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BOUNDARY LAYER PHENOMENA IN THE PLASTIC

ZONE NEAR A RAPIDLY PROPAGATING CRACK TIP

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Abstract

The steady-state dynamic fields near a rapidly propagating crack tip in an elastic perfectly-plastic material have been investigated for the case of Mode-III fracture. For arbitrary values of the dimensionless crack-tip speed (M) the inner solution consists of a central-fan field ahead of the crack tip and a uniform field in its wake. It is shown that the inner solution is valid in a "boundary layer" which shrinks on the crack tip in the limit of vanishing M. For small M the outer solution was found as a regular perturbation expansion in M, with the quasi-static solution as its first term. A uniform expansion over the polar angle θ measured from the plane of the crack was also obtained; its first term displays the connection between the inner and outer solutions.

1. Introduction

In a recent paper the authors have investigated dynamic effects on the fields of stress and strain near a rapidly propagating crack tip in an elastic perfectly-plastic material [1]. For the steady-state case it was found that the dynamic near-tip fields can be expressed as simple-wave solutions of the governing system of hyperbolic partial differential equations. These solutions are independent of the dimensionless distance to the crack tip, r/r_p, but they do depend specifically on the dimensionless crack-tip speed M. For Mode-III crack propagation the simple-wave solution in the near-tip field is a combination of a centered-fan field and a uniform field. Explicit expressions have been presented in Refs.[1] and [2].

The solutions that were obtained in Ref.[1] show some anomalies in the transition from the dynamic to the quasi-static solution. As the crack-tip speed, M, decreases the expressions for the stresses reduce to the ones for the corresponding quasi-static solution, as might be expected on the basis of intuitive reasoning. This is however not true for the strains, which become unbounded in the limit of vanishing M. In Ref.[1] it was speculated that the transition from dynamic to quasi-static conditions with decreasing crack-tip speed is effected because the dynamic solution is asymptotically valid in a small edge zone, which shrinks on the crack tip in the limit of vanishing crack-tip speed.

In this paper this non-uniform transition has been investigated in detail in the plane of the crack for the case of crack propagation in anti-plane strain (Mode-III). It is shown that for small crack-tip speeds the complete near-tip solution consists of the outer solution, which is a regular perturbation expansion in M, with the quasi-static solution as the first term, and the inner solution, which is of completely different nature with a strong influence of dynamic effects. The inner solution is valid in an edge-zone which is analogous to a boundary layer. The first term in a uniform expansion over the polar angle θ measured from the plane of the crack displays the connection between the inner and outer solutions.

The governing equations and the boundary conditions are stated in Section 2. The outer solution, its relation to the quasi-static solution, as well as its inadequacy in the immediate vicinity of the crack tip have been discussed in Section 3. The results for the inner solution have been summarized in Section 4. It is shown in Section 5, that certain functions which define the fields for small polar angle from the plane of the crack, satisfy a system of coupled ordinary differential equations. An investigation of the singular points in the phase plane and an inspection of the trajectories of the solutions of these equations as the distance to the crack tip decreases, reveals that the expressions of Section 4 do indeed provide the solution in an edge zone. Exact solutions in implicit form for the coupled ordinary differential equations mentioned above have also been obtained in Section 5, and these solutions reproduce the inner solution as well as the outer solution. Finally the coupled equations have been solved numerically, and the results have been plotted vs. r/r_p and M, to show the transition of the inner solution in the edge zone to the outer solution in the "far-field".

2. Governing Equations

A coordinate system (x,y,z) is attached to the moving crack tip as shown in Fig. 1. The assumption that a steady-state has been established relative to the moving crack tip, implies that absolute time derivatives take the following forms relative to the moving coordinate system:

$$\partial_t(\ ) = -v\ \partial_x(\ ); \qquad \partial_{tt}(\ ) = v^2\partial_{xx}(\ ) \quad . \tag{2.1a,b}$$

In the moving coordinate system steady-state deformation in anti-plane strain is defined by a displacement in the z-direction, which is denoted by w(x,y). The corresponding stress components are $\sigma_{xz}(x,y)$ and $\sigma_{yz}(x,y)$.

In view of (2.1b) the equation of motion becomes

$$\partial_x\sigma_{xz} + \partial_y\sigma_{yz} = \rho v^2\partial_{xx}w \tag{2.2}$$

The Tresca yield condition is

$$\sigma_{xz}^2 + \sigma_{yz}^2 = k^2 \ , \tag{2.3}$$

where k is the yield stress in shear. By virtue of (2.1a), the Prandtl-Reuss equations for an elastic perfectly-plastic solid reduce to the forms

$$\partial_{xx}w = \mu^{-1}\partial_x\sigma_{xz} - \Lambda\sigma_{xz} \tag{2.4}$$

$$\partial_{xy}w = \mu^{-1}\partial_x\sigma_{yz} - \Lambda\sigma_{yz} \ , \qquad \Lambda \geq 0 \tag{2.5}$$

Here μ is the elastic shear modulus, and Λ is a non-negative proportionality factor, which may vary in space.

In the region of plastic deformation the yield condition is identically satisfied by

$$\sigma_{xz} = -\ k\ \sin\omega \ , \qquad \sigma_{yz} = k\ \cos\omega \tag{2.6a,b}$$

Elimination of Λ from (2.4) and (2.5), and the use of (2.2) and (2.6a,b) then yields the following system of equations

$$\cos\omega\ \partial_x\omega + \sin\omega\ \partial_y\omega + M^2\frac{\mu}{k}\partial_x\omega_x = 0 \tag{2.7}$$

$$\cos\omega\ \partial_x w_x + \sin\omega\ \partial_y w_x + \frac{k}{\mu}\partial_x\omega = 0 \tag{2.8}$$

where

$$w_x = \partial_x w\ , \qquad \text{and } w_y = \partial_y w \tag{2.9a,b}$$

$$M = v/(\mu/\rho)^{\frac{1}{2}} \tag{2.10}$$

The condition of vanishing σ_{yz} on the crack faces yields by virtue of (2.6b)

$$y = 0,\ x < 0: \qquad \omega = \pi/2 \tag{2.11}$$

Displacement anti-symmetry relative to $y = 0$ implies that $w = 0$ for $y = 0$. Hence $w_x = 0$, and thus by (2.6a)

$$y = 0,\ x > 0: \qquad \omega = 0 \tag{2.12}$$

In the polar coordinate system shown in Fig. 1, (2.7) and (2.8) have the forms

$$\cos(\omega-\theta)\ \partial_r\omega + \frac{1}{r}\sin(\omega-\theta)\ \partial_\theta\omega + \frac{\mu M^2}{k}(\cos\theta\ \partial_r w_x - \frac{1}{r}\sin\theta\ \partial_\theta w_x) = 0 \tag{2.13}$$

$$\cos(\omega-\theta)\ \partial_r w_x + \frac{1}{r}\sin(\omega-\theta)\ \partial_\theta w_x + \frac{k}{\mu}(\cos\theta\partial_r\omega - \frac{1}{r}\sin\theta\ \partial_\theta\omega) = 0 \tag{2.14}$$

3. Far-Field Solution

Let us consider regular series expansions for ω and w_x with respect to the "Mach number" M defined by (2.10)

$$\omega = \omega_o + M\omega_1 + \ldots \tag{3.1}$$

$$w_x = w_{xo} + Mw_{x1} + \ldots \tag{3.2}$$

For the leading terms ω_o and w_{xo} we obtain the following equations

$$\cos(\omega_o-\theta)\ \partial_r\omega_o + \frac{1}{r}\sin(\omega_o-\theta)\partial_\theta\omega_o = 0 \tag{3.3}$$

$$\cos(\omega_o-\theta)\ \partial_r w_{xo} + \frac{1}{r}\sin(\omega_o-\theta)\ \partial_\theta w_{xo} +$$

$$+\frac{k}{\mu}\left(\cos\theta\ \partial_r\omega_o - \frac{1}{r}\sin\theta\partial_\theta\omega_o\right) = 0 \tag{3.4}$$

These equations govern the quasi-static problem, which was discussed by Chitaley and McClintock [3]. For small values of r/r_p (where r_p defines the boundary of the plastic domain) we obtain (see Ref. 3)

$$\omega_o = \theta \tag{3.5}$$

$$w_{xo} = \frac{k}{\mu}\sin\theta\ \ln\left(\frac{r}{r_p}\right) + f(\theta) \tag{3.6}$$

where $f(\theta)$ is an arbitrary function of θ.

It is of interest to examine the magnitude of the inertia term, $M^2\partial_x w_x$, which would correspond to the quasi-static solution. By the use of (3.5) and (3.6) we find

$$M^2\partial_x w_x \simeq \frac{k}{2\mu}\frac{M^2}{r}\left[1-\ln\left(\frac{r}{r_p}\right) - \frac{f'(\theta)}{\cos\theta}\right]\sin 2\theta \tag{3.7}$$

On the other hand, if the stress derivatives appearing in Eq.(2.2) are computed on the basis of (3.5) and (3.6), it is found that they vanish identically. Equation (3.7) then suggests that the quasi-static approximation is not valid in a small neighborhood of the crack-tip defined by $r/r_p \sim O(M^2)$ (we shall make a more accurate estimate in a later Section). The subsequent terms in the series (3.1) and (3.2) do not remove this nonuniformity of the approximation.

Thus the regular expansion with respect to M cannot be accepted in the immediate vicinity of the crack-tip. It appears that this solution represents an outer or "far-field" expansion.

4. Near-Field Solution

In a recent paper Achenbach and Dunayevsky [1] have shown that the solution (ω, w_x) to Eqs.(2.7) and (2.8) is either singular near the crack tip, or it is represented by a centered fan-field in combination with a uniform field. Since it was not possible to obtain a singular solution, the centered fan + uniform field solution was considered in some detail. The solution was obtained in the following form [1 , see also 2]:

$0 \leq \theta \leq \theta^*$

$$w_x = -\frac{k}{\mu M} \cos^{-1}\left[M\sin^2\theta + (1-M^2\sin\theta)^{\frac{1}{2}}\cos\theta\right] \tag{4.1}$$

$$\sigma_{xz} = -k\left[(1-M^2\sin^2\theta)^{\frac{1}{2}} - M\cos\theta\right]\sin\theta \tag{4.2}$$

$$\sigma_{yz} = k\left[(1-M^2\sin^2\theta)^{\frac{1}{2}}\cos\theta + M\sin^2\theta\right] \tag{4.3}$$

$\theta^* \leq \theta \leq \pi$

$$\sigma_{xz} = -k\ , \quad \sigma_{yz} = 0\ , \quad w_x = -k\pi/\mu M \tag{4.4a,b,c}$$

where

$$\theta^* = \tan^{-1}(1/M) \tag{4.5}$$

In the limit $M \rightarrow 0$ the stress fields reduce to the corresponding quasi-static fields. This is, however, not true for the strains, which become unbounded as $M \rightarrow 0$. Difficulties of this kind in the transition from the dynamic to the quasi-static solution as $M \rightarrow 0$ could have been anticipated from the structure of the governing equations. As $M \rightarrow 0$ the two distinct families of characteristic curves of Eqs.(2.6) and (2.7), which are defined by

$$\frac{dx}{dy} = \cos\omega \pm \frac{M}{\sin\omega} \tag{4.6}$$

merge into one family, defined by

$$\frac{dx}{dy} = \cos\omega \tag{4.7}$$

Such a degeneracy usually leads to a non-uniform transition, and the appearance of a "boundary layer". Some examples are given by Cole [4].

Another indication of a non-uniform transition is given by the form of the equations in the hodograph plane. To transform (2.7) and (2.8) to the hodograph plane we introduce the following changes of variables [5].

$$\frac{\partial x}{\partial \omega} = J^{-1} \frac{\partial w_x}{\partial y} , \quad \frac{\partial y}{\partial \omega} = - J^{-1} \frac{\partial w_x}{\partial x} \tag{4.8a,b}$$

$$\frac{\partial x}{\partial w_x} = - J^{-1} \frac{\partial \omega}{\partial y} , \quad \frac{\partial y}{\partial w_x} = J^{-1} \frac{\partial \omega}{\partial x} \tag{4.9a,b}$$

where J is the Jacobian

$$J = \frac{\partial \omega}{\partial x} \frac{\partial w_x}{\partial y} - \frac{\partial w_x}{\partial x} \frac{\partial \omega}{\partial y} \tag{4.10}$$

In the hodograph plane Eqs.(2.7) and (2.8) take the form

$$\cos\omega \frac{\partial y}{\partial w_x} - \sin\omega \frac{\partial x}{\partial w_x} - M^2 \frac{k}{\mu} \frac{\partial y}{\partial \omega} = 0 \tag{4.11}$$

$$-\cos\omega \frac{\partial y}{\partial \omega} + \sin\omega \frac{\partial x}{\partial \omega} + \frac{k}{\mu} \frac{\partial y}{\partial w_x} = 0 \tag{4.12}$$

This linear system of equations can be reduced to a single equation of the second order

$$\left(\frac{k}{\mu}\right)^2 \frac{\partial^2 y}{\partial w_x^2} - M^2 \frac{\partial^2 y}{\partial \omega^2} + M^2 \cot\omega \frac{\partial y}{\partial \omega} - \frac{1}{\sin\omega} \frac{\partial y}{\partial w_x} = 0 \tag{4.13}$$

When $M \to 0$ the second order derivative $\partial^2 y/\partial\omega^2$ disappears, which indicates that an asymptotic expansion with respect to M cannot be uniform.

On the basis of the foregoing observations it may be assumed that the field near the crack tip is of different forms in two zones. The outer solution in the "far field" is a regular expansion in M as given by (3.1) and (3.2), where the leading term represents the quasi-static solution. The effect of inertia is relatively small in this far-field. Inertia is, however, important in the inner solution in the near field, represented by (4.1)-(4.5). The near-field can be thought of as a "boundary layer". In the present geometry the terminology "edge zone" is, however, more appropriate. In the edge-zone the inertia effects appear to remove the singularity of the quasi-static strain component.

In the next section we consider the matching of the inner solution and the outer solution, and the shrinking of the edge-zone on the crack tip as $M \to 0$.

5. Solution in the Plane of the Crack

Matching of the far-field and near-field solutions has not been accomplished for arbitrary values of the polar angle θ. It is, however, possible to investigate the matching for small values of θ, by seeking a solution in the following form

$$\omega = \omega_1(r)\ \theta + \omega_3(r)\ \theta^3 + \ldots \tag{5.1}$$

$$w_x = w_{x1}(r)\ \theta + w_{x3}(r)\ \theta^3 + \ldots \tag{5.2}$$

Substitution of (5.1) into (2.3) and (2.4), and collecting the terms of equal powers of θ , gives for the first approximation the following equations

$$\frac{d\omega_1}{dr} + \frac{\omega_1(\omega_1 - 1)}{r} + \frac{\mu M^2}{k}\left(\frac{dw_{x1}}{dr} - \frac{w_{x1}}{r}\right) = 0 \tag{5.3}$$

$$\frac{dw_{x1}}{dr} + \frac{w_{x1}(\omega_1 - 1)}{r} + \frac{k}{\mu}\left(\frac{d\omega_1}{dr} - \frac{\omega_1}{r}\right) = 0 \tag{5.4}$$

At this stage it is useful to introduce new variables

$$J_+ = Mw_{x1} + \frac{k}{\mu}\,\omega_1\ , \qquad J_- = Mw_{x1} - \frac{k}{\mu}\,\omega_1 \tag{5.5a,b}$$

Inserting (5.5) into (5.3) and (5.4) we arrive at

$$(1+M)\ \frac{dJ_+}{d\alpha} + (1+M)\ J_+ - \frac{\mu}{2k}\ (J_+ - J_-)J_+ = 0 \tag{5.6}$$

$$(1-M)\ \frac{dJ_-}{d\alpha} + (1-M)J_- - \frac{\mu}{2k}\ (J_+ - J_-)J_- = 0 \tag{5.7}$$

where the following change of variables has been used

$$\alpha = -\ell n(r/r_p) \tag{5.8}$$

Let us consider the solutions to Eqs.(5.6) and (5.7) in the phase plane

(J_+, J_-). The singular points are the solutions to the following equations

$$(1+M)J_+ - \frac{\mu}{2k}(J_+ - J_-)J_+ = 0 \tag{5.9}$$

$$(1-M)J_- - \frac{\mu}{2k}(J_+ - J_-)J_- = 0 \tag{5.10}$$

Equations (5.9) and (5.10) define three singular points, whose position and character are given by

1) $J_+ = J_- = 0$; stable focus (5.11)

2) $J_- = 0$; $J_+ = (1+M)\,\frac{2k}{\mu}$; unstable node (5.12)

3) $J_+ = 0$; $J_- = -(1-M)\,\frac{2k}{\mu}$; saddle point (5.13)

The nature of the singular points given by (5.11)-(5.13) determines the phase flow pattern, i.e., the trajectories of the solutions as α varies, shown in Fig. 2 (see e.g. Ref. [6]).

Since no limit cycle or center point exists, the trajectories beneath the separatrix line AA′ run towards infinity as $\alpha \to \infty$ $(r \to 0)$. Along these lines J_+ and J_- , and consequently ω_1 and w_{x1}, become unbounded. However infinite growth of ω_1 leads to an oscillation of the stress field (with increasing frequency) which is not admissible from the physical point of view. Thus this domain of the phase plane falls outside our consideration.

The phase flow above the separatrix line AA′ tends to the origin $J_+ = J_- = 0$ so that for $\alpha \to \infty$ $(r \to 0)$ we have $J_+ \to 0$, $J_- \to 0$, and consequently $\omega_1 \to 0$, $w_{x1} \to 0$. This result would correspond to a uniform zero field ahead of the crack tip. The hyperbolicity of the governing equations (2.7) and (2.8) then would imply a zero field for the whole loading domain, which does, however, not satisfy the boundary condition on the crack faces and therefore has to be rejected.

Thus we have no other alternative but the trajectory along the separatrix AA′. There is however a "forbidden" region in the phase plane which is defined by the condition $\Lambda \geq 0$. It follows from (2.4), (2.5) and (5.1), (5.2) that

$$k\Lambda = -\frac{w_{x1}}{r} \tag{5.14}$$

The requirement $\Lambda \geq 0$ leads to the inequality

$$-w_{x1} = -\frac{1}{2M}(J_+ + J_-) \geq 0 \tag{5.15}$$

which implies that we have to consider the part of the separatrix which lies to the left of the bisector $J_+ = -J_-$.

Thus, when $\alpha \to \infty$ $(r \to 0)$ the solution moves along the line AA′ toward the point 3. Let us now find the solution in the neighborhood of the point 3. Linearization of Eqs.(5.6) and (5.7) near this point leads to the following equations

$$\frac{dJ_+}{d\alpha} = -\frac{2M}{1+M} J_+ \tag{5.16}$$

$$\frac{dJ_-}{d\alpha} = -J_+ + J_- + (1-M)\frac{2k}{\mu} \tag{5.17}$$

In terms of the variable r the solution has the form

$$J_+ = C\,\frac{1+3M}{2M}\,(r/r_p)^{2M/(1+M)} \tag{5.18}$$

$$J_- = -(1-M)\frac{2k}{\mu} + C(r/r_p)^{2M/(1+M)} \tag{5.19}$$

where C is an arbitrary constant and (5.8) has been used. Hence by virtue of (5.5) we obtain

$$w_{x1} = -\frac{k}{M}(1-M) + C\,\frac{1+5M}{4M}\,(r/r_p)^{2M/(1+M)} \tag{5.20}$$

$$\omega_1 = 1-M + C\,\frac{\mu}{2k}\,\frac{1+M}{2M}\,(r/r_p)^{2M/(1+M)} \tag{5.21}$$

Hence we have

$$\omega \simeq \omega_1\theta = (1-M)\theta + O\,[(r/r_p)^{2M/(1+M)}] \tag{5.22}$$

$$w_x \simeq w_{x1}\theta = -\frac{k}{\mu M}(1-M)\theta + O\,[(r/r_p)^{2M/(1+M)}] \tag{5.23}$$

The dominant terms of the expressions (5.22) and (5.23) coincide with those of the expansion of the near field, (4.1) and (4.4), for small θ. This result confirms the validity of the simple wave solution near the crack tip.

Equations (5.6) and (5.7) can be integrated to yield the following solution in implicit form

$$J_+ = A\,(r/r_p)^{2M/(1+M)}\,|J_-|^{(1-M)/(1+M)} \tag{5.24}$$

$$F\left(1, \frac{1+M}{2M}, 1 + \frac{1+M}{2M}, -B\left(\frac{r}{r_p}\right)^{2M/(1+M)} |J_-|^{-2M/(1-M)}\right) = -\frac{1}{1-M}\,\frac{\mu}{2k}\,J_- + B\,J_-\left(\frac{r}{r_p}\right)^{-1} \tag{5.25}$$

Here A and B are constants of integrations and F(p,q,r,s) is the hypergeometrical function. The trajectory along the separatrix AA′ corresponds to B = 0. This follows from the observation that the separatrix crosses the line $J_+ = 0$ at the point 3, which is defined by (5.13)

It can now be verified that for $(r/r_p) \to 0$, (5.24) and (5.25) yield $J_+ \to 0$ and $J_- \to (2k/\mu)(1-M)$, which correspond to $\omega_1 \to 1-M$ and $w_{x1} \to -(k/\mu M)(1-M)$. These results agree with (5.20) and (5.21).

Next we consider (5.24) and (5.25) in the limit $M \to 0$. It is noted from (5.5) that for M = 0 we have $J_+ = -J_-$. Since $J_- < 0$, the constant A in (5.24) must equal unity: A = 1. For $M \neq 0$, the combination of (5.5a,b) and (5.24) then yields

$$\ell n(Mw_{x1} + \frac{k}{\mu}\omega) = \frac{2M}{1+M}\ell n\ (\frac{r}{r_p}) + \frac{1-M}{1+M}\ell n\ |Mw_{x1} - \frac{k}{\mu}\omega| \tag{5.26}$$

To expand (5.26) for small M we require $r/r_p > \delta$ where δ is such that

$$-\ \ell n\ (\frac{r}{r_p}) \sim O(\frac{1}{M}) \tag{5.27}$$

Equation (5.26) then yields

$$w_{x1} = \frac{k}{\mu}\omega\ [\ell n(\frac{r}{r_p}) - \omega\ \ell n(\frac{k}{\mu}\omega)] + O(M) \tag{5.28}$$

In deriving this expression, it was taken into account that $J_- < 0$.

For the hypergeometrical function in Eq.(5.25) we find for small M

$$F(1, \frac{1+M}{2M}, 1 + \frac{1+M}{2M}, -(\frac{r}{r_p})^{2M/1+M}|J_-|^{-2M/1+M}) \approx$$

$$F(1, \frac{1+M}{2M}, 1 + \frac{1+M}{2M}, -1) + O(M^2) = \frac{1+M}{2M}\beta(\frac{1+M}{2M}) + O(M^2) \tag{5.29}$$

Here $\beta(x)$ is the β-function

$$\beta(x) = \frac{1}{2}\sum_{n=0}^{\infty}\frac{n!}{x(x+1)\ldots(x+k)}\frac{1}{2^n} \tag{5.30}$$

By virtue of (5.29), (5.30) and (5.25) we obtain for $M \to 0$

$$\frac{1}{2} + O(M) = -(1+M)\frac{\mu}{2k}(Mw_{x1} - \frac{k}{\mu}\omega) \tag{5.31}$$

For $(r/r_p) > \delta$ the formulas (5.26) and (5.31) imply

$$\omega_1 \sim 1\ ,\ w_{x1} \sim \frac{k}{\mu}\ell n\ (\frac{r}{r_p}) + \frac{k}{\mu}\ell n(\frac{k}{\mu}) \tag{5.32}$$

This result agrees with the quasi-static solution given by (3.5)-(3.6), which coincides with the dominant terms of the outer solution for small θ, as given by (3.1)-(3.2).

To follow in some detail the phase flow when $M \to 0$ it is convenient to consider the (w_x,ω) phase plane shown in Fig. 3. The point of intersection of the separatrix AA′ with the ω-axis should be located between 1-M and 1+M. The angle γ of intersection of the J_+- and J_--lines, is given by

$$\gamma = \pi - 2 \tan^{-1}(1/M),$$

which tends to zero when $M \to 0$. The abscissa of the saddle point 3 is $\omega_3 = 1-M$ and the abscissa of the node 2 is $\omega_2 = 1+M$, which both tend to 1 as $M \to 0$. Thus the singular points 2 and 3 shift to infinity as $M \to 0$. For $M \equiv 0$ the phase flow has degenerated as shown in Fig. 4. The separatrix has turned into a straight line which is parallel to the w_x-axis and has the abscissa $\omega = 1$. The magnitude w_{x1} has become unbounded along this line.

In summary, for small values of the angle θ the solution given by (5.24) and (5.25) matches the near field (4.1) and (4.3) to the far field (3.5) - (3.6) for small values of the angle θ . The region defined by $\ln(r/r_p) \sim O(M^{-1})$ can be identified as an edge zone.

The implicit form of (5.24) and (5.25) is not convenient for the computation of curves. Equations (5.6) and (5.7) have, therefore, been solved numerically. The function

$$w_{x1} = (J_+ + J_-)/2M$$

has been plotted in Fig. 5. The corresponding quasi-static solution, which is indicated by $M = 0$, has also been plotted. The quasi-static solution for w_{x1} is singular at $r/r_p = 0$, while the dynamic solution remains bounded. The curve for $M = 0.01$ is very close to the one for $M \equiv 0$, i.e., to $\ln(r/r_p)$, in the region $r/r_p > \delta$, where δ is the "length" of the edge zone in the plane of the crack. An estimate of the length of the edge zone is indicated in Fig. 5.

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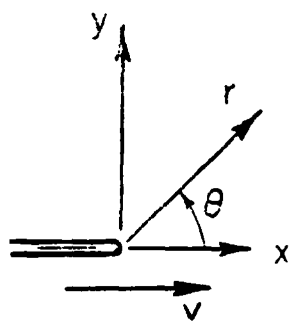

Fig. 1 Propagating crack tip (velocity v) with moving coordinate system

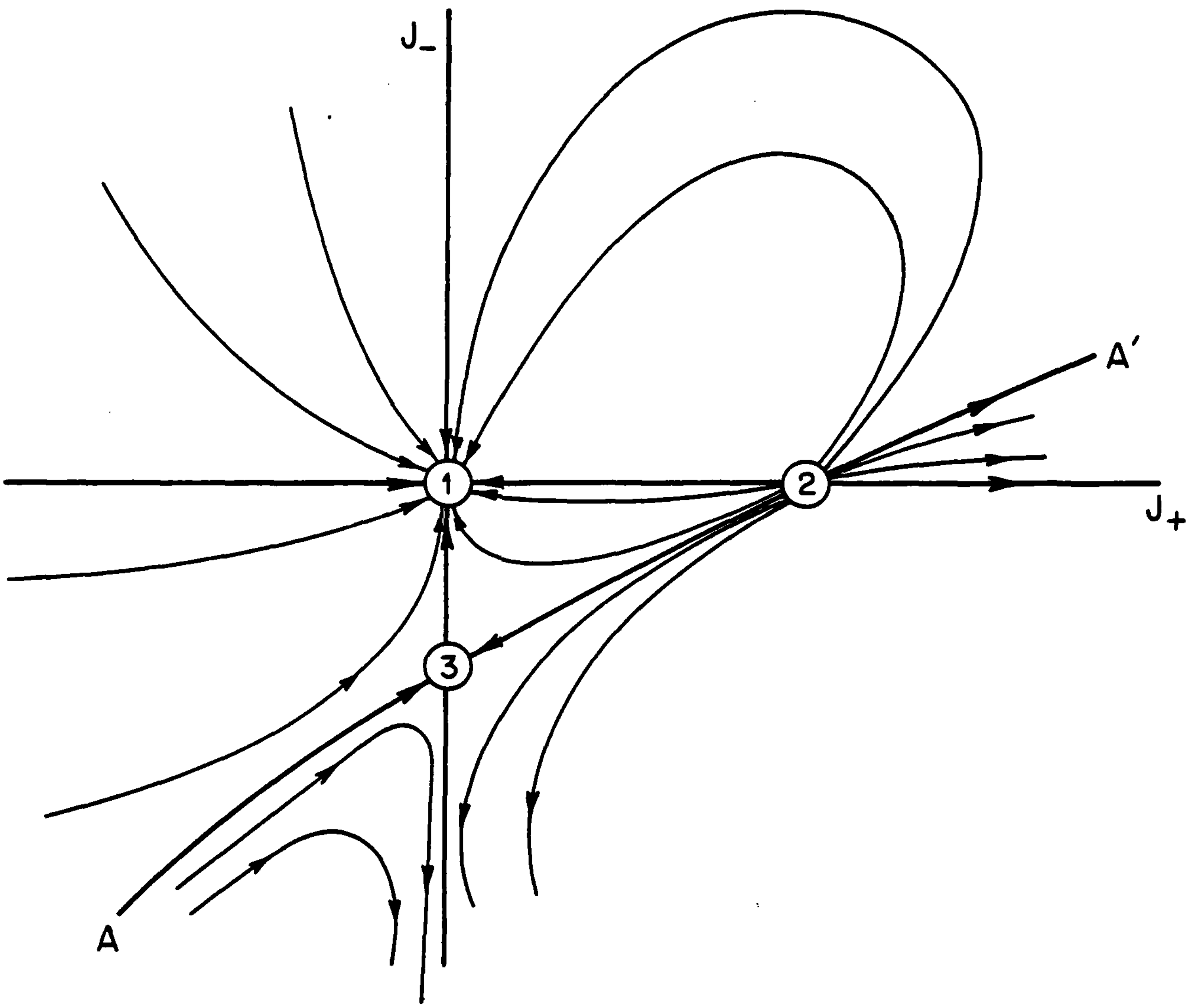

Fig. 2 Phase plane with singular points, for Eqs. (5.6) and (5.7)

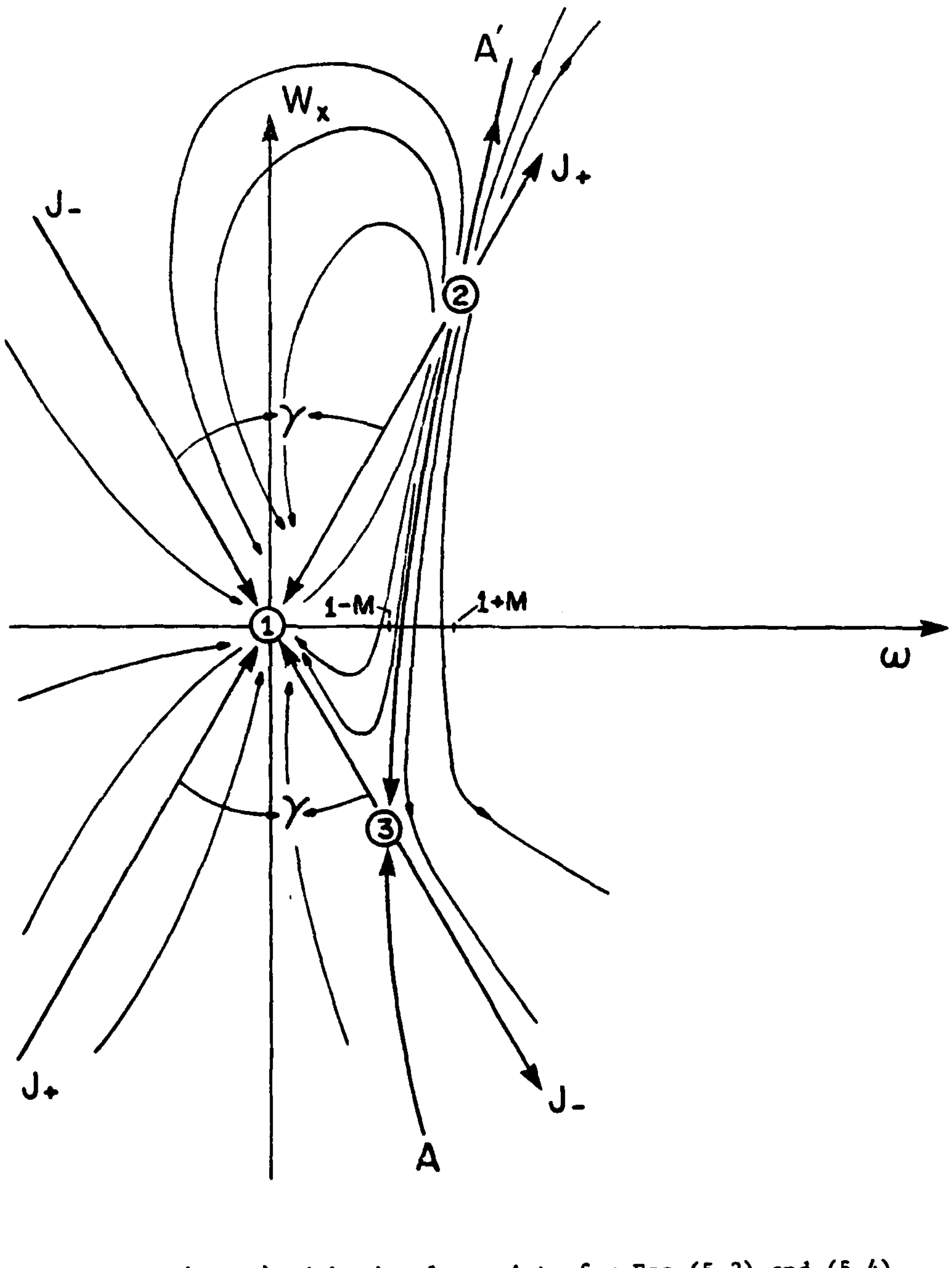

Fig. 3 Phase plane (ω, w_{x1}) with singular points for Eqs.(5.3) and (5.4)

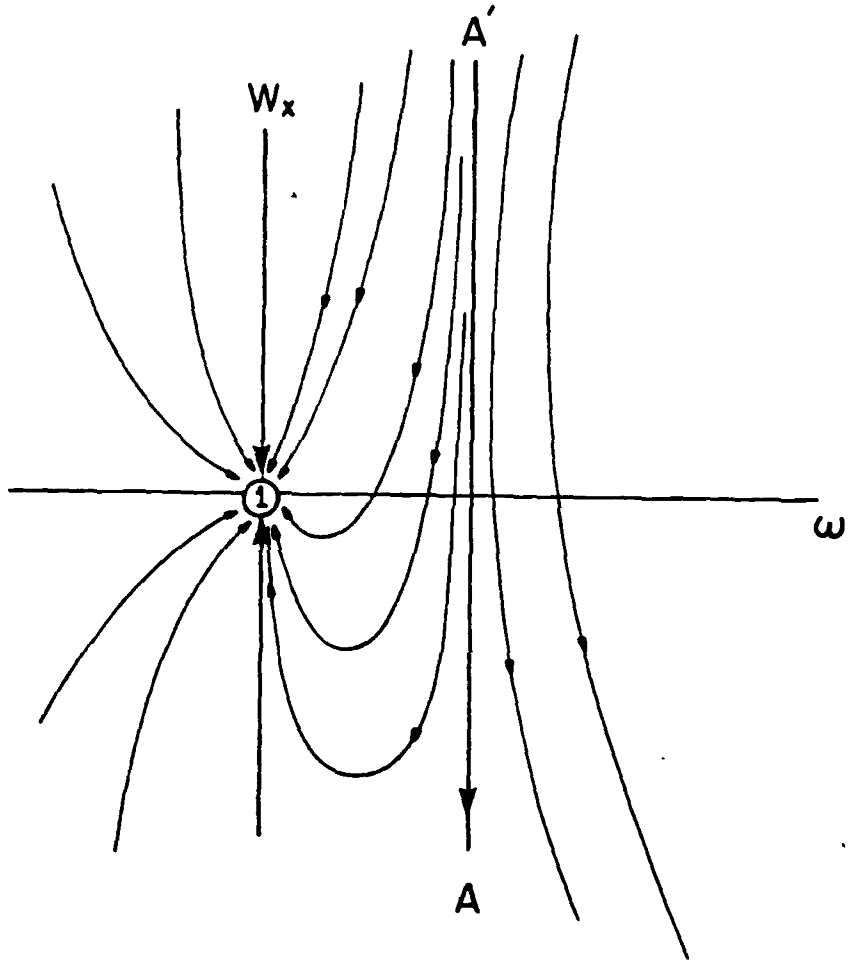

Fig. 4 Degenerated phase plane for $M \equiv 0$

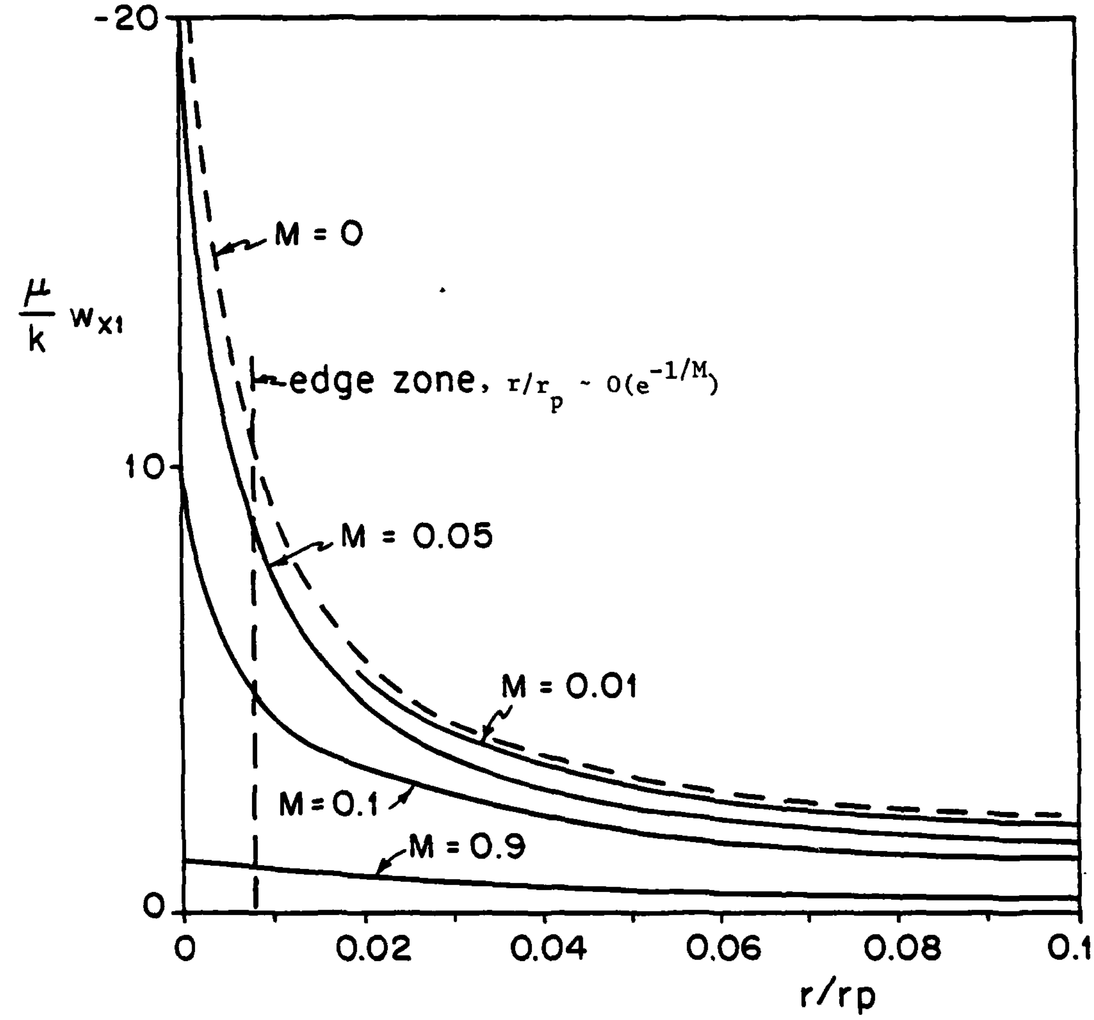

Fig. 5 Shear strain, $(\mu/k)w_{x1}$ versus r/r_p for various crack tip speeds, with estimate of edge zone

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The steady-state dynamic fields near a rapidly propagating crack tip in an elastic perfectly-plastic material have been investigated for the case of Mode-III fracture. For arbitrary values of the dimensionless crack-tip speed (M) the "near-field" consists of a central-fan field ahead of the crack tip and a uniform field in its wake. It is shown that this field is valid in a "boundary layer" which shrinks on the crack tip in the limit of vanishing M. For small M the "far-field" was found as a regular perturbation

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expansion in M, with the quasi-static solution as its first term. A uniform expansion over the polar angle θ measured from the plane of the crack was also obtained; its first term displays the connection between the near-field and the far-field.

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