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Task NR 062-297

LEVEL III

FOIL BEARING INVESTIGATION
1. SUMMARY OF RESULTS
2. TECHNICAL REPORT: ANALYSIS OF HYDROSTATIC FOIL BEARINGS INCLUDING THE EFFECTS OF FLUID INERTIA

Ampex Corporation
Advanced Technology Division
401 Broadway
Redwood City, CA 94063

30 November 1980

Final Report, 1 October 1976 – 30 September 1980

Approved for public release; distribution unlimited.

Prepared for

Mathematical and Information Science Division
Office of Naval Research,
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Lubrication, bearings, foil bearings, externally pressurized, hydrostatic, inertia effects.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) of Technical Report:

Analysis is made of planar hydrostatic foil bearings with the inclusion of fluid inertia effects. It is shown that the double valuedness of the pressure-flow characteristics observed in earlier works is attributable to the neglect of fluid inertia effects. These effects alter the bearing behavior qualitatively at high flow rate.

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FOIL BEARING INVESTIGATION

Summary of Results

1 Oct. 1976 - 30 Sept. 1980

PAPERS/REPORTS COMPLETED

1. A. Eshel, "Numerical Solution of Planar Hydrostatic Foil Bearing", ONR Report RR 77-07 and ASME Trans. J. Lub Tech 101, 1979, 86-91.
2. A. Eshel, "Analysis of Hydrostatic Foil Bearings Including the Effects of Fluid Inertia" Included in this report.
3. A. Eshel and T. G. Kennedy "Numerical Simulation of Longitudinal Vibrations in Foils Including the Effects of Dry Friction", Proc. 1979 Summer Simulation Conference, Toronto, Canada, July 1979.
4. A. Eshel and T. G. Kennedy, "A Foil Bearing Model for High Speed Rotating Magnetic Heads", Ampex ONR report RR 78-10 8/29/78.

ACCOMPLISHMENTS

Hydrostatic Foil Bearings

Hydrostatic planar foil bearings have been analyzed using lubrication theory. The results have been summarized in [1].

The effects of fluid inertia in this type of bearing have been analyzed and shown to be of essential importance. The results are given in a technical report included herewith in part 2 of this report. (See Item [2] above).

Stick Slip Vibrations of Foils

A model has been developed which allows simulation of longitudinal vibrations in foils under the influence of Coulomb friction with variable and discontinuous friction coefficient. Self-excited vibrations and stick-slip phenomena have been described. The results are summarized in item [3] above. Coupling with transverse vibrations has been studied but not completed.

Foil Response to Bumps

Analysis has been made of the effect of a rotating bump (representing a magnetic head) on a foil floated on a cylindrical drum. The effect of several parameters have been studied and in particular the ratio of bump velocity to the speed of propagation of transverse disturbances in the foil. The results are given in item [4] above. Additional analyses and experiments including the effects of cutouts have been performed but have not reached publication.

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FOIL BEARING INVESTIGATION

Technical Report: Analysis of Hydrostatic Foil Bearings Including the Effects of Fluid Inertia

INTRODUCTION

This report explains a pecurliarity found in earlier analyses of externally pressurized foil bearings. Ref. [1] summarizes some of the preceding studies [2-5].

In the above mentioned paper, the problem of a planar hydrostatic foil bearing was analyzed on the basis of lubrication theory. Among the results, the paper presented charts of foil bearing inlet pressure vs. flow rate. An interesting feature of these charts (Figure 5)* is that for the same inlet pressure a dual value of flow rate is predicted. This result is not physically impossible (since the restrictor characteristics would generally select the proper unique physical solution), but nevertheless, it is unexpected. The motivation for the present work was the hypothesis that the source of the phenomenon is fluid inertia effects which were not included in the analysis. Indeed, as the results of the present study show, inertia effects alter drastically the behavior at higher flow rates and resolve the puzzle associated with the earlier results.

The model under consideration is shown in Figure 1. It describes a planar flow through a restrictor feeding into a cylindrical region covered by a tensioned foil. The pressure vs flow characteristics of the configuration comprise the objective of the analysis.

Basic Equations

The starting point of the analysis is the compressible Navier-Stokes equations in polar coorindates:

*See Fig. 2 of the present report, for A = 0.

$$\rho\left[v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r}\right]$$
$$= -\frac{\partial p}{\partial r} + \mu\left[2\frac{\partial^2 v_r}{\partial r^2} - \frac{2}{3}\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}\right)\right\}\right.$$
$$\left. + \frac{1}{r}\frac{\partial}{\partial \theta}\left\{\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right\} + \frac{2}{r}\left(\frac{\partial v_r}{\partial r} - \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r}\right)\right] \tag{1}$$

$$\rho\left[v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r}\right]$$
$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{2}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}\right) - \frac{2}{3r}\frac{\partial}{\partial \theta}\left\{\frac{1}{r}\frac{\partial r v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}\right\}\right. \tag{2}$$
$$\left. + \frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right\} + \frac{2}{r}\left\{\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right\}\right]$$

The continuity equation is

$$\frac{\partial(\rho v_r r)}{\partial r} + \frac{\partial(\rho v_\theta)}{\partial \theta} = 0 \tag{3}$$

The perfectly flexible foil equation is

$$p - p_a = \frac{T}{R} \tag{4}$$

The isothermal equation of state is

$$\frac{p}{p_a} = \frac{\rho}{\rho_a} \tag{5}$$

The curvature in polar coordinates is

$$\frac{1}{R} = \frac{2\left(\frac{dr_1}{d\theta}\right)^2 - r\frac{d^2 r}{d\theta^2} + r^2}{\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]^{3/2}} \tag{6}$$

and the clearance

$$h = r - r_0 \tag{7}$$

The pressure boundary conditions are that the pressure on the foil becomes negligible sufficiently far from the spindle. No velocity boundary conditions are prescribed as explained in [6].

Simplification

We introduce a dimensionless small parameter ϵ to be defined later whose purpose is to indicate how open the bearing is to flow. Qualitatively, it is expected to be directly related to M, the mass flow rate and inversely related to the tension. In addition we define

$$C = \frac{p_a}{T/r_o} \text{ compressibility parameter} \tag{8}$$

and the dimensionless variables

$$\hat{v} = \frac{v_r}{M/(\rho_a r_o)} \epsilon^k \tag{9}$$

$$\hat{u} = \frac{v_\theta}{M/(\rho_a r_o)} \epsilon^n \tag{10}$$

$$\pi = \frac{p - p_a}{T/r_o} \tag{11}$$

$$\xi = \theta \epsilon^{-m} \tag{12}$$

$$\eta = \frac{y}{r_o} \epsilon^{-n} \tag{13}$$

$$H = \frac{h}{r_o} \epsilon^{-n} \tag{14}$$

where k, m, n are constants to be determined.

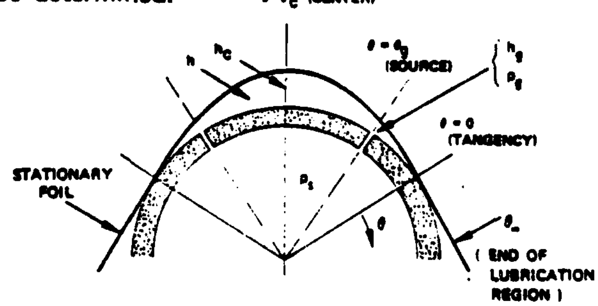

Figure 1 Cylindrical Hydrostatic Foil Bearing

Substituting the notation (8) - (14) in Eqns. (1) - (7) two dimensionless parameters arise

$$\epsilon = \frac{12\mu M}{P_a r_o T} \qquad \text{Foil Bearing Parameter} \qquad (15)$$

$$I = \frac{M^2}{P_a r_o T}\,\epsilon^{-2n} \qquad \text{Inertia Parameter} \qquad (16)$$

In accordance with conventional foil bearing theory [7], the following requirements are imposed on the equations for the purpose of the determination of k, m, n as the limit $\epsilon \to 0$ is pursued.

1. The expansion of equation (6) has to retain the capability of satisfying the condition that the tape becomes flat at infinity.

2. The purpose of this work is to study cases in which inertia effects are significant. It will be required, therefore, that at least some inertia term will be of the order of some viscous term.

3. Finally, it is required that the lateral and longitudinal terms in the continuity equation will be of the same order.

These three requirements results in the conditions

$$n = \frac{2}{5}\ ; \quad m = \frac{1}{5}\ ; \quad k = \frac{1}{5} \qquad (17)$$

With these results, the equations can be shown to degenerate, to the zeroth approximation, into Prandtl's boundary-layer equations.

$$I\left(1 + \frac{\pi}{C}\right)\left[\hat{u}\frac{\partial \hat{u}}{\partial \xi} + \hat{v}\frac{\partial \hat{u}}{\partial \eta}\right] = -\frac{1}{2}\frac{d\pi}{d\xi} + \frac{1}{12}\frac{\partial^2 \hat{u}}{\partial \eta^2} \qquad (18)$$

$$\frac{\partial}{\partial \eta}\{(C + \pi)\hat{v}\} + \frac{\partial}{\partial \xi}\{(C + \pi)\hat{u}\} = 0 \qquad (19)$$

$$\pi = 1 - H'' \qquad (20)$$

We assume that the velocity profile is parabolic. The profile which satisifes the boundary conditions

$$\hat{u} = 0 \qquad \eta = 0 \tag{21}$$

$$\hat{u} = 0 \qquad \eta = H \tag{22}$$

$$\hat{u} = f\left[\left(\frac{\eta}{H}\right) - \left(\frac{\eta}{H}\right)^2\right] \tag{23}$$

where f, a measure of local velocity, is to be determined.

The three unknown functions π, H, f will be determined from the momentum and continuity integrals between $\eta = 0$ and $\eta = H$ and from the elastic equation.

The momentum integral is

$$\frac{I}{30c}\frac{d}{d\xi}\left\{(\pi + c)f^2 H\right\} = -\frac{d\pi}{d\xi}H - \frac{f}{6H} \tag{24}$$

Noting that the integral of Eqn. (3) with respect to dr yields the mass flow rate M, the continuity integral becomes

$$\frac{(C+\pi)fH}{6} = C \tag{25}$$

Eliminating f and π from Eqns. (20), (24), (25) the formulation is

$$H''' = \frac{1 - 1.2\,I H'}{H^3\left(1 + \frac{1 - H''}{C}\right) - \frac{1.2(I/C)H}{1 + \frac{1 - H''}{C}}} \tag{26}$$

together with the boundary conditions:

$H' \sim \xi$ outside the lubrication zone; (27)

$H'' \sim 1$ origin of ξ is at point of tengency (28)

and

$$\left.\frac{dH}{d\xi}\right|_{\xi=\xi_g} = (1-\Pi_g)(\xi_g - \xi_c) \qquad (29)$$

(at the supply line boundary; $\xi = \xi_g$)

$$\left.\frac{d^2H}{d\xi^2}\right|_{\xi=\xi_g} = 1 - \Pi_g \qquad (30)$$

The solution is expected to depend on I, C, θ_g/θ_c, ξ_c . i.e., the problem depends on four parameters. In the incompressible ($C \rightarrow \infty$) inertialess (I = 0) limit, the formulation degenerates to that of [1].

It is useful to present the results as dimensionless pressure vs flow rate. Therefore, an alternative parameter will be defined which eliminates flow rate

$$A = I^{1/6}\xi_c = \theta_c\left(\frac{\sqrt{\rho_a}\, r_o T}{12\mu}\right)^{1/3} \qquad (31)$$

Hence, the solution will be of the form

$$H = F\left(\xi; A, \frac{I}{C}, \frac{\theta_g}{\theta_c}, \xi_c\right) \qquad (32)$$

$$\Pi_g = F\left(A, \frac{I}{C}, \frac{\theta_g}{\theta_c}, \xi_c\right) \qquad (33)$$

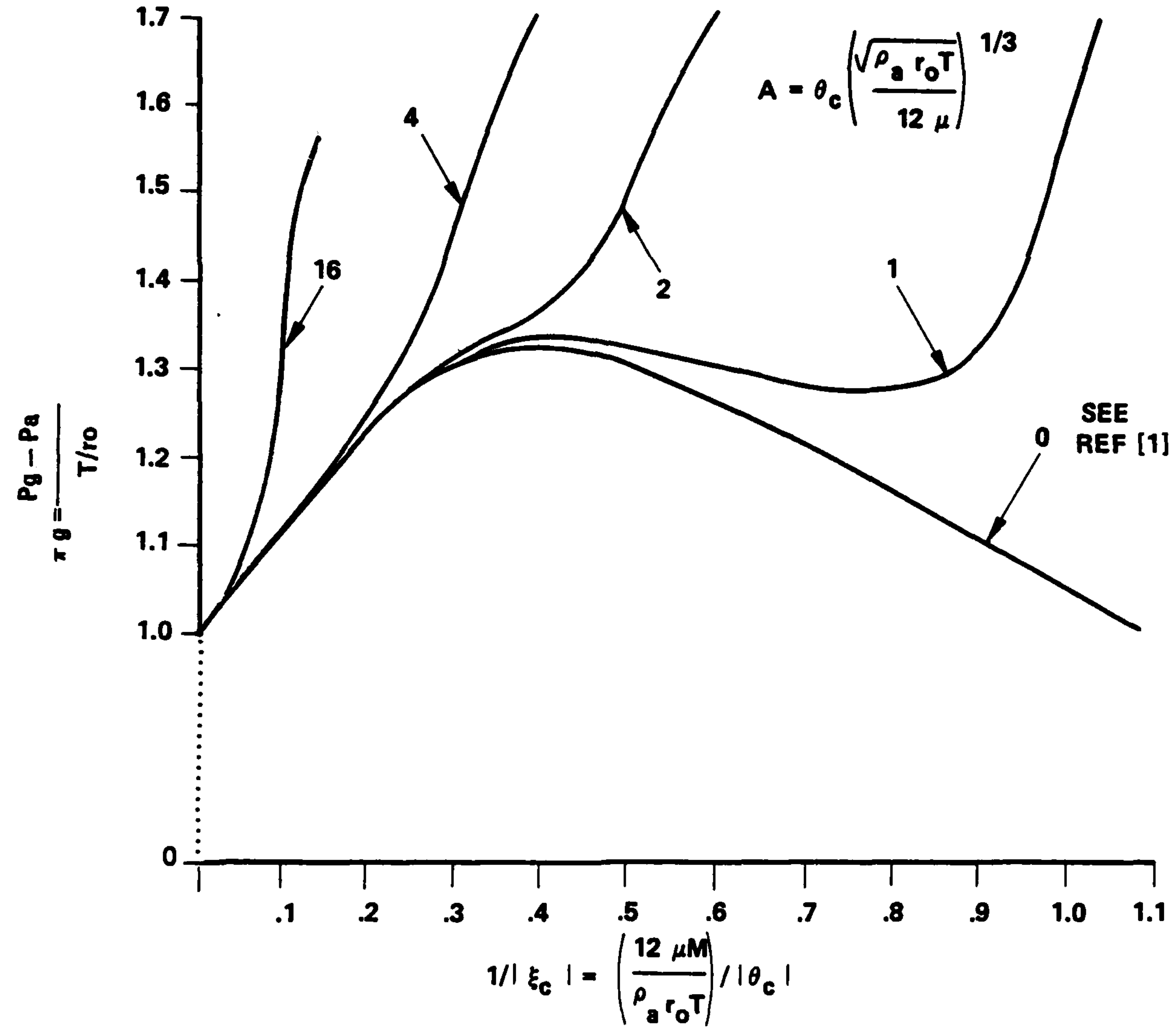

Figure 2 Bearing Characteristics for the Case of $\frac{\theta_g}{\theta_c} = 1, \frac{1}{C} = 0$ for Various Values of $A = I^{1/6} |\xi_c|$

RESULTS

Due to extraneous constraints the results described here are limited to some particular cases. A desirable extensive parametric investigation has not been made. Nevertheless, the main point of the study, i.e., the essential importance of inertia effects is well demonstrated in Figure 2. It is seen that for low flow rate the solution tends to be identical with that of Ref [1]. For higher flow rates the behavior is completely different, showing that increased groove pressure is required, as one may expect, in order to overcome additional inertia effect.

APPENDIX

The solution procedure is described below. Let equations (26)-(28) be reformulated in terms of alternative variables $\hat{H}(\hat{\xi})$, in order to permit solution as an initial value problem: (Later the solution for $\hat{H}$ will be transformed back into $H(\xi)$. After studying the procedure the reader may verify that the following formulation may indeed be recast into the original coordinates using transformation formula given below.)

$$\hat{H}''' = \frac{1 - \alpha_3 \hat{H}'}{\hat{H}^3\left(\alpha_R - \alpha_2 \hat{H}''\right) - \dfrac{\alpha_2 \alpha_3 \hat{H}}{\alpha_R - \alpha_2 \hat{H}''}} \tag{33}$$

Subject to the initial conditions, that at $\hat{\xi} = 0$

$$\hat{H} = \hat{H}_g \tag{34}$$

$$\hat{H}' = \hat{H}_g' \tag{35}$$

$$\hat{H}'' = \hat{H}_g'' \tag{36}$$

where the parameters $\alpha_R, \alpha_2, \alpha_3, \hat{H}_g, \hat{H}_g', \hat{H}_g''$ are inputs. Integrating numerically from $\hat{\xi} = 0$, then depending on the selection of inputs, the initially positve $\hat{H}$ may become positively unbounded for large $\hat{\xi}$ or it may decrease to zero causing the denominator of (33) to vanish. Only the first event is of physical interest. In this case, $\hat{H}''$ tends to reach an asymptotic value denoted by $\hat{H}''_\infty$. Once the numerical value of this quantity has reached sufficiently close to its asymptotic value, the solution may be terminated (at a somewhat arbitrary point) with the known values of $\hat{H}_\infty$, $\hat{H}'_\infty$, $\hat{\xi}_\infty$ retained. Now the physical parameters of the problem

$$H = \hat{H}\, \hat{H}''^{\,3/5}_\infty\, \alpha^{2/5} \tag{37}$$

$$\xi = \left(\hat{\xi} - \hat{\xi}_\infty + \hat{H}'_\infty / \hat{H}''_\infty\right) \hat{H}''^{\,4/5}_\infty\, \alpha^{1/5} \tag{38}$$

$$C = \frac{\alpha_R}{\hat{H}''_\infty \alpha_2} - 1 \tag{39}$$

$$I = \frac{\alpha_3\, \hat{H}''^{\,1/5}_\infty}{1.2\, \alpha^{1/5}} \tag{40}$$

$$\pi_g = 1 - \hat{H}''_g / \hat{H}''_\infty \tag{41}$$

$$\xi_g = -\left(\hat{\xi}_\infty - \hat{H}'_\infty / \hat{H}''_\infty\right) \hat{H}''^{\,4/5}_\infty\, \alpha^{1/5} \tag{42}$$

$$\xi_c = \xi_g - \left(\hat{H}'_g / \hat{H}''_g\right) \hat{H}''^{\,4/5}_\infty\, \alpha^{1/5} \tag{43}$$

$$H_c = H_g - \frac{1}{2}\left(1 - \pi_g\right)\left(\xi_g - \xi_c\right)^2 \tag{44}$$

where

$$\alpha = \alpha_R - \hat{H}''_\infty\, \alpha_2 \tag{45}$$

While the formulation (26) - (30) depends on four parameters shown in Eqn. (31), the equivalent formulation (33) - (36) depends on six inputs. Thus, two of the six parameters are expected to be arbitrary, and to lead to redundant solutions.

The iterative approach selected is based on consideration of the following limiting cases. By comparing Eqns. (33) and (26) one may see that with $\alpha_R = 1$, the limit $\alpha_2 = 0$ leads to the incompressible case, and the limit $\alpha_3 = 0$ leads to the inertia-less case. Finally, the case of a groove located at the center of the lubrication zone ($\xi_g = \xi_c$) requires (Eqn. 29) $\hat{H}_g' = 0$. Thus, one may expect that to a first approximation $\hat{H}_g'$, α_2, α_3 control θ_g/θ_c, $1/C$, A, respectively. To determine the fourth parameter e.g., ξ_c (or flow rate) we choose $\hat{H}_g''$. Thus we arbitrarily fix α_R=1 and $\hat{H}_g = 1$, then for each selected value of $\hat{H}_g''$, we iterate on $\hat{H}_g'$, α_2, α_3 to generate a solution with desired values of $\frac{\theta_g}{\theta_c}$, $\frac{1}{C}$, A and some point on the $\hat{\pi}_g - \xi_c$ curve.

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