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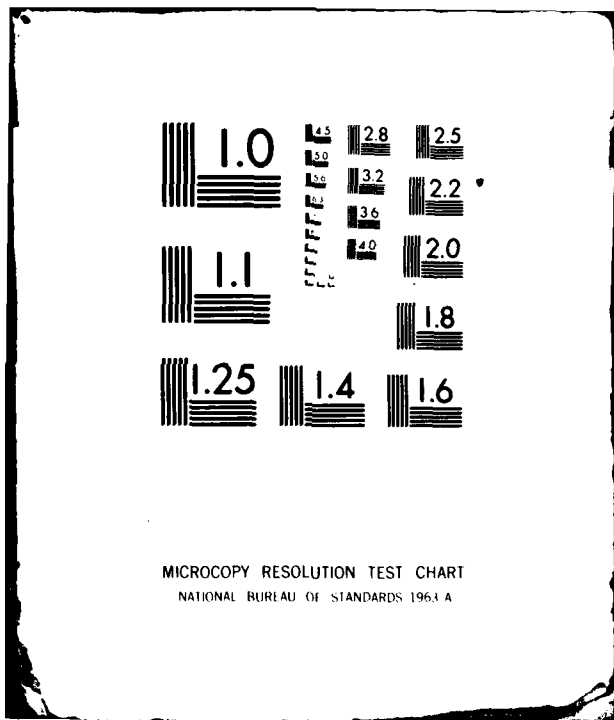
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A PRIMAL SIMPLEX CODE FOR COMPUTING THE EFFICIENCY OF DECISION --ETC(U)
OCT 80 A BESSENT, J KENNINGTON N00014-80-C-0242
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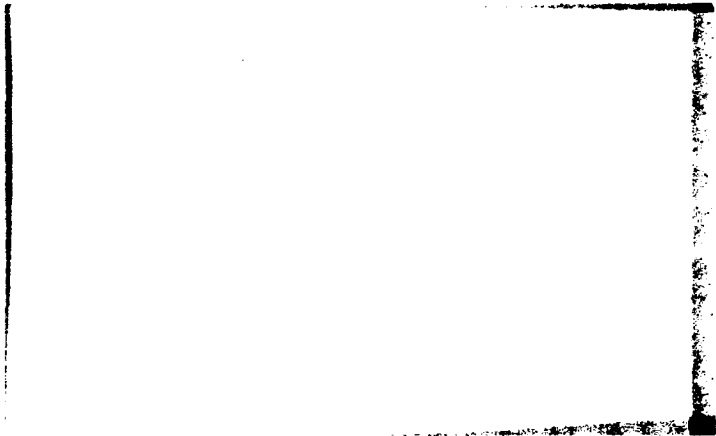


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9 Research Report, ECS 383

6 A PRIMAL SIMPLEX CODE FOR COMPUTING THE EFFICIENCY OF DECISION MAKING UNITS (VERSION 2.0)

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11 Revised
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I. BASIC DESCRIPTION

The computer code (Version 2.0 of SMU1.0) described in this technical report was developed as part of research projects under the direction of Dr. A. Charnes (The University of Texas at Austin), Office of Naval Research Contract N00014-80-C-0242, and Dr. W.W. Cooper (The University of Texas at Austin), Office of Naval Research Contract N00014-76-C-0932. Dr. Authella Bessent (The University of Texas at Austin), Dr. Wailand Bessent (The University of Texas at Austin), and Dr. J. Kennington (Southern Methodist University) joined the group during 1979 and designed and tested SMU1.0 during the last six months of that year.

Given a set of decision making units (DMU's) with common inputs and outputs, SMU1.0 will compute the efficiency of each of the DMU's. The underlying theory is described in Charnes, Cooper, and Rhodes [1].

SMU1.0 is written in FORTRAN and was tested on SMU's CDC Cyber 73 using the MNF compiler. The code employs a modified primal simplex code which was originally developed by Iqbal Ali (The University of Texas at Austin) as a part of his dissertation work. The input format and output reports were designed by Authella and Wailand Bessent.

II. MODEL AND SOFTWARE DESIGN

To determine the efficiency of any DMU requires the solution of a particular linear program. In this section, we present the mathematical model, we give the initial feasible basis used to initiate the primal simplex algorithm, and we briefly discuss the design of the system.

A. Notation

- m = number of DMU's
 n_1 = number of outputs
 n_2 = number of inputs
 A = an $n_1 \times m$ matrix of outputs
 C = an $n_2 \times m$ matrix of inputs
 $A(j)$ = the j^{th} column of A
 $C(j)$ = the j^{th} column of C
 a_{ij} = the $(i, j)^{\text{th}}$ element of A
 c_{ij} = the $(i, j)^{\text{th}}$ element of C
 \underline{e}_i = a column vector having i^{th} component equal +1 and all other components zero.

B. Assumptions

- $a_{ij} \neq 0$ for all i, j .
 $c_{ij} \neq 0$ for all i, j .

C. Model

Consider the following linear program:

$$P(j) \left\{ \begin{array}{l} \max \quad z \\ \text{s.t.} \quad -A(j)z + A\lambda + s_1 = 0 \\ \quad \quad \quad C\lambda + s_2 = C(j) \\ \quad \quad \quad z, \lambda, s_1, s_2 \geq 0 \end{array} \right.$$

where λ is an m vector, s_1 is an n_1 vector of slacks, and s_2 is an n_2 vector of slacks. Let $(z^*, \lambda^*, s_1^*, s_2^*)$ denote an optimum for $P(j)$.

If $\begin{cases} z^* = 1, & \text{then DMU } j \text{ is said to be efficient;} \\ z^* > 1, & \text{then DMU } j \text{ is said to be inefficient.} \end{cases}$

D. Initial Feasible Basis

The following matrix is a feasible basis for $P(j)$ with $z = \lambda_j = 1$ and all other variables equal zero:

$$\left[\begin{array}{c|c|c|c|c|c|c|c|c|c} -A(j) & A(j) & e_2 & e_3 & \dots & e_{n_1} & 0 & 0 & \dots & 0 \\ \hline 0 & c(j) & 0 & 0 & \dots & 0 & e_2 & e_3 & \dots & e_{n_2} \end{array} \right] \quad (1)$$

E. Software Design

SMUL.0 solves $P(1)$ through $P(m)$ via the revised primal simplex method using the starting bases (1) for each problem, $P(j)$. The basis inverse is maintained in product form and only the non-zeroes are stored. The reinversion routine is based on the work of Hellerman and Rarick [4]. This routine incorporates a technique known as "splitting the bump" (see Kalan [5] and Orchard-Hays [6]) and uses the "spike swapping theory" of Helgason and Kennington [2]. A good description of the computational procedures may be found in Helgason [3]. The arrays and working files require approximately $(3m + 13)(n_1 + n_2) + 4m$ words of core storage.

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III. INPUT FORMAT

<u>Card Group</u>	<u>Source File</u>	<u>Composition</u>	<u>Description and Format</u>
1	Tape 1	1 card	"Problem Title" (13A6, A2)
2	Tape 1	1 card	"Number of DMU's", "Number of Outputs", "Number of Inputs" (3I5)
3	Tape 1	card group	"Output 1 Label", "Output 2 Label", ..., "Output Last Label", "Input 1 Label", "Input 2 Label", ..., "Input Last Label" (8A10)
4	Tape 1	1 card	"Blank Card"
5	Tape 1	card group for each DMU	Outputs for the DMU followed by Inputs for the DMU (10F8.0)

Card groups 6 and 7 are optional. If included, each DMU's name will appear on its output report.

6	Tape 1	1 card	"Blank Card" (Use only if card Group 7 follows)
7	Tape 1	card group for each DMU (in same order as DMU data)	"DMU1 name" (up to 80 characters) "DMU2 name" ' ' ' "DMUm name"

Card group 8 is optional. A blank card defaults to an analysis of all DMU's.

8	Input	1 card	"First DMU to be evaluated in this run", "Last DMU to be evaluated in this run" (free format, i.e., the two numbers may be in any columns as long as they are in the correct sequence and separated by either a blank or a comma)
---	-------	--------	---

IV. SET CODE DIMENSIONS

To set the dimension for a problem one should use a text editor which has the capability of replacing all occurrences of a specified character string by a number.

A. The Main Working Storage

Replace "ZMAT" with "----". (Should be at least $(m + n_1 + n_2 + 3)$
 $(n_1 + n_2 + 2) + (n_1 + n_2 + 2)^2$)

B. For The Number of Inefficient DMU's

Replace "ZINEFF" with "-----". (m is sufficiently large)

C. For Column Length Arrays

Replace "ZCOL" with "----". (Should be at least $m + n_1 + n_2 + 3$)

D. For Row Length Arrays

Replace "ZROW" with "----". (Should be at least $n_1 + n_2 + 2$)

E. For The Length Of The ETA File

Replace "ZETA" with "----". (Should be at least 2m)

WARNING: For each of the above replacements, replace the words with a number having the same number of digits. Use leading zeroes if necessary. Suppose $m = 100$. Then replace ZETA with 0200 and replace ZINEFF with 000100.

SAMPLE SCHOOL DISTRICT ANALYSIS

Jul 2 Jan
Aug 2 35
Mar 2 4200
Apr 2 100
May 2 00
27 5 1

RUN FOR UNO 1 1MMU UNO 10

RUN FOR UNO 1 1MMU UNO 10

NCE HEAD NCE ENG NCE MATH PHUP/100 S/PUPIL AMURMIN *PAY LUNLM *EXP.TEACH STU ATTEND TEACH.ATT

SAMPLE SCHUUL DISTRICT ANALYSIS

DECISION MAKING UNIT 1 SCHUUL 1 NAME

EFFICIENCY = 1.000

INEFFICIENCY = 1.000

RIGHT-HAND SIDE RANGES

 MINIMUM VALUE

 MAXIMUM VALUE

VALUE IF EFFICIENT

 SLACK

 DUAL

 * OUTPUTS *

NCE READ	56.1000	56.1000	0.0000	0.0100	-4.4046	0.0000
NCE ENG	57.4000	57.4000	0.0000	0.0000		0.0000
NCE MATH	54.7000	54.7000	0.0000	0.0000	0.0000	0.0000

 * INPUTS *

PROF/100	6.3000	6.5000	0.0000	0.0000		
S/PUPIL	970.4100	970.4100	0.0000	0.0000		
NON-MIN	87.0000	87.0000	0.0000	0.0000		
PAY LUNCH	95.0000	95.0000	0.0000	0.0000		
EXP. TEACH	39.0000	39.0000	0.0000	0.2500	0.0000	39.0000
STU ATTEND	48900.0000	98900.0000	0.0000	0.0000		
TEACH.ATI	90.2000	90.2000	0.0000	0.0000		

SAMPLE SCHUUL DISTRICT ANALYSIS

DECISION MAKING UNIT 2 SCHUUL 2 NAME

EFFICIENCY = .870

INEFFICIENCY = 1.149

	VALUE MEASURED	VALUE IF EFFICIENT	SLACK	DUAL	RIGHT-HAND SIDE RANGES
					MINIMUM VALUE
					MAXIMUM VALUE

 * OUTPUTS *

NCE READ	50.5000	60.3443	10.3560	0.0000		
NCE ENG	55.3000	67.4543	3.6990	0.0000		
NCE MATH	54.2000	62.2960	0.0000	.0185	-02.2400	3.0215

 * INPUTS *

PROF/100	5.3000	5.2210	.0790	0.0000		
S/PUPIL	1614.0000	959.6402	654.3598	0.0000		
NON-MIN	7.5000	7.5000	0.0000	.0065	.1010	22.5090
UPAY LUNCH	91.0000	70.5582	20.4418	0.0000		
EXP. TEACH	64.0000	40.7443	15.0057	0.0000		
STU ATTEND	91.4000	90.5142	1.0000	0.0000		
TEACH.A1T	95.2000	95.2000	0.0000	.0120	13.6605	96.3450

SAMPLE SCHUUL DJSIMICI ANALYSIS

DECISION MAKING UNIT 10 SCHUUL ID NAME

EFFICIENCY = .726
 INEFFICIENCY = 1.578

RIGHT-HAND SIDE RANGES

 MINIMUM VALUE

 MAXIMUM VALUE

VALUE MEASURED

 VALUE IF EFFICIENT

 SLACK

 DUAL

 * OUTPUTS *

	VALUE MEASURED	VALUE IF EFFICIENT	SLACK	DUAL	MINIMUM VALUE	MAXIMUM VALUE
NCE READ	40.4000	69.0524	2.5695	0.0000		
NCE ENG	49.5000	68.1464	0.0000	.0062	-1.5412	.6712
NCE MATH	46.3000	63.7890	0.0000	.0150	-.8149	1.4415

 * INPUTS *

PROF/100	7.5000	5.3630	2.1370	0.0000		
S/PUPIL	2062.0000	1071.1072	984.8928	0.0000		
NON-MIN	40.4000	17.0660	24.5134	0.0000		
PAY LUNCH	95.6000	15.9161	19.6813	0.0000		
EXP. TEACH	67.0000	55.0724	14.5276	0.0000		
STU ATTEND	93.5000	92.8124	.6876	0.0000		
TEACH.AIT	97.1000	97.1000	0.0000	.0142	0.0000	97.0194

EFFICIENCY/INEFFICIENCY SUMMARY

DMU	H0	L0
1	1.00000	1.00000
2	.87004	1.1437
3	1.00000	1.00000
4	.54909	1.02110
5	.70090	1.42674
6	.50105	1.71072
7	.53675	1.06515
8	.96925	1.03175
9	.75425	1.52502
	1.502	1.5175

SUMMARY OF SLACK VALUES--INEFFICIENT UNITS ONLY

OUTPUT/INPUT	2	4	5	6	7	8
NCE READ	10.35540055	4.06100005	0.41501497	7.55155517	7.52000072	3.95521235
NCE ENG	3.09902092	0.00000000	0.00000000	0.00000000	0.00000000	5.02000000
NCE MATH	0.00000000	0.0552540	1.15000214	0.32010003	3.00195551	0.00000000
PRIP/100	0.7401001	2.01130094	0.4554545	0.95541037	0.50000000	0.00000000
S/PUPIL	050.35970074	1230.55057023	702.11000002	057.02053001	700.50000000	073.00025117
AMOR-MIN	0.00000000	2.00030010	00.74750011	0.20795010	0.00000000	05.70000130
DAY LUNCH	20.00100372	15.72505034	27.00003102	20.0000796	22.02000000	12.50022052
DEP. TEACH	15.00500000	00.05070100	19.00070000	9.37102057	0.50000000	19.11700005
STU ATTEND	1.00077070	0.00000000	0.5037433	0.00000000	0.30000000	2.35070102
TEACH.ATT	0.00000000	1.71002721	0.00000000	0.20100073	1.20000000	0.00000000

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SUMMARY OF SLACK VALUES--INEFFICIENT UNITS ONLY

OUTPUT/INPUT	9	10
NCE HEAD	9.51000000	2.50449010
NCE ENG	0.00000000	0.00000000
NCE MATH	1.04342039	0.00000000
PROF/100	.66909091	2.13097144
S/PUPIL	772.24792513	904.04205040
NON-MIN	23.09743310	24.51342550
PAY LUNCH	24.14561497	19.00120503
EXP. TEACH	10.23200056	14.52704251
STU ATTEND	4.55004270	.00704075
TEACH.ATT	0.00000000	0.00000000

SUMMARY OF UAL VALUES--INEFFICIENT UTILS ONLY

OUTPUT/INPUT	2	4	5	6	7	8
ICE READ	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
ICE LMS	.00000000	.02/39/25	.0200/005	.0252/253	.0274/276	.00000000
ICE MAIN	.01045010	.00000000	.00000000	.00000000	.00000000	.01039304
PROF/140	.00000000	.00000000	.00000000	.00000000	.00000000	.10000000
S/PUPIL	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
BOOK-MIN	.00007009	.00000000	.00000000	.00000000	10.63120002	.00000000
APAY LUNCH	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
GRIP-TEACH	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
STU ATTEND	.00000000	.020/10/4	.00000000	.01000000	.00000000	.00000000
TEACH-ATT	.01203051	.00000000	.01009200	.00000000	.00000000	.00070019

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SUMMARY OF DUAL VALUES--INEFFICIENT UNITS ONLY

OUTPUT/INPUT	9	10
NCE REAU	0.00000000	0.00000000
NCE ENG	0.01937484	0.00018810
NCE MATH	0.00000000	0.01499104
PROF/100	0.00000000	0.00000000
S/PUPIL	0.00000000	0.00000000
NON-MIN	0.00000000	0.00000000
PAY LUNCH	0.00000000	0.00000000
EXP. TEACH	0.00000000	0.00000000
STU ATTEND	0.00000000	0.00000000
TEACH.ATT	0.01382498	0.01418894

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This technical report describes the computer code (Version 2.0 of SMU1) developed to compute the efficiency of a set of decision making units (DMU's) with common inputs and outputs. The code, written in FORTRAN and tested on a CDC Cyber 73 using an MNF compiler, employs a modified primal simplex code. A sample of the computer generated solution is included.		

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