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ON MULTICHANNEL (MULTIVARIATE) MAXIMUM ENTROPY SPECTRAL ANALYSIS--ETC(U)  
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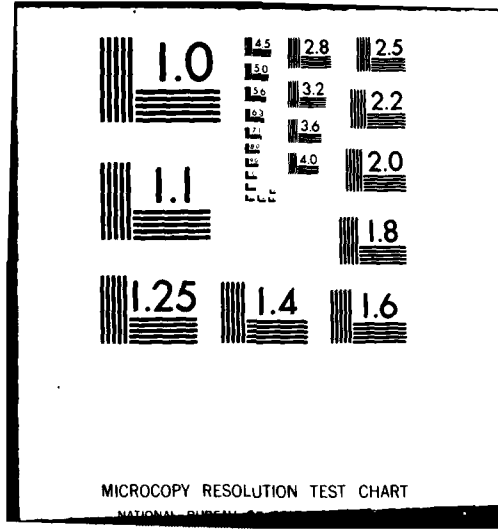
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Contract Number N00014-79-C-0494

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January 23, 1981

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# On Multichannel (Multivariate) Maximum Entropy Spectral Analysis

C.H. Chen and Chihung Yen

## 1. Introduction

The univariate maximum entropy spectral analysis has now been well developed and applied to many defense research areas such as radar, sonar and geophysics. There has been some work done to extend the maximum entropy spectral analysis to multivariate case. Whittle [1] and Robinson [2][3] generalized the Levinson-Durbin recursion to the multivariate case by fitting both forward and backward autoregressions in a stepwise fashion. In this thesis, Burg [4] has mentioned about the multichannel case. However the computer programs for both multichannel and multivariate maximum entropy spectral analysis were only recently developed successfully. Morf et.al [5] developed an algorithm for direct estimation of the normalized reflection coefficients from the observed data for maximum entropy spectral analysis. They also compared the spectral estimation with the methods of Jones [6], Nuttall [7] and Strand [8], which are more of a direct extension of Burg's work to the multichannel (multivariate) case. Burg's algorithm does not generalize directly since the forward and backward autoregression matrices are not the same in the multivariate case, and the forward and backward one-step prediction error covariance matrices are different, although they have the same determinant. In this report, the programs developed by Strand and Jones are applied to real multichannel data and imagery data in addition to a set of test signals. The merits of these methods are closely examined. In spite of programming complexity the multichannel and multivariate maximum entropy spectral analysis will have increased application as the real data are almost always gathered in several channels. Data from several channels form a vector for multivariate study.

## 2. Brief Mathematical Analysis

Let  $x_1, x_2, \dots, x_n$  denote  $n$  zero mean vectors of dimension  $d$  each. The sample estimate of covariance sequence for lag  $j$  is

$$\hat{R}_j = \frac{1}{n} \sum_{t=1}^{n-j} x_{t+j} x_t' \quad (1)$$

where the prime denotes the transposed vector. The forward and backward predicting autoregressions of order  $p$  are given, respectively, as

$$\begin{aligned} \hat{x}_t^{(f)} &= \sum_{k=1}^p A_k^{(p)} x_{t-k} \\ \hat{x}_t^{(b)} &= \sum_{k=1}^p B_k^{(p)} x_{t+k} \end{aligned} \quad (2)$$

where  $A_k^{(p)}$  and  $B_k^{(p)}$  are  $d \times d$  matrices, and can be determined recursively [6] by making use of the estimated covariance matrix in Eq. (1). The recursion starts with

$$S_0^{(f)} = S_0^{(b)} = R_0 \quad (3)$$

The one-step forward and backward prediction error covariance matrices are

$$\begin{aligned} S_p^{(f)} &= (I - A_p^{(p)} B_p^{(p)}) S_{p-1}^{(f)} \\ S_p^{(b)} &= (I - B_p^{(p)} A_p^{(p)}) S_{p-1}^{(b)} \end{aligned} \quad (4)$$

The forward and backward residuals are, respectively,

$$\begin{aligned} e_t^{(f)} &= x_t - \sum_{k=1}^p A_k^{(p)} x_{t-k}, \quad t = p+1, \dots, n \\ \beta_t^{(b)} &= x_t - \sum_{k=1}^p B_k^{(p)} x_{t+k}, \quad t = 1, \dots, n-p \end{aligned} \quad (5)$$

The recursive equations are then given by

$$\begin{aligned} e_t^{(p)} &= e_t^{(p-1)} - A_p^{(p)} \beta_{t-p}^{(p-1)}, \quad t = p+1, \dots, n \\ \beta_t^{(p)} &= \beta_t^{(p-1)} - B_p^{(p)} e_{t+p}^{(p-1)}, \quad t = 1, \dots, n-p \end{aligned} \quad (6)$$

The least squares estimates for the forward and backward autoregression matrices are

$$\begin{aligned} A_p^{(p)} &= UV^{-1} \\ B_p^{(p)} &= U'W^{-1} \end{aligned} \quad (7)$$

where  $U$  is the sum of cross products of forward and backward residuals at lag  $p$ ,

$$U = \sum_{t=1}^{n-p} e_{t+p}^{(p-1)} (\beta_t^{(p-1)})', \quad (8)$$

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and  $V$  and  $W$  are estimates of  $(n - p)S_{p-1}^{(b)}$ ,  $(n - p)S_{p-1}^{(f)}$  respectively,

$$V = \sum_{t=1}^{n-p} \beta_t^{(p-1)} (\beta_t^{(p-1)})', \quad (9)$$

$$W = \sum_{t=1}^{n-p} e_{t+p}^{(p-1)} (e_{t+p}^{(p-1)})', \quad (10)$$

Although the forward and backward autoregression matrices and the prediction error covariance matrices are different, the multivariate spectra should be identical when calculated from the forward and backward fits by

$$S(f) = h[A(f)]^{-1} S_p^{(f)} [A^*(f)]^{-1}$$

or by

$$S(f) = h[B(f)]^{-1} S_p^{(b)} [B^*(f)]^{-1}$$

where

$$A(f) = I - \sum_{k=1}^p A_k^{(p)} e^{2\pi i k h f}$$

$$B(f) = I - \sum_{k=1}^p B_k^{(p)} e^{-2\pi i k h f}$$

$h$  is the sampling period and  $*$  denotes complex conjugate transpose.

The above approach based on the work of Jones [6] does not guarantee stability and does not generally produce a non-negative definite spectrum as has been pointed out by Nuttall [7]. Subsequently, Nuttall [7] and Strand [8] applied a weighted arithmetic mean error criterion in order to provide model stability and to ensure positive definite stationary spectra. Another procedure suggested by Morf, et.al. [5] that also meets the spectral requirements is to compute the spectrum from the normalized reflection coefficient matrix  $\rho$ . To obtain this matrix,  $W$  and  $V$  are factored by using Cholesky decomposition into the product of lower triangular matrices times their transposes. A new recursive procedure for  $S_p^{(f)}$  and  $S_p^{(b)}$  by using  $\rho$  in place of Eqs. (4) can then be obtained. Other recursive algorithm has been proposed [9] for the solution of the normal equations for both single and multichannel data.

### 3. Spectral Analysis of Multichannel Sinusoids

To test his multichannel maximum entropy spectral analysis program. Strand [8] defined a two-channel complex sinusoid with frequency 0.0625 Hz. Each channel contains 128 complex numbers plus a small amount of additive Gaussian noise. Morf, et. al. employed the same test signals and demonstrated that their multichannel algorithm performs better than that of Strand. The complete data set is tabulated in Appendix I. In our study the data set is considered as four-channel real sinusoids. Fig. 1 is a plot of the first two channels. The computer program developed in our study is based on the mathematical analysis of the previous section and a subroutine due to Strand. At 9 lags, our spectral peak value is 2100 as compared to 2000 at the frequency of 0.0625 Hz, obtained by Morf. For 9 lags, Fig. 2a is the spectral plot in linear scale while Fig. 2b uses the logarithmic scale. In Fig. 2c the upper photo shows the phase difference while the lower photo is the coherence function between the two channels. If all four channels are considered simultaneously, the spectral peaks will be proportionally reduced while the shapes of the spectra remain almost unchanged.

A comparison is also made with the Jones' method by using the computer program listed in the Appendix of Ref. 6. The optimum order (i.e. the number of lags) is found to be 7 for the four-channel test data studied. Fig. 3a is the spectral plot of channels 1 & 2 in linear scale (left) and in logarithmic scale (right). Similarly Fig. 3b is the spectral plot of channels 3 & 4. Fig. 3c is a plot of phase differences between two-channels. Fig. 3d is a plot of coherence functions between two-channels. Although there is little change in the spectral shapes, the Jones' method provides much smaller spectral peaks (Note that all figures are normalized plots). Thus the computer program given in the Appendix provides somewhat better results than those available in other methods.

Fig. 2a

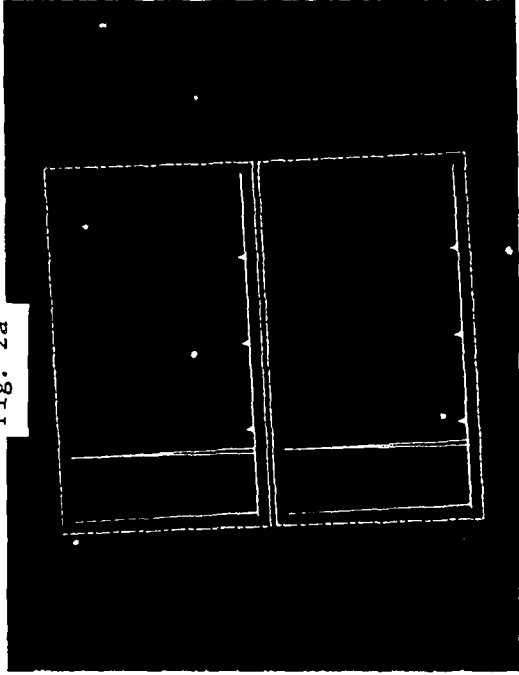


Fig. 2c

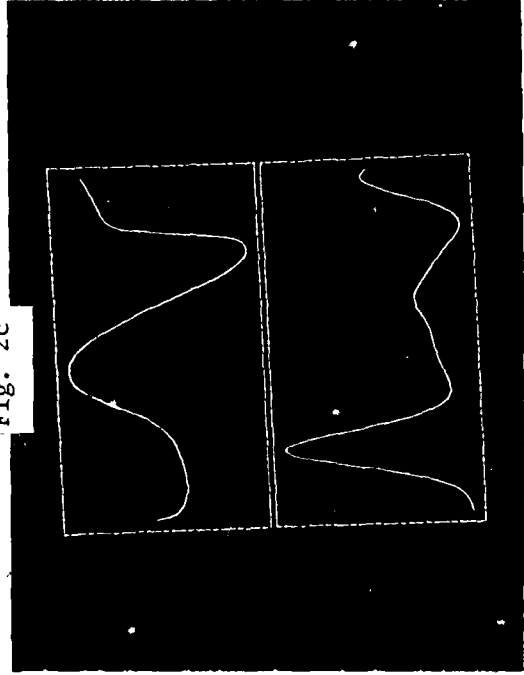


Fig. 1

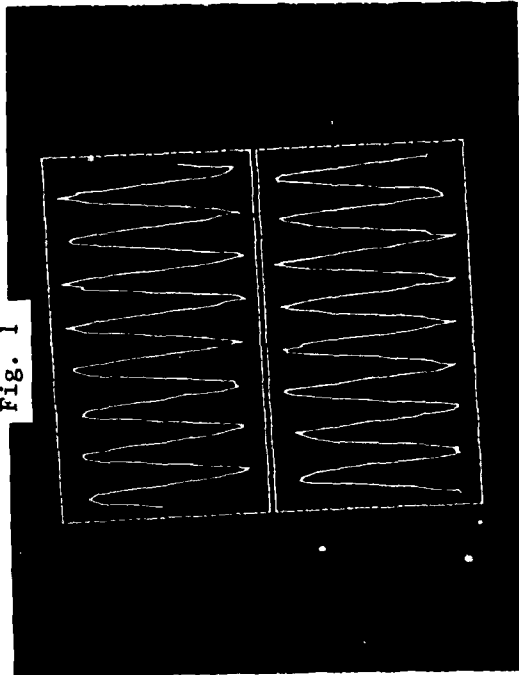


Fig. 2b

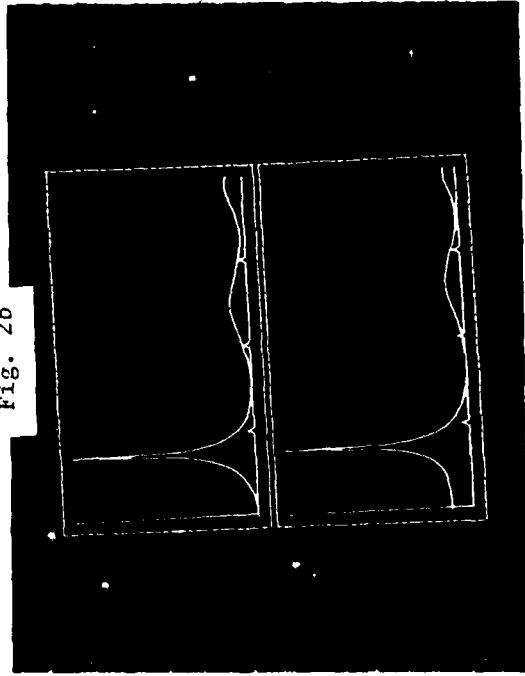


Fig. 3b

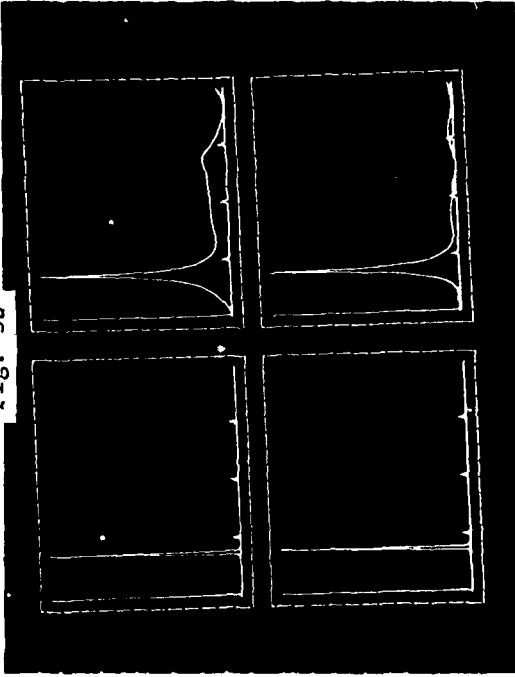


Fig. 3d

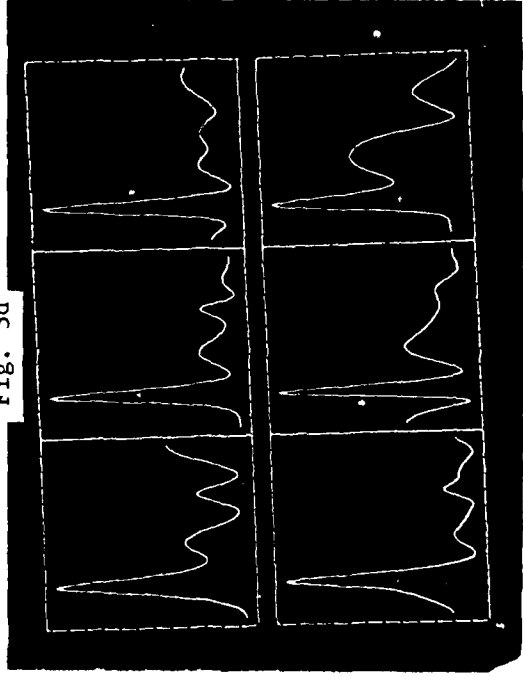


Fig. 3a

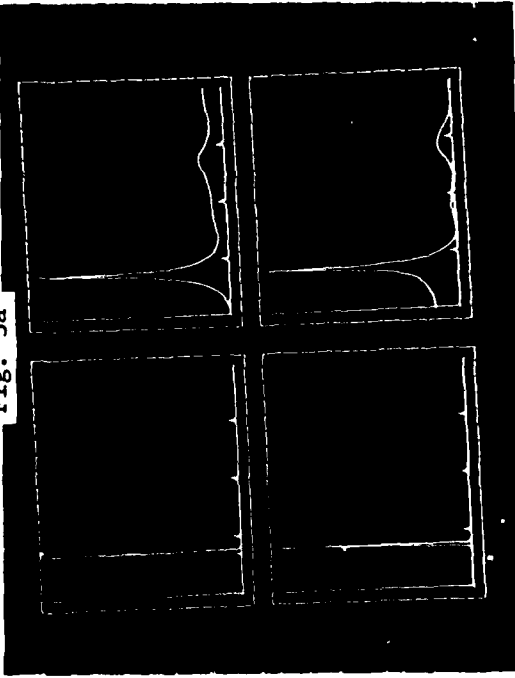
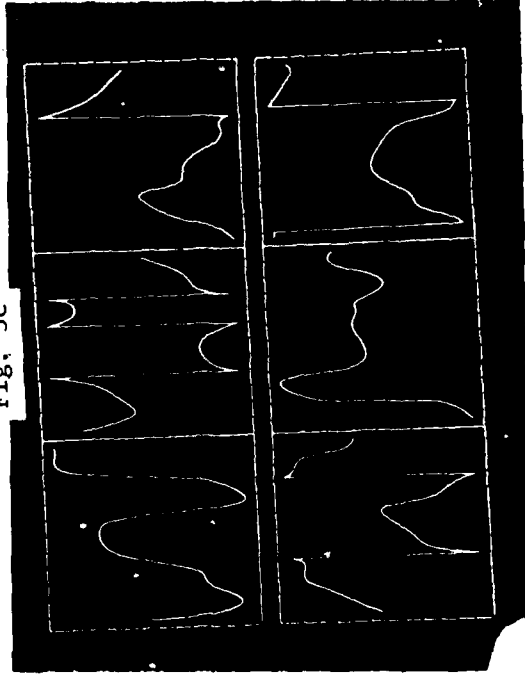


Fig. 3c



#### 4. Spectral Analysis of Geophysical Data

A three-channel geophysical data set is provided by Robinson [10] that includes sunspot numbers, northern light activity and earthquake activity with 100 data points each. Fig. 4b is the spectral plot of each channel in linear scale, while Fig. 4c is the same plot in logarithmic scale. Fig. 4d shows the phase differences between two-channels. The number of lags is 17 which is determined as the optimum order. It is interesting to note that spectral peak for the sunspot numbers is determined accurately.

#### 5. Image Segmentation Via Multivariate Spectral Analysis

The use of multivariate autoregressive analysis for texture classification and segmentation was considered by Gambotto and Gueguen [11]. Their segmentation result however is much to be desired. We have employed the multivariate autoregression estimation program of Jones [6] on the infrared image as shown in Fig. 5a. A small area of  $64 \times 64$  is considered in the computer study. The subimage is processed by small squares of  $2 \times 10$  pixels each, making use of the two-channel multivariate autoregression analysis. Then the mean-square prediction error (defined by Eq. 14 of Ref. 11) is computed and compared with a threshold. The idea is that the prediction error tends to be large for pixels near the object boundary and it is small for homogeneous region. Segmentation is performed by pixel classification using the prediction error. The classified result superimposed on the binary image of original picture is shown in Fig. 5b for orders 4, 5, 6 and 7. These results are acceptable but not as good as compared with the segmentation using Fisher's linear discriminant [12]. Furthermore the amount of computation required for the multivariate autoregression analysis is also much larger. Further study of image analysis using multivariate autoregression is much needed.

Fig. 4b

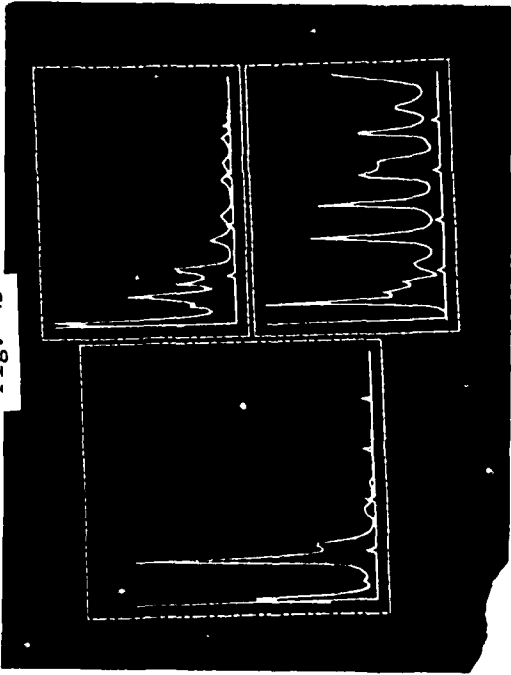


Fig. 4d

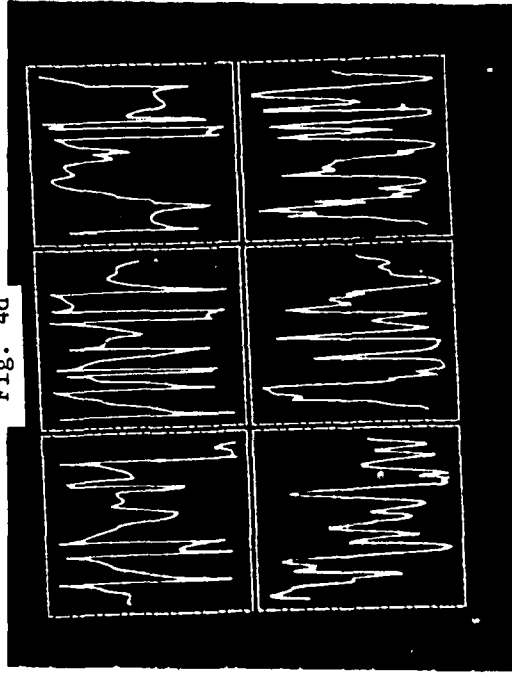


Fig. 4a

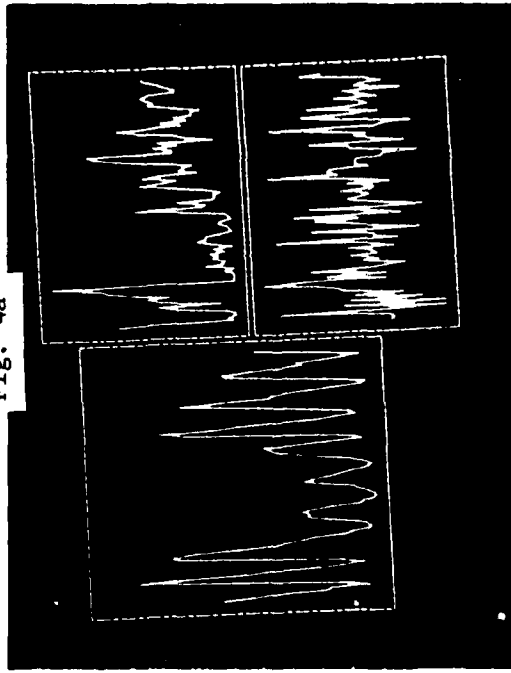
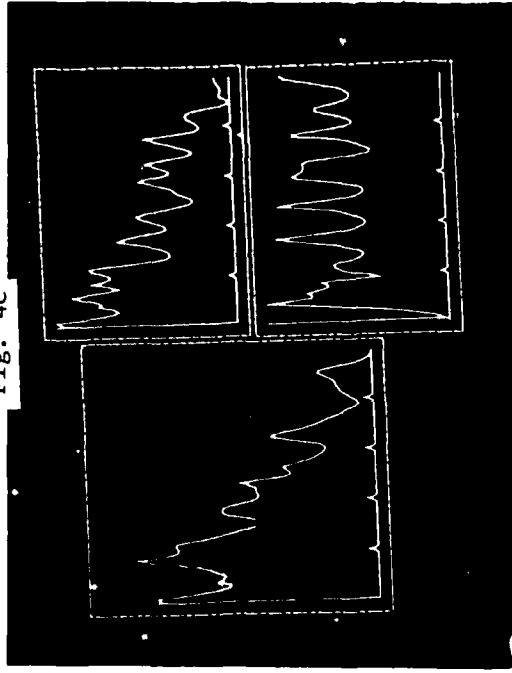


Fig. 4c



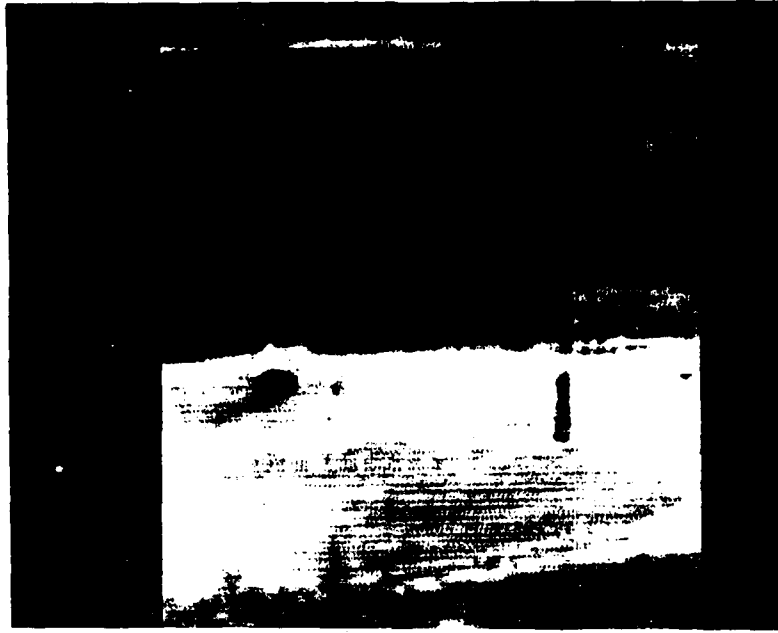


Fig. 5a

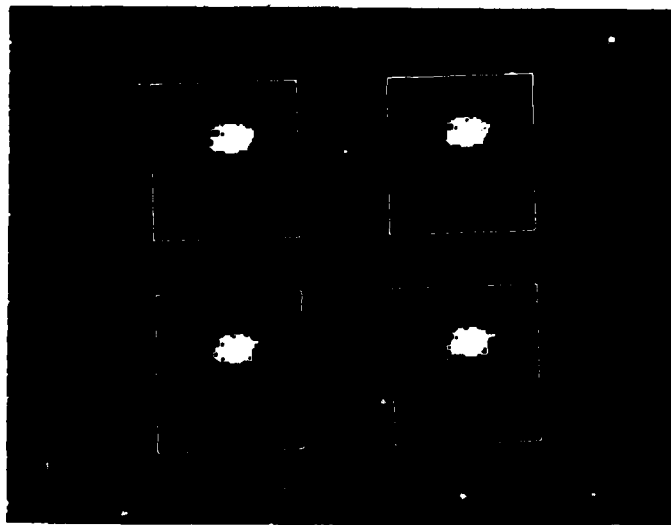


Fig. 5b

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Appendix I Four-channel Data

0.816	0.357	0.293	0.912	1.052	-0.056	0.531	0.950
0.809	0.627	-0.317	0.923	0.806	0.787	0.188	0.969
0.460	1.018	-0.456	0.900	0.674	0.612	-0.296	1.076
0.045	0.985	-0.895	0.530	0.489	0.811	-0.589	0.878
-0.261	0.928	-0.950	0.112	0.003	1.029	-0.928	0.643
-0.722	0.781	-0.873	-0.150	-0.485	0.901	-0.930	0.143
-0.970	0.445	-0.852	-0.546	-0.804	0.601	-0.999	-0.120
-1.054	-0.053	-0.663	-0.723	-0.843	0.282	-0.874	-0.529
-1.018	-0.287	-0.058	-1.045	-1.081	-0.101	-0.447	-0.933
-0.635	-0.830	0.197	-1.039	-1.018	-0.418	-0.142	-1.074
-0.448	-0.844	0.643	-0.732	-0.740	-0.734	0.716	-0.900
0.004	-1.063	0.956	-0.589	-0.295	-0.988	0.686	-0.848
0.317	-0.816	0.981	-0.063	0.111	-0.596	0.856	-0.565
0.817	-0.720	0.901	0.098	0.316	-0.916	1.069	-0.136
0.900	-0.365	0.845	0.465	0.725	-0.805	0.997	0.328
0.959	0.100	0.418	0.913	0.816	-0.413	0.839	0.449
0.983	0.472	0.701	0.954	1.022	-0.026	0.429	0.771
0.674	0.807	-0.247	1.093	0.901	0.347	0.059	0.847
0.350	0.897	-0.643	0.803	0.790	0.777	-0.173	0.863
-0.101	1.074	-0.908	0.558	0.446	0.911	-0.484	0.924
-0.471	0.811	-1.088	0.158	-0.068	0.911	-0.738	0.576
-0.828	0.700	-0.908	-0.285	-0.293	0.957	-0.860	0.181
-1.017	0.346	-0.870	-0.458	-0.656	0.772	-0.990	-0.136
-0.935	-0.122	-0.523	-0.744	-1.047	0.291	-0.762	-0.681
-0.800	-0.373	-0.261	-1.008	-1.076	0.115	-0.472	-0.830
-0.670	-0.671	0.203	-1.093	-0.959	-0.302	-0.160	-0.901
-0.426	-0.906	0.534	-0.773	-0.737	-0.600	0.740	-1.086
0.061	-0.955	0.860	-0.545	-0.434	-0.845	0.623	-0.732
0.365	-0.912	1.004	-0.271	-0.042	-0.901	0.852	-0.584
0.740	-0.601	1.053	0.210	0.264	-0.931	0.934	-0.177
0.800	-0.290	0.849	0.650	0.815	-0.773	0.935	0.121
1.101	0.114	0.571	0.841	1.030	-0.336	0.920	0.454
0.895	0.377	0.247	0.928	1.028	-0.078	0.444	0.965
0.599	0.706	-0.274	1.035	0.897	0.301	0.297	1.004
0.314	1.040	-0.446	0.720	0.608	0.697	-0.177	1.031
-0.062	1.096	-0.913	0.604	0.497	0.976	-0.520	0.917
-0.372	0.954	-0.998	0.097	-0.087	0.970	-0.719	0.516
-0.595	0.747	-0.910	-0.196	-0.275	0.846	-1.069	0.118
-0.969	0.469	-0.753	-0.607	-0.620	0.654	-1.061	-0.270
-1.115	0.121	-0.571	-0.739	-0.973	0.340	-0.754	-0.592
-0.930	-0.411	-0.075	-0.904	-0.888	0.072	-0.440	-0.966
-0.702	-0.747	0.105	-0.892	-0.812	-0.383	-0.104	-0.876
-0.331	-0.911	0.450	-0.872	-0.710	-0.640	0.103	-0.921
0.091	-1.072	0.814	-0.463	-0.502	-0.940	0.536	-0.828
0.420	-0.875	0.932	-0.104	0.026	-0.940	0.808	-0.634
0.797	-0.830	1.040	0.147	0.461	-0.914	1.106	-0.191
0.852	-0.421	0.711	0.472	0.751	-0.754	1.004	0.148
1.070	-0.083	0.527	0.840	1.003	-0.774	0.805	0.679
0.993	0.363	0.287	0.937	0.952	-0.064	0.487	0.755
0.603	0.761	-0.281	1.061	1.018	0.357	0.173	0.876
0.461	1.025	-0.487	0.939	0.808	0.652	-0.303	0.897
0.050	1.090	-0.787	0.617	0.445	1.013	-0.600	0.778
-0.488	0.808	-0.891	0.161	-0.111	0.990	-0.736	0.499
-0.713	0.739	-1.036	-0.134	-0.463	0.866	-0.859	0.183
-0.906	0.398	-0.918	-0.593	-0.747	0.610	-0.967	-0.308
-1.074	-0.020	-0.531	-0.816	-0.819	0.330	-0.832	-0.680
-0.891	-0.326	-0.173	-1.105	-0.971	0.044	-0.503	-0.717
-0.622	-0.674	0.140	-0.895	-0.955	-0.391	-0.099	-1.107
-0.459	-0.820	0.464	-0.910	-0.812	-0.793	0.327	-0.875
-0.032	-1.050	0.873	-0.474	-0.318	-0.891	0.561	-0.859
0.417	-0.931	0.039	-0.125	0.074	-0.909	0.728	-0.608
0.660	-0.828	0.988	0.163	0.453	-0.825	0.946	-0.104
0.261	-0.352	0.135	0.120	0.568	-0.654	0.854	0.135
				0.949	-0.373	0.924	-0.581
				0.994	-0.037	0.421	0.907

Appendix I Four-channel Data

0.816	0.357	0.293	0.912	1.052	-0.056	0.531	0.950
0.809	0.627	-0.312	0.923	0.806	0.287	0.188	0.969
0.460	1.018	-0.454	0.900	0.674	0.612	-0.296	1.076
0.045	0.985	-0.895	0.530	0.489	0.811	-0.589	0.878
-0.261	0.928	-0.950	0.112	0.003	1.029	-0.828	0.643
-0.722	0.781	-0.873	-0.150	-0.485	0.901	-0.930	0.143
-0.970	0.445	-0.852	-0.546	-0.804	0.601	-0.999	-0.120
-1.054	-0.053	-0.663	-0.723	-0.843	0.282	-0.874	-0.529
-1.018	-0.287	-0.058	-1.045	-1.081	-0.101	-0.447	-0.933
-0.635	-0.830	0.197	-1.039	-1.018	-0.418	-0.142	-1.074
-0.448	-0.844	0.643	-0.732	-0.740	-0.734	0.216	-0.900
0.004	-1.063	0.956	-0.589	-0.295	-0.988	0.686	-0.848
0.317	-0.816	0.981	-0.063	0.111	-0.998	0.854	-0.565
0.817	-0.720	0.901	0.098	0.316	-0.916	1.069	-0.136
0.900	-0.365	0.845	0.465	0.725	-0.805	0.997	0.328
0.959	0.100	0.418	0.913	0.816	-0.413	0.839	0.449
0.983	0.472	0.201	0.994	1.022	-0.076	0.429	0.771
0.674	0.807	-0.247	1.093	0.901	0.347	0.059	0.867
0.350	0.897	-0.643	0.803	0.790	0.727	-0.173	0.863
-0.101	1.074	-0.908	0.559	0.446	0.911	-0.484	0.924
-0.471	0.811	-1.088	0.158	-0.068	0.911	-0.738	0.576
-0.828	0.700	-0.908	-0.285	-0.293	0.957	-0.860	0.181
-1.017	0.346	-0.870	-0.458	-0.656	0.777	-0.990	-0.136
-0.935	-0.122	-0.523	-0.744	-1.047	0.291	-0.762	-0.681
-0.800	-0.373	-0.261	-1.008	-1.076	0.115	-0.472	-0.830
-0.670	-0.671	0.203	-1.098	-0.959	-0.302	-0.160	-0.901
-0.426	-0.906	0.534	-0.778	-0.737	-0.600	0.240	-1.086
0.061	-0.955	0.860	-0.545	-0.434	-0.845	0.623	-0.732
0.365	-0.912	1.004	-0.271	-0.042	-0.901	0.352	-0.584
0.740	-0.601	1.053	0.210	0.264	-0.931	0.934	-0.177
0.800	-0.290	0.849	0.650	0.815	-0.773	0.935	0.171
1.101	0.114	0.571	0.841	1.030	-0.336	0.920	0.454
0.895	0.327	0.247	0.928	1.028	-0.028	0.444	0.965
0.599	0.706	-0.224	1.035	0.897	0.301	0.297	1.004
0.314	1.040	-0.446	0.720	0.608	0.697	-0.127	1.031
-0.062	1.096	-0.913	0.604	0.497	0.976	-0.520	0.917
-0.372	0.954	-0.998	0.097	-0.087	0.970	-0.719	0.516
-0.599	0.747	-0.910	-0.196	-0.275	0.846	-1.069	0.118
-0.969	0.469	-0.753	-0.607	-0.620	0.654	-1.061	-0.270
-1.115	0.121	-0.591	-0.739	-0.973	0.340	-0.754	-0.592
-0.930	-0.411	-0.075	-0.904	-0.888	0.072	-0.440	-0.966
-0.702	-0.747	0.105	-0.892	-0.812	-0.383	-0.104	-0.876
-0.331	-0.911	0.450	-0.872	-0.710	-0.640	0.103	-0.921
0.091	-1.072	0.814	-0.463	-0.502	-0.940	0.536	-0.828
0.420	-0.825	0.932	-0.104	0.026	-0.940	0.808	-0.634
0.797	-0.830	1.040	0.147	0.461	-0.914	1.106	-0.191
0.852	-0.421	0.711	0.472	0.751	-0.734	1.004	0.148
1.070	-0.083	0.522	0.840	1.003	-0.274	0.805	0.679
0.993	0.363	0.287	0.937	0.952	-0.064	0.487	0.755
0.603	0.761	-0.281	1.061	1.018	0.357	0.173	0.876
0.461	1.025	-0.487	0.939	0.808	0.652	-0.303	0.897
0.050	1.090	-0.782	0.617	0.445	1.015	-0.600	0.778
-0.488	0.808	-0.891	0.161	-0.111	0.990	-0.736	0.499
-0.713	0.739	-1.036	-0.134	-0.463	0.866	-0.869	0.183
-0.906	0.398	-0.918	-0.593	-0.747	0.610	-0.967	-0.308
-1.074	-0.020	-0.531	-0.816	-0.819	0.330	-0.832	-0.680
-0.891	-0.326	-0.173	-1.105	-0.921	0.064	-0.503	-0.717
-0.622	-0.674	0.140	-0.895	-0.955	-0.291	-0.099	-1.107
-0.459	-0.820	0.464	-0.910	-0.812	-0.293	0.327	-0.875
-0.032	-1.050	0.873	-0.474	-0.318	-0.831	0.561	-0.859
0.417	-0.931	1.039	-0.125	0.074	-0.909	0.728	-0.608
0.660	-0.828	0.988	0.163	0.453	-0.825	0.946	-0.104
0.841	-0.322	0.175	-0.800	0.568	-0.654	0.854	0.135
				0.969	-0.325	0.924	-0.581
				0.994	-0.037	0.421	0.902

Appendix II Program MUL1 (Multi-channel Maximum Entropy Spectral Analysis)

```

C      MUL1
      REAL ET(4,128),BT(4,128),UU(4)
      COMMON /BLK0/ET,BT
      COMMON INDEX
      DEFINE FILE 7(128,8,U,LINE)
      READ(6,20)N,NP,NSPE,NPRS,NFREQ
20     FORMAT(5I6)
      NOUT=128
      NDAT=128
      DT=1.
      DO 2 K=1,NDAT
      LINE=K
      READ(7,LINE)UU
      DO 3 I=1,NP
      ET(I,K)=UU(I)
3      BT(I,K)=UU(I)
2      CONTINUE
      END FILE 7
      DEFINE FILE 1(16,256,U,INDEX)
      CALL STRAND(NOUT,NP,NDAT,N,DT,NFREQ,NSPE,NPRS)
      CALL EXIT
      END

      SUBROUTINE CPRINT(NP,NDAT,I9,P,PP,CN,CNP,CF,CB)
      REAL P(4,4),PP(4,4),CN(4,4),CNP(4,4),CF(4,4,10),CB(4,4,10)
      REAL ET(4,128),BT(4,128)
      COMMON /BLK0/ET,BT
      I9P1=I9+1
      NDATM9=NDAT-I9
      PRINT 2,I9,I9P1
2      FORMAT(1H1,///' RESULTS BELOW FOR',I5,' LAGS, I. E. MATRIX
      1 OF BLOCK DIMENSION',I5,/)
      PRINT 6
6      FORMAT(//1X,' FORWARD RESIDUALS',60X,' BACKWARD RESIDUALS',/)
      DO 7 J=1,NDATM9
      PRINT 8,(ET(K,J),K=1,NP),(BT(K,J),K=1,NP)
7      CONTINUE
      PRINT 19
19     FORMAT(//1X,' FORWARD POWER BELOW',60X,' BACKWARD POWER BELOW',/)
      DO 18 I=1,NP
      PRINT 8,(P(I,K),K=1,NP),(PP(I,K),K=1,NP)
18     CONTINUE
      PRINT 3
3      FORMAT(//1X,' FORWARD REFLECT COEFF',60X,' BACKWARD REFLECT COEFF
      DO 4 I=1,NP
      PRINT 8,(CN(I,K),K=1,NP),(CNP(I,K),K=1,NP)
4      CONTINUE
8      FORMAT(1X,4E12.5,10X,4E12.5)
      PRINT 69
69     FORMAT(//1X,' FORWARD FILTER BELOW',60X,' BACKWARD FILTER BELOW',/)
      DO 74 K=1,I9P1
      DO 73 I=1,NP
      PRINT 8,(CF(I,J,K),J=1,NP),(CB(I,J,K),J=1,NP)
73     CONTINUE
      WRITE(5,77)
77     FORMAT(//)
74     CONTINUE
      RETURN
      END

```

```

SUBROUTINE CMINV(A, N, B)
DIMENSION IPIVOT(16), A(16, 16), B(16), INDEX(16, 2), PIVOT(16)
20  DO 20 J=1, N
    IPIVOT(J)=0
    DO 550 I=1, N
    AMAX=0.
    DO 101 J=1, N
    IF(IPIVOT(J).EQ.1)GO TO 101
    DO 100 K=1, N
    IF(IPIVOT(K)-1)80, 100, 740
80  A11=ABS(AMAX)
    A12=ABS(A(J, K))
    IF(A11.GE. A12)GO TO 100
85  IROW=J
    ICOLUM=K
    AMAX=A(J, K)
100  CONTINUE
101  CONTINUE
    IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
    IF(IROW.EQ. ICOLUM)GO TO 260
    DO 200 L=1, N
    SWAP=A(IROW, L)
    A(IROW, L)=A(ICOLUM, L)
200  A(ICOLUM, L)=SWAP
    SWAP=B(IROW)
    B(IROW)=B(ICOLUM)
    B(ICOLUM)=SWAP
260  INDEX(I, 1)=IROW
    INDEX(I, 2)=ICOLUM
    PIVOT(I)=A(ICOLUM, ICOLUM)
    A(ICOLUM, ICOLUM)=1.
    DO 350 L=1, N
350  A(ICOLUM, L)=A(ICOLUM, L)/PIVOT(I)
    B(ICOLUM)=B(ICOLUM)/PIVOT(I)
    DO 550 L1=1, N
    IF(L1.EQ. ICOLUM)GO TO 550
400  T=A(L1, ICOLUM)
    A(L1, ICOLUM)=0.
    DO 450 L=1, N
450  A(L1, L)=A(L1, L)-A(ICOLUM, L)*T
    B(L1)=B(L1)-B(ICOLUM)*T
550  CONTINUE
    DO 710 I=1, N
    L=N+1-I
    IF(INDEX(L, 1).EQ. INDEX(L, 2))GO TO 710
    JROW=INDEX(L, 1)
    JCOLUM=INDEX(L, 2)
    DO 705 K=1, N
    SWAP=A(K, JROW)
    A(K, JROW)=A(K, JCOLUM)
    A(K, JCOLUM)=SWAP
705  CONTINUE
710  CONTINUE
740  CONTINUE
    RETURN
    END

```

```

SUBROUTINE STRAND(NOUT, NP, NDAT, N, DT, NFREQ, NSPE, NPRS)
DIMENSION BT(4, 128), ET(4, 128), CF(4, 4, 10), CB(4, 4, 10), FPE(10)
1, E(4, 4), B(4, 4), G(4, 4), P(4, 4), PP(4, 4), Q1(4, 4), V1(4, 4)
2, V3(4, 4), CN(4, 4), CNP(4, 4), U5(4, 4, 10), U6(4, 4, 10)
REAL Q2(4, 4), TI(4, 4), AA(4, 4), BB(4, 4), CC(4, 4)
COMMON INDEX
COMMON /BLKO/ET, BT
NP1=NP+1
DO 11 I=1, NP
DO 11 J=1, NP
DO 11 K=1, NP1
CB(I, J, K)=0.
11 CF(I, J, K)=0.
DO 31 I=1, NP
CB(I, I, 1)=1.
31 CF(I, I, 1)=1.
DO 22 I=1, NP
DO 22 J=1, NP
P(I, J)=0.
DO 21 K=1, NDAT
21 P(I, J)=P(I, J)+ET(I, K)*ET(J, K)
P(I, J)=P(I, J)/FLOAT(NDAT)
22 PP(I, J)=P(I, J)
CALL AKAIK(P, NP, NDAT, 0., FPE, 1)
DO 12 L=2, N
LM1=L-1
NDATM9=NDAT-LM1
DO 9 K=1, NDATM9
KP1=K+1
DO 9 I=1, NP
9 ET(I, K)=ET(I, KP1)
DO 1 I=1, NP
DO 1 J=1, NP
E(I, J)=0.
B(I, J)=0.
G(I, J)=0.
DO 2 K=1, NDATM9
E(I, J)=E(I, J)+ET(I, K)*ET(J, K)
B(I, J)=B(I, J)+BT(I, K)*BT(J, K)
2 G(I, J)=G(I, J)+BT(I, K)*ET(J, K)
E(I, J)=E(I, J)/FLOAT(NDATM9)
B(I, J)=B(I, J)/FLOAT(NDATM9)
G(I, J)=-2.*G(I, J)/FLOAT(NDATM9)
Q1(I, J)=P(I, J)
1 Q2(I, J)=PP(I, J)
CALL INVERS(Q1, NP)
CALL INVERS(Q2, NP)
CALL ACCUM(NP, AA, Q2, B)
CALL ACCUM(NP, BB, Q1, E)
CALL ACCUM(NP, CC, Q2, G)
CALL GETCNN(AA, BB, CC, NP, CN)
DO 25 I=1, NP
DO 25 J=1, NP
V1(I, J)=0.
DO 25 K=1, NP

```

```

25      V1(I, J)=V1(I, J)-CN(K, I)*PP(K, J)
        CALL ACCUM(NP, CNP, Q1, V1)
        DO 27 I=1, NP
          DO 27 J=1, NP
            DO 27 K=1, L
              U5(I, J, K)=CF(I, J, K)
27      U6(I, J, K)=CB(I, J, K)
          DO 4 K=1, L
            INDEX=L-K+1
            DO 4 I=1, NP
              DO 4 J=1, NP
                DO 4 KK=1, NP
                  CF(I, J, K)=CF(I, J, K)+U6(I, KK, INDEX)*CN(KK, J)
                  CB(I, J, K)=CB(I, J, K)+U5(I, KK, INDEX)*CNP(KK, J)
4      CONTINUE
          DO 5 K=1, NDATM9
            DO 8 I=1, NP
              V3(I, 1)=ET(I, K)
8      V3(I, 2)=BT(I, K)
          DO 5 I=1, NP
            DO 5 KK=1, NP
              ET(I, K)=ET(I, K)+CN(KK, I)*V3(KK, 2)
5      BT(I, K)=BT(I, K)+CNP(KK, I)*V3(KK, 1)
          CALL ACCUM(NP, V1, PP, CN)
          CALL ACCUM(NP, V3, P, CNP)
          DO 7 I=1, NP
            DO 7 J=1, NP
              DO 7 K=1, NP
                P(I, J)=P(I, J)-CN(K, I)*V1(K, J)
7      PP(I, J)=PP(I, J)-CNP(K, I)*V3(K, J)
          S=FLOAT(NP*LM1)
          CALL AKAIK(P, NP, NDAT, S, FPE, L)
          IF(LM1. LT. NPRS)GO TO 82
          CALL CPRINT(NP, NDAT, LM1, P, PP, CN, CNP, CF, CB)
82      IF(LM1. LT. NSPE)GO TO 12
          CALL SPECIAL(NOUT, LM1, NDAT, NP, CF, P, FPE, DT, NREQ)
12      CONTINUE
          RETURN
          END

```

```

SUBROUTINE GETCNN(A, B, C, N, CN)
REAL A(4, 4), B(4, 4), C(4, 4), CN(4, 4), BB(16), AB(16, 16)
N2=N*N
DO 1 I=1, N
  DO 1 J=1, N
    I1=(I-1)*N+J
1  BB(I1)=C(I, J)
  CALL KRONPR(A, B, AB, N)
  CALL CMINV(AB, N2, BB)
  DO 2 I=1, N
    DO 2 J=1, N
      I1=(I-1)*N+J
2  CN(I, J)=BB(I1)
  RETURN
END

```

```

SUBROUTINE SPECAL(NOUT, I9, NDATA, NP, CF, P, FPE, DT, NFREQ)
REAL CF(4, 4, 10), P(4, 4), FPE(10), T1(4, 4), E(4, 4), B(4, 4), G(4, 4)
1, V1(4, 4), V3(4, 4), Q1(4, 4), Q2(4, 4)
REAL SS(16, 128)
COMMON /BI K1/SS
COMMON INDEX
I9P1=I9+1
ANGLE=3. 1415926/FLOAT(NFREQ)
FREQ1=1. /(2. *FLOAT(NFREQ))
DEGREE=360. /(2. *3. 1415926)
C=1.
S=0.
C1=COS(ANGLE)
S1=SIN(ANGLE)
PRINT 2001
2001 FORMAT(///1X, ' FREQ SPEC11 SPEC22 SPEC33 SPEC44
1 PH12 COH12 PH13 COH13 PH14 COH14 PH23 COH23
2 PH24 COH24 PH34 COH34', /)
DO 99 KK=1, NOUT
FREQ=FLOAT(KK)*FREQ1
C2=C1*C-S1*S
S=C1*S+S1*C
C=C2
CN1=1.
SN1=0.
DO 90 I=1, NP
DO 90 J=1, NP
E(I, J)=0.
90 B(I, J)=0.
DO 91 L=1, I9P1
DO 92 I=1, NP
DO 92 J=1, NP
E(I, J)=E(I, J)+CN1*CF(J, I, L)
92 B(I, J)=B(I, J)+SN1*CF(J, I, L)
CN2=CN1*C-SN1*S
SN1=CN1*S+SN1*C
CN1=CN2
91 CONTINUE
DO 93 I=1, NP
DO 93 J=1, NP
93 V1(I, J)=E(I, J)
CALL INVERS(V1, NP)
CALL ACCUM(NP, V3, V1, B)
CALL ACCUM(NP, G, B, V3)
DO 95 I=1, NP
DO 95 J=1, NP
95 Q1(I, J)=F(I, J)+G(I, J)
CALL INVERS(Q1, NP)
CALL ACCUM(NP, V3, B, V1)
CALL ACCUM(NP, Q2, Q1, V3)
DO 96 I=1, NP
DO 96 J=1, NP
96 Q2(I, J)=-Q2(I, J)
CALL ACCUM(NP, V1, Q1, P)
CALL ACCUM(NP, V3, Q2, P)

```

```

      DO 30 I=1, NP
      DO 30 J=1, NP
      B(I, J)=0.
      G(I, J)=0.
      DO 30 K=1, NP
      B(I, J)=B(I, J)+V1(I, K)*Q1(J, K)+V3(I, K)*Q2(J, K)
30    G(I, J)=G(I, J)+V3(I, K)*Q1(J, K)-V1(I, K)*Q2(J, K)
      DO 31 I=1, NP
      DO 31 J=1, NP
      B(I, J)=2. *DT*B(I, J)
31    G(I, J)=2. *DT*G(I, J)
      CALL PHCO(G, B, NP, KK, FREQ)
99    CONTINUE
      CALL KBP
      RETURN
      END

```

```

SUBROUTINE PHCO(G, B, NP, KK, FREQ)
REAL G(4, 4), B(4, 4), COH(6), PHAS(6), SS(16, 128)
COMMON /BLK1/SS
DEG=360. / (2. *3. 14159265)
I=0
NPP=NP-1
DO 10 J=1, NPP
DO 10 K=2, NP
IF(K LE. J)GO TO 10
I=I+1
PHAS(I)=DEG*ATAN2(G(J, K), B(J, K))
COH(I)=(B(J, K)**2+G(J, K)**2)/(B(J, J)*B(K, K))
10  CONTINUE
DO 20 J=1, NP
20  SS(J, KK)=B(J, J)
DO 30 J=1, I
30  SS(NP+J, KK)=PHAS(J)
SS(NP+I+J, KK)=COH(J)
2001 PRINT 2001, FREQ, (B(I1, I1), I1=1, NP), (PHAS(I), COH(I), I=1, 6)
FORMAT(1X, F9. 4, 4E9. 2, 12F7. 1)
RETURN
END

```

```

SUBROUTINE KRONPR(A, B, AB, N)
DIMENSION A(4, 4), B(4, 4), AB(16, 16)
N2=N*N
DO 1 I=1, N2
DO 1 J=1, N2
1  AB(I, J)=0.
DO 2 I=1, N
DO 2 J=1, N
DO 2 K=1, N
I1=(I-1)*N+K
I2=(J-1)*N+K
2  AB(I1, I2)=A(I, J)
DO 3 I=1, N
DO 3 J=1, N
DO 3 K=1, N
I1=(I-1)*N+J
I2=(I-1)*N+K
3  AB(I1, I2)=AB(I1, I2)+B(K, J)
RETURN
END

```

```

FUNCTION DETERM(ARRAY, NORDER)
DIMENSION ARRAY(4, 4)
DETERM=1.
DO 50 K=1, NORDER
IF(ARRAY(K, K). NE. 0.) GO TO 41
DO 23 J=K, NORDER
IF(ARRAY(K, J). NE. 0.) GO TO 31
23 CONTINUE
DETERM=0.
RETURN
31 DO 34 I=K, NORDER
SAVE=ARRAY(I, J)
ARRAY(I, J)=ARRAY(I, K)
34 ARRAY(I, K)=SAVE
DETERM=-DETERM
41 DETERM=DETERM*ARRAY(K, K)
IF(K. GE. NORDER) GO TO 50
K1=K+1
DO 46 I=K1, NORDER
DO 46 J=K1, NORDER
46 ARRAY(I, J)=ARRAY(I, J)-ARRAY(I, K)*ARRAY(K, J)/ARRAY(K, K)
50 CONTINUE
RETURN
END

```

```

SUBROUTINE INVERS(AINV, NP)
DIMENSION AINV(4, 4), EINV(16, 16), BBB(16)
DO 30 I=1, NP
BBB(I)=0.
DO 30 J=1, NP
30 EINV(I, J)=AINV(I, J)
CALL CMINV(EINV, NP, BBB)
DO 31 I=1, NP
DO 31 J=1, NP
31 AINV(I, J)=EINV(I, J)
RETURN
END

```

```

SUBROUTINE ACCUM(NP, A, B, C)
DIMENSION A(4, 4), B(4, 4), C(4, 4)
DO 11 I=1, NP
DO 11 J=1, NP
A(I, J)=0.
DO 11 K=1, NP
11 A(I, J)=A(I, J)+B(I, K)*C(K, J)
RETURN
END

```

```

SUBROUTINE KBP
REAL SS(16, 128)
COMMON INDEX
COMMON /BLK1/SS
DO 10 I=1, 16
INDEX=I
WRITE(1, INDEX)(SS(I, J), J=1, 128)
10 CONTINUE
CALL BELL
RETURN
END

```

```
1  SUBROUTINE AKAIK(P, NP, NDAT, S, FPE, INDEX)
    DIMENSION P(4, 4), FPE(10), DUM(4, 4)
    DO 1 I=1, NP
    DO 1 J=1, NP
    DUM(I, J)=P(I, J)
    DETP=DETERM(DUM, NP)
    FACT1=FLOAT(NDAT)+1. +S
    FACT2=FLOAT(NDAT)-1. -S
    FACT=FACT1/FACT2
    FPE(INDEX)=DETP*(FACT**NP)
    RETURN
    END
```

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