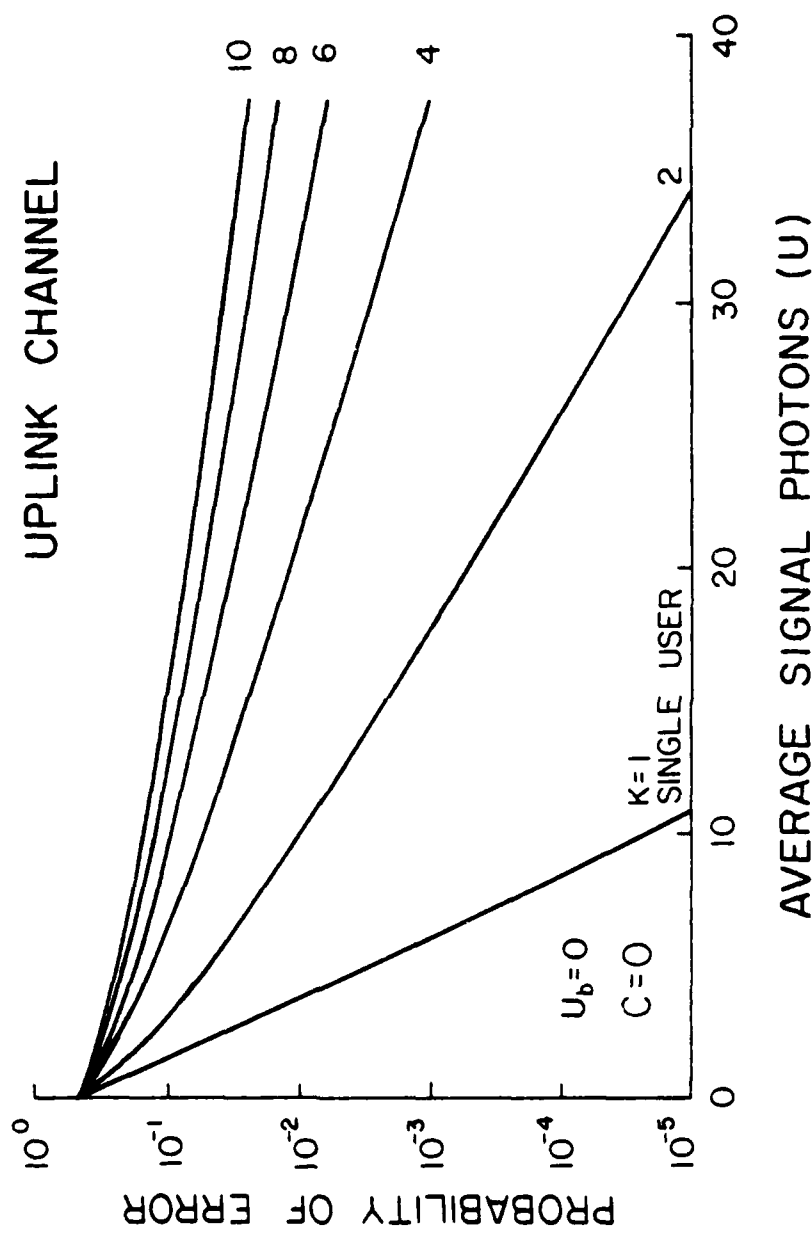


Figure 5. Probability of error vs. average signal photons for the uplink channel. Curves show the limiting error rate for up to 10 simultaneous users. K is the number of users.

As the code sequences get longer, their properties more closely approach the properties of the random codes. Thus, we expect the longer code sequences, with correspondingly low cross-correlations, to improve system performance. Figure 6 shows the effects of increasing code length on the probability of error for a system of two users. In this curve the background noise is set to zero to highlight the effects of increasing the code length. The plots are for code lengths 31, 127, and infinite. The curve for infinite code lengths corresponds to setting the cross-correlation to zero, which is the case for purely random codes. As the code length increases, the error performance correspondingly improves and error rates close to the irreducible error rate (that for infinite code lengths) can be achieved using moderately long code sequences. For instance, the performance of length 127 code sequences is very close to that for infinite codes.

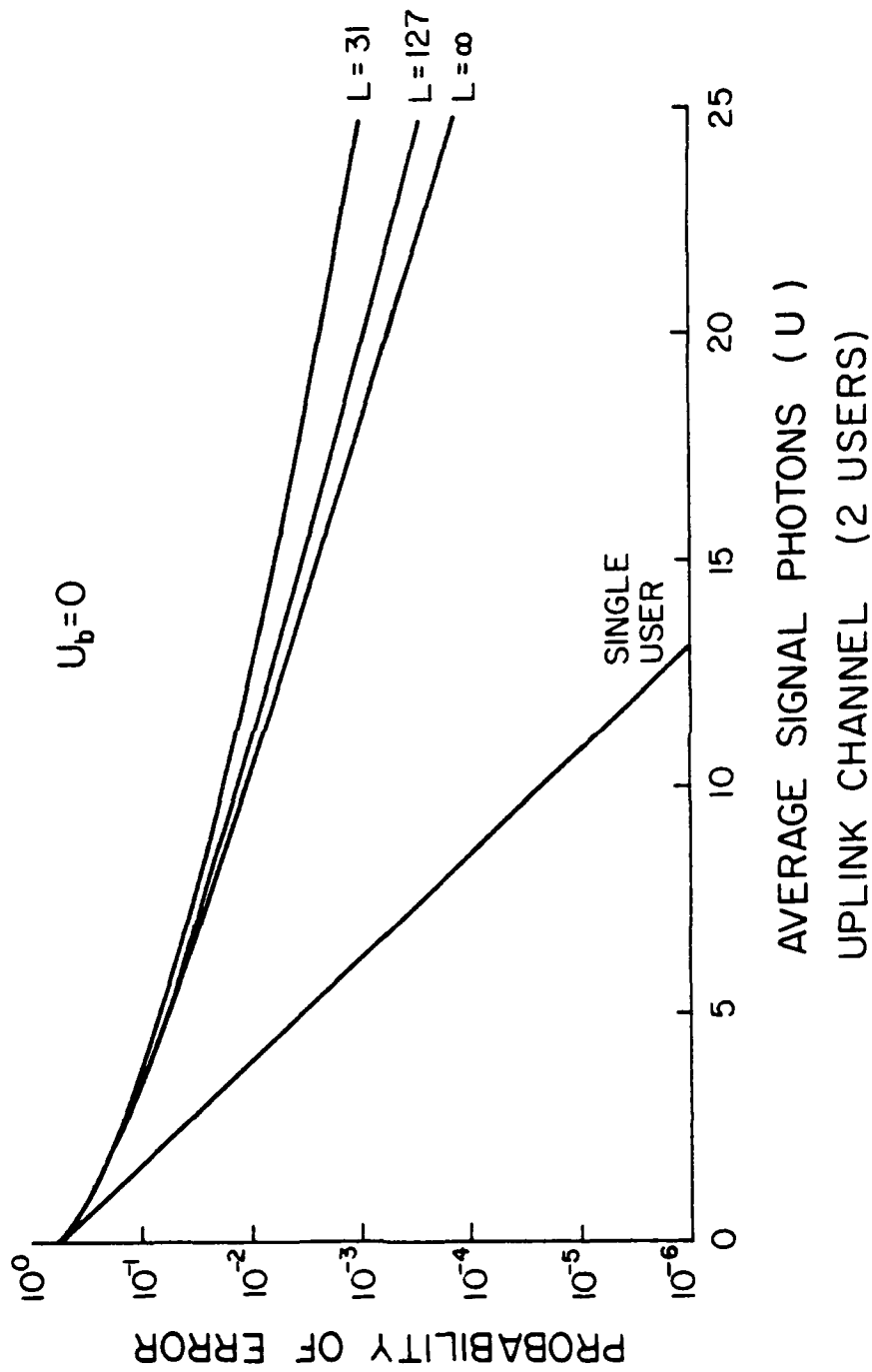


Figure 6. Probability of error vs. average signal photons for the uplink channel. Curves show the effects of code length L on the error rate.

CHAPTER 4

DOWNLINK CHANNEL ANALYSIS

4.1 Conditional Probability of Error

Since the receivers on the downlink channel perform basically the same demodulation process as is performed in the uplink channel, the conditional error probability is obtained from the error expression (3.8) and determined by substituting the mean and variance of the receiver output resulting from the downlink modulation input $S(t)$ defined in (2.2). From the derivation in Appendix B, the conditional mean and the variance are

$$E[R | \hat{C}] = b_1 U + U \sum_{i=2}^K \hat{C}_i \quad (4.1)$$

where

$$\hat{C} = \{ \hat{C}_2, \hat{C}_3, \dots, \hat{C}_K \} \quad (4.2)$$

and

$$\text{Var}(R) = 2U_b + K'U \quad (4.3)$$

where \hat{C}_i is defined by

$$\hat{C}_i = 1/T_d \int_0^{T_d} a_1(t) a_i(t-\tau_i) \hat{b}_i(t-\tau_i) dt \quad (4.4)$$

and

$$K' = 1/T_d \int_0^{T_d} \left| \sum_{i=1}^K a_i(t-\tau_i) \hat{b}_i(t-\tau_i) \right| dt \quad (4.5)$$

For K users the conditional error probability from Appendix B is

$$\begin{aligned} P(E | \underline{\hat{C}}) &= 1/2 [1 - Q(b', a') + Q(d', c')] \\ &+ 1/4 \exp[-(2U_b + K'_+ U)] I_0(a' b') \\ &- 1/4 \exp[-(2U_b + K'_- U)] I_0(c' d') \end{aligned} \quad (4.6)$$

where

$$\begin{aligned} a' &= [2U_b + U(K'_+ - K) + U \sum_{i=2}^K (1 - \hat{C}_i)]^{1/2} \\ b' &= [2U_b + 2U + U(K'_+ - K) + U \sum_{i=2}^K (1 + \hat{C}_i)]^{1/2} \end{aligned}$$

$$c' = [2U_b + 2U + U(K'_+ + K) + U \sum_{i=2}^K (1 - \hat{C}_i)]^{1/2}$$

$$d' = [2U_b + U(K'_- + K) + U \sum_{i=2}^K (1 + \hat{C}_i)]^{1/2} \quad (4.7)$$

The designations K'_+ and K'_- denote the conditional values of K' when $\hat{b}_1(t) = +1$ and $\hat{b}_1(t) = -1$, respectively.

In the case of no interfering signals, the term K' is

$$K' = 1/T_d \int_0^{T_d} |a_1(t - \tau_1) b_1(t - \tau_1)| dt \quad (4.8)$$

K' reduces to one since the product $|a_1(t - \tau_1) \hat{b}_1(t - \tau_1)|$ is identically one for all values of $a_1(t - \tau_1) \hat{b}_1(t - \tau_1)$. The error probability for this case is

$$P(E) = 1/2[1 - Q(b, a) + Q(a, b)] \quad (4.9)$$

where the arguments a and b of the Q -function are the same as those given (3.16). This result, as we expected, is identical to the uplink, single user case.

4.2 Special Case: Single Interferer

For an arbitrary number of users, K' defines a complex statistical relationship between the code sequences $a_i(t)$ and the cross-correlation C_i ; however, we can obtain simplified expressions for K' under some limiting conditions. For example, for two users we have shown in Appendix C that K' reduces to the particularly simple form:

$$K' = 1 + \hat{b}_1 \hat{C}_2 \quad (4.10)$$

The resulting error probability is

$$\begin{aligned} P(E|\hat{C}_2) &= 1/2[1 - Q(b', a') + Q(a', c')] \\ &\quad + 1/4 \exp\{-[2U_b + (1 + \hat{C}_2)U]\} I_0(a'b') \\ &\quad - 1/4 \exp\{-[2U_b + (1 - \hat{C}_2)U]\} I_0(a'c') \end{aligned} \quad (4.11)$$

where

$$a' = [2U_b]^{1/2}$$

$$b' = [2U_b + 2U(1 + \hat{C}_2)]^{1/2}$$

$$c' = [2U_b + 2U(1 - \hat{C}_2)]^{1/2} \quad (4.12)$$

Now to obtain the irreducible limit to cross-correlation interference effects for two users, we evaluate the expression above for $\hat{C}_2 = 0$. The error probability reduces to the same result obtained for the single user case:

$$P(E) = 1/2[1 - Q(b,a) + Q(a,b)] \quad (4.13)$$

where the arguments, a and b, of the Q-function are given by (3.16). This implies that two users employing code sequences in which $\hat{C}_2 = 0$, can theoretically communicate in the downlink channel without interfering with each other. However, this is a fortuitous consequence of the modulation technique and does not hold for more than two users.

For $\hat{C}_1 = 0$, K' is no longer coupled to the cross-correlation values, as is shown explicitly for the single user case, and may be treated as a random variable which is simply the integral over the absolute value of the sum of the product terms $a_i b_i$. The probability of error for this case is

$$P(E) = 1/2[1 - Q(b',a') + Q(a',b')] \quad (4.14)$$

where

$$\begin{aligned}
 a' &= [2U_b + U(K' - 1)]^{1/2} \\
 b' &= [2U_b + U(K' + 1)]^{1/2}
 \end{aligned}
 \tag{4.15}$$

4.3 Probability of Error: Large Signal Approximation

We can obtain an estimate of the minimum achievable error rate for the downlink channel by setting, as before, $\underline{c} = 0$ and assuming large signal intensities. Under these assumptions the background noise is negligible, and K' is independent of the cross-correlation. Using the asymptotic form of the Q-function we can approximate the error probability by

$$P(E) \approx 1/2 \operatorname{erfc} \{ \sqrt{2U} / (\sqrt{K'+1} + \sqrt{K'-1}) \}
 \tag{4.16}$$

or for large K'

$$P(E) \approx 1/2 \operatorname{erfc} \{ U / \sqrt{2K'U} \}
 \tag{4.17}$$

K' simplifies considerably when evaluated under the assumption that the cross-correlation is zero, because as we previously stated for this limiting case, we can treat K' as an independent random variable. The product terms in (4.5) are mutually independent, therefore, the general expression for K' has the form of a time average of the absolute value of the sum of finitely many independent random variables which take on the values $+1$ and -1 . Assuming that the system is stationary, we can model the summation in the integrand of K' as resulting from the chance outcome of a random walk process [Papoulis, 1965], and take K' as the statistical average of the absolute value of the path length of the random walk after K random steps have been taken, where K represents the number of simultaneous users in the system. The details of this derivation are contained in Appendix C; the results are

$$K' = K \binom{K}{K/2} (1/2)^K \quad \text{for } K \text{ even} \quad (4.18)$$

and

$$K' = K \binom{K}{(K-1)/2} (1/2)^{K-1} \quad \text{for } K \text{ odd} \quad (4.19)$$

For a large number of users, we can apply Stirling's approximation [Abramowitz, 1965], to the binomial coefficients above and simplify K' to

$$K' = \sqrt{2K/\pi} \quad (4.20)$$

Comparing the large signal approximation of the error probability for the uplink channel with that for the downlink, we note that both systems degrade in proportion to the increase in users; but since K' increases in proportion to $K^{1/2}$, the downlink channel will invariably perform significantly better; particularly for large K .

In Table 2, we give the generalized error expression for the downlink channel, with some special cases provided for comparison.

TABLE 2

SPECIAL CASES OF THE PROBABILITY OF ERROR EXPRESSION FOR THE DOWNLINK CHANNEL

GENERALIZED ERROR EXPRESSION

$$P(E|\hat{c}) = 1/2 [1 - Q(b', a') + Q(d', c')] + 1/4 \exp[-(2u_b + \kappa_+^2 u)] I_0(a' b')$$

$$- 1/4 \exp[-(2u_b + \kappa_-^2 u)] I_0(c' d')$$

GENERAL

$$a' = [2u_b + u(\kappa_+^2 - \kappa) + u \sum_{i=2}^K (1 - \hat{c}_i)]^{1/2}$$

$$b' = [2u_b + 2u + u(\kappa_+^2 - \kappa) + u \sum_{i=2}^K (1 + \hat{c}_i)]^{1/2}$$

$$c' = [2u_b + 2u + u(\kappa_-^2 + \kappa) + u \sum_{i=2}^K (1 - \hat{c}_i)]^{1/2}$$

$$d' = [2u_b + u(\kappa_-^2 + \kappa) + u \sum_{i=2}^K (1 + \hat{c}_i)]^{1/2}$$

CROSS-CORRELATION, $\hat{c}=0$

$$a' = [2u_b + u(\kappa^2 - 1)]^{1/2}$$

$$b' = [2u_b + u(\kappa^2 + 1)]^{1/2}$$

$$c' = b'$$

$$d' = a'$$

LARGE SIGNAL APPROXIMATION

$$P(E) \approx 1/2 \operatorname{erfc}(1/\sqrt{2}) / (\sqrt{\kappa^2 + 1} + \sqrt{\kappa^2 - 1})$$

LARGE SIGNAL, LARGE USER APPROXIMATION

$$P(E) \approx 1/2 \operatorname{erfc}(u/\sqrt{2\kappa}u)$$

u: signal count

u_b : background noise count

\hat{c}_i : cross-correlation

4.4 Numerical Results

To further test the conjecture that the magnitude of the second moment of the cross-correlation values is a significant factor in determining the error rate performance, we evaluated (4.11) using the same length 31, Gold code pairs that were used in the uplink channel analysis. The probability of error was evaluated for the two code pairs having the respective largest and smallest second moment. The results are shown in Figure 7. The average error performance, calculated using the probability distribution for the average over all possible, length 31 Gold code pair combinations is also shown for comparison. As with the uplink, the best case is the code pair with the smallest second moment and the worst case is the code pair with the largest second moment. Hence, for this random selection of length 31 Gold codes, the dependence of the error rate performance on the magnitude of the second moment of the cross-correlation is consistent for both the uplink and downlink channels.

Comparing Figure 7, which gives the performance of the downlink, with Figure 4, the corresponding performance for the uplink, we note that the downlink channel's performance is significantly better than that of the uplink. This is not unexpected, for as we showed earlier in the asymptotic error expressions, the equivalent SNR in the uplink is asymptotically proportional to $1/K$, while in the downlink it is asymptotically proportional to $1/K^{1/2}$. The performance comparison in Figure 8 illustrates the wide disparity in the uplink and downlink channel performance. From a system's point of view, this suggests that for every K users on the uplink, the downlink has the capability of

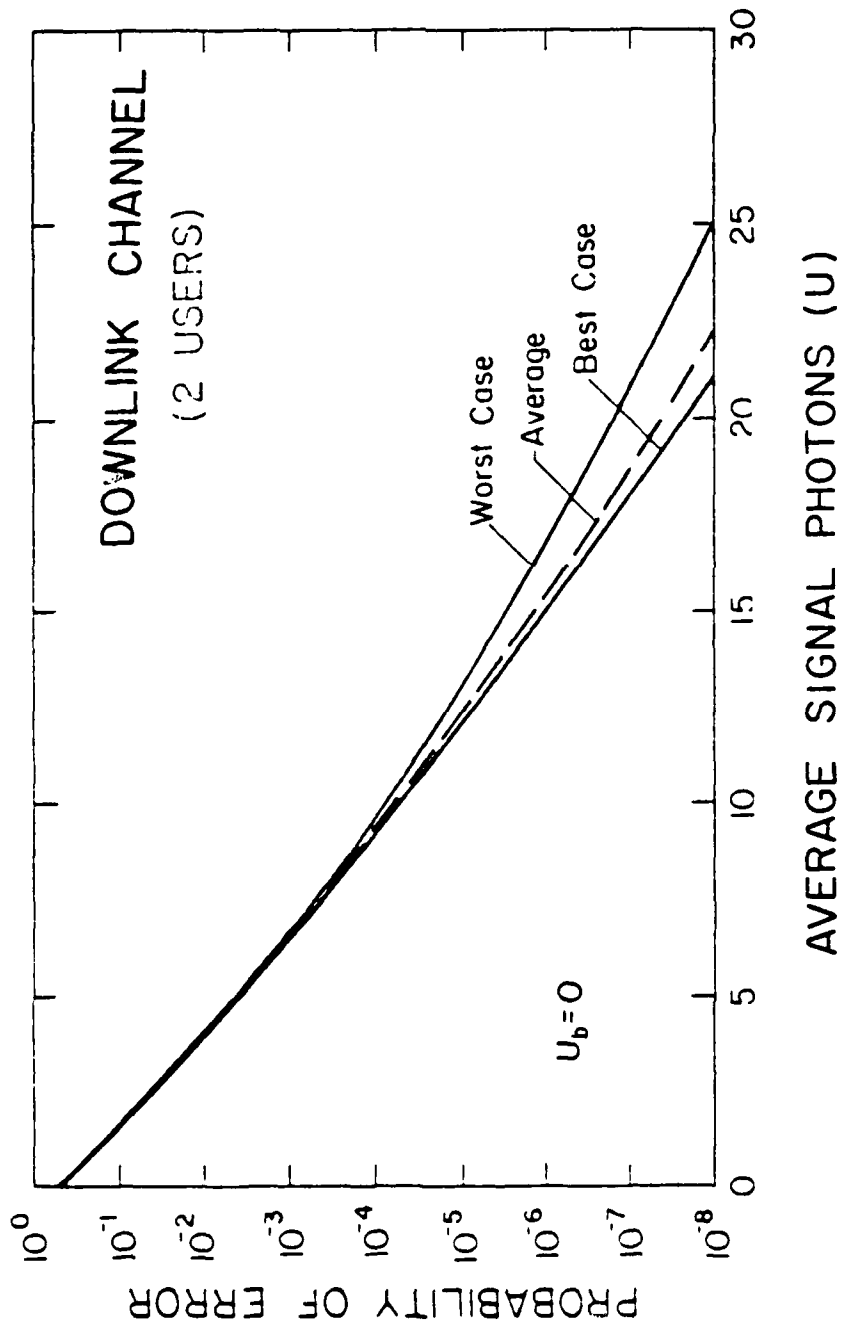


Figure 7. Probability of error vs. average signal photons for the downlink channel. Best and worse case plots are for length 31 Gold code pairs having smallest and largest normalized second moment, respectively. Average plot represents average performance over all length 31 Gold code pairs.

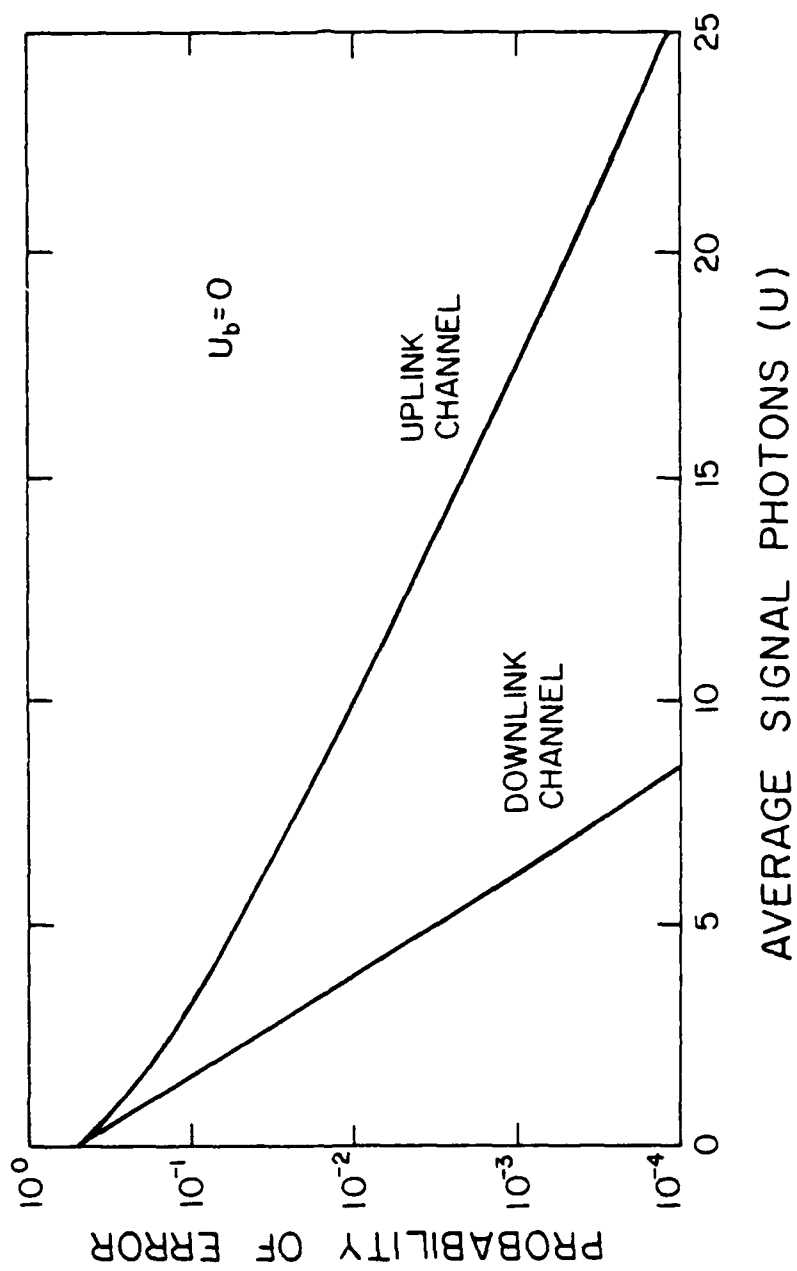


Figure 8. Performance comparison of the uplink and downlink channels. Data are given for systems of two users employing uncorrelated code sequences.

supporting K^2 users at a comparable error rate. This of course assumes that we are operating under low error rate conditions in which we can neglect the processing errors onboard the satellite.

The limiting error rate, as a function of the number of active users, was obtained for the downlink by evaluating (4.14) for the ideal case in which the background noise and cross-correlation were set to zero. Results for systems with up to 20 users are given in Figure 9 for the downlink channel. This plot shows the degrading effects of increasing the number of users in the downlink channel. However, in comparison to the uplink, the downlink performs substantially better. The curve shows that, for comparable photon counts and error rates, the downlink channel can support substantially more users than the uplink. At very large intensities, we have shown previously that for every K users on the uplink, the downlink has the capability of supporting K^2 users at a comparable error rate.

We also evaluated the downlink error performance for length 31 and 127 code sequences, to compare with the corresponding results for the uplink channel. We used the same probability densities for the cross-correlation values that were used in the uplink analysis. The results are shown in Figure 10. The plot for an infinite code length corresponds to setting the cross-correlation to zero, which is a property of the purely random codes. As shown in the figure, the error performance improves as the code length increases. This behavior is consistent with that for the uplink, except at smaller error rates. Note also that error performance close to that for the random codes can be achieved by using moderately long codes.

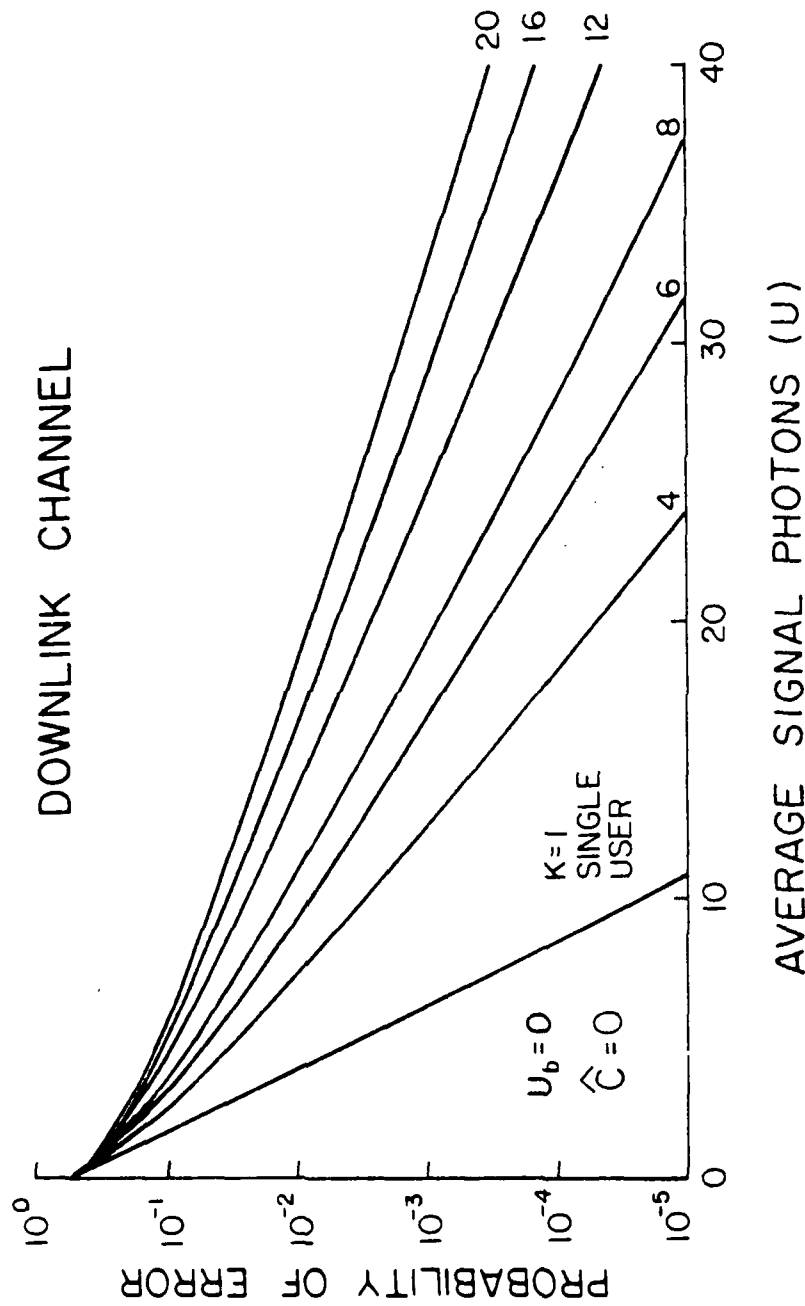


Figure 9. Probability of error vs. average signal photons for the downlink channel. Curves show the limiting error rate for up to 20 simultaneous users. K is the number of users.

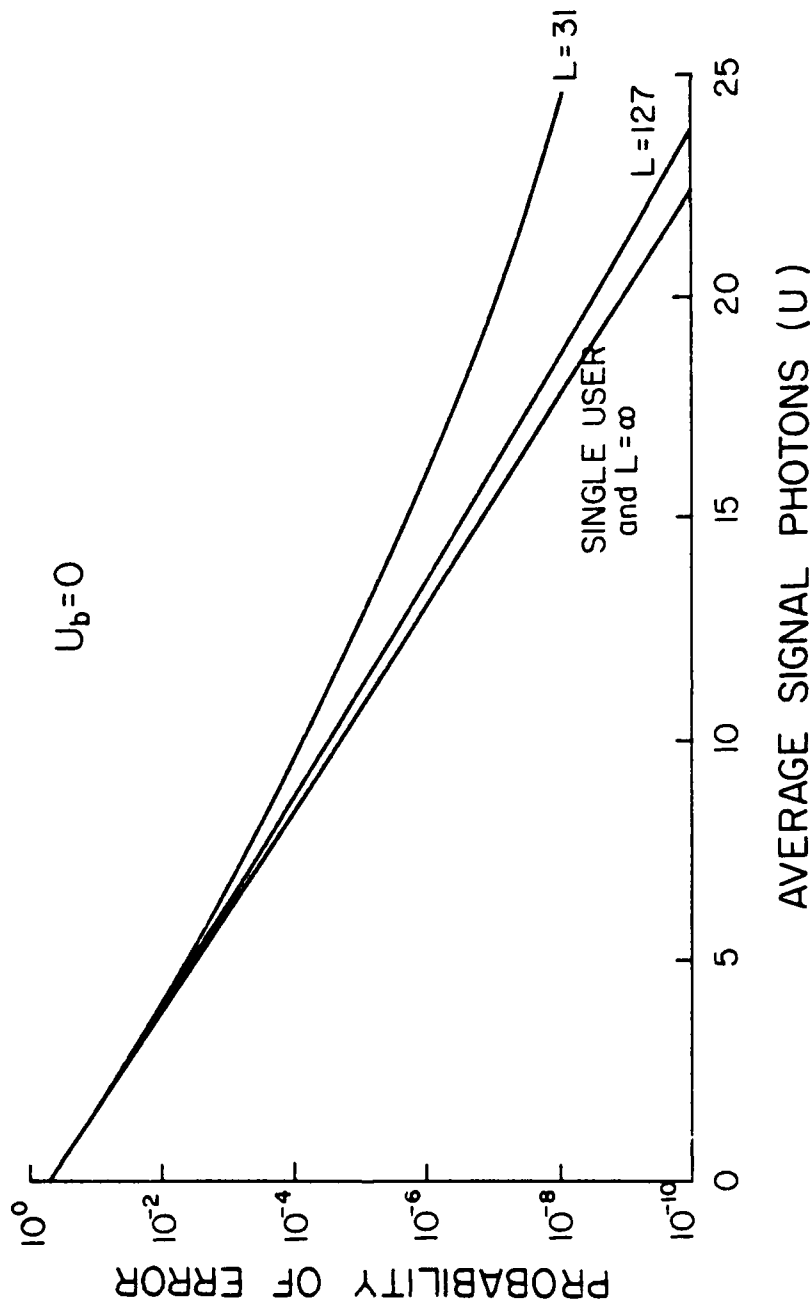


Figure 10. Probability of error vs. average signal photons for the downlink channel. Curves show the effects of code length L on the error rate.

CHAPTER 5

SYSTEM ANALYSIS FOR THE FADING CHANNEL

5.1 Model for the Fading Channel

An optical signal propagating through the earth's atmosphere can be severely degraded by atmospheric turbulence [Lawrence, 1970]. This turbulence is caused by random variations in the refractive index of the atmosphere. The interaction of the optical signal with the turbulent medium leads to random amplitude and phase variations in the optical signal [Kerr, 1970]. These effects produce a multipath phenomenon that gives rise to random fading of the received optical intensity. Theoretical and experimental studies of wave propagation in the atmosphere have shown that fading at optical frequencies is governed by the lognormal probability density [Tatarski, 1961]. The strength of fading is defined in terms of the log-intensity variance, which increases as the strength of fading and length of the optical path increase. Measured data [Tatarski, 1961] on terrestrial optical paths yield values of the log-intensity variance ranging from .042 for a 2000 meter path to .0078 for a 250 meter path. These results are typical of earth-to-satellite links where the primary disturbing medium is the first few kilometers of the lower atmosphere [Kennedy, 1969]. Optical fading generally occurs at rates less than 500Hz [Churnside, 1980].

If the optical field incident on the photodetectors in an optical receiver fades randomly in time, the photodetector output is described as a conditional or doubly stochastic Poisson process [Snyder, 1975]. In other words, the photodetector output is Poisson distributed but with an intensity which is itself a random process. The counting statistics for a conditional Poisson process can be obtained by taking the expectation of the conditional Poisson count with respect to the statistics of the optical intensity. For example, in a stationary system, the probability of obtaining $n = 0, 1, 2, \dots$, photon counts in an interval $(t, t + \tau)$ is

$$P(n) = E_{\lambda} [(n!)^{-1} \left\{ \int_t^{t+\tau} \lambda(x) dx \right\}^n \exp \left\{ - \int_t^{t+\tau} \lambda(x) dx \right\}] \quad (5.1)$$

where $E_{\lambda} [\cdot]$ is the expectation with respect to λ . $\lambda(t)$ is given by the formula [Pratt, 1969],

$$\lambda(t) = \eta I(t)/hf \quad (5.2)$$

where $I(t)$ is the randomly fading optical intensity, h is Planck's constant, f is the optical frequency and η , ($\eta < 1$) is the photodetector quantum efficiency.

A well-known statistical model of fading signals is the Nakagami m -distribution [Nakagami, 1960]. If the magnitude of a fading optical field is m -distributed, the distribution of the intensity variations is the familiar gamma or χ^2_{2m} distribution. This distribution is used as a statistical model of speckle noise in laser systems [Gardner, 1977], [Goodman, 1975]. The photon counting probability corresponding to the gamma distributed intensity is the negative binominal distribution. The negative binominal distribution includes as special cases, the Bose-Einstein distribution, $m = 1$ and the Poisson distribution, $m \rightarrow \infty$. Under conditions when the gamma distribution is used to approximate the lognormal distribution [Yoshikawa, 1978], the negative binominal is also a good approximation to the counting statistics for a lognormally distributed intensity. In this analysis we use the Nakagami m -distribution as a generalized statistical model of the fading channel. We assume that fading is due solely to the properties of the channel, as opposed to the nature of the optical source, i.e., the optical field originates from a highly coherent source but is perturbed by random variations in the channel properties. We also assume that the background noise and cross-correlation interference are negligible so as to facilitate the mathematical analysis.

5.2 Probability Density for the Optical Intensity

If the magnitude of the optical field is Nakagami m -distributed, the optical intensity (magnitude-squared) is gamma distributed with probability density function [Goodman, 1975]

$$p(I) = (m / \langle I \rangle)^m \{ I^{m-1} / \Gamma(m) \} \exp \{-m(I/\langle I \rangle)\} \quad (5.3)$$

where I is the instantaneous optical intensity, $\langle I \rangle$ is the average intensity and the factor m is defined as the inverse normalized variance of I ,

$$m = \langle I \rangle^2 / \langle (I - \langle I \rangle)^2 \rangle \quad (5.4)$$

The m -distribution is completely specified by the two parameters $\langle I \rangle$ and m . For a given average intensity $\langle I \rangle$, m can be selected to describe the strength of fading. The strength of fading is inversely proportional to m . The photon counting probability corresponding to the gamma distributed optical intensity is the negative binominal distribution [Saleh, 1978],

$$P(k) = \Gamma(m+k) / k! \Gamma(m) [1 + \langle I \rangle / m]^{-m} [1 + m / \langle I \rangle]^{-k} \quad (5.5)$$

The mean and variance of (5.5) are given by

$$E[k] = \langle I \rangle$$

and

$$(5.6)$$

$$\text{Var}(k) = \langle I \rangle + \langle I \rangle^2 / m$$

In the limit as $m \rightarrow \infty$,

$$\Gamma(m+k)/\Gamma(m) \approx m^k$$

and

(5.7)

$$[1 + \langle I \rangle / m]^{-m} \approx \exp(-\langle I \rangle)$$

Substituting these approximations into the limiting form ($m \rightarrow \infty$) of (5.5), we get

$$P(k) = \exp(-\langle I \rangle) \langle I \rangle^k / k! \quad (5.8)$$

which is the well-known Poisson counting model for the no fading case analyzed in Chapter 3. For the special case, $m = 1$, the negative binominal distribution reduces to the Bose-Einstein distribution

$$P(k) = [1 + \langle I \rangle]^{-1} [\langle I \rangle / (1 + \langle I \rangle)]^k \quad (5.9)$$

which is a statistical model of a narrow band Gaussian optical field [Saleh, 1978].

5.3 Uplink Channel Analysis

5.3.1 Correlated fading

It appears that the simplest approach to deriving the average probability of error for the fading case is to generalize the error expressions derived in Chapter 3 to probability of error expressions conditioned on the fading intensities and then remove the conditioning by averaging over the fading statistics of the individual users. The average probability of error is obtained by evaluating

$$P(E) = \int \int \dots \int p(U_1, U_2, \dots, U_K) P(E|U_1, U_2, \dots, U_K) dU_1 dU_2 \dots dU_K \quad (5.10)$$

where $P(E|U_1, U_2, \dots, U_K)$ is the conditional probability of error and $p(U_1, U_2, \dots, U_K)$ is the joint density for U_1, U_2, \dots, U_K , the conditional photon counts of the K system users.

However, if we have equal power users, the background noise and cross-correlation interference are neglected and the fading is correlated between users, the average probability of error is obtained with a single integration over the probability density for the optical intensity

$$P(E) = \int p(U)P(E|U)du \quad (5.11)$$

where, in this case, we set

$$U_1 = U_2 = \dots = U_K = U$$

and use the error expression derived in Chapter 3 for $P(E|U)$. The probability density for the optical intensity is $p(U)$.

However for uncorrelated fading, a condition under which each user fades independently, it is difficult to derive a useful form of the probability of error using this approach. The difficulty arises from the fact that this approach requires K -fold integrations involving the joint statistics describing the random fading. To circumvent this problem, we resorted to an alternative formulation of the error expression. First, we analyze the simple case of correlated fading on the uplink, under the conditions of equal power users, neglecting the background noise and cross-correlation interference. For this case, we generalize the probability of error in (3.18) to a conditional probability of error

$$P(E|U) = 1/2[1 - Q(b,a) + Q(a,b)] \quad (5.12)$$

where the arguments of the Q -function are

$$a = [(K - 1)U]^{1/2}$$

and

$$b = [(K + 1)U]^{1/2} \quad (5.13)$$

The photon count U reflects the equal power, correlated user case, for which

$$U_1 = U_2 = \dots = U_K = U$$

The average probability of error is obtained from

$$P(E) = \int P(E|U) p(U) dU \quad (5.14)$$

where $p(U)$ is the gamma probability density in the parameter U . In Appendix D we give the mathematical operations required to simplify the error expression, the final result is

$$\begin{aligned}
 P(E) &= [1 + K\langle U \rangle/m]^{-m} \\
 &\cdot \sum_{p=0}^{\infty} [(K - 1)/2K]^p \delta^p [(m)_p/p!] \\
 &\cdot \sum_{j=0}^p [(K + 1)/2K]^j \delta^j [(m+p)_j/j!] \gamma_{j,p}
 \end{aligned} \quad (5.15)$$

where

$$\delta = [1 + m/K\langle U \rangle]^{-1}$$

m is the fading parameter, $\langle U \rangle$ is the average number of photons, K the number of simultaneous users and $\gamma_{j,p} = 1/2$ for $j = p$ and is 1, otherwise.

5.3.2 Uncorrelated fading

For the case of uncorrelated fading, we are confronted with the difficulties involved in deconditioning the error expression with respect to the fading statistics of each individual user. To recast the problem in a more convenient form, recall from the analysis in Chapter 3 that the receiver output R (Figure 2b), can be written as the difference between two photon counts, R^+ and R^- . If a +1 is transmitted, R^+ is the sum of the desired signal count S (which is synchronized to the data bit) and an interference count I^+ .

$$R^+ = S + I^+ \tag{5.16}$$

I^+ represents spurious photons counted on the upper photodetector due to the presence of interfering users which are not synchronized to the data bit. On the other hand, R^- is simply the corresponding count which appears on the lower photodetector, also due to the interferers

$$R^- = I^- \quad (5.17)$$

For no fading, S and I^+ are independent Poisson random variables; therefore, the sum $R^+ = S + I^+$ is a Poisson random variable. I^- is also a Poisson random variable, independent of S and I^+ . Consequently, the receiver output R is analyzed as the difference between two independent Poisson random variables. However, if we allow each user to fade independently and if the fading is slow enough so that we may consider it essentially constant over the bit period, then we must account for the fact that the interference on the upper and lower photodetectors is correlated by virtue of the fading channel. As a consequence, we cannot treat the receiver output as the difference of independent Poisson random variables.

To derive an alternative expression we note that since it is equally likely that $b_1 = \pm 1$, the average probability of error, defined at the receiver output R , can be written

$$P(E) = \Pr(R^+ < R^-) + 1/2 \Pr(R^+ = R^-) \quad (5.18)$$

which, using (5.16) and (5.17), can also be expressed in terms of the desired signal and interference

$$P(E) = \Pr(S < Z) + 1/2 \Pr(S = Z) \quad (5.19)$$

where Z , given by

$$Z = I^- - I^+ \quad (5.20)$$

is the difference between the interference counts on the upper and lower photodetectors. The last terms in the error expressions above account for equality in the decision test, whereby we make a random decision with a probability of 1/2 being incorrect.

In terms of counting probabilities, the average probability of error is

$$P(E) = \sum_{k=-\infty}^{\infty} P_Z(k) \sum_{j=0}^k P_S(j) \gamma_{j,k} \quad (5.21)$$

where P_S and P_Z are the counting probabilities for the desired signal count S and the interference count Z .

The factor

$$\gamma_{j,k} = \begin{cases} 1/2 & j=k \\ 1 & \text{otherwise} \end{cases} \quad (5.22)$$

accounts for equality in the decision test. At this point we must determine the unconditioned counting statistics for the desired signal and the interference.

To determine P_s we note that, since the system is synchronized to the code of the desired signal, S is Poisson distributed given the fading optical field

$$P_s | \lambda(j) = (j!)^{-1} \left\{ \int_0^{T_d} \lambda(x) dx \right\}^j \exp \left[- \int_0^{T_d} \lambda(x) dx \right] \quad (5.23)$$

where the counting rate $\lambda(x)$ is the random process proportional to the intensity and T_d is the length of the data bit interval. If the fading rate is slow when compared to the data bit interval, as will be the case for data rates > 500 Hz, we can assume $\lambda(x)$ is approximately constant over the integration period and remove it from within the integral. If we let

$$U_1 = \int_0^{T_d} \lambda(x) dx \approx \lambda T_d \quad (5.24)$$

then, U_1 is a random variable which reflects the fluctuating behavior in the optical intensity of the desired signal. The counting probability P_s is then determined by integrating the conditional Poisson count over the gamma probability density $p(U_1)$. From (5.5) this gives

$$P_s(j) = \Gamma(j+m)/j! \Gamma(m) [1 + \langle U_1 \rangle / m]^{-m} [1 + m/\langle U_1 \rangle]^{-j} \quad (5.25)$$

for $j \geq 0$.

where $\langle U_1 \rangle$ is the average number of photons/bit and m is the Nakagami fading parameter.

The interferers, which are not synchronized to the desired signal, generate spurious photon counts on both the upper and lower photodetectors. The interference count, conditioned on both the fading statistics and the cross-correlation values for each user, is distributed as the difference of the Poisson random variables, I^+ and I^- . The conditional probability distribution of the interference count is [Pratt, 1969]

$$P_{Z|\underline{U}, \underline{C}}(k) = \exp\{-(U_+ + U_-)\} (U_+ / U_-)^{-k/2} I_{|k|}(\sqrt{2U_+U_-}) \quad (5.26)$$

where

$$\underline{U} = \{ U_2, U_3, \dots, U_K \}$$

are the random variables defining the average photon count for the interferers and

$$\underline{C} = \{ C_2, C_3, \dots, C_K \}$$

are the cross-correlations between the desired signal and the interferers. U_+ and U_- are the conditional average values of the total interference counts on I^+ and I^- , respectively.

$$U_+ = E[I^+ | \underline{C}, \underline{U}]$$

and

$$U_- = E[I^- | \underline{C}, \underline{U}] \quad (5.27)$$

To define $P_z | \underline{u}, \underline{c}$, we must determine the conditional values of U_+ and U_- . This can be determined from a simple analysis of the receiver output for the no fading case i.e., the case for which \underline{U} is deterministic. For K users, in the absence of background noise, we can express (3.11) as the conditional mean and variance

$$E[R | \underline{U}, \underline{C}] = b_1 U_1 + \sum_{i=2}^K C_i U_i$$

and

$$\text{Var}(R | \underline{U}) = \sum_{i=1}^K U_i \quad (5.28)$$

where b_1 is the data bit of the desired signal and U_1 is the corresponding average photon count. U_i is the average photon count of the i -th interferer and C_i is the cross-correlation between the desired signal and the i -th interferer.

From results in Appendix A we can write

$$E[R^+ | \underline{C}, \underline{U}] = 1/2 \{ \text{Var}(R | \underline{U}) + E[R | \underline{C}, \underline{U}] \}$$

and

(5.29)

$$E[R^- | \underline{C}, \underline{U}] = 1/2 \{ \text{Var}(R | \underline{U}) - E[R | \underline{C}, \underline{U}] \}$$

By substituting (5.28), evaluated for $b_1 = +1$, into (5.29), we get

$$E[R^+ | \underline{C}, \underline{U}] = U_1 + \sum_{i=2}^K 1/2(1 + C_i)U_i$$

and

$$E[R^- | \underline{C}, \underline{U}] = \sum_{i=2}^K 1/2(1 - C_i)U_i \quad (5.30)$$

Since the desired signal has an average count U_1 , the average counts for U_+ and U_- are

$$U_+ = 1/2 \sum_{i=2}^K (1 + C_i)U_i$$

and

$$U_- = 1/2 \sum_{i=2}^K (1 - C_i)U_i \quad (5.31)$$

To uncondition the counting probability $P_{z|U,q}$, we must determine the statistics for U_+ and U_- . This is an intractable problem without first applying some limiting conditions. For example, consider the expression for U_+ which is the sum of $K-1$ independent random variables. For the simple case of a single interferer in which U_i is gamma distributed, the characteristic function for the probability density of U_+ , conditioned on C_i is

$$M_i(j\omega) = [1 - j\omega(1+C_i)\langle U_i \rangle / 2m_i]^{-m_i} \quad (5.32)$$

where $\langle U_i \rangle$ and m_i are the parameters of the gamma distribution. For N interferers, the characteristic function is

$$M_+(j\omega) = \prod_{i=1}^N [1 - j\omega(1+C_i)\langle U_i \rangle / 2m_i]^{-m_i} \quad (5.33)$$

Note that in the expression for $M_+(j\omega)$, the C_i 's are all independent random variables and the m_i 's and $\langle U_i \rangle$'s are different for each interferer. As a consequence, it is difficult to invert $M_+(j\omega)$ to obtain the probability density function of U_+ . The same is true for U_- . However, we can simplify the problem considerably if we assume that the C_i 's are very small, as is the case for long code sequences, and the fading on each user is identically distributed. This latter assumption is the case when the system users are geographically located in a region characterized by similar atmospheric conditions. Under these conditions, we can let the C_i 's = 0 and

$$\langle U_1 \rangle = \langle U_2 \rangle = \dots = \langle U_K \rangle = \langle U \rangle$$

and

$$m_1 = m_2 = \dots = m_K = m$$

In this case, U_+ is equal to U_- , so that

$$M_+(j\omega) = M_-(j\omega) = [1 - j\omega\langle U \rangle/2m]^{mN} \quad (5.34)$$

where $N = K-1$.

The probability density which corresponds to the characteristic function given in (5.34) is the gamma density with parameters $N\langle U \rangle/2$ and Nm

$$p(U) = (2m/\langle U \rangle)^{Nm} \{U^{Nm-1} / \Gamma(Nm)\} \exp\{-2mU/\langle U \rangle\} \quad (5.35)$$

for $U \geq 0$.

Under these simplifying conditions, (5.26) becomes

$$P_z|u(k) = \exp\{-2U_+\} I_{|k|}(2U_+) \quad (5.36)$$

Finally, the unconditioned counting probability P_z is determined by averaging (5.36) over the gamma probability density given by (5.35). The integration gives [Gradshteyn, 1965]

$$P_z(k) = \Gamma(|k|+Nm)/(|k|)! \Gamma(Nm) [1 + \langle U \rangle / m]^{-Nm} \\ \cdot (\beta/2)^k {}_2F_1\left\{\frac{(|k|+Nm)/2}, \frac{(|k|+Nm+1)/2}; |k|+1; \beta^2\right\} \quad (5.37)$$

where

$$\beta = [1 + m/\langle U \rangle]^{-1} \quad (5.38)$$

m is the Nakagami fading parameter and ${}_2F_1$ is a hypergeometric function defined by

$${}_2F_1(a, b; c; t) = \sum_{j=0}^{\infty} \frac{(a)_j (b)_j}{(c)_j} \frac{t^j}{j!} \quad (5.39)$$

where $(a)_j$ is the Pochhammer symbol for the product

$$(a)_j = a(a+1)(a+2)\dots(a+j-1); (a)_0 = 1$$

By substituting the counting probabilities P_s and P_z into (5.21), we have shown in Section D.2 of Appendix D that the average probability of error equals

$$P(E) = [1 + \langle U \rangle / m]^{-m(N+1)} \sum_{r=0}^{\infty} [(Nm)_r / r!] (\beta/2)^r$$

$$\cdot \sum_{p=0}^r [(r+Nm)_p / p!] (\beta/2)^p \sum_{s=0}^{r-p} [(m)_s / s!] (\beta)^s \gamma_{s,r-p} \quad (5.40)$$

where

$$\beta = [1 + m/\langle U \rangle]^{-1}$$

N is the number of interferers and m is the fading parameter. $\gamma_{s,r-p}$ is $1/2$ for $s = r-p$ and is 1 otherwise.

5.4 Downlink Channel Analysis

On the downlink channel, the multiple access users are combined asynchronously and coded onto a single output signal; therefore, the fading is correlated between each user. Consequently, the average probability of error for users on the downlink can be determined by simply deconditioning the error expression obtained for the no fading case in Chapter 4.

For the conditions where 1) the cross-correlation $\underline{C} = 0$, 2) the background radiation and detector dark currents are negligible and 3) the fading on the intensity is slow with respect to the data bit, the error expression for the downlink (4.7), generalized to a conditional probability of error, is

$$P(E|U) = 1/2 [1 - Q(b',a') + Q(a',b')] \quad (5.41)$$

where $Q(a',b')$ is the Marcum's Q-function [Marcum, 1960], and

$$\begin{aligned} a' &= [(K' - 1)U]^{1/2} \\ b' &= [(K' + 1)U]^{1/2} \end{aligned} \quad (5.42)$$

The factor K' , as defined in (4.5), is

$$K' = K \begin{matrix} \left[\begin{matrix} K \\ K/2 \end{matrix} \right] \end{matrix} (1/2)^K \quad K \text{ even}$$

and

$$K' = K \binom{K-1}{[K-1]/2} (1/2)^{K-1} \quad K \text{ odd} \quad (5.43)$$

Notice that the conditional error expression for the downlink is identical to that for correlated fading on the uplink, with K replaced by K' . Hence, the average probability of error is the same form as that for the uplink with K replaced by K'

$$P(E) = [1 + K' \langle U \rangle / m]^{-m} \cdot \sum_{k=0}^{\infty} \binom{m}{k} [(K'-1)/2K']^k \alpha^k (k!)^{-1} \cdot \sum_{j=0}^k \binom{k+m}{j} [(K'+1)/2K']^j \alpha^j (j!)^{-1} \gamma_{j,k} \quad (5.44)$$

where

$$\alpha = [1 + m/K' \langle U \rangle]^{-1}$$

and $\gamma_{j,k} = 1/2$ for $j = k$ and is 1, otherwise.

5.5 Numerical Results

Error probabilities showing the effects of fading on optical CDMA systems are presented in Figures 11-19 for various values of the Nakagami fading parameter and numbers of interfering users. The results are computed for negligible background noise, $U_b = 0$, and negligible cross-correlation interference, $\underline{C} = 0$. The data were generated by numerically evaluating the error expressions for correlated (5.15) and uncorrelated (5.40) fading on the uplink and for correlated fading (5.44) on the downlink. Uncorrelated fading is not a consideration for the downlink because the satellite transmits a single optical carrier on which all of the active, uplink users have been modulated. Hence, fading of the optical carrier is equivalent to each user undergoing the same instantaneous fade; therefore, the fading is correlated between each user. On the other hand, correlated fading on the uplink is unrealistic for most systems; since this implies that the users are so close geographically that they all experience the same instantaneous fades. However, this case was easy to solve and the result, with a simple change of variable, is directly applicable to the downlink. In all of the numerical evaluations, series expressions were truncated and considered a satisfactory approximation after summing over a sufficient number of terms to insure accuracy to within 10^{-7} . For greater accuracies, the uplink expression for uncorrelated fading which contains the triple summations, converges very poorly; especially for small values of the fading parameter m and large signal intensities.

Figures 11-13 examine the case of a single interferer under strong fading conditions. These conditions range from $m = 1$, the Bose-Einstein case to $m = 10$. The Bose-Einstein case characterizes narrow band Gaussian optical fields. The magnitude of the optical field is Rayleigh distributed; a well-known fading model in RF systems. The error rates for the no fading case, labeled NF, are included in each figure for comparison. As m increases, the error rates for all fading conditions improve as expected. The uncorrelated fading on the uplink (Figure 12) creates a greater degree of degradation to the system than correlated fading on either the uplink or downlink. This is not totally unexpected because an uncorrelated fading condition here implies that the desired signal and interferer are fading independent of each other. The instantaneous strength of the fade on the interferer is not likely to be the same as that on the desired signal. This randomness is an additional form of noise and is not completely averaged out in the detection process, even though the cross-correlation between the signals is negligible. Whereas in the case of correlated fading, the interferer and desired signal fade instantaneously and the net noise effect is likely to be less after the averaging in the detection process. Over the performance range plotted, the downlink exhibits a better error rate than the uplink under either correlated or uncorrelated fading conditions. This is noted by the wide disparity in the average numbers of photons required to support a given error rate for a specified value of m . However, the system as a whole, suffers serious degradation from strong fading. Much larger signal intensities are required to obtain reasonable error rates for small values of m .

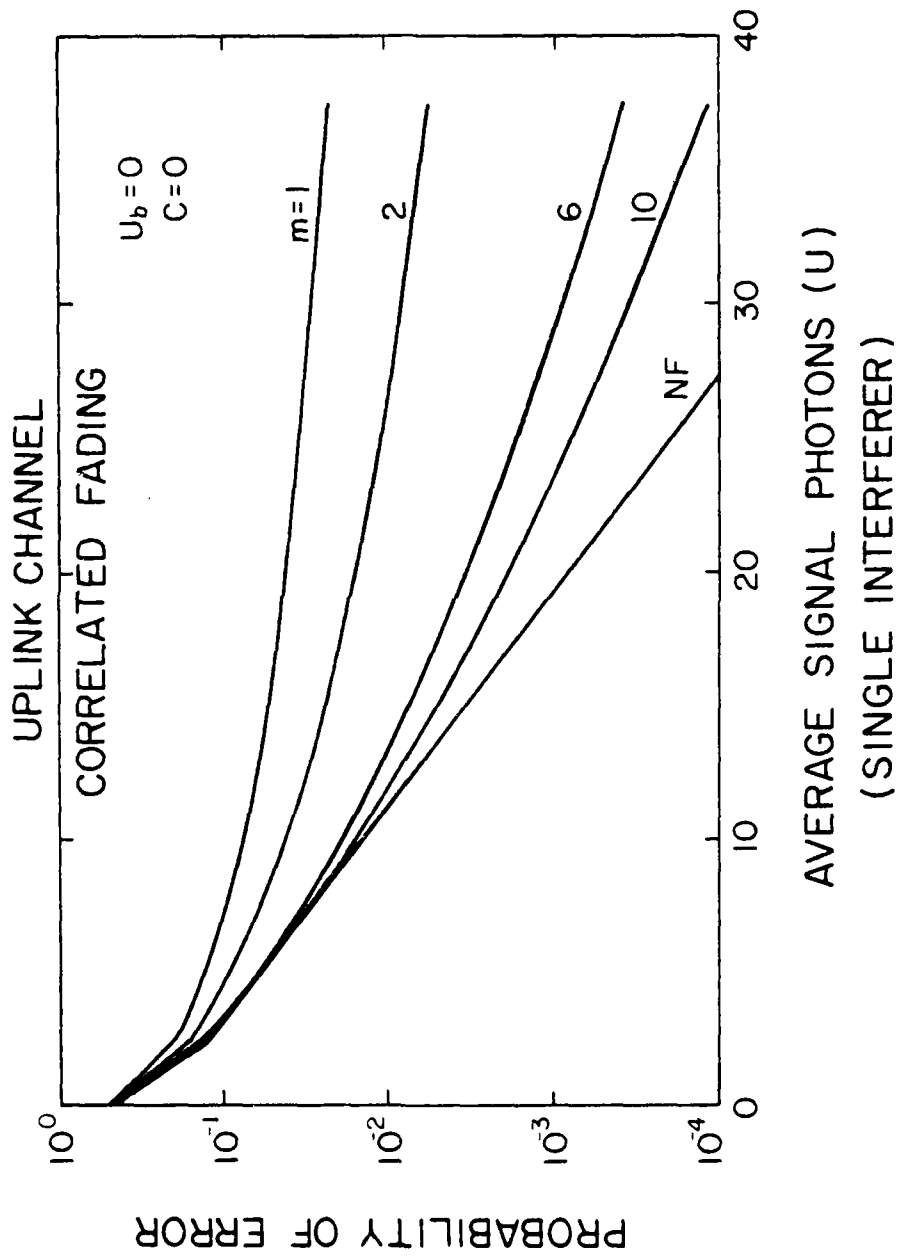


Figure 11. Probability of error vs. average signal photons for correlated fading on the uplink channel for a single interferer. m is the Nakagami fading parameter. Background noise and cross-correlation are zero.

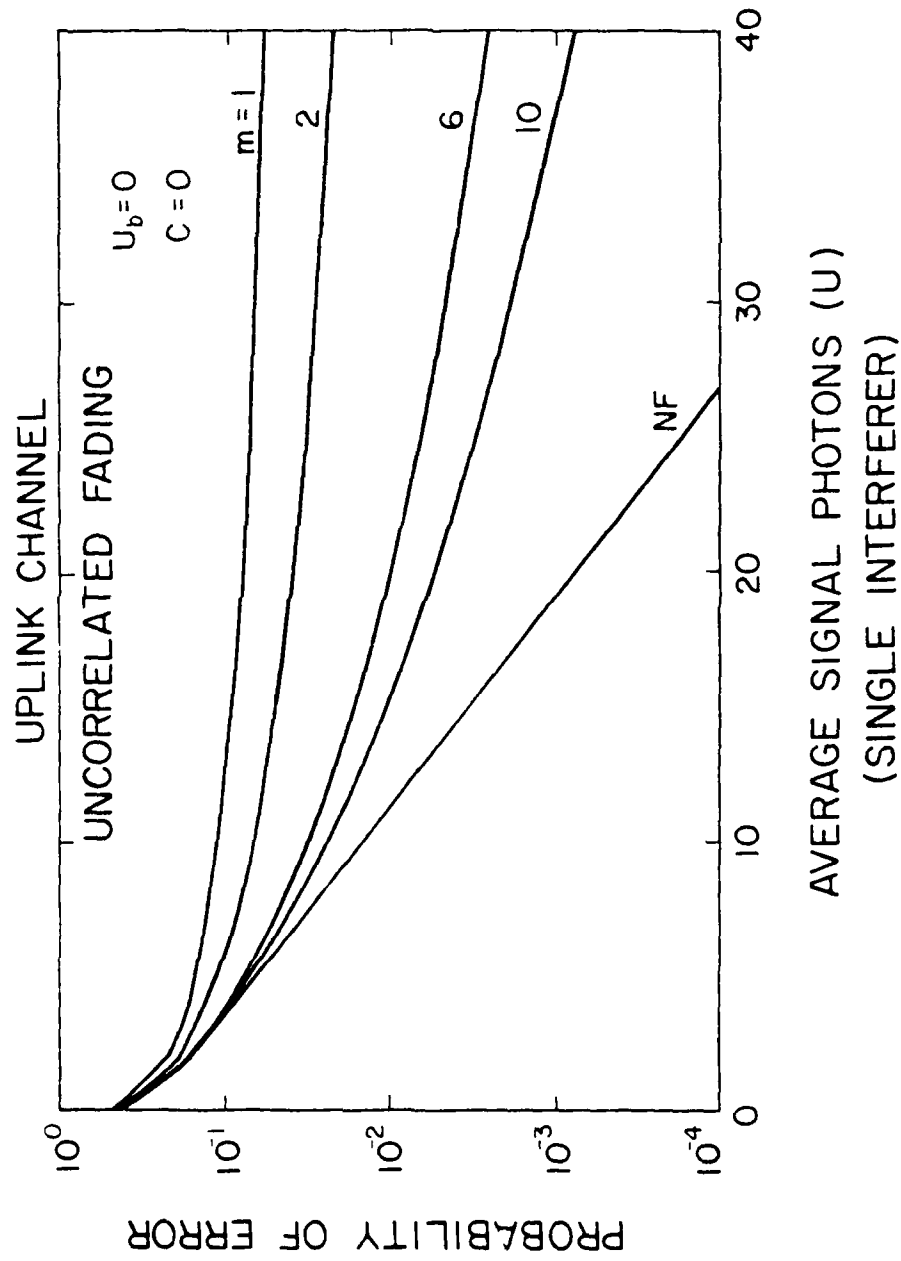


Figure 12. Probability of error vs. average signal photons for uncorrelated fading on the uplink channel for a single interferer. m is the Nakagami fading parameter. Background noise and cross-correlation are zero.

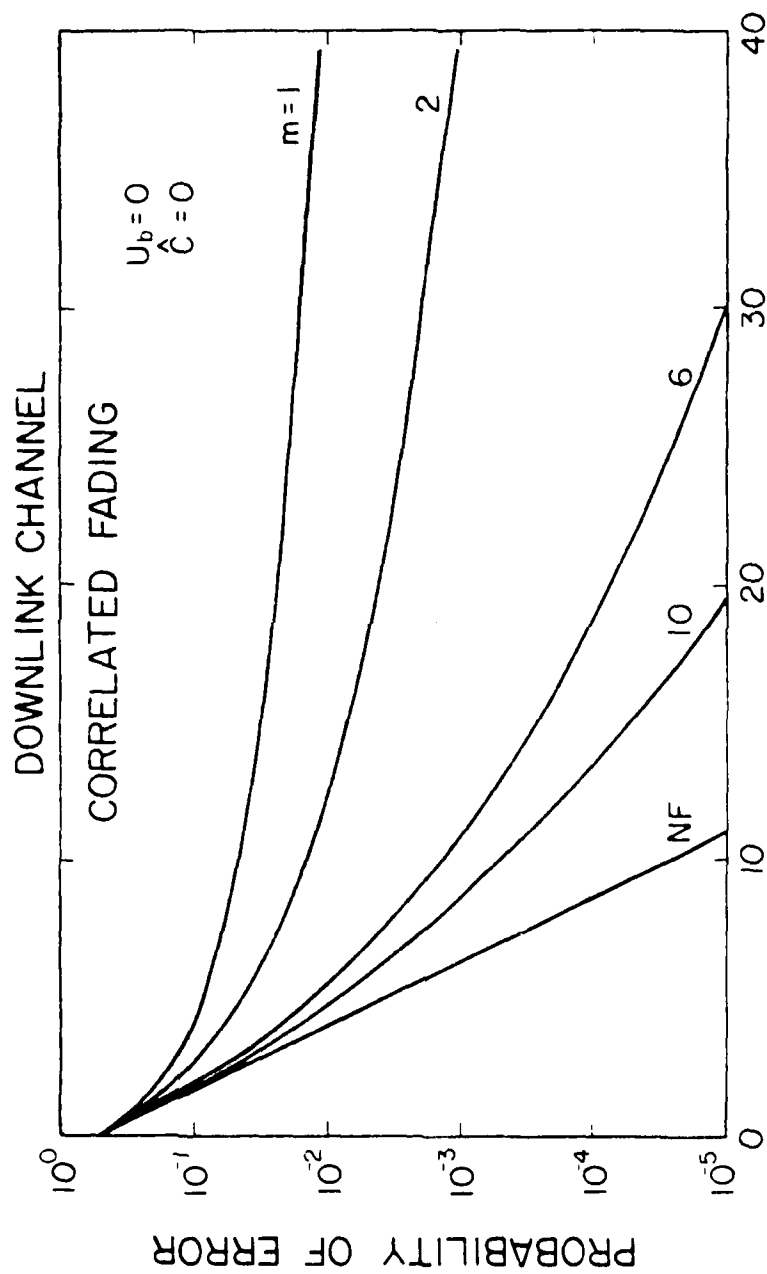


Figure 13. Probability of error vs. average signal photons for correlated fading on the downlink channel for a single interferer. m is the Nakagami fading parameter. Background noise and cross-correlation are zero.

For $m \gg 1$, the gamma distribution can be used to approximate the lognormal distribution. Using the transformations

$$I_0 = 10 \log_{10}(I)$$

and

(5.45)

$$\langle I_0 \rangle = 10 \log_{10}(\langle I \rangle)$$

Yoshikwa (1978) has shown that the probability density for I_0 can be approximated by the form

$$p(I_0) = \frac{1}{\sqrt{2\pi} \sigma_0} \exp \left[-\frac{(I_0 - \langle I_0 \rangle)^2}{2 \sigma_0^2} \right] \quad (5.46)$$

where

$$\sigma_0^2 = 4.343 / m$$

is the normalized variance in terms of the fading parameter m . Hence, with I_0 in the form of a normal distribution, I , the intensity, takes the form of a lognormal distribution.

Figures 14-16 are plots showing the effects of fading on the multiple user capabilities of the uplink and downlink channels, for $\sigma_0 = .3$ ($m=210$). This fading condition is characteristic of the moderate turbulence measured on 1000 meter terrestrial links by Tatarski, (1961) and Axford, (1979). Figure 14 gives the performance of the uplink channel for up to 15 interferers, under correlated fading conditions. The uncorrelated fading condition is given in Figure 15 for 8 interferers. Overall, the performance of the uplink seriously degrades when the number of interferers is increased; especially for uncorrelated fading. For the uplink to provide reasonable error rates under fading conditions in a multiuser environment, very large signal intensities are required. Comparing these results with the corresponding error rates for the downlink (Figure 16) we note that the downlink error rate is significantly better than the uplink under these moderate fading conditions, as a much larger number of active users can be supported at a given error rate and photon count. This is advantageous, because the users on the uplink (ground stations) can potentially boost their signal levels until a lower error rate is attained. Whereas, the downlink (satellite) is power-limited. The additional margin of performance in the downlink may help offset this limitation.

Figures 17-19 give the error rate performance for strong, lognormal fading, $\sigma_0 = .9$. By comparison, the downlink continues to perform better than the uplink. Overall, increased fading causes increased degradation to the system. Under strong fading conditions, such as the lognormal condition in Figures 17-19, it is unlikely that satisfactory error rates from end-to-end can be achieved without boosting the signal intensities on the uplink to unreasonable output levels, especially to support a large number of users.

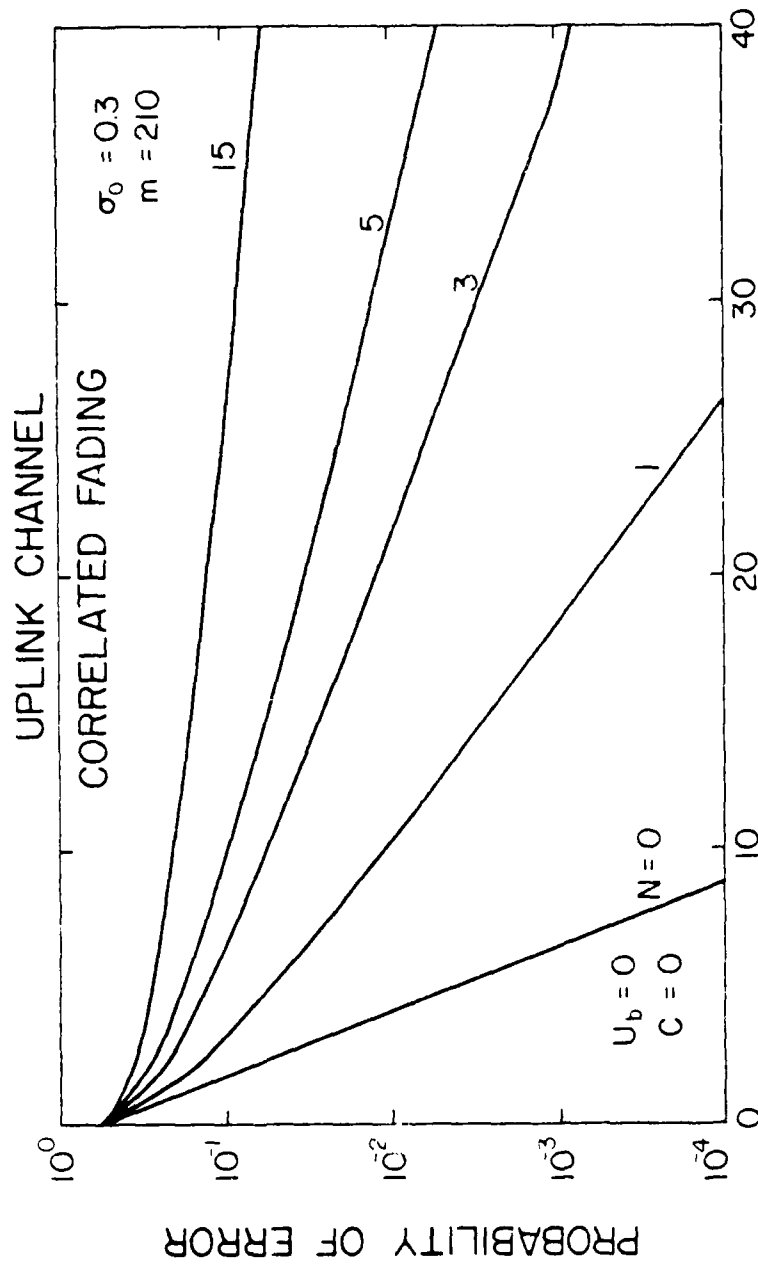


Figure 14. Probability of error vs. average signal photons for correlated fading on the uplink channel for multiple users in moderate lognormal fading. N is the number of interferers and σ_0 is the normalized variance of the log-intensity. Background noise and cross-correlation are zero.

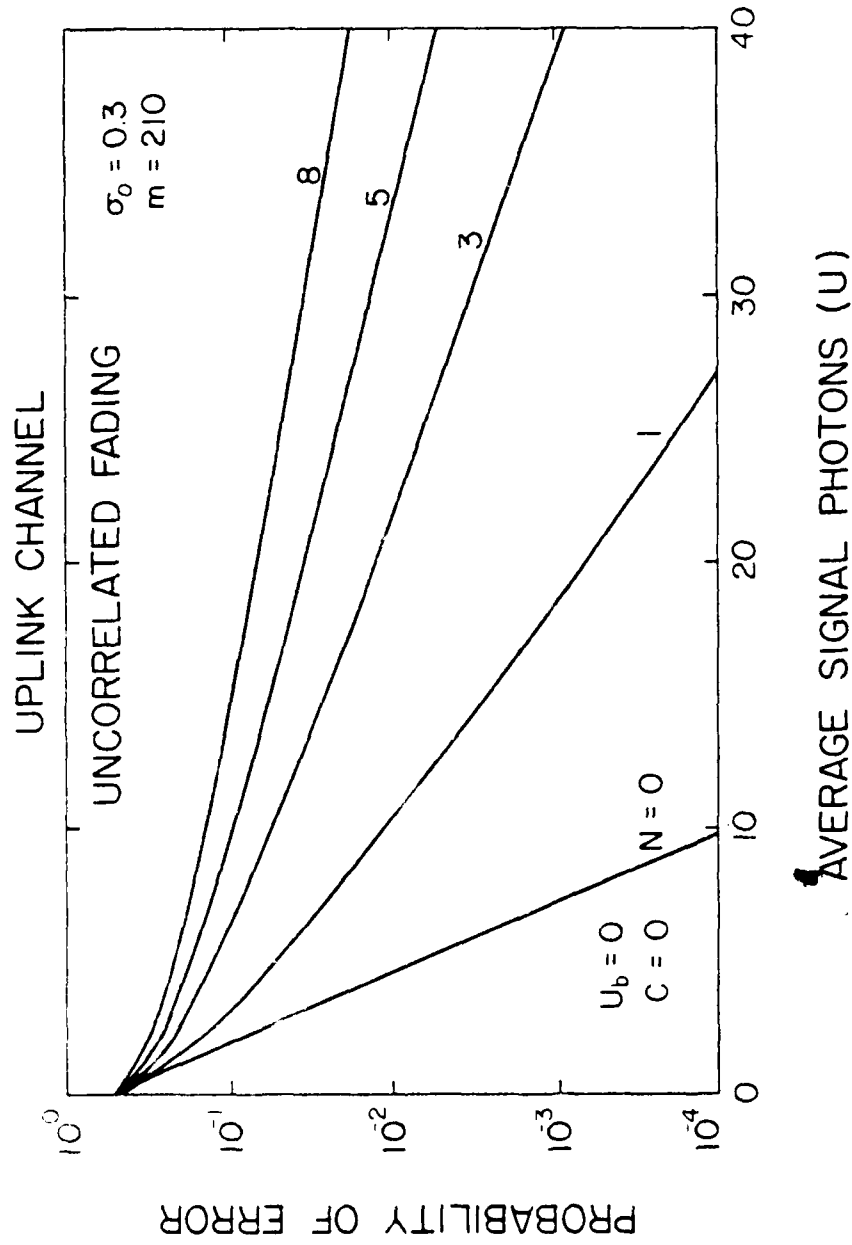


Figure 15. Probability of error vs. average signal photons for uncorrelated fading on the uplink channel for multiple users in moderate lognormal fading. N is the number of interferers and σ_0 is the normalized variance of the log-intensity. Background noise and cross-correlation are zero.

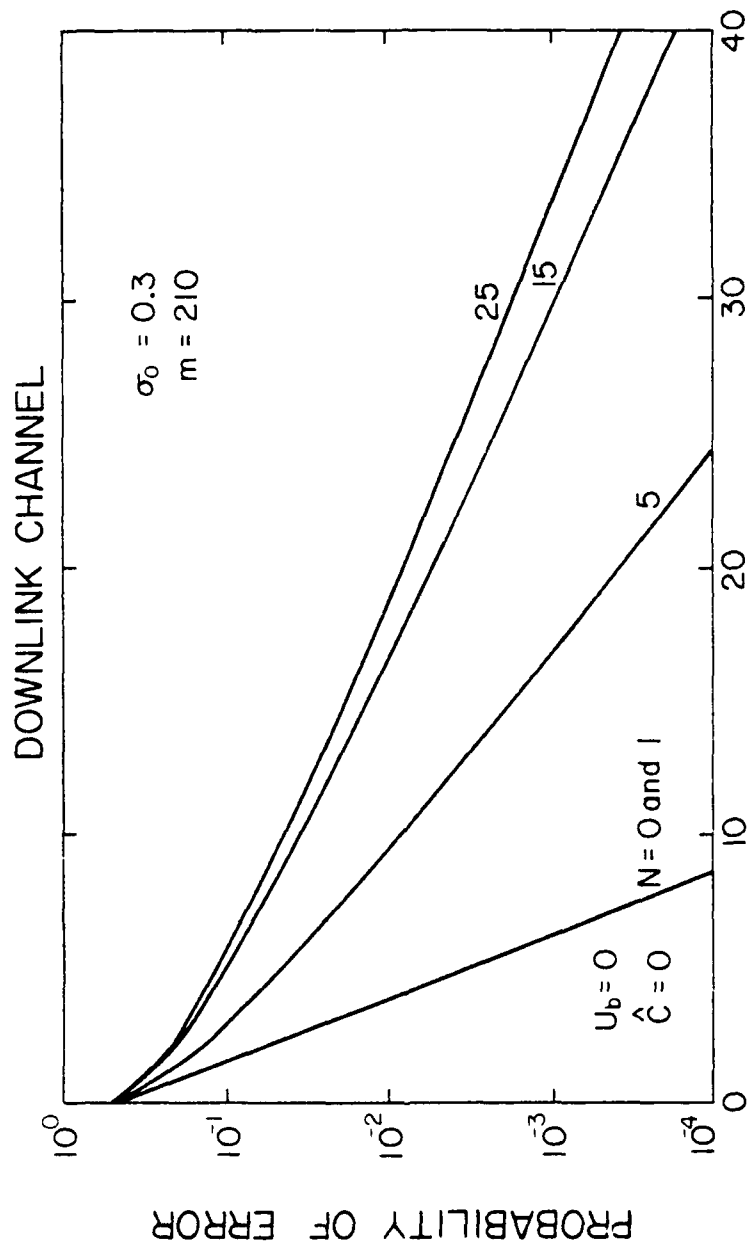


Figure 16. Probability of error vs. average signal photons for correlated fading on the downlink channel for multiple users in moderate lognormal fading. N is the number of interferers and σ_0 is the normalized variance of the log-intensity. Background noise and cross-correlation are zero.

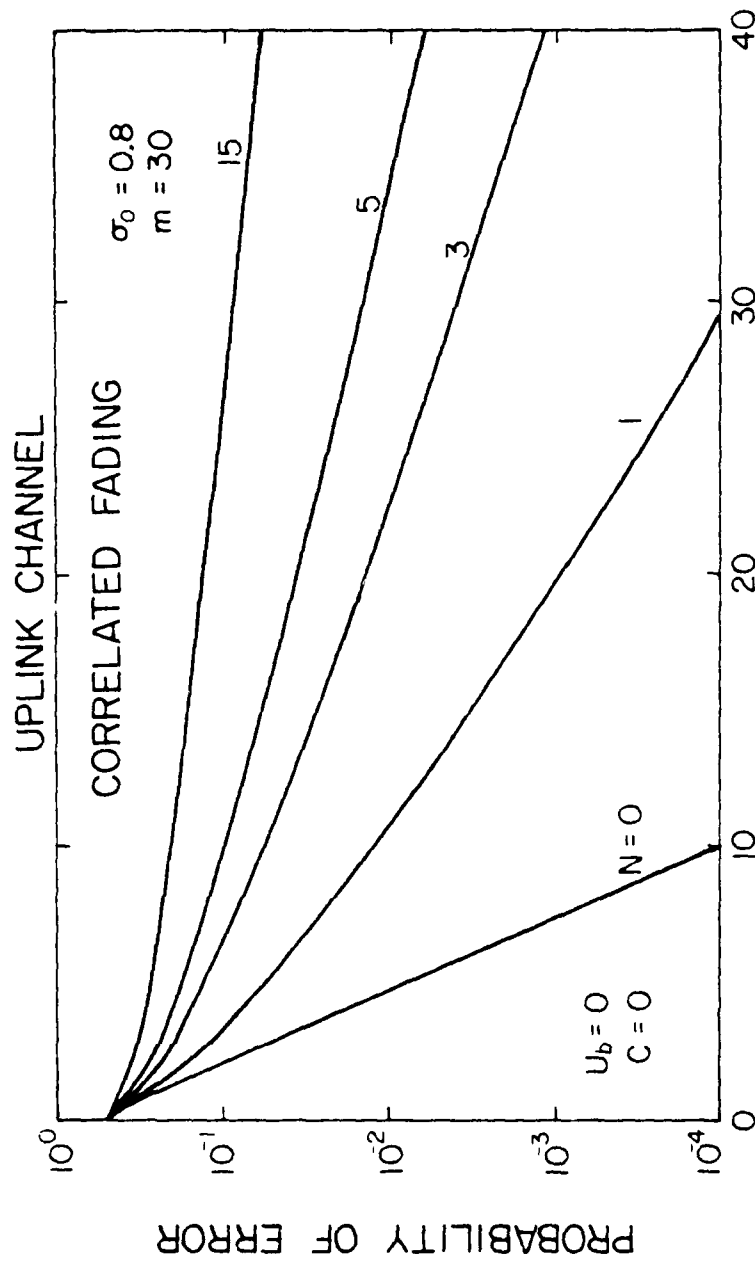


Figure 17. Probability of error vs. average signal photons for correlated fading on the uplink channel for multiple users in strong lognormal fading. N is the number of interferers and σ_0 is the normalized variance of the log-intensity. Background noise and cross-correlation are zero.

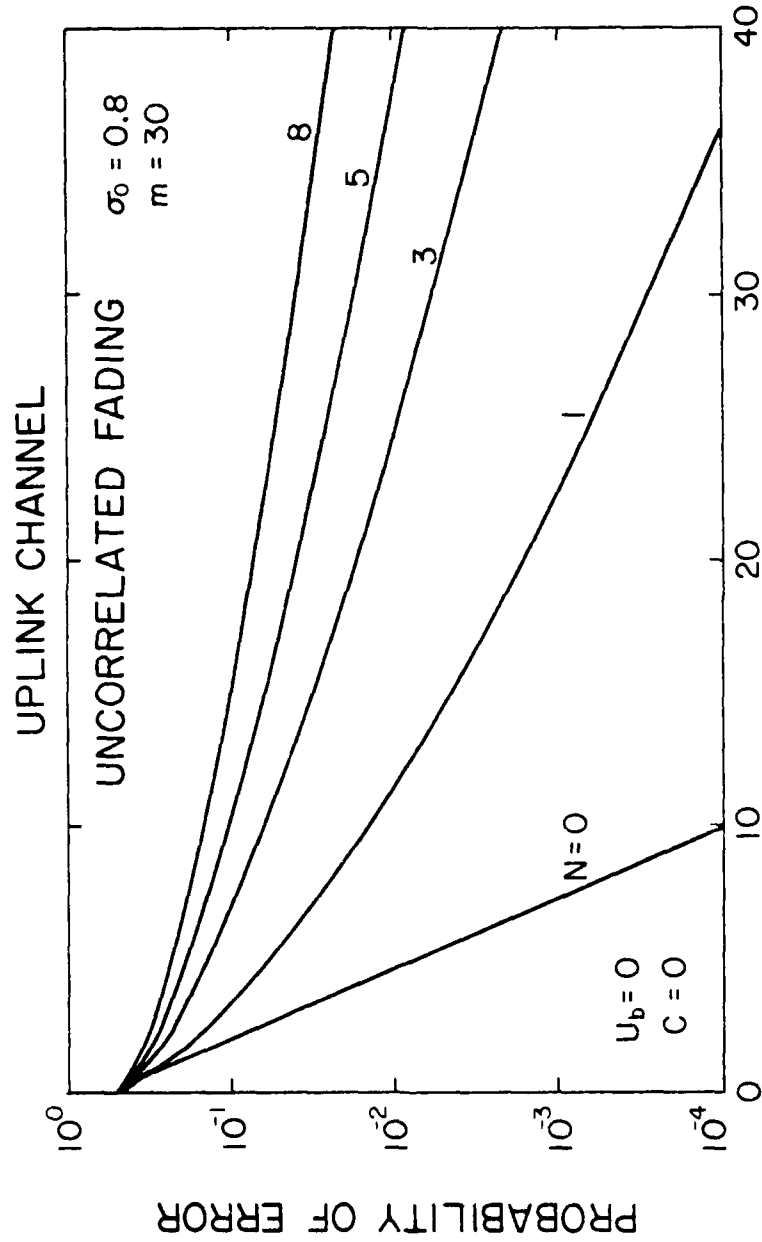
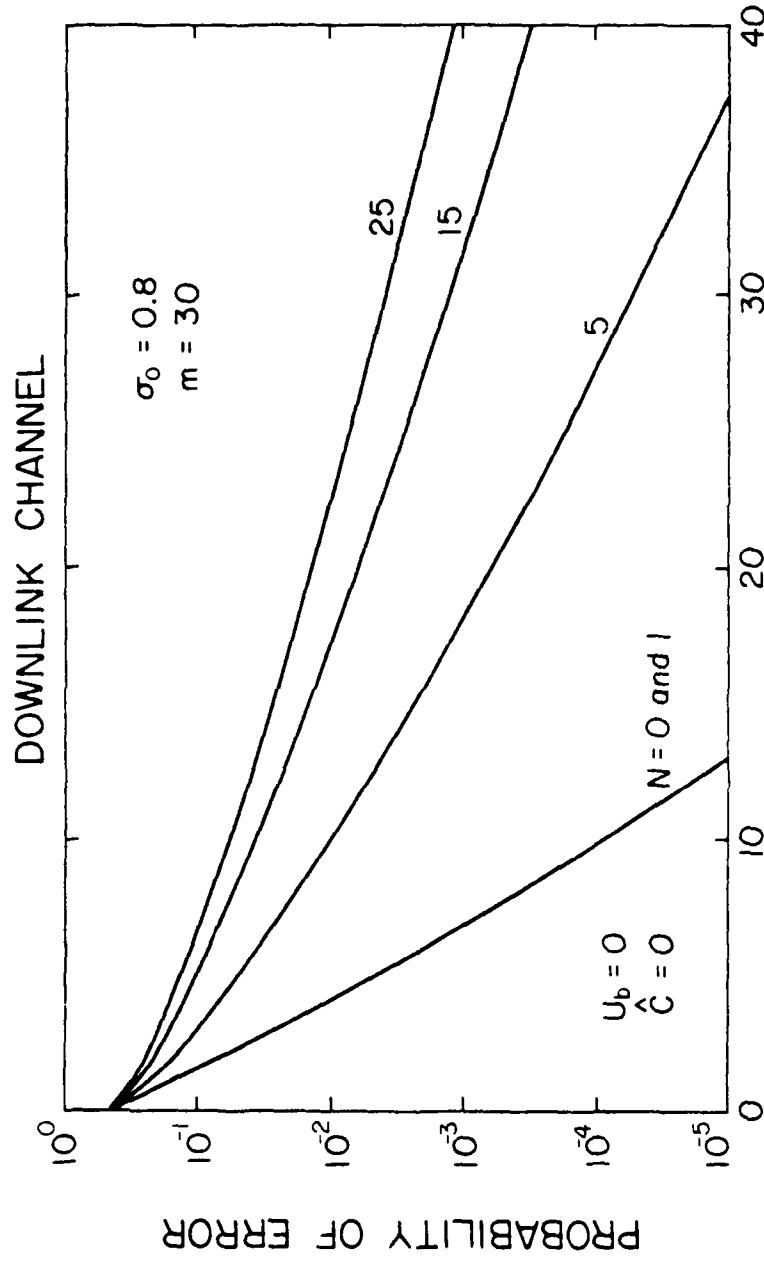


Figure 18. Probability of error vs. average signal photons for uncorrelated fading on the uplink channel for multiple users in strong lognormal fading. N is the number of interferers and σ_0 is the normalized variance of the log-intensity. Background noise and cross-correlation are zero.



AVERAGE SIGNAL PHOTONS (U)

Figure 19. Probability of error vs. average signal photons for correlated fading on the downlink channel for multiple users in strong lognormal fading. N is the number of interferers and σ_0 is the normalized variance of the log-intensity. Background noise and cross-correlation are zero.

CHAPTER 6

CONCLUSIONS

6.1 Conclusions/Summary of Results

In this study we have investigated the effects of cross-correlation interference and intensity fading on the error rate performance of an optical CDMA communication system. The analysis is restricted to direct detection, optical polarization modulation systems employing direct sequence modulation. We have developed generalized expressions for the conditional probability of error for the uplink and downlink channels of a multiaccess satellite repeater system. The expressions are given in terms of the number of active users, signal strength and the normalized cross-correlation between user codes. Numerical results were obtained for systems of two users, for both length 31 and 127 Gold codes. We have shown, from the analysis of a random sampling of the length 31 codes, that performance is significantly affected by the magnitude of the normalized second moment of the cross-correlation between codes. The smallest error rate was obtained from the code pair having the smallest second moment. Thus, in a multiaccess system in which the code length is fixed, interference can be minimized by using only those code pair combinations which give small second moments.

We have also shown, in comparing the performance of length 31 codes to length 127 codes, that the error probability decreases with increasing code length. This is not unexpected; for as the code length increases the code

properties more closely approximate those of the purely random codes for which the cross-correlation between codes is zero. This suggests that the effects of cross-correlation interference can be made increasingly small by using longer length codes.

The multiuser capabilities of the uplink and downlink channels were shown to differ significantly. Specifically, it was shown that the SNR in the uplink is asymptotically proportional to $1/K$, whereas in the downlink it is asymptotically proportional to $1/K^{1/2}$. This means that for K users at a given error rate in the uplink, K^2 users can be supported at the same error rate in the downlink. Therefore, the overall system capacity is limited by the capacity of the uplink. However, this problem can be solved, to some extent, by dividing the uplink coverage area into geographically separated cells. These cells could be spatially multiplexed in the satellite receiver by imaging each cell onto a photodiode element of an array of photodiodes. Then, by multiplexing K cells of K users, we could extend the capacity of the uplink to K^2 users. This effectively matches the capacity of the downlink for a comparable error rate. However, in this case "dead spots" between adjacent elements of the photodiode array would introduce some degradation.

In the analysis of fading effects on optical CDMA systems, we have used the Nakagami m -distribution as a generalized statistical model for fading on the received optical intensity. The average probability of error was derived for both correlated and uncorrelated fading in terms of the number of interfering users, the Nakagami fading parameter and the average number of photons counted in a bit interval. The results are valid for a noise-free system in which the

cross-correlation interference is negligible. Numerical results were obtained for both strong and moderate lognormal fading, using the Nakagami m -distribution to approximate the lognormal distribution. The results show a significant difference between the multiuser capabilities of the uplink and downlink channels under fading conditions. It was shown that the downlink performs consistently better than the uplink under all of the fading conditions evaluated. This margin of performance derives from the SNR advantage of the downlink, i.e., the SNR of the downlink is asymptotically proportional to $1/K^{1/2}$ while in the uplink is asymptotically proportional to $1/K$. As a consequence, the overall system capacity, under fading conditions, is limited by the capacity of the uplink.

Strong fading, which is characterized by small values of the fading parameter ($m < 5$), seriously degrades the performance of this system. However, moderate lognormal fading with intensity variances comparable to the measured data in the references [Tatarski, 1961] and [Axford, 1979], introduces moderate degradation and reasonable error rates for multiple users can be obtained by boosting the signal power.

6.2 Recommendations for Future Study

The general study of cross-correlation interference in optical CDMA systems offers many logical extensions. Some which closely parallel this study are,

1) Development of methods to improve the multiuser capability of the uplink channel.

The foregoing analysis showed that the weak link in the end-to-end performance of this system is the uplink channel. In spite of setting the cross-correlation to zero, each additional user to the uplink contributed a shot noise count equal to $U/2$. This noise count is directly proportional to the chip period of the multiple access code sequence. Therefore, an approach to reducing this noise contribution is to transmit shorter pulses; that is, instead of transmitting a constant carrier for the full chip period, transmit over only a fraction of the period. The receiver, gated to correlate over only a fraction of the chip period, would less likely be gated on at the same time that the interfering signal is present at the receiver input. We expect that if the carrier is gated to 1/100-th of a chip period, for example, the net effect would be a 1/100-th reduction in the shot noise contribution.

2) Development of a simple, easy to evaluate, approximation to the probability of error in terms of the moments for the cross-correlation values.

From a system's point of view, it would be invaluable to have a simple expression relating the gross behavior of the error probability as a function of the number of users, moments for the cross-correlation values and the signal intensity. System designers could then obtain an order of magnitude feel for the code lengths and signal powers required to support a given number of users at a specified error rate.

3) Analysis of the error rate performance of optical heterodyne detection systems.

Optical heterodyning is accomplished by adding the received intensity to a strong locally generated reference intensity and photodetecting the sum. The prime objective, as with conventional heterodyning, is to generate a frequency-translated version of the received field at a frequency which optimizes detection. If the local reference is strong enough, photoelectrons are emitted from the detectors at rates which are so high that the detector output signal is, in effect, a replica of the received field plus an additive Gaussian noise contribution from the reference intensity. In this case, the primary source of detection noise is that induced by the reference intensity; the shot noise generated by the signal and detector dark currents can be neglected. SNR comparisons of heterodyne and direct detection systems show that for certain frequency ranges heterodyning is preferred to direct detection [Fried, 1967]. Extending this study to include an analysis of optical heterodyne systems would provide a performance comparison between direct detection and heterodyne CDMA systems in the presence of cross-correlation interference and form a basis for choosing the appropriate receiver for a specific application.

4) Extension of the analysis technique to the study of cross-correlation interference in optical ranging systems which use direct sequence modulation.

Range measurements are made by determining the time required for a signal to propagate to an object and return. Optical ranging using DS modulation can be accomplished by modulating the polarization of the transmitted signal with a pseudorandom code sequence (ranging code). The return signal is a time-delayed version of the transmitted signal plus noise. The receiver, using correlation techniques, extracts the relative time delay and, indirectly, the range information. The analysis procedure applied to cross-correlation interference is also applicable to an optical DS ranging system. However, in this case, the autocorrelation properties of a particular ranging code, as opposed to the cross-correlation between different codes, is an important system consideration. By appropriately modeling the photoelectron emission rate of the return signal, we can analyze the effects of atmospheric fading, doppler shift, signal depolarization, or background radiation on range resolution and accuracy.

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APPENDIX A

COUNTING STATISTICS FOR THE RECEIVER OUTPUT

In this appendix we derive an alternative expression for the probability distribution of the photon count at the receiver output (3.4) in Chapter 3. The receiver output is analyzed as the difference between two independent Poisson random variables.

Let $R = R^+ - R^-$ be the difference between two independent Poisson random variables (R^+ and R^-). Then the probability distribution of R is given by Pratt (1969)

$$P_R(R=n) = \exp[-(E[R^+] + E[R^-])] \{E[R^-]/E[R^+]\}^{-n/2}$$

$$I_{|n|} \{2 E[R^+] E[R^-]\} \quad (A1)$$

where $E[\cdot]$ denotes expectation. To express the distribution of R in terms of its mean and variance, we note that

$$E[R] = E[R^+] - E[R^-] \quad (A2)$$

and

$$\text{Var}(R) = \text{Var}(R^+) + \text{Var}(R^-) \quad (A3)$$

where we have used the fact that the Poisson random variables are independent. Since the mean and variance of Poisson random variables are equal, (A3) can be written:

$$\text{Var}(R) = E[R^+] + E[R^-] \quad (\text{A4})$$

Solving (A2) and (A4) for $E[R^+]$ and $E[R^-]$, we get

$$E[R^+] = 1/2 \{ \text{Var}(R) + E[R] \} \quad (\text{A5})$$

$$E[R^-] = 1/2 \{ \text{Var}(R) - E[R] \} \quad (\text{A6})$$

The probability distribution of R can now be written:

$$P_R(R=n) = \exp \{-\text{Var}(R)\} \{ (\text{Var}(R) - E[R]) / (\text{Var}(R) + E[R]) \}^{-n/2} \\ I_{|n|} \{ [\text{Var}(R)^2 - E[R]^2]^{1/2} \} \quad (\text{A7})$$

APPENDIX B

DERIVATION OF THE CONDITIONAL PROBABILITY OF ERROR

In this appendix we derive the conditional probability of error for an arbitrary number of independent users with different signal intensities. The error expression is conditioned on the cross-correlation C_1 , which is defined as the cross-correlation between the code of the desired user and the code-modulated data signal of the i -th interferer. This derivation is facilitated by the fact that the counting statistics of the receiver output can be expressed in terms of its mean and variance. This is shown in Appendix A. Therefore, as a first step, we derive the mean and variance of the receiver output, defined by (3.1). We designate the desired signal as $b_1(t)$ with code sequence $a_1(t)$, and to simplify notation, use the designation $E[R]$ to represent the conditional mean $E[R|C_1]$.

B.1 Mean and Variance/Uplink Channel

For K independent users, the photodetector outputs can be written

$$Y(t) = \sum_{i=1}^K Y_i(t) \quad (B1)$$

and

$$X(t) = \sum_{i=1}^K X_i(t) \quad (B2)$$

where $Y_i(t)$ and $X_i(t)$ are the photodetector emissions due to the i -th user. The time-varying rate parameters of the emission processes are proportional to the instantaneous polarity of the coded signals. These rate parameters are

$$\lambda_{y_i} = \begin{cases} \lambda_i a_i(t-\tau_i) b_i(t-\tau_i) + \lambda_b/K & a_i b_i > 0 \\ \lambda_b/K & a_i b_i < 0 \end{cases}$$

$$\lambda_{x_i} = \begin{cases} \lambda_b/K & a_i b_i > 0 \\ -\lambda_i a_i(t-\tau_i) b_i(t-\tau_i) + \lambda_b/K & a_i b_i < 0 \end{cases} \quad (B3)$$

where λ_i is the photon counting rate of the i -th user and λ_b is the rate parameter for the background noise, shown equally distributed among the K users. The conditional mean of (3.1) is

$$E[R] = \sum_{i=1}^K \int a_i(t) \{E[Y_i] - E[X_i]\} dt \quad (B4)$$

Since the mean value of a Poisson process is equal to the rate parameter, we have

$$E[R] = \sum_{i=1}^K \int_0^{T_d} a_1(t) [\lambda_{y_i} - \lambda_{x_i}] dt \quad (B5)$$

Substituting (B3) into (B5), we get

$$E[R] = \sum_{i=1}^K \lambda_i \int_0^{T_d} a_1(t) a_i(t-\tau_i) b_i(t-\tau_i) dt \quad (B6)$$

This is simplified by noting that $a_1^2(t)$ is identically one and b_1 is constant over the integration period. The conditional mean reduces to

$$E[R] = b_1 U_1 + \sum_{i=2}^K U_i C_i \quad (B7)$$

where

$$C_i = 1/T_d \int_0^{T_d} a_1(t) a_i(t-\tau_i) b_i(t-\tau_i) dt \quad (B8)$$

is the normalized cross correlaton and $U_1 = \lambda_1 T_d$.

The variance is obtained in a similar manner

$$\begin{aligned} \text{Var}(R) = & \sum_{j=1}^K \sum_{i=1}^K \int_0^{T_d} \int_0^{T_d} a_1(t_1) a_1(t_2) \{ E[\tilde{Y}_i(t_1) \tilde{Y}_j(t_2)] \\ & + E[\tilde{X}_i(t_1) \tilde{X}_j(t_2)] - E[\tilde{X}_i(t_1) \tilde{Y}_j(t_2)] \\ & - E[\tilde{X}_j(t_2) \tilde{Y}_i(t_1)] \} dt_1 dt_2 \end{aligned} \quad (B9)$$

where for convenience we have defined

$$\tilde{Y}_i(t) = Y_i(t) - E[Y_i(t)]$$

and

$$\tilde{X}_i(t) = X_i(t) - E[X_i(t)] \quad (\text{B10})$$

Under the assumption that the user signals are mutually independent and also independent of the background noise, (B9) simplifies to

$$\text{Var}(R) = \sum_{i=1}^K \int_0^{T_d} \int_0^{T_d} a_1(t_1) a_1(t_2) [C_Y(t_1, t_2) + C_X(t_1, t_2)] dt_1 dt_2 \quad (\text{B11})$$

$C_Y(t_1, t_2)$ and $C_X(t_1, t_2)$ are the covariance functions associated with the Poisson processes $Y_i(t)$ and $X_i(t)$. They are given by [Papoulis, 1965]

$$C(t_1, t_2) = \lambda(t_2) \delta(t_1 - t_2) \quad (\text{B12})$$

Substituting (B12) into (B11), we get

$$\text{Var}(R) = \sum_{i=1}^K \int_0^{T_d} [\lambda_{y_i} + \lambda_{x_i}] dt \quad (\text{B13})$$

From (B3) we obtain

$$\lambda_{y_i} + \lambda_{x_i} = 2\lambda_b / K + \lambda_i |a_i(t-\tau_i)b_i(t-\tau_i)| \quad (\text{B14})$$

The factor $|a_i(t-\tau_i)b_i(t-\tau_i)|$ is identically one for all values of $a_i b_i$; therefore, the variance reduces to

$$\text{Var}(R) = 2U_b + \sum_{i=1}^K U_i \quad (\text{B15})$$

where $U_i = \lambda_i T_d$ and $U_b = \lambda_b T_d$.

B.2 Mean and Variance/Downlink Channel

For the downlink channel, the rates at which photoelectrons are emitted from the photodetectors in the receiver are proportional to the instantaneous magnitude of the multiplexed signals. These rates are

$$\lambda_y = \begin{cases} \lambda \sum_{i=1}^K a_i(t-\tau_i) \hat{b}_i(t-\tau_i) + \lambda_b & \sum a_i b_i > 0 \\ \lambda_b & \sum a_i b_i < 0 \end{cases}$$

$$\lambda_x = \begin{cases} \lambda_b & \sum a_i b_i > 0 \\ -\lambda \sum_{i=1}^K a_i(t-\tau_i) \hat{b}_i(t-\tau_i) + \lambda_b & \sum a_i b_i < 0 \end{cases} \quad (\text{B16})$$

where λ is a constant photon counting rate which is multiplied by the instantaneous magnitude of the multiplexed signals to form the magnitude of the transmitted signal.

The conditional mean of R is determined in a manner similar to that of the previous case. The result is

$$E[R] = b_1 U + U \sum_{i=2}^K C_i \quad (\text{B17})$$

where $U = \lambda T_d$.

If we apply the same independence assumptions that were used in the previous case, the variance reduces to

$$\text{Var}(R) = \int_0^{T_d} [\lambda_y(t) + \lambda_x(t)] dt \quad (\text{B18})$$

Substituting (B16) into (B18), we get

$$\text{Var}(R) = \int_0^{T_d} [2\lambda_b + \lambda \left| \sum_{i=1}^K a_i(t-\tau_i) \hat{b}_i(t-\tau_i) \right|] dt \quad (\text{B19})$$

or

$$\text{Var}(R) = 2U_b + K'U \quad (\text{B20})$$

where

$$K' = 1/T_d \int_0^{T_d} \left| \sum_{i=1}^K a_i(t-\tau_i) \hat{b}_i(t-\tau_i) \right| dt \quad (B21)$$

B.3 Derivation of the Error Probability

If $b_i = \pm 1$ are equally likely, the conditional probability of error, defined at the receiver output R is

$$\begin{aligned} P(E|C_i) &= 1/2 \Pr(R>0|C_i, -1) + 1/2 \Pr(R<0|C_i, +1) \\ &\quad + 1/4 \Pr(R=0|C_i, -1) + 1/4 \Pr(R=0|C_i, +1) \end{aligned} \quad (B22)$$

where C_i is the cross-correlation given in (B8). Denote the conditional counting probability of the receiver output by $P_R(k|C_i, b_i)$, then the conditional probability of error can be written

$$\begin{aligned} P(E|C_i) &= 1/2 \left[1 + \sum_{k=0}^{\infty} P_R(k|C_i, -1) - \sum_{k=0}^{\infty} P_R(k|C_i, +1) \right. \\ &\quad \left. + 1/2 P_R(0|C_i, +1) - 1/2 P_R(0|C_i, -1) \right] \end{aligned} \quad (B23)$$

From Appendix A, P_R can be expressed

$$P_R(k|C_1) = \exp\{-\text{Var}(R)\} \{(\text{Var}(R)-E[R])/(\text{Var}(R)+E[R])\}^{-k/2}$$

$$I_{|k|} \{ \sqrt{\text{Var}(R)^2 - E[R]^2} \} \quad (\text{B24})$$

Consider first the derivation of the uplink probability of error. By substituting the mean (B7) and the variance (B15) of R into (B24), we obtain the receiver counting statistics for the uplink

$$P(k|C_1, +1) = \exp\{-(a^2 + b^2)/2\} (b/a)^k I_{|k|}(ab) \quad (\text{B25})$$

and

$$P(k|C_1, -1) = \exp\{-(c^2 + d^2)/2\} (d/c)^k I_{|k|}(cd) \quad (\text{B26})$$

where

$$\begin{aligned} a &= [2U_b + \sum_{i=2}^K U_i (1-C_i)]^{1/2} \\ b &= [2U_b + 2U_1 + \sum_{i=2}^K U_i (1+C_i)]^{1/2} \\ c &= [2U_b + 2U_1 + \sum_{i=2}^K U_i (1-C_i)]^{1/2} \\ d &= [2U_b + \sum_{i=2}^K U_i (1+C_i)]^{1/2} \end{aligned} \quad (\text{B27})$$

When (B25) and (B26) are substituted into (B23) and we use the series expansion of the Marcum's Q-function [Marcum, 1960] given by

$$Q(a,b) = \exp\{-(a^2 + b^2)/2\} \sum_{j=0}^{\infty} (a/b)^j I_j(ab) \quad (\text{B28})$$

to simplify, we get

$$P(E|C_1) = 1/2[1 - Q(b,a) + Q(a,b)] \\ + 1/4 \exp\{-(2U_b + \sum_{i=1}^K U_i)\} \{I_0(ab) - I_0(cd)\} \quad (\text{B29})$$

where the arguments of the Q-function are given in (B27).

The probability of error for the downlink is derived in a similar manner but using the mean (B17) and the variance (B20) of the receiver output for the downlink. The significant difference however, is that we must account for the fact that K' is also conditioned on $b_1(t)$ and C_1 . Hence, in the generalized expressions for the conditional probability of error for the downlink we denote this conditioning by defining

$$K'_+ \equiv K' \text{ given } b_1 = +1$$

and

$$K'_- \equiv K' \text{ given } b_1 = -1$$

This dependence is shown explicitly in the evaluation of K' for some special cases in Appendix C. The conditional probability of error for the downlink is

$$\begin{aligned}
 P(E|\hat{C}_i) &= 1/2[1 - Q(b', a') + Q(d', c')] \\
 &\quad + 1/4 \exp[-(2U_b + K'_+ U)] I_0(a' b') \\
 &\quad - 1/4 \exp[-(2U_b + K'_- U)] I_0(c' d')
 \end{aligned} \tag{B30}$$

where

$$\begin{aligned}
 a' &= [2U_b + U(K'_+ - K) + U \sum_{i=2}^K (1 - \hat{C}_i)]^{1/2} \\
 b' &= [2U_b + 2U + U(K'_+ - K) + U \sum_{i=2}^K (1 + \hat{C}_i)]^{1/2} \\
 c' &= [2U_b + 2U + U(K'_- + K) + U \sum_{i=2}^K (1 - \hat{C}_i)]^{1/2} \\
 d' &= [2U_b + U(K'_- + K) + U \sum_{i=2}^K (1 + \hat{C}_i)]^{1/2}
 \end{aligned} \tag{B31}$$

APPENDIX C

SIMPLIFICATION OF THE EXPRESSION FOR K'

In this appendix, we derive two simplified expressions for K': 1) for a single interferer and 2) for an arbitrary number of interferers when the cross-correlation equals zero.

C.1 Single Interferer

For a single interferer, K' is given by

$$K' = 1/T_d \int_0^{T_d} |a_1(t) \hat{b}_1(t) + a_2(t-\tau_2) \hat{b}_2(t-\tau_2)| dt \quad (C1)$$

where, since time delays are referenced to $a_1(t)$, we have set $\tau_1 = 0$. If $a_1 \hat{b}_1$ and $a_2 \hat{b}_2$ are of the same polarity

$$a_1 \hat{b}_1 + a_2 \hat{b}_2 = 2 \quad (C2)$$

On the other hand, if $a_1 \hat{b}_1$ and $a_2 \hat{b}_2$ are of opposite polarity

$$a_1 \hat{b}_1 + a_2 \hat{b}_2 = 0 \quad (C3)$$

We can conveniently express these relations with the single form

$$1 + \hat{a}_1 \hat{b}_1 \hat{a}_2 \hat{b}_2 \quad (C4)$$

Substituting the above into (C1), we get

$$K' = 1/T_d \int_0^{T_d} [1 + \hat{a}_1(t) \hat{b}_1(t) \hat{a}_2(t-\tau_2) \hat{b}_2(t-\tau_2)] dt \quad (C5)$$

Since $\hat{b}_1(t)$ is constant over the bit interval, it can be brought outside the integral. Performing the integration gives

$$K' = 1 + \hat{b}_1 [1/T_d \int_0^{T_d} \hat{a}_1(t) \hat{a}_2(t-\tau_2) \hat{b}_2(t-\tau_2) dt] \quad (C6)$$

The factor in brackets is the normalized cross-correlation \hat{C}_2 between \hat{a}_1 and $\hat{a}_2 \hat{b}_2$ for delay τ_2 . Therefore, K' takes the particularly simple form

$$K' = 1 + \hat{b}_1 \hat{C}_2 \quad (C7)$$

C.2 Multiple Interferers

In the limit as $\hat{C}_i \rightarrow 0$, the general expression for K' takes the form of a time average of the absolute value of the sum of finitely many independent random variables which take on the values +1 and -1. The chance fluctuations in

the value of the summation in the integrand of K' can be modelled analytically as a random walk process with the integrand taken to be the absolute value of the path length from the origin to the final position of a man or particle performing a random walk [Feller, 1966]. Assuming a statistically stationary system, K' can then be analyzed as the expectation of the absolute value of the path length after K random steps, where K corresponds to the number of simultaneous users in the downlink channel.

Let Z_k be the path length after K steps then the probability distribution of $|Z_k|$ is given by

$$\Pr(|Z_k| = n) = \epsilon_n \binom{K}{(K+n)/2} (1/2)^K \quad (C8)$$

where $n = 0, 1, 2, \dots, K$, and the binomial coefficient is taken to be zero unless $(K+n)/2$ is an integer $< K$. The Neumann factor ϵ_n is 1 if $n=0$ and 2 otherwise.

For K even we have

$$K' = E[|Z_k|] = \sum_{n=1}^{K/2} 4n \binom{K}{n+K/2} (1/2)^K \quad (C9)$$

which by a simple change of variable can be written as

$$K' = 4 \sum_{n=K/2+1}^K \binom{K}{n} (1/2)^K - 2K \sum_{n=K/2+1}^K \binom{K}{n} (1/2)^K \quad (C10)$$

Using the properties of the binomial distribution to simplify the summations in the above equation, we get

$$K' = K \binom{K}{K/2} (1/2)^K \quad (C11)$$

The derivation for K odd follows in a similar manner; the result is

$$K' = K \begin{pmatrix} K-1 \\ (K-1)/2 \end{pmatrix} (1/2)^{K-1} \quad (C12)$$

APPENDIX D

THE PROBABILITY OF ERROR FOR CORRELATED AND UNCORRELATED FADING

In this appendix, we derive simplified expressions for the average probability of error for multiple users on the uplink channel, for correlated and uncorrelated fading.

D.1 Correlated Fading

The average probability of error for correlated fading on the uplink channel is obtained by by integrating the conditional probability of error (3.18) over the probability density for the fading intensity

$$P(E) = \int_0^{\infty} P(E|U) p(U) dU \quad (D1)$$

where $p(U)$ is the probability density for the fading intensity, given by

$$p(U) = (m/\langle U \rangle)^m \{U^{m-1} / \Gamma(m)\} \exp\{-m(U/\langle U \rangle)\} \quad (D2)$$

for $U \geq 0$ and

$$P(E|U) = 1/2 [1 - Q(b,a) + Q(a,b)] \quad (D3)$$

where $Q(a,b)$ is the Marcum's Q-function [Marcum, 1960], with arguments

$$a = [(K-1)U]^{1/2}$$

and

$$b = [(K+1)U]^{1/2} \quad (D4)$$

Using the symmetry relation [Helstrom, 1960]

$$Q(a,b) + Q(b,a) = 1 + \exp\{- (a^2 + b^2)/2\} I_0(ab) \quad (D5)$$

the series expansions for

$$Q(a,b) = \exp\{- (a^2 + b^2)/2\} \sum_{k=0}^{\infty} (a^2/2)^k / k! \sum_{j=0}^k (b^2/2)^j / j! \quad (D6)$$

and the modified Bessel function

$$I_0(a) = \sum_{k=0}^{\infty} (a/2)^{2k} / k! k! \quad (D7)$$

in (D3), we get

$$P(E U) = \exp(-KU) \sum_{p=0}^{\infty} [U(K-1)/2]^p / p! \\ \cdot \sum_{j=0}^p [U(K+1)/2]^j / j! \gamma_{j,p} \quad (D8)$$

where $\gamma_{j,p} = 1/2$ for $j=p$ and 1 otherwise.

If (D8) and (D2) are substituted into (D1) the required integration becomes

$$P(E) = (m/\langle U \rangle)^m / \Gamma(m) \sum_{p=0}^{\infty} \sum_{j=0}^p [(K-1)/2]^p / p! [(K+1)/2]^j / j! \gamma_{j,p} \\ \cdot \int_0^{\infty} U^{j+p+m-1} \exp\{-(K+m/\langle U \rangle)U\} dU \quad (D9)$$

Using the integral identity [Gradshteyn, 1965]

$$\int_0^{\infty} x^{p-1} \exp(-qx) dx = \Gamma(p) q^{-p} \quad (D10)$$

we get

$$P(E) = [1 + K\langle U \rangle/m]^{-m} \\ \cdot \sum_{p=0}^{\infty} [(K-1)/2K]^p \delta^p [(m)_p / p!] \\ \cdot \sum_{j=0}^p [(K+1)/2K]^j \delta^j [(m+p)_j / j!] \gamma_{j,p} \quad (D11)$$

where

$$\delta = [1 + m/K\langle U \rangle]^{-1} \quad (D12)$$

D.2 Uncorrelated Fading

The equation for the average probability of error, expressed in terms of the counting probabilities for the desired signal S and the interference Z, is (5.20)

$$P(E) = \sum_{k=-\infty}^{\infty} P_Z(k) \sum_{j=0}^k P_S(j) \gamma_{j,k} \quad (D13)$$

where, $\gamma_{j,k}$ is 1/2 for $j=k$ and 1 otherwise. P_S is the desired signal counting probability (5.24)

$$P_S(j) = \Gamma(m+j)/\Gamma(m) j! [1 + \langle U \rangle / m]^{-m} \beta^j$$

$$\text{for } j \geq 0. \quad (D14)$$

where $\langle U \rangle$ is the mean signal count, m is the Nagakami fading parameter and

$$\beta = [1 + m/\langle U \rangle]^{-1} \quad (D15)$$

Since P_S is zero for $j < 0$, the summation in (D13) is reduced to a summation over the positive range of the index k . P_Z is the counting probability for the interference, which for the positive integers is (5.31)

$$P_2(k) = \Gamma(k+Nm)/\Gamma(Nm) k! [1 + \langle U \rangle/m]^{-Nm} (\beta/2)^k \cdot {}_2F_1\{(k+Nm)/2, (k+Nm+1)/2; k+1; \beta^2\} \quad (D16)$$

where N is the number of interferers

$$\beta = [1 + m/\langle U \rangle]^{-1}$$

and ${}_2F_1$ is a hypergeometric function defined by Abramowitz, (1965)

$${}_2F_1(a,b;c;t) = \sum_{p=0}^{\infty} \{(a)_p (b)_p / (c)_p\} t^p/p! \quad (D17)$$

The notation $(a)_p$ is the Pochhammer symbol for the product

$$(a)_p = a(a+1)(a+2)\dots(a+p-1); \quad (a)_0 = 1$$

The Nakagami fading parameter m and the mean signal count $\langle U \rangle$ are the same as those for the desired signal.

By substituting the counting probabilities, P_s and P_z , into the error expression, we get

$$P(E) = [1 + \langle U \rangle/m]^{-m(N+1)} \sum_{p=0}^{\infty} (Nm)_p/p! (\beta/2)^p \cdot {}_2F_1\{(p+Nm)/2, (p+Nm+1)/2; p+1; \beta^2\} \sum_{j=0}^p (m)_j/j! \beta^j \gamma_{j,p} \quad (D18)$$

where we have used the identity [Hansen, 1975]

$$(m)_k = \Gamma(m+k)/\Gamma(m) \quad (D19)$$

to simplify notation.

Using the relation [Abramowitz, 1965]

$$\pi^{1/2} \Gamma(2p) = 2^{2p-1} \Gamma(p)\Gamma(p+1/2) \quad (D20)$$

we can write the hypergeometric function in (D18) as follows,

$$\begin{aligned} {}_2F_1\left\{\frac{(p+Nm)}{2}, \frac{(p+Nm+1)}{2}; p+1; \beta^2\right\} = \\ \frac{1}{\Gamma(p+Nm)} \sum_{r=0}^{\infty} \left[\frac{\Gamma(p+Nm+2r)}{(p+1)_r r!} \right] (\beta/2)^{2r} \end{aligned} \quad (D21)$$

Substituting (D21) into (D18) and applying the identity [Hansen, 1975]

$$\sum_{p=0}^{\infty} \sum_{r=0}^{\infty} f(r,p) = \sum_{p=0}^{\infty} \sum_{r=0}^p f(r,p-r) \quad (D22)$$

to reorder the summations, we obtain

$$\begin{aligned} P(E) = [1 + \langle U \rangle / m]^{-m(N+1)} \sum_{r=0}^{\infty} \sum_{p=0}^r \sum_{j=0}^{r-p} \gamma_{j,r-p} \\ \cdot \beta^{r+p+j} (\beta/2)^{r+p} \left[\frac{\Gamma(p+Nm+r)}{\Gamma(Nm)} \right] \left[\frac{(m)_j}{j! p! r!} \right] \end{aligned} \quad (D23)$$

This can be reduced further using the identity (D19); the resulting average probability of error is

$$\begin{aligned}
 P(E) &= [1 + \langle U \rangle / m]^{-(N+1)m} \sum_{r=0}^{\infty} (Nm)_r / r! (\beta/2)^r \\
 &\cdot \sum_{p=0}^r (r+Nm)_p / p! (\beta/2)^p \sum_{j=0}^{r-p} (m)_j / j! \beta^j \gamma_{j,r-p} \quad (D24)
 \end{aligned}$$

VITA

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