AMSAA

(12)

TECHNICAL REPORT NO. 309

LEVEL II

A092835

MOVING TARGET LOCATION ERRORS FOR GROUND VEHICLES

DTIC
ELECTE
DEC [illegible]
S
E

JULIAN A. CHERNICK

Best Available Copy

AUGUST 1980

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

U.S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
ABERDEEN PROVING GROUND, MARYLAND

80 12 10 0

DISPOSITION

Destroy this report when no longer needed. Do not return it to the originator.

DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so specified by other official documentation.

WARNING

Information and data contained in this document are based on the input available at the time of preparation. The results may be subject to change and should not be construed as representing the DARCOM position unless so specified.

TRADE NAMES

The use of trade names in this report does not constitute an official endorsement or approval of the use of such commercial hardware or software. The report may not be cited for purposes of advertisement.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

A092835

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER Technical Report No. 309	2. GOVT ACCESSION NO. AD-A092 835	3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)

(6) Moving Target Location Errors for Ground Vehicles.

5. TYPE OF REPORT & PERIOD COVERED

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)

(10) Julian A. Chernick

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS

Director
US Army Materiel Systems Analysis Activity
Aberdeen Proving Ground, MD 21005

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

(16) DA Project No. 1R765706M541

11. CONTROLLING OFFICE NAME AND ADDRESS

Commander
US Army Materiel Development & Readiness Command
5001 Eisenhower Ave., Alexandria, VA 22333

(11) 12. REPORT DATE

AUGUST 1980 (12) 41

13. NUMBER OF PAGES

40

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

(14) AMSAA-TR-309

15. SECURITY CLASS. (of this report)

UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Moving Targets
Target Location Errors
Indirect Fire
Prediction Errors

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report documents AMSAA's current methodology and data base for estimating the target location errors associated with moving ground vehicles. Starting from basic principles, the equations for the heading and cross-heading components of the moving target location error are derived. Reviewing the available data provides estimates for the parameters of the problem. The speed variability error component is found to increase with the mean target speed while the direction errors are found to decrease as the mean speed increases. The target location error, and its two orthogonal components, show

→ next page

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED LB
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

403910

20. Abstract - Continued

a nearly linear increase with prediction time.

CONTENTS

ACKNOWLEDGEMENT

The author acknowledges the helpful discussions on some of the mathematical points in this report with Arthur Groves, Tactical Operations Analysis Office, and Richard Sandmeyer, Ground Warfare Division, both of AMSAA.

MOVING TARGET LOCATION ERRORS FOR GROUND VEHICLES

1. BASIC CONSIDERATIONS

This report documents the available data base and methodology for estimating target location errors for ground vehicles. It is an update of a letter response given in August 1978 to a request for such information from the Project Manager CAWS COPPERHEAD Office. While the basic results and trends have not changed significantly from what was already provided, a more general expression of the problem has been developed. This provides a clearer understanding of both the underlying factors that drive the results and the limitations of the current data base. This effort also provides results that are readily extendable to systems other than COPPERHEAD.

The target location error (TLE) for moving targets is the difference between a target's predicted location and its actual location at a particular time. This time is generally associated with the arrival of the first round in the target's vicinity. The total time from target acquisition until first round impact is usually referred to as the response time. The principal relevant components of the response time are shown in Figure 1. Estimates of the total response time are required for computing the predicted target intercept time and point, so that guns may be properly aimed. For this reason, part of the TLE may be due to the error in predicting the correct total response time, owing for instance, to unusual delays in the transmittal of targeting data.

Other components of TLE are due to errors in estimating target speed and direction during the observation time; errors induced by the variations of target speed and direction during the prediction time; and finally, map errors associated with the observer location.

2. GENERAL EXPRESSION FOR THE MOVING TARGET LOCATION ERROR

In the most basic description of the problem, the target is considered to be moving with a speed S_1, and in a direction D_1, during the observation period from which estimates of target speed and direction ($\tilde{S}$ and $\tilde{D}$) are derived. These estimates include the effects of measurement noise. During the prediction time, the target's speed and direction change to S_2 and D_2. The ideal situation from an analytical viewpoint would be to have sufficiently accurate test data from which both the variability component (S_2-S_1)* and the estimation component $(S_1-\tilde{S})$ could

---

*Although the speed errors are used in the example, the same arguments apply to the direction errors.

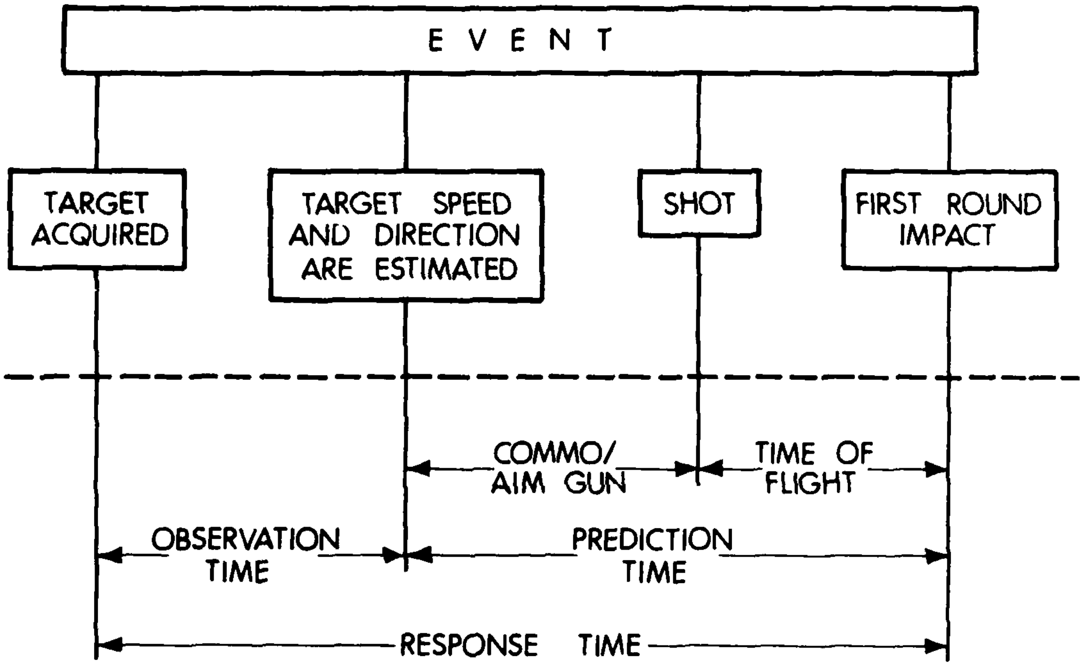


Figure 1. Components of Response Time.

be obtained individually. Unfortunately, none of the data currently available (References 1 through 4) allow this degree of refinement. Instead, the best that can be done is to combine the estimation errors with the variability errors.

To the extent to which the estimation and variability errors are independent of one another, the total error variance is given by:

$$\sigma^2(S_2-\tilde{S}) \qquad [1]$$

This is easily shown since $\sigma^2(S_2-\tilde{S}) = \sigma^2\left[(S_2-S_1) + (S_1-\tilde{S})\right]$ and it is known that $\sigma^2(U+V) = \sigma^2(U) + \sigma^2(V)$ when U, V are independent random variables.

The geometry of the situation is depicted in Figure 2, where the subscript in Equation [1] has been dropped for simplicity, and $\tilde{T}$ is the estimated prediction time. The estimated direction of movement is taken to be along the X-axis so that the coordinates of the predicted target intercept point are given by:

$$X_p = \tilde{S}\,\tilde{T} \qquad [2a]$$

$$Y_p = 0 \qquad [2b]$$

---

[1] Horley, G., W. Dousa, Jr., and J. Lenoci; HELBAT 2-Forward Observers (U); TM-17-72, June 1972, US Army Human Engineering Laboratory, Aberdeen Proving Ground, MD, CONFIDENTIAL report.

[2] Horley, G., and W. Dousa, Jr.; HELBAT 4-Automated Fire Direction on Moving Targets (U); TM-19-76, May 1976, US Army Human Engineering Laboratory, Aberdeen Proving Ground, MD, CONFIDENTIAL report.

[3] Horley, G., et.al.; HELBAT 5-Automated Fire Direction Techniques (U); TR 1-77, March 1978, US Army Human Engineering Laboratory, Aberdeen Proving Ground, MD, CONFIDENTIAL report.

[4] Chernick, Julian and M. Chernick; Forecasting Ground Target Speeds; Technical Report No. 111, August 1974, US Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, MD, UNCLASSIFIED report.

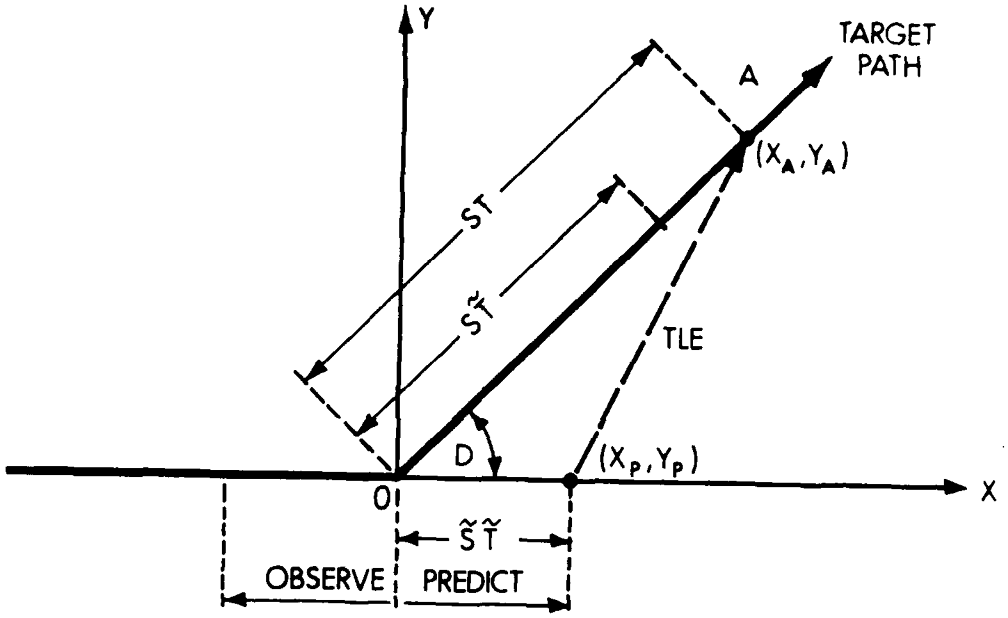


Figure 2. Target Location Error Geometry.

However, the combination of target movement variability and measurement error result in the target moving along the line $\overline{OA}$ which makes an angle D with respect to the predicted direction, and with speed S not necessarily equal to S. At the expected time of first round impact, the target is not at the position (X_p, Y_p) but rather at a distance S $\tilde{T}$ along the line $\overline{OA}$. Similarly, by the actual time of first round impact, the target has progressed a distance S T along $\overline{OA}$, reaching the point A whose coordinates (X_A, Y_A) are given by:

$$X_A = S\ T \cos D \qquad [3a]$$

$$Y_A = S\ T \sin D \qquad [3b]$$

The relative target location error is therefore, the difference between these actual coordinates and the predicted coordinates, and is given by:

$$\sigma^2\ (X_A - X_p) = \sigma^2\ (S\ T \cos D - \tilde{S}\ \tilde{T}) \qquad [4a]$$

$$\sigma^2\ (Y_A - Y_p) = \sigma^2\ (S\ T \sin D) \qquad [4b]$$

By adding the map error variance to the relative target location error variances shown in Equations [4a] and [4b], the total target location errors are obtained.

In order to evaluate Equations [4a] and [4b], it is first necessary to evaluate the quantity $\tilde{S}\ \tilde{T}$. Now, $\tilde{T}$ will most likely be taken to be the mean prediction time, μ_T, since the distribution of prediction times will presumably be known in advance.

The determination of the predicted speed $\tilde{S}$, however, raises some questions. If the observation time is relatively long, then the observed speed would approach a value equal to the mean speed, μ_S, assuming target speeds are random fluctuations about a mean speed (see Reference 4 on this subject). On the other hand, if the observation time is relatively short, the observed speed would be just another random speed sample drawn independently from the same distribution as the target speed during the prediction time.* The above arguments lead to two different estimates for the speed error. When the predicted speed equals the mean speed, the following expression results:

$$\sigma^2(S\mu_T - \tilde{S}\mu_T) = \sigma^2(S\mu_T) + \sigma^2(\mu_S\mu_T) = \mu_T{}^2\ \sigma_S{}^2$$

***This is assuming measurement errors are small compared to variability errors for this calculation. This also assumes the prediction time is long enough that correlation between the speeds during the observation time and prediction time is not significant.**

However, when the predicted speed is an independent random sample, the result is:

$$\sigma^2(S\mu_T - \tilde{S}\mu_T) = \sigma^2(S\mu_T) + \sigma^2(S\mu_T) = 2\mu_T{}^2\sigma_S{}^2$$

The net result of the above considerations means that the predicted target intercept point (X_p, Y_p) can be taken to be equal to ($\mu_S\mu_T$,0) for both cases above. When the observation times are short however, the predicted speed errors will be found to be larger by a factor of up to $\sqrt{2}$, **than** when the observation times are long.

Combining the above arguments with the map error variance gives:

$$\sigma^2{}_{TLE,X} = \sigma^2{}_{MAP} + \sigma^2(S\ T\ \cos D) \qquad [5a]$$

$$\sigma^2{}_{TLE,Y} = \sigma^2{}_{MAP} + \sigma^2(S\ T\ \sin D) \qquad [5b]$$

In order to evaluate Equations [5a] and [5b], it is necessary to work out the variance of products of random variables. Using the method of Goodman (Reference 5) allows this to be accomplished in closed form. First, applications of this method will be shown for three general expressions which will be helpful in evaluating Equations [5a] and [5b]. These expressions involve products and squares of the independent random variables U, V, W and a multiplier constant C. The results are:

$$\sigma^2(CUV) = C^2(\mu^2{}_U\sigma^2{}_V + \mu^2{}_V\sigma^2{}_U + \sigma^2{}_U\sigma^2{}_V) \qquad [6]$$

$$\begin{aligned}\sigma^2(CUVW) = C^2\Big[&\mu^2{}_U(\mu^2{}_W\sigma^2{}_V + \mu^2{}_V\sigma^2{}_W + \sigma^2{}_V\sigma^2{}_W) + \mu^2{}_V \qquad [7]\\ &(\mu^2{}_W\sigma^2{}_U + \sigma^2{}_U\sigma^2{}_W) + \mu^2{}_W(\sigma^2{}_U\sigma^2{}_V) + \sigma^2{}_U\sigma^2{}_V\sigma^2{}_W\Big]\end{aligned}$$

$$\sigma^2(CU^2) = 4C^2\mu^2{}_U\sigma^2{}_U + 2C^2\sigma^4{}_U \qquad [8]$$

---

[5] Goodman, L.A.; On The Exact Variance of Products; pages 708-713, December 1960, American Statistical Association Journal (also see same journal, Corrigenda, page 917, December 1962).

Applying Equation [7] to Equations [3a] and [3b] and taking the mean direction error to be 0 degrees (hence $\mu_{\cos D} = 1$; $\mu_{\sin D} = 0$) gives:

$$\sigma^2_{XA} = \sigma^2(S\ T \cos D) = \sigma^2_S \sigma^2_T + \mu^2_T \sigma^2_S + \mu^2_S \sigma^2_T + \sigma^2_{\cos D} (\sigma^2_S \sigma^2_T + \mu^2_S \sigma^2_T + \mu^2_T \sigma^2_S + \mu^2_S \mu^2_T) \qquad [9]$$

and

$$\sigma^2_{YA} = \sigma^2(S\ T \sin D) = \sigma^2_{\sin D} (\mu^2_S \mu^2_T + \mu^2_S \sigma^2_T + \mu^2_T \sigma^2_S + \sigma^2_S \sigma^2_T) \qquad [10]$$

Now assuming the direction errors are within about ±0.50 radians (~ 30 degrees) the small angle approximations

$$\cos D = 1 - \frac{1}{2} D^2 \qquad [11a]$$

$$\sin D = D \qquad [11b]$$

can be used to evaluate the variance of the sines and cosines, giving

$$\sigma^2 (\cos D) = \sigma^2 (1 - \frac{1}{2} D^2) = \sigma^2 (- \frac{1}{2} D^2) \qquad [12]$$

$$\sigma^2 (\sin D) = \sigma^2 (D) = \sigma^2_D \qquad [13]$$

Equation [12] can be evaluated using Equation [8] with C = -1/2 giving

$$\sigma^2 (\cos D) = \mu^2_D \sigma^2_D + \frac{1}{2} \sigma^4_D$$

For $\mu_D = 0$ this gives

$$\sigma^2 (\cos D) = \frac{1}{2} \sigma^4_D \qquad [14]$$

By substituting Equations [13] and [14] into Equations [9] and [10] and after rearrangement, the following expressions for the variance of the actual target location are obtained:

$$\sigma^2_{XA} = \sigma^2 (S\ T \cos D) = \mu^2_S\ \mu^2_T \left[(1+K_1)(1+K_2)(1+K_3)-1\right] \qquad [15a]$$

$$\sigma^2_{YA} = \sigma^2 (S\ T \sin D) = \mu^2_S\ \mu^2_T\ \sigma^2_D\ (1+K_1)(1+K_2) \qquad [15b]$$

where

$$K_1 = (\sigma_S/\mu_S)^2$$

$$K_2 = (\sigma_T/\mu_T)^2$$

$$K_3 = \frac{1}{2}\sigma^4_D$$

From Equations [15a] and [15b], it is seen that K_1 constitutes a speed error, K_2 a time error, K_3 a direction error, and that these errors act through multiplication in a compounding manner.

Finally, substituting Equation [15] into Equation [5] gives:

$$\sigma^2_{TLE,X} = \sigma^2_{MAP} + \mu^2_S\ \mu^2_T \left[(1+K_1)(1+K_2)(1+K_3)-1\right] \qquad [16a]$$

$$\sigma^2_{TLE,Y} = \sigma^2_{MAP} + \mu^2_S\ \mu^2_T\ \sigma^2_D\ (1+K_1)(1+K_2) \qquad [16b]$$

The following sections of this report will attempt to develop the current best estimates for the various quantities that constitute Equation [16] and justify some of the assumptions made above.

3. ESTIMATES OF THE MAP ERROR

Estimates of the component of the TLE due to map error are obtained by combining data on the ability of the forward observer to locate his own position in relation to reference points in the surrounding terrain with estimates on the accuracy with which the reference points are plotted on standard Army maps. In HELBAT 2, a number of techniques using the laser designator/rangefinder were tried and found to significantly reduce the forward observer location error compared with the standard map spot techniques. Results are summarized in Table 1 below:

TABLE 1 ABILITY OF FORWARD OBSERVER TO LOCATE HIS POSITION (FROM REFERENCE 1)*

FO Location Technique	σ_{MAP}(M)	Sample Size
Laser (2 Reference Points)	2	10
Laser (1 Reference Point and Gyro Compass)	6	12
Laser (White Phosphorus Rounds)	46	19
Laser (Illuminating Rounds)	42	20
Map Spot	74	42

*Converting from reported mean radial errors to standard deviations for the circular normal distribution (see Appendix A).

Estimates of reference point map errors from AMSAA analysts experienced with the use of CLASS A maps (1:50,000 scale) for parts of the world with fairly frequent terrain contours or that have been fairly well surveyed (like most of W. Europe) indicate that these errors are equal to about 30m (1 σ). The total map error is the root mean square value of these two components. This is about 30-31m for the laser reference point methods. In parts of the world where terrain is very flat and survey points are scarce, map errors asociated with lasing the position of an illumination or white phosphorus round would be appropriate, which from Table 1 is about 42-46m.

4. TARGET DIRECTION ERROR

Discussions with the US Army Human Engineering Laboratory personnel indicated that data were available from the HELBAT-5 files from which target direction errors could be estimated. Data from 224 missions were received and analyzed by AMSAA. A listing of the data is included in Appendix B.

These data are based on three successive lasings of target position. The line connecting the first and second lased positions determines the nominal heading. The line from the second and third lased positions is then compared to the nominal heading to obtain the direction error.

Naturally, measurement errors (or noise) in this case due to the laser ranging error, also contributes to the direction errors obtained in this manner. Measurement errors of one type or another, however, will exist in any real target sensing system, and the use of the laser for this purpose is currently a strong possibility.

The existence of a laser ranging error contribution can be deduced by examining the standard deviation of the direction error (σ_D) as a function of the time between the second and third target lasing. This relation is shown in Table 2. An intuitive concept of target movement paths would hold that they might be similar to a random walk process about a moving mean. The general increase in σ_D between about 30 seconds and 70 seconds would appear to be consistent with such an intuitive target movement concept.

On the other hand, the rise in σ_D for times less than 30 seconds would not fit this picture, except for either of two possiblities: confounding of the data associated with the short time intervals by being cross-correlated with other supposedly "independent" variables, or the presence of measurement noise. The first possibility can be discounted since the other independent variable, the target mean velocity, is nearly the same for times less than 30 seconds, as seen in Table 2.

TABLE 2 RELATION BETWEEN DIRECTION ERRORS AND THE TIME BETWEEN TARGET LASINGS

Mean Time Between Lasings (Sec)	σ_D (Deg)	Mean Velocity (M/S)	Sample Size
8	33	5.4	31
22	14	5.1	71
33	7	5.3	56
44	21	5.1	35
56	17	5.9	13
69	22	5.0	8
77	21	4.0	6
92	9	4.5	2

This leaves the measurement noise as the likely culprit. The manner in which shortening the time between measurements serves to increase the relative importance of the measurement noise can be quickly grasped through the use of a simplified diagram as depicted in Figure 4. The target is predicted to travel along the line $\overline{P_1P_2}$ but after passing P_2 the target changes direction along $\overline{P_2P_3}$. The uncertainty in measuring target position at time T_3 is represented by the circle around P_3 and the resulting estimated direction is shown by the longer dashed line. However, at a later time, T_4, the same amount of measurement uncertainty (same size circle about P_4) does not lead to such a large error in estimating the actual direction $\overline{P_2P_4}$, as seen by the short dashed line.* To put it another way, what is important is the ratio of the size of the measurement error to the distance between measurements. As this ratio increases the accuracy of the measurements becomes poorer. Since some amount of measurement error will probably always exist, this sets a lower limit on the time between measurements in order for resulting predictions to be reasonably accurate.** Based on the plot of data in Figure 3, the time between target lasing should not be any shorter than about 15-20 seconds, if possible. This would insure reasonably accurate predictors, given the current magnitude of laser measurement errors for target speeds of about 5 m/s. Slower target speeds would cause this required observation (waiting) time to increase. Naturally, requiring a deliberate delay of any type in engaging a moving target runs counter to the basic desire of shortening the response time as much as possible, indicating that a comprehensive trade-off study of this problem is needed before definitive answers can be obtained.

For times between 40 seconds and 80 seconds, the data of Figure 3 indicate σ_D levels off at about 20 degrees. For times greater than 80 seconds, the data gives an indication that the σ_D may be decreasing but the sample size is so meager that this trend may not be real.

A complete lack of data exists for prediction times larger than about 90 seconds. The significance of this data void for long prediction times can be critical as the following example shows. Assuming the decrease in σ_D between 80 and 90 seconds is not real but instead σ_D remains at about 20 degrees for all prediction times greater than 40 seconds, then for a target speed of 5 m/s and a prediction time of 3 minutes the σ_{TLE} in the direction perpendicular to the predicted heading would be about 315m based on Equation [16b]. However, if the decrease is real and a σ_D of 9 degrees is appropriate the error would amount to only 140m. Naturally even smaller direction errors would result if the decreasing trend is real and is extrapolated even beyond 90 seconds.

---

*The true situation is more involved in that there is measurement error involved in the positions P_1 and P_2 but the conclusions are the same.

**The idential argument and conclusion also apply when speed estimates are considered.

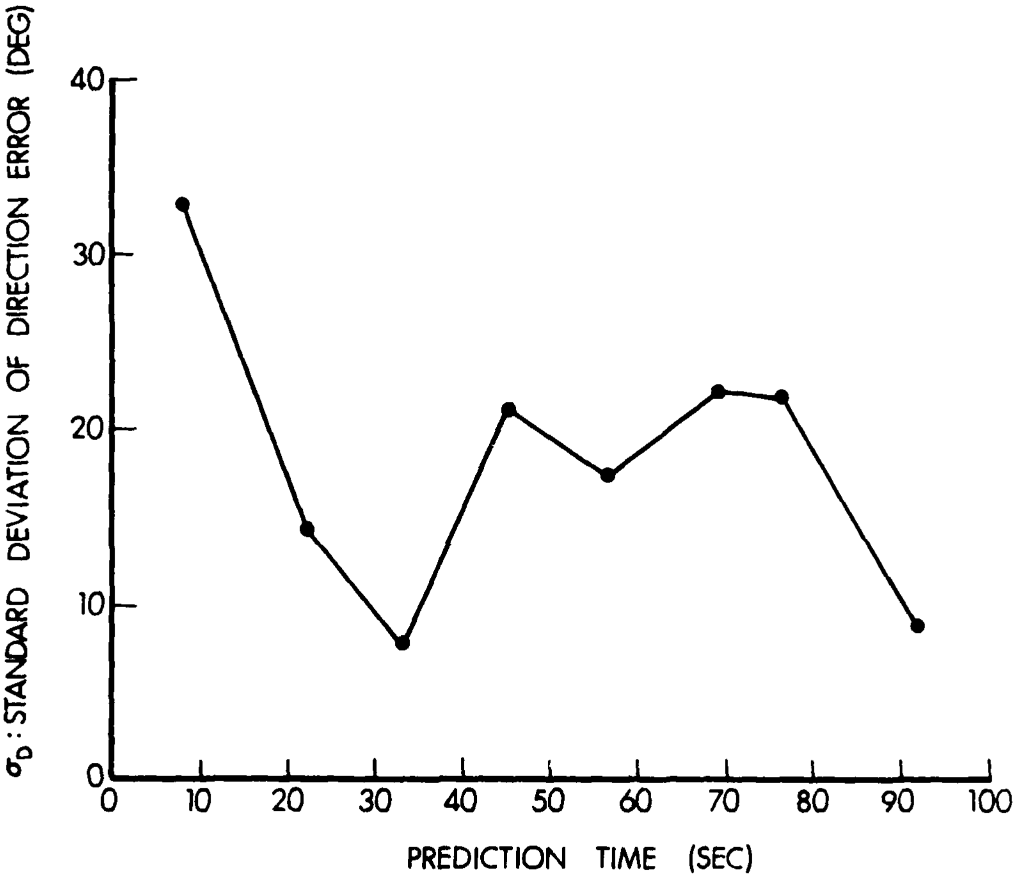
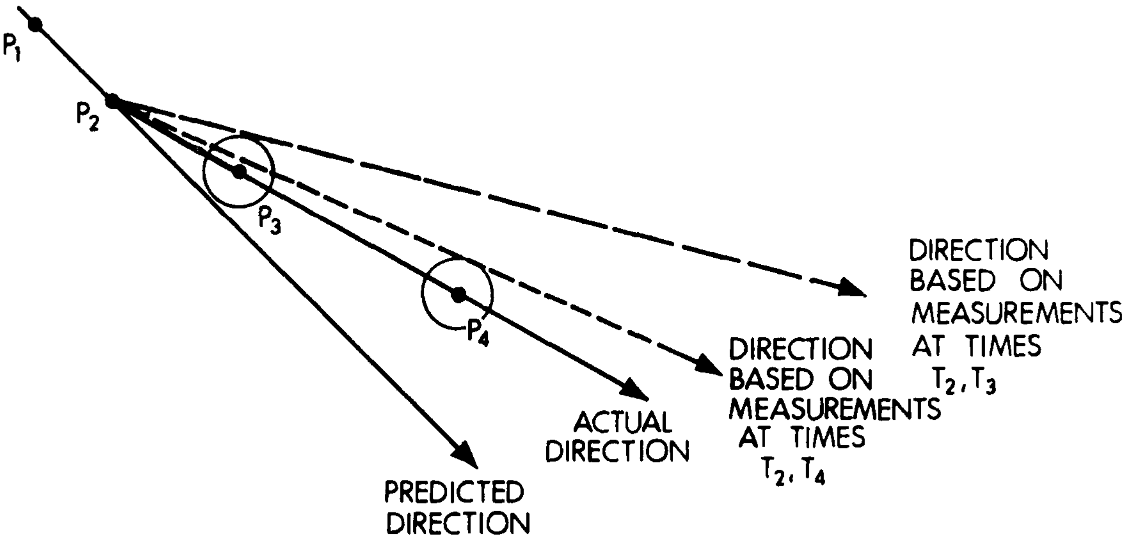


Figure 3. Direction Error vs. Prediction Time.



P_i = POSITIONS OF MOVING TARGET

T_i = TIMES OF MEASURING TARGET POSITION

○ = UNCERTAINTY IN POSITION DUE TO MEASUREMENT ERROR

Figure 4. Effect Time Between Measurements has on Measurement Error.

Obviously, data appropriate for longer prediction times are badly needed, particularly for analyses of second-generation guided projectiles which are more sensitive to target location errors than are the current generation projectiles, due to their relatively smaller acquisition radii.

Next, the relation between the σ_D and the mean target speed is examined. In order to reduce the effect of the measurement noise somewhat, data for prediction times less than a parametric minimum time, T_{MIN}, have been deleted. The results are shown in Figure 5 and Table 3, for T_{MIN} = 0, 20, 30 seconds.

The σ_D is found to be very sensitive to the mean target speed regardless of the value of T_{MIN}. As the target speed increases from 3 m/s to 8 m/s, the σ_D decreases from about 27 degrees to 2 degrees. The decrease is basically monotonic although noise is superimposed on the curves. Some noise is expected even for the cases where T_{MIN} = 20 and 30 seconds, because the short times between the first two target lasings were not filtered out (since they were not part of the HELBAT-5 data package), but the reduction in the σ_D for these cases compared to the T_{MIN} = 0 case is apparent, particularly for the 4 and 7 m/s target speeds.

TABLE 3 RELATION BETWEEN TARGET DIRECTION ERROR AND MEAN TARGET SPEED

Mean Target Speed (m/s)	σ_D (Deg)	Mean Time Between Lasings (Sec)	Sample Size
3	27	46	5
4	19	41	37
5	19	38	67
6	9	36	53
7	5	39	8
8	2	31	5

Examination of Table 3 for a cross-correlation between the mean target speed and time between lasings indicates that such a cross-correlation does exist. As the speeds increases the average time between lasings happens to show a general decrease. Looking back to Figure 3 indicates that this cross-correlation could be significant.

To determine whether this (indirect) time effect alone is sufficient to account for the trend of Figure 5, the σ_D values corresponding to the times shown in Table 3 were estimated through the use of Figure 3. These results are plotted in Figure 6 along with a re-plot of the T_{MIN} = 20 sec. (least noisy) data from Figure 5. The indication from Figure 6 is that the time effect accounts for a significant portion of the decrease in σ_D with target speeds, but not all of the effect.

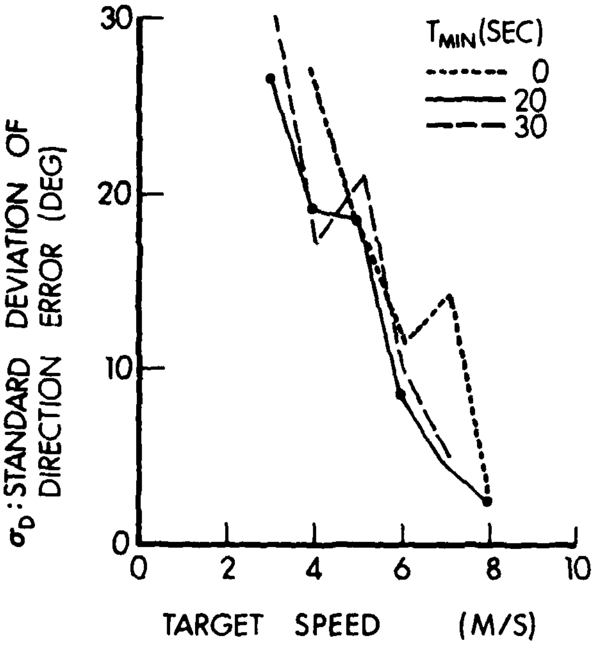


Figure 5. Direction Error vs. Target Speed.

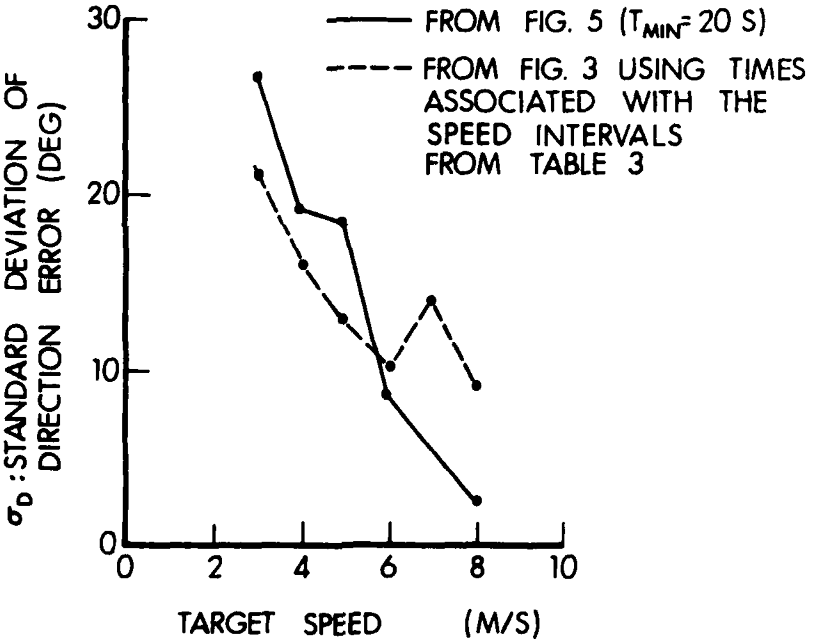


Figure 6. Comparison of Indirect Time Effect With Speed Effect on Direction Errors.

The direction errors appear to be slightly larger for the lower velocities than what would be obtained by the time effect alone, while for the higher speeds, the direction errors are significantly smaller than what would be estimated based only on the times associated with the higher speeds.

The fact that a strong inverse relation appears to exist between direction error and mean target speed is indeed logical. If a driver is fairly certain of where he is going with few obstacles or hills in the terrain ahead of him, he would tend to keep up his speed and turn less often. But, if the terrain becomes more difficult and he is less sure about the area immediately in front of his vehicle, he would tend to slow down and more turning would be expected.

A mathematical expression that appears to fit the data of Table 3, within the noise, for the interval $3 \leq \mu_S \leq 8$ m/s is given by:

$$\sigma_D = 0.70 \exp\left(- \left[\mu_S/5\right]^2\right) \qquad [17]$$

for σ_D in radians

μ_S in m/sec

5. TARGET SPEED ERRORS

While many estimates of the speed of ground vehicles are available, including the manner in which the average speed depends on the terrain characteristics, data on the variability of target speed over the course of time is less readily available. A calculation based on Equation [16], however, shows that even a 1 m/s speed error can result in an error in target location of 180m when maintained over a 3 minute prediction time.

In Reference 4, some data on target speed variability is presented, based on a measurement procedure which involved clocking a vehicle's odometer readings. The vehicle was part of a mixed convoy of tanks and armored personnel carriers in a road march on hard surfaced tank trails at Fort Hood, Texas.

Various mathematical techniques of predicting future target speeds based on past data were employed and it was found that for prediction times greater than about 1 minute, the use of the mean value of all past speed estimates gave the smallest prediction errors, while the use of only the most recent speed estimates yielded the worst prediction errors.

As seen in Table 4, the differences in these two sets of speed errors are nearly equal to the factor $\sqrt{2}$, for the reasons previously discussed in Section 2. Other prediction methods generally were found to give results between these two extremes.

TABLE 4 STANDARD DEVIATIONS OF TARGET SPEED ERRORS (M/S) (FROM REFERENCE 4, TABLE 4.1)*

Prediction Technique	Prediction Time (Minutes) 2.5	5.0	10.0	15.0
Most Recent Speed Estimate	1.1 - 1.5	1.1 - 1.5	1.0 - 1.4	1.1 - 1.5
Mean of All Past Speed Estimates	0.9 - 1.2	0.9 - 1.0	0.9 - 0.9	0.8 - 0.9

*Converting from displayed mean radial errors to standard deviations for the univariate normal distribution (see Appendix A).

The spread of speed errors within each category results from using two different series of target movement where each series lasted between 40-50 minutes. The different speed series were separated by an interval during which the vehicles stopped. The reason for this stopping period was not known to the operations analyst recording the movement times.

The speed predictions were based on a part of only one of the series. When these predictions were compared with the remainder of the same series, the smaller set of prediction errors resulted, while the larger set of prediction errors occurred when the speed estimates from one series were compared to the actual speeds for the other series.

The overall trend from the data in Table 4 indicates that the standard deviation of speed error (σ_S) remains fairly constant even for relatively long prediction times. For example, for the most recent speed estimate prediction technique, the σ_S only varies between 1.0 - 1.1 m/s when predicting the same series, and between 1.4 - 1.5 m/s when predicting a different series.

Since the mean target speeds for either series were nearly the same, and equal to about 7.5 m/s, the ratio of σ_S to the mean speed varies from about 0.13 for predicting the same series to about 0.20 for predicting a different series.

A very rough estimate of the variability of speed for the case of a maneuvering vehicle, as opposed to the previous data on vehicles in a convoy, can be made using the HELBAT-5 data reported in Reference 3. By comparing the previously derived HELBAT-5 direction error and map error variances with the reported aimpoint-to-target miss distance variance, the component of the error due to speed variation can be determined. The results indicate a σ_S of between 1.1 - 1.5 m/s depending on the type of fire control used. While this is very close to the previous values σ_S for convoy movements, when considered in relation to the average target speed of 5.2 m/s for the HELBAT test, the ratios are actually quite larger. These ratios are summarized in Table 5 below.

TABLE 5 STANDARD DEVIATION OF TARGET SPEEDS

	Convoy Movements (Ref. 4)	Maneuvering Targets (Ref. 3)
σ_S (m/s)	1.0 - 1.5	1.1 - 1.5
μ_S (m/s)	7.5	5.2
σ_S/μ_S	0.13 - 0.20	0.21 - 0.28

Currently there is an insufficient data base to determine how much of the change in the ratio (σ_S/μ_S) from Table 5, is due to the type of vehicle movement as opposed to the difference in the average target speeds. Perhaps these two factors are inevitably confounded together. These relationships are expected to be terrain and weather dependent, as well.

6. RESULTS

In section 2, the general expressions for the σ_{TLE} were derived (Equation [16]) and in sections 3 through 5 the current best available estimates of the various components that contribute to the σ_{TLE} were presented. No data were presented on the time variability error, σ_T, since this is highly dependent on the particular system being considered, including the communications equipment and the scenario in which the system is to be employed. This quantity will be treated parametrically, as will the target mean speed, μ_S, and mean prediction time, μ_T.

In Tables 6 and 7, the σ_{TLE} components in the directions parallel and perpendicular to the nominal target heading are displayed for the convoy and maneuvering target cases, respectively. The variability of the prediction time has been assumed to be 15 seconds, a map error of 30m was assumed, and the higher estimates of the speed variability have been used. In all cases the σ_{TLE}'s for the maneuvering targets are higher than for the convoy targets, but in some cases only slightly so.

It is clear that the σ_{TLE}'s are strong functions of the mean prediction time. The σ_{TLE} in the direction parallel to the nominal heading increases with mean target speed.

This occurs because the direction error term, (the K_3 term in Equation [16a]) which decreases with the mean speed, is never large enough to make up for the other parts of the expression which increase with the mean speed. The σ_{TLE} in the direction perpendicular to the nominal heading decreases markedly with mean speed, due to the relatively greater significance of the direction error term on this axis.

At the bottom of Tables 6 and 7, the circularized σ_{TLE} are shown, where σ_{TLE} is computed by averaging the two component σ_{TLE}'s. Since one component increases while the other decreases with the mean speed, the circularized σ_{TLE} is a fairly weak function of the mean speed. It is important to realize that in certain applications the use of the circularized TLE can result in larger than desirable computation errors, when the ratio of the two σ's differs significantly from unity. For further discussion of this point, see Reference 6.

Finally in Table 8, the sensitivity of the σ_{TLE} to the variation of prediction time, σ_T, is displayed for the maneuvering target case when the mean speed = 5 m/s. It is apparent that the component of σ_{TLE} in the nominal heading direction is very sensitive to the time error, while the perpendicular component is nearly independent of this error, over the range of values considered.

---

[6] Groves, A.D.; Handbook on the Use of the Bivariate Normal Distribution in Describing Weapon Accuracy; MR 1372, September 1961, US Army Ballistic Research Laboratory, UNCLASSIFIED report.

TABLE 6 CONVOY TARGET TLE'S (σ_T = 15 SEC; $\sigma_{FO/MAP}$ = 30m; MOST RECENT SPEED ESTIMATE)

$\sigma_{TLE,X}$ = Standard Deviation of TLE in Nominal Heading Direction (M)

	μ_T = Mean Prediction Time (Sec)				
Target Mean Speed μ_S (m/s)	100	150	200	250	300
3	134	191	250	309	369
5	160	221	285	351	418
8	206	275	350	427	505

$\sigma_{TLE,Y}$ = Standard Deviation of TLE in Perpendicular Direction (M)

	μ_T = Mean Prediction Time (Sec)				
Target Mean Speed μ_S (m/s)	100	150	200	250	300
3	154	227	301	375	450
5	136	200	265	330	396
8	54	73	93	115	136

σ_{TLE}* = Standard Deviation of Circularized TLE (M)

	μ_T = Mean Prediction Time (Sec)				
Target Mean Speed μ_S (m/s)	100	150	200	250	300
3	144	209	276	342	410
5	148	211	275	341	407
8	130	174	222	271	321

* $\sigma_{TLE} = (\sigma_{TLE,X} + \sigma_{TLE,Y})/2$.

TABLE 7 MANEUVERING TARGET TLE'S (σ_T = 15 SEC; $\sigma_{FO/MAP}$ = 30m; MOST RECENT SPEED ESTIMATE)

$\sigma_{TLE,X}$ = Standard Deviation of TLE in Nominal Heading Direction (M)

	μ_T = Mean Prediction Time (Sec)				
Target Mean Speed μ_S(m/s)	100	150	200	250	300
3	148	213	279	346	414
5	189	267	348	431	514
8	260	363	470	580	691

$\sigma_{TLE,Y}$ = Standard Deviation of TLE in Perpendicular Direction (M)

	μ_T = Mean Prediction Time (Sec)				
Target Mean Speed μ_S(m/s)	100	150	200	250	300
3	157	231	307	382	458
5	138	204	270	336	403
8	54	74	95	117	138

σ_{TLE}* = Standard Deviation of Circularized TLE (M)

	μ_T = Mean Prediction Time (Sec)				
Target Mean Speed μ_S(m/s)	100	150	200	250	300
3	153	222	293	364	436
5	164	236	309	384	459
8	157	219	283	349	415

* $\sigma_{TLE} = (\sigma_{TLE,X} + \sigma_{TLE,Y})/2$

TABLE 8 VARIATION OF σ_{TLE} WITH σ_T

(MANEUVERING TARGET; μ_S = 5 m/s; σ_{MAP} = 30m; MOST RECENT SPEED ESTIMATE)

$\sigma_{TLE,X}$ = Standard Deviation of TLE in Nominal Heading Direction (M)

Prediction Time Deviation σ_T(sec)	μ_T = Mean Prediction Time (Sec) 100	150	200	250	300
0	171	255	339	423	508
10	180	261	343	427	510
20	201	276	355	436	519
30	233	300	374	452	532

$\sigma_{TLE,Y}$ = Standard Deviation of TLE in Perpendicular Direction (M)

Prediction Time Deviation σ_T(sec)	μ_T = Mean Prediction Time (Sec) 100	150	200	250	300
0	137	203	269	336	402
10	138	203	269	336	402
20	140	205	270	337	403
30	143	207	272	338	404

σ_{TLE}* = Standard Deviation of Circularized TLE (M)

Prediction Time Deviation σ_T(sec)	μ_T = Mean Prediction Time (Sec) 100	150	200	250	300
0	154	229	304	380	455
10	159	232	306	382	456
20	171	241	313	387	461
30	188	254	323	395	468

* $\sigma_{TLE} = (\sigma_{TLE,X} + \sigma_{TLE,Y})/2$

7. RECOMMENDATIONS

As discussed previously, considerable improvements can be made in the data base which drives the current estimates of moving target location errors. The availability of improved estimates is of relatively greater importance for studies of "second generation" guided projectiles such as SADARM, which are more sensitive to TLE due to smaller acquisition radii, than for studies of the first generation projectiles (i.e., COPPERHEAD).

First, it is imperative to obtain data on target movements for longer periods of time than were available from the HELBAT-5 missions. This is of particular concern in the estimates of target direction errors. The 10-20 degree direction errors for prediction times less than 90 seconds may not be maintained over longer times, as attacking targets may find they are able to maneuver around relatively small terrain obstacles and "return" more or less to the intended heading.

Secondly, better quality data are needed, in order to better separate components of errors due to noisy measurement procedures from the inherent vehicle movement variabilities. This would allow more confidence to be placed in TLE estimates, and a greater ability to extrapolate results to other types of target observation/measurement systems.

Third, new data are required in order to substantiate correlations between the variables discussed in this paper, such as the relation between speed variability and mean speed, only very roughly estimated at the current time.

Finally, the effects on target location error of terrain type, weather, vehicle mobility, evasive maneuvers, and tactical considerations, while intuitively of importance, are currently a long way from being quantified.

With regard to these considerations, target movement data from the CDEC RMS Range at Hunter Liggett, ought to be examined. These data are of far greater quality than anything examined in this paper. Data from various tests (TMAWS, TASVAL, and ARMVAL) should have sufficiently long time histories to be pertinent for the indirect fire problem. The quantities of data that exist should allow a stratified approach which could yield more of the possible interactive effects of the variables. Some insights into the effects of terrain and tactics may be developed from such an effort, particularly as more data becomes available on vehicle movements under simulated battlefield conditions.

APPENDIX A

RELATION BETWEEN BIVARIATE DISTRIBUTION PARAMETERS

APPENDIX A

RELATION BETWEEN BIVARIATE DISTRIBUTION PARAMETERS

In this appendix, relations are presented between parameters of the bivariate distribution which were found to be useful in converting from one type of error to another (i.e., mean radial errors and standard deviation errors). References to the derivation of some of these relations were found while others were not and are therefore derived here. The required relations are summarized in a two-way classification shown in Table A.1 below.

TABLE A.1 RELATIONS BETWEEN BIVARIATE DISTRIBUTION PARAMETERS

Type of Bivariate Distribution	**Mean** Radius ($\bar{r}$)	**Median** Radius*(r_m)
Circular ($\sigma_{MIN} = \sigma_{MAX} = \sigma$)	$\bar{r} = \sqrt{\pi/2}\,\sigma$	$r_m = \sqrt{2 \ln 2}\,\sigma$
Univariate ($\sigma_{MIN} = 0$)	$\bar{r} = \sqrt{2/\pi}\,\sigma_{MAX}$	$r_m = 0.6745\,\sigma_{MAX}$

*Sometimes called circular probable error (CPE or CEP)

Derivation of Mean Radius Relations

Circular Distribution ($\sigma_{MIN} = \sigma_{MAX} = \sigma$)

Starting with the known density of the circular normal distribution:*

$$f(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}$$

The mean radius $\bar{r}$ is defined by:

$$\bar{r} = \int_0^\infty rf(r)dr = \int_0^\infty \frac{r^2}{\sigma^2} e^{-r^2/2\sigma^2} dr$$

---

*For derivation to this point, see Reference 6, page 10.

Let $t = r/\sigma$, $dr = \sigma dt$ gives:

$$\bar{r} = \sigma \int_0^\infty t^2 e^{-t^2/2} dt = \sigma I_1$$

Now I_1, is a known definite integral (see for example Reference 7, 860.13) with the value

$$I_1 = \sqrt{\pi}/4K^3$$

where $K^2 = 1/2$ in this case.

Therefore, the resulting relation is: $\bar{r} = \sqrt{\pi/2}\,\sigma$

Univariate Distribution ($\sigma_{MIN} = 0$)

Since the radius is always a positive quantity $\left(r = +\sqrt{(\pm X)^2 + (\pm Y)^2}\right)$ for the univariate distribution, the following change of variable is made:

$$r = |X|$$

Now, $\bar{r} = \int_0^\infty r f(r) dr$,

but $f(r)dr = 2f(x)dx$ for $r = |X|$ on the interval $(0 \leq r < \infty)$ where

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma_X} e^{-x^2/2\sigma_X^2}$$

This gives

$$\bar{r} = \frac{2}{\sqrt{2\pi}\,\sigma_X} \int_0^\infty X e^{-x^2/2\sigma_X^2} d_X$$

Let $t = x^2/2\sigma_X^2$, $dt = (x/\sigma_X^2)dx$ giving the desired relation

$$\bar{r} = \left(\frac{2}{\sqrt{2\pi}\,\sigma_X}\right) \sigma_X^2 \int_0^\infty e^{-t} dt = \sqrt{2/\pi}\,\sigma_X \left[-e^{-t}\int_0^\infty\right] = \sqrt{2/\pi}\,\sigma_X$$

Median Radius

The derivations of the median radii (i.e., radius containing 50 percent of the distribution within it) can be found in Reference 6.

---

[7] Dwight, H.B.; Tables of Integrals and Other Mathematical Data; 4th Edition, 1968, MacMillan Co., New York.

Next page is blank.

APPENDIX B

HELBAT-5 DIRECTION ERRORS

HEL ID.	PREDICTION TIME (SEC)	TARGET SPEED (M/SEC)	DISTANCE (M)	HEADING ERROR (DEG)
3030004	4.0	6.0	24.0	-12.8
2020003	4.0	7.0	28.0	-40.7
27140004	4.0	6.0	24.0	.3
9040013	5.0	4.0	20.0	-76.0
5100012	5.0	6.0	30.0	-17.0
9030010	5.0	4.0	20.0	50.5
5100014	5.0	6.0	30.0	36.0
27140005	5.0	5.0	25.0	10.0
27130001	6.0	4.0	24.0	43.1
3030005	6.0	6.0	36.0	1.9
3110010	6.0	8.0	48.0	1.0
27130004	6.0	6.0	36.0	-8.3
25190004	7.0	5.0	35.0	-2.6
4090012	7.0	5.0	35.0	-4.2
5100007	8.0	5.0	40.0	38.7
9090007	8.0	6.0	48.0	-38.8
2140003	8.0	5.0	40.0	-6.8
2140006	8.0	5.0	40.0	-2.0
9060010	8.0	3.0	24.0	-117.6
9090015	8.0	4.0	32.0	.6
3120002	9.0	6.0	54.0	8.5
5080006	9.0	5.0	45.0	6.9
10060003	10.0	6.0	60.0	.1
2030005	11.0	4.0	44.0	-.5
4090002	11.0	6.0	66.0	-8.3
4040003	11.0	4.0	44.0	-21.7
27140003	11.0	5.0	55.0	.8
4090007	12.0	4.0	48.0	9.8
3090003	13.0	8.0	104.0	.8
25190001	14.0	7.0	98.0	3.5
9090005	14.0	6.0	84.0	36.8
10050006	16.0	4.0	64.0	-4.3
4090001	16.0	5.0	80.0	3.5
4050003	16.0	5.0	80.0	-2.8
27130003	16.0	5.0	80.0	2.9
25190003	16.0	3.0	48.0	-7.1
9100010	17.0	3.0	51.0	2.0
9040011	17.0	4.0	68.0	-3.7
9040012	18.0	4.0	72.0	-1.2
5100010	18.0	6.0	108.0	.6
2020004	18.0	5.0	90.0	-18.1
4040004	18.0	5.0	90.0	-1.4
9060006	18.0	3.0	54.0	3.3
10110007	19.0	5.0	95.0	9.2
27150004	19.0	6.0	114.0	4.4
27150002	19.0	5.0	95.0	-5.8
5080004	20.0	5.0	100.0	-23.2
10120003	20.0	6.0	120.0	.4
27100003	20.0	7.0	140.0	-2.2
11030006	20.0	6.0	120.0	1.2

HEL ID.	PREDICTION TIME (SEC)	TARGET SPEED (M/SEC)	DISTANCE (M)	HEADING ERROR (DEG)
3120003	20.0	5.0	100.0	-10.6
9090013	20.0	4.0	80.0	-62.5
2140005	20.0	4.0	80.0	1.7
4090008	21.0	5.0	105.0	16.6
27150001	21.0	5.0	105.0	6.7
4040008	21.0	8.0	168.0	5.6
9060002	21.0	5.0	105.0	-16.3
9040006	21.0	5.0	105.0	-5.1
9100002	22.0	6.0	132.0	-3.2
3110007	22.0	4.0	88.0	-2.6
9100007	22.0	6.0	132.0	3.7
3030006	22.0	4.0	88.0	2.6
5080002	22.0	5.0	110.0	-4.0
4090005	22.0	5.0	110.0	-5.7
9030002	22.0	5.0	110.0	-6.5
11050007	22.0	6.0	132.0	-.1
2140008	23.0	5.0	115.0	.0
5100011	23.0	5.0	115.0	-42.7
3110003	23.0	6.0	138.0	-5.3
5080009	23.0	5.0	115.0	4.7
6070002	23.0	5.0	115.0	.0
5100006	23.0	5.0	115.0	-2.9
8080008	24.0	6.0	144.0	2.8
27150003	24.0	6.0	144.0	4.7
9090002	24.0	3.0	72.0	-3.0
8080002	24.0	4.0	96.0	-.8
9090012	24.0	5.0	120.0	7.0
2140002	24.0	5.0	120.0	2.2
3090005	24.0	8.0	192.0	2.3
9070002	24.0	6.0	144.0	7.3
2020001	24.0	6.0	144.0	5.5
2030007	24.0	5.0	120.0	-1.5
9030003	24.0	4.0	96.0	-5.9
9040005	25.0	4.0	100.0	41.3
9050007	25.0	5.0	125.0	2.6
3110002	25.0	6.0	150.0	1.0
5080008	25.0	4.0	100.0	7.9
27150005	25.0	6.0	150.0	-.9
11040006	25.0	5.0	125.0	-.4
3110005	25.0	5.0	125.0	-52.0
27050001	26.0	8.0	208.0	3.9
8080006	26.0	6.0	156.0	-1.2
9040008	26.0	4.0	104.0	-23.6
6080004	26.0	6.0	156.0	.8
4040007	26.0	6.0	156.0	7.6
11040004	26.0	5.0	130.0	-3.3
9100005	26.0	5.0	130.0	-26.0
25190002	26.0	6.0	156.0	-6.2
8080003	26.0	6.0	156.0	-.2
9090003	27.0	4.0	108.0	-9.6

HEL ID.	PREDICTION TIME (SEC)	TARGET SPEED (M/SEC)	DISTANCE (M)	HEADING ERROR (DEG)
9040014	27.0	5.0	135.0	2.4
8070001	27.0	5.0	135.0	2.8
4030007	28.0	6.0	168.0	2.3
5100004	28.0	5.0	140.0	-4.0
10120002	28.0	5.0	140.0	-6.7
10050005	28.0	4.0	112.0	-2.0
5100005	28.0	5.0	140.0	10.9
10050004	29.0	4.0	116.0	1.0
4050001	29.0	6.0	174.0	.1
3090004	29.0	8.0	232.0	2.1
9090001	29.0	3.0	87.0	-.4
10120006	29.0	6.0	174.0	4.9
9100008	30.0	5.0	150.0	-7.0
2030001	30.0	5.0	150.0	1.6
9030009	30.0	6.0	180.0	6.1
5080005	30.0	6.0	180.0	-5.0
5080007	30.0	4.0	120.0	3.1
5100002	31.0	7.0	217.0	-7.0
9070003	31.0	5.0	155.0	-1.1
2020002	31.0	7.0	217.0	.4
8070003	32.0	5.0	160.0	1.6
5080003	32.0	4.0	128.0	7.1
4030003	32.0	5.0	160.0	13.4
4040002	32.0	6.0	192.0	9.4
10110003	32.0	6.0	192.0	1.6
9090006	32.0	6.0	192.0	-2.6
3110009	32.0	3.0	96.0	6.9
9040007	32.0	5.0	160.0	-21.0
9100003	33.0	4.0	132.0	-8.3
4100009	33.0	4.0	132.0	3.3
11040001	33.0	7.0	231.0	4.6
4090004	33.0	4.0	132.0	-7.7
9040003	34.0	6.0	204.0	4.3
9100004	34.0	5.0	170.0	13.7
9040001	34.0	5.0	170.0	1.2
27150006	34.0	6.0	204.0	9.0
4050004	34.0	6.0	204.0	-1.6
10110006	34.0	5.0	170.0	14.3
4100002	35.0	6.0	210.0	.3
4090009	35.0	5.0	175.0	9.3
4100008	35.0	6.0	210.0	-3.3
3030002	35.0	6.0	210.0	-4.1
11050002	35.0	5.0	175.0	-3.5
27130002	35.0	5.0	175.0	-1.6
4090010	35.0	6.0	210.0	2.4
10120001	35.0	6.0	210.0	-4.4
9040004	36.0	4.0	144.0	-7.9
3030003	36.0	6.0	216.0	-.6
9060003	36.0	4.0	144.0	-.9
4090006	36.0	4.0	144.0	-4.6

HEL ID.	PREDICTION TIME (SEC)	TARGET SPEED (M/SEC)	DISTANCE (M)	HEADING ERROR (DEG)
9090004	36.0	5.0	180.0	5.5
4100003	37.0	4.0	148.0	-8.0
5100008	37.0	6.0	222.0	-10.2
10110005	38.0	5.0	190.0	-.2
9100006	38.0	6.0	228.0	1.6
4100005	38.0	4.0	152.0	-22.4
3030007	39.0	6.0	234.0	6.5
10110002	39.0	6.0	234.0	-.1
4100007	40.0	4.0	160.0	-.8
2030006	40.0	4.0	160.0	-49.3
4040006	40.0	6.0	240.0	.6
4030002	41.0	6.0	246.0	-22.6
11040003	41.0	5.0	205.0	1.6
10110004	42.0	6.0	252.0	47.2
9090014	42.0	5.0	210.0	1.3
9100001	43.0	6.0	258.0	5.8
10060002	43.0	5.0	215.0	-2.2
5100001	43.0	7.0	301.0	3.4
9060005	43.0	4.0	172.0	14.7
9070001	43.0	5.0	215.0	4.0
5100013	43.0	6.0	258.0	3.7
3120001	44.0	6.0	264.0	-5.0
2140004	44.0	5.0	220.0	-4.3
27100001	44.0	6.0	264.0	-2.2
3110006	44.0	4.0	176.0	19.4
10050001	44.0	4.0	176.0	-37.5
25190008	45.0	5.0	225.0	-18.6
10060001	45.0	5.0	225.0	4.1
10050002	45.0	5.0	225.0	56.2
9090009	45.0	4.0	180.0	-18.7
27140001	46.0	5.0	230.0	3.2
9030008	46.0	5.0	230.0	-55.6
4050002	46.0	6.0	276.0	-.3
25190005	46.0	5.0	230.0	10.9
11050001	46.0	5.0	230.0	-6.4
8060001	46.0	4.0	184.0	-1.9
2140007	47.0	5.0	235.0	-6.7
9030004	47.0	5.0	235.0	-9.6
27140002	48.0	5.0	240.0	-1.7
27050002	48.0	7.0	336.0	2.0
4030001	49.0	6.0	294.0	-2.6
4090011	49.0	4.0	196.0	.9
9090008	49.0	5.0	245.0	23.5
4050005	52.0	6.0	312.0	1.7
6070004	52.0	7.0	364.0	9.9
9040009	53.0	5.0	265.0	-1.1
8080005	54.0	6.0	324.0	1.0
27100002	54.0	8.0	432.0	-.6
3110004	56.0	7.0	392.0	1.7
11050004	56.0	6.0	336.0	-4.5

HEL ID.	PREDICTION TIME (SEC)	TARGET SPEED (M/SEC)	DISTANCE (M)	HEADING ERROR (DEG)
4040005	56.0	6.0	336.0	-1.7
4090003	57.0	4.0	228.0	3.5
2030002	58.0	5.0	290.0	-1.7
10060004	59.0	5.0	295.0	-4.8
4030006	59.0	5.0	295.0	-58.6
4040001	61.0	6.0	366.0	-16.9
2140001	65.0	5.0	325.0	2.4
5080001	65.0	5.0	325.0	8.8
3110006	66.0	3.0	198.0	-56.1
11040002	66.0	6.0	396.0	.8
4100001	68.0	6.0	408.0	-4.4
9060001	71.0	5.0	355.0	6.9
9060007	74.0	4.0	296.0	11.5
9030005	74.0	6.0	444.0	-1.0
9030006	75.0	4.0	300.0	-35.4
4100004	76.0	4.0	304.0	7.6
3090001	77.0	3.0	231.0	-33.9
25190007	77.0	4.0	308.0	16.5
3030001	79.0	5.0	395.0	-5.3
2030003	79.0	4.0	316.0	-4.9
9030001	91.0	4.0	364.0	-4.6
10030003	92.0	5.0	460.0	-17.0
→ 4030004	93.0	15.0	1395.0	-81.9
10110001	121.0	5.0	605.0	64.8

REFERENCES

1. Horley, G., W. Dousa, Jr., J. Lenoci; HELBAT 2- Forward Observers (U); TM-17-72, June 1972, US Army Human Engineering Laboratory, Aberdeen Proving Ground, MD, CONFIDENTIAL report.

2. Horley, G., W. Dousa, Jr.; HELBAT 4- Automated Fire Direction on Moving Targets (U); TM-19-76, May 1976, US Army Human Engineering Laboratory, Aberdeen Proving Ground, MD, CONFIDENTIAL report.

3. Horley, G., et. al.; HELBAT 5- Automated Fire Direction Techniques (U); TM-1-77, March 1978, US Army Human Engineering Laboratory, Aberdeen Proving Ground, MD, CONFIDENTIAL report.

4. Chernick, Julian, M. Chernick; Forecasting Ground Target Speeds; Technical Report No. 111, August 1974, US Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, MD 21005, UNCLASSIFIED report.

5. Goodman, L.A.; On the Exact Variance of Products; pages 708-713, December 1960, American Statistical Association Journal, (also see same journal, "Corrigenda", page 917, December 1962).

6. Groves, A.D.; Handbook on the Use of the Bivariate Normal Distribution in Describing Weapon Accuracy; MR-1372, September 1961, US Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, UNCLASSIFIED report.

7. Dwight, H.B.; Tables of Integrals and Other Mathematical Data; 4th Edition, 1968, Macmillan Co., New York.

DISTRIBUTION LIST

No. of Copies	Organization
12	Commander The Defense Technical Information Center ATTN: TCA Cameron Station Alexandria, VA 22314
1	Commander US Army Materiel Development & Readiness Command ATTN: DRCPA-S 5001 Eisenhower Avenue Alexandria, VA 22333
1	Commander US Army Materiel Development & Readiness Command ATTN: DRCBSI-L 5001 Eisenhower Avenue Alexandria, VA 22333
1	Commander US Army Electronics R&D Command ATTN: DRDEL-AP-OA 2800 Powder Mill Road Adelphi, MD 20783
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SL White Sands Missile Range, NM 88002
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-T White Sands Missile Range, NM 88002
2	Chief Defense Logistics Studies Information Exchange US Army Logistics Management Center ATTN: DRXMC-D FT Lee, VA 23801
1	Commander US Army Concepts Analysis Agency 8120 Woodmont Avenue Bethesda, MD 200 4
1	Reliability Analysis Center ATTN: Mr. I.L. Krulac Griffiss AFB, NY 13441
	Aberdeen Proving Ground
1	Director, BRL, Bldg. 328
1	Director, BRL ATTN: DRDAR-TSB-S Bldg. 305
1	Director, HEL, Bldg. 520