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A STUDY OF STATE RATE FEEDBACK IMPLICIT
MODEL FOLLOWING CONTROL FOR
VSTOL AIRCRAFT

by

Lawrence Ernest Epley

September 1980

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A Study of
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for VSTOL Aircraft

by

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Submitted in partial fulfillment of the
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ABSTRACT

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LIST OF SYMBOLS

A   matrix of coefficients of the closed loop controller

A*   modal transformation of the matrix of coefficients of the closed loop controller

A(s)   SRFIMF controller compensator transfer function

B   matrix of control input coefficients

B*   modal transformation of the matrix of input coefficients

b   damping term in aircraft rigid-body second order modes

c   stiffness term in aircraft rigid-body second order modes

C*   modal transformation of the SRFIMF output matrix

C   SRFIMF output matrix

CPG   constant plant gain or control power gradient depending upon use

F   implicit model following feedback matrix

f   value of the highly damped root of a SRFIMF system

G   matrix of coefficients of the noise inputs to the SRFIMF system

G(s)   Laplace transform of airplane rigid-body modes

H(s)   transfer function representing the dynamic behavior of control force or moment application

h   height in feet of aircraft center of gravity

K   forward loop gain of SRFIMF controller

KRL   SRFIMF controller gain parameter

K_x position feedback gain

$K_{\dot{x}}$ rate feedback gain

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L   matrix of coefficients of the model

L_w characteristic length of atmospheric turbulance

NASA   National Aeronautics and Space Administration

P   matrix of SRFIMF system covariances

Q   matrix of SRFIMF exponentially corellated white noise
           strengths

RTA   Research and Technology Aircraft

s   Laplace transform variable

SRFIMF State Rate Feedback Implicit Model Following

T   transformation matrix between state variable coordinates
           and modal coordinates

T_r correlation time of exponentially correlated sensor
           noises

T_g correlation time of exponentially correlated wind gust
           noise

U   Vector of SRFIMF system inputs

V_{wind} nominal surface wind

$W(s)$ SRFIMF control law

$W_t(t)$ white noise

$X(s)$ controlled variable

X   vector of SRFIMF state variables

X*   vector of SRFIMF modes

X_c commanded input

X_a^* acceleration error mode

X_p^* position error mode

X_v^* velocity error mode

Y   SRFIMF output vector

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Z	vector of model states
δ	general transfer function control input
δ_g	gust input
δ_e	control stick input
ζ	second order response damping ratio
θ	pitch attitude angle
μ	white noise strength
ξ	exponentially correlated noise
σ	standard deviation
σ_a	standard deviation of acceleration error
σ_g	standard deviation of gust input
σ_p	standard deviation of position error
σ_v	standard deviation of velocity error
τ	control actuator time constant
ω_n	natural frequency of second order response

ACKNOWLEDGMENT

I. INTRODUCTION

A State Rate Feedback Implicit Model Following (SRFIMF) flight controller has been proposed as a possible approach to improving the handling qualities of Vertical and Short Field Take Off and Landing (VSTOL) aircraft by Merrick at the NASA Ames Aeronautical Research Laboratory [1, 2]. The SRFIMF concept has potential applications in various types of control problems encountered in aircraft design, among which are attitude, guidance, and engine control. The concept has not been used in actual flight tests, however several detailed simulations [2, 3] have been conducted at NASA Ames in which SRFIMF control was applied. The most notable features of the control scheme are that:

1. The Input-output relationship of a system using SRFIMF control is insensitive to changes in airframe and propulsion dynamic characteristics.

2. The dynamic relationship of the output to the input is approximately that of a second order system whose frequency and damping is chosen by the designer.

3. The system is self trimming and the commanded output variable is independent of external disturbances.

4. The system has good gust alleviation.

This study presents a detailed analysis of the SRFIMF control concept as applied to the attitude control of VSTOL aircraft.

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The following discussion is given to clarify the meaning of State Rate Feedback Implicit Model Following as used in this study. Model following refers to the ability of a control scheme to impart specified dynamic characteristics, given by a model, to the closed loop system. The model being considered here is a second order response in which the parameters of natural frequency and damping ratio are chosen by the designer. Typically a second order response is mathematically defined in terms of the states position and velocity. The state rate of the second order model is the acceleration. The State Rate Feedback Implicit Model Following controller, studied here, achieves model following by measurement and feedback of the system's state rate, acceleration. The result is that a priori knowledge of the plant is not required to produce model following.

To illustrate the use of state rate feedback, consider a plant of second order. With the states of the system X_1 and X_2, defined as position and velocity, the representation of the plant in matrix notation is

$$\begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -c & -b \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} U(t) \qquad 1\text{-}1$$

The acceleration, $\dot{X}_2$, equation is thus

$$\dot{X}_2 = -cX_1 - bX_2 + U(t) \qquad 1\text{-}2$$

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The values of b and c define the dynamic behavior of the plant. Design of a control system would in general require that b and c be known. The feedback scheme or control law of the SRFIMF controller is formulated so that the plant dependent quantities of b and c are only involved in the total quantity of $(-cX_1-bX_2)$. With this arrangement the quantity, $(-cX_1-bX_2)$, can be obtained by measurement of the state rate, acceleration, minus the input, U. Model following by measurement of the state rate, as in this illustration, is the basic concept of the SRFIMF controller.

The intent of this study is to provide a detailed analysis of the SRFIMF controller from a modern and classical control viewpoint. Particular emphasis will be placed on a basic description of the control scheme and the effects of measurement errors on the output of the closed loop system. The first two sections deal with a classical analysis of the controller as applied to the attitude position control of a general VSTOL aircraft. The third section considers the effect of measurement errors on the system from a modern control viewpoint. From that analysis, the steady state covariance of the state variables, as a result of measurement uncertainties, will be found. Finally, the previously developed analysis technique will be applied to an example where SRFIMF is used for pitch control of the Harrier aircraft. The effect of sensor errors will be examined and the response of the Harrier to gust inputs will be determined.

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The following assumptions are made in this study:

1. The system is linear. Non-linear effects such as control saturation and actuator hysteresis are not considered.

2. The dynamic response of the plant will be represented at a single point by linearized, rigid-body, transfer functions.

3. Measurement uncertainties are represented as exponentially correlated noise and the effect of bias error is not considered. The basic description and development of the SRFIMF controller will be considered in section II.

II. DESCRIPTION OF THE SRFIMF CONTROLLER

A. THE BASIC SCHEME

It is desirable for a flight controller to impart specific dynamic characteristics to the closed loop response of an aircraft system. Often the desired response is given in terms of a natural frequency and damping ratio. Piloted simulations at NASA Ames [2] have indicated that a desirable response from the pilot's stick to the aircraft attitude is a second order response whose natural frequency, ω_n, is approximately 2 rad/sec and damping ratio, ζ, of 0.75. One possible approach to the design of an attitude controller is to apply model following techniques. Mathematically the model for attitude control dynamics can be represented by a transfer function in the frequency domain. If, as an example we let $\theta(s)$ represent the aircraft pitch angle and $\delta(s)$ the elevator control input, then the transfer function of a model for pitch attitude control can be written as

$$\frac{\theta(s)}{\delta(s)} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad 2\text{-}1$$

In this example, if the desired response has a natural frequency of 2 rad/sec and damping ratio of 0.75, and equation 2-1 becomes

$$\frac{\theta(s)}{\delta(s)} = \frac{1}{s^2 + 3s + 4} \qquad 2\text{-}2$$

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This transfer function will be used to illustrate model following techniques in the development of the SRFIMF concept. The desired model response, given by 2-1, can also be expressed in state variable notation with states Z_1 of position and Z_2 velocity. The matrix of coefficients of the model is given the symbol L and equation 2-1 can be written as

$$\dot{Z} = [L]Z + [B]U$$

$$= \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} Z + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} U \qquad 2-3$$

Given somewhat arbitrary open loop plant dynamics, the object of model following is to produce a closed loop system whose dynamic characteristics are given by equation 2-1 or equation 2-3.

For a general system

$$\dot{X} = AX + BU$$
$$Y = CX \qquad\qquad 2-4$$

the object of implicit model following is to force the output of the system to follow the model equation

$$\dot{Z} = LZ + BU \qquad 2-5$$

That is to say, the output, Y, should approximate Z ($Y \approx Z$) so that

$$\dot{Y} \approx LY + BU \qquad 2-6$$

By proper choice of the feedback gain F, as shown in figure 2-1, the output is forced to follow equation 2-6. The

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implied model shown in the upper portion of the figure, is
not actually generated but implied by the behavior of Y.
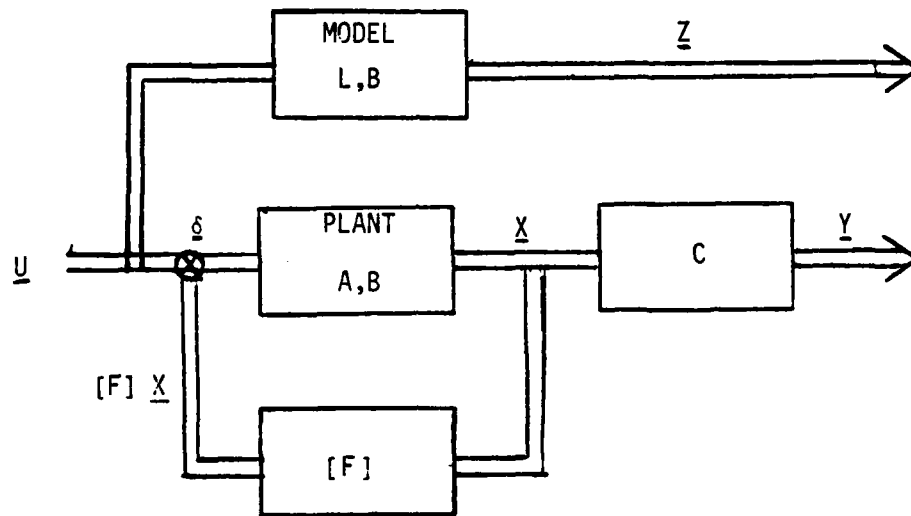
Figure 2-1 Implicit Model Following scheme

The formulation of the feedback law required in model
following has been determined by Taylor [4] using optimal
control techniques. In addition, Erzberger [5] has defined
algebraic methods for determining if perfect model following
can be achieved. This algorithm will be examined later in
this section. In aircraft applications certain physical
facts about the system allow a form of implicit model follow-
ing to be obtained by simple intuitive reasoning.

Consider the problem of stabilizing the pitch attitude
of a VSTOL aircraft. The following assumptions can be made
and will lead to a simplified SRFIMF controller:

1. The desired model response is of second order with frequency, ω_n, of 2 rad/sec and damping ratio, ζ, of 0.75 from equation 2-1.

2. Measurements of angular position, angular rate and angular acceleration are available.

3. The control U is either a thrust or control deflection whose net result is to produce angular acceleration of the vehicle.

4. The open loop plant is arbitrary and the transfer function for θ, $\theta(s)/U(s) = G(s)$, may be unknown.

5. The control law which produces model following will be developed so that it represents the difference between the vehicle's angular acceleration, as measured, and the angular acceleration which would be implied by the model given in 1, above.

From assumption 1 we obtain the desired closed loop response as that of the model and express it as

$$\frac{\theta(s)}{U(s)} = \frac{1}{s^2 + K_{\dot{x}}s + K_x} \qquad 2\text{-}7$$

where $\theta(s)$ is the pitch attitude and the constants K_x and $K_{\dot{x}}$ are defined for convenience and will be used throughout this study as

$$K_{\dot{x}} = 2\zeta\omega_n = 3 \qquad 2\text{-}8$$

$$K_x = \omega_n^2 = 4 \qquad 2\text{-}9$$

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By requiring the model, equation 2-7, to hold we observe that
the implied angular acceleration is

$$s^2\theta(s) = -K_x\theta(s) - K_{\dot{x}}s\theta(s) + U(s) \qquad 2\text{-}10$$

$\theta(s)$ and $s\theta(s)$ are angular position and angular rates which,
by assumption 2, are available from measurements. The
quantity $U(s)$ is the input to the open loop plant. Given
$\theta(s)$, $s\theta(s)$ and $U(s)$, the quantity $s^2\theta(s)$ can be calculated
from equation 2-10.

We will define $W(s)$ as the control law for the system.
$W(s)$ will be taken as the difference between the implied model
acceleration $s^2\theta(s)$ and the actual measured acceleration

$$W(s) = s^2\theta(s) - (\text{measured acceleration}) \qquad 2\text{-}11$$

Using equation 2-10 and substituting for $s^2\theta(s)$ one has

$$W(s) = -K_x\theta(s) - K_{\dot{x}}s\theta(s) + U(s) - \\ (\text{measured acceleration}) \qquad 2\text{-}12$$

Further the quantities $\theta(s)$ and $s\theta(s)$ will be determined by
sensors so that

$$W(s) = -K_x \cdot (\text{measured position}) - K_{\dot{x}} \cdot (\text{measured} \\ \text{rate}) + U(s) - (\text{measured} \qquad 2\text{-}13 \\ \text{angular acceleration})$$

The quantity U(s) could be considered pilot or other control
input to the plant. Since we also defined W(s) as the control
there is some ambiguity in the notation. The symbol U(s) will
be used consistently for the control feedback quantity as
indicated in figure 2-2.
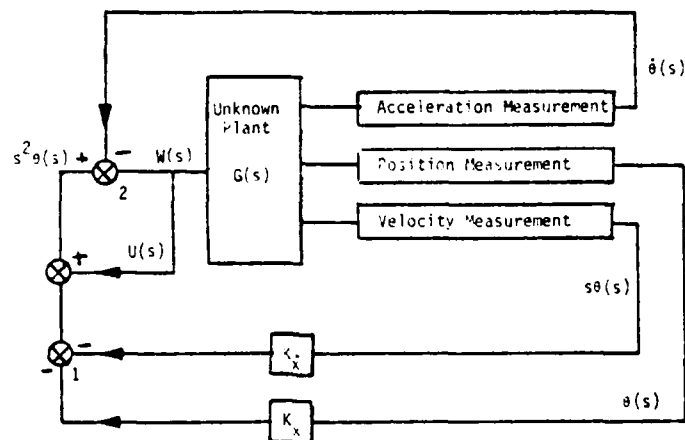
Figure 2-2 SRFIMF position controller developed
by intuitive arguments.

Figure 2-2 shows schematically the control law and unknown
plant G(s). It can also be seen that the plant input, U(s)
is a feedback quantity and the implied acceleration, $s^2\theta(s)$
is compared to the measured acceleration at point 2 of the
figure. The term state rate feedback is applied to this type
of control because of acceleration feedback. It is this
measurement which brings to the controller information about
the plant making a priori knowledge of the transfer function
for $\theta(s)$ unnecessary. This aspect of the SRFIMF controller
is most clearly seen by application of the Erzberger criterion
mentioned earlier.

The Erzberger criterion for exact implicit model follow-ing is obtained by analysis of the system in state space representation where the system is given by

$$\dot{X} = AX + BU \qquad\qquad 2\text{-}14$$

$$Y = CX \qquad\qquad 2\text{-}15$$

For implicit model following the output is approximated by

$$\dot{Y} = LY \qquad\qquad 2\text{-}16$$

Taking the derivative of equation 2-15, one has

$$\dot{Y} = C\dot{X} \qquad\qquad 2\text{-}17$$

Substituting from equation 2-14

$$\dot{Y} = C(AX + BU)$$

or $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2\text{-}18$

$$\dot{Y} = CAX + CBU$$

but from equations 2-15 and 2-16 one has

$$\dot{Y} = LY = LCX \qquad\qquad 2\text{-}19$$

so that one has, on equating 2-18 and 2-19,

$$CAX + CBU = LCX$$

or $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2\text{-}20$

$$CBU = (LC - CA) \cdot X$$

Solving for the control, U, from 2-20 we have that

$$U = \left[(CB)^+ \cdot (LC - CA)\right] X \qquad\qquad 2\text{-}21$$

where $(CB)^+$ is the pseudo, or generalized, inverse of (CB).
Eliminating U from 2-20 and 2-21, the condition for perfect
model following becomes

$$[(CB)(CB)^+ - I][(LC) - (CA)] \cdot X = 0 \qquad\qquad 2\text{-}22$$

The use of the pseudo inverse is based on the property of the
pseudo inverse that $(CB)(CB)^+$ is an orthogonal projection
operator on the range of (CB). It then follows that if 2-22
holds for all X, the range of CB must contain the range of
$(LC-CA)$. This implies that 2-16 is valid which has already
been assumed to be the case.

We will now apply equation 2-22 to the pitch attitude
controller given earlier. In this case $\theta(s)/U(s)$ is assumed
to be the transfer function of an arbitrary second order
system.

$$\frac{\theta(s)}{U(s)} = \frac{1}{s^2 + bs + c}$$

The desired closed loop performance is given by the model as in
equation 2-7. For this example we initially take the feedback
quantities to be angular position and angular rate. Defining
the quantities in state variable representation we have the
plant where

$$X_1 = \theta(t)$$

$$X_2 = \dot{\theta}(t)$$

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$$\dot{X} = [A] \ X + BU$$

$$= \begin{bmatrix} 0 & 1 \\ -c & -b \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \cdot \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} U \qquad\qquad 2\text{-}23$$

and

$$Y = CX$$

or

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \qquad\qquad 2\text{-}24$$

Calculation of the pseudo inverse depends upon the relative rank of C and B in this case, following Noble [6], the pseudo inverse of (CB) is given as

$$(CB)^+ = [(CB)^+ \cdot (CB)]^{-1} \cdot (CB)^T$$

From 2-23 and 2-24

$$(CB) = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

therefore

$$(CB)^+ = [0 \ 1]$$

To determine if perfect model following is possible, we substitute into 2-22 using 2-23, 2-24, 2-8, 2-9 and the above for (CB) and $(CB)^+$

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$$[(CB)(CB)^+ - I] = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \text{2-25}$$

$$[(LC) - (CA)] = \begin{bmatrix} 0 & 0 \\ -(K_x + c) & -(K_{\dot{x}} + b) \end{bmatrix} \qquad \text{2-26}$$

The result is that

$$[(CB)(CB)^+ - I][(LC) - (CA)] \equiv 0 \qquad \text{2-27}$$

Equation 2-27 shows that perfect model following is possible for all X using position and rate feedback only. The control can be determined by equation 2-21 with the result

$$U_c = [(-K_x + c)(-K_{\dot{x}} + b)] \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \qquad \text{2-28}$$

$$= -K_x X_1 - K_{\dot{x}} X_2 + c X_1 + b X_2 \qquad \text{2-29}$$

The control law given in equation 2-29 requires that the constants c and b be known in order to produce the desired model following. In the previous discussion we stated that the addition of acceleration feedback provided the needed information about the plant. To show this, note that the latter two terms of equation 2-29, $c X_1 + b X_2$, can be interpreted in terms of the acceleration of the system, X_2, since

$$\dot{X}_2 = -c X_1 - b X_2 + U(s) \qquad \text{2-30}$$

one has

$$\dot{X}_2 - U(s) = -c X_1 + -b X_2 \qquad \text{2-31}$$

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or rewriting equation 2-29 one has

$$U_c = -K_x X_1 - K_{\dot{x}} X_2 + U(s) - \dot{X}_2 \qquad 2\text{-}32$$

We can compare this to equation 2-13 rewritten in the same notation

$$W(s) = -K_x X_1 - K_{\dot{x}} X_2 + U(s) - \text{(measured} \qquad 2\text{-}33$$
$$\text{angular acceleration)}$$

Note, X_1 and X_2 are the measured angular position and measured angular rate. This comparison shows that the measured acceleration supplies the terms needed for perfect model following without a need for a knowledge of the plant dynamics.

The significance of this result is that state rate feedback can be used to provide information about the plant in applications where the plant has unknown and changing dynamic characteristics. This conclusion was reached based upon the assumption that the plant was of second order. We shall now consider a more general form of the SRFIMF controller and we will show that model following can be achieved by measurement of state rate for a higher order plant.

B. THE GENERAL FORM OF THE SRFIMF CONTROLLER

The preceding discussion presented an intuitive description of the principal operation of the SRFIMF controller and the use of state rate feedback. It is the intent of this section to develop a general form of state rate feedback and

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to show again that model following can be achieved without a priori knowledge of the plant by using measurement of the state rate. We begin by examining the basic SRFIMF controller as developed by Merrick at NASA Ames. The block diagram of this controller is shown in figure 2-3.
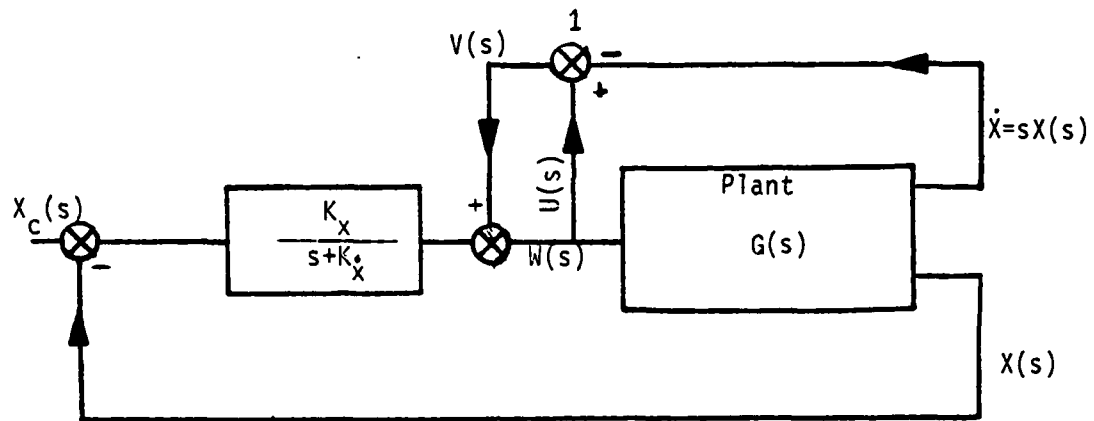
Figure 2-3 The general form of the SRFIMF controller.

Assume that figure 2-3 represents a velocity controller, then the feedback quantities $X(s)$ and $sX(s)$ are velocity and acceleration respectively. It can be seen that at position 1 we are summing $\left(-\text{ acceleration } + U(s)\right)$. It will be shown that the transfer function between the input $X_c(s)$ and the output $X(s)$ does not depend upon the plant transfer function $G(s)$ and that the closed loop response is that of a second order system whose damping and natural frequency are determined by the choice of feedback constants K_x and $K_{\dot{x}}$.

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From figure 2-3 W(s) is given by

$$W(s) = U(s) - sX(s) + [X_c(s) - X(s)] \cdot \left[\frac{K_x}{s + K_{\dot{x}}} \right] \quad 2\text{-}34$$

and for V(s) we write

$$V(s) = U(s) - sX(s) \qquad 2\text{-}35$$

from the definition of the transfer function G(s) we have that

$$X(s) = W(s) \, G(s) \qquad 2\text{-}36$$

Combining 2-34 and 2-36 we have that

$$X(s) = \left\{ U(s) - sX(s) + \frac{(X_c(s) - X(s)) K_x}{s + K_{\dot{x}}} \right\} G(s) \quad 2\text{-}37$$

Since U(s) = W(s) as seen from figure 2-3, we have

$$X(s) = U(s) \cdot G(s) \qquad 2\text{-}38$$

and equation 2-37 can be rewritten as

$$X(s) = X(s) - sX(s)G(s) + \frac{(X_c(s) - X(s)) K_x}{s + K_{\dot{x}}} G(s) \quad 2\text{-}39$$

and

$$sX(s)G(s) = \frac{\left(X_c(s) - X(s) \right) K_x}{s + K_{\dot{x}}} \cdot G(s) \qquad 2\text{-}40$$

G(s) on both sides of equation 2-40 cancels and the result becomes

$$\frac{X(s)}{X_c(s)} = \frac{K_x}{s^2 + K_{\dot{x}}s + K_x} \qquad 2\text{-}41$$

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Equation 2-41 shows that the closed loop response of the SRFIMF is identically that of a second order system whose natural frequency and damping ratio is completely determined by the constant feedback gains which are given by

$$\omega_n = K_x \qquad \qquad \zeta = \frac{K_{\dot{x}}}{2\sqrt{K_x}}$$

The results of this analysis indicate that the SRFIMF controller is an excellent candidate for VSTOL aircraft applications. The form of the controller shown in figure 2-2 could be used to control a position such as attitude while the general form shown in figure 2-3 might be used to control rates or velocity. It will now be necessary to examine the requirements necessary to impliment a SRFIMF controller in an aircraft. In particular, the exact relationship between U(s) and W(s) must be considered. The next section will examine practical examples of SRFIMF controllers in a realistic aircraft environment.

C. PRACTICAL EXAMPLES OF THE SRFIMF CONTROLLER

Two types of controllers are illustrated in this section, a position controller, figure 2-4, and a velocity controller, figure 2-5. A first order actuator is included in the plant of each controller. Also included is a compensator in the control feedback loop involving U(s). The system is not realizable without the compensator. These controllers will be used in the analysis of this report.

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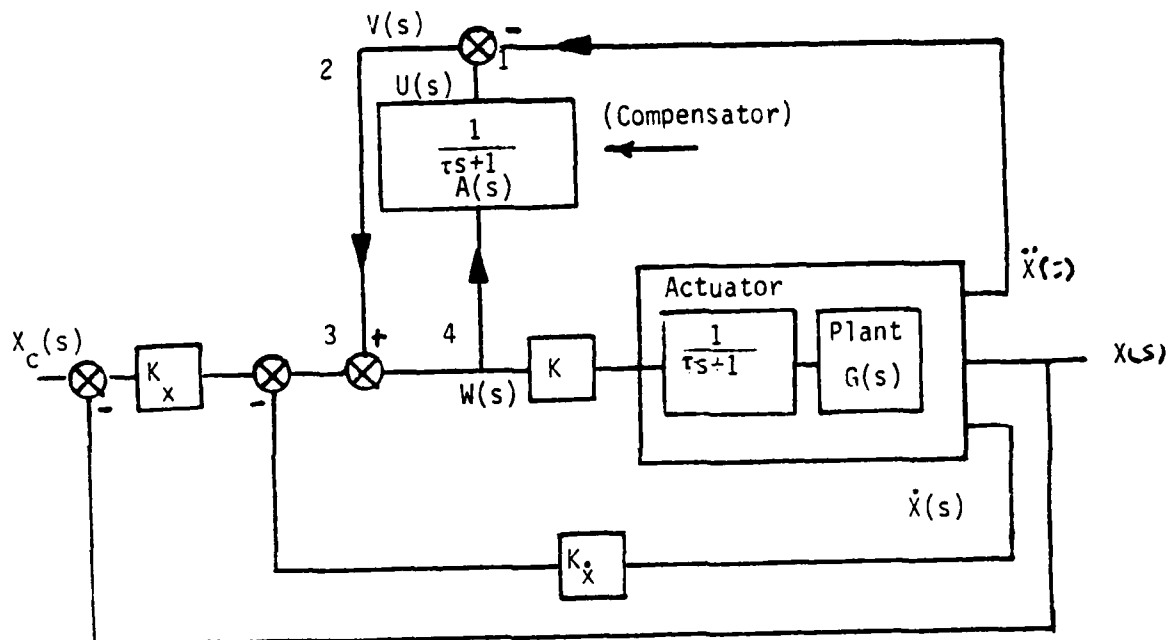
Figure 2-4 Physically realizable SRFIMF
position controller.
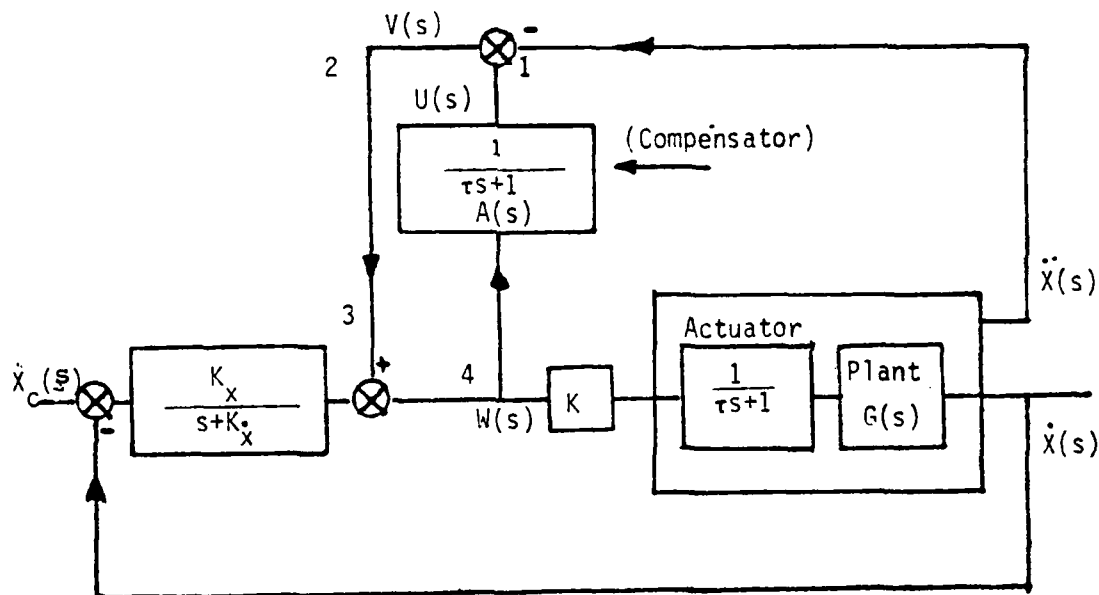
Figure 2-5 Physically realizable SRFIMF
rate controller.

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The plant is assumed to be driven by a control actuator and because of that a phase lag can be expected to exist between the commanded control signal, W(t) and the input of the plant. The effect of the actuator will be modeled by using the first order transfer function

$$\frac{H(s)}{W(s)} = \frac{1}{(\tau s + 1)} \qquad\qquad 2\text{-}42$$

where H(s) is the output of the actuator, W(s) is the input and τ is the actuator time constant assumed to be 0.1 sec for all of the examples considered in this work. Recall from a previous section that SRFIMF feedback contained a term U(t) which represented the control input to the plant and that W(t) was assumed to equal U(t). Because of the fact that U(t) was equal to W(t) the output signal of the controller could be used to cancel the U(t) term in the acceleration feedback. It was shown that

$$\text{acceleration} = -cX_1 -bX_2 + U(t) \qquad\qquad 2\text{-}43$$

and it was assumed that

$$U(t) = W(t) \qquad\qquad 2\text{-}44$$

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In order for equation 2-44 to be valid, the system would need to have instantaneous response to any input. Because of the action of the control actuator this is not possible and a compensator is placed in the feedback loop between W(s) and the summing junction at 1, as shown in figures 2-4 and 2-5. The compensator transfer function is A(s). A(s) is chosen so that the output of the compensator is dynamically identical to the input of the plant, after the control actuator. When the control is represented by a first order lag, as in this case, A(s) is the same as the transfer function of the actuator. In cases of higher order actuator dynamics the result is more complicated. An algorithm for determining the necessary transfer function, A(s), which must be used in the feedback is given by Merrick [2]. We will assume a first order actuator and a compensator of the form $1/(\tau s + 1)$ as shown in figures 2-4 and 2-5, for the remainder of this work.

Actual controllers used at NASA Ames are illustrated in figures 2-6 and 2-7. Figure 2-6 is a speed controller. The quantities VD and VDD represent the measured speed and accel-acceleration. VC is the commanded speed and W(s) is the output of the controller. Figure 2-7 is a SRFIMF position controller used to control the pitch attitude, θ, of the RTA vehicle. The limiters seen in the figures are not included in the model studied here. The purpose of the limiter is to prevent the control feedback loop from acting as an integrater when the plan control is saturated. This condition could occur

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when a large difference between the commanded variable, X,
and the input variable, X_c, existed and the plant control
was saturated. In this condition $W(s)$ would increase with-
out bound and a reversal of the input would be delayed because
of the very high value of $W(t)$ at the time the control reversal
was applied. The limiter was not considered here because of
the assumption of linearlity.

Figure 2-6 SRFIMF controller employed on the RTA
for speed control.

Figure 2-7 SRFIMF controller employed on RTA for
pitch attitude control.

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III. TIME AND FREQUENCY DOMAIN ANALYSIS OF THE SRFIMF

A. REPRESENTATIVE AIRCRAFT PLANT DYNAMICS

The State Rate Feedback Implicit Model Follower controller presented in section II, figure 2-4 will now be examined using Root Locus, Bode Analysis and Time Simulations. Root Locus analysis will show the asymptotic behavior of the closed loop poles and Bode analysis will show the filter characteristics of the controller. Simulated time response of the controller will be shown in order to demonstrate the ability of the SRFIMF to follow the given model and to show graphically the self trimming feature of the controller. The model and representative plant will next be defined.

The model for the system is taken as a second order system with a transfer function given by

$$\frac{M(s)}{U(s)} = \frac{K_x}{s^2 + K_{\dot{x}}s + K_x} \qquad\qquad 3\text{-}1$$

The gains K_x and K_x are selected to yield two different models, one a position controller and the other a rate controller as shown in the following table.

	ω_n	ζ	K_x	$K_{\dot{x}}$	Pole Location	
Attitude Control	2 rad/sec	.75	4	3	-1.5	1.32i
Rate Control	1.23 rad/	.7023	1.57	1.76	-.88	.89i

TABLE 3-1 Assumed Model Constants.

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The values listed in table 3-1 are based upon piloted simulations at NASA Ames [2] and reflect what has been judged to be representative of good handling qualities.

The plant transfer function is also taken as a second order system

$$G(s) = \frac{CPG}{s^2 + bs + c} \qquad 3\text{-}2$$

While initially this assumption may seem unrepresentative of a real aircraft, proper choice of b and c can represent the dynamic behavior of the individual modes of any aircraft, provided that first order modes are taken two at a time. To illustrate, consider the general transfer function

$$C(s) = \frac{K \prod\limits_{i=1}^{M} (s + z_i)}{s \prod\limits_{J=1}^{q} (s + P_J) \prod\limits_{K=1}^{r} (s^2 + 2\zeta_K \omega_K s + \omega_K^2)} \qquad 3\text{-}3$$

If the poles are distinct then equation 3-3 can be expanded into partial fractions as follows

$$C(s) = \frac{a}{s} + \sum\limits_{J=1}^{q} \frac{a_J}{s + P_J} + \sum\limits_{K=1}^{r} \frac{b_K(s + \zeta_K \omega_K) + C_K W_n \sqrt{1 - \zeta_K^2}}{s^2 + 2\zeta_K \omega_K s + \omega_K^2}$$

$$3\text{-}4$$

It can be seen from the above that the response of a higher order system is composed of a summation of first and second order terms. When the first order terms are taken two at a

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time, the total system response can be expressed as the sum of only second order terms with constant numerators. The assumption of linearity allows us to analyze the modes independently. It remains to choose values of b and c which represent typical aircraft modes. To do this, it will be necessary to survey several representative aircraft. From this survey a number of modes will be selected as typical of VSTOL applications. It will be assumed that acceptable control of each of the modes will lead to acceptable control of the system. This method of analysis leads to considerable simplification since it allows the plant to be taken as a second order system. While this simplified plant will serve the majority of the analysis, a more complex plant will be examined in section V.

Table 3-2 is a listing of pole locations for selected aircraft. Figure 3-1 is a sketch of these poles in the complex plane and an assumed envelope of VSTOL pole locations. Table 3-3 lists points which will be considered by time simulation. From this list, four representative modes will be examined by root locus.

AIRCRAFT	FLIGHT CONDITION	FREQUENCY/DAMPING		OSCILLATORY POLES		REAL POLES	
		ω_n	ζ	Re.	Im.	$1/T_1$	$1/T_2$
F106	S.L. M=.2	2.42	.62	-1.5	1.9	-.169	-.59
Lateral/Directional	20,000', M=.9	3.01	.159	-.48	3.0	-1.84	+.006
A4	S.L. M=.2 short period	1.56	.31	-.48	1.5		
Longitudinal	long period	1.52	.087	-.013	.15		
	15,000', M=.9 short period	.623	.344	-.20	.585		
	long period	.12	-.073	+.01	.12		
A4	S.L. M=.2	1.89	.05	-.09	1.89	-.065	-.56
Lateral/Directional	15,000', M=.9	6.61	.096	-.641	6.6	-2.48	-.006
VZ4	0 kts.	.731	-.439	+.32	.66	-.82	-.137
Longitudinal	26.5 kts.	2.16	.4	-.086	.20		
	75.6 kts. short period	3.4	.374	-.127	3.4		
	long period	.316	.346	-.11	.30		
VZ4	0 kts.	.669	-.347	+.30	.63	-.65	-.90
Lateral/Directional	75 kts.	1.59	.421	-.67	1.4		
H13 (helo)	0 kts.	.43	-.250	+.10	.42	-.69	-.87
	115 kts.	.38	-.043	+.016	.40	-.9	-1.05
Harrier	0 kts.	.31	-.48	+.148	.27	-.33	-.02
Longitudinal	60 kts.	.32	-.91	+.30	.13	-.073	-1.0
Harrier	0 kts.	.52	-.50	+.26	.45	-.015	-.58
Lateral/Directional	60 kts.	1.2	-.28	+.336	1.15	-.068	-1.26
	120 kts.	1.8	-.15	+.27	1.78	-.056	-1.73

Table 3-2 Typical aircraft pole locations where the characteristic equation has the form

$$(s + 1/T_1)(s + 1/T_2)(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0$$

Source: References [4 & 6]

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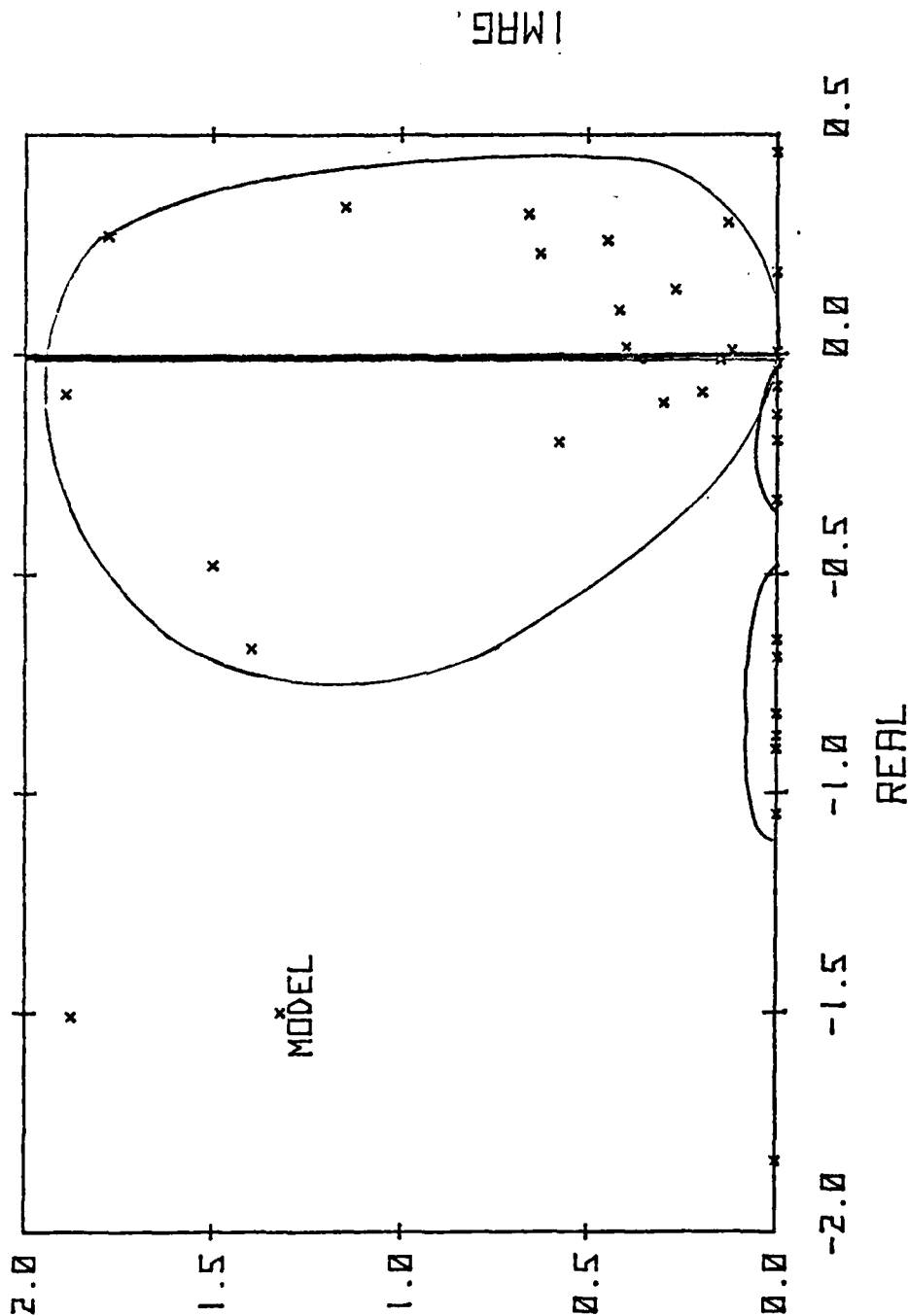
Figure 3-1 Typical aircraft pole locations.

Mode #	Real	Im.	b	c
1	-.6	3.	1.2	8.64
2	-.15	2.15	0.3	6.23
3	-.3	1.0	0.6	0.91
4	-0.1	0.3	0.2	0.08
5	-0.2/0.		0.2	0.0
6	0.3	1.5	-0.6	2.16
7	0.3	0.0	-0.6	0.0
8	0.1	0.14	-0.2	0.03
9	0.0	0.0	0.0	0.0
10	-1.0/ 0.6		1.6	0.6

Table 3-3 Aircraft modes used in simulations.
The modes were choosen to be typical of
VSTOL aircraft as seen in figure 3-1.

B. FREQUENCY RESPONSE AND BODE ANALYSIS

The object of Bode Analysis is to examine the frequency response characteristics of the SRFIMF controller. It will be necessary to develop the transfer function of the position controller shown in figure 2-4. The plant transfer function is given in general as

$$G(s) = \frac{X(s)}{W(s)} = \frac{CPG}{s^2 + bs + C}$$

As in earlier sections the control actuator transfer function, H(s), is assumed to be of the form

$$H(s) = \frac{1}{(\tau s + 1)} \qquad\qquad 3-5$$

The transfer function of the plant is given by G(s), the output position, defined as X(s) and the commanded input is $X_c(s)$. The compensator shown in figure 2-4, and described in section 2-3 is

$$A(s) = \frac{1}{(\tau s + 1)} \qquad\qquad 3-6$$

From figure 2-4 we can write at point 1, the equation

$$V(s) = A(s)W(s) - s^2 X(s)$$

where $s^2 X(s)$ is now the acceleration feedback. We write the control quantity, W(s), from figure 2-4 as

$$W(s) = U(s) - K_{\dot{x}} s X(s) - K_x [-X(s) - X_c(s)] \qquad 3-7$$

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and also note

$$X(s) = K \cdot G(s) \cdot H(s) \cdot W(s) \qquad \text{3-8}$$

Combining and rearranging the above expressions we have that

$$X_c(s) K_x = \left(\frac{(1 - A(s))}{K} \cdot \frac{1}{G(s) H(s)} + s^2 + K_{\dot{x}} s + K_x \right) \cdot X(s)$$

and further rearrangement leads to the transfer function for the input-output relation

$$\frac{X(s)}{X_c(s)} = \frac{K_x}{\left(\left[\frac{1 - A(s)}{K} \right] \cdot \frac{1}{G(s) \, H(s)} \right) + s^2 + K_{\dot{x}} s + K_x} \qquad \text{3-9}$$

It will be convenient to separate the constant plant gain, CPG, from the plant transfer function. The remaining transfer function, defined as $G'(s)$, has a unity constant gain. Stated another way

$$G(s) = CPG \cdot G'(s) \qquad \text{3-10}$$

The parameter KRL is thus defined as

$$KRL = K \cdot CPG / \tau$$

Substituting for $A(s)$, $H(s)$, and KRL and, rearranging 3-9 we are left with

$$\frac{X(s)}{X_c(s)} = \frac{KRL \cdot K_x}{s \left(\frac{1}{G'(s)} \right) + KRL \, (s^2 + K_{\dot{x}} s + K_x)} \qquad \text{3-12}$$

Equation 3-12 shows the effect of the SRFIMF gain parameter, KRL. As KRL increases, the model term $(s^2 + K_x s + K_x)$,

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becomes more dominant in the closed loop transfer function.
Using equation 3-12, we can examine the frequency response of
the controller.

Assuming the plant transfer function to be a second order
system given by G(s), equation 3-2, equation 3-12 can be
rewritten as

$$\frac{X(s)}{X_c(s)} = \frac{KRL \cdot K_x}{s^3 + (KRL + b)s^2 + (KRL\ K_{\dot{x}} + c)s + K_x \cdot KRL} \qquad 3\text{-}13$$

The dominant effect of KRL in equation 3-13 is again seen in
the coefficients of the s and s^2 terms. From table 3-2 we see
that a likely range in the values of b and c are $-0.6 \leq b \leq 1.6$
and $0.0 \leq c \leq 8.64$. If KRL is of the order of 25 and b and c are
in the range of 2 and 8 respectively, then KRL will dominate
the terms of equation 3-13 which contain b and c. Equation
3-13 can be simplified by neglecting b and c with resulting
transfer function for the closed loop system written as

$$\frac{X(s)}{X_c(s)} = \frac{KRL \cdot K_x}{s^3 + KRL(s^2 + KRL\ K_{\dot{x}}s + K_x)} \qquad 3\text{-}14$$

From this later representation it is clear that the plant
dynamics which were determined by the coefficients b and c no
longer play a role in the frequency response analysis.
Equation 3-14 expressed in Bode form becomes

$$\frac{X(s)}{X_c(s)} = \frac{KRL \cdot K_x}{(i\omega)^3 + KRL(i\omega)^2 + KRL\ K_{\dot{x}}(i\omega) + KRL\ K_x} \qquad 3\text{-}15$$

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Equation 3-15 was plotted for frequencies from .01 to 100
and for KRL values of 1, 10, 25, 50 and 100. Figures 3-2 to
3-6 are the plots of the frequency response of the closed loop
system. It can be seen that the frequency response of the
system is that of a low pass filter with a break frequency of
around 3.0 and a 40 db/Dec roll off. It is also seen that the
response is very nearly in phase with the input for frequencies
lower than 3.0. The results of the frequency response analysis
show that the controller possesses good frequency response
characteristics from the input to the output in that it has a
flat response for all input frequencies of interest and little,
if any phase shift. It has also been shown by Merrick [2] that
the controller attenuates plant disturbances in the form of
applied accelerations. The question of control disturbances
will be examined from a different point of view in section V.
We shall now consider the requirements for the gain, KRL, by
root locus analysis.

C. ROOT LOCUS ANALYSIS

The intent of the root locus analysis is to determine the
magnitude of the SRFIMF controller gain parameter, KRL, neces-
sary for acceptable model following. Plots of the root locus
of the oscillatory pole will show that the desired closed loop
pole location is approached asymptotically as KRL is increased.
Acceptable performance is determined by the specific applica-
tion for which the SRFIMF controller is used. We begin by
examining the equations for the closed loop system developed
in the preceeding section.

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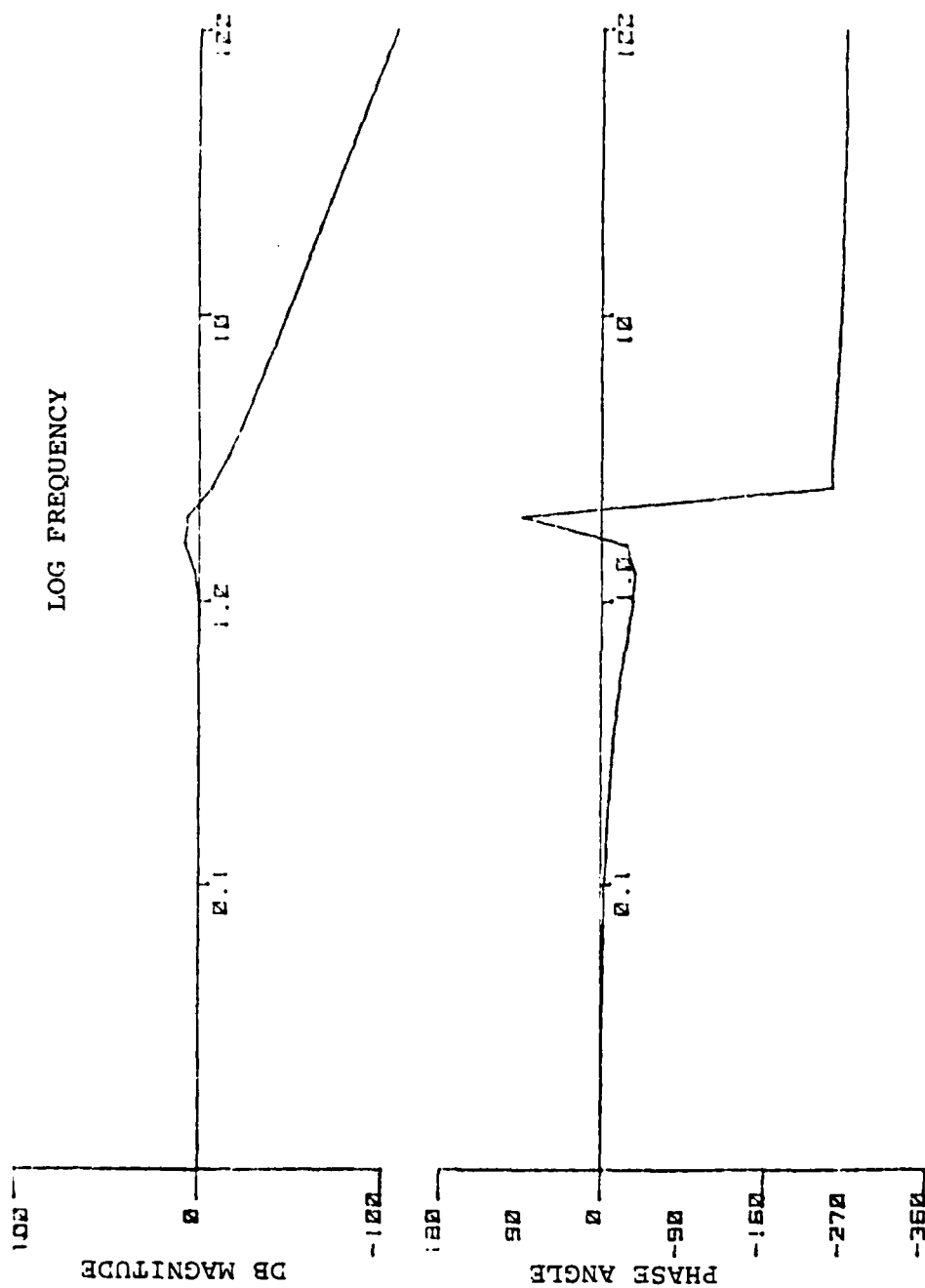
Figure 3-2 Frequency response of SRFIMF position controller, KRL = 1.
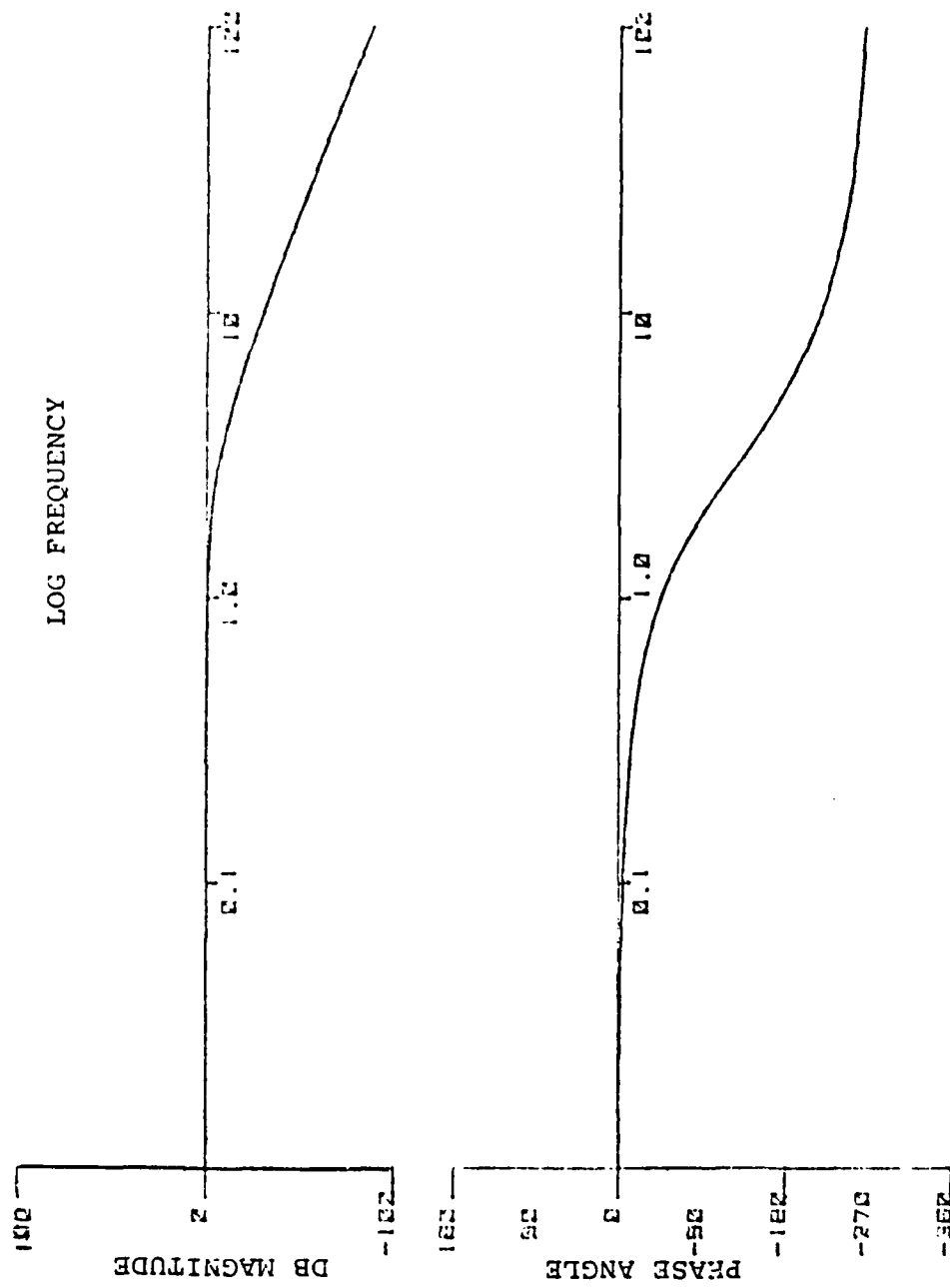
Figure 3-3 Frequency response of SRFIMF position controller, KRL = 10.

49
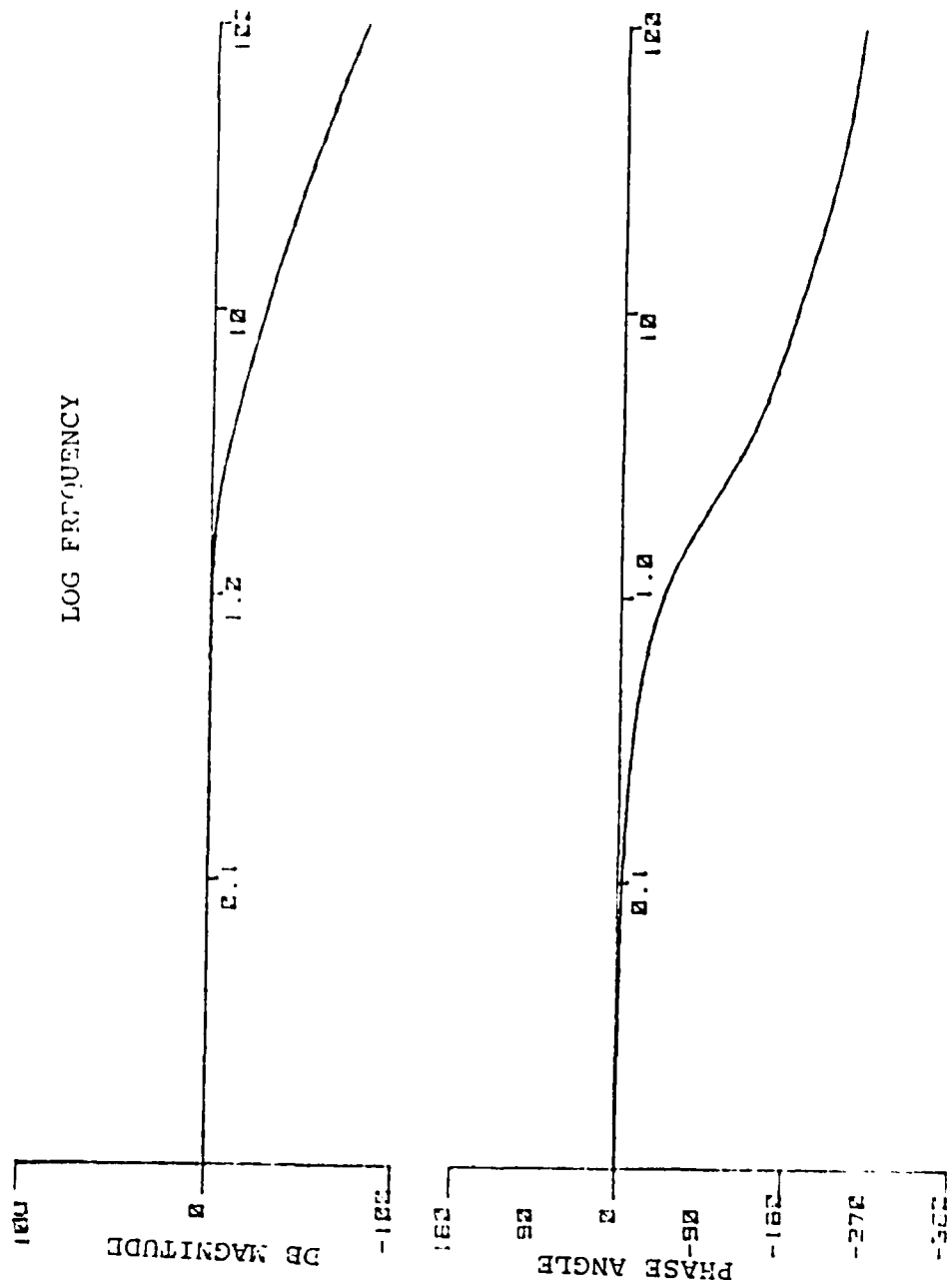
Figure 3-4 Frequency response of the SRFIMF position controller, KRL = 25.

50

LOG FREQUENCY

DB MAGNITUDE

PHASE ANGLE
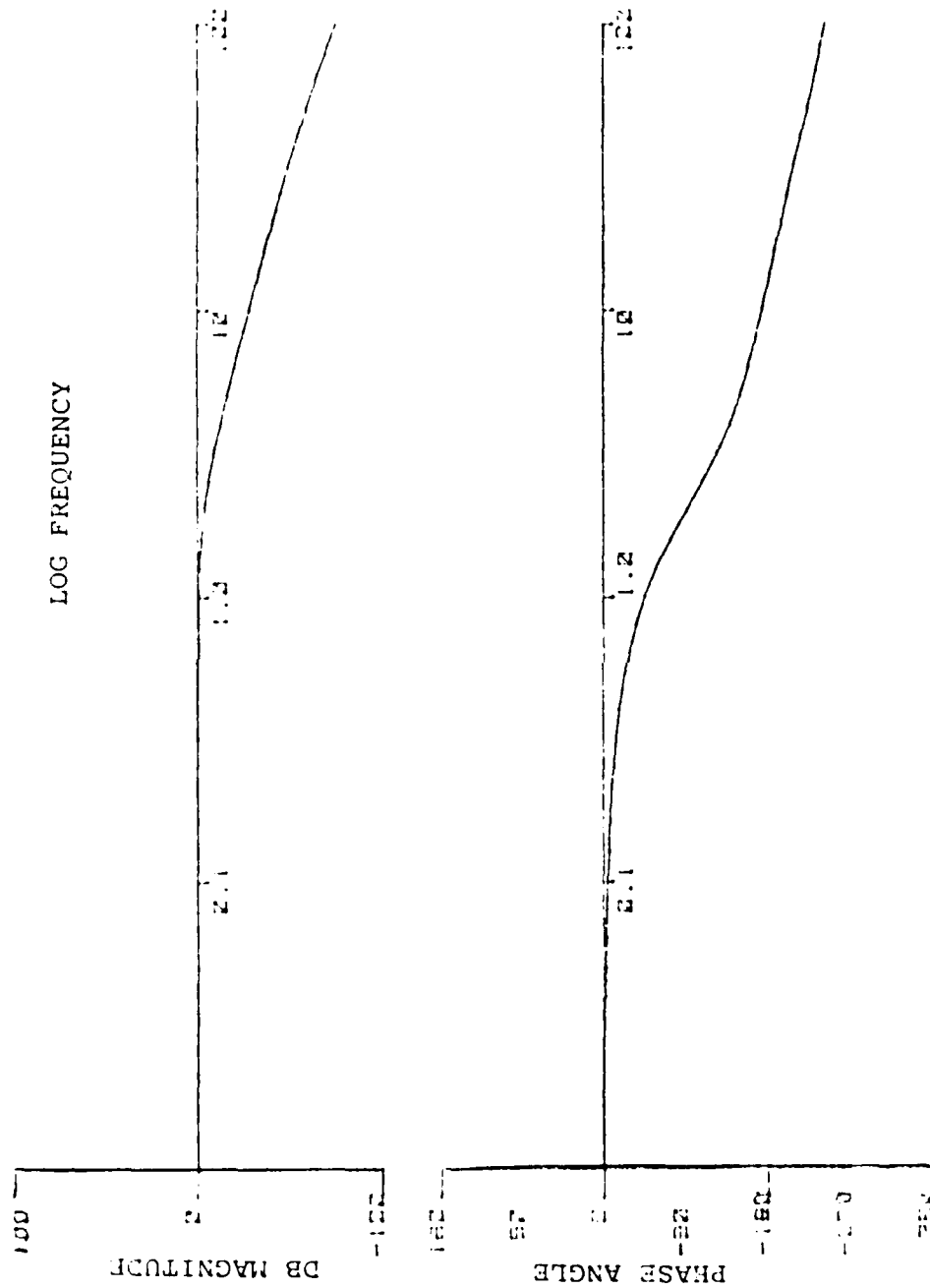
Figure 3-5 Frequency response of SRFIMF position controller, KRL = 50.
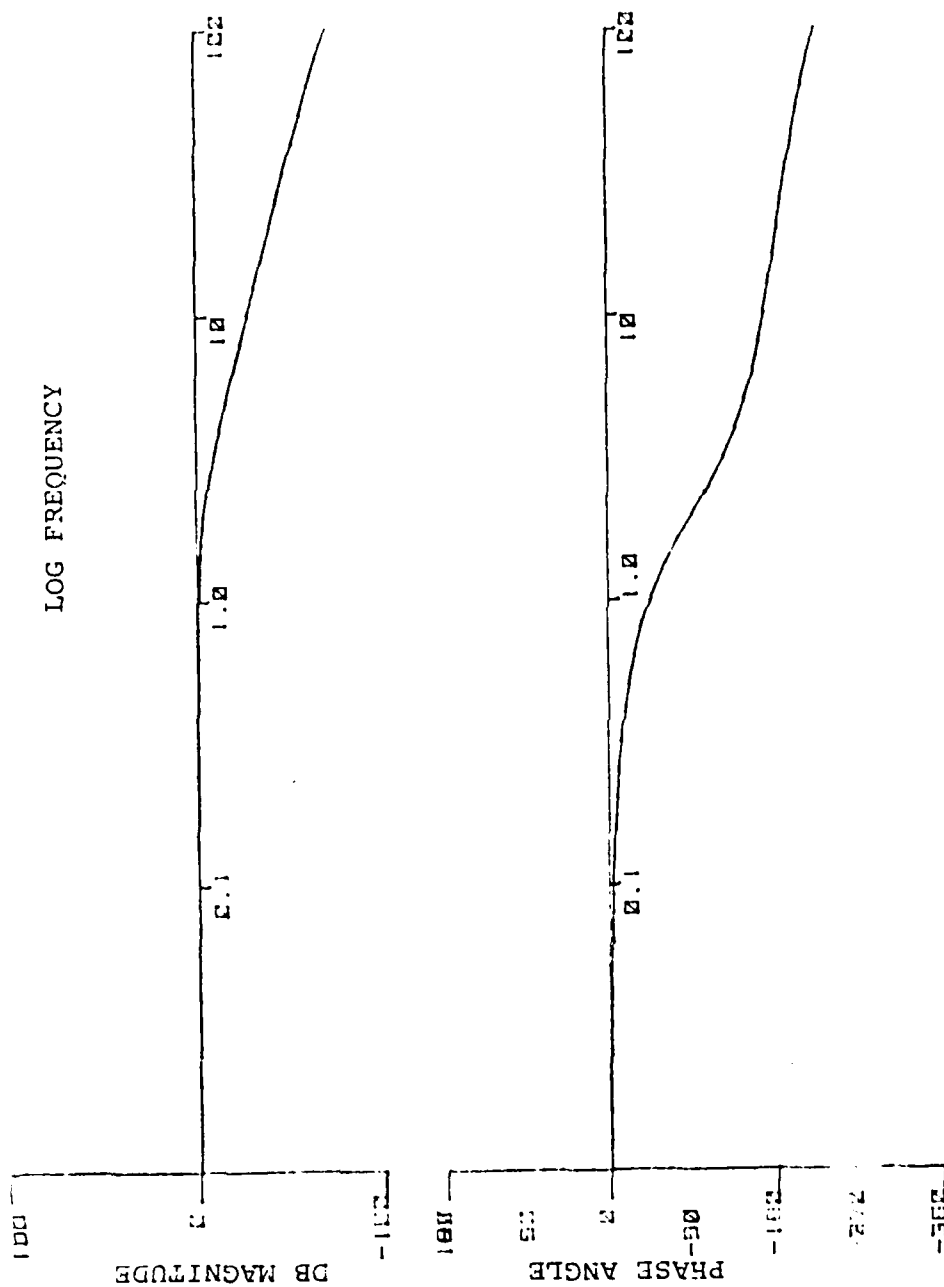
Figure 3-6 Frequency response of SRIMF position controller, KRL = 100.

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The characteristic equation for the system can be obtained by setting the denominator of equation 3-13 to zero. The resulting equation is

$$s^3 + (b + KRL)s^3 + (c + K_{\dot{x}} KRL)s + KRL \cdot K_x = 0 \qquad 3\text{-}16$$

At least one root of 3-16 must be real and the factor corresponding to the real root is defined as $(s + f)$. Assuming that the system described by the transfer function, equation 3-13, represents a model following system, then the second factor of 3-16 must be given by the model. In other words, assuming perfect model following, equation 3-16 must factor as

$$(s + f)\,(s^2 + K_{\dot{x}}s + K_x) = 0 \qquad 3\text{-}17$$

As KRL becomes large, the SRFIMF closed loop system response does approach perfect model following, and it can be stated that as KRL increases the condition is approached where

$$s^3 + (b + KRL)s^2 + (c + K_{\dot{x}}KRL)s + KRL \cdot K_x \longrightarrow$$
$$(s + f) \cdot (s^2 + K_{\dot{x}}s + K_x) \qquad 3\text{-}18$$

In other words, the dynamic behavior of the system's oscillatory mode is approximately given by the model parameters $K_{\dot{x}}$ and K_x. Expanding the right hand side of 3-18 we have for increasing KRL

$$s^3 + (b + KRL)s^2 + (c + K_{\dot{x}}KRL)s + KRL \cdot K_x \longrightarrow$$
$$s^3 + (K_{\dot{x}} + f)s^2 + (K_x + K_{\dot{x}}f)s + f \cdot K_x \qquad 3\text{-}19$$

Comparing coefficients reveals that for increasing KRL

$$b + KRL \gg K_{\dot{x}} + f \qquad\qquad 3\text{-}20a$$

$$c + K_{\dot{x}}KRL \gg K_{\dot{x}}f + K_x \qquad\qquad 3\text{-}20b$$

$$KRL \gg f \qquad\qquad 3\text{-}20c$$

The relationships of 3-20 are not equalities, but they indicate that while KRL is increasing, the difference between KRL and f remains finite.

Therefore, it can be said that one real pole is on the negative real axis and its location is approximated by the value of KRL since f is approaching KRL as KRL becomes large relative to b or c. To examine the behavior of the oscillatory pole as KRL is increased, it will be necessary to rearrange equation 3-9 into root locus form. Setting the denominator of 3-9 equal to zero yields

$$\left[\frac{1 - A(s)}{K}\right] \cdot \left[\frac{1}{H(s)\,G(s)}\right] + s^2 + K_{\dot{x}}s + K_x = 0 \qquad 3\text{-}21$$

Substituting for A(s), H(s) and G(s) and rearranging, the general relation for the rool locus is

$$0 = 1 + \frac{KRL\,(s^2 + K_{\dot{x}}s + K_x)}{s}\,G'(s) \qquad\qquad 3\text{-}22$$

where G'(s) is an arbitrary plant transfer function divided by its constant gain, CPG, as was shown by equation 3-10.

Equation 3-21 will now be used to determine the root locus for the position controller where G'(s) is given by one of four of the modes assumed to be typical of VSTOL aircraft as listed in table 3-4. The cases considered are

	Mode # from Table 3-2	b	c
Case 1	6	-.6	2.16
Case 2	3	.6	.91
Case 3	9	0	0
Case 4	10	1.6	.6

Table 3-4 Plant Parameters for Root Locus Analysis

The root locus computer program developed by Melsa and Jones [7] was used to evaluate and plot the root locus given by equation 3-21. The gain constant, KRL, was varied from 0 to 100. The resulting root locus trajectory of the oscilla-tory poles for the four mode cases are shown in figures 3-7 to 3-10. From the figures it can be seen that in all cases the oscillatory pole approaches the desired value given by the model, in this case -1.5, 1.32i. Because of scaling, the real pole described earlier is not shown in the figures. It can also be seen that the pole location of the closed loop system is within 5 percent of the desired value for KRL of between 25 and 50. Although a value of KRL equal to 25 is normally sufficient, a KRL value of 50 will be used for the remainder of this work.
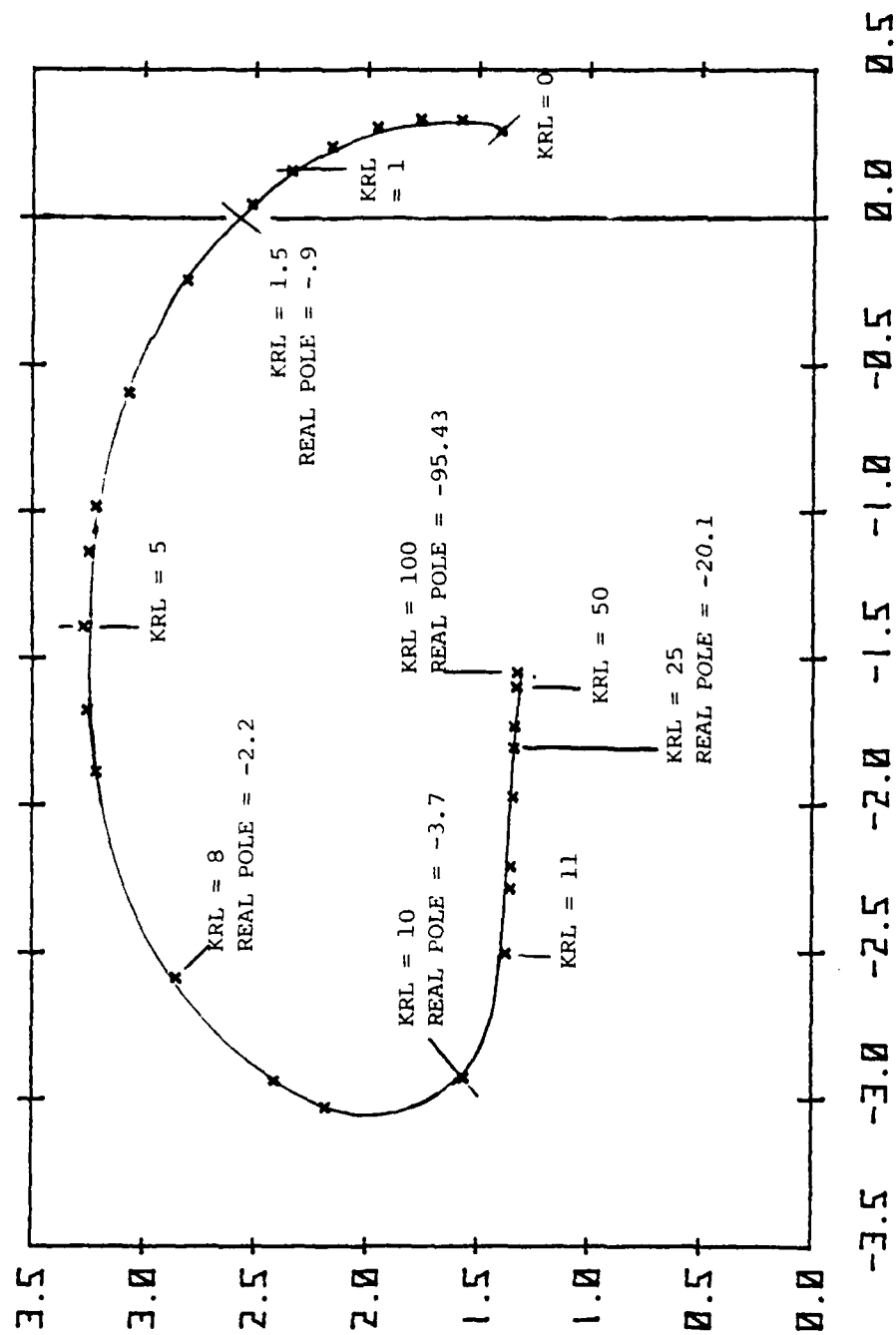
Figure 3-7 Root locus plot of oscillatory pole of a SRFIMF position controller. The open loop plant has a natural frequency of 1.47 rad/sec and a damping ratio of -.441.

56
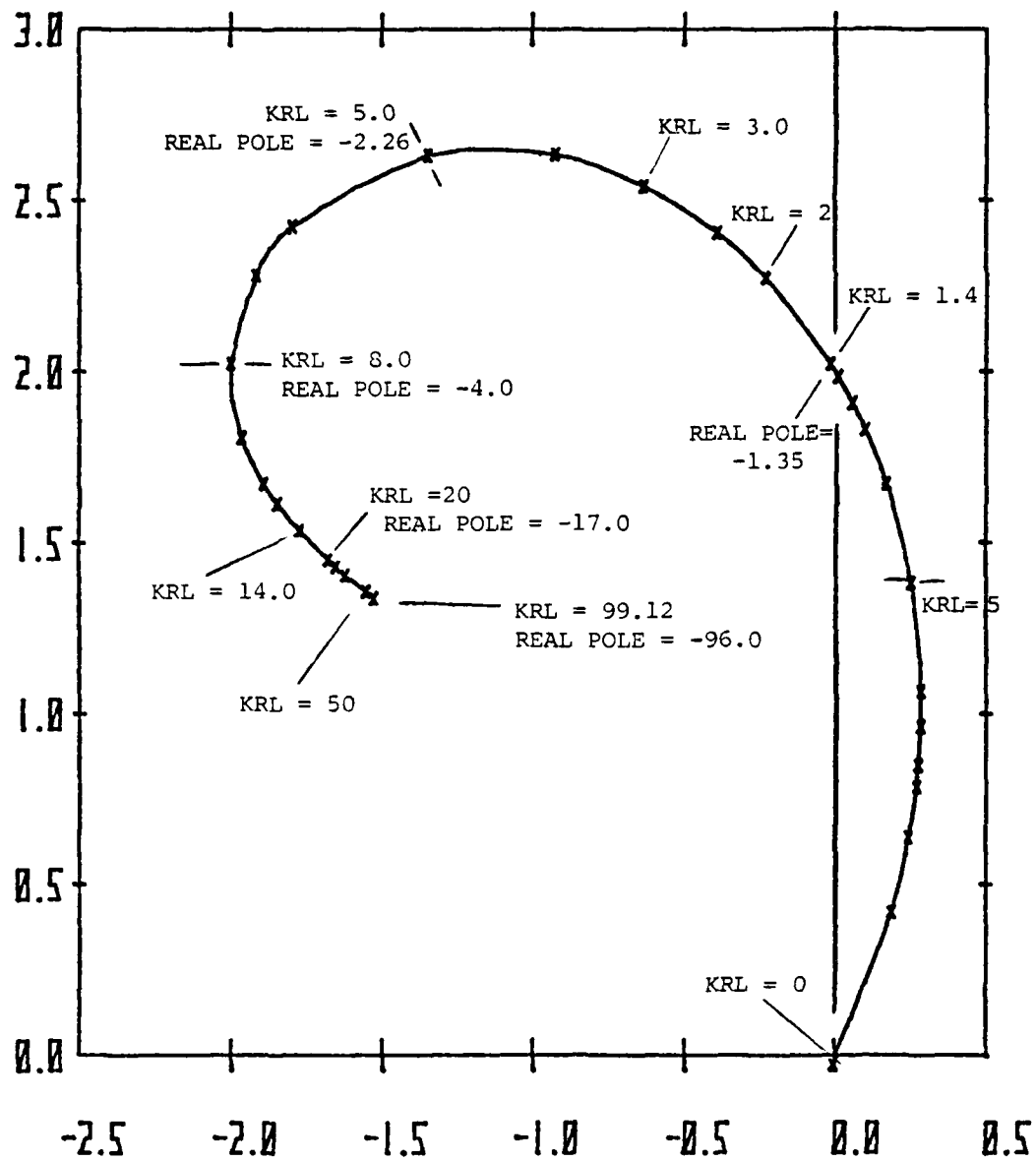
Figure 3-8 Root locus of the oscillatory pole of a SRFIMF position controller. The open loop plant has two poles at the origin.

57
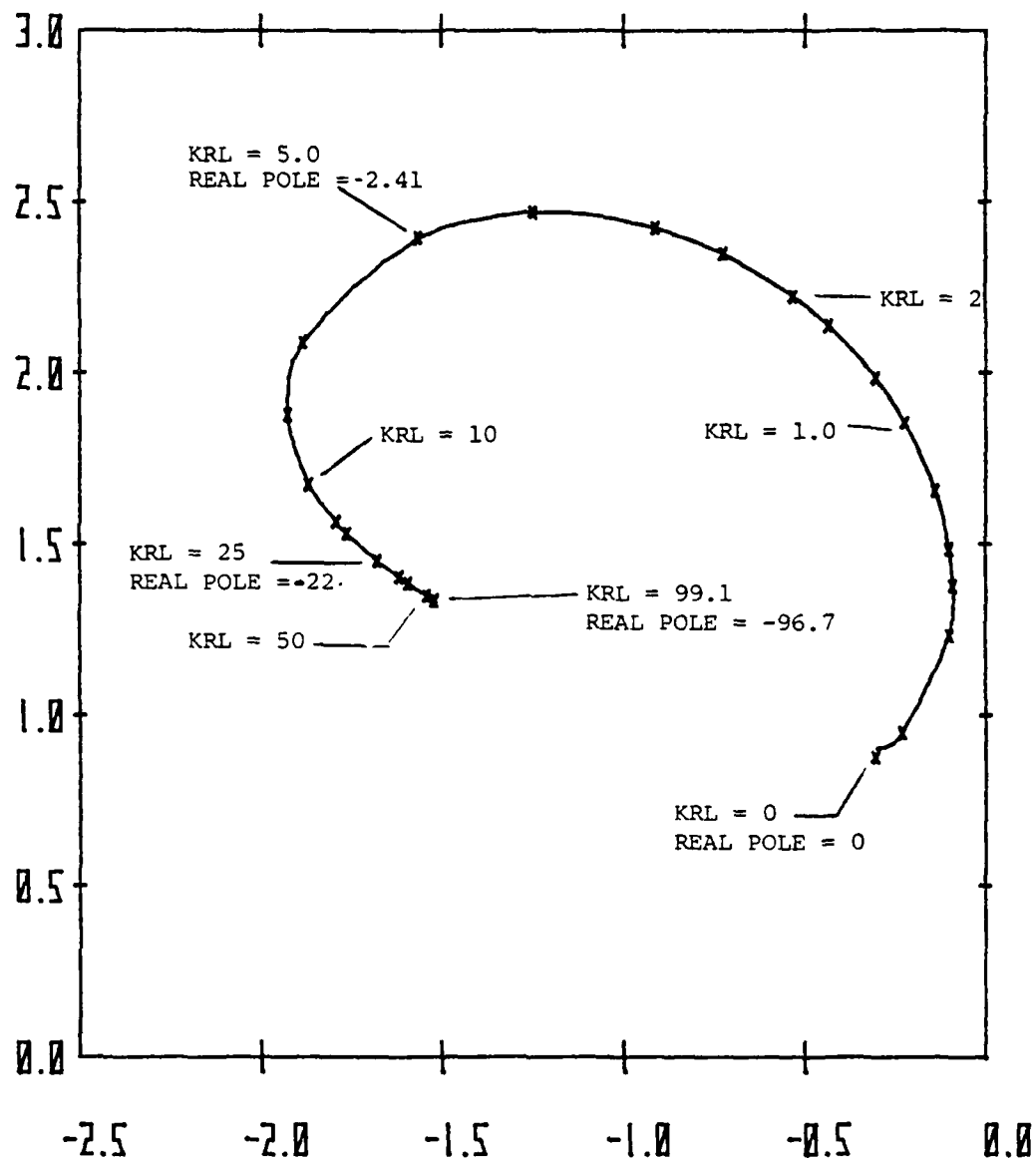
Figure 3-9 Root locus of the oscillatory pole of a SRFIMF controller. The open loop plant has a natural frequency of .95 rad/sec and damping ratio of 0.475.

58
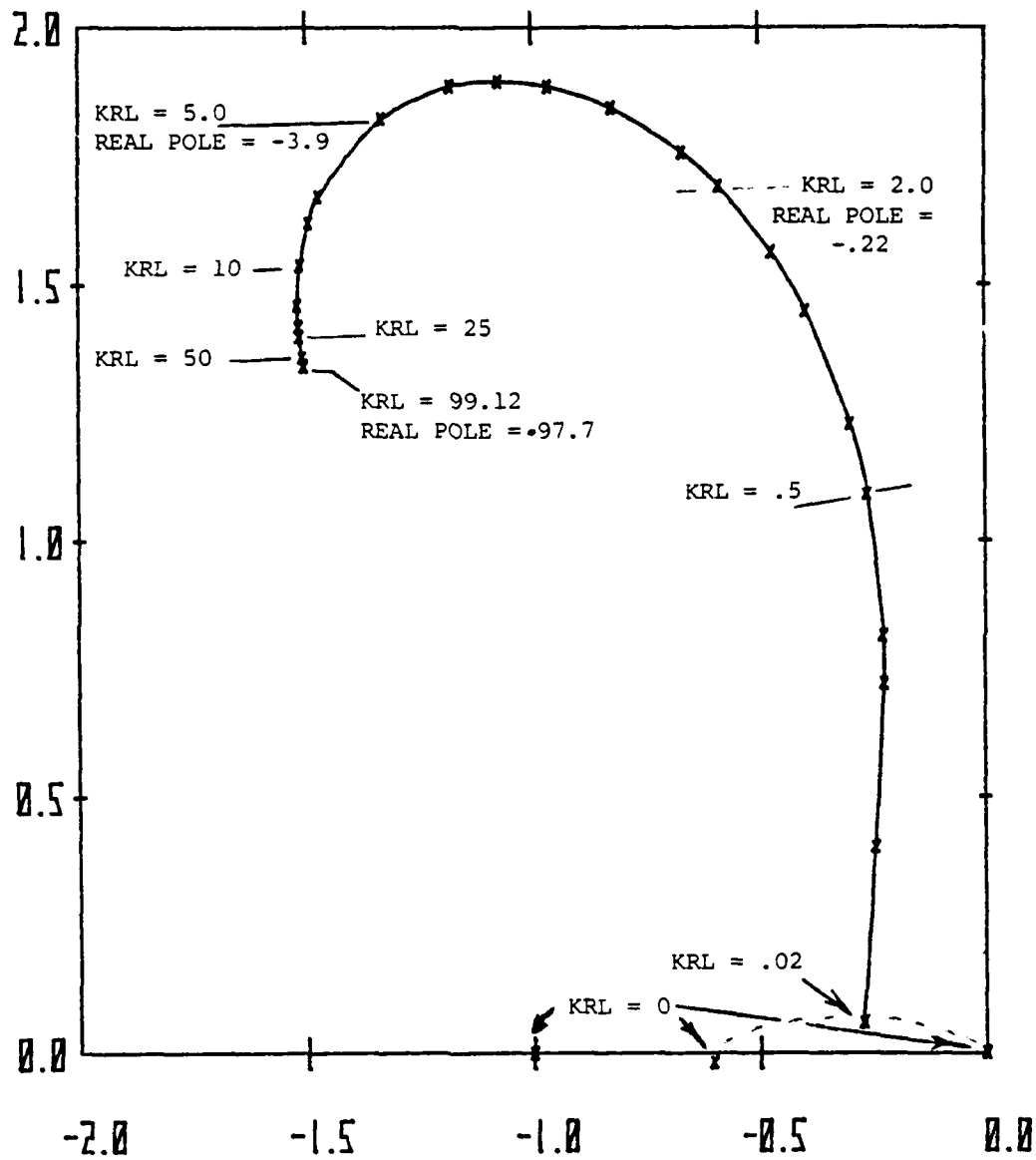
Figure 3-10 Root locus of the oscillatory pole of a SRFIMF position controller. The open loop plant has real poles at -1.0 and -.6.

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The root locus behavior of a more complex plant will be shown in section V where the SRFIMF controller will be applied to a transfer function of the Harrier aircraft. The system will next be simulated for each of the sample VSTOL aircraft modes listed in table 3-2.

D. SRFIMF SIMULATIONS

The simulated response of a plant under the control of SRFIMF position controller was done for all ten of the typical VSTOL aircraft modes listed in table 3-2. The time histories were generated using the CSMP program, reference [6]. The open loop plant for each of the ten systems was assumed to be of the form

$$G(s) = \frac{X(s)}{\delta(s)} = \frac{CPG}{s^2 + bs + c}$$

with CPG = 1 and b and c given in table 3-2.

The model was as assumed in section II, a second order with a natural frequency of 2 rad/sec and damping ratio, of 0.75. The constants $K_{\dot{x}}$ and K_x were 3 and 4 respectively as determined by equations 2-8 and 2-9. The simulation of the model response is shown in figure 3-9. The rise time of the modeled response is approximately 1.2 seconds. The model response has a slight overshot (2.9 percent) with a peak time at 2.4 seconds. Settling to within 1 percent occurs immediately after the peak response. For the simulations, KRL was taken to be 50 and the actuator time constant, τ, was 0.10 seconds. The

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input to the system and to the open loop plant was a unit step function, representing X_c, at $t = 0$. A sample listing of the CSMP source code is given in Appendix A.

Figures 3-11 to 3-21 show the time histories of the open and closed loop response of each of the ten simulated systems. It can be seen from the figures that the closed loop response of each system is indistinguisable from the model response. Figures 3-11 to 3-21 graphically show that the response of the closed loop system is approximated by the model response regardless of the plant being controlled. In addition, the self trimming feature of the controller is seen. In all cases the steady state value of the output of the closed loop system is the same as the input, X_c, namely 1.

TIME
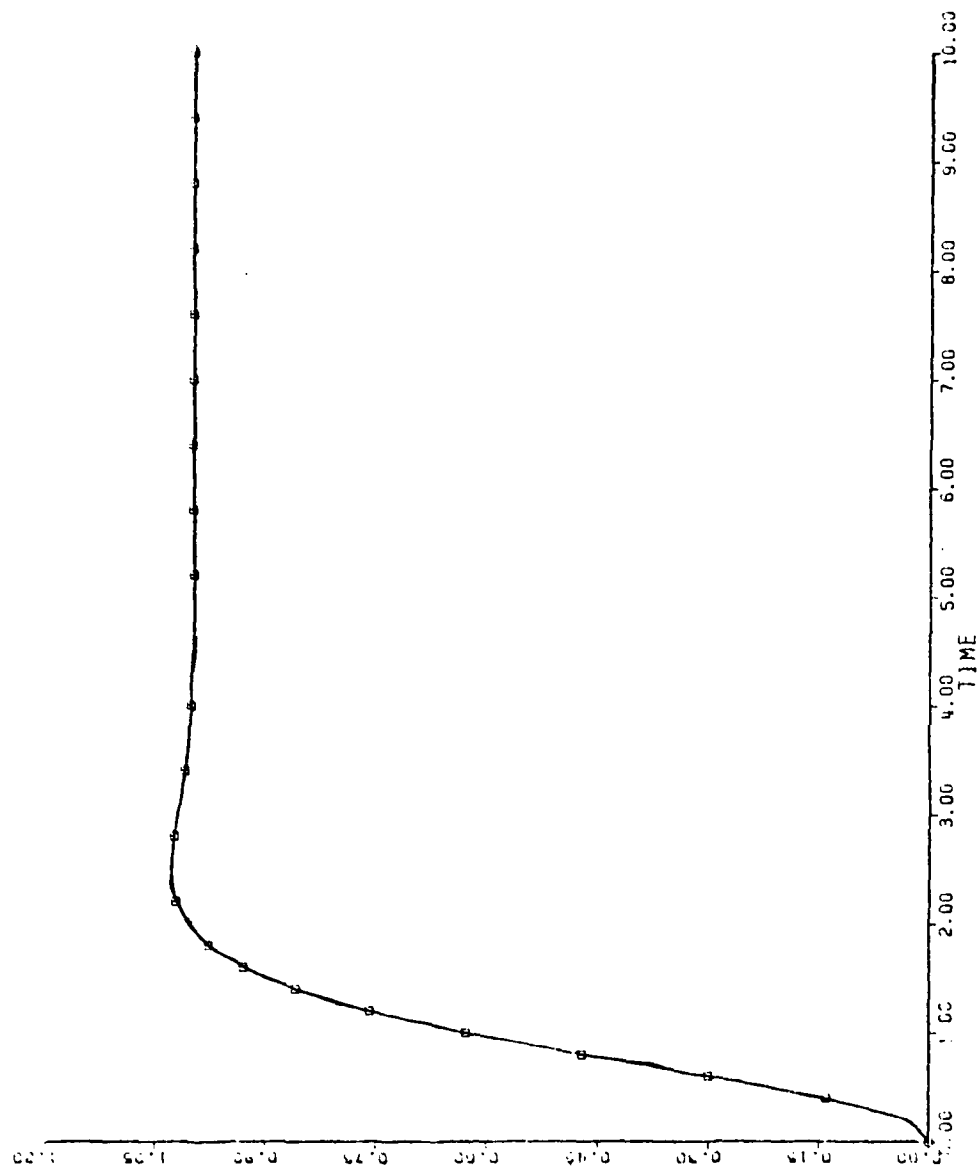
Figure 3-11 Perfect model response. $K_x = 4$, $K_{\dot{x}} = 3$, $\omega_n = 2$, $\zeta = 0.75$.

62
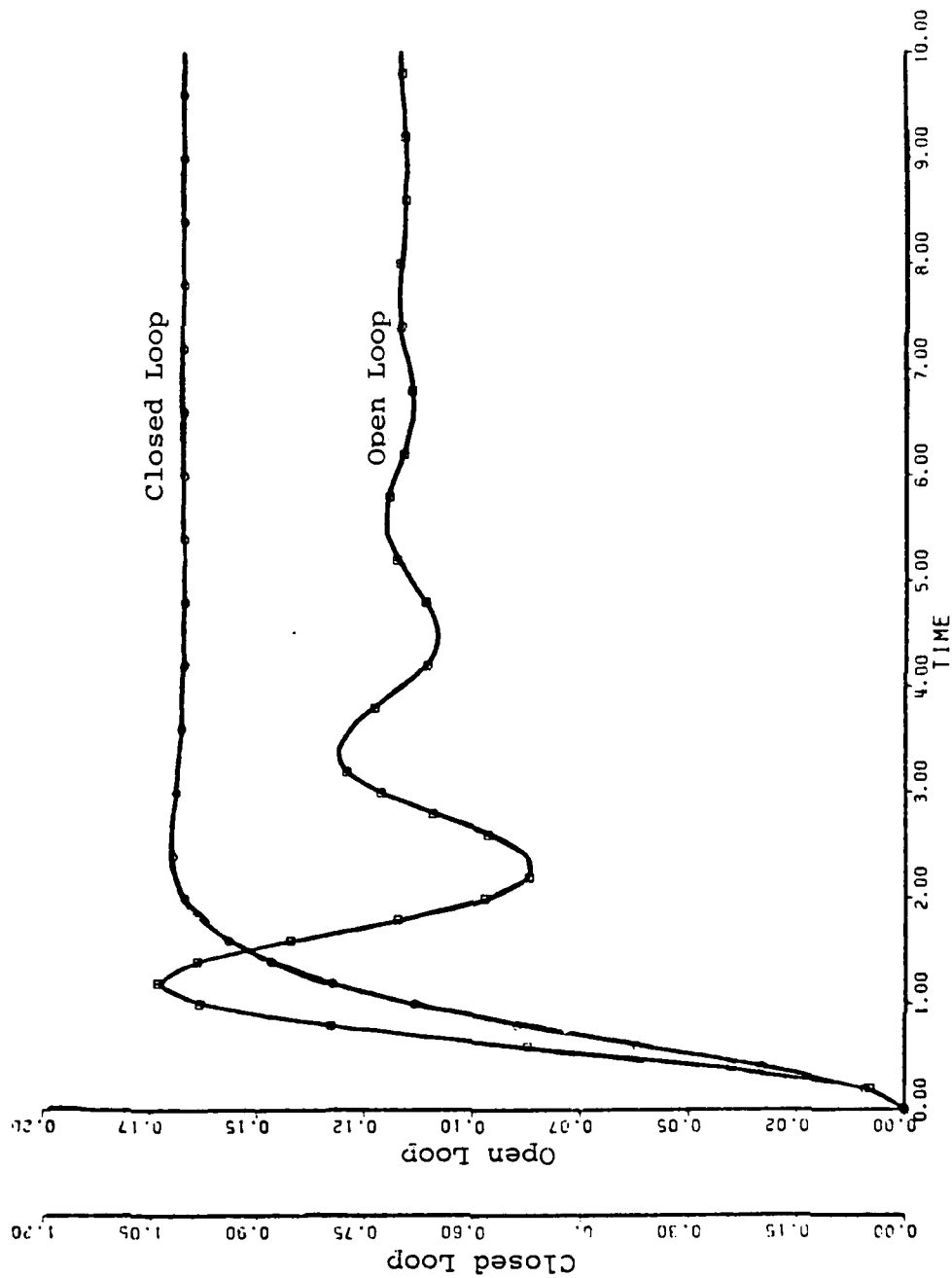
Figure 3-12 Simulation of SRFIMF position control for aircraft mode 1, table 3-3. The open loop plant has a natural frequency of 2.94 rad/sec and damping of .2.

63
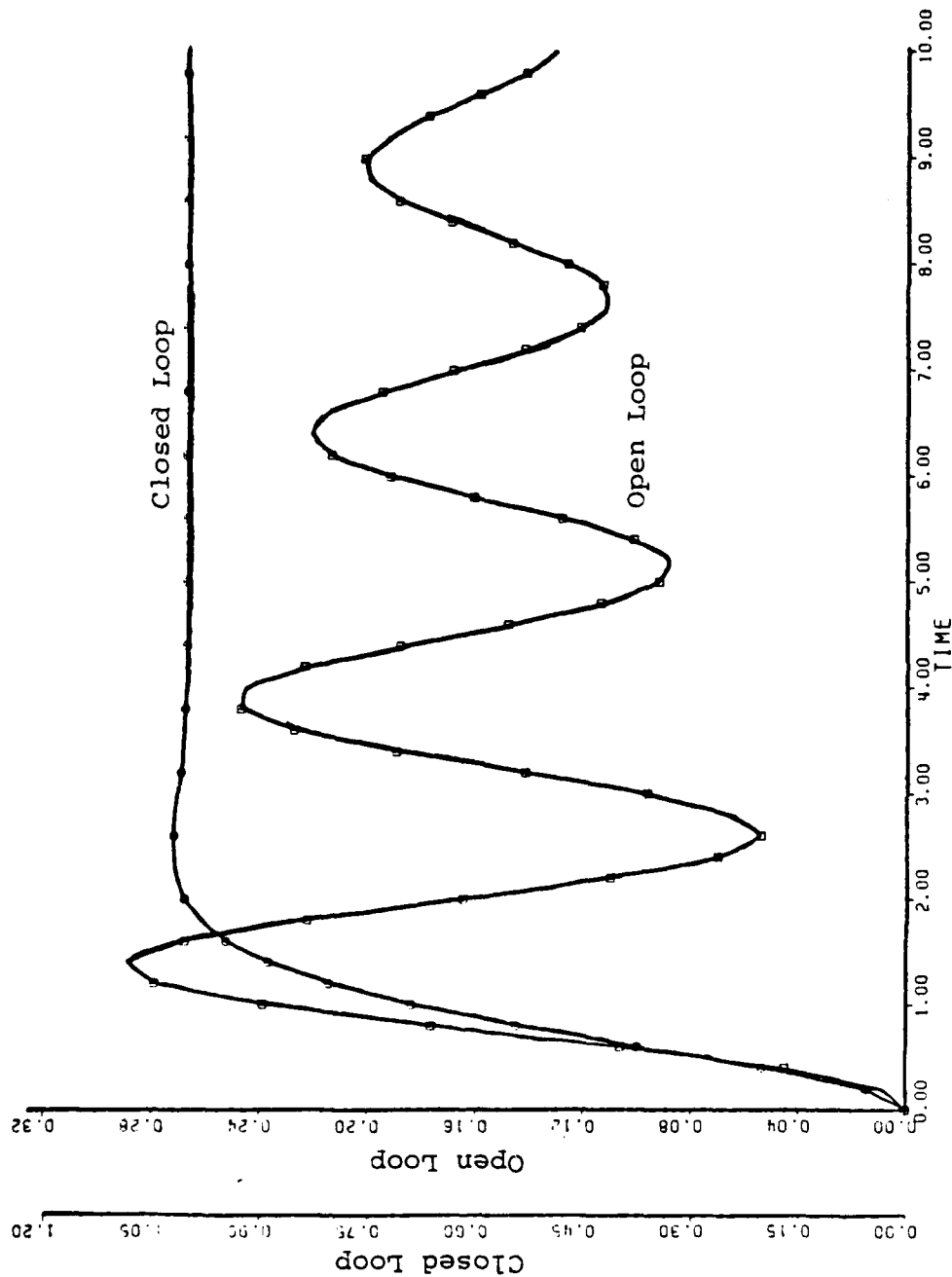
Figure 3-13 Simulation of SRFIMF position control for aircraft mode 2, table 3-3. The open loop plant has a natural frequency of 2.5 rad/sec and damping ratio of 0.06.

64
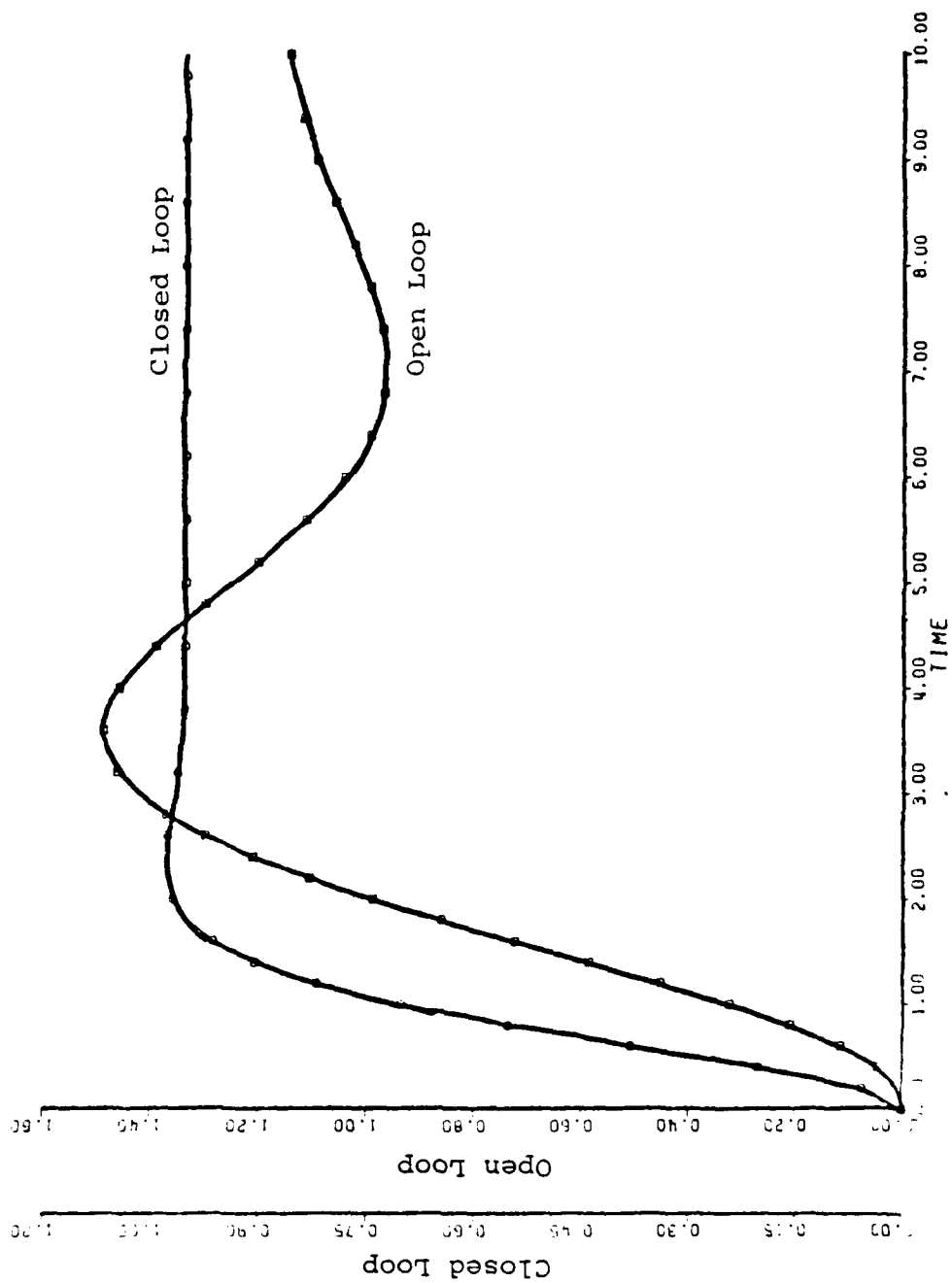
Figure 3-14 Simulation of SRFIMF position control for aircraft mode 3, table 3-3. The open loop plant has a natural frequency of 0.954 rad/sec and a damping ratio of 0.314.

65
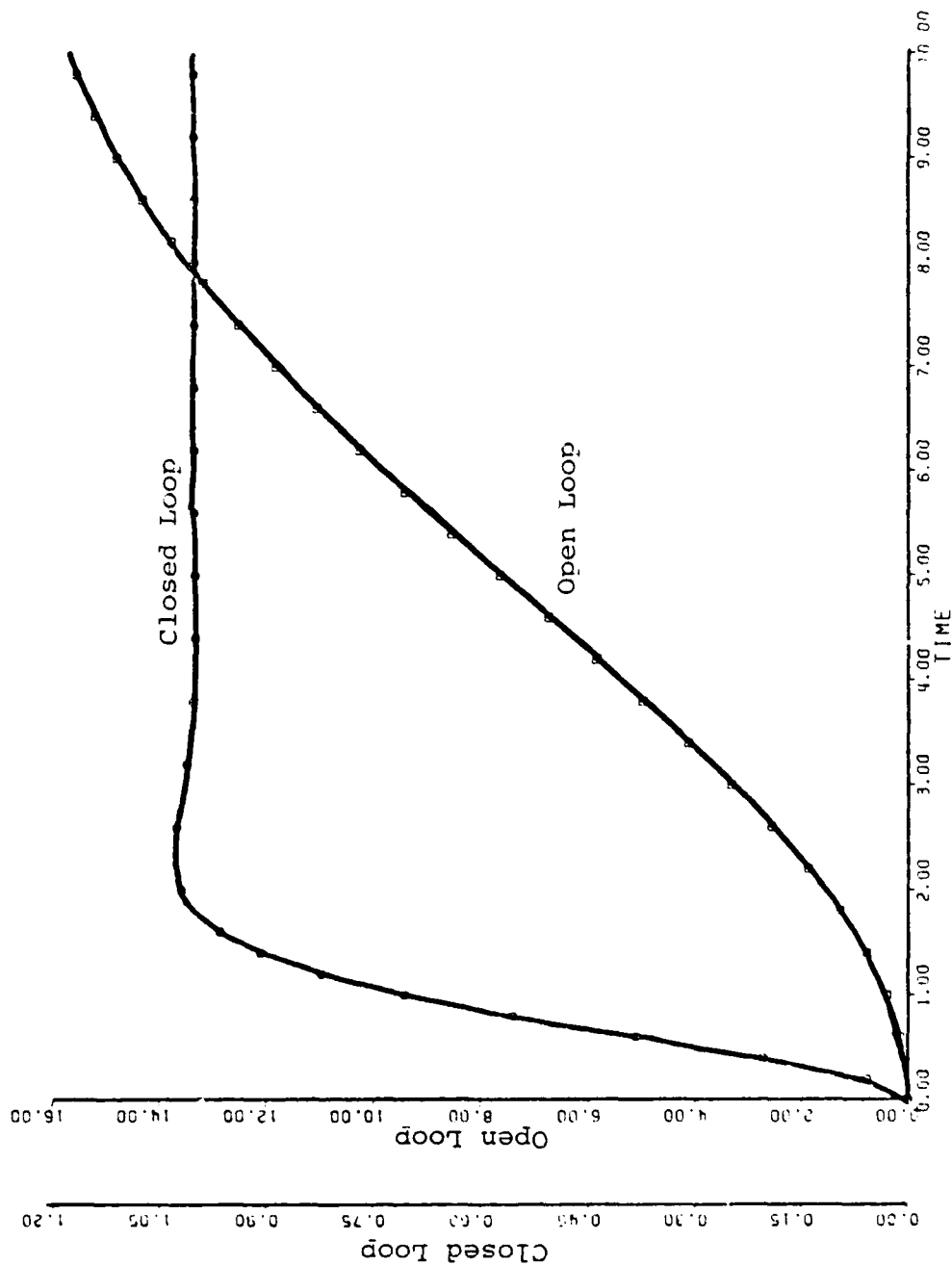
Figure 3-15 Simulation of SRFIMF position control for aircraft mode 4, table 3-3. The open loop plant has a natural frequency of 0.283 rad/sec and damping ratio of 0.353.

66
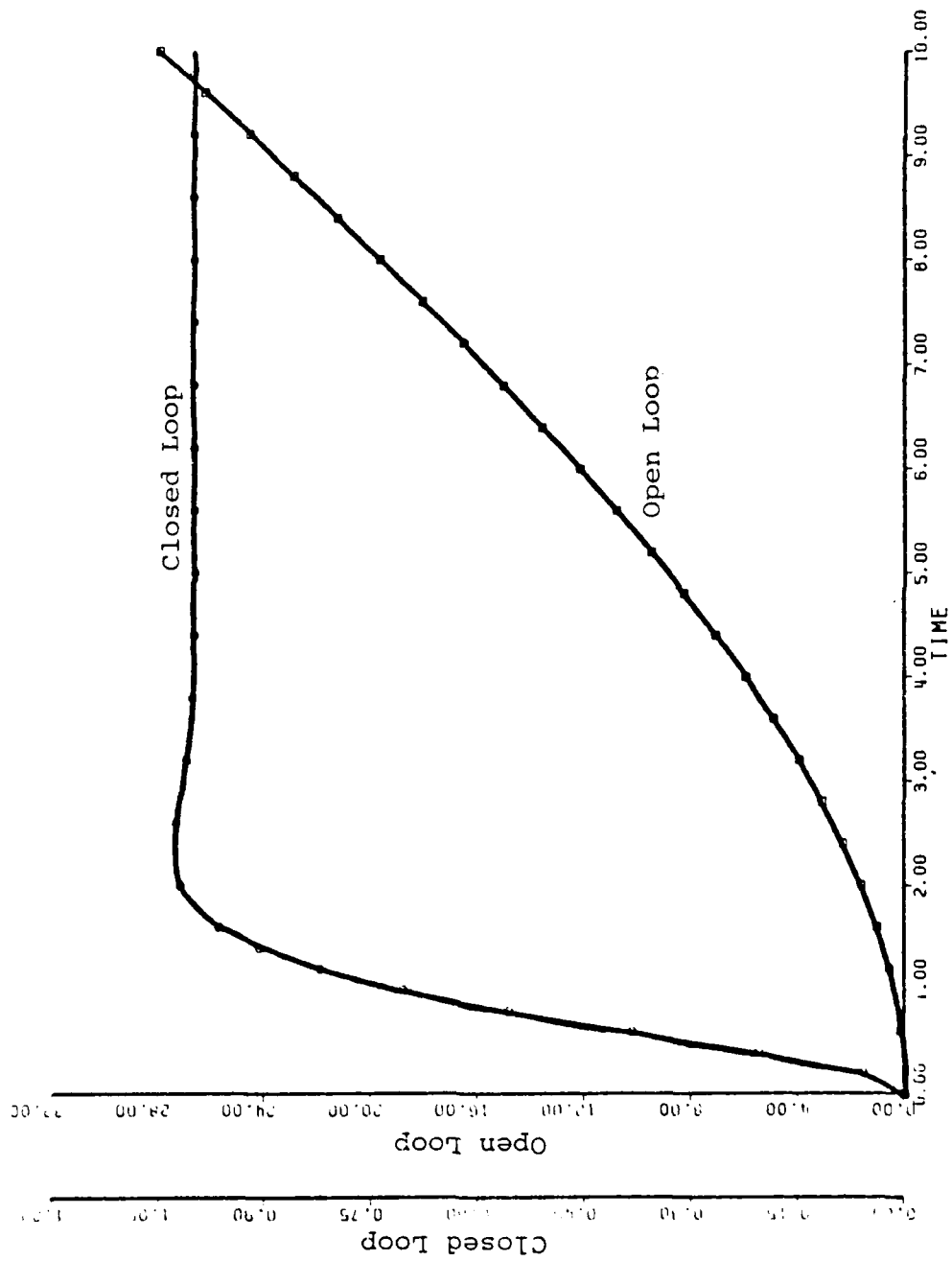
Figure 3-16 Simulation of SRFIMF position control for aircraft mode 5, table 3-3. The open loop plant has real poles at -0.2 and 0.

67
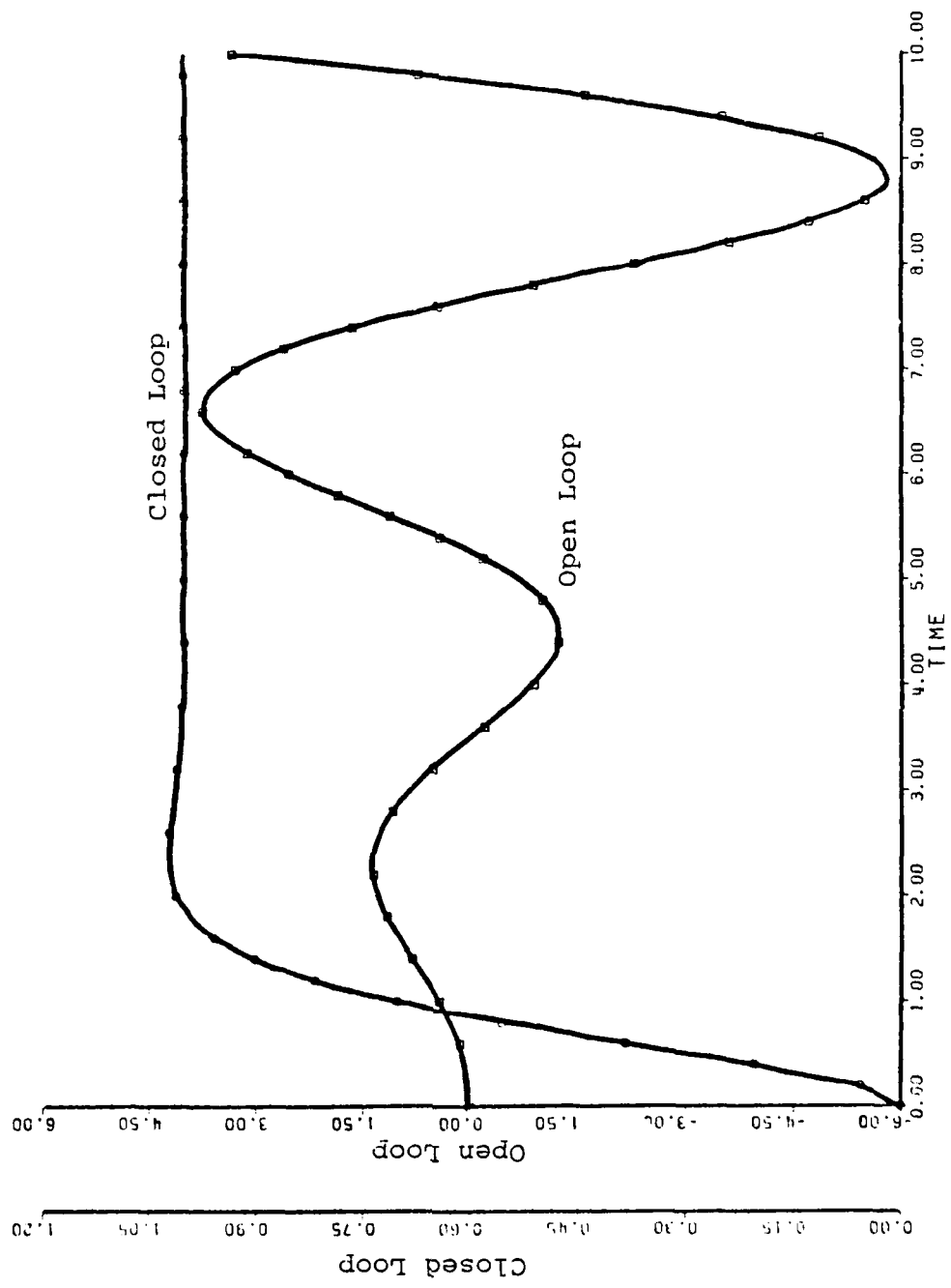
Figure 3-17 Simulation of SRFIMF position control for aircraft mode 6, table 3-3. The open loop plant has a natural frequency of 1.47 rad/sec and damping ratio of -.204.

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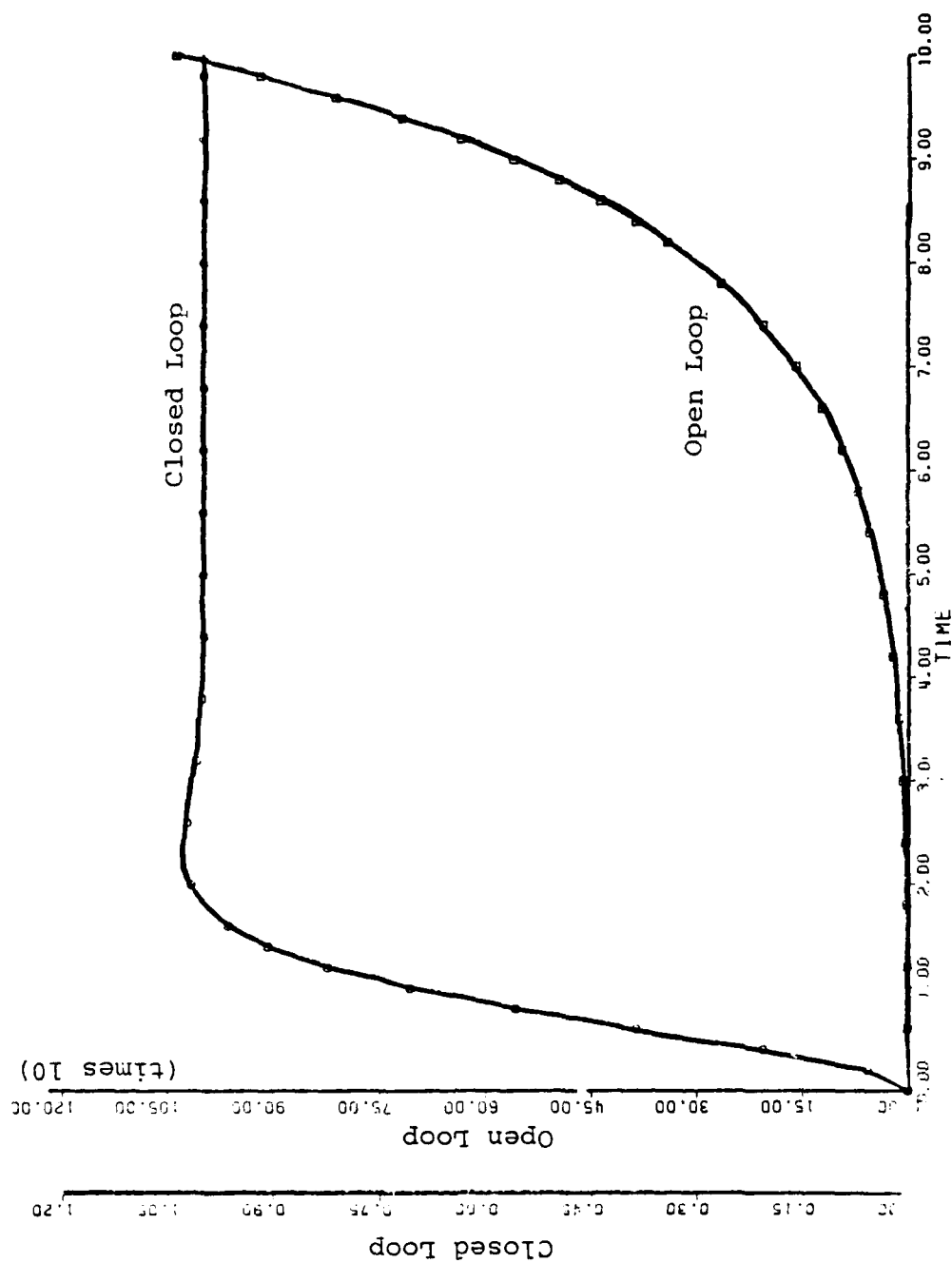
Figure 3-18 Simulation of SRFIMF position control for aircraft mode 7, table 3-3. The open loop plant has real poles at 0.30 and 0.

69
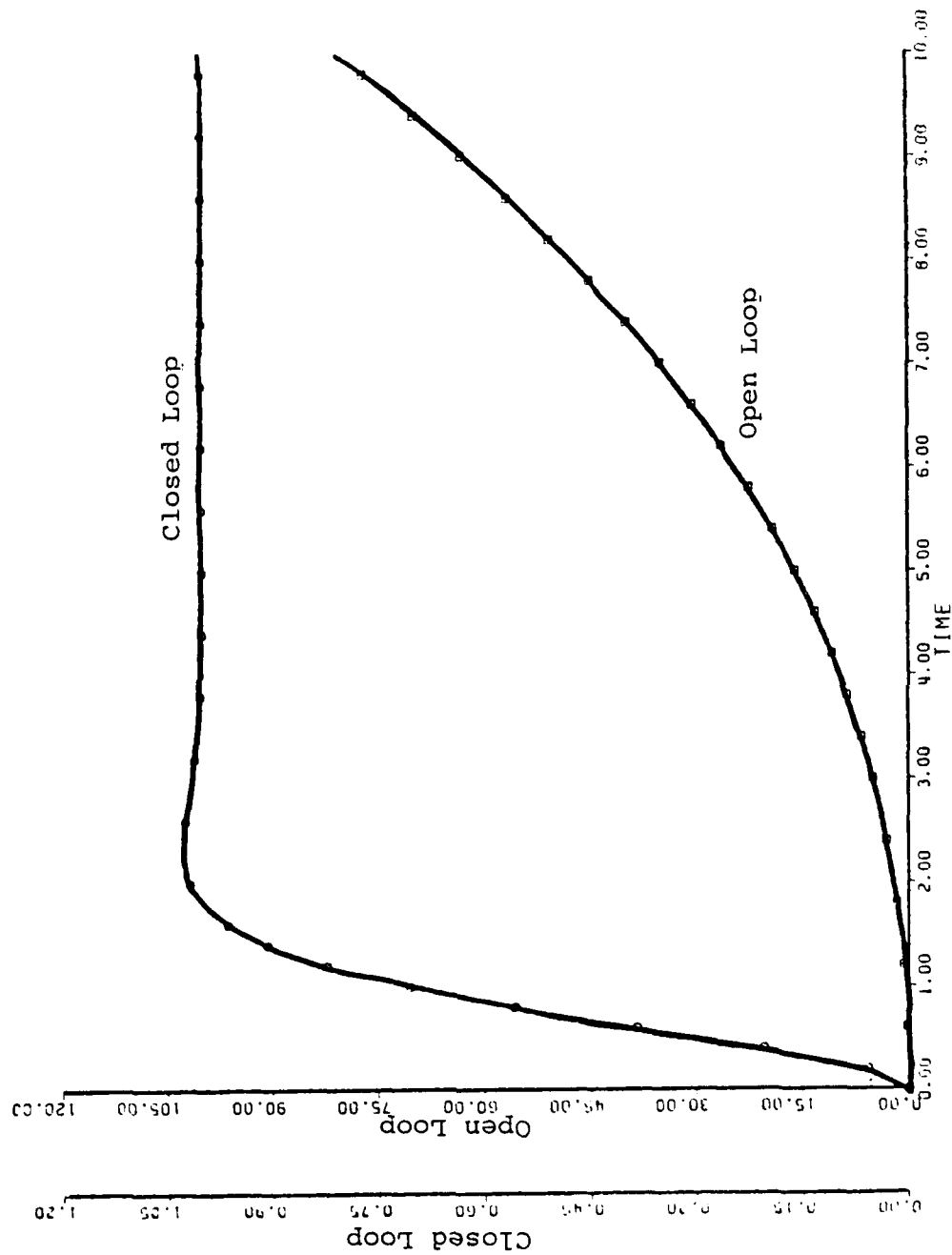
Figure 3-19 Simulation of SRFIMF position control for aircraft mode 8, table 3-3. The open loop plant has a natural frequency of 0.17 rad/sec and damping ratio of 0.590.

70
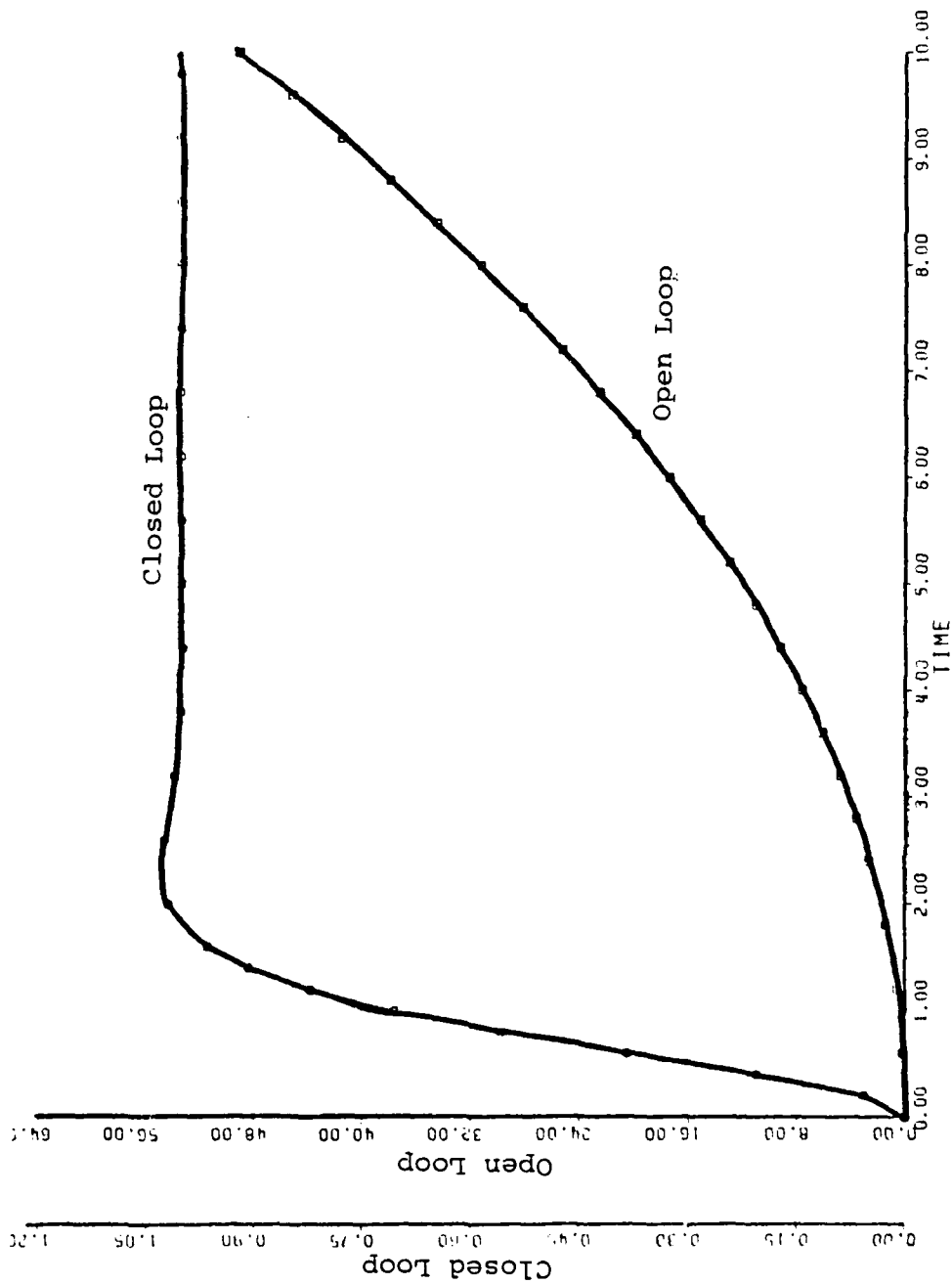
Figure 3-20 Simulation of SRFIMF position control of aircraft mode 9, table 3-3. The open loop plant has two poles at the origin.

71
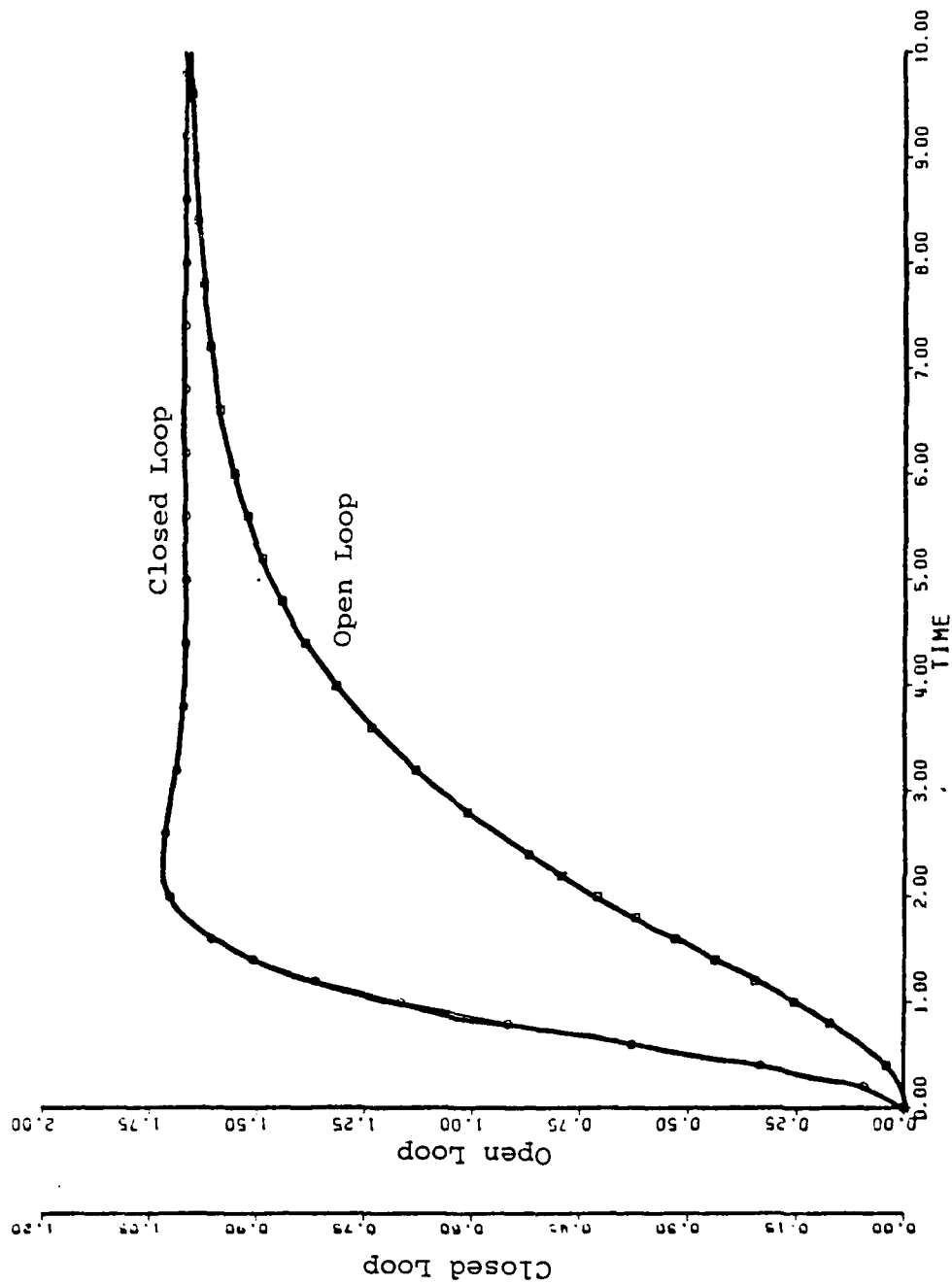
Figure 3-21 Simulation of SRFIMF position control of aircraft mode 10, table 3-3. The open loop plant has real poles at -1.0 and -0.6.

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IV.   SENSOR NOISE ANALYSIS

A.   SRFIMF STOCHASTIC MODEL

The SRFIMF controller relies upon measurements to produce model following. We shall now examine the effect of measurement uncertainty on the operation of the closed loop system. In order to accomplish this we will consider a position controller with a representative second order plant given by

$$G(s) = \frac{CPG}{s^2 + bs + c} \qquad\qquad 4\text{-}1$$

The position controller will be augmented with sensor noise sources and the observability and controllability of the closed loop system with these noise sources will be analyzed. Errors in the state variables will be determined by covariance analysis using the Lyapunov equation.

The measured quantities that are considered to be contaminated by sensor noise are the attitude position, attitude rate and attitude acceleration measurements. Two types of sensors will be considered. One is of high accuracy and typical of good quality inertial navigation system measurements. The second is of lower accuracy and might be considered typical of strapdown sensors. Sensor errors are assumed to be of two types, bias and high frequency. Bias errors are usually small relative to measured quantities and they are of constant value. Bias errors have little effect on the dynamic behavior of the control system and will not be considered in this study.

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High frequency errors can be modeled in various ways. It is assumed that the sensor noise sources can be modeled by exponentially correlated noise. Exponentially correlated noise is obtained by passing Gaussian white noise through a first order shaping filter. A first order filter is represented by the differential equation

$$\dot{\xi} = -1/T_r \xi + W_t(t) \qquad\qquad 4-2$$

where $W_t(t)$ is a scalar white, zero mean, Gaussian noise of constant strength, μ. μ is chosen so that the steady state value produced by the filter is the square of the sensor error standard deviation, σ^2. Following Maybeck [9], the required value of μ as the input to the filter is given by

$$\mu = 2\sigma^2/T_r \qquad\qquad 4-3$$

where T_r is the correlation time of the exponentially correlated noise. Figure 4-1 is a schematic representation of a typical shaping filter, in Laplace transform notation. $\xi(s)$ is resulting exponentially correlated noise.
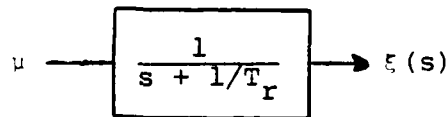
Figure 4-1 Exponentially correlated noise
shaping filter

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The standard deviation, σ, of the sensor error is a function of the sensors. The values of σ and T_r were obtained from a study by Analytical Mechanics Association (AMA), reference [10], in which an aircraft using a SRFIMF controller was studied. Table 4-1 is a list of the standard deviation estimates which were obtained from the AMA study. Also based upon the AMA study, we assume a value of T_r to be 10 seconds for both sensor models.

	Strapdown	Inertial
Attitude position, σ_p	1°	$0.1^{\circ}/$
Attitude rate, σ_v	$0.5^{\circ}/sec$	$0.5^{\circ}/sec$
Attitude acceleration, σ_a	$1^{\circ}/sec^2$	$0.1^{\circ}/s^2$

$$(T_r = 10 \text{ seconds in all cases})$$

Table 4-1 Sensor error model parameters

The position controller shown in figure 2-4 is augmented with the sensor error sources and the resulting system is shown in figure 4-2.

To analyze the closed loop controller, we first obtain the state space representation of the plant given by equation 4-1. Taking position and velocity as the state variable of the plant we write

Position	$= X_1$	4-4a
Velocity	$= \dot{X}_1 = X_2$	4-4b
Acceleration	$= \dot{X}_2 = -cX_1 - bX_2 + CPGX_3$	4-4c
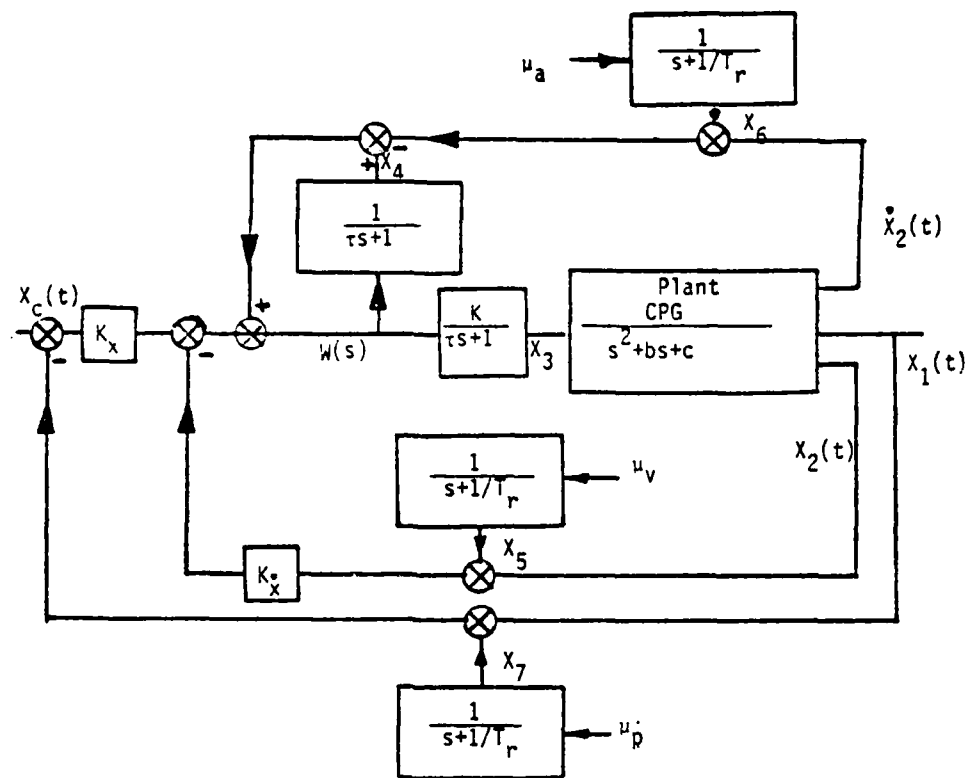
Figure 4-2 High frequency error model,
position controller

The additional state variables required for analysis of the

controller as shown in figure 4-2 are

X_3 = output of the actuator which is also input to
the plant

X_4 = output of the actuator compensator

X_5 = state variable representing the error in the
velocity measurement

X_6 = state variable representing the error in
acceleration measurement

X_7 = state variable representing the error in
position measurement

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From the figure we can write the control law W(s), with $X_c = 0$ as

$$W(t) = -K_x(X_1 + X_7) - K_{\dot{x}}(X_2 + X_5) - (-cX_1 - bX_2 + CPGX_3 + X_6) + X_4$$

$$= (-K_x + c)X_1 + (-K_{\dot{x}} + b)X_2 - CPGX_3 + X_4 - K_{\dot{x}}X_5 - X_6 - K_xX_7 \qquad 4-5$$

The remaining first order differential equations, obtained from figure 4-1, are given as

$$\dot{X}_3 = -1/\tau X_3 + K/\tau W(t) \qquad \text{4-6a}$$

$$\dot{X}_4 = -1/\tau X_4 + 1/\tau W(t) \qquad \text{4-6b}$$

$$\dot{X}_5 = -1/T_r X_5 + 2\sigma_v^2/T_r \qquad \text{4-6c}$$

$$\dot{X}_6 = -1/T_r X_6 + 2\sigma_a^2/T_r \qquad \text{4-6d}$$

$$\dot{X}_7 = -1/T_r X_7 + 2\sigma_p^2/T_r \qquad \text{4-6e}$$

Combining equation 4-4, 4-5 and 4-6 and expressing the system in matrix form,* we have the closed loop system whose inputs are the random sensor noise quantities given by

$$X = [A] \, X + [B][Q]$$

$$Y = [C] \, X \qquad 4-7$$

where σ_p, σ_v, and σ_a are the standard deviation of the position measurement error of the velocity measurement error and of acceleration measurement error respectively and $[Q]$ is the matrix of input white noise powers given by

$$Q_{ii} = \mu_i = 2\sigma_i^2/T_r$$

*Matricies A, B, Q and C are shown on following page.

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$$A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
(c - K_x)\dfrac{K}{\tau} & (b - K_x)\dfrac{K}{\tau} & -(CPG\cdot K + 1)/\tau & \dfrac{K}{\tau} & -\dfrac{K_x\cdot K}{\tau} & -\dfrac{K}{\tau} & -\dfrac{K\cdot K_x}{\tau} \\
(c - K_x)/\tau & (b - K_x)/\tau & -CPG/\tau & 0 & -\dfrac{K_x}{\tau} & -\dfrac{1}{\tau} & -\dfrac{K_x}{\tau} \\
0 & 0 & 0 & 0 & -1/Tr & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1/Tr & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1/Tr
\end{bmatrix}$$

$$B = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}$$

$$C = \begin{bmatrix}
-c & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & -b & 0 & 0 & 0 & 1 & 0 \\
-c & -b & CPG & 0 & 0 & 1 & 0
\end{bmatrix}$$

$$Q = \begin{bmatrix}
2\sigma_v/Tr & 0 & 0 \\
0 & 2\sigma_a/Tr & 0 \\
0 & 0 & 2\sigma_p/Tr
\end{bmatrix}$$

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Having the matrix representation of the closed loop controller being driven by the sensor noise inputs, we can analyze the effect of the sensor error by considering the controllability and observability of the system with respect to the noise inputs and outputs of position, velocity and acceleration.

The linear control system computer programs, developed by Melsa and Jones [7], were first used with the result that the system is both observable and controllable. In order to gain a better understanding of the controllability and observability of the system, a second approach, following a method suggested by Bryson [11], was used. In the later method we decouple the system of equations by diagonalizing the system given by equation 4-7. This produces a system of equations in modal coordinates. The diagonalization procedure requires that we calculate the eigenvalues and eigenvectors of the A matrix. The transformation matrix, T, from the original coordinates to the new coordinates, is obtained by normalizing, in a complex sense, the matrix of eigenvectors. The matrix quantities corresponding to the modal coordinates are denoted with a superscript, *. The system, in modal coordinates is expressed as

$$X^* = A^*X^* + B^*Q \qquad \qquad 4\text{-}8a$$
$$Y = C^*X^* \qquad \qquad 4\text{-}8b$$

and the transformation is as follows

$$X^* = T^{-1}X \qquad \qquad 4\text{-}9a$$
$$A^* = T^{-1}AT \qquad \qquad 4\text{-}9b$$
$$B^* = T^{-1}B \qquad \qquad 4\text{-}9c$$
$$C^* = CT \qquad \qquad 4\text{-}9d$$

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We will now apply the above coordinate transformation to examine the observability and controllability of the system when the plant is a specific second order system.

For this example we chose one of the representative VSTOL second order plants given by mode number 6 of table 3-2. The transfer function for this plant is given by

$$G(s) = \frac{1}{s^2 + .6s + 2.16}$$

4-10

We will assume the following additional parameters

$$\tau = .1 \text{ sec, } K_x = 4, \ K_x = 3, \ K = 5, \ T_r = 10 \text{ sec}$$

with these choices the gain parameter KRL is

$$KRL = K \ CPG/\tau = 50$$

Because the system is linear with respect to the noise inputs we can solve the problem in general assuming a unit value for the individual noise standard deviations, or

$$\sigma_p = \sigma_v = \sigma_a = 1 \quad .$$

where σ_p, σ_v and σ_a is the standard deviation of the position, velocity and acceleration measurement error. The original system and the resulting diagonalized system was determined by a fortran computer code called MODAL, listed in Appendix A. The results are given

$$X = AX + BQ \qquad\qquad X^* = A^*X^* + B^*Q$$
$$Y = CX \qquad\qquad\qquad Y = C^*X^*$$

Matricies A, B, C, A*, B* and C* are shown on the following pages.

$$A = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -2.16 & -0.6 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -92.0 & -120.0 & -60.0 & 50.0 & -150.0 & -50.0 & -200.0 \\ -18.4 & -24.0 & -10.0 & 0.0 & -30.0 & -10.0 & -40.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 1.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ -2.16 & -0.6 & 1.0 & 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

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$$A^* = \begin{bmatrix}
-1.56 & 1.34 & 0.0 & -0.0 & 0.0 & -0.0 & 0.0 \\
-1.34 & -1.56 & -0.0 & 0.0 & -0.0 & -0.0 & -0.0 \\
-0.0 & -0.0 & -47.48 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.0 & 0.0 & -0.0 & -10.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1
\end{bmatrix}$$

$$B^* = \begin{array}{ccc}
Q_v & Q_a & Q_p \\
\begin{bmatrix}
15.48 & 5.16 & 20.64 \\
17.91 & 5.97 & 23.88 \\
-3.41 & -1.14 & -4.54 \\
0.0 & 0.0 & 0.0 \\
2.17 & 0.0 & 0.0 \\
0.0 & 1.19 & 0.0 \\
0.0 & 0.0 & 2.75
\end{bmatrix}
\end{array}$$

$$C^* = \begin{array}{ccccccc}
 & & & & x_v^* & x_a^* & x_p^* \\
\begin{bmatrix}
0.05 & 0.0 & -0.0 & 0.01 & -0.87 & -0.23 & -0.03 \\
0.08 & 0.07 & 0.02 & -0.08 & 0.5 & 0.02 & 0.04 \\
0.03 & -0.22 & -0.99 & 0.82 & -0.0 & 0.84 & -0.0
\end{bmatrix}
\end{array}$$

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It is instructive to observe the A matrix of the system. In the upper right corner, rows and columns 1 and 2 represent the plant. Row 3 is derived from the controller terms, $K \cdot W(t)/\tau$. These terms dominate the matrix.

The diagonal elements of A* are the eigenvalues of the original system. All of the eigenvalues are negative, indicating that the system is stable. The system has one coupled oscillatory mode, -1.56, 1.34 in the upper right corner of A*. This mode corresponds to the model and will be referred to as the model mode throughout the remainder of this study. The eigenvalue -47.48, row 3 column 3 of A*, is the mode which represents the output of the controller. We refer to this as the controller mode. It physically represents the real pole whose location corresponds to the value of KRL as discussed in the root locus analysis of section III-C. Similarly the eigenvalue -10.00, row 4 column 4 of A*, is the compensator mode. The other three eigenvalues correspond to the noise filter time constant, $-1/T_r$. From the transformed system, the controllability and observability of each of the previously mentioned modes with respect to the noise input can be determined.

In the B* matrix the Q_v above the first column indicates that column 1 is the input vector corresponding to velocity error measurement input. The other columns are annotated similarly. The first two rows of the B* matrix indicate that the model and controller modes are affected by all three

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measurement errors and that acceleration measurement error is least significant. The zeroes in row 4 of the B* matrix indicate that the compensator mode is unaffected by measurement noise. This indicates that the compensator does not contribute to the uncertainty of the system.

The C* matrix has been annotated in a way similar to B* so the X_v^*, X_a^*, and X_p^* correspond to the velocity, acceleration, and position error modes respectively. From the C* matrix it is seen that the observation of position, corresponding to the first row of the matrix, is affected by all three measurement errors but that velocity and acceleration are only slightly affected. We conclude from the C* matrix that measurement errors will affect the position, which is the quantity being controlled, but that the dynamic behavior, velocity and acceleration are only slightly affected. This result is perhaps due to the low frequency error model which results from the choise of T_r equal to 10 seconds.

From the modal analysis it is concluded that sensor errors can affect the model and controller mode of the closed loop system and that the errors can be observed in the measurement of the position. The exact relationship between the output quantities of position, velocity and acceleration will be determined in the next section where the variances of state variable X_1, X_2, and X_3 will be found by solving the Lyapunov equation for the system.

B. COVARIANCE ANALYSIS

The object of the covariance analysis is to determine the standard deviations of the error in the states X_1, X_2, and X_3 of the system developed in the previous section. The standard deviation of X_1 and X_2 are of interest because they represent position and velocity tracking accuracy. X_3 is of interest because it is the input to the plant. A large deviation in X_3 due to measurement error would mean unnecessarily high amounts of control energy lost because of measurement errors. The state variable representation of the system developed in section 4-1, where the inputs to the system are noise sources representing position, velocity and acceleration measurement error, will again be used.

The problem of determining the covariance of the states of a system in the presence of disturbances is typically encountered in the design of optimal estimators, the Kalman filter. In the estimator problem the system is expressed in the following way

$$\dot{X} = AX + BU + GW_t(t) \qquad\qquad 4-11$$

where A is the matrix representation of the plant, B the control input matrix, $W_t(t)$ white noise disturbance to the plant states with

$$E[W_t(t)] = 0 \ , \qquad E[W_t(t), W_t^T(t)] = \mu$$

and G the input matrix of the disturbances. In estimation problems one assumes that measurements of the system are made

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which can be expressed in terms of the states of the system
by the relationship

$$\underline{Z}_N = H\underline{X} + \underline{V}_N$$

Here N quantities are measured with an expected error of V_N.

In the developement of the SRFIMF controller, sensor
noise was defined as a state of the system and the A matrix
was augmented to include these states. The covariance analysis
assumes quiescent operation of the controller, therefore, the
only input to the system is white noise of strength, μ, given
by equation 4-3. The white noise acts only to disturb the
sensor error states. The governing equation for the covariance
of the states is given by the Lyapunov differential equation

$$P = AP + PA^T + GQG^T \qquad\qquad 4\text{-}12$$

where P is the matrix of the state variable covariances defined
as

$$P_{ij} = \sigma_i \sigma_j$$

and Q is as before, the matrix of white noise input whose power
is μ. We have chosen to rename the matrix G of equation 4-11
to B because the white noise sources are considered to be the
inputs to the system.

Again a unit value of standard deviations was assumed and,
because the system is linear with respect to the noise inputs,
we can apply later the set of sensors. We will consider the

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same plant as used in section 4-1. The transfer function
for the plant is given by

$$\frac{G(s)}{U(s)} = \frac{CPG}{s^2 + .6s + 2.16}$$

<div align="right">4-13</div>

At time equal to zero no uncertainties exist in the system,
therefore the initial condition on P is zero.

A computer program, Vary, listed in Appendix A, was
developed to solve the Lyapunov equation. The program uses
an International Mathematical and Statistical Library (IMSL)
subroutine called DVERK. This subroutine solves a system of
first order differential equations using a Runge-Kutta
Algorithm. The resulting covariance matrix, P, is printed by
the program as a function of time, at a number of discrete
times. The diagonal elements of the P matrix are interpreted
as the square of the standard deviation or RMS value (σ^2_{ii}) of
each of the state variable uncertainties as a result of the
noise input. The data obtained from the computation of the P
matrix is plotted in figures 4-3 to 4-5 and tabulated in
Appendix B, table B-1. The figures show the standard deviation
(sigma) of the states X_1, X_2 and X_3 as functions of time from
zero to ten seconds. When necessary, the steady state value
is shown by a "+" on the figure.

Figures 4-3 to 4-5 can be interpreted in the following
way. Each individual curve represents the contribution to the
total state variable error as a result of one measurement

source. For example, in figure 4-3 we see that a unit value
of standard deviation error in position measurement results
in a standard deviation error in X_1 of approximately 0.8° at
six seconds, and a steady state error of 0.98°. The total
expected value of the uncertainty in X_1 is the sum of the
errors resulting from position, velocity and acceleration
measurements as given by

$$\sigma_i = \sigma_{ip} + \sigma_{iv} + \sigma_{ia} \qquad 4\text{-}14$$

Where the notation σ_{ij} refers to the ith state variable and
j refers to a noise source. Thus, σ_{ip} refers to the standard
deviation of X_1 as a result of position measurement error.
Table 4-2 summarizes the steady state values of the components
of the state variable errors.

	X_1	X_2	X_3
Position measurement error	.980	.360	2.13
Velocity measurement error	.735	.270	1.60
Acceleration measurement error	.245	.089	0.53

Table 4-2 Steady state errors as a result of
measurement errors

The actual measurements are made either by inertial
navigation sensors or by strapdown sensors whose standard
deviations are given by table 4-1. We can now apply the
results obtained from the covariance analysis to obtain an
estimate of the state variable errors for actual measurement

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cases. For example, the steady state standard deviation of the error in X_1 when the measurements are made by an inertial system is given as

$$\sigma_1 = q_p\sigma_{1p} + q_v\sigma_{1v} + q_a\sigma_{1a} \qquad\qquad 4\text{-}15$$

where q_p, q_v and q_a are the standard deviations of the inertial sensor measurements given by table 4-1 as

$$q_p = 0.1^{\circ}$$
$$q_v = 0.5^{\circ}/sec$$
$$q_a = 0.1^{\circ}/sec^2$$

and σ_{1p}, σ_{1v} and σ_{1a} are given in table 4-2. The resulting expected error in X_1 in steady state is found from equation 4-15 to be 0.49°. In a similar way, the steady state errors in X_1, X_2 and X_3 for both inertial measurements and strapdown measurements can be evaluated. The results are shown in table 4-3.

	X_1	X_2	X_3
Inertial sensors	0.49°	$0.18^{\circ}/sec$	$1.07^{\circ}/sec^2$
Strapdown sensors	1.6°	$0.58^{\circ}/sec$	$3.46^{\circ}/sec^2$

Table 4-3 Steady state tracking errors produced
by strapdown and inertial navigation sensors

The result of the covariance analysis indicates that the expected uncertainity in X_1 and X_2 as a result of measurement errors are not significant. For example, in the case of strapdown sensors, the expected error in position, X_1, is 1.6°

while measurement errors had standard deviation values of 1°, $0.5^o/sec$ and $1^o/sec^2$.

To interpret the significance of the expected value of the error in X_3, the value of X_3 will be determined when a 1° input is applied to the controller. Refer to figure 4-2 and, for this example, disregard the effect of the control actuator at the input to the plant. Assuming that initially the controller is in steady operation with feedbacks equal to zero, at the instant the 1° input is applied the value of X_3 is determined to be

$$X_3 = X_c \cdot K_x \cdot K = 20 \qquad\qquad 4\text{-}16$$

Comparing the value of the standard deviation in X_3 to the value of X_3 when a 1° unit step input is applied to the controller, we conclude that the value of the standard deviation error in X_3 is only 17.3 percent of the value of X_3 when a 1° input is applied to the system.

From both the modal analysis and the variance analysis it can be concluded that high quality acceleration measurements are not necessary and that errors in the measurement of position and velocity are not amplified. The errors in the states are most affected by position measurement error. This result might have been expected because the controller is attempting to track to the commanded position and errors in the position measurement should dominate the uncertainty in the other states. Figures 4-3 thru 4-5 show that the uncertainty in the states reaches a
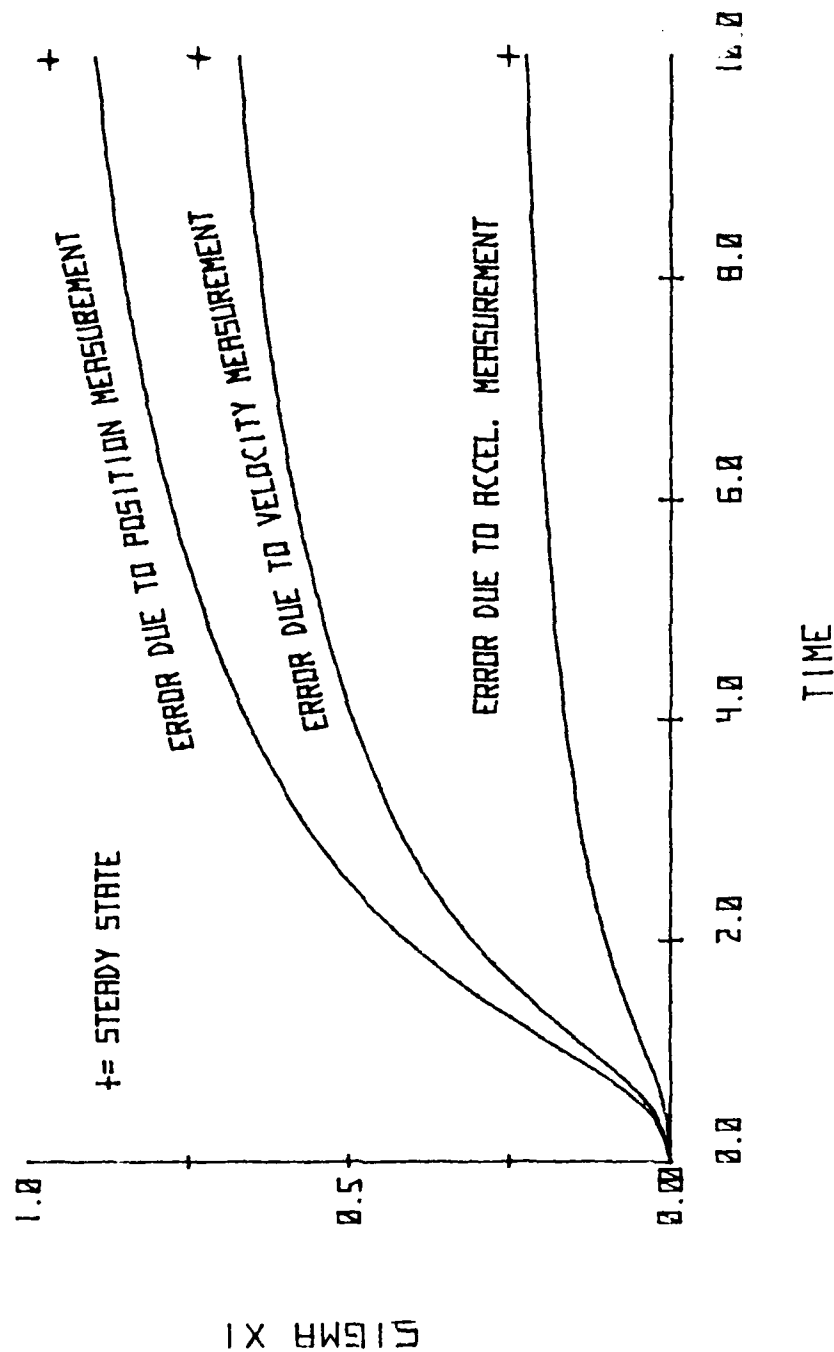
Figure 4-3 Tracking errors in X_1 as a result of measurement error in position, velocity and acceleration.

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SIGMA X2 VS. TIME

ERROR DUE TO POSITION MEASUREMENT

ERROR DUE TO VELOCITY MEASUREMENT

ERROR DUE TO ACCEL. MEASUREMENT
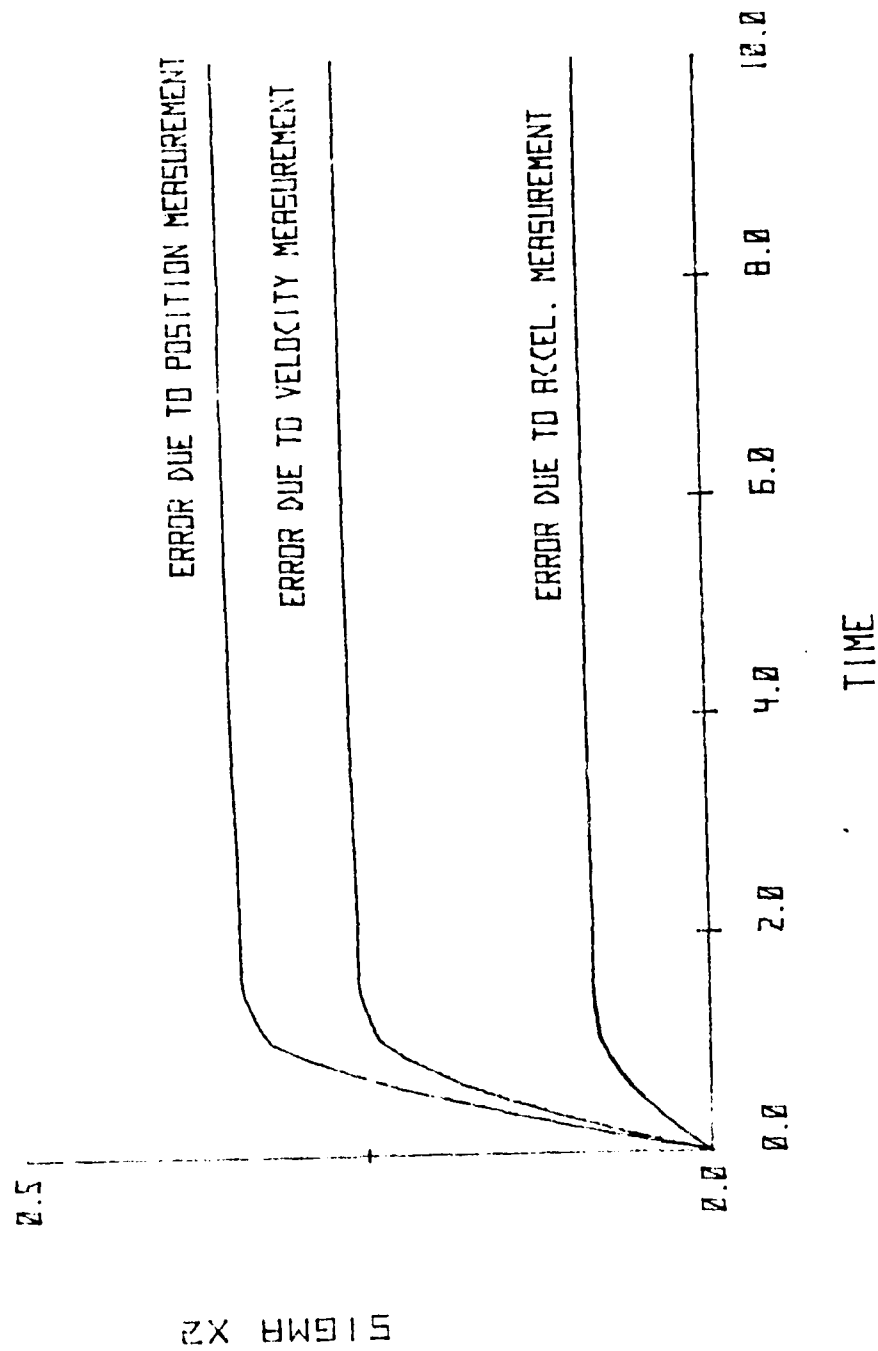
TIME

SIGMA X2

Figure 4-4 Tracking errors in X_2 as a result of measurement error position, velocity and acceleration.

Figure 4-5 Tracking errors in X_3 as a result of measurement error in position, velocity and acceleration.

near steady state value in a short time, particularly in the case of X_2. The rise time of the errors, seen in the figures, should not be interpreted as a lag in the response, since the figures represent the expected error in each of the states.

C. ALTERNATE MEASUREMENT SCHEMES

Two possible methods of reducing the number of measurements required by the SRFIMF controller will be considered. The first case is to measure acceleration and integrate to obtain velocity and position. In the second scheme, position and velocity will be measured and acceleration will be estimated from knowledge of the plant. The second case will be used to obtain an estimate of the added uncertainty caused by the acceleration measurement. Figure 4-6 is a schematic representation of the first case. As before the state representation of the system is obtained.
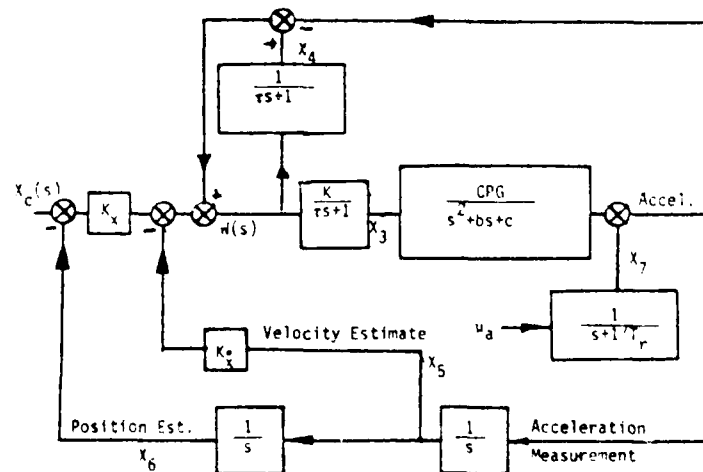
Figure 4-6 First alternate measurement scheme. Measured acceleration and implied position and velocity.

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The state variables are defined as

X_1 = position

X_2 = velocity

X_3 = output of the control actuator

X_4 = output of the compensator

X_5 = velocity estimate

X_6 = position estimate

X_7 = acceleration error

The control input, from figure 4-5 is written as

$$W(s) = cX_1 + bX_2 - CPGX_3 + X_4 - K_x X_5 - X_7 \qquad \text{4-16}$$

Following a development analogous to that used in section 2-3 we obtain the state representation of the system to be

$$\dot{X} = AX + BQ$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c & -b & CPG & 0 & 0 & 0 & 0 \\ K \cdot c/\tau & b \cdot K/\tau & -(CPG \cdot K+1)/\tau & K/\tau & -K \cdot K_x/\tau & -KK_x/\tau & -K/\tau \\ c/\tau & b/\tau & -CPG/\tau & 0 & -KX/\tau & -KX/\tau & -1/\tau \\ -c & -b & CPG & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/T_r \end{bmatrix}$$

$$B = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \qquad\qquad Q = \begin{bmatrix} \dfrac{2\sigma_a}{T_r} \end{bmatrix}$$

We apply the modal computer program to this system using the plant defined by equation 4-10 in order to determine the observability and controllability. The resulting system in modal coordinates is

$$X^* = A^* \, X^* + B^* Q$$

$$A^* = \begin{bmatrix} -47.48 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.56 & 1.34 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.34 & -1.56 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -10.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 \end{bmatrix}$$

$$B^* = \begin{bmatrix} -1.07 \\ .17 \\ -.20 \\ -211.0 \\ 9989.68 \\ 0.0 \\ 237.0 \end{bmatrix}$$

$$C^* = \begin{bmatrix} 0.0 & -0.03 & -0.05 & 0.46 & 0.0 & -0.01 & -0.42 \\ 0.02 & 0.12 & 0.04 & 0.0 & 0.0 & 0.08 & 0.04 \\ -0.99 & -0.24 & 0.09 & 0.0 & 0.0 & -0.82 & 0.0 \end{bmatrix} \begin{matrix} X_a^* \\ \\ \\ \end{matrix}$$

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From the A* matrix we can see that this system has two zero eigenvalues meaning that the modes of the system which correspond to the open loop integrators are neutrally stable. The modal control vector B* indicates that the velocity and position estimate modes of this system are strongly affected by the noise input. In particular, mode five (row 5 of B*) which represents the position estimate mode has a control coefficient from the noise source four orders of magnitude greater than the model mode (rows 2 and 3 of B*). We expect that in this case acceleration sensor noise significantly affects the performance of the controller.

Applying the algorithm used in the previous section to this case we obtain the covariance estimate. Figure 4-7 is a plot of the standard deviation (sigma) of X_1, X_2 and X_3 assuming
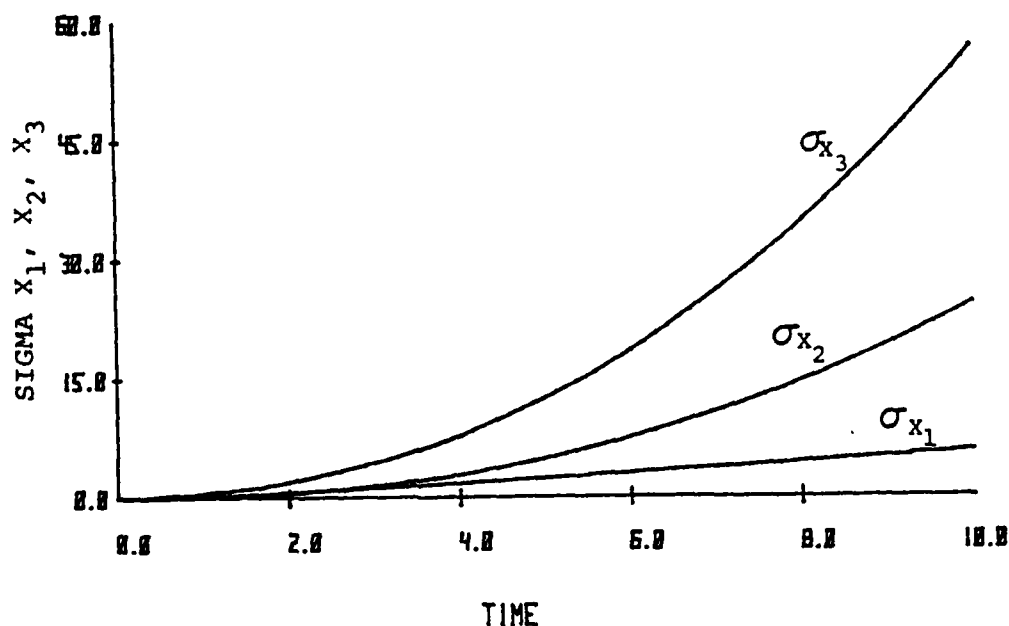
Figure 4-7 Error in X_1, X_2, and X_3 as a result of measurement errors in acceleration.

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a unit standard deviation noise source as the acceleration measurement error. Data used to plot figure 4-7 is also tabluated in Appendix B, table B-2. It is seen from the figure that acceleration measurement error causes unbounded errors in X_1 and X_3. Unbounded errors in any of the states are unacceptable in the controller and unless we can improve the method of estimating the position and velocity, we will be required to measure these quantities. A Luenberger observer could be used to estimate position and velocity, however the design of the observer would requre use of knowledge of the plant. We wish to avoid using detailed knowledge of the plant.

The second alternate measurement scheme is the case of measured position and velocity with acceleration estimated from the plant parameters b and c. The estimation is given by

$$\text{Acceleration} = -cX_1 - bX_2 + CPG\ X_3 \qquad\qquad 4\text{-}17$$

This scheme is shown in figure 4-8.

Figure 4-8 Second alternate measurement scheme, measured position and velocity. Estimated acceleration.

Following the same procedures as before we define the state variable as

X_1 = position

X_2 = velocity

X_3 = output of the control actuator

X_4 = output of the compensator

X_5 = position measurement error

X_6 = velocity measurement error

From figure 4-8 we obtain the control law W(t) as

$$W(t) = (-K_x + c) X_1 + (-K_{\dot{x}} + b)X_2 - CPG\ X_3 + X_4$$
$$+ (-K_x + c)X_5 + (-K_{\dot{x}} + b)X_6 \qquad 4\text{-}18$$

Assuming the same second order plant given by equation 4-10 we obtain the modal transformation of the system.[1]

$X = AX + BQ$

$Y = CX$

$X* = A*X* + B*Q$

$Y = C*X*$

---

[1]Matricies A, B, C, A*, B* and C* are shown on the following pages.

$$A = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -2.16 & -.6 & 1.0 & 0.0 & 0.0 & 0.0 \\ -92.0 & -120.0 & -60.0 & -50.0 & -92.0 & -120.0 \\ -18.0 & 24.0 & -10.0 & 0.0 & -18.0 & -24.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ -2.16 & -.6 & 1.0 \end{bmatrix}$$

$$A^* = \begin{bmatrix} -1.56 & 1.34 & 0.0 & 0.0 & 0.0 & 0.0 \\ -1.34 & -1.56 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -47.48 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -10.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -.1 \end{bmatrix}$$

$$B^* = \begin{array}{cc} Q_v & Q_p \\ \begin{bmatrix} 9.49 & 12.38 \\ 10.99 & 14.33 \\ -2.09 & -2.73 \\ 0.0 & 0.0 \\ 1.55 & 0.0 \\ 0.0 & 1.83 \end{bmatrix} \end{array}$$

$$C^* = \begin{array}{cccccc} & & & & x_v^* & x_p^* \\ \begin{bmatrix} 0.05 & 0.0 & 0.0 & 0.01 & -0.32 & -0.35 \\ -0.08 & 0.07 & 0.02 & -0.08 & 0.03 & 0.04 \\ 0.03 & -0.22 & -0.99 & 0.82 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

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The results of the modal analysis are similar to those obtained when acceleration was a measured quantity rather than calculated as in this case. The model and controller modes are controllable from the noise sources. The position and velocity outputs (rows 1 and 2 of the C* matrix) contain sensor noise terms.

The covariance analysis results are shown in figures 4-9 to 4-11 and listed in Appendix B, table B-3. Here it is seen that the standard deviation of X_1, X_2 and X_3 is slightly less than in the case when acceleration was measured. This is because the quantity taken to be acceleration does not contain the additional error of actual acceleration measurement. In this case the system requires only measurements of position and velocity.

The second alternate measurement scheme can be used to compare a state rate feedback control scheme to a state variable feedback controller, from the viewpoint of increased uncertainty in the state variable resulting from the additional measurement of acceleration. To make the comparison, assume that the measurements are made by strapdown type sensors whose measurement errors are given in table 4-1. Table 4-4 lists the total uncertainty of X_1, X_2 and X_3 for both measurement schemes. It can be seen from table 4-4 that the overall uncertainty of X_1, X_2 and X_3 is increased when acceleration is measured, as compared to the second alternate measurement scheme when acceleration is obtained without measurement. In a practical
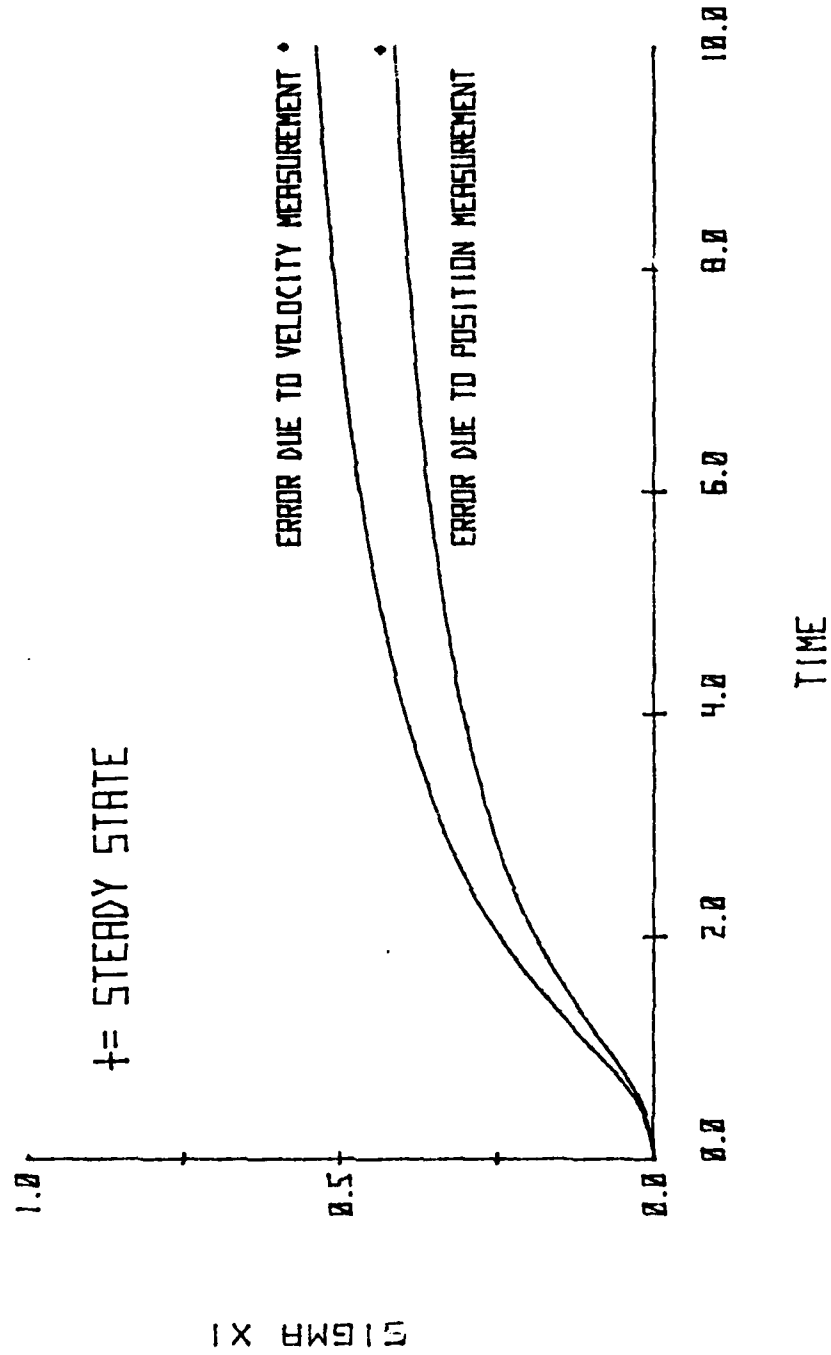
Figure 4-9 Tracking errors of X_1 as a result of measurement errors in position and velocity.

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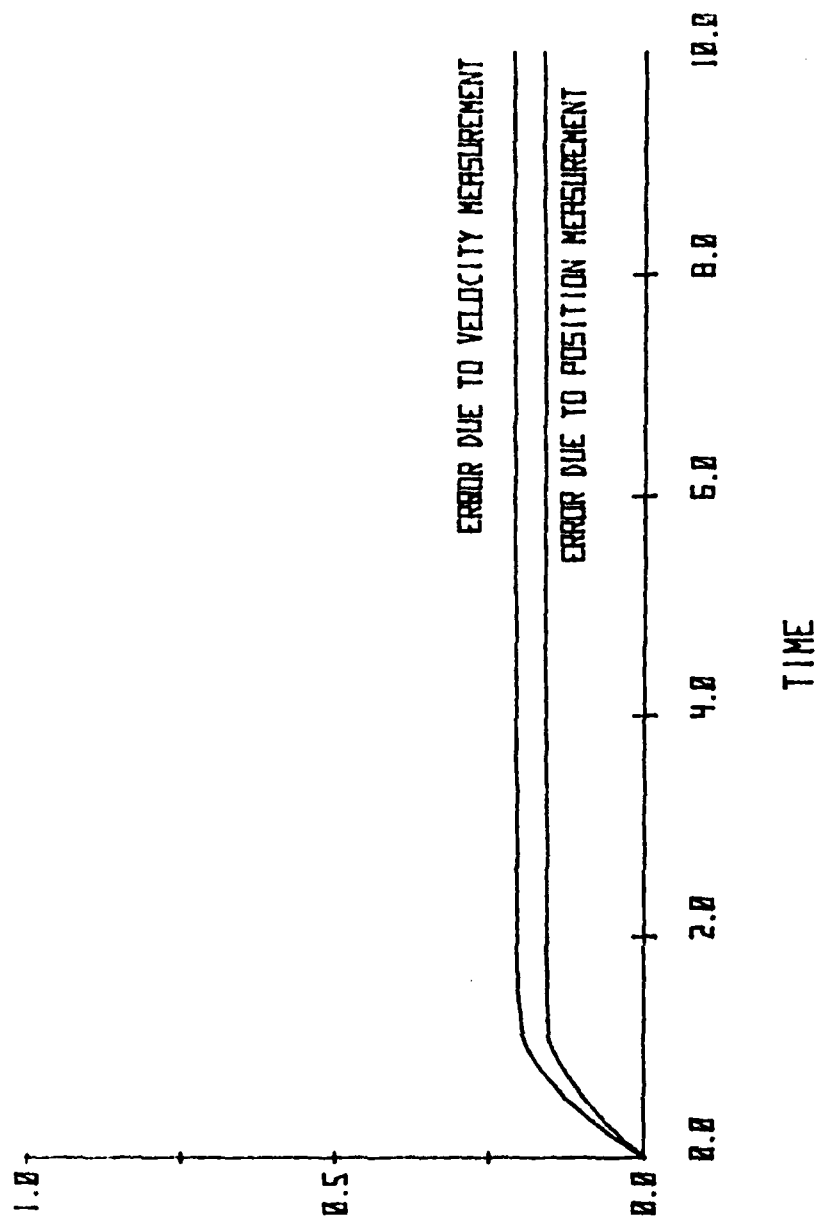
Figure 4-10 Tracking errors in X_2 as a result of measurement errors in position and velocity.

104
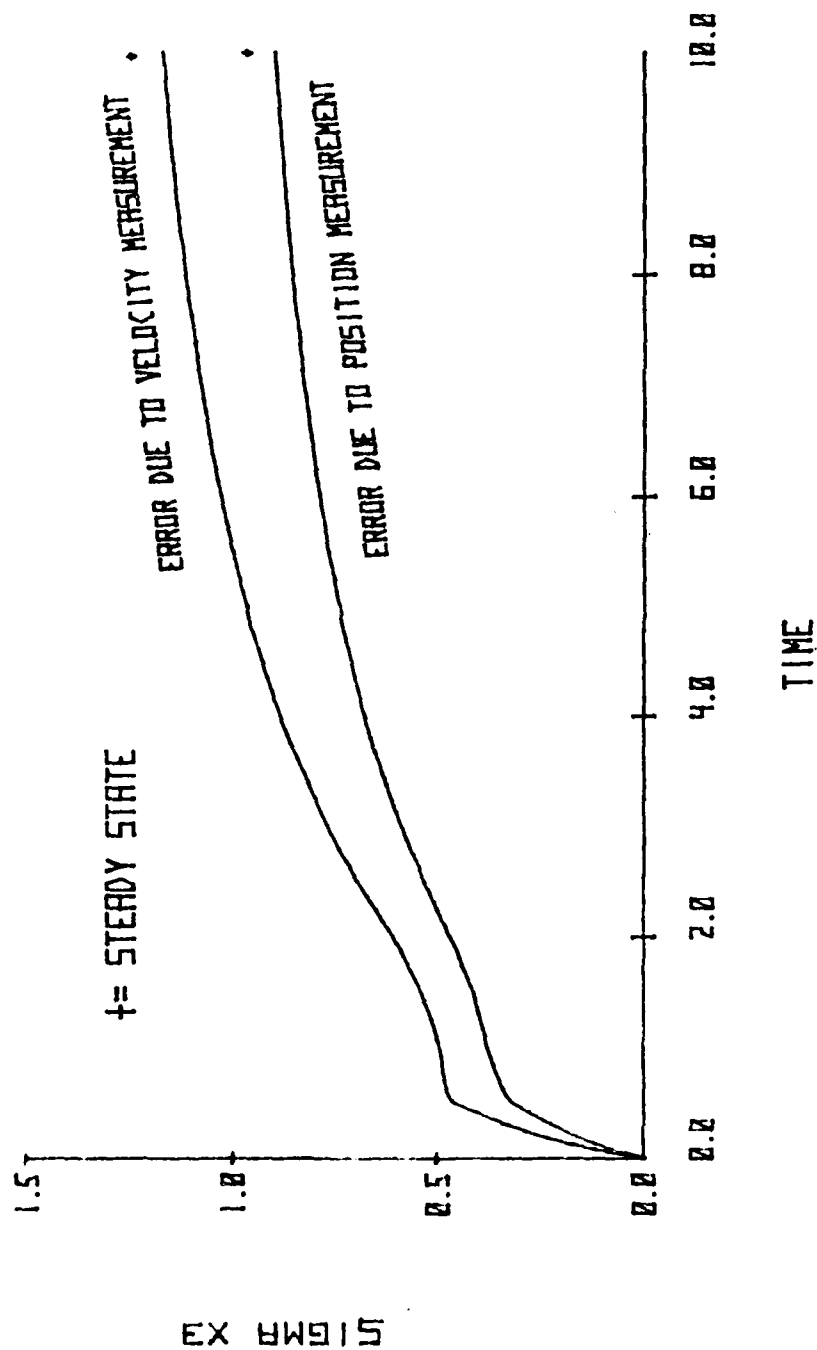
Figure 4-11 Tracking errors in X_3 as a result of measurement errors in position and velocity.

	X_1	X_2	X_3
Second Alternate Measurement Scheme, Calculated Acceleration	0.75˚	0.267˚/s	1.6˚/s^2
Measured Acceleration	1.6˚	0.58˚/s	3.46˚/s^2
Percent increase of expected error	113%	117%	116%

Table 4-4 Steady state error in X_1, X_2 and X_3 for measurements made by strapdown sensors.

system, uncertainty would exist in the knowledge of constants b and c and it could be expected that this would result in additional uncertainty in the value of the state variables. It might be possible that the uncertainty in the knowledge of the plant could negate the advantage gained by estimating acceleration as in the second alternate measurement scheme.

In conclusion, it has been shown that sensor noise does not adversely affect the SRFIMF position controller. It was also shown that acceleration measurement alone is not sufficient for acceptable operation of the controller. The analysis technique used in this section will next be applied to the analysis of a SRFIMF controller when the plant is assumed to be the longitudinal axis of the Harrier aircraft.

V. APPLICATION OF THE SRFIMF CONCEPT TO THE HARRIER AIRCRAFT

A. PITCH ATTITUDE CONTROL

The preceeding discussion dealt with the SRFIMF controller with the plant assumed to be a general second order system. In this section, the closed loop system will be modeled using the transfer function of a VSTOL aircraft. For this purpose the AV-8A Harrier, a jet-lift type VTOL aircraft, was chosen. The stability derivatives and transfer functions were obtained from reference [12]. The SRFIMF controller concept will be applied to the pitch attitude control of the aircraft and the analysis will be similar to that done earlier when the plant was assumed to be that of a second order system. As before, we will examine the root locus, time response and the effect of measurement errors. The effect of gust inputs to the plant will also be considered.

The transfer function between the pilot's stick and the pitch attitude of the Harrier, at 60 kts, 100 ft/sec, as given by reference [12] is

$$\frac{\theta(s)}{\delta_e(s)} = \frac{0.25\ (s^2 + .246s + .00756)}{s^4 + .4896s^3 - .4495s^2 + .06736s + .00747} \quad \text{5-1a}$$

$$= \frac{0.25\ (s + .21)(s + .036)}{(s + 1)(s + .073)(s^2 - .1864s + .1024)} \quad \text{5-1b}$$

$\theta(s)$ is given in radians and $\delta_e(s)$ in inches of stick displacement. The above transfer function does not include the effect of the control actuator which we include, as before, in the model of the controller.

The denominator factors of equation 5-1b indicate that at 60 kts the Harrier has an unstable oscillatory mode with a natural frequency, of 0.32 and damping ratio of -0.91, and that the constant plant gain, CPG, is 0.25. The plant, given by equation 5-1, is used to compute the root locus for the system by applying equation 3-21, developed for the controller in section III and rewritten here for convenience

$$1 + \frac{KRL(s^2 + K_{\dot{x}}s + K_x)}{s} G'(s) = 0 \qquad\qquad 3\text{-}21$$

where $G'(s)$ is the open loop plant given by

$$G'(s) = \frac{G(s)}{CPG}$$

For the root locus evaluation, KRL was varied from 1 to 100. The value of the actuator time constant, τ, was chosen to be 0.1 sec and $K_{\dot{x}}$ and K_x were 3 and 4 respectively.

The root locus of the oscillatory pole is given in figure 5-1. The trajectory of this pole is similar to those given in section III where the plant was assumed to be a second order system. As before, the closed loop system oscillatory poles are near those of the model (-1.5, 1.32) at KRL values of about 25 and greater. Because of the scale of the figure,

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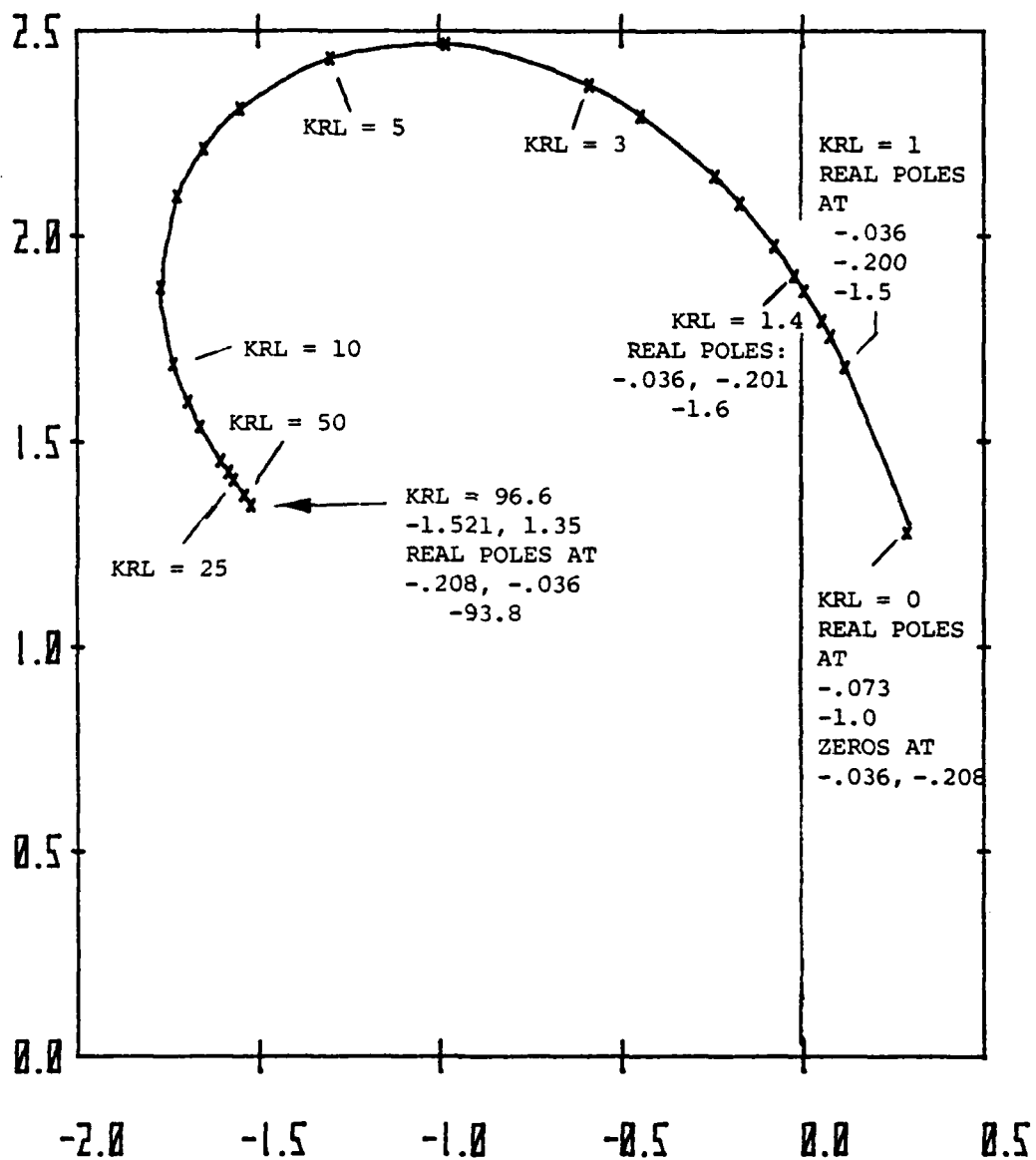
Figure 5-1 Root locus of the oscillatory poles of the longitu-
dinal axis of the Harrier using SRFIMF position control. Note
that the open loop zeros are nearly canceled at KRL = 1.0.

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the real poles are not plotted, however their values are listed on the figure for various values of KRL. At the final value of KRL (96.6), the system's real poles were located at -93.82, -.2079 and -.036. The pole at -93.28 corresponds to the controller mode and its location is nearly equal to the value of KRL. This result was discussed in section three. The two other poles located at -.036 and -.2079 cancel the open loop zeroes of the plant located at -.036 and -.21 as seen in the open loop transfer function, equation 5-1b. Zero cancellation by a pole could be very detrimental to the response if the open loop zeroes are located in the right half of the complex plane, becaue it would be unreasonable to expect perfect pole-zero cancellation. This is a problem typically encountered in model following techniques when the plant has zeroes in the right half plane.

From the root locus, figure 5-1, it can be seen that the dynamic behavior of the SRFIMF controller is unchanged by the introduction of a more complicated plant. We shall now consider the time response of the closed loop system.

The controller and plant system was simulated using the CSMP program discussed in section III. The source code is listed in Appendix A. Simulation of the Harrier transfer function is illustrated using a signal flow graph shown in figure 5-2. The outputs of position, velocity and acceleration are shown in the figure as part of the overall transfer function simulation. For the simulation, KRL was chosen to be 50, τ was

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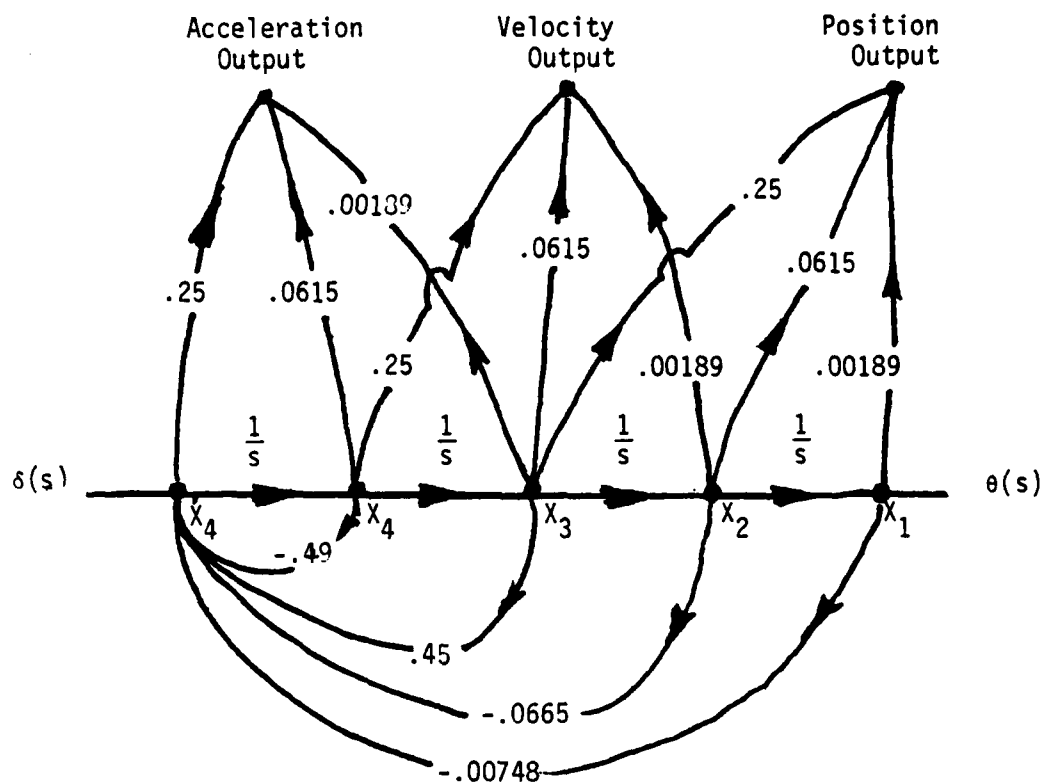
Figure 5-2 Harrier longitudinal signal flow graph transfer
function simulation.

0.1 sec, K_x and K_x were 3 and 4. Plots of the time histories
of the closed and open loop position, $\theta(t)$, as a result of
the simulation, are given in figure 5-3. The figure indicates
that the closed loop dynamic response is nearly identical to
that of the model as shown in figure 3-5.

To evaluate the effect of the sensor noise, the state
space representation of the system is required. Four states
represent the plant. Defining X_1 and position and X_2 as
velocity we write the transfer function for the Harrier, given

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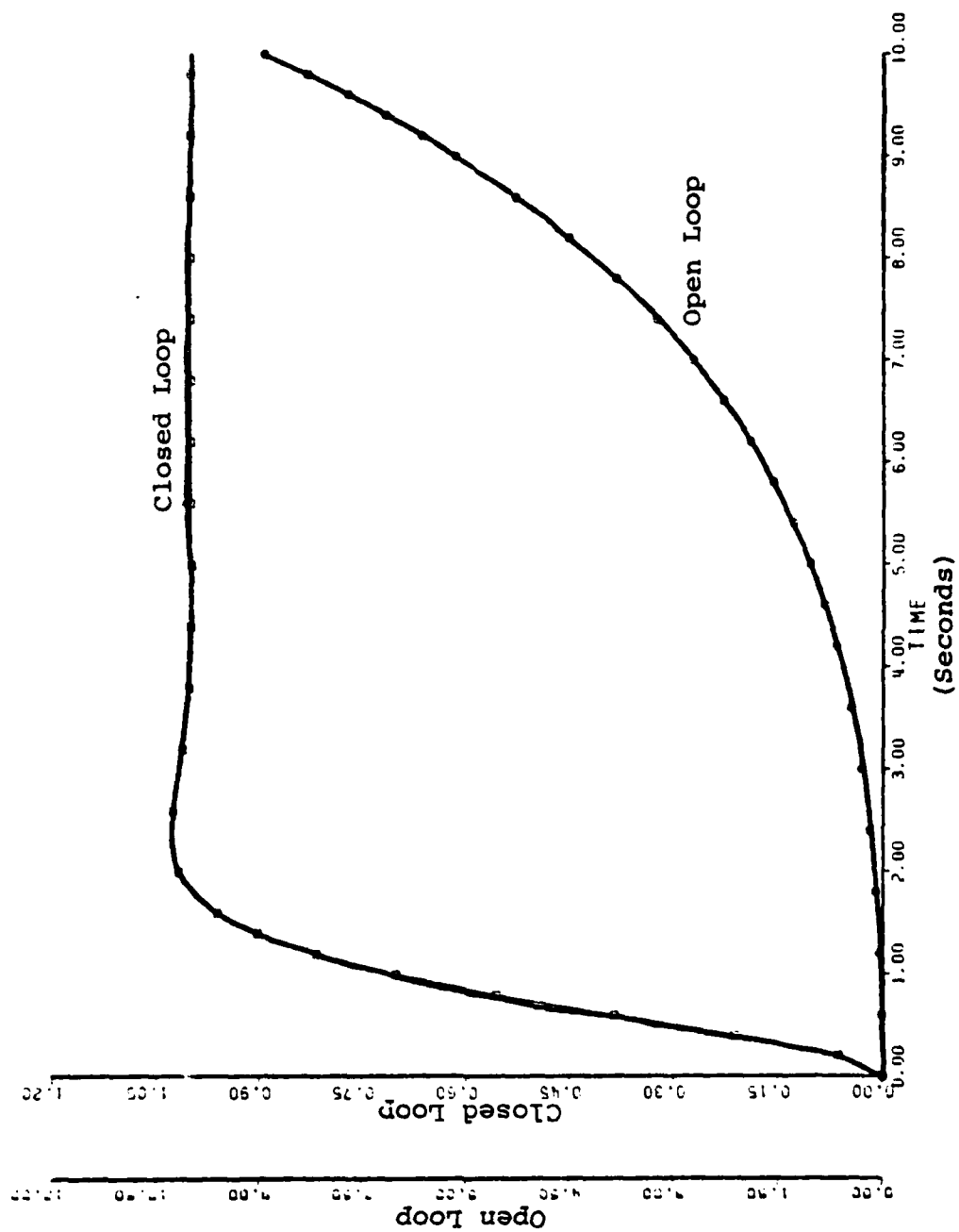
Figure 5-3 Simulation of the unit step response of the pitch attitude
of the AV-8A Harrier with SRFIMF control.

in equation 5-1 in matrix notation as

$$
\begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -.00747 & -.06736 & +.4495 & -.4896 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} + \begin{Bmatrix} 0 \\ .25 \\ -.061 \\ .1443 \end{Bmatrix} \delta_e
$$

$$5\text{-}2$$

The control vector of equation 5-2 is determined by the numerator of equation 5-1 so that the states X_1 and X_2 are position and velocity. The algorithm for determining the required control vector is given by Ogata, reference [13].

The remaining state variables are defined in the following way

X_5 = output of the control actuator, input to the plant

X_6 = output of the compensator

X_7 = state variable representing velocity sensor error

X_8 = state variable representing acceleration sensor error

X_9 = state variable representing position sensor error

Figure 5-4 is the schematic representation of the system and from the figure the control law is obtained as

$$W(t) = -K_x X_1 - K_x X_2 - X_3 - CPG\ X_5 + X_6 - K_x X_7 - X_8 - K_x X_9$$

$$5\text{-}3$$

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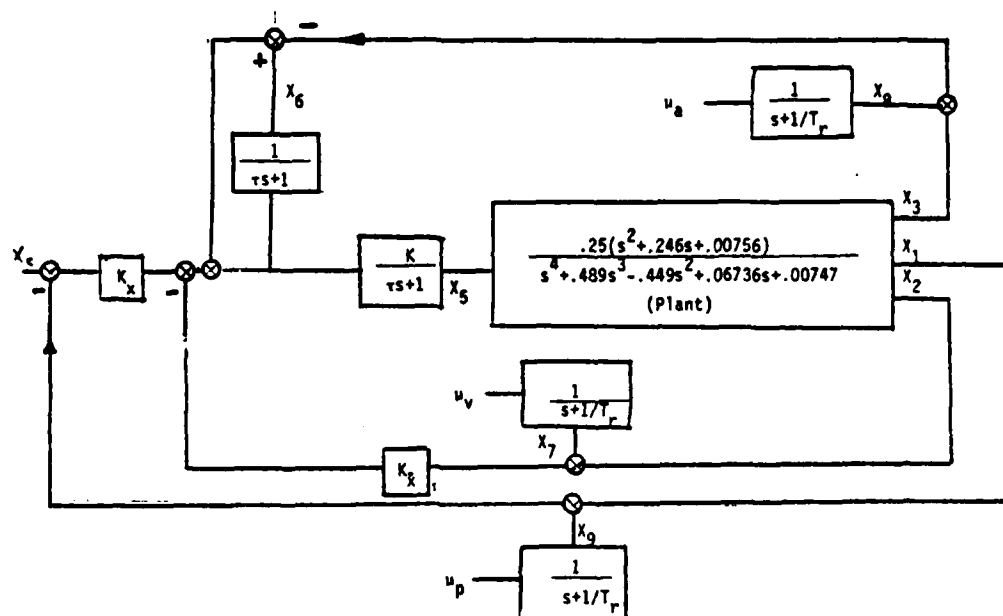
Figure 5-4 Schematic representation of the AV-8A Harrier pitch axis with the SRFIMF controller. Including sensor noise.

Combining and expressing the system in matrix form, the closed loop system is given by

$$\dot{X} = AX + BQ \qquad\qquad 5\text{-}4$$

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & .25 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -.06115 & 0 & 0 & 0 & 0 \\
.00748 & -.0675 & .45 & -.4906 & .1443 & 0 & 0 & 0 & 0 \\
-KK_x/\tau & -KK_{\dot{x}}/\tau & -K/\tau & 0 & -(.25K+1)/\tau & K/\tau & -KK_{\dot{x}}/\tau & -K/\tau & -KK_x/\tau \\
-K_x/\tau & -K_{\dot{x}}/\tau & -1/\tau & 0 & -.25/\tau & 0 & -K_{\dot{x}}/\tau & -1/\tau & -K_x/\tau \\
0 & 0 & 0 & 0 & 0 & 0 & -1/T_r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/T_r & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/T_r
\end{bmatrix}
$$

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$$
B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\qquad
Q = \begin{bmatrix} 2\sigma_v^2/T_r & 0 & 0 \\ 0 & 2\sigma_a^2/T_r & 0 \\ 0 & 0 & 2\sigma_p^2/T_r \end{bmatrix}
$$

Having the system, represented in state space, the technique used in the previous section can be applied. The general solution will be obtained for a unit value of sensor error standard deviation and later applied to the specific sensor suits assumed earlier. The constants required to obtain the numerical solution are: $K = 20$; $\tau = 0.1$; $K_x = 4$; $K_{\dot{x}} = 3$; $KRL = K \cdot CPG/\tau = 50$. The resulting system is

$$\dot{X} = AX + BQ \qquad\qquad\qquad\qquad 5\text{-}5$$

$$Y = CX$$

$$\dot{X}^* = A^*X^* + B^*Q \qquad\qquad\qquad 5\text{-}6$$

$$Y = C^*X^*$$

(Matricies A, B, C, A*, B* and C* are shown on the following pages.)

The result of the modal transformation is that the noise sources have about the same controllability and observability as in the

$$A = \begin{bmatrix}
0.0 & 0.0 & 0.0 & -0.01 & -800.0 & -40.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 0.0 & 0.0 & -0.07 & -800.0 & -30.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 0.0 & 0.45 & -200.0 & -10.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0 & -0.49 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.25 & -0.06 & 0.14 & -60.0 & -2.5 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 200.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & -600.0 & -30.0 & -0.1 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & -200.0 & -10.0 & 0.0 & -0.1 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & -800.0 & -40.0 & 0.0 & 0.0 & -0.1
\end{bmatrix}$$

$$B = \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0
\end{bmatrix}$$

$$C = \begin{bmatrix}
1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\
0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0
\end{bmatrix}$$

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$$A^* = \begin{bmatrix} -47.17 & 0.0 & 0.01 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.54 & 1.37 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.37 & -1.54 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.21 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.03 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -10.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 \end{bmatrix}$$

$$B^* = \begin{matrix} Q_v & Q_a & Q_p \end{matrix}$$
$$B^* = \begin{bmatrix} -13.55 & -4.52 & -18.07 \\ 10.18 & 3.39 & 13.57 \\ -7.15 & -2.38 & -9.53 \\ 5.41 & 1.80 & 7.21 \\ 1.25 & 0.42 & 1.67 \\ 0.0 & 0.0 & 0.0 \\ 2.26 & 0.0 & 0.0 \\ 0.0 & 1.21 & 0.0 \\ 0.0 & 0.0 & 2.88 \end{bmatrix}$$

$$\begin{matrix} & & & & & & x_v^* & x_a^* & x_p^* \end{matrix}$$
$$C^* = \begin{bmatrix} 0.0 & 0.01 & -0.1 & 0.0 & 0.0 & 0.0 & -0.36 & -0.22 & -0.03 \\ 0.01 & 0.11 & 0.16 & 0.0 & 0.0 & -0.03 & 0.48 & 0.02 & 0.04 \\ 0.0 & -0.01 & -0.1 & -0.24 & 0.24 & 0.01 & 0.19 & 0.95 & 0.20 \end{bmatrix}$$

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earlier cases. It can be seen from the A* matrix that the
closed loop system has two additional eigenvalues. These
correspond to the shifted poles of the system. Variance
analysis was done in the same way as in the earlier examples
and the results are shown plotted in figures 5-5 through 5-8
and tabulated in Appendix B, table B-4. Table 5-1 lists the
steady state values of the expected error in states X_1, X_2
and X_5.

	X_1	X_2	X_5
Position measurement error	0.98	0.36	3.85
Velocity measurement error	0.73	0.27	2.90
Acceleration measurement error	0.25	0.089	0.96

Table 5-1 Steady state values of σ_1, σ_2 and σ_5
as a result of position, velocity and
acceleration measurement error.

Comparison of table 5-1 with table 4-2 indicates that the
values of the standard deviation of the X_1 and X_2 errors are
identical in both cases even though the two plants are very
different. The standard deviation of the plan input error
(X_5 here, X_3 in section IV) is slightly different in the two
cases. This should be expected because of the difference in
the value of control gain K, required to yield a value of KRL
equal to 50 and the large difference between the two plants.
From the variance analysis, we obtain an estimate of the
expected errors in X_1, X_2 and X_5 which result from the sensor
suit assumed in section IV. These values are listed in table
5-2.

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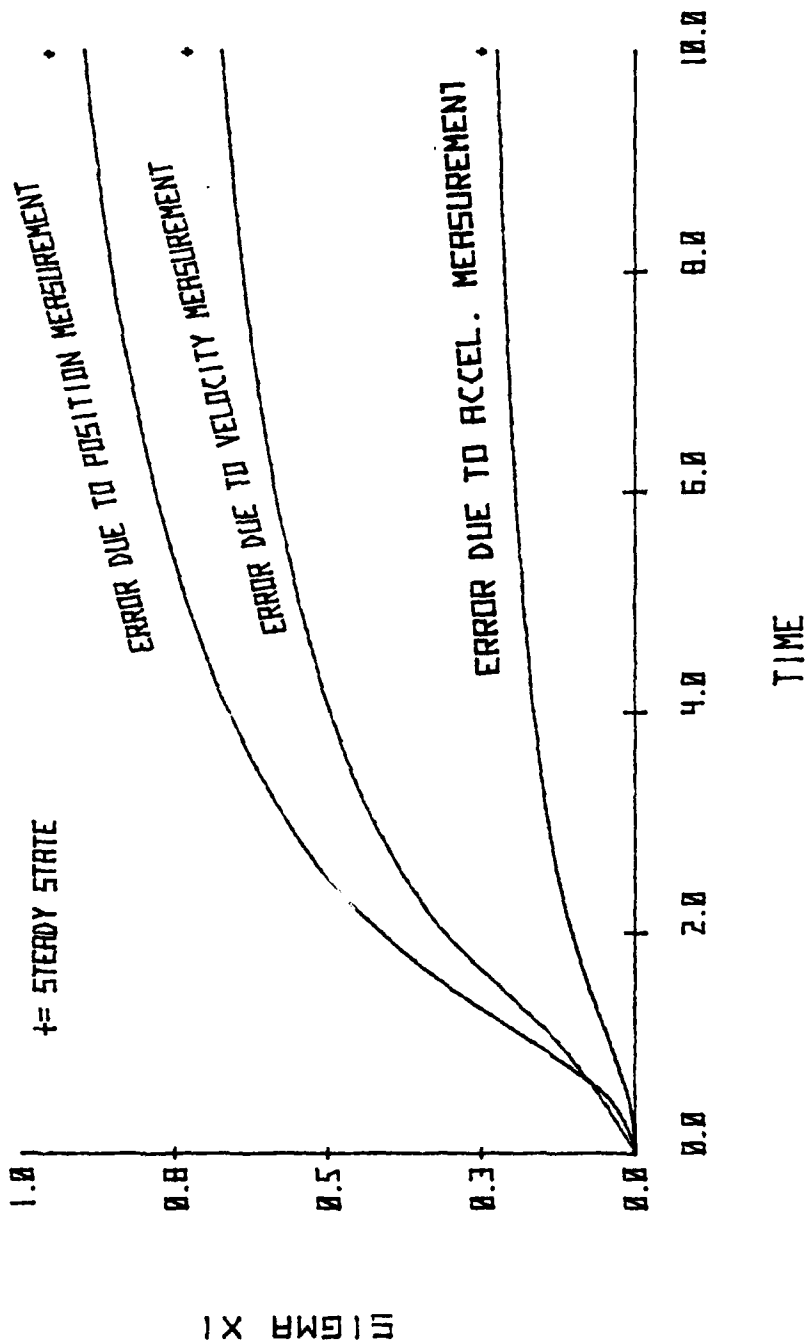
Figure 5-5 Tracking error in X_1 as a result of measurement error in position, velocity and acceleration for the Harrier longitudinal axis position control.

SIGMA X2 VS. TIME

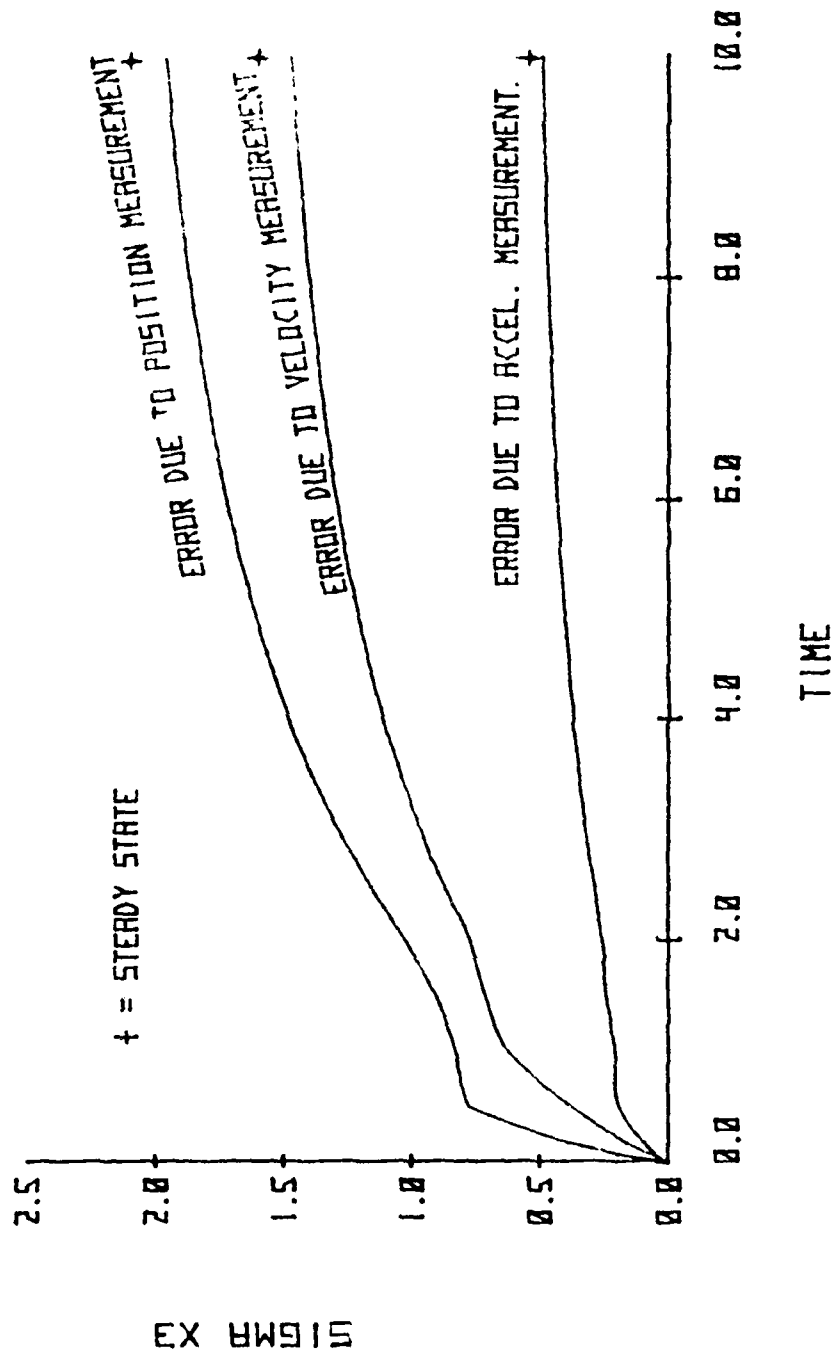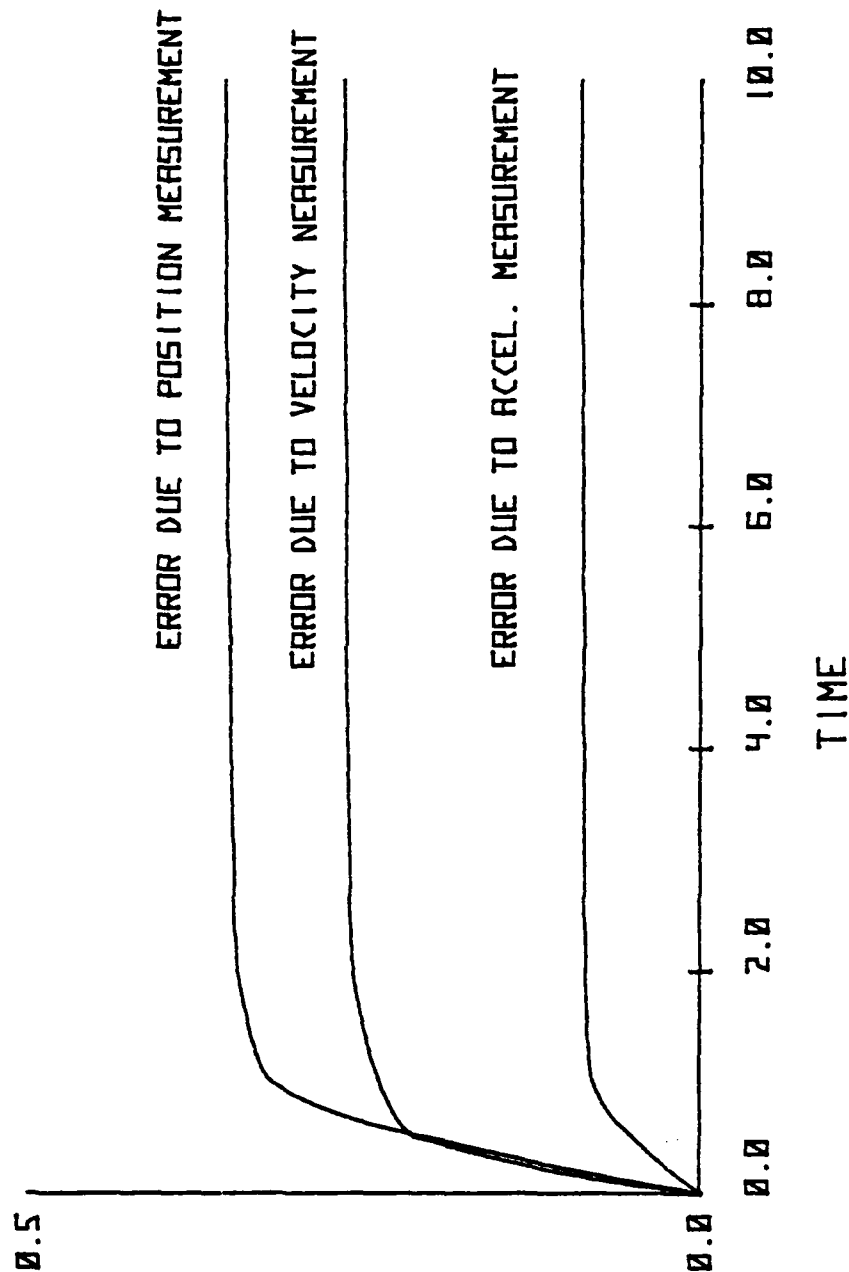Figure 5-6 Tracking error in X_2 as a result of measurement error in position, velocity and acceleration for the Harrier longitudinal axis positon control.

SIGMA X5 VS. TIME

ERROR DUE TO POSITION MEASUREMENT.

ERROR DUE TO VELOCITY MEASUREMENT

ERROR DUE TO ACCEL. MEASUREMENT

TIME

SIGMA X5

5.0 4.0 3.0 2.0 1.0 0.0

0.0 2.0 4.0 6.0 8.0 10.0
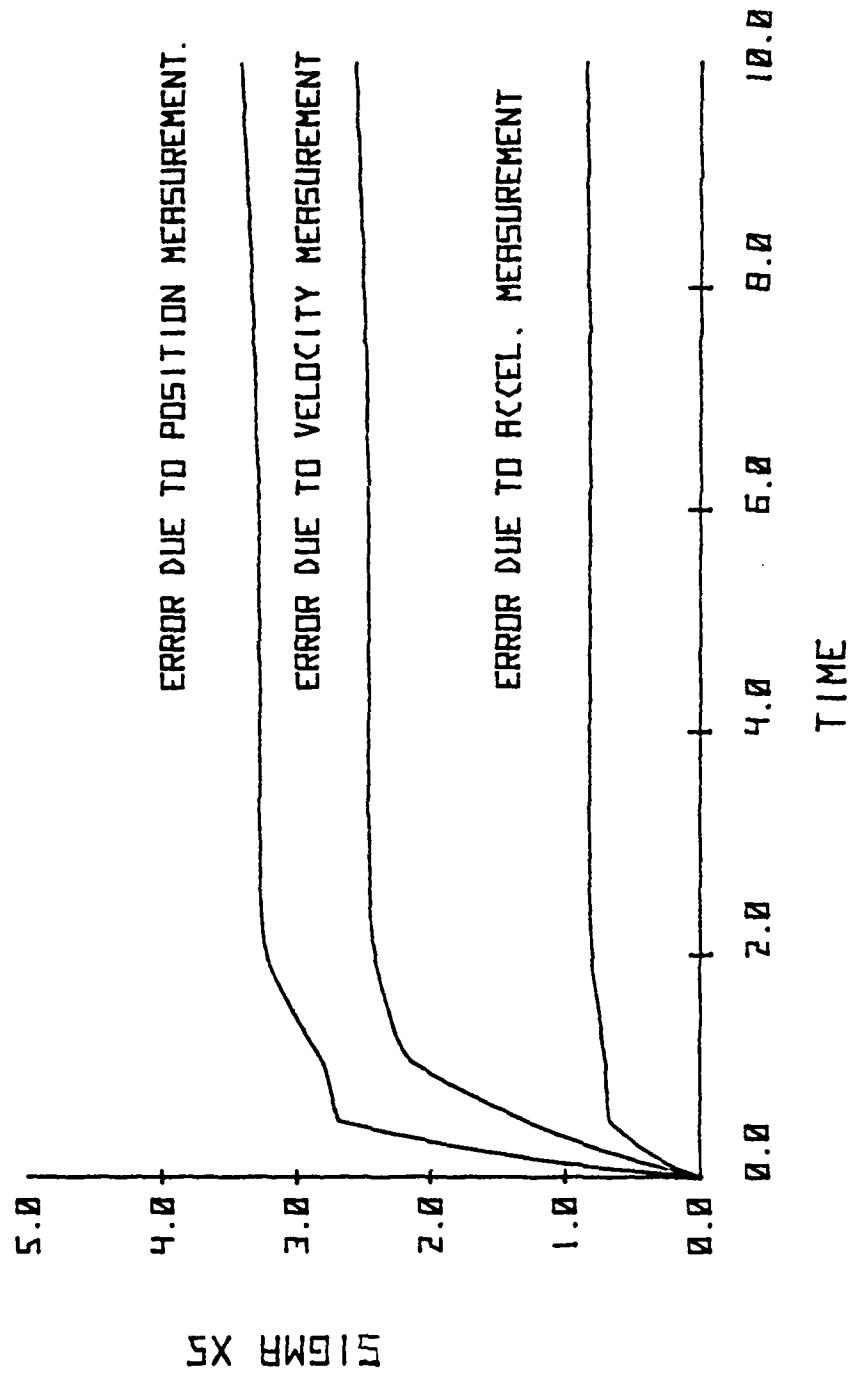
Figure 5-7 Tracking error in X_5 as a result of measurement error in position, velocity and acceleration for the Harrier longitudinal axis position control.

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	X_1	X_2	X_5
Inertial sensors	0.49	0.18/sec	1.93/sec^2
Strapdown sensors	1.575	0.584/sec	6.26/sec^2

Table 5-2 Standard deviation of steady state tracking errors in X_1, X_2 and X_5 as a result of sensor noise in the Harrier aircraft.

The Significance of the total tracking error in X_5 is determined by comparing the standard deviation of the error to the value of X_5 when a 1° input is applied to the system. In quiescent operation, a 1° step input results in an instantaneous value of X_5 of

$$X_5 = .0175 \text{rad} \cdot 1^\circ K_x \quad K = 1.4 \text{ rad/sec}^2$$
$$= 80^\circ/\text{sec}^2$$

The value of X_5 given in table 5-2 for strapdown sensors whose expected error is in the order of 1° is not large compared to a 1° input. We conclude that sensor measurement errors do not adversely affect the performance of the SRFIMF Harrier pitch controller.

B.   PITCH ATTITUDE GUST RESPONSE

The pitch attitude response of the Harrier from a gust input will be considered by applying the same type of analysis used earlier. For this purpose assume that the gust acts as an additional, uncontrolled input to the system. The symbol δ_g is used to denote this input. The gust input will be

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modeled as an exponentially correlated noise source in the same way that the sensor noise was modeled. The schematic representation of an external exponentially correlated noise source was shown in figure 4-1. In this case the output of the noise source, $\xi(s)$, is the gust input to the system and $W_t(s)$ is the white noise input of strength μ_g as given by equation 4-12. The standard deviation and correlation time for the gust can be determined by examining the Dryden wind model used in simulation by NASA. At altitudes of 500 ft. and above the Dryden model assumes that the RMS value of atmospheric turbulence, σ_g, is given as

$$\sigma_g = 0.2 \ V_{wind}$$

The corrleation time, T_g, is determined by a characteristic length, L_w, divided by the vehicle speed, V, or

$$T_g = L_w/V$$

The Dryden model gives the characteristic length of the turbulence as

$$150 \ \text{ft.} + h = L_w$$

where h is the altitude. Choosing 500 ft. as the flight altitude, a flight speed of 60 kts and 15 kts as the value of the wind, the standard deviation and correlation time is

$$\sigma_g = 5 \ \text{ft/sec}, \ T_g = 6.5 \ \text{sec}$$

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The gust input is represented schematically by figure 5-8.

$$\frac{2\sigma_{\subsetsym}}{T_g} \longrightarrow \boxed{\frac{1}{s + 1/T_g}} \longrightarrow \delta_g \quad (ft./sec)$$
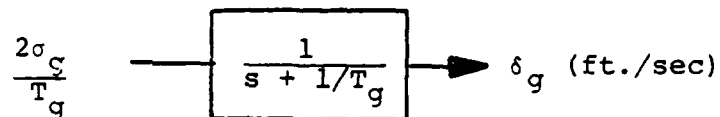
Figure 5-8 Gust shaping filter.

The transfer function between the gust and position, velocity and acceleration of the vehicle can be determined from the stability derivatives given by reference [14]. For This example we chose pitch attitude position control as was done in the previous example therefore the denominator of the gust transfer function is the same as the denominator of equation 5-1. The numerator is given by McRuer [14] for a vertical gust as

$$\frac{U_o N_g^W}{s} = (M_q - M_\alpha)s^2 + (M_\alpha - Z_w (M_q - M_\alpha)s +$$

$$X_u (M_\alpha + Z_w Mq) - X_w (V \cdot M_u + Z_u Mq) \qquad 5-5$$

The stability derivatives for the Harrier at 60 kts are (6)

$M_q = -2.6$

$M_\alpha = V \cdot M_w = .54$

$Z_w = -.190$

$Z_u = -.036$

$X_u = -0.43$

$X_w = -0.27$

$M_u = .022$

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From equation 5-5 and 5-1, we obtain the transfer function
for the pitch attitude from a gust input

$$\frac{\theta(s)}{\delta_g(s)} = \frac{-.0026s\,[s^2 + 2.31s + .073]}{s^4 + .4896s^3 - .4495s^2 + .0636s + .00747} \qquad 5\text{-}6$$

The transfer function from the pilot's stick to the pitch
angle is given by equation 5-1 where the input to the plant
is δ_e. Expressing the plant, with multiple inputs of stick and
gust load, in matrix notation with states X_1 and X_2 defined as
position and velocity we have

$$\begin{Bmatrix} \ddot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -.00747 & -.06736 & +.4495 & -.4896 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_2 \\ X_4 \end{Bmatrix} + \begin{bmatrix} 0 & -.0026 \\ .25 & -.00473 \\ -.061 & .00306 \\ .1443 & .00345 \end{bmatrix} [\delta_e \; \delta_g]$$

The B matrix has been determined as before so that states
X_1 and X_2 represent position and velocity.

Having the plant transfer function defined by 5-7 we can
develope the equation to represent the closed loop controller.
The additional state variables required are defined

X_5 = output of the control actuator, input to plant

X_6 = output of the compensator

X_7 = gust input to the plant, $\delta_g(s)$, figure 5-5.

The schematic of the closed loop system is shown in figure 5-9.

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Figure 5-9 Schematic representation of the AV-8A Pitch
control using a SRFIMF controller with wind gust input.

With $X_c = 0$, the control law $W(t)$ is
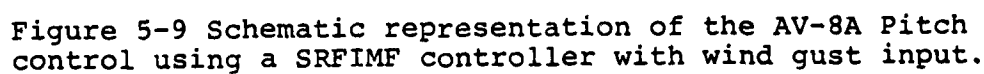
$$W(t) = -K_x X_1 - K_{\dot{x}} X_2 - X_3 - .25X_5 + X_6 + (K_{\dot{x}} \cdot .0026 + .00473)X_7$$

Rearranging in a manner similar to earlier work, we write the

closed loop equation in matrix notation as

$$\dot{X} = AX + BQ \qquad\qquad 5\text{-}8$$

$$A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & -.0026 \\
0 & 0 & 1 & 0 & .25 & 0 & -.00473 \\
0 & 0 & 0 & 1 & -.061 & 0 & .00306 \\
-.00748 & -.0675 & .45 & -.49 & .1443 & 0 & .90346 \\
-K \cdot K_x/\tau & -KK_{\dot{x}}/\tau & -K/\tau & 0 & -(.25 \cdot K+1)/\tau & K/\tau & (K_{\dot{x}} \cdot .0026+.00473)K/\tau \\
-K_x/\tau & -K_{\dot{x}}/\tau & -1/\tau & 0 & -.25/\tau & -K_{\dot{x}}/\tau & (K_{\dot{x}} \cdot .0026+.00473)/\tau \\
0 & 0 & 0 & 0 & 0 & 0 & -1/T_g
\end{bmatrix}$$

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$$B = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \qquad\qquad Q = \begin{bmatrix} 2\sigma_g/T_g \end{bmatrix}$$

The covariance of the states were calculated for the system described above as was done in previous examples. The results of the covariance analysis was that the RMS values of X_1 and X_2 are very small and quickly reach the steady state values of

$$\sigma_p = \sigma_{X_1} = .0017 \qquad\qquad \sigma_v = \sigma_{X_2} = .0134$$

In the case of the gust we are interested in the pitch acceleration of the vehicle as a result of the gust input. Acceleration is given as $\dot{X}_2$ and from equation 5-8 is

$$\text{Acceleration} = X_3 + .25X_5 - .00473X_7 \qquad\qquad 5\text{-}9$$

Recalling that the elements of the covariance matrix, P, are the squares of the standard deviations of the state variables, we write from equation 5-9 that

$$\sigma_{accel} = P_{33}^{\frac{1}{2}} + .25P_{55}^{\frac{1}{2}} - .00473P_{77}^{\frac{1}{2}}$$

The standard deviation in pitch acceleration as a result of a gust input was plotted along with the gust input and the RMS value of X_5 as a function of time in figure 5-7. The results are also listed in Appendix B table B-5. The gust is shown for comparison of response time.

It is seen from the figure that the rise time in the RMS value of the vehicle acceleration is much slower than the gust itself. This result can be interpretated as a smoothing of the gust by the aircraft. This is primarily due to the slow response of the aircraft to the gust. The controller has a relatively fast reaction time and can maintain the output X_1 with only very small errors as a result of a gust input.

The analysis of the SRFIMF controller applied to the Harrier has indicated that the qualities of the controller found by analysis when the plant is assumed to be a second order system apply equally well when the plant is an actual VSTOL aircraft. We have also seen that the controller does not produce undesireable dynamic response when the vehicle is subjected to gust inputs and that the dynamic response of the aircraft and controller is due to the response of the aircraft alone to the gust.
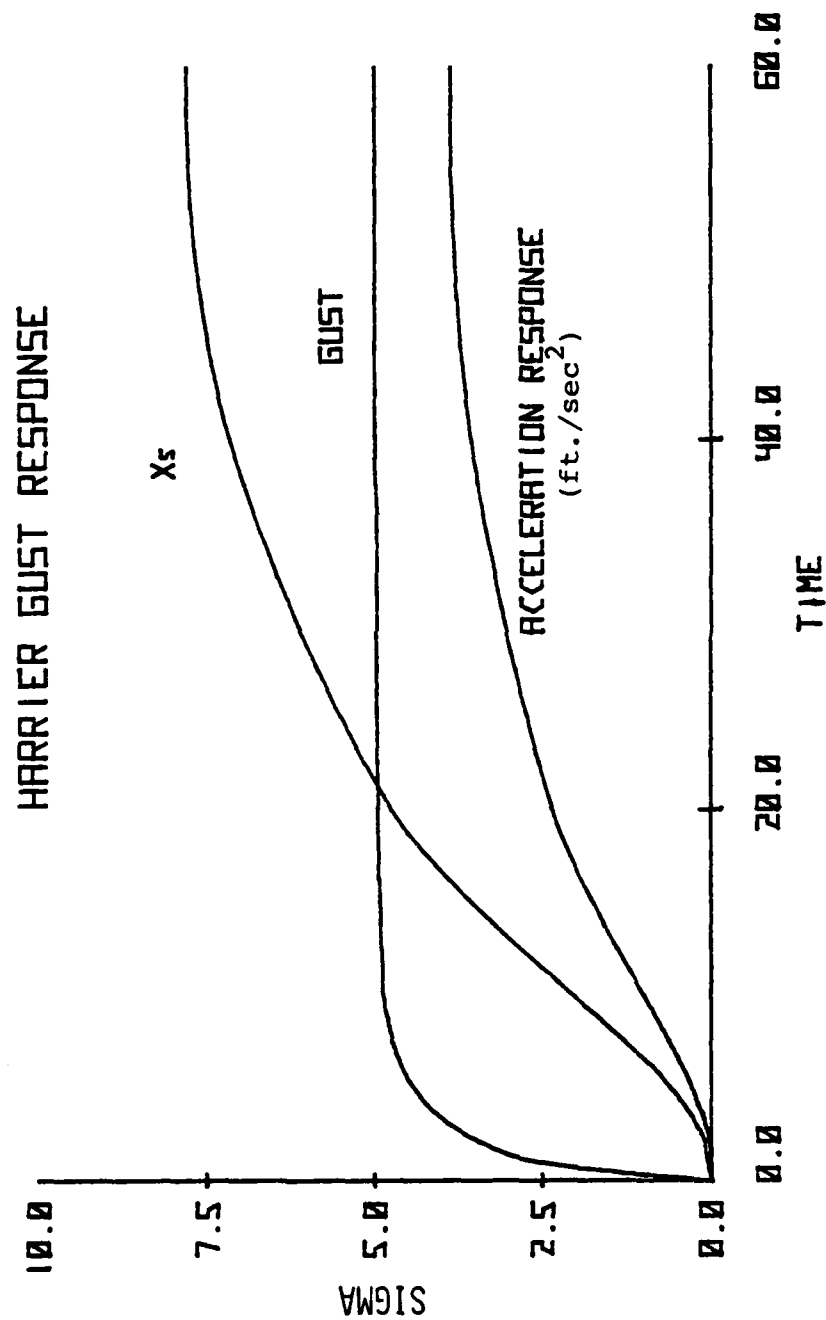
Figure 5-10 Standard deviation of vehicle pitch acceleration and plant input, x_5, as a result of a gust.

VI. CONCLUSIONS

The use of State Rate Feedback Implicit Model Following,
for the attitude control of VSTOL aircraft, has been studied.
The SRFIMF control scheme is simple, easy to implement and
can be expected to be reliable. It was shown that the
dynamic response of the closed loop system is a second order
response and that the natural frequency and damping are free
choices of the designer. It was also shown that sensor noise
does not adversely affect the operation of the system. Detailed
conclusions of this study are:

1. Model following was shown in section III-D by simula-
tions to be very good. Despite changing plant dynamics,
representing a variety of flight conditions, the closed loop
system had a response characteristic of that of the model
and that the output of the closed loop system was driven to
the value of the input.

2. The frequency response of the system is that of a low
pass filter and that there was no phase shift between the
output and the input at low frequencies.

3. Non-minimum phase system, pole-zero cancellation can
lead to unstable performance.

4. High quality sensors are not required. It was shown
by the covariance analysis in section IV-B that the error in
position is approximately equal to the error in position

measurement plus 75 percent of the error in velocity measurement plus 25 percent of the error in acceleration measurement. The general result is that sensor errors do not adversely affect the performance of the system.

5. Estimation of position and velocity by integration of acceleration was shown to lead to unstable performance because of neutrally stable modes resulting from the integration. It was shown in section IV-C that acceleration sensor noise disturbs the neutrally stable modes with the result that the system diverges. It is then concluded that the quantities of position and velocity must be measured in order to have a stable system which does not require knowledge of the plant.

6. The system is capable of compensating for gust inputs. The analysis of section V-C showed that the error in position output was small as a result of the gust. Secondly, the response of the system to the gust is smoothed by the action of the uncontrolled plant.

APPENDIX A

COMPUTER LISTINGS

```
********************************************************************
*
*            ***   SRFIMF SIMULATION SOURCE CODE   ***
*
*       CSMP SOURCE CODE USED TO SIMULATE THE RESPONSE OF
*       THE SRFIMF AND AN ARBITRARY SECOND ORDER PLANT.
*       THE FUNCTIONS INTGRL AND REALPL ARE CSMP FUNCTIONS
*       WHICH PREFORM INTEGRATION OR SIMULATE A FIRST ORDER
*       RESPONSE.THE ORDER OF THE STATEMENTS IS UNIMPORTANT IN
*       CSMP.
*
********************************************************************
// EXEC CSMPXV
//X.COMPRINT DD DUMMY
//X.SYSPRINT DD DUMMY
//X.SYSIN DD *
        AS=REALPL(0,TOU,WS)
        X=INTGRL(0,VX)
        VX=INTGRL(0,AX)
        AX1=REALPL(0,TOU,IS)
        AX=AX1*CPG-C*X-B*VX
        IS=WS*K
        WS=-KXD*VX+(XC-X)*KX+VS
        VS=AS-AX
*
*       SIMULATION OF THE OPEN LOOP PLANT
*
        XDD=CPG*AX2-C*XP-VXP*B
        AX2=REALPL(0.,TOU,XC)
        VXP=INTGRL(0,XDD)
        XP=INTGRL(0,VXP)
        XC=G*STEP(0)
        G=1.
        CPG=4.
        TOU=.1
        K=5.
        KXD=3.
        KX=4.
        B=.6
        C=2.16
TIMER FINTIM=10,OUTDEL=.2
OUTPUT X,XP
OUTPUT TIME,XP
PAGE XYPLOT
        END
        STOP
```

```
C***********************************************************************  MOD00040
C                                                                        MOD00050
C                          MAIN 1                                        MOD00060
C                                                                        MOD00070
C      MAIN 1 IS THE CODE USED TO GENERATE THE A,B AND C MATRICIES FOR    MOD00080
C      THE BASIC STOCHASTIC SRFIMF.POSITION,VELOCITY AND ACCELERATION     MOD00090
C      ARE ASSUMED TO BE MEASURED AND THE PLANT IS A SECOND ORDER         MOD00100
C      SYSTEM.THE PROGRAM CALLS VARY TO DETERMINE THE SOLUTION TO         MOD00110
C      THE LYAPUNOV EQUATION AND MODAL FOR THE MODAL ANALYSIS.            MOD00120
C***********************************************************************  MOD00130
      DIMENSION B(8,4),TR(4),H(3,8)                                       MOD00140
      REAL K,KX,KXD                                                       MOD00150
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(64),Q(8,8),A(8,8)                MOD00160
      WRITE(6,100)                                                        MOD00170
100   FORMAT(//,20X,90('*'),//,40X,'MODEL 1   POSITION,VELOCITY AND ',    MOD00180
     1'ACCELERATION MEASURED',//,20X,90('*'))                            MOD00190
      N=8                                                                 MOD00200
      NCOUNT=0                                                            MOD00210
      M=3                                                                 MOD00220
      L=4                                                                 MOD00230
      BP=.6                                                               MOD00240
      CP=2.15                                                             MOD00250
      K=5.                                                                MOD00260
      KXD=3.                                                              MOD00270
      KX=4.                                                               MOD00280
      TR(1)=10.                                                           MOD00290
      TR(2)=10.                                                           MOD00300
      TR(3)=10.                                                           MOD00310
      TR(4)=10.                                                           MOD00320
      SIGMA1=1.                                                           MOD00330
      TOU=.1                                                              MOD00340
      CPG=1.                                                              MOD00350
      SIGMA2=1.                                                           MOD00360
      SIGMA3=1.                                                           MOD00370
C                                                                         MOD00380
C     COMPUTE A,B,C                                                       MOD00390
C                                                                         MOD00400
      DO 9 I=1,N                                                          MOD00410
      DO 9 J=1,N                                                          MOD00420
9     A(I,J)=0.0                                                          MOD00430
      DO 5 I=1,N                                                          MOD00440
      DO 5 J=1,L                                                          MOD00450
      B(I,J)=0.0                                                          MOD00460
      A(1,2)=1.                                                           MOD00470
5     A(2,1)=-CP                                                          MOD00480
                                                                         MOD00490
                                                                         MOD00500
                                                                         MOD00510
```

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```
      A(2,2)=-BP
      A(2,3)=CPG
      A(3,1)=(CP-KX)*K/TOU
      A(3,3)=(BP-KXD)*K/TOU
      A(3,4)=-(CPG*K+1.)/TOU
      A(3,5)=K/TOU
      A(3,6)=-KXD*K/TOU
      A(3,7)=-K/TOU
      A(3,8)=A(3,7)
      A(4,1)=A(3,1)/K
      A(4,2)=A(3,2)/K
      A(4,3)=-CPG/TOU
      A(4,5)=-KXD/TOU
      A(4,6)=-1./TOU
      A(4,7)=-KX/TOU
      A(4,8)=KX/TOU
      A(5,5)=-1./TR(1)
      A(6,6)=-1./TR(2)
      A(7,7)=-1./TR(3)
      A(8,8)=-1./TR(4)
C
      B(5,1)=1.
      B(6,2)=1.
      B(7,3)=1.
C
      DO 1 I=1,M
      DO 1 J=1,N
    1 H(I,J)=0.0
      H(1,1)=1.
      H(1,7)=1.
      H(2,2)=1.
      H(3,1)=-CP
      H(3,2)=-BP
      H(3,3)=CPG
      H(3,6)=1.
C
      DO 4 I=1,L
      DO 4 J=1,L
   20 Q(I,J)=0.0
      Q(2,2)=(2./TR(1))*SIGMA1**2
      Q(2,2)=(2./TR(2))*SIGMA2**2
      Q(3,3)=(2./TR(3))*SIGMA3**2
      Q(4,4)=(2./TR(4))*SIGMA4**2
C
      NPT=3
      T1=0.0
```

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```
      T2=60.
      CALL ECHO(A,B,Q,H,TR,N,M,L)                              MOD01000
      CALL VARY(B,L,NPT,T1,T2)                                 MOD01010
      CCALL MODAL(A,B,H,N,M,L)                                 MOD01020
      STOP                                                     MOD01030
      END                                                      MOD01040
C*********************************************                 MOD01050
C                                                              MOD01060
C                  MAIN 2                                      MOD01070
C                                                              MOD01080
C*********************************************                 MOD01090
C     MAIN 2 IS THE CODE USED TO GENERATE THE A,B AND C        MOD01100
C     MATRICIES FOR THE FIRST ALTERNATE MEASUREMENT SCHEME.    MOD01110
C     IN THIS CASE ACCELERATION IS MEASURED,POSITION           MOD01120
C     AND VELOCITY ARE ESTIMATED.                              MOD01130
C*********************************************                 MOD01140
      DIMENSION B(8,4),H(3,8),TR(2)                            MOD01150
      REAL K,KX,KXD                                            MOD01160
      COMMON BP,CP,C,KX,KXD,TOU,CPG,N,PD(64),Q(8,8),A(8,8)     MOD01170
      WRITE(6,100)                                             MOD01180
100   FORMAT(20X,90('*'),//,40X,'MODEL 2','MEASURED ACCELERATION',  MOD01190
     L' IMPLIED POSITION AND VELOCITY',//,20X,90('*'))         MOD01200
      N=8                                                      MOD01210
      M=3                                                      MOD01220
      L=2                                                      MOD01230
      NCOUNT=0                                                 MOD01240
C                                                              MOD01250
C     INITIALIZE MATRICIES TO ZERO                             MOD01260
C                                                              MOD01270
      DO 1 I=1,N                                               MOD01280
      DO 1 J=1,N                                               MOD01290
1     A(I,J)=0.0                                               MOD01300
      DO 2 I=1,N                                               MOD01310
      DO 2 J=1,L                                               MOD01320
2     B(I,J)=0.0                                               MOD01330
      DO 3 I=1,L                                               MOD01340
      DO 3 J=1,L                                               MOD01350
3     Q(I,J)=0.0                                               MOD01360
      DO 4 I=1,M                                               MOD01370
      DO 4 J=1,N                                               MOD01380
4     H(I,J)=0.0                                               MOD01390
C                                                              MOD01400
C     CALCULATE THE A,B,C MATRICIES                            MOD01410
C                                                              MOD01420
      A(1,2)=1.                                                MOD01430
      A(2,1)=-CP                                               MOD01450
      A(2,2)=-BP                                               MOD01460
      A(2,3)=CPG                                               MOD01470
```

```
      A(3,1)=CP*K/TOU
      A(3,2)=BP*K/TJJ
      A(3,3)=-(CPG*K+1.)/TOU
      A(3,4)=-K/TOU
      A(3,5)=-K*KXD/TOU
      A(3,6)=-K*KX/TOU
      A(3,7)=-K/TJJ
      A(3,8)=K*KX/TJJ
      A(4,1)=CP/TOU
      A(4,2)=BP/TOU
      A(4,3)=-CPG/TOU
      A(4,5)=-KXD/TOU
      A(4,6)=-KX/TOU
      A(4,7)=-1./TOU
      A(4,8)=KX/TOU
      A(5,1)=-CP
      A(5,2)=-BP
      A(5,3)=-CPG
      A(6,5)=1.
      A(7,7)=-1./TR(1)
      A(8,8)=-1./TR(2)
C
      B(7,1)=1.
      B(8,2)=0.
      H(1,1)=1.
      H(2,2)=1.
      H(3,1)=-CP
      H(3,2)=-BP
      H(3,3)=-CPG
      SIGMA1=1.
      SIGMA2=1.
      H(3,7)=1.
C
      Q(1,1)=(2./TR(1))*SIGMA1**2
      Q(2,2)=(2./TR(2))*SIGMA2**2
      NPT=1
      T1=0.0
      T2=.5
      CALL ECHO(A,B,Q,H,TR,N,M,L)
      CALL VARY(B,L,NPT,T1,T2)
      CALL MODAL(A,B,H,N,M,L)
      STOP
1151  END
C***************************************************
C                    MAIN 3
C     MAIN 3 IS THE CODE USED TO GENERATE THE A,B AND C MATRICIES
```

```
MODO1480
MODO1490
MODO1500
MODO1510
MODO1520
MODO1530
MODO1540
MODO1550
MODO1560
MODO1570
MODO1580
MODO1590
MODO1600
MODO1610
MODO1620
MODO1630
MODO1640
MODO1650
MODO1660
MODO1670
MODO1680
MODO1690
MODO1700
MODO1710
MODO1720
MODO1730
MODO1740
MODO1750
MODO1760
MODO1770
MODO1780
MODO1790
MODO1800
MODO1810
MODO1820
MODO1830
MODO1840
MODO1850
MODO1860
MODO1870
MODO1880
MODO1890
MODO1900
MODO1910
MODO1920
MODO1930
MODO1940
MODO1950
```

```
C    FOR THE SECOND ALTERNATE MEASUREMENT SCHEME.IN THIS                  MOD01960
C    CASE POSITION AND VELOCITY ARE MEASURED AND ACCELERATION IS          MOD01970
C    ESTIMATED.                                                           MOD01980
C*****************************************************************        MOD01990
     DIMENSION B(7,3),TR(3),H(3,7)                                        MOD02000
     REAL K,KX,KXD                                                        MOD02010
     COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(49),Q(7,7),A(7,7)                 MOD02020
     WRITE(6,100)                                                         MOD02030
100  FORMAT(20X,90('*'),//,40X,'MODEL 3 MEASURED POSITION AND VELOCI'     MOD02040
    1,'TY IMPLIED ACCELERATION',//,20X,90('*'))                          MOD02050
     N=7                                                                  MOD02060
     M=3                                                                  MOD02070
     L=3                                                                  MOD02080
     NCOUNT=0                                                             MOD02090
     WRITE(6,502)                                                         MOD02100
502  FORMAT(//,10X,' ERROR DUE TO POSITION MEASUREMENT',//)              MOD02110
     SIGMA1=1.                                                            MOD02120
     SIGMA2=1.                                                            MOD02130
     SIGMA3=1.                                                            MOD02140
C                                                                         MOD02150
C    COMPUTE A,B,C,Q                                                      MOD02160
C                                                                         MOD02170
C    INITIALIZE ALL MATRICIES                                             MOD02180
C                                                                         MOD02190
     DO 1 I=1,N                                                           MOD02200
     DO 1 J=1,N                                                           MOD02210
1    A(I,J)=0.0                                                           MOD02220
     DO 2 I=1,N                                                           MOD02230
     DO 2 J=1,L                                                           MOD02240
2    B(I,J)=0.0                                                           MOD02250
     DO 3 I=1,N                                                           MOD02260
     DO 3 J=1,L                                                           MOD02270
3    Q(I,J)=0.0                                                           MOD02280
     DO 4 I=1,M                                                           MOD02290
     DO 4 J=1,N                                                           MOD02300
4    H(I,J)=0.0                                                           MOD02310
C                                                                         MOD02320
     A(1,2)=1.                                                            MOD02330
     A(2,1)=-CP                                                           MOD02340
     A(2,2)=-BP                                                           MOD02350
     A(2,3)=CPG                                                           MOD02360
     A(3,1)=K*(CP-KX)/TOU                                                 MOD02370
     A(3,2)=K*(BP-KXD)/TOU                                                MOD02380
     A(3,3)=-(K*CPG+1.)/TOU                                               MOD02390
     A(3,4)=K/TOU                                                         MOD02400
     A(3,5)=A(3,1)                                                        MOD02410
     A(3,6)=A(3,2)                                                        MOD02420
```

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```
      A(3,7)=K*KX/TOU                                                   MOD02440
      A(4,1)=(CP-KX)/TOU                                                MOD02450
      A(4,2)=(BP-KXD)/TOU                                               MOD02460
      A(4,3)=-CPG/TOU                                                   MOD02470
      A(4,5)=A(4,1)                                                     MOD02480
      A(4,6)=A(4,2)                                                     MOD02490
      A(4,7)=KX/TOU                                                     MOD02500
      A(5,5)=-1./TR(1)                                                  MOD02510
      A(6,6)=-1./TR(2)                                                  MOD02520
      A(7,7)=-1./TR(3)                                                  MOD02530
      B(5,1)=1.                                                         MOD02540
      B(6,2)=0.                                                         MOD02550
      B(7,3)=0.                                                         MOD02560
      H(1,1)=1.                                                         MOD02570
      H(2,2)=1.                                                         MOD02580
      H(3,1)=-CP                                                        MOD02590
      H(3,2)=-BP                                                        MOD02600
      H(3,3)=CPG                                                        MOD02610
C                                                                       MOD02620
      Q(1,1)=(2./TR(1))*SIGMA1**2                                       MOD02630
      Q(2,2)=(2./TR(2))*SIGMA2**2                                       MOD02640
      Q(3,3)=(2./TR(3))*SIGMA3**2                                       MOD02650
      NPT=40                                                            MOD02660
      T1=0.                                                             MOD02670
      T2=20.                                                            MOD02680
      CALL ECHO(A,B,Q,H,TR,N,M,L)                                       MOD02690
      CALL VARY(B,L,NPT,T1,T2)                                          MOD02700
      CALL MODAL(A,B,H,N,M,L)                                           MOD02710
      STOP                                                              MOD02720
      END                                                              MOD02730
C***********************************************************************MOD02740
C***********************************************************************MOD02750
C                             HARRIER 1                                 MOD02760
C  HARRIER 1 IS THE CODE USED TO GENERATE THE A,B AND C MATRICIES       MOD02770
C  FOR THE COVARIANCE CALCULATIONS OF THE SRFIMF CONTROLLER             MOD02780
C  AND THE LONGITJDNAL MODE OF THE HARRIER.                             MOD02790
C***********************************************************************MOD02800
C***********************************************************************MOD02810
      DIMENSION B(9,31),H(3,9),TR(3)                                    MOD02820
      REAL K,KX,KXD                                                     MOD02830
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(81),Q(9,9),A(9,9)              MOD02840
      WRITE(6,100)                                                      MOD02850
  100 FORMAT(//,20X,90('*'),//,40X,'HARRIER MODEL ',                    MOD02860
     1. ALL QUANTITIES MEASURED',//,20X,90('*'),//)                     MOD02870
      N=9                                                               MOD02880
      L=3                                                               MOD02890
      M=3                                                               MOD02900
      K=20.                                                             MOD02910
```

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```
      KX=4.
      KXD=3.
      TOU=.1
      SIGMA1=1.
      SIGMA2=1.
      SIGMA3=1.
      TR(1)=10.
      TR(2)=10.
      TR(3)=10.
C
C     CALCULATE THE A MATRIX
C
      DO 1 I=1,N
      DO 1 J=1,N
1     A(I,J)=0.0
      A(1,2)=1.
      A(2,3)=1.
      A(2,5)=.25
      A(3,4)=1.
      A(3,5)=-.06115
      A(4,1)=-.00748
      A(4,2)=-.0675
      A(4,3)=.45
      A(4,4)=.4906
      A(4,5)=-.14428
      A(5,1)=-K*KX/TOU
      A(5,2)=-K*KXD/TOU
      A(5,3)=-K/TOU
      A(5,5)=-(.25*K+1.)/TOU
      A(5,6)=K/TOU
      A(5,7)=-K*KXD/TOU
      A(5,8)=-K/TOU
      A(5,9)=-K*KX/TOU
      A(6,1)=-K*KX/TOU
      A(6,2)=-KXD/TOU
      A(6,3)=-1./TOJ
      A(6,5)=-.25/TOU
      A(6,7)=-KXD/TOU
      A(6,8)=-1./TOJ
      A(6,9)=-Kx/TOU
      A(7,7)=-1./TR(1)
      A(8,8)=-1./TR(2)
      A(9,9)=-1./TR(3)
C
C     B MATRIX
C
      DO 2 I=1,N
      DO 2 J=1,L
```

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```
    2 B(I,J)=0.0                                                    MODD03400
      B(7,1)=1.                                                     MODD03410
      B(8,2)=1.                                                     MODD03420
      B(9,3)=1.                                                     MODD03430
   20 CONTINUE                                                      MODD03440
C                                                                   MODD03450
C     Q MATRIX                                                      MODD03460
C                                                                   MODD03470
      DO 3 I=1,M                                                    MODD03480
      DO 3 J=1,M                                                    MODD03490
    3 Q(I,J)=0.0                                                    MODD03500
      Q(1,1)=(2./TR(1))*SIGMA1**2                                   MODD03510
      Q(2,2)=(2./TR(2))*SIGMA2**2                                   MODD03520
      Q(3,3)=(2./TR(3))*SIGMA3**2                                   MODD03530
C                                                                   MODD03540
C     C MATRIX                                                      MODD03550
C                                                                   MODD03560
      DO 4 I=1,M                                                    MODD03570
      DO 4 J=1,N                                                    MODD03580
    4 H(I,J)=0.0                                                    MODD03590
      H(1,1)=1.0                                                    MODD03600
      H(1,9)=1.                                                     MODD03610
      H(2,2)=1.0                                                    MODD03620
      H(2,7)=1.00                                                   MODD03630
      H(3,3)=1.0                                                    MODD03640
      H(3,8)=1.                                                     MODD03650
      NPT=40                                                        MODD03660
      T1=0.                                                         MODD03670
      T2=20.                                                        MODD03680
      CALL ECHO(A,B,Q,H,TR,N,M,L)                                   MODD03690
      CALL MODAL(A,B,H,N,M,L)                                       MODD03700
      CALL VARY(B,L,NPT,T1,T2)                                      MODD03710
      STOP                                                          MODD03720
      END                                                           MODD03730
C*********************************************************          MODD03740
C                                                                   MODD03750
C           HARRIER G                                               MODD03760
C                                                                   MODD03770
C     HARRIER G IS THE CODE USED TO GENERATE THE A,B AND C          MODD03780
C     MATRICIES FOR THE COVARIANCE CALCULATION OF THE HARRIER       MODD03790
C     LONGITUDNAL AXIS SRFIMF CONTROL WITH A GUST INPUT.            MODD03800
C*********************************************************          MODD03810
      DIMENSION B(10,4),H(3,10),TR(4)                               MODD03820
      REAL K,KX,KXD                                                 MODD03830
      COMMON BP,CP,K,KX,KXD,TDU,CPG,N,PD(100),Q(10,10),A(10,10)     MODD03840
      WRITE(6,100)                                                  MODD03850
  100 FORMAT(//,20X,90('*'),//,40X,'HARRIER MODEL ',                MODD03860
                                                                    MODD03870
```

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```
      1 ' ALL QUANTITIES MEASURED',//,20X,90('*'),//)                  MOD03880
        N=10                                                           MOD03890
        L=4                                                            MOD03900
        M=3                                                            MOD03910
        K=20.                                                          MOD03920
        KX=4.                                                          MOD03930
        KXD=3.                                                         MOD03940
        TOU=.1                                                         MOD03950
        SIGMA1=1.                                                      MOD03960
        SIGMA2=1.                                                      MOD03970
        SIGMA3=1.                                                      MOD03980
        SIGMA4=1.                                                      MOD03990
        TR(1)=10.                                                      MOD04000
        TR(2)=10.                                                      MOD04010
        TR(3)=10.                                                      MOD04020
        TR(4)=6.5                                                      MOD04030
C                                                                      MOD04040
C     CALCULATE THE A MATRIX                                           MOD04050
C                                                                      MOD04060
        DO 1 I=1,N                                                     MOD04070
        DO 1 J=1,N                                                     MOD04080
      1 A(I,J)=0.0                                                     MOD04090
        A(1,2)=1.                                                      MOD04100
        A(1,10)=-.0026                                                 MOD04110
        A(2,3)=1.25                                                    MOD04120
        A(2,5)=-.00473                                                 MOD04130
        A(3,4)=1.                                                      MOD04140
        A(3,5)=-.06115                                                 MOD04150
        A(3,10)=-.00306                                                MOD04160
        A(4,1)=-.00748                                                 MOD04170
        A(4,2)=-.0675                                                  MOD04180
        A(4,3)=-.45                                                    MOD04190
        A(4,4)=-.4906                                                  MOD04200
        A(4,5)=-.14428                                                 MOD04210
        A(4,10)=-.00346                                                MOD04220
        A(5,1)=-K*KX/TOU                                               MOD04230
        A(5,2)=-K*KXD/TOU                                              MOD04240
        A(5,3)=-K/TOU                                                  MOD04250
        A(5,5)=-(.25*K+1.)/TOU                                         MOD04260
        A(5,6)=K/TOU                                                   MOD04270
        A(5,7)=-K*KXD/TOU                                              MOD04280
        A(5,8)=-K/TOU                                                  MOD04290
        A(5,9)=-K*KX/TOU                                               MOD04300
        A(5,10)=(KXD*.0026+.00473)*K/TOU                               MOD04310
        A(6,1)=-KX/TOU                                                 MOD04320
        A(6,2)=-KXD/TOU                                                MOD04330
        A(6,3)=-1./TOU                                                 MOD04350
```

```
      A(6,5)=-.25/TOU
      A(6,7)=-KXD/TOU
      A(6,8)=-1./TOU
      A(6,9)=-KX/TOU
      A(6,10)=A(5,10)/K
      A(7,7)=-1./TR(1)
      A(8,8)=-1./TR(2)
      A(9,9)=-1./TR(3)
      A(10,10)=-1./TR(4)
C
C     B MATRIX
C
      DO 2 I=1,N
      DO 2 J=1,L
      B(I,J)=0.0
      B(7,1)=1.
      B(8,2)=1.
      B(9,3)=1.
      B(10,4)=1.
C
C     Q MATRIX
C
      DO 3 I=1,L
      DO 3 J=1,L
      Q(I,J)=0.0
      Q(1,1)=(2./TR(1))*SIGMA1**2
      Q(2,2)=(2./TR(2))*SIGMA2***2
      Q(3,3)=(2./TR(3))*SIGMA3***2
      Q(4,4)=(2./TR(4))*SIGMA4**2
C
C     C MATRIX
C
      DO 4 I=1,M
      DO 4 J=1,N
      H(I,J)=0.0
      H(1,1)=1.00
      H(2,2)=1.00
      H(3,3)=1.0
      NPT=60
      T1=0.
      T2=60.
      CALL ECHO(A,B,Q,H,TR,N,M,L)
C     CALL MODAL(A,B,H,N,M,L)
      CALL VARY (B,L,NPT,T1,T2)
      STOP
      END
```

MOD04360
MOD04370
MOD04380
MOD04390
MOD04400
MOD04410
MOD04420
MOD04430
MOD04440
MOD04450
MOD04460
MOD04470
MOD04480
MOD04490
MOD04500
MOD04510
MOD04520
MOD04530
MOD04540
MOD04550
MOD04560
MOD04570
MOD04580
MOD04590
MOD04600
MOD04610
MOD04620
MOD04630
MOD04640
MOD04650
MOD04660
MOD04670
MOD04680
MOD04690
MOD04700
MOD04710
MOD04720
MOD04730
MOD04740
MOD04750
MOD04760
MOD04770
MOD04780
MOD04790
MOD04800
MOD04810

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```
C***********************************************************************  MODO00040
C                                                                        MODO00050
C           SJBROUTINE MODAL (A,B,H,N,M,L)                               MODO00060
C                                                                        MODO00070
C     THIS SUBROUTINE IS USED TO TRANSFORM A SYSTEM OF THE FORM          MODO00080
C                                                                        MODO00090
C           XDOT=AX+BU                                                   MODO00100
C                                                                        MODO00110
C           Y=CX                                                         MODO00120
C                                                                        MODO00130
C     TO MODAL COORDINATES.                                              MODO00140
C                                                                        MODO00150
C***********************************************************************  MODO00160
      SUBROUTINE MODAL(A,B,H,N,M,L)                                      MODO00170
      DIMENSION A(N,N),B(N,L),H(M,N),AC(10,10),C(10,10),D(10,10)         MODO00180
     1,G(4,10),T(10,10)                                                  MODO00190
     2COMPLEX Z(10,10),W(10),WK(99),ENORM,VNORM                          MODO00200
      REAL K,KX,KXD                                                      MODO00210
      DO 1 I=1,N                                                         MODO00220
      DO 1 J=1,N                                                         MODO00230
    1 AC(I,J)=A(I,J)                                                     MODO00240
      WRITE(6,210)                                                       MODO00250
  210 FORMAT(//,20X,70('*'),//,20X,'*',/,20X,'*',78('*'),/,//)   MODAL ANALYSIS  MODO00260
    1FOR THE SYSTEM',/,20X,'*',/,20X,'*',78('*'),/,//)                   MODO00270
      IA=N                                                               MODO00280
      IZ=N                                                               MODO00290
      IJOB=1                                                             MODO00300
      CALL EIGRF(A,N,N,IJOB,W,Z,IZ,WK,IER)                              MODO00310
C                                                                        MODO00320
C     WRITE OUT THE EIGENCVALUES                                         MODO00330
C                                                                        MODO00340
      WRITE(6,201)                                                       MODO00350
  201 FORMAT(//,10X,'THE EIGENVALUES ARE',//)                            MODO00360
      WRITE(6,101) (W(I),I=1,N)                                          MODO00370
  101 FORMAT(1X,16F6.2,/)                                                MODO00380
C                                                                        MODO00390
C     WRITE THE EIGENVECTORS_                                            MODO00400
C                                                                        MODO00410
      WRITE(6,202)                                                       MODO00420
  202 FORMAT(//,10X,'THE EIGENVECTORS ARE',//)                           MODO00430
      WRITE(6,101) ((Z(I,J),J=1,N),I=1,N)                                MODO00440
C                                                                        MODO00450
C     EIGENVECTOR NORMALIZATION                                          MODO00460
C                                                                        MODO00470
      J=1                                                                MODO00480
   10 CONTINUE                                                           MODO00490
```

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```
      FNORM=0.0
      IF(AIMAG(W(J)).NE.0) GO TO 4
C
C     EIGENVECTOR IS REAL
C
      DO 2 I=1,N
    2 FNORM=FNORM+REAL(Z(I,J)**2)
      DO 3 I=1,N
    3 T(I,J)=REAL(Z(I,J))/SQRT(FNORM)
      GO TO 7
C
C     COMPLEX NORMALIZATION
C
    4 DO 5 I=1,N
      TAT=REAL((Z(I,J))**2
      IF(TAT.LE.FNORM) GO TO 5
      IMAX=I
      FNORM=TAT
    5 CONTINUE
C
C     THE I'TH ROW WILL BE NORMALIZED TO 1+0I
C
      XN1=REAL(Z(IMAX,J))
      XN2=AIMAG(Z(IMAX,J))
      ENORM=CMPLX(XN1/(XN1+XN2**2),-XN2/(XN1+XN2**2))
      DO 6 I=1,N
      VNORM=Z(I,J)*ENORM
      T(I,J)=REAL(VNORM)
    6 T(I,J+1)=AIMAG(VNORM)
      J=J+1
    7 J=J+1
      IF(J.LE.N) GO TO 10
C
C     WRITE THE NORMALIZED MATRIX
C
      WRITE(5,203)
  203 FORMAT(//,10X,'THE NORMALIZED TRANSFER MATRIX IS',//)
      WRITE(6,180) ((T(I,J),J=1,N),I=1,N)
C
C     INVERT T
C
      IDGT=0
      CALL LINV2F(T,N,IA,A,IDGT,WK,IER)
      WRITE(6,204)
  204 FORMAT(//,10X,'THE INVERSE OF THE TRANSFORM IS',//)
      WRITE(6,180) ((A(I,J),J=1,N),I=1,N)
C
C     TAKE T INV*A*T
```

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```
C
      CALL VMULFF(A,AC,N,N,N,IA,C,N,IER)                      MOD01000
      CALL VMULFF(C,T,N,N,N,IA,IA,AC,N,IER)                   MOD01010
      WRITE(6,205)                                            MOD01020
205   FORMAT(//,10X,'THE DIAGINAL MATRIX IS',//)              MOD01030
      WRITE(6,180) ((AC(I,J),J=1,N),I=1,N)                    MOD01040
C                                                             MOD01050
C     CALCULATE THE CONTROL MATRIX                            MOD01060
C                                                             MOD01070
      WRITE(6,206)                                            MOD01080
206   FORMAT(//,10X,'THE CONTROL MATRIX IS',//)               MOD01090
      CALL VMULFF(A,B,N,N,L,N,N,D,N,IER)                      MOD01100
      DO 30 I=1,N                                             MOD01110
30    WRITE(6,104) (D(I,J),J=1,L)                             MOD01120
104   FORMAT(/,8(2X,F7.2,2X))                                 MOD01130
C                                                             MOD01140
C     CALCULATE Y=CI                                          MOD01150
C                                                             MOD01160
      WRITE(6,207)                                            MOD01170
207   FORMAT(//,10X,'THE MEASUREMENT VECTOR IS',//)           MOD01180
      CALL VMULFF(H,T,M,N,M,IA,G,M,IER)                       MOD01190
      WRITE(6,100) ((G(I,J),J=1,V),I=1,M)                     MOD01200
100   FORMAT(/,10(2X,F7.2))                                   MOD01210
      RETURN                                                  MOD01220
      END                                                     MOD01230
C*****************************************************        MOD01240
C*****************************************************        MOD01250
C*****************************************************        MOD01260
C*****************************************************        MOD01270
C*****************************************************        MOD01280
C            SUBROUTINE VARY (B,L,NPT,T1,T2)                  MOD01290
C                                                             MOD01300
C     VARY IS USED TO CALCULATE THE COVARIANCE OF THE STATES OF MOD01310
C     A SYSTEM.THE COVARIANCE IS FOUND BY SOLVING THE LYAPUNOV  MOD01320
C     DIFFERENTIAL EQUATION USING THE ISML SUBROUTINE DVERK.    MOD01330
C*****************************************************        MOD01340
C*****************************************************        MOD01350
      SUBROUTINE VARY (B,L,NPT,T1,T2)                         MOD01360
      DIMENSION B(10,4),P(100),PDOT(100),D(10,10),W(100,10),  MOD01370
     1G(10,10),C(24)                                          MOD01380
      EXTERNAL FCN                                            MOD01390
      COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(100),Q(10,10),A(10,10) MOD01400
      WRITE(6,200)                                            MOD01410
200   FORMAT(//,20X,90('*'),//,40X,'VARIANCE ANALYSIS',       MOD01420
     1//,20X,90('*'),/)                                       MOD01430
C                                                             MOD01440
C     START CLACULATIONS                                      MOD01450
```

```
C        TST=.00005
         NSQ=N*N
         NW=NSQ
         TIME=T1
         TOL=.001
         XNPT=NPT
         STEP=(T2-T1)/XNPT
         DO 1 I=1,N
         DO 1 J=1,N
         G(I,J)=0.0
         DO 1 K=1,L
    1    G(I,J)=G(I,J)+B(I,K)*Q(K,J)
C
         G=B*QA
C
         DO 2 I=1,N
         DO 2 J=1,N
         Q(I,J)=0.0
         DO 2 K=1,L
    2    Q(I,J)=Q(I,J)+G(I,K)*B(J,K)
C
         Q=B*Q*BT
C
C        NOW SET INITIAL VALUES OF P
C
         DO 3 I=1,N
         DO 3 J=1,N
    3    G(I,J)=0.0
C
C        G IS THE MATRIX OF THE P VECTOR SET TO 0.0
C
C        SET UP TO CALL DVERK
C
         DO 4 I=1,24
    4    C(I)=0.0
         WRITE(6,700) TIME
  700    FORMAT(/,10X,'AT TIME= ',F5.2,/)
         CALL MTV(N,NSQ,G,P)
         TIMEE=TIME+STEP
         IND=1
         CALL DVERK (NSQ,FCN,TIME,P,TIMEE,TOL,IND,C,NW,W,IER)
         WRITE(6,750) IND
  750    FORMAT(10X,'IND=',I3,' AFTER CALL TO DVERK')
         CALL VTM (N,NSQ,G,P)
         WRITE (6,700) TIME
         WRITE(6,800)
  800    FORMAT(//,20X,'THE P MATRIX IS',//)
```

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```
            WRITE(6,100) ((G(I,J),J=1,N),I=1,N)                        MOD001960
            X5=SQRT(G(5,5))                                            MOD001970
      C     CALL VTM (N,NSQ,D,PD)                                      MOD001980
  801       WRITE(6,801)                                              MOD001990
      C     FORMAT(//,20X,'P DOT IS',[/)                               MOD002000
            WRITE(6,100) ((D(I,J),J={,N),I=1,N)                        MOD002010
            SUM=0.0                                                    MOD002020
            DO 5 I=1,NSQ                                               MOD002030
  5         SUM=SU4+ABS(PD(I))                                         MOD002040
            WRITE(6,802) SJM                                           MOD002050
  802       FORMAT(10X,'THE SUM OF THE P DOT TERMS IS ',E15.5)         MOD002060
            IF(SUM.LE.†ST) GO TO 50                                    MOD002070
            IF(TIMEE.GE.T2) GO TO 51                                   MOD002080
            GO TO 20                                                   MOD002090
  50        WRITE(6,710)                                               MOD002100
  710       FORMAT(//,20X,'THE LAST P IS A STEADY STATE',//)           MOD002110
            RETURN                                                     MOD002120
  51        CALL VTM (N,NSQ,G,PD)                                      MOD002130
            WRITE (6,720)                                              MOD002140
  720       FORMAT(//,20X,'END OF RUN THE FINAL PDOT IS',//)           MOD002150
            WRITE(6,100) ((G(I,J),J=1,N),I=1,N)                        MOD002160
  100       FORMAT(10(2X,E10.4),/)                                     MOD002170
            RETURN                                                     MOD002180
            END                                                        MOD002190
                                                                       MOD002200
      C           SUBROUTINE FCN (NSQ,TIME,P,PDOT)                     MOD002210
      C                                                                MOD002220
      C     FCN IS USED BY DVERK.IT PROVIDES THE FORM OF THE           MOD002230
      C     DIFFERENTIAL EQUATION.                                     MOD002240
      C                                                                MOD002250
            SUBROUTINE FCN (NSQ,TIME,P,PDOT)                           MOD002260
            DIMENSION D(10,10),G(10,10),P(NSQ),PDOT(NSQ)               MOD002270
            COMMON BP,CP,K,KX,KXD,TOU,CPG,N,PD(100),Q(10,10),A(10,10)  MOD002280
            CALL VTM(N,NSQ,G,β)                                        MOD002290
            DO 1 I=1,N                                                 MOD002300
            DO 1 J=1,N                                                 MOD002310
            D(I,J)=Q(I,J)                                              MOD002320
            DO 1 K=1,N                                                 MOD002330
  1         D(I,J)=D(I,J)+A(I,K)*G(K,J)+G(I,K)*A(J,K)                  MOD002340
            CALL MTV (N,NSQ,D,PDOT)                                    MOD002350
            DO 2 I=1,NSQ                                               MOD002360
  2         PD(I)=PDOT(I)                                             MOD002370
            RETURN                                                     MOD002380
            END                                                        MOD002390
                                                                       MOD002400
      C     SUBROUTINES MTV AND VTM                                    MOD002410
      C                                                                MOD002420
            SUBROUTINE MTV (N,NSQ,A,B)                                 MOD002430
```

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```
      DIMENSION A(N,N),B(NSQ)
      K=1
      DO 1 I=1,N
      DO 1 J=1,N
    1 B(K)=A(I,J)
      K=K+1
      DEBUG SUBCHK
      RETURN
      END
C
      SUBROUTINE VTM (N,NSQ,A,B)
      DIMENSION A(N,N),B(NSQ)
      K=1
      DO 1 I=1,N
      DO 1 J=1,N
    1 A(I,J)=B(K)
      K=K+1
      DEBUG SUBCHK
      RETURN
      END
```

```
MODO2440
MODO2450
MODO2460
MODO2470
MODO2480
MODO2490
MODO2500
MODO2510
MODO2520
MODO2530
MODO2540
MODO2550
MODO2560
MODO2570
MODO2580
MODO2590
MODO2600
MODO2610
MODO2620
MODO2630
```

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```
***********************************************************
*
*      **    HARRIER LONDITUDNAL SRFIMF CONTROL SIMULATION   **
*
***********************************************************
// EXEC CSMPXV
//X.COMPRINT DD DUMMY
//X.SYSPRINT DD DUMMY
//X.SYSIN DD *
*
*       SIMULATION OF OPEN LOOP PLANT
*
X1P=INTGRL(0.0,X2P)
X2P=INTGRL(0.0,X3P)
X3P=INTGRL(0.0,X4P)
X4P=INTGRL(0.0,X4PDOT)
X4PDOT=.25*XC-.49*X4P+.45*X3P-.0665*X2P-.00748*X1P
ACCELP=-.00189*X1P-.016625*X2P+.1144*X3P-.061*X4P+.25*XC
POSITP=.00189*X1P+.0615*X2P+.25*X3P
VELP=.00189*X2P+.0615*X3P+.25*X4P
*
*
*       CLOSED LOOP SYSTEM SIMULATION
*
        X1=INTGRL(0.0,X2)
        X2=INTGRL(0.0,X3)
        X3=INTGRL(0.0,X4)
        X4=INTGRL(0.0,X4DOT)
        X4DOT=X5-.49*X4+.45*X3-.0665*X2-.00748*X1
        ACCEL=-.00189*X1-.016625*X2+.1144*X3-.061*X4+.25*X5
        POSIT=.00189*X1+.0615*X2+.25*X3
        VEL=.00189*X2+.0615*X3+.25*X4
        X5=REALPL(0.0,TOU,IS)
        XC=STEP(0.0)
        WS=-KXD*VX+KX*(XC-X)+VS
        IS=K*WS
        VS=AS-AX
        AS=REALPL(0.0,TOU,WS)
        AX=ACCEL
        VX=VEL
        X=POSIT
        K=100.
        KX=4.
        KXD=3.
        TOU=.1
TIMER FINTIM=10,OUTDEL=.2
OUTPUT POSIT,POSITP
OUTPUT TIME,POSIT,POSITP
PAGE XYPLOT
        END
        STOP
```

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TABULATED DATA

Time (sec)	σ_{1p}	σ_{1v}	σ_{1a}	σ_{2p}	σ_{2v}	σ_{2a}	σ_{3p}	σ_{3v}	σ_{3a}
0	0	0	0	0	0	0	0	0	0
.5	.045	.033	.011	.191	.149	.050	.764	.573	.191
1.0	.164	.123	.074	.315	.236	.079	.825	.619	.206
2.0	.410	.307	.102	.342	.256	.085	1.022	.767	.256
3.0	.564	.423	.141	.344	.258	.086	1.283	.962	.321
4.0	.660	.495	.165	.347	.260	.087	1.475	1.106	.369
5.0	.728	.546	.182	.348	.261	.087	1.613	1.210	.403
6.0	.780	.585	.195	.349	.262	.087	1.717	1.288	.429
7.0	.819	.615	.205	.350	.262	.087	1.797	1.348	.449
8.0	.851	.638	.213	.351	.263	.088	1.861	1.395	.465
9.0	.875	.657	.219	.351	.263	.088	1.911	1.433	.478
10.0	.895	.671	.224	.352	.264	.088	1.950	1.464	.482
20.0	.968	.726	.242	.354	.265	.088	2.100	1.576	.525
40.0	.979	.734	.245	.354	.265	.089	2.123	1.592	.531
60.0	.979	.734	.245	.354	.266	.089	2.123	1.592	.531
S.S.	.979	.734	.245	.354	.266	.089	2.123	1.592	.531

Table B-1 Tabulated Data
for figures 4-4 through 4-6.

Time (sec)	σ_{1a}	σ_{2a}	σ_{3a}
0	0	0	0
2.0	.530	.675	2.005
4.0	2.858	1.781	7.745
6.0	8.070	3.060	18.515
8.0	14.649	4.412	34.742
10.0	24.399	5.791	56.569

The rate of change in P, $\dot{P}$ after 10 seconds was

	$\sigma_{\dot{P}1}$	$\sigma_{\dot{P}2}$	$\sigma_{\dot{P}3}$
10.0	16.422	2.826	37.350

Table B-2
Tabulated data for figure 4-7.

Time (sec)	σ_{1p}	σ_{1v}	σ_{2p}	σ_{2v}	σ_{3p}	σ_{3v}
0	0	0	0	0	0	0
.5	.020	.026	.091	.119	.351	.458
1.0	.076	.099	.149	.189	.379	.495
2.0	.189	.246	.157	.205	.470	.613
3.0	.259	.338	.158	.206	.590	.770
4.0	.304	.396	.160	.208	.679	.885
5.0	.335	.437	.160	.209	.740	.968
6.0	.359	.468	.160	.209	.790	1.030
7.0	.377	.492	.161	.210	.827	1.078
8.0	.391	.510	.161	.210	.856	1.116
9.0	.403	.525	.162	.211	.879	1.147
10.0	.412	.537	.162	.211	.898	1.171
20.0	.445	.581	.163	.212	.966	1.261
S.S	.450	.590	.163	.212	.970	1.300

Table B-3 Tabulated data for figures 4-9 through 4-11.

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Time (sec)	σ_{1p}	σ_{1v}	σ_{1a}	σ_{2p}	σ_{2v}	σ_{2a}	σ_{5p}	σ_{5v}	σ_{5a}
0	0	0	0	0	0	0	0	0	0
.5	.440	.081	.011	.201	.210	.050	2.672	2.010	.668
1.0	.166	.125	.042	.319	.239	.080	2.807	2.106	.702
2.0	.413	.310	.103	.345	.259	.086	3.237	2.428	.809
3.0	.566	.425	.142	.348	.261	.087	3.286	2.465	.822
4.0	.662	.496	.165	.350	.263	.088	3.288	2.466	.822
5.0	.730	.547	.182	.351	.264	.088	3.291	2.468	.822
6.0	.781	.586	.195	.353	.264	.088	3.300	2.475	.825
7.0	.820	.615	.205	.353	.265	.088	3.320	2.490	.830
8.0	.852	.639	.213	.354	.266	.089	3.345	2.511	.837
9.0	.876	.657	.219	.355	.266	.089	3.384	2.538	.846
10.0	.896	.672	.224	.355	.266	.089	3.425	2.570	.856
20.0	.969	.727	.242	.357	.268	.089	3.821	2.866	.955
S.S.	.980	.734	.245	.360	.270	.089	3.900	2.900	.970

Table B-4 Tabulated data for figures 5-4 through 5-6.

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Time (sec)	σ_{1g}	σ_{2g}	σ_{5g}	σ_{acc_g}	σ_g
0	0	0	0	0	0
1.0	.0013	.006	.076	.013	2.573
2.0	.0014	.009	.110	.040	3.390
3.0	.0014	.011	.227	.102	3.882
4.0	.0015	.011	.402	.191	4.207
5.0	.0015	.012	.622	.301	4.431
6.0	.0015	.012	.874	.427	4.588
7.0	.0016	.013	1.150	.564	4.701
8.0	.0016	.013	1.442	.701	4.782
9.0	.0016	.013	1.743	.860	4.840
10.0	.0016	.013	2.050	1.013	4.884
20.0	.0016	.013	4.750	2.360	4.959
40.0	.0016	.013	7.169	3.567	5.00
60.0	.0017	.013	7.783	3.874	5.00
s.s.	.0017	.013	7.800	4.00	4.00

Table B-5 Tabulated data for figure 5-7.

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