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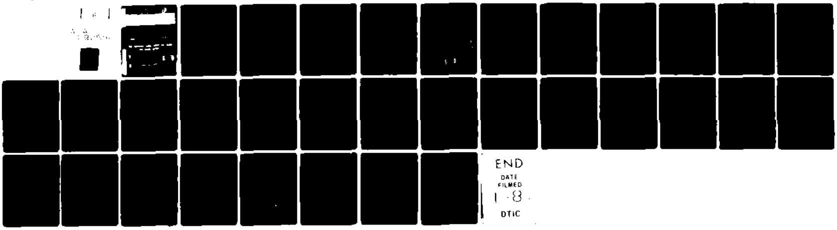

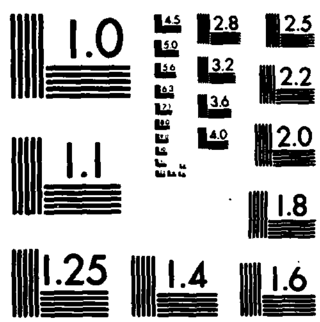

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John F. Zingg

JOHN F. ZINGG
Project Engineer
Fire Control Technology Branch

Lester McFawn

LESTER MCFAWN, Chief
Fire Control Technology Group
Fire Control Branch

FOR THE COMMANDER

Marvin Spector

MARVIN SPECTOR, Chief
Fire Control Branch
Reconnaissance & Weapon Delivery Div.

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The missile-aircraft pursuit-evasion problem is formulated by a three-dimensional linearized kinematic model with bounded control. The formulation is valid both for the optimal control (against a known adversary strategy) and the zero sum differential game versions. Assuming perfect information the linearized kinematic model yields for both versions a solution which can be implemented in real-time for airborne application. The avoidance of a known pursuer by an evader who has no state information is solved by a stochastically optimal periodical maneuver. Other examples of imperfect information are briefly (CONT)

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THREE DIMENSIONAL OPTIMAL PURSUIT AND EVASION WITH BOUNDED CONTROLS*

J. Shinar** and S. Gutman***

Technion - Israel Institute of Technology

Haifa, Israel

T.A.E. Report No. 381

* Research partially sponsored by AFSC Contract No. F49620-79-C-0135

** Associate Professor, Department of Aeronautical Engineering

*** Senior Lecturer, Faculty of Mechanical Engineering.

ABSTRACT

The missile-aircraft pursuit-evasion problem is formulated by a three-dimensional linearized kinematic model with bounded control. The formulation is valid both for the optimal control (against a known adversary strategy) and the zero sum differential game versions. Assuming perfect information the linearized kinematic model yields for both versions a solution which can be implemented in real-time for airborne application. The avoidance of a known pursuer by an evader who has no state information is solved by a stochastically optimal periodical maneuver. Other examples of imperfect information are briefly discussed.

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1. INTRODUCTION

The research effort to solve the missile-aircraft pursuit-evasion problem has been directed towards two important applications: (a) optimization of the control law for guided missiles; (b) enhancement of aircraft survivability by missile avoidance. The problem can be formulated either as a one-sided optimization, i.e., optimal pursuit[1,2] and optimal evasion,[3-6] or as a zero-sum differential game.[7-11] Regardless of the formulation the problem is inherently complex. Thus the solution has to be attempted in 3 consecutive phases:

1. *Modelling,* which preserves the essential features of the original problem;
2. *Analytical solution* of the simplified, but representative mathematical model;
3. *Implementation* of the results derived from the analysis.

The objective of this paper is to discuss all 3 phases of the solution and to present some recently obtained results.

The missile-aircraft engagement is characterized - unlike other pursuit-evasion problems (as the "homicidal chauffer," the game of two cars,[12] etc.) - by a "fast" pursuer ($V_P/V_E > 1$), which also has a higher lateral acceleration capability ($a_P/a_E > 1$). Analyses based on pure kinematic models (neglecting vehicle dynamic response) have demonstrated[13,14] that, due to such superiority of the pursuer, *point capture* of the evader can be guaranteed. Yet whenever pursuer dynamics has been taken into account,[3-6,9] it has been shown that a *non-zero miss distance* can be achieved by appropriate evasive maneuvering.

Moreover, a recent study[6] clearly indicated that optimal missile avoidance requires a three-dimensional strategy.

Thus, two attractive and frequently used simplifying assumptions as: (i) neglecting pursuer dynamics,[7-9,13,14] (ii) two-dimensional analysis,[1-5,9,10,13] are not made in the mathematical model used in this paper. Fortunately, the very nature of the missile-aircraft engagement does justify the trajectory linearization used in many studies.[1-4,6-11]

As a consequence of these major modelling considerations the missile-aircraft pursuit-evasion is formulated and solved in the following sections as a 3-D, linear differential game. These sections also include brief comparisons with the one-sided optimal control formulation.

The assumptions used in the analysis can be summarized as:

1. Both pursuer and evader are considered as point-mass vehicles.
2. The speed of each vehicle is constant, the pursuer being the faster ($V_P/V_E > 1$).
3. The relative motion is three-dimensional (See Fig. 1).
4. Gravity, having no effect on the relative trajectory, is neglected.
5. The initial conditions of the pursuit are near to a collision course (See Fig.2).
6. Relative trajectory can be linearized around the initial line of sight vector.
7. The lateral acceleration commands of both vehicles are bounded with circular vectograms perpendicular to the respective velocity vectors ($a_P/a_E > 1$).

8. The pursuer's response to its acceleration command is approximated by single time constant τ_p.
9. Evader dynamics is neglected for sake of simplicity. (It has been shown[4,6] that evader dynamics has only a non-qualitative secondary effect on missile avoidance).
10. The performance index of the problem is the miss distance (distance of closest approach).
11. There exists perfect (complete and instantaneous information on the state variables and the parameters of the problem. This assumption is mandatory to obtain a solution in the deterministic sense.

Assumptions 7 and 8 present a new formulation of the three-dimensional pursuit-evasion game compared to previous linear quadratic mathematical models neglecting pursuer dynamics.[7,8] The validity of the other assumptions is discussed in detail in previous papers dealing with optimal evasion control.[4,6]

The generalized analytical solution for this class of problems, which has been developed in recent papers,[11,15] is briefly summarized in Section 2 and applied to the three-dimensional pursuit evasion problem of interest, defined by the above listed assumptions, in Section 3.

In Section 4 the characteristics of this entirely new solution of the deterministic linearized differential game are discussed and in the sequel the implementation of the optimal strategies is considered.

Since implementation depends on the available information, the difficulties created by non-complete information are examined. In Section 5 an example of a recent stochastically optimal solution is presented for missile avoidance without state information. Other cases of imperfect information are also discussed briefly.

2. MATHEMATICAL ANALYSIS

A. Two-Person Zero-Sum Linear Differential Games.

It is assumed that the reader is familiar with basic results in two-person zero-sum differential games.[12 16] Let a game of fixed duration be defined by

$$\left.\begin{aligned} &\dot{x} = A(t)x + B(t)u + C(t)v,\ x(0) = x_0 \\ &u \in U \quad , \quad v \in V \\ &J = \|Dx(T)\| \end{aligned}\right\} \tag{1}$$

where matrices A, B, C, D, have proper dimensions and $A(\cdot)$, $B(\cdot)$, $C(\cdot)$ are continuous.

It is required to find among all admissible strategies $\{p(\cdot), e(\cdot)\}$, such that $u(t) = p(x(t),t)$, $v(t) = e(x(t),t)$, an optimal strategy pair $\{p^*(\cdot), e^*(\cdot)\}$ satisfying the saddle point inequality

$$J(x,t,p^*,e) \leq J(x,t,p^*,e^*) \triangleq J^*(x,t) \leq J(x,t,p,e^*) \tag{2}$$

In order to simplify (1), let

$$\left.\begin{aligned} &y = D\ (T,t)x \\ &\mathcal{B}(T,t) = D\Phi(T,t)B \\ &\mathcal{C}(T,t) = D\Phi(T,t)C \end{aligned}\right\} \tag{3}$$

where $\Phi(T,t)$ satisfies

$$\frac{d}{dt}\,\Phi(T,t) = -\,\Phi(T,t)A(t) \tag{4}$$

By (3) the game (1) is transformed to

$$\left.\begin{aligned} &\dot{y} = \mathcal{B}(T,t)u + \mathcal{C}(T,t)v \quad , \quad y(0) = y_0 \\ &u \in U \quad , \quad v \in V \\ &J = \|y(T)\| \end{aligned}\right\} \tag{5}$$

For this formulation a candidate optimal strategy pair is given[11] by

$$\left.\begin{aligned} &\underset{u \in U}{\mathrm{Min}}\ \xi^{*\prime}\mathcal{B}(T,t)u = \xi^{*\prime}\mathcal{B}(T,t)p^* \\ &\underset{v \in V}{\mathrm{Max}}\ \xi^{*\prime}\mathcal{C}(T,t)v = \xi^{*\prime}\mathcal{C}(T,t)e^* \end{aligned}\right\} \tag{6}$$

where both the vector ξ^* and the value of the game J^* are determined by

$$J^* = \underset{\|\xi\|=1}{\mathrm{Sup}} \left\{\xi' y + \int_t^T \left[\underset{u \in U}{\mathrm{Min}}\ \xi'\ \mathcal{B}(T,\tau)u + \underset{v \in V}{\mathrm{Max}}\ \xi'\ \mathcal{C}(T,\tau)v\right]d\tau\right\} \tag{7}$$

Now a sufficiency theorem on the existence of the saddle point solution is presented.

Definition: A tube π is defined by

$$\left.\begin{aligned} \pi &= \{(y,t) : J^*(y,t) = c, c > 0 \\ \pi_i &= \{(y,t) : J^*(y,t) < c, c > 0 \\ \pi_0 &= \{(y,t) : J^*(y,t) > c, c > 0 \end{aligned}\right\}$$

Based on this definition, the following theorem has been proved.[11]

Theorem 1. *Consider game (5). Let* $y(t) \in \pi$ *and suppose that* π *is* C^1. *Then* *(i)* $\{p^*(\cdot), e^*(\cdot)\}$ *given by (6)-(7) is an optimal strategy pair;*

(ii) $p^*(\cdot)$ *guarantees the minimizer a cost not higher than* c, *while* $e^*(\cdot)$ *guarantees the maximizer a cost not lower than* c;

(iii) $y_0 \in \pi_i$ *implies that a solution* $y(\cdot)$, *generated by* $\{p^*, e\}$ *remains in* π_i; *likewise,* $y_0 \in \pi_0$ *implies that a solution* $y(\cdot)$ *generated by* $\{p, e^*\}$ *remains in* π_0.

A direct consequence of this theorem is that along an optimal trajectory, ξ^* is a *constant* unit vector.

Remark 1. In case π is only piece-wise smooth, Eq.(7) still determines the optimal cost J^* and the vector ξ^* at every point along those optimal trajectories which do not intersect the region of non-smoothness. (For a more detailed discussion on this subject, see Refs. 16 and 17).

B. Linear Optimal Control

If one of the participants has a fixed strategy known by his adversary the problem reduces to a one sided optimization. In this subsection the optimal control problem is formulated from the maximizer's viewpoint (a similar formulation for the minimizer will not be repeated). Assuming that the minimizer has a known linear feedback control law $u = K(t)x$ (without hard constraints being imposed), the one sided version of (1) becomes

$$\left.\begin{array}{ll} \dot{x} = \bar{A}(t)x + C(t)v & x(0) = x_0 \\ v \in V & \\ J = \|Dx(T)\| & \end{array}\right\} \qquad (8)$$

with $\bar{A}(t) = A(t) + B(t)K(t)$.

A transformation similar to (3) simplifies (8) to

$$\left.\begin{aligned} &\dot{y} = \overline{C}(T,t)v \qquad y(0) = y_0 \\ &v \in V \\ &J = \|y(T)\| \end{aligned}\right\} \tag{9}$$

For this formulation the optimal control $v^*(t)$ is given by

$$\underset{v \in V}{\text{Max}}\ \xi'\overline{C}(T,t)v = \xi^*\overline{C}(T,t)v^* \tag{10}$$

where ξ^* and the optimal cost J^* are determined by

$$J^*(y,t) = \underset{\|\xi\|=1}{\text{Sup}} \left\{\xi' y + \int_t^T \left[\underset{v \in V}{\text{Max}}\ \xi'\overline{C}(T,\tau)v\right] d\tau\right\} \tag{11}$$

Along an optimal solution ξ^* is a constant unit vector. It is important to note that the one sided optimization is valid (unlike the differential game version) without any further assumption on the smoothness of $J(\cdot)$.

3. APPLICATION TO 3-D PURSUIT-EVASION

A. Differential Game Version

In this sub-section the missile-aircraft pursuit-evasion is solved based on the set of assumptions 1-11 as a zero-sum perfect information linear differential game in three-dimensional space (Fig. 1). By Ass. 5 the analysis is restricted to the neighborhood of collision course (Fig. 2) where the relative trajectory can be linearized (Ass. 6). As a consequence of the linearization, the relative motion in the line of sight direction (the X axis) is of constant speed and the duration of the game T is determined. The dynamics to be considered is perpendicular to the line of sight. In this coordinate system the control vectograms, which are by Ass. 7 circular, perpendicular to the respective velocity vectors, become elliptic (See Fig. 3).

Accordingly, the equations of motion of the three-dimensional pursuit-evasion[6] have the form of (1) with

$$U = \{u : u'Ru \leq a_P^2\}$$
$$V = \{v : v'Sv \leq a_E^2\} \tag{12}$$

where the matrices R, S are symmetric and of proper dimensions. By defining (see Fig. 1)

$$Y \triangleq Y_P - Y_E \quad , \quad Z \triangleq Z_P - Z_E \tag{13}$$

the state vector x is of 6 components

$$x' = [Y, \dot{Y}, \ddot{Y}_P \;\vdots\; Z, \dot{Z}, \ddot{Z}_P] \tag{14}$$

The matrices in (1), (4), and (12) are

$$A = \begin{bmatrix} A_1 & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & A_1 \end{bmatrix} \quad ; \quad \Phi = \begin{bmatrix} \Phi_1 & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & \Phi_1 \end{bmatrix}$$

with

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\tau_p \end{bmatrix} \quad ; \quad D = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \quad ;$$

$$B' = 1/\tau_p \left[\begin{array}{ccc:ccc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad ;$$

$$C' = \left[\begin{array}{ccc:ccc} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{array}\right]$$

$$R = \begin{bmatrix} 1/\cos^2\chi_p(0) & 0 \\ 0 & 1 \end{bmatrix} \quad ; \quad S = \begin{bmatrix} 1/\cos^2\chi_E(0) & 0 \\ 0 & 1 \end{bmatrix}$$

Note that $V_p/V_E > 1$ (Ass. 2) implies that $\cos\chi_p(0) > \cos\chi_E(0)$ (See Fig. 2).

Equations (3) have the form

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} Y + (T-t)\dot{Y} + \phi_{13}\ddot{Y}_p \\ Z + (T-t)\dot{Z} + \phi_{13} Z_p \end{bmatrix} \tag{15}$$

$$B = (\phi_{13}/\tau_p) I_2 \quad ; \quad C = -\phi_{12} I_2 \tag{16}$$

with

$$\phi_{12} = \tau_p\theta \quad , \quad \phi_{13} = \tau_p^2\psi(\theta) \tag{17}$$

where

$$\theta \triangleq \frac{T-t}{\tau_P} \tag{18}$$

is the normalized "time-to-go" and

$$\psi(\theta) \triangleq \theta + e^{-\theta} - 1 > 0 \quad \forall\, \theta > 0 \tag{19}$$

Let $R^{-1} = M_P' M_P$ and $S^{-1} = M_E' M_E$;

$$M_P = \begin{bmatrix} \cos\chi_P(0) & 0 \\ 0 & 1 \end{bmatrix} \quad ; \quad M_E = \begin{bmatrix} \cos\chi_E(0) & 0 \\ 0 & 1 \end{bmatrix}$$

Then equation (6) is satisfied (using θ rather than t) by

$$\left.\begin{aligned} p^*(y,\theta) &= -\, a_P M_P' \frac{M_P\xi^*}{\| M_P\xi^* \|} \\ e^*(y,\theta) &= -\, a_E M_E' \frac{M_E\xi^*}{\| M_E\xi^* \|} . \end{aligned}\right. \tag{20}$$

where ξ^* and the value of the game are determined by

$$J^*(y,\theta) = \sup_{\|\xi\|=1} \{\xi' y - \alpha(\theta)\, M_P\xi + \beta(\theta)\| M_E\xi \|\} \tag{21}$$

with

$$\left.\begin{aligned} \alpha(\theta) &\triangleq a_P\tau_P^2 \int_0^\theta \psi(\eta)d\eta = a_P\tau_P^2 \left[\frac{\theta^2}{2} - \psi(\theta)\right] \\ \beta(\theta) &\triangleq a_E\tau_P^2 \int_0^\theta \eta d\eta = \frac{1}{2}\, a_E\tau_P^2\, \theta^2 \end{aligned}\right\} \tag{22}$$

It can be directly concluded from (21), (22) that at $y = 0$, $\xi^{*\prime} = (0, \pm 1)$; i.e., the optimal control vectors are perpendicular to the nominal collision plane. Moreover, the same conclusion holds for $\{y : y_1 = 0\}$.

The existence of such a saddle point solution can be proved (based on the definition of the tube π) via Theorem 1 of the previous section, if all tubes are smooth (see also Remark 1).

For this particular problem a simple test on (21) shows that at some subregions of $\{y; y_2 = 0\}$ (containing $y = 0$), ξ^* is not unique; hence tubes π in these regions are not smooth. Since the investigation of piecewise smooth tubes is not the aim of the present paper (Refs. 16 and 17 deal with this topic), we slightly modify the control set U, V, in order to avoid this phenomenon. In a usual situation $a_P \cos \chi_P(0) > a_E$ (see Fig. 3). Based on this observation, we reformulate the problem from the pursuer's point of view. Replacing in (12) the matrices R and S by the identity matrix yields circular vectograms and consequently,

$$M_P = M_E = I_2 \tag{23}$$

If we further replace in (12) a_P by

$$\hat{a}_P \triangleq a_P \cos \chi_P(0) \ , \tag{24}$$

the "worst case" version of the original problem is obtained from the pursuer's point of view ($\hat{a}_E = a_E$). For this formulation

$$\hat{\xi}^* = \frac{y}{\|y\|} \tag{25}$$

and consequently the optimal strategies and the Value of the game are given by

$$\hat{p}^*(y,\theta) = - \hat{a}_P \frac{y}{\|y\|}$$

$$\hat{e}^*(y,\theta) = - a_E \frac{y}{\|y\|} \tag{26}$$

$$\hat{J}^*(y,\theta) = \|y\| - \hat{\alpha}(\theta) + \beta(\theta) \tag{27}$$

with $\hat{a}_P$ replacing a_P in (22).

Analysis of the tubes shows that there exists some c_m such that a tube $\pi(c = c_m)$ is tangent to $\{(y,\theta) : y = 0\}$ at $\theta = \theta_s$. Such tube is called the "minimal tube."[11] The point of tangency $\theta = \theta_s$ can be easily computed by setting in (27) $\hat{J}^* = c_m$,

$$\|y\| = c_m + \hat{\alpha}(\theta) - \beta(\theta) \tag{28}$$

θ_s is now defined by

$$\frac{d\|y\|}{d\theta} = 0 \tag{29}$$

yielding

$$\psi(\theta_s) = \frac{a_E}{\hat{a}_P} \theta_s \tag{30}$$

The minimal cost c_m is obtained by substituting (30) and $y = 0$ into Eq.(28)

$$c_m = a_E \tau_P^2 \left[\theta_s + \frac{\theta_s^2}{2}\left(1 - \frac{\hat{a}_P}{a_E}\right)\right] \tag{31}$$

The dependence of θ_s and $\tilde{c}_m \triangleq c_m/a_E\tau_P^2$ on the maneuver ratio $(a_E/\hat{a}_P)$ is depicted in Fig. 4.

It is easy to see from (27) that all tubes π ($c \geq c_m$) are smooth. As a consequence of the existence of the "minimal tube" the (y,θ) state space can be decomposed into $\mathcal{D}$ and $\mathcal{D}^c$, being defined as

$$\mathcal{D}^c \triangleq \{(y,\theta) \in \pi_j : c_j \geq c_m\} \cup \{(y,\theta) : \theta < \theta_s\} \tag{32}$$

The optimal strategy pair $\hat{p}^*(\cdot), \hat{e}^*(\cdot)$ is given by (26) in $\mathcal{D}^c$ and by any arbitrary admissible pair in $\mathcal{D}$. The Value of the game $\hat{J}^*$ (i.e., the miss distance guaranteed to players using the optimal strategy pair) is determined by (27) in $\mathcal{D}^c$ and by c_m = const. (31) for every point in $\mathcal{D}$.

B. Optimal Control Version

In the investigation of the avoidance of a proportionally guided missile,[4,6] one deals with a model of the form given in (8). Following the notation of the previous sub-section, the solution has the form

$$e^*(y,\theta) = -a_E M_E' \frac{M_E \xi^*}{\| M_E \xi^* \|} \operatorname{sgn} \bar{\Phi}_{12}(\theta) \tag{33}$$

where the *constant* vector ξ^* and the optimal cost $\bar{J}^*$ are given by

$$\bar{J}^*(y,\theta) = \sup_{\|\xi\|=1} \{\xi' y + \bar{\beta}(\theta) \| M_E \xi \| \} \tag{34}$$

$$\bar{\beta}(\theta) = a_E \tau_P \int_0^\theta |\bar{\Phi}_{12}(\eta)| d\eta \tag{35}$$

As in the game version, it is clear that at $y = 0$, $\xi^{*\prime} = (0, \pm 1)$, i.e., the optimal control is directed perpendicular to the initial collision plane. Eq. (34) is an extension of the solution obtained in Ref. 6, which was derived for $y_0 = 0$.

It is very important to observe that $\overline{\phi}_{12}(\theta)$ is not the same as ϕ_{12} in (17) and as a consequence it is no longer positive for all θ. As a matter of fact $\phi_{12}(\theta)$ is a *switching function* (computed in Ref. 4).

4. DISCUSSION OF THE DETERMINISTIC RESULTS

A. Interpretation

The deterministic solution of the differential game version gives the optimal strategies and the resulting miss distance as a function of the "reduced state" y and the normalized "time-to-go" θ. The vector y can be interpreted as the "predicted miss distance," being composed (see Eq. (15)) of the "zero-effort miss" and an acceleration term compensating for pursuer dynamics.

The decomposition of the (y,θ) state space is of major significance. The domain $\mathcal{D}$ is characterized by "long" pursuit times $\theta > \theta_s$ and small or moderate deviations from collision course. This region (where according to Ass. 5 most trajectories begin) is dominated by the pursuer, who can reduce, for any trajectory initiated in $\mathcal{D}$, the predicted miss distance to zero at $\theta = \theta_s$ against any admissible evasive maneuver. Though the pursuer strategy can also be an arbitrary admissible one in $\mathcal{D}$, an adaptation of a linear time varying feedback control law as proposed for the 2-D case[10] may be an attractive choice. This strategy has some similarity with the guidance laws obtained by one sided linear quadratic optimization.[1,2]

As every trajectory initiated in $\mathcal{D}$ passes through the point ($y = 0$, $\theta = \theta_s$), they all have the same strategy for $\theta \leq \theta_s$ and consequently an outcome independent of the initial conditions. In the original formulation (Eqs. (12)-(22)), the terminal strategy is given by $\xi_1^* = 0$, i.e.: hard maneuvering perpendicular to the initial collision plane. This verifies the recently obtained result for optimal missile avoidance,[6] and confirms the necessity of 3-D analysis. The direction chosen is the one where the evader-pursuer maneuver ratio is maximal (see Fig. 3), being the consequence of (21) for $y = 0$.

The main limitation of the present analysis lays in the trajectory linearization (see Ass. 6). In a previous paper,[6] dealing with the optimal control version of the same problem, it was concluded that trajectory lineariztion is justified if two conditions are satisfied:

1. The dynamic similarity parameter of the problem,[18] which can be interpreted as the maximum direction change of the evader during the period of the pursuer's time constant, is small.

2. The optimal solution does not predict excessively long maneuvers in any specific direction.

The first condition can be observed before the linearized trajectory assumption is adopted. The second one, however, requires an "a posteriori" verification. The total change of interception geometry has to be computed and examined to decide upon the validity of the linearization.

In order to put the present analysis in the proper context with some "classical" missile avoidance concepts the following remark seems to be of relevance. V[illegible] superiority of the pursuer ($V_P > V_E$) and the constant speed hypothesis (See Ass. 2), used in the mathematical model, clearly indicate that the initial conditions of the engagement has been chosen well inside the effective firing envelop of the missile, ruling out the so called fleeing tactics of the evader. Such tactics, having the objectve to convert a head-on engagement into a tail-chase, hope to profit from eventual missile slow down. They can be effective only against missile systems which use an "ill determined" (constant target direction) firing zone concept.

B. Implementation

Optimal strategies are based on perfect knowledge of the reduced state variables and the "time-to-go". The computation of "time-to-go" requires range and range-rate measurements. The "zero-effort miss" can be computed if the rate of turn of the line of sight is also measured. These measurements, as well as of the actual missile acceleration, are of common practice in many guidance systems. Thus the implementation of the optimal pursuer strategy given by (25) does not seem to present difficulties. The efficiency of such scheme will however depend on the accuracy of the measurements or the estimation process.

By contrast, the implementation of the corresponding evasion strategy is a rather formidable, if not impossible, task. Although some recent efforts have been made to estimate the "zero-effort miss" and the "time-to-go" onboard of aircraft by appropriate filtering,[12] the estimation of missile acceleration is beyond the state of art. Moreover, the optimal strategy assumes perfect knowledge of the missile parameters. It has to be based on previous intelligence as well as on real-time identification of the particular type of missile to be avoided. If intelligence is available, it may also include data on the actual guidance law, reducing the problem to a one sided optimization,[3-6] where the evader can achieve (assuming perfect information) larger miss distances than guaranteed by (26) and (27).

It can be concluded that for missile avoidance applications the hypothesis of perfect information is a most critical one and in many cases it cannot be justified. Therefore, the problem has to be re-examined with non complete information.

5. MISSILE AVOIDANCE WITH IMPERFECT INFORMATION

In the optimal missile avoidance problem there are several sources of information imperfections:

a) Lack of intelligence or identification (parameter uncertainty);

b) Unaccessible state variables (partial observability);

c) Unaccurate state variables (measurement errors or noise);

d) Non existent threat warning (no initial conditions).

Each of these topics deserves a separate analysis and a detailed discussion. In the limited scope of this paper it seems, however, more productive to present an example which illustrates the relative importance of perfect state information to successfull missile avoidance.

For this purpose it is assumed that the evading airplane has *no information* on the relative state (including initial conditions), but the pursuer parameters are known. This problem has been recently solved[20] by determining a periodical random phase maneuver which maximizes the R M.S. value of the miss distance. It has been shown that such periodical strategy is much superior to the Random Telegraph Maneuver mentioned in previous works[20] [21] and it is indeed of optimal structure. The optimal frequency ω^* and the resulting miss distance distribution can be computed as functions of pursuer and evader parameters.

Comparing the results of such stochastic optimization to the case of perfect information (for an example of two-dimensional missile guided by proportional navigation, $N' = 4.0$) reveals that the R.M.S. miss distances rech 60-80% of the optimal deterministic value. This comparison indicates that the degradation of missile avoidance capability due to imperfect information may not be as serious as it could have been estimated. If the avoidance efficiency could be partially retained in the stochastic sense even for a total absence of state information, there is a definite hope that better (and probably satisfactory) performance could be achieved if partial or noise corrupted state measurements do exist.

6. CONCLUDING REMARKS

In this paper the 3-D missile-aircraft pursuit-evasion is solved in a closed form as a linear zero-sum *perfect information* differential game. It is the first time that such solution of this problem with a realistic mathematical model, taking into account hard control constraints and pursuer dynamics, is presented. The compact closed form solution is an application of a recently developed methodology,[11, 15] which can be equally used for the one sided optimal control formulation of the problem. The solution of the game has a very clear geometrical interpretation creating a new insight for the missile-aircraft pursuit-evasion. As a consequence, the applicability of the results can be directly discussed.

The optimal strategy for the pursuer, proposed by the closed form solution, seems very attractive for guided missile applications. On the contrary, the implementation of the *deterministic* optimal missile avoidance strategy presents serious difficulties due to incomplete available information.

In the paper the sources of imperfect information are discussed and for the "worst case" example of *total absence of state information* a stochastically optimal solution[20] is given. Comparison to relevant deterministic results shows that the degradation of avoidance efficiency, due to lack of information, is not overwhelming. It implies a potential of better results for the case of partial or noisy state measurements and parameter uncertainties. These problems require an extensive future research.

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LIST OF FIGURES

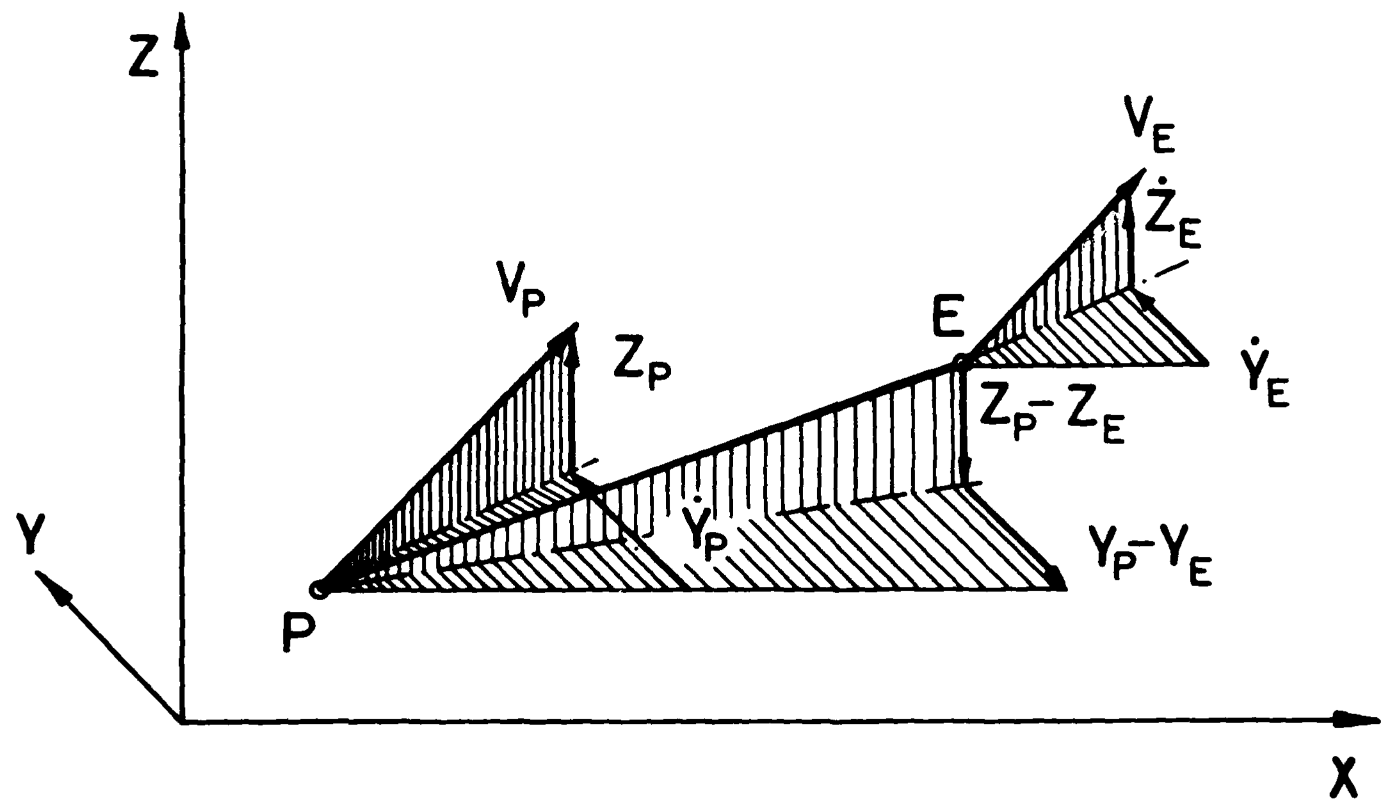

FIGURE 1. 3-D Pursuit-Evasion Geometry.

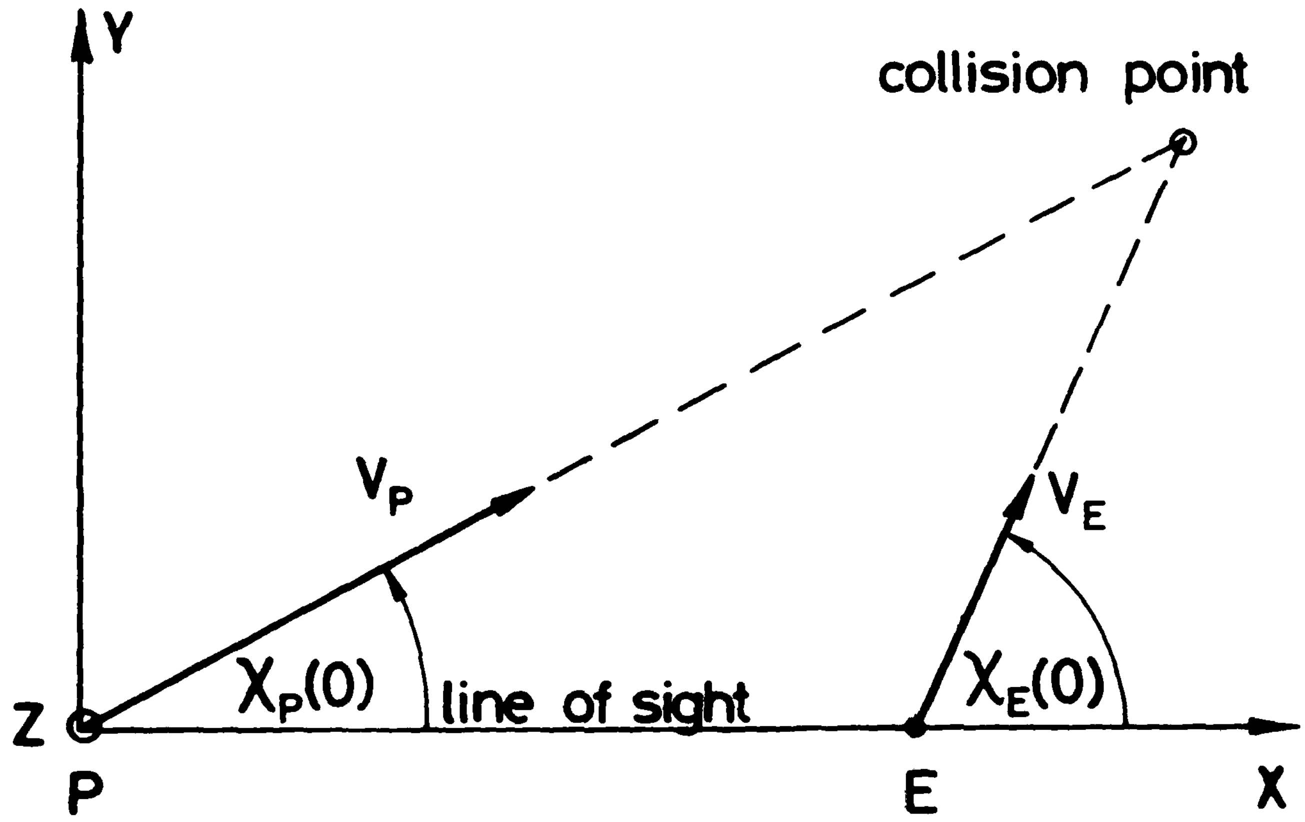

FIGURE 2. Collision Course Geometry.

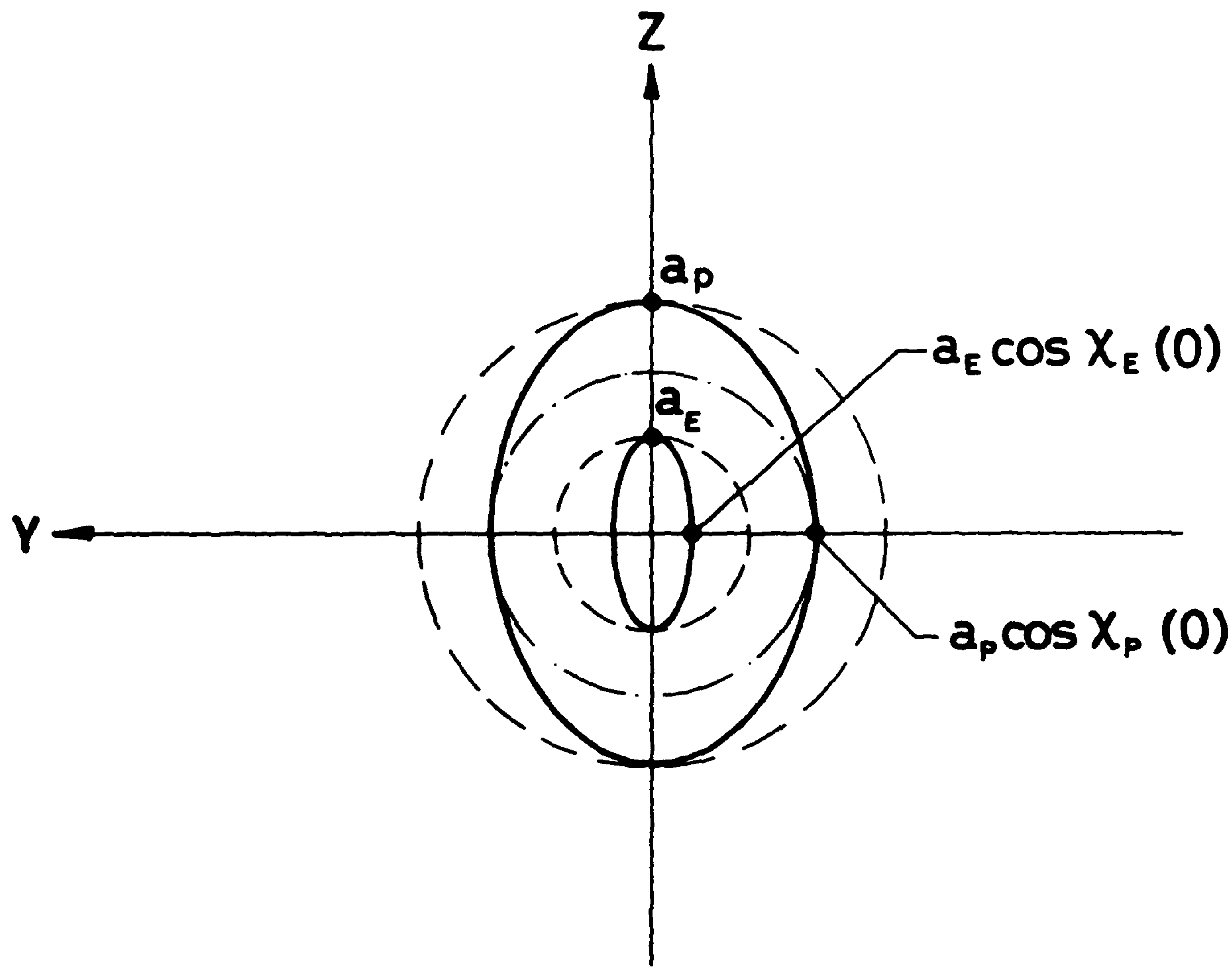

FIGURE 3. Acceleration Vectograms Perpendicular to the Initial Line of Sight.

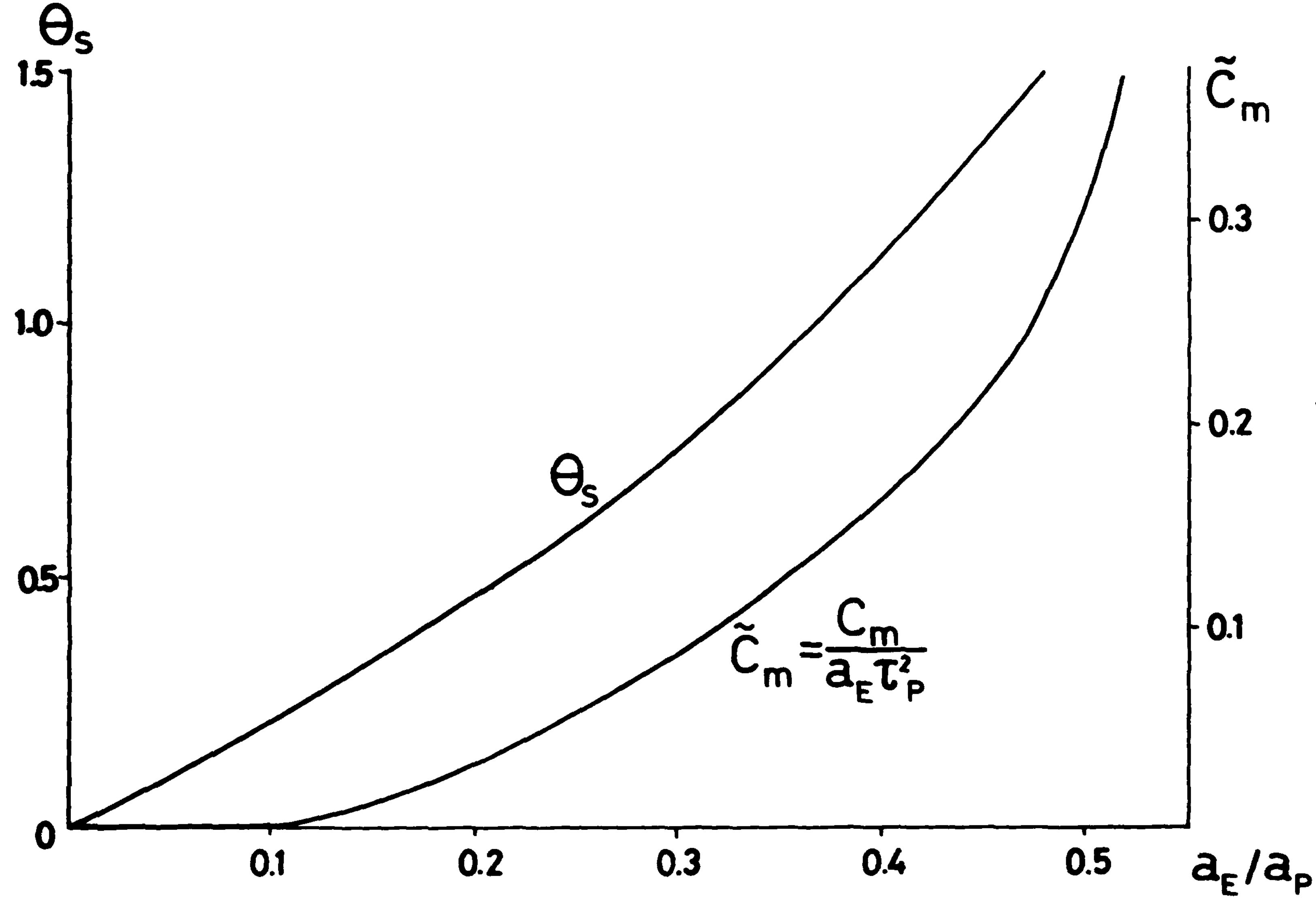

FIGURE 4. Dependence of θ_s and the Normalized Minimal Cost $\tilde{c}_m$ on the Evader-Pursuer Maneuver Ratio.