

AD-A094 614

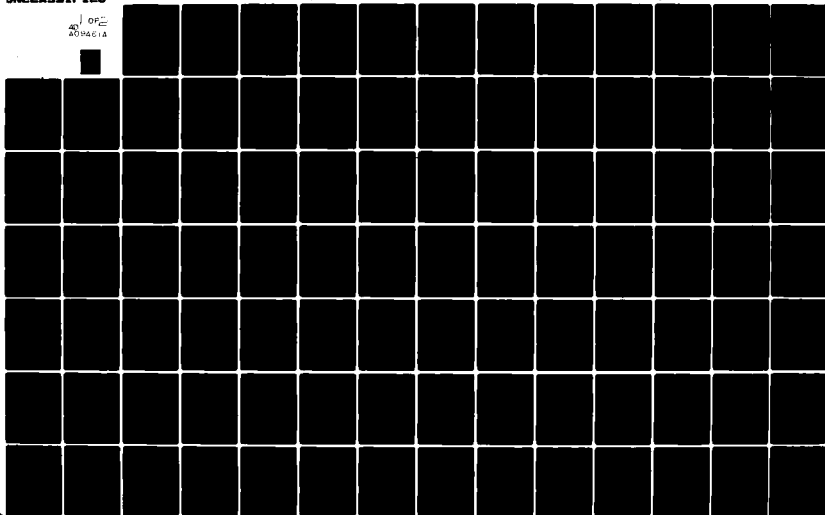
NAVAL POSTGRADUATE SCHOOL MONTEREY CA
NUMERICAL SOLUTION OF STEADY AND PERIODICALLY PULSED TWO-DIMENS--ETC(U)
APR 80 J C LAI, J M SIMMONS

F/S 20/4

UNCLASSIFIED

ML

1 OF 2
20 PAGES



14
NPS67-80-015

AD A094614

NAVAL POSTGRADUATE SCHOOL
Monterey, California

2
LEVEL 2



DTIC
ELECT
S FEB 5 1981
F

6
Numerical Solution of Steady and
Periodically Pulsed Two-Dimensional
Turbulent Free Jets,

by

10
Joseph C. S. Lai

and

J. M. Simmons

11 April 1980

Approved for public release; distribution
unlimited

Prepared for
Naval Air Systems Command
Washington, D.C. 20361

DDC FILE COPY

81 2 04 044

NAVAL POSTGRADUATE SCHOOL

Monterey, California

Rear Admiral J. J. Ekelund
Superintendent

Jack R. Borsting
Provost

The work reported herein was supported by the Naval Air
Systems Command, Washington, DC.

Reproduction of all or part of this report is authorized.

This report was prepared by:

Joseph C. S. Lai
Joseph C. S. Lai
Visiting Research Associate

J. M. Simmons
J. M. Simmons
Senior Lecturer of
Mechanical Engineering
University of Queensland
Brisbane, Australia

Reviewed by:

Released by:

M. F. Platzner
M. F. Platzner, Chairman
Department of Aeronautics

William M. Tolles
W. M. Tolles
Dean of Research

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS67-80-015	2. GOVT ACCESSION NO. AD-A094 614	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Numerical Solution of Steady And Periodically Pulsed Two-Dimensional Turbulent Free Jets		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Joseph C. S. Lai and J. M. Simmons		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS NAVAL POSTGRADUATE SCHOOL Monterey, CA 93940		10. PROGRAM ELEMENT PROJECT TASK AREA & WORK UNIT NUMBERS 61153N N00019-80-WR-01199
11. CONTROLLING OFFICE NAME AND ADDRESS NAVAL POSTGRADUATE SCHOOL Monterey, CA 93940		12. REPORT DATE April 1980
		13. NUMBER OF PAGES 110
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) NAVAL POSTGRADUATE SCHOOL Monterey, CA 93940		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release, distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) turbulent jets, viscous/inviscid interactions, shear flows, turbulence modelling, computational fluid dynamics, Cebeci-Keller box method		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The flow fields of a steady and a periodically pulsed two-dimensional turbulent free jet have been studied by solving the thin shear layer equations by the Keller Box method in transformed variable form. A constant eddy-viscosity formulation was used to model the Reynolds shear stress term. For the steady jet, calculations agree well with documented experimental data. Computed results of the unsteady jet indicate that the mean flow characteristics follow closely those of the steady jet and compare well with		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

available experimental data. For sufficiently high frequency and amplitude of pulsation or at large streamwise distance, significant unsteady effects occur in the instantaneous quantities.

Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

DD Form 1473
1 Jan 73
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

TABLE OF CONTENTS

	PAGE
Abstract	v
Acknowledgement	vi
Nomenclature	vii
1.0 Introduction	1
2.0 Governing Equations	4
2.1 Governing Equations in Non-dimensional Form	7
2.2 Turbulence Modelling	8
2.3 Governing Equations in Transformed Variable Form	12
3.0 Method of Solution	14
3.1 Finite Difference Form of the Governing Equations	15
3.2 Time-Varying Velocity Profile at the Nozzle Exit	19
3.3 Solution of the Finite Difference Equation	22
3.4 Convergence Criterion	24
3.5 Criterion for the Spreading of Jet	25
3.6 Criterion for the Attainment of Steady State Solution	27
3.7 Mean Momentum Flux, Phase Angle and Peak to Peak Oscillation	29
3.7.1 Mean Momentum Flux	29
3.7.2 Phase Angle	29
3.7.3 Peak to Peak Oscillation	30

	PAGE
4.0 Results and Discussions	32
4.1 Steady Jet	33
4.2 Unsteady Jet	37
4.2.1 Sensitivity of the Solution to the Convergence Criterion	37
4.2.2 Validity of the Criterion for the Attainment of Steady State Solution	38
4.2.3 Sensitivity of the Solution to Time Step	38
4.2.4 Relaxation of the Criterion for Jet Spreading	38
4.2.5 Results	39
5.0 Concluding Remarks	43
6.0 References	45
Appendix A Structure and Listing of Computer Program	48
Appendix B Figures	79
Initial Distribution List	80

ABSTRACT

The flow fields of a steady and a periodically pulsed two-dimensional turbulent free jet have been studied by solving the thin shear layer equations by the Keller Box method in transformed variable form. A constant eddy-viscosity formulation was used to model the Reynolds shear stress term. For the steady jet, calculations agree well with documented experimental data. Computed results of the unsteady jet indicate that the mean flow characteristics follow closely those of the steady jet and compare well with available experimental data. For sufficiently high frequency and amplitude of pulsation or at large streamwise distance, significant unsteady effects occur in the instantaneous quantities.

ACKNOWLEDGEMENT

This work was supported by the Australian Research Grants Committee under Reference No. F77/15026 and by the Naval Air Systems Command, Code AIR 310 under contract No. N0001980WR01199, Job Order No. 57763 Segment 1385/4. The first author (JCSL) is grateful to Professor M. F. Platzer for his encouragement and support during his 6-months visit at the Naval Postgraduate School. Also, thanks are extended to Professor T. Cebeci for his valuable comments.

NOMENCLATURE

Unless otherwise stated, the symbols used in the text have the following meanings.

a	constant defined in Eq. (15a)
b	constant defined in Eq. (15b)
c	constant defined in Eq. (11)
F	% peak-to-peak oscillation of centre-line velocity
f	stream function in (ζ, t, η) coordinates defined in Eq. (15b)
h	nozzle width
M	mean momentum flux in streamwise direction
p^*	mean pressure
p^*	instantaneous pressure
Q	mean mass flow at any streamwise station
Q_E	mean mass flow at nozzle exit
T	period of pulsation
t	non-dimensional time = $U_{ci}^* t^* / h$
U	non-dimensional mean x-component velocity = U^* / U_{ci}^*
$U_o(y)$	non-dimensional mean velocity profile at the nozzle exit
u	non-dimensional instantaneous x-component velocity = u^* / U_{ci}^*
u'	non-dimensional x-component velocity fluctuation = u'^* / U_{ci}^*
V	non-dimensional mean y-component velocity = V^* / U_{ci}^*
v	non-dimensional instantaneous y-component velocity = v^* / U_{ci}^*

v'	non-dimensional y-component velocity fluctuation = v^{*}/U_{ci}^{*}
x	non-dimensional streamwise distance - x^{*}/h
y	non-dimensional transverse distance - y^{*}/h
$y_{\frac{1}{2}}$	value of y at which $U = \frac{1}{2}U_c$

GREEK SYMBOLS

Δ	difference between two quantities
ϵ	amplitude of pulsation
ν	non-dimensional kinematic viscosity = $\nu^{*}/(U_{ci}^{*}h)$
ν_t	eddy-viscosity defined in Eq. (10)
ν_{eff}	Effective eddy viscosity defined in Eq. (13)
ψ	stream function in (ζ, t, y) coordinates defined in Eq. (15b)
ϕ	phase angle
Φ	function defined in Eq. (28)
η	transformed variable defined in Eq. (15b)
ω	non-dimensional angular frequency of pulsation = $\omega^{*}h/U_{ci}^{*}$
ζ	non-dimensional distance identical to x
ζ_0	constant defined in Eq. (15a)

SUPERSCRIPT

$*$	dimensional quantities
-----	------------------------

SUBSCRIPTS

c	centerline
i	initial
∞	jet edge
Overbar	time-average quantities

1.0 Introduction

The steady two-dimensional turbulent jet has long been the subject of many theoretical and experimental investigations because of its important role in many different types of engineering applications such as fluidic and combustion systems and because of its significance in providing fundamental understanding to the physics of flow mechanisms such as turbulence and vortex structures. Owing to its simplicity in configuration, the steady two-dimensional turbulent jet has been studied in detail experimentally and is regarded as one of the most well-documented flows in the literature where sufficient data are available for many practical engineering purposes. However, despite such extensive investigations, for example Heskestad (1965), Gutmark and Wygnanski (1976) and Everitt and Robins (1978), considerable scatter is found to exist between the results of various workers even in the mean flow parameters such as centre-line velocity decay rate and jet spreading rate. Such discrepancies have been casually attributed to different effects such as Reynolds number, aspect ratio, nozzle geometry, initial conditions, upstream turbulence intensities and the uncertainties involved in the hot wire results in regions where reversed flow may occur. Nevertheless, no unified agreement on the effects of such factors and other flow mechanisms on the flow development has been reached. A

comprehensive review and evaluation of the experiemntal data on steady turbulent jets was given by Harsha (1971) and Rodi (1975).

The analytical solution of the fully-developed steady turbulent jet was first sought by Tollmien (1926) followed by Goertler (1942) and Schlichting (1965). With the advent of computer technology and the rapid development of numerical techniques, the steady turbulent jets have commonly been used for turbulence modelling and as standard test cases of turbulence models. Many numerical predictions of steady turbulent jets using various turbulence models have been attempted such as Rodi and Spalding (1970), Launder et al (1972) and Chen and Nikitopoulos (1979) to calculate the flow properties and to complement the experimental results.

Although almost inevitably unsteadiness of varying degrees occurs in practice either desirably to achieve certain favorable characteristics or undersirably due to the fluctuations in the surrounding fluid, very few results appear to have been reported on unsteady turbulent jets both theoretically and experimentally. Only until recently, because of the growing realization of the fundamental and practical implications of an improved understanding of unsteady effects, the excitation of turbulent jets by acoustic (Fiedler and Korschelt (1979)), mechanical (Bremhorst and Harch (1979)) and fluidic (Piatt and Viets (1979)) means has received considerable attention. On the other hand,

closed form solutions of the unsteady jets which adequately describe the flow development can hardly be obtained despite the efforts of Pai (1965) and McCormack et al (1966).

Numerical solution of unsteady laminar jets was obtained by Kent (1973). Although turbulence models have been developed to give sufficiently accurate predictions of a wide variety of steady flows, the applicability of such turbulence models to unsteady flows is uncertain.

The objectives of this study are to apply the transformation developed by Lai and Simmons (1978) to compute steady and unsteady turbulent jets, to add to the understanding of the steady and unsteady two-dimensional turbulent free jets, to investigate the validity of quasi-steady approximations and to evaluate the suitability of using turbulence models established for steady flows in unsteady flow calculations. The unsteady jet considered consists of an initially steady, two-dimensional, turbulent free jet with a sinusoidal mass flow variation superimposed on it at the nozzle exit. The steady-state oscillatory flow characteristics at any location downstream of the nozzle are obtained by solving the thin shear layer equations in two spatial (ζ, η) and one time (t) transformed coordinates.

2.0 Governing Equations

The Navier-Stokes equations for two-dimensional incompressible flow in tensor notations are given by (see, e.g., Hinze (1975))

$$\frac{\partial U_i^*}{\partial t^*} + u_j^* \frac{\partial U_i^*}{\partial x_j^*} = - \frac{1}{\rho} \frac{\partial p^*}{\partial x_i^*} + \nu^* \frac{\partial^2 U_i^*}{\partial x_j^* \partial x_j^*} \quad (1)$$

with the continuity equation being given by

$$\frac{\partial u_i^*}{\partial x_i^*} = 0 \quad (2)$$

By applying Reynolds decomposition

$$u_i^* = U_i^* + u_i^{*'} , \text{ etc.}$$

to equation (1) and taking time average with a time scale large compared with that of the turbulent motions but small compared with the periodicity of the flow, the time-averaged Reynolds equations can then be given by

$$\begin{aligned} \frac{\partial U_i^*}{\partial t^*} + U_j^* \frac{\partial U_i^*}{\partial x_j^*} = & \frac{1}{\rho} \frac{\partial}{\partial x_j^*} (-P^* + \frac{\partial U_i^*}{\partial x_j^*} + \frac{\partial U_j^*}{\partial x_i^*} \\ & - \overline{\rho u_i^{*'} u_j^{*'}} \end{aligned} \quad (3)$$

Consider an unsteady, two-dimensional, constant property, turbulent free jet issuing into a stationary medium. The instantaneous configuration is shown schematically in Fig. 1.

Assuming the thin shear layer approximations as in Cebeci and Bradshaw (1977), the governing equations can be obtained from equations (2) and (3) to yield in rectangular coordinate system

$$\frac{\partial U^*}{\partial x^*} + \frac{\partial V^*}{\partial y^*} = 0 \quad (4a)$$

$$\text{and} \quad \frac{\partial U^*}{\partial t^*} + U^* \frac{\partial U^*}{\partial x^*} + V^* \frac{\partial U^*}{\partial y^*} = \frac{\partial}{\partial y^*} (v^* \frac{\partial U^*}{\partial y^*} - \overline{u^{*'} v^{*'}}) \quad (4b)$$

The boundary conditions on the centre-line and at the edge of the jet can be expressed respectively by

$$t^* \geq 0 \quad \begin{cases} y^* = 0 & \frac{\partial U^*}{\partial y^*} = V^* = 0 \\ y^* = y_\infty & U^* = 0 \end{cases} \quad (5a)$$

$$(5b)$$

The time-varying boundary condition at the nozzle exit is given by

$$t^* \geq 0 \quad x^* = 0 \quad U^* = U_0^*(y^*) (1 + \epsilon \sin \omega^* t^*) \quad (6a)$$

where $U_0^*(y^*)$ is the mean velocity profile at the nozzle exit, ϵ is the amplitude of pulsation and ω^* is the angular frequency of pulsation.

For all locations downstream of the nozzle, the initial conditions are given by

$$t^* = 0 \quad x^* \geq 0 \quad U^* = U_i^*(x^*, y^*) \quad (6b)$$

where $U_i^*(x^*, y^*)$ is the steady state solution of equation (4) with the boundary condition at the nozzle exit given by equation (6a) with $t^* = 0$.

2.1 Governing Equations in Non-Dimensional Form:

In order to enable the solution obtained to be valid for a family of velocity profiles which have the same normalized shape at the nozzle exit, the following non-dimensional variables are used: -

$$U(x,y,t) = \frac{U^*(x^*,y^*,t^*)}{U_{ci}^*}, \quad V(x,y,t) = \frac{V^*(x^*,y^*,t^*)}{U_{ci}^*}$$

$$x = \frac{x^*}{h} \quad y = \frac{y^*}{h} \quad t = \frac{U_{ci}^*}{h} t^* \quad \omega = \frac{h}{U_{ci}^*} \omega^*$$

$$v = \frac{v^*}{U_{ci}^* h} \quad \overline{u'v'} = \frac{\overline{u'^* v'^*}}{U_{ci}^{*2}}$$

where U_{ci}^* is the centre-line velocity at the nozzle exit at $t^* = 0$.

Equation (4) can now be written as

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (7a)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} \left(v \frac{\partial U}{\partial y} - \overline{u'v'} \right) \quad (7b)$$

subject to the following boundary and initial conditions

$$t \geq 0 \quad \begin{cases} y = 0 & \frac{\partial U}{\partial y} = V = 0 \\ y = \infty & U = 0 \end{cases} \quad (8a)$$

$$(8b)$$

$$t \geq 0 \quad x = 0 \quad U = U_0(y) (1 + \epsilon \sin \omega t) \quad (9a)$$

$$t = 0 \quad x \geq 0 \quad U = U_i(x,y) \quad (9b)$$

2.2 Turbulence Modelling

The Navier-Stokes equations constitute a complete set of equations of motion which can in theory be solved to yield a solution for any laminar or turbulent flow field. However, turbulence comprises a wide range of length scales bounded from above by the dimensions of the flow field and bounded from below by the diffusive action of molecular viscosity. The resolution scale of the smaller eddies which are responsible for the decay of turbulence is too small that it precludes the use of any existing computer. Furthermore, very often only the time-averaged properties are of interest in engineering applications even if the flow is time dependent. Hence, the Navier-Stokes equations are time-averaged first before being solved. This avoids not only the difficulty in representing all the characteristic turbulence scales but also unnecessary computations of transients if only the time-averaged properties are required. The process of time-averaging such as the Reynolds time-averaging described in section 2.0 causes the loss of certain information contained in the original equations and results in more unknowns than the governing equations through the introduction of statistical correlations of fluctuating velocities such as the $\overline{u'v'}$ term in equation (7b) which are known as apparent Reynolds stresses and are responsible for the actual momentum

transfer. Attempts to derive additional equations for those Reynolds stresses will only result in additional unknowns. Thus the time-averaging process presents a closure problem which is to reduce the number of unknowns to equal the number of governing equations. In order to achieve this, the additional unknown quantities must be modelled or approximated in terms of known quantities through a set of equations which, when solved with the mean-flow equations, simulate the actual flow situation. This process is generally termed "turbulence modelling."

Turbulence models can broadly be classified into "first-order" models in which the mean flow equations are solved without additional partial differential equations for the velocity fluctuation terms and "higher-order" models in which transport equations for higher-order velocity correlations are solved with the mean flow equations. Since in the "first-order" models additional partial differential equations are not solved for the turbulence quantities which are expressed in terms of known quantities through certain algebraic formulation, they are known as zero-equation models and "higher-order" models which involve at least one partial differential equation for the turbulence quantities can thus be termed one-equation or two-equation models, etc. A description of the various types of turbulence models can be found in Launder and Spalding (1972) and a comprehensive review of the state-of-art is given by Reynolds and Cebeci (1978).

Zero-equation models which are based on empirical correlations of the extensive available experimental data and mostly on the eddy-viscosity and mixing-length concepts such as the Cebeci-Smith eddy viscosity model (1974) have been widely used and proved to be very successful in obtaining very accurate predictions of non-separating flows. For more complex flows which involve recirculation and separation, the more refined two-equation $k-\epsilon$ model described by Launder and Spalding (1976) is the most well-developed among all other "high-order" models. It has been recognized that although "higher-order" models contain more information and hence simulate the flow situation more realistically than the "first-order" models, they are more difficult to solve and require generally an order-of-magnitude more computing storage and time while in non-separating and simple flow situations, the first-order models can yield predictions to the same degree of accuracy as the "higher-order" models. It is in the light of this philosophy that in this study, a constant-eddy viscosity model due to Prandtl (1942) is used.

By employing the eddy-viscosity concept of Boussinesq, the fluctuating-velocity correlation term $\overline{u'v'}$ in equation (7b) is related to the mean velocity gradient through the eddy viscosity ν_t as follows:

$$\overline{u'v'} = -\nu_t \frac{\partial U}{\partial y} \quad (10)$$

The eddy viscosity ν_t can then be obtained from the constant eddy-viscosity model as

$$\nu_t(t) = c y_{1/2} U_c(t) \quad (11)$$

where c is a constant, and $y_{1/2}$ is the jet half-width.

In this model, two constants c_1 and c_2 are used for the free jet as it emerges from developing to fully developed form. Hence c is given by

$$c = \begin{cases} c_1 & \text{if } \bar{U}_c \geq 0.95 \\ c_2 & \text{if } \bar{U}_c < 0.95 \end{cases} \quad (12)$$

By defining an effective eddy viscosity ν_{eff} such that

$$\nu_{eff} = \nu_t + \nu \quad (13)$$

and using equation (10), equation (7b) can then be written as

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} (\nu_{eff} \frac{\partial U}{\partial y}) \quad (14)$$

Hence the flow development of a periodically pulsed turbulent free jet is governed by equations (7a) and (14) subject to the boundary and initial conditions of equations (8) and (9) with the constant eddy viscosity model of (11)

2.3 Governing Equation in Transformed Variable Form

It has been shown by Lai and Simmons (1978) that transformations based on the similarity solutions are successful in reducing substantially the rate of spread of an unsteady laminar jet, hence enabling the use of variable grid sizes over predetermined regions. In this study, similar transformations are employed. A dimensionless transverse distance η and a dimensionless stream function ψ are defined by

$$\eta = ay/(\zeta + \zeta_0) \quad (15a)$$

$$\psi(\zeta, y, t) = b(\zeta + \zeta_0)^{1/2} f(\zeta, \eta, t) \quad (15b)$$

where a , b and ζ_0 are arbitrary constants which can be varied to facilitate computation and x is renamed as ζ .

The function f automatically satisfies the continuity equation (7a) and equation (14) can be re-written in the transformed coordinates as

$$\begin{aligned} (\zeta + \zeta_0)^{-1/2} [v_{eff} f'']' + (f')^2 + ff'' = 2(\zeta + \zeta_0) [f' \frac{\partial f'}{\partial \zeta} \\ - f'' \frac{\partial f}{\partial \zeta} + \frac{(\zeta + \zeta_0)^{1/2}}{ab} \frac{\partial f'}{\partial t}] \end{aligned} \quad (16)$$

where prime denotes differentiation with respect to η ,

$$f' = (\zeta + \zeta_0)^{1/2} U/ab \quad (17)$$

and a and b are chosen for convenience such that $a/b = \frac{1}{2}$.

The boundary conditions in equation (8) become

$$t > 0 \quad \begin{cases} \eta = 0 & f'' = 0 & f + 2(\zeta + \zeta_0) \frac{\partial f}{\partial \zeta} = 0 \\ \eta = \eta_\infty & f' = 0 \end{cases} \quad (18a)$$

$$(18b)$$

The initial conditions at $\zeta = 0$ in the (η, t) plane is obtained by writing equation (9a) with equation (17) as

$$f' = f'_0(\eta)(1 + e \sin \omega t) \quad (19a)$$

The initial conditions at $t = 0$ in the (ζ, η) plane are generated by the solutions $U_1(\zeta, \eta)$ of the following steady jet equation obtained from equation (16)

$$(\zeta + \zeta_0)^{-1/2} [v_{eff} f'']' + (f')^2 + ff'' = 2(\zeta + \zeta_0) [f' \frac{\partial f'}{\partial \zeta} - f'' \frac{\partial f}{\partial \zeta}] \quad (19b)$$

The effective eddy-viscosity can be obtained from equation (13) using equations (11) and (15) to give

$$v_{eff}(t) = cb(\zeta + \zeta_0)^{1/2} \eta_{1/2} f'_c(t) + v \quad (20)$$

where c is subject to the conditions specified in equation (12).

3.0 Method of Solution

Since the initial conditions in the (ζ, η) plane are generated by solving the steady jet equation (19b) subject to the boundary conditions in equation (18), the flow development of the steady jet is first studied. For computer programme development purpose, two separate computer programs were written for the steady and unsteady jet respectively with the solutions of the steady jet stored on disk for use in the unsteady jet program. However, if necessary, the steady jet program can easily be incorporated into the unsteady one. Both equations (16) and (19b) are parabolic and can be solved by a marching procedure. The finite difference scheme employed is the Box Method developed by Keller (1970), and described in detail by Cebeci and Bradshaw (1977). The scheme has been applied successfully to a wide range of boundary-layer type of flows including both time-dependent and separating flows by Cebeci and his co-workers (e.g., see Bradshaw et al (1980)).

3.1 Finite Difference Form of the Governing Equations

Equation (16) is rewritten as a system of first order partial differential equations -

$$f' = g \quad (21a)$$

$$g' = q \quad (21b)$$

$$\begin{aligned} (\zeta + \zeta_0)^{-1/2} [v_{\text{eff}} q]' + g^2 + fq = 2(\zeta + \zeta_0) \left[g \frac{\partial g}{\partial \zeta} - q \frac{\zeta f}{\partial \zeta} \right. \\ \left. + \left(\frac{\zeta + \zeta_0}{ab} \right)^{1/2} \frac{\partial g}{\partial t} \right] \end{aligned} \quad (21c)$$

Consider the net cube shown in Fig. 2 and denote the grid points by

$$\zeta_0 = 0 \quad \zeta_{i+1} = \zeta_i + r_i \quad i = 1, 2, \dots, I$$

$$t_0 = 0 \quad t_n = t_{n-1} + k_n \quad n = 1, 2, \dots, N$$

$$\eta_0 = 0 \quad \eta_j = \eta_{j-1} + h_j \quad j = 1, 2, \dots, J$$

$$\eta_J = \eta_\infty$$

where i, n, j are just sequence numbers and the variable net spacings r_i , k_n and h_j are completely arbitrary.

In the study of jets, a coarse grid spacing h_j can be used in the vicinity of the centre line and the edge of the jet as the transverse (η) gradients of f are small in those regions. The quantities (f, g, q) at points (ζ_i, t_n, η_j) are approximated by the grid functions $(f_j^{i,n}, g_j^{i,n}, q_j^{i,n})$. Hence by using central differencing and averaging about the mid-point $(\zeta_i, t_n, \eta_{j-1/2})$, Eqs. (21a) and (21b) can be written in the following finite difference forms

$$(f_j^{i,n} - f_{j-1}^{i,n})/h_j = g_{j-\frac{1}{2}}^{i,n} \quad (22a)$$

$$(g_j^{i,n} - g_{j-1}^{i,n})/h_j = q_{j-\frac{1}{2}}^{i,n} \quad (22b)$$

where, for example, the shorthand notation $g_{j-\frac{1}{2}}^{i,n}$ has been used for $\frac{1}{2}(g_j^{i,n} + g_{j-1}^{i,n})$

Shorthand notations are introduced, for example,

$$\bar{f}_i = \frac{1}{2}(f_j^{i,n} + f_j^{i,n-1} + f_{j-1}^{i,n} + f_{j-1}^{i,n-1}) = \frac{1}{2}(f_{j-\frac{1}{2}}^{i,n} + f_{j-\frac{1}{2}}^{i,n-1})$$

$$\bar{f}_j = \frac{1}{4}(f_j^{i,n} + f_j^{i-1,n} + f_j^{i,n-1} + f_j^{i-1,n-1}) = \frac{1}{4}(f_j^{i,n} + f_j^{234})$$

$$\bar{f}_n = \frac{1}{4}(f_j^{i,n} + f_j^{i-1,n} + f_{j-1}^{i,n} + f_{j-1}^{i-1,n}) = \frac{1}{2}(f_{j-\frac{1}{2}}^{i,n} + f_{j-\frac{1}{2}}^{i-1,n})$$

$$f_j^{234} = f_j^{i,n-1} + f_j^{i-1,n} + f_j^{i-1,n-1}$$

Eq. (21c) can be approximated using central differencing at $(\zeta_{i-\frac{1}{2}}, t_{n-\frac{1}{2}}, \eta_{j-\frac{1}{2}})$ by

$$\begin{aligned} (Bq)_j - (Bq)_{j-1} + h_j \{ (g^2)_{j-\frac{1}{2}} + (fq)_{j-\frac{1}{2}} - \alpha [g_{j-\frac{1}{2}}(g_{j-\frac{1}{2}} + \\ g_{j-\frac{1}{2}}^{i,n-1} + g_{j-\frac{1}{2}}^{234} - 2\bar{g}_{i-1}) - q_{j-\frac{1}{2}}(f_{j-\frac{1}{2}} + \\ f_{j-\frac{1}{2}}^{i,n-1} - 2\bar{f}_{i-1}) - f_{j-\frac{1}{2}}q_{j-\frac{1}{2}}^{234}] - \\ \alpha_i g_{j-\frac{1}{2}} \} = \tau_{j-\frac{1}{2}}^{i-1,n-1} \end{aligned} \quad (22c)$$

where $B = (\zeta + \zeta_0)^{-\frac{1}{2}} v_{eff}$

$$\alpha = \frac{m_1^{i-\frac{1}{2}}}{2r_i}$$

$$m_1 = 2(\zeta + \zeta_0)$$

$$\alpha_1 = \frac{4[(\zeta + \zeta_0)^{3/2}]^{i-\frac{1}{2}}}{ab k_n}$$

$$T_{j-\frac{1}{2}}^{i-1,n-1} = (Bq)_{j-1}^{234} - (Bq)_j^{234} + h_j \{ - (g^2)_{j-\frac{1}{2}}^{234} - (fq)_{j-\frac{1}{2}}^{234} + \\ a[\beta_2 g_{j-\frac{1}{2}}^{234} - \beta_1 q_{j-\frac{1}{2}}^{234}] + \alpha_1 \beta_3 \}$$

$$\beta = g_{j-\frac{1}{2}}^{i,n-1} + g_{j-\frac{1}{2}}^{234} - 2 \bar{g}_{i-1}$$

$$\beta_1 = f_{j-\frac{1}{2}}^{i,n-1} + 2 \bar{f}_{i-1}$$

$$\beta_2 = g_{j-\frac{1}{2}}^{i,n-1} - 2 \bar{g}_{i-1}$$

$$\beta_3 = g_{j-\frac{1}{2}}^{i-1,n} - 2 \bar{g}_{n-1}$$

In the above equation (22c), the superscripts i,n have been dropped for simplicity.

It has been pointed out by Keller (1978) that in approximating non-linear terms such as $(fq)_{j-\frac{1}{2}}$ in equation (22c) with the Box scheme, there exists several choices which should not have serious effect on accuracy or stability as long as the proper centering is maintained. However, in jet calculations, it has been found in this study that the approximation of terms such as (fq) in the form of $(f)_{j-\frac{1}{2}}(q)_{j-\frac{1}{2}}$

results in better numerical stability at the jet edge.

The boundary conditions in Eq. (18) become

$$q_0^{i,n} = 0 \quad (23a)$$

$$f_0^{i,n} = \left(1 + \frac{2m_1}{r_i}\right)^{-1} \left(\frac{2m_1}{r_i} - 1\right) f_0^{i-1,n} \quad (23b)$$

$$g_J^{i,n} = 0 \quad (23c)$$

The initial conditions at $\zeta = 0$ in the (η, t) plane in Eq (19a) are given by

$$(f')_j^n = (f'_0)_j^n [1 + \epsilon \sin \omega(n\Delta t)] \quad (24)$$

where Δt is the temporal grid size.

The initial conditions at $t = 0$ in the (ζ, η) plane correspond to the steady jet solutions of equation (19b). The finite difference equations can be obtained similar to the above derivation for the unsteady jet except terms in equation (22) at $t = t_{n-1}$ are equal to those at $t = t_n$ and $\alpha_1 = 0$.

3.2. Time Varying Velocity Profile at the Nozzle Exit

Because of symmetry, only half of the jet needs to be computed. The time-varying velocity profile considered at the nozzle exit is the commonly assumed "top-hat" profile which is given by

$$U(y,t) = \begin{cases} 1 + \epsilon \sin \omega t & \text{for } 0 \leq y \leq 1/2 \\ 0 & \text{for } 1/2 < y \end{cases} \quad (25)$$

In terms of the transformed coordinates, equation (25) can be rewritten using equation (17) as

$$f' = \begin{cases} \frac{(\zeta + \zeta_0)^{1/2}}{ab} (1 + \epsilon \sin \omega t) & \text{for } 0 \leq \eta \leq (\eta_\infty)_i \\ 0 & \text{for } (\eta_\infty)_i \leq \eta \end{cases}$$

$$\text{where } (\eta_\infty)_i = 0.5 a/\zeta_0 \quad (26)$$

Since a discontinuity exists in f' at $\eta = (\eta_\infty)_i$ and will cause computational problems, the top-hat profile has to be approximated by

$$f' = \frac{(\zeta + \zeta_0)^{1/2}}{ab} \phi_1(\eta) (1 + \epsilon \sin \omega t) \quad (27)$$

where

$$\phi_1(\eta) = \begin{cases} 1 & \text{for } 0 \leq \eta \leq \eta_a \\ (1 - \eta_N^2)(1 + 2\eta_N) & \text{for } \eta_a \leq \eta \leq (\eta_\infty)_i \\ 0 & \text{for } (\eta_\infty)_i \leq \eta \end{cases} \quad (28)$$

$$\eta_N = \frac{\eta - \eta_a}{(\eta_\infty)_i - \eta_a}$$

and η_a is some point in the interval $[0, (\eta_\infty)_i]$.

The function $(1 - \eta_N^2)(1 + 2\eta_N)$ was obtained by matching the velocity and velocity gradient profiles at $\eta = (\eta_\infty)_i$ and $\eta = \eta_a$ through the following boundary conditions to ensure continuity:

$$\eta_N = 0 \quad \phi_1 = 1 \quad \phi_1' = 0 \quad (29a)$$

$$\eta_N = 1 \quad \phi_1 = 0 \quad \phi_1' = 0 \quad (29b)$$

f and f'' can be obtained by respectively integrating and differentiating equation (27) with respect to η to yield

$$f = \frac{(\zeta + \zeta_0)^{1/2}}{ab} \phi_0(\eta)(1 + \epsilon \sin \omega t) \quad (30)$$

$$\text{and } f'' = \frac{(\zeta + \zeta_0)^{1/2}}{ab} \phi_2(\eta)(1 + \epsilon \sin \omega t) \quad (31)$$

$$\phi_0(\eta) = \begin{cases} 1 & \text{for } 0 \leq \eta \leq \eta_a \\ [(\eta_\infty)_i - \eta_a] \eta_N (1 - \eta_N^2 + 0.5 \eta_N^3) & \text{for } \eta_a \leq \eta \leq (\eta_\infty)_i \\ 0 & \text{for } (\eta_\infty)_i \leq \eta \end{cases} \quad (32)$$

$$\text{and } \phi_2(\eta) = \begin{cases} 1 & \text{for } 0 < \eta < \eta_a \\ -6\eta_N (1 - \eta_N) / [(\eta_\infty)_i - \eta_a] & \text{for } \eta_a \leq \eta \leq (\eta_\infty)_i \\ 0 & \text{for } (\eta_\infty)_i \leq \eta \end{cases} \quad (33)$$

The value of η_a can be determined to match the experimental top-hat profile as close as possible and in this study it is chosen to be the point such that the velocity profile is uniform over 97.5% of the nozzle.

Adopting similar procedure given in Lai and Simmons (1978),
a and ζ_0 are given the values 0.3 and 3.75 respectively
such that $(\eta_\infty)_i = 0.04$ and $\eta_a = 0.039$.

3.3 Solution of the Finite Difference Equation

With $(f_j^{n-1}, g_j^{n-1}, q_j^{n-1})$ known from the solution of Eq (19b) and $(f_j^{i-1}, g_j^{i-1}, q_j^{i-1})$ specified by Eqs. (27), (30) and (31), Eq.(22) for $1 \leq j \leq J$ and Eq. (23) yield an implicit non-linear algebraic system of $3J + 3$ equations. This system is linearized by Newton's Method by introducing the perturbed quantities $(\delta f, \delta g, \delta q)$ to yield after considerable algebra

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}(\delta g_j + \delta g_{j-1}) = (r_1)_j \quad (34a)$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2}(\delta q_j + \delta q_{j-1}) = (r_3)_{j-1} \quad (34b)$$

$$\begin{aligned} (s_1)_j \delta q_j + (s_2)_j \delta q_{j-1} + (s_3)_j \delta f_j + (s_4)_j \delta f_{j-1} \\ + (s_5)_j \delta g_j + (s_6)_j \delta g_{j-1} = (r_2)_j \end{aligned} \quad (34c)$$

where

$$(s_1)_j = B_j + \frac{h_j}{2} (f_j + \alpha f_{j-\frac{1}{2}} + \alpha \beta_1)$$

$$(s_2)_j = B_{j-1} + \frac{h_j}{2} (f_{j-1} + \alpha f_{j-\frac{1}{2}} + \alpha \beta_1)$$

$$(s_3)_j = \frac{h_j}{2} (q_j + \alpha q_{j-\frac{1}{2}} + \alpha q_{j-\frac{1}{2}}^{234})$$

$$(s_4)_j = \frac{h_j}{2} (q_{j-1} + \alpha q_{j-\frac{1}{2}}^{234})$$

$$(s_5)_j = h_j (g_j - \alpha g_{j-\frac{1}{2}} - \frac{\alpha \beta}{2} - \frac{\alpha_1}{2})$$

$$(s_6)_j = h_j (g_{j-1} - \alpha g_{j-\frac{1}{2}} - \frac{\alpha \beta}{2} - \frac{\alpha_1}{2})$$

$$(r_1)_j = f_{j-1} - f_j + h_j g_{j-\frac{1}{2}}$$

$$(r_3)_{j-1} = g_{j-1} - g_j + h_j q_{j-\frac{1}{2}}$$

$$\begin{aligned} (r_2)_j &= T_{j-\frac{1}{2}}^{i-1,n-1} + (Bq)_{j-1} - (Bq)_j - h_j \{g_{j-\frac{1}{2}}^2 (fq)_{j-\frac{1}{2}} \\ &\quad - \alpha [\frac{1}{2}(g_{j-\frac{1}{2}}^2 + g_j g_{j-1}) + \beta g_{j-\frac{1}{2}} - f_{j-\frac{1}{2}} g_{j-\frac{1}{2}} \\ &\quad - \beta_1 q_{j-\frac{1}{2}} - q_{j-\frac{1}{2}}^{234} f_{j-\frac{1}{2}}] - \alpha_1 g_{j-\frac{1}{2}} \} \end{aligned}$$

The boundary conditions are given by

$$\begin{aligned} \delta f_0 &= 0 \\ \delta q_0 &= 0 \\ \delta g_j &= 0 \end{aligned} \tag{35}$$

The above derivation is similar to that outlined in Appendix C in Lai & Simmons (1978). The linear system Eq. (3.10) is then solved very effectively by the Block Elimination Method discussed by Cebeci and Bradshaw (1977).

The linearized form of the steady jet equation can similarly be obtained and solved with quantities at $t = t_{n-1}$ equal to those at $t = t_n$ and $\alpha_1 = 0$.

3.4 Convergence Criterion

As the governing equations for both the steady and unsteady jet are parabolic, they are solved by marching along the t -direction in the case of the unsteady jet and along the ζ -direction in the case of the steady jet. At a given stream-wise (ζ) station, the linearized system Eq. (34) is solved by iterating at each t -station until some convergence criterion is satisfied. Iterations are terminated at each t -station if

$$|f^{(i+1)} - f^{(i)}| < \epsilon_1, \text{ at } \eta = \eta_{\text{con}} \quad (36)$$

where the value of ϵ_1 is prescribed;

$f^{(i)}$ and $f^{(i+1)}$ are the i th and $(i+1)$ th iterates of f' respectively; and η_{con} is some point where the convergence criterion is applied. In practice, it is adequate to set ϵ_1 to be 10^{-3} .

3.5 Criterion for the Spreading of Jet

Since the mass flow varies sinusoidally with time, and because of the initially steep velocity gradient, the jet width will be changing with time at a given streamwise (ζ) station. A criterion must therefore be set to determine η_∞ at each t -station, noting that $\eta_\infty^{(i)} \geq \eta_\infty^{(i-1)}$. Here $\eta_\infty^{(i)}$ and $\eta_\infty^{(i-1)}$ are the jet width for the i th and $(i-1)$ th iterations. The edge of the jet is defined by the following two conditions -

$$|f'_{J-1}| \leq \epsilon_2 \quad (37a)$$

$$|f''_j| \leq \epsilon_3 \quad (37b)$$

where the values of ϵ_2 and ϵ_3 are prescribed and J denotes the point at the jet boundary. Experience indicates that it is sufficient to choose

$$\epsilon_2 = 10^{-2} \quad \text{and} \quad \epsilon_3 = 10^{-1}$$

If the criteria set out in Eq. (37) are satisfied then $\eta_\infty^{(i+1)} = \eta_\infty^{(i)}$. Otherwise n_g points have to be added so that $J_{\text{new}} = J_{\text{old}} + n_g$ and the values of $(f_j^{i,n}, g_j^{i,n}, q_j^{i,n})$ for the new n_j points are obtained as follows -

$$f_j^{i,n} = (\eta_j - \eta_\infty) g_J^{i,n} + f_J^{i,n} \quad (38a)$$

$$g_j^{i,n} = g_J^{i,n} \quad (38b)$$

$$q_j^{i,n} = 0 \quad (38c)$$

$$B_j^{i,n} = B_J^{i,n} \quad (38d)$$

The same procedure is also applied to $f_j^{i,n-1}$, $g_j^{i,n-1}$,
 $q_j^{i,n-1}$, $B_J^{i,n-1}$, $f_j^{i-1,n}$, $g_j^{i-1,n}$, $q_j^{i-1,n}$, $B_j^{i,n-1}$, $f_j^{i-1,n-1}$,
 $q_j^{i-1,n-1}$, $B_j^{i-1,n-1}$

3.6 Criterion for the Attainment of Steady State Solution

As the mass flow at the nozzle varies sinusoidally with time, the steady state solution at any streamwise (ζ) station downstream must vary periodically and can be expressed in the form

$$f'(\zeta, t, \eta) = f'_0(\zeta, t, \eta) + \lim_{L \rightarrow \infty} \sum_{\ell=1}^L \varepsilon_{\ell} f_{\ell}(\zeta, \eta) \sin(\ell \omega t + \phi_{\ell}) \quad (39)$$

In general, for a periodically pulsed flow, the steady state solution at a given streamwise (ζ_i) station, is said to be reached if

$$f'(\zeta, t + nT, \eta) = f'(\zeta, t, \eta) \quad \text{for } n = 1, 2, \dots$$

where T is the period of oscillation.

This entails velocity profiles to agree over a few cycles and thus requires unnecessary and uneconomic computations extending for a few periods in order to ascertain that steady state has been attained. However, if all the three parameters, f , f' and f'' are considered, it is adequate to regard that the steady state solution has been reached if at some point η_s , the following criteria are satisfied -

$$\begin{aligned} | [f(\zeta, t_n + T, \eta_s) - f(\zeta, t_n, \eta_s)] / f(\zeta, t_n, \eta_s) | &\leq \varepsilon_4 \\ | [f'(\zeta, t_n + T, \eta_s) - f'(\zeta, t_n, \eta_s)] / f'(\zeta, t_n, \eta_s) | &\leq \varepsilon \quad n = 1, 2 \quad (40) \\ | [f''(\zeta, t_n + T, \eta_s) - f''(\zeta, t_n, \eta_s)] / f''(\zeta, t_n, \eta_s) | &\leq \varepsilon_6 \end{aligned}$$

where the values of ϵ_4 , ϵ_5 and ϵ_6 are prescribed and $t_2 - t_1 = \Delta t$.

The sensitivity of the steady-state test varies with the point η_s to which it is applied. In practice, the point η_s should be so chosen such that reasonable sensitivity is achieved without excessive computations.

Since f' and f'' are the first and second order derivatives of f , discrepancies between results separated by one period T are more pronounced in f'' . Hence, in general ϵ_4 , ϵ_5 and ϵ_6 must be so chosen that $\epsilon_4 \leq \epsilon_5 \leq \epsilon_6$. Otherwise, if $\epsilon_6 > \epsilon_5 > \epsilon_4$ and if ϵ_6 is very small, the criteria in Eq. (40) might never be satisfied although the steady state solution has long been attained within practical limits.

As a further check of the attainment of the steady-state condition, the following condition is imposed on f' at the centre-line -

$$|[f'(\zeta, t_n + T, 0) - f'(\zeta, t_n, 0)]/f'(\zeta, t_n, 0)| \leq \epsilon_7 \quad (41)$$

for $n = 1, 2$

where the value of ϵ_7 is prescribed.

It has been chosen that $\epsilon_4 = \epsilon_5 = \epsilon_6 = \epsilon_7 = 10^{-2}$

3.7 Mean Momentum Flux, Phase Angle and Peak-to-Peak Oscillation

3.7.1 Mean Momentum Flux

The instantaneous momentum at a specified streamwise (ζ_i) station is given by

$$M_i = 2hU_{ci}^2 \int_0^{\eta_\infty} [f'(\zeta, t_i, \eta)]^2 d\eta \quad (42)$$

M_i is evaluated by Simpson's rule for unequally spaced points derived in Appendix D in Lai & Simmons (1978).

The mean momentum flux in the streamwise direction is defined by

$$M = \frac{1}{T} \int_{t_0}^{t_0 + T} M_i dt \quad (43)$$

and the integral is approximated by

$$M = \frac{1}{N} \sum_{i=1}^N M_i \quad (44)$$

The quantity M can be served as an additional check on the overall results by testing for its constancy with ζ .

3.7.2. Phase Angle

The phase angle between the fundamental component of the centre-line velocity at any downstream station and that at the nozzle can readily be obtained by cross-correlating the steady-state instantaneous centre-line velocity at that station with a reference sine and cosine signal respectively.

Consider

$$S_1 = \sin \omega t \quad (45)$$

$$S_2 = \cos \omega t \quad (46)$$

From Eq. (39),

$$f'_C \equiv f'(\zeta, t, 0) = f'_0(\zeta, t, 0) + \lim_{L \rightarrow \infty} \sum_{\ell=1}^L \epsilon^\ell f_\ell(\zeta, 0) \sin(\ell\omega + \phi_\ell) \quad (47)$$

Multiplying Eq. (45) with Eq. (47) and taking time average yields

$$\overline{S_1 f'_C} = A \cos \phi_1 \quad (48)$$

where A is a constant.

Multiplying Eq. (46) with Eq. (47) and taking time average gives

$$\overline{S_2 f'_C} = A \sin \phi_1 \quad (49)$$

From Eqs. (48) and (49),

$$\phi_1 = \tan^{-1} \frac{\overline{S_2 f'_C}}{\overline{S_1 f'_C}} \quad (50)$$

3.7.3. Peak-to-Peak Oscillation

The percentage peak to peak variation of the centre-line velocity at any downstream station is defined by

$$F = \frac{(f'_C)_{\max} - (f'_C)_{\min}}{\overline{f'_C}} \times 100\% \quad (51)$$

The quantities $(f'_c)_{\max}$ and $(f'_c)_{\min}$ are obtained by quadratic interpolation of the form

$$f'_c = a_0 t^2 + a_1 t + a_2 \quad (52)$$

through three points $(f'_c(t_1), t_1)$, $(f'_c(t_2), t_2)$ and $(f'_c(t_3), t_3)$

where for $(f'_c)_{\max}$, $f'_c(t_1) \leq f'_c(t_2)$

$$f'_c(t_3) \leq f'_c(t_2)$$

and for $(f'_c)_{\min}$ $f'_c(t_1) \geq f'_c(t_2)$

$$f'_c(t_3) \geq f'_c(t_2)$$

Eq. (52) can be solved for a_0 , a_1 and a_2 by substituting $f'_c(t_1)$, $f'_c(t_2)$ and $f'_c(t_3)$. With a_0 , a_1 and a_2 known, $(f'_c)_{\max}$ or $(f'_c)_{\min}$ can be obtained from Eq. (52) with t given by $-a_1/2a_0$.

4.0 Results and Discussions

The structure and listing of the computer programs for both the steady and unsteady jet are described in Appendix A. Calculations were performed on the University of Queensland PDP1055 and the U.S. Naval Postgraduate School IBM 360/67 computers. The computer program for the steady jet occupies a core memory of 16K words and that of the unsteady jet occupies a core memory of 67K words. It must be pointed out here that the unsteady jet program can actually be reduced to about 25K words of core memory since at any one time instant, only calculations involving two time levels are required in core whereas all the quantities at other time levels can be stored on disk and retrieved when required. However, in this case, the computer core storage is not a problem with the computing facility available whereas the computing time is important and as such a trade-off is made such that quantities at all time levels are retained in core in order to save computing time of writing to and reading from disk. Moreover, although a constant eddy viscosity model was used, the programs were written to accept variable eddy viscosity. A very straightforward modification of the program by incorporating a routine to read from and write to a disk can reduce its size to enable it to be run on any mini-computer with available core memory of about 25K words.

4.1 Steady Jet

Owing to the nature of the initially top-hat velocity profile, the initial velocity gradient is very steep and the velocity profile changes vary appreciably over a very short streamwise distance. Since the solution of the finite difference equations is obtained through linearization by Newton's method as described in section 3.3, an initial guess at any streamwise station must be close to the solution. Consequently, a fine grid has also to be used initially but as the computation proceeds and as the velocity profiles start to appear in similar form, a coarser grid can be used by dropping every other point. The initial grid used in the transverse (η) direction is specified as follows:

$$\begin{array}{rcl}
 & 0.005 & 0 \leq \eta \leq 0.01 \\
 & 0.002 & 0.01 \leq \eta \leq 0.036 \\
 & 0.0002 & 0.036 \leq \eta \leq 0.039 \\
 \Delta \eta = & 0.0001 & 0.039 \leq \eta \leq 0.04 \\
 & 0.00025 & 0.04 \leq \eta \leq 0.045 \\
 & 0.001 & 0.045 \leq \eta \leq 0.06 \\
 & 0.002 & 0.06 \leq \eta \leq 0.02 \\
 & 0.01 & \eta \leq 0.12 \\
 (\eta_{\infty})_i = & 0.04 &
 \end{array} \tag{53}$$

The grid-sizes in the streamwise (ζ) direction are specified as follows:

$\Delta \zeta =$	0.0005	$0 \leq \zeta \leq 0.007$	
	0.005	$0.007 \leq \zeta \leq 0.032$	
	0.025	$0.032 \leq \zeta \leq 0.132$	
	0.1	$0.132 \leq \zeta \leq 1.032$	
	1	$1.032 \leq \zeta \leq 20.032$	(54)
	2	$20.032 \leq \zeta \leq 40.032$	
	5	$40.032 \leq \zeta$	

The constants c_1 and c_2 in equation (12) in the constant eddy viscosity model are varied to match the experimental data. The sensitivity of the solutions to various values of c_1 and c_2 was tested. No significant difference was found when c_1 was varied from 0.009 to 0.012 and c_2 from 0.032 to 0.037. The final values of $c_1 = 0.009$ and $c_2 = 0.034$ were chosen.

As shown in Fig. 3, the non-dimensional self-preserved velocity profiles of various workers agree very well with each other and with the Goertler solution except near the jet edge where conventional hot-wire measurements are dubious. However, the results of Heskestad (1965) follow very closely the Goertler solution and those of Robins (1971) which have been recommended by Rodi (1975) as reliable agree very well with the results of Heskestad (1965). The computed development of the non-dimensional velocity profiles of the steady jet with y normalized with respect to the jet half-width $y_{1/2}$ are depicted in Fig. 4 and at streamwise station $\zeta = 22.032$, the mean velocity profile differs insignificantly from

the Goertler solution, indicating that self-preservation in mean velocity profile is attained.

The variation of the non-dimensional centre-line velocity with streamwise distance is plotted in Fig. 5. The results agree well with the experimental data of Lai and Simmons (1979) and Zijnen (1958). The variation of the jet half-width with streamwise distance is compared with the result of Kotsovinos (1976) and Lai and Simmons (1979) in Fig. 6 and is found to give reasonably good agreement for streamwise distance less than about 40 nozzle widths. The discrepancy which exists at larger streamwise distance is not as significant as it appears because the results of Kotsovinos (1976) were obtained by fitting a third-order polynomial through the scattered data in the literature and most of the experimental data were not available for streamwise distance larger than 50 nozzle widths. Furthermore, uncertainty in the hot-wire data increases as the streamwise distance increases because of the decay of the velocity profile and possible three-dimensional and reversed flow effects. The rate of decay of the centre-line velocity $d \bar{U}_C^2 / d(x/h)$ of 0.165 and the spreading rate $d(y_{1/2}/h) / d(x/h)$ of 0.106 obtained from Figs. 5 and 6 respectively fall within the range of values reported in the literature. As pointed out by Goldschmidt and Bradshaw (1980), exact self-preservation in mean quantities requires the kinematic virtual origin, obtained from the centre-line velocity decay curve, be equal to the geometric virtual origin obtained from the jet spreading curve. However, most

experimental data do not confirm this. In this study, the computed kinematic virtual origin agrees with the geometric virtual origin and is found to be $0.8h$.

The development of the normalized shear stress profiles is shown in Fig. 7. The agreement between the computed shear stress profile at $\zeta = 95.032$ and the experimental results of Gutmark and Wagnanski (1976) is good considering that the shear stress term is the most difficult to be measured with sufficient accuracy and a scatter of at least 20% exists in the available experimental data in the literature. The maximum value of the computed non-dimensional shear stress term $\overline{u'v'}/U_c^2$ is 0.023 which agrees well with most measured values reported in Rodi (1975).

In jet computations, numerical errors introduced at the jet edge normally have little overall influence on the calculations as noted by McGuirk and Rodi (1979). The momentum integral in this study varies by less than 1% over a streamwise distance of 100 nozzle widths.

4.2 Unsteady Jet

The constant eddy-viscosity model which was found to give good agreement with experimental data was used in the computation of the preiodically pulsed jet. Results were obtained for three frequencies of pulsation, $\omega = 0.000871$, 0.00871 and 0.0871 which for a jet exit Reynolds number of 10^4 and a jet width h of 5mm correspond to 1 , 10 and 100 Hz respectively. Two values of amplitude of pulsation were studied, namely $\epsilon = 0.1$ and 0.15 . The results were compared with the experimental data of Lai and Simmons (1979) and solutions for steady jets. The grid-sizes used in the streamwise (ζ) and transverse (η) directions are the same as those of the steady jet given in section 4.1.

4.2.1 Sensitivity of the Solution to the Convergence Criterion

Results show that solutions are very sensitive to the convergence criterion applied to points very near to the edge of the jet. Normally, the closer the point η_{con} is to the edge of the jet, the more difficult it will be for the solution to converge. It was found by Lai and Simmons (1979) that the convergence criterion can be applied to a point in the jet which varies with streamwise distance and is defined by

$$\eta_{\text{con}} = \begin{cases} (\eta_{\infty})_i - 3 & \text{for } \zeta \leq 0.1 \\ 5(\eta_{\infty})_{\text{min}}/10-1 & \text{for } \zeta \geq 0.1 \end{cases} \quad (55)$$

This allows a comparatively sensitive measure of the convergence of the solution and yet maintains a reasonably

fast convergence without affecting the overall accuracy of the solution.

A fixed point arbitrarily defined by $\eta_{\text{con}} = \eta_{\infty} - 6$ may also be sufficient.

4.2.2 Validity of the Criterion for the Attainment of Steady State Solution

The criterion for the attainment of steady state solution described in section 3.6 is found to be adequate. The point η_s to which the steady state tests are applied has been chosen to coincide with η_{con} specified in equation (53).

4.2.3 Sensitivity of the Solution to Time Step

The sensitivity of the solution to time step has been tested for various frequencies. Calculations have been carried out with the period of pulsation divided into 12, and 49 intervals respectively. Results indicate that although division of a period into 12 time intervals seems to be too coarse, they differ insignificantly from each other. It is therefore sufficient to use 12 time intervals in the present study.

4.2.4 Relaxation of the Criterion for Jet Spreading

As pointed out in section 3.5, the jet spreads rapidly initially due to a steep velocity gradient. As the computations proceed downstream, the velocity profile becomes wider with a long tail at the jet edge. Because of the very low velocities at the jet edge, small numerical errors introduced there might cause instabilities and the criteria set out in equation (37) for the definition of the jet edge will not be satisfied resulting in continuous addition of unnecessary grid points. Furthermore, the number n_g of grid points which can

be added each time imposes a trade-off between the accuracy of the solution and the computing time required. This is because less computing time will be required if more points are added but since the quantities at the new points are obtained by extrapolation and the jet edge is particularly sensitive to small numerical errors, instability or inaccurate solution may result. The program provides choices of two different number of added grid points as the calculations are marched downstream and if the jet spreads beyond a certain n value which can be obtained from the Goertler solution, a new definition for the jet edge is applied to terminate the jet at the point where the velocity starts increasing or changes sign.

4.2.5 Results

Non-dimensional mean and instantaneous velocity profiles at various time delay intervals are plotted in Fig. 8 for the various frequencies and amplitudes of pulsation and $\zeta = 40.032$. The mean velocity profiles follow closely the steady jet curve. The instantaneous profiles also collapse into the steady jet curve except in Figs. 8(e) and (f) where a slight discrepancy occurs for $\omega = 0.00871$ and 0.0871 and $\epsilon = 0.15$. This agrees with the trend observed in the experimental data of Lai and Simmons (1979), in which a slight departure of the non-dimensional instantaneous velocity profiles from the steady jet curve exists.

The mean centre-line velocity U_c obtained by time-averaging the steady state instantaneous centre-line velocity U_c over a period is shown in Fig. 9 to agree well with the centre-line velocity

decay curve for the steady jets for all the computed cases. Fig. 10 shows the variation of the jet half-width of the mean velocity profile with streamwise distance. For all cases, the mean jet half-width of the unsteady jet collapses on the steady jet spreading curve. A plot of Q/Q_E , a measure of mean entrainment, versus ζ for various frequencies and amplitudes of pulsation indicates that the mean entrainment rate does not differ from that of the steady jet, which is consistent with the trend associated with the mean centre-line velocity decay and jet spreading. In all the computed cases, the mean momentum flux M is conserved to within 1% over a streamwise distance of 100 nozzle widths, thus confirming the accuracy of the solutions.

With reference to Fig. 12, the non-dimensional mean shear stress profiles of the unsteady jet at $\zeta = 40.032$ agrees very well with that of the steady jet. The non-dimensional instantaneous shear stress profiles can also be collapsed into the steady jet profile except for $\omega = 0.0871$ in Figs. 12(c) and (f) where significant discrepancies are noted. This is consistent with the behavior of the instantaneous velocity profiles where a slight disagreement will be amplified in the velocity gradient term which is shown up in the shear stress profile.

Fig. 13 shows the variation of the steady-state instantaneous centre-line velocity with time for various frequencies and amplitudes of pulsation at three different streamwise distances. The reference signal is given by

$\bar{U}_c (1 + \epsilon \sin \omega t)$. It can be noted that as the frequency and amplitude increases and for large streamwise distance, distortions in the waveform of the centre-line velocity exist, an indication of the importance of higher harmonics. Such a trend was also observed by Lai and Simmons (1979) in their experimentally pulsed jet. The variation of the phase angle ϕ between the steady-state fundamental component of the centre-line velocity at any streamwise station and that at the nozzle exit with ζ is depicted in Fig. 14. The phase angle is a lag which increases with both frequency and streamwise distance. However, the amplitude of pulsation does not have any significant effect on the phase lag variation. For a region sufficiently close to the nozzle, the flow varies with time in a quasi-steady manner. That is, the solution over the region of interest can be approximated by a sequence of steady jet solutions, each of which corresponds to the instantaneous conditions at the nozzle. The region over which quasi-steady approximations can be applied to an unsteady jet can readily be determined by using the phase plot in Fig. 14. It is taken arbitrarily here that a phase lag of 5 degrees defines the downstream limit to quasi-steady solutions. This definition enables the quasi-steady approximations to $\bar{U}_c$ to be accurate to within 1%. From Fig. 14 it is apparent that the region over which quasi-steady approximations can be used decreases with increasing frequency. The quasi-steady region for the computed frequencies and for $\epsilon = 0.1$ or 0.15 is as follows -

$$\omega = 0.000871 \quad 0 \leq \zeta \leq 60$$

$$\omega = 0.00871 \quad 0 \leq \zeta \leq 10$$

$$\omega = 0.0871 \quad 0 \leq \zeta \leq 1$$

The variation of the percentage peak-to-peak oscillation, F , of the steady-state instantaneous centre-line velocity expressed as a percentage of the mean value with streamwise distance is plotted in Fig. 15. Although for $\omega = 0.0871$, F differs quite significantly from its initial value of 20% or 30%, it varies very little for other tested frequencies and amplitudes of pulsation. In the experimental data of Lai and Simmons (1979), it was shown that for frequencies between 1 and 10 Hz, which correspond to $\omega = 0.000871$ and 0.00871 here, the general trend indicates that F increases to a maximum at 10 nozzle widths and drops off beyond that point and F increases by almost 40% over the initial value even for 1Hz case. The present computations using the constant eddy viscosity model fail to predict this trend.

5.0 Concluding Remarks

A method which employs a transformation to solve the thin shear layer equations for the steady and unsteady, two-dimensional turbulent free jet issuing into stationary air has been presented. A Prandtl type constant eddy viscosity formulation was used to model the Reynolds shear stress term. The transformation reduces the rate of spread of the jet and enables the use of variable grid sizes over predetermined regions leading to accurate predictions.

In the steady jet, the mean velocity profiles, the mean centre-line velocity decay rate, the spreading rate of the jet and the normalized shear stress profiles all show good agreement with experimental data.

In the unsteady jet, the mean flow characteristics, such as the mean velocity and shear stress profiles, the mean centre-line velocity decay, the mean jet spreading and entrainment all follow closely those of the steady jet for the range of tested frequencies and amplitudes of pulsation. This is in agreement with the only available experimental data of this type which are available in Lai & Simmons (1979). However, unsteady effects are apparent in the distortion of the waveform for the variation of the instantaneous centre-line velocity with time and in the increase in the phase lag with streamwise distance. For high frequencies and amplitude of pulsation or at large streamwise distance from the nozzle, instantaneous quantities depart from quasi-steady values. The flow region over which quasi-steady approximations are applicable has been established for various frequencies.

Although the constant-eddy viscosity model used gives good predictions for the steady jet and the mean quantities for the unsteady jet, it fails to predict accurately the instantaneous quantities especially the percentage peak to peak oscillation of the steady-state instantaneous centre-line velocity. This suggests that if only mean flow quantities are required and if the frequencies and amplitudes of pulsation are of the order of those in this study, the constant eddy-viscosity model is adequate. However, if instantaneous quantities are critical and if frequencies and amplitudes of pulsation are an order of magnitude higher than those in this study, a more refined turbulence model has to be sought and tested.

6.0 References

1. BRADBURY, L. J. S., 1965, "The Structure of a Self-Preserving Turbulent Plane Jet" J. Fluid Mech., vol. 23, part 1, pp. 31-64.
2. BRADSHAW, P., CEBECI, T. and WHITELAW, J. H., 1980, "Lecture Series on Engineering Calculation Methods for Turbulent Flows." Calif State Univ, Long Beach, CA.
3. BREMHORST, K. and HARCH, W. H., 1979, "Near Field Velocity Measurements in a Fully Pulsed Subsonic Air Jet," in Turbulent Shear Flows I, ed., by Durst, F., Launder, B. E., Schmidt, F. W. and Whitelaw J. H., Springer-Verlag, Berlin, Heidelberg, pp. 37-54.
4. CEBECI, T. and BRADSHAW, P., 1977, "Momentum Transfer in Boundary Layers", McGraw Hill, New York.
5. CEBECI, T. and SMITH, A. M. O., 1974, "Analysis of Turbulent Boundary Layers", Applied Math. and Mech., 15, Academic Press, New York.
6. CHEN, J. C. and NIKITOPOULOS, C. P., 1979, "On the Near Field Characteristics of Axisymmetric Turbulent Buoyant Jets in a Uniform Environment" Int. J. Heat Mass Transfer, vol 22, pp. 245-255.
7. EVERITT, K. W. and ROBINS, A. G., 1978, "The Development and Structure of Turbulent Plane Jets", J. Fluid Mech., vol. 88, part 3, pp. 563-583.
8. FIEDLER, H. and KORSCHULT, D., 1979, "The Two-Dimensional Jet with Periodic Initial Condition," 2nd Symposium on Turbulent Shear Flows, Imperial College, July.
9. GOERTLER, H., 1942, "Beeschnung von Aufgaben der freien Turbulenz auf Grund eines neuen Naherungsansatzes" Z.A.M.M., 22, pp. 244-254.
10. GOLDSCHMIDT, V. W. and BRADSHAW, P., 1980, "Upstream Effects on the Widening Rate of Plane Free Jets", To appear.
11. GUTMARK, E. and WYGNANSKI, 1976, "The Planar Turbulent Jet", J. Fluid Mech., vol. 73, part 3, pp. 465-495.
12. HARSHA, P. T., 1971, "Free Turbulent Mixing: A Critical Evaluation of Theory and Experiment" Arnold Engineering Development Center, Report No. AEDC-TR-71-36.

13. HESKESTAD, G., 1965, "Hot-Wire Measurements in a Plane Turbulent Jet", J. Applied, Mech. ASME, Dec., pp. 721-734.
14. HINZE, J. O., 1975, "Turbulence" 2nd Edition, McGraw-Hill, New York.
15. KELLER, H. B., 1970, "A New Difference Scheme for Parabolic Problems", in Numerical Solutions of Partial Differential Equations, vol. II, ed. by J. Bramble, Academic Press, New York.
16. KELLER, H. B., 1978, "Numerical Methods in Boundary-Layer Theory", Ann Rev. Fluid Mech., vol. 10, pp. 417-433.
17. KENT, J. C., 1973, "Unsteady Viscous Jet Flow into Stationary Surroundings" Computers and Fluids, vol. 1, pp. 101-117.
18. KOTSOVINOS, N. E., 1976, "A Note on the Spreading Rate and Virtual Origin of a Plane Turbulent Jet", J. Fluid Mech., vol 77, part 2, pp. 305-311.
19. LAI, J. C. S., and SIMMONS, J. M., 1978, "Numerical Solution of the Steady Two-Dimensional Laminar Free Jet" Univ. Queensland, Brisbane, Australia, Department of Mechanical Engineering, Rep. No. 5/78.
20. LAI, J. C. S. and SIMMONS, J. M., 1978, "Numerical Solution of a Periodically Pulsed, two-Dimensional Laminar Free Jet", Univ. Queensland, Brisbane, Australia, Dept. of Mech. Eng., Rep. No. 8/78.
21. LAI, J. C. S., and SIMMONS, J. M., 1979, "Instantaneous Velocity Measurements in a Periodically Pulsed Two-Dimensional Turbulent Jet", Univ. Queensland, Brisbane, Australia, Dept. of Mech. Eng. Rep. No. 13/79.
22. LAUNDER, B. E. and SPALDING, D. B., 1972, "Lectures in Mathematical Models of Turbulence", Academic Press, London.
23. LAUNDER, B. E., MORSE, A., RODI, W. and SPALDING, D. B., 1972, "Prediction of Free Shear Flows-A Comparison of the Performance of Six Turbulence Models", Free Turbulent Shear Flows, vol. 1, NASA SP-321, pp. 361-426.
24. LAUNDER, B. E. and SPALDING, D. B., 1974, "The Numerical Computation of Turbulent Flows" Computer Methods in Applied Mech. and Eng., vol. 3, pp. 269-289.
25. McCORMACK, P. D., COCHRAN, D. and CRANE, L., 1966, "Periodic Vorticity and Its Effect on Jet Mixing" Physics of Fluids, vol. 9, pp. 1555-1560.

26. MCGUIRK, J. J. and RODI, W., 1979, "Mathematical Modelling of Three-Dimensional Heated Surface Jets", J. Fluid Mech., vol. 95, part 4, pp. 609-633.
27. PAI, S. I., 1965, "Unsteady Three-Dimensional Laminar Jet Mixing of a Compressible Fluid", AIAA J., March, pp. 617-621.
28. PIATT, M. and VIETS, H., 1979, "Conditional Sampling in an Unsteady Jet", Paper No. 79-1857, AIAA Aircraft Systems and Technology Meeting, August 20-22.
29. PRANDTL, L., 1942, "Bemerkungen zur Theorie der freien Turbulenz" ZAMM 22, pp. 241-243.
30. REYNOLDS, W. C and CEBECI, T., 1978, "Calculation of Turbulent Flows" in Turbulence, Topics in Applied Physics, vol. 12, ed. by Bradshaw, P., 2nd edition, Springer-Verlag, New York.
31. ROBINS, A. G., 1973, Ph.D. Thesis, London University (As quoted in Rodi, W., 1975).
32. RODI, W., 1975, "A Review of Experimental Data of Uniform Density Free Turbulent Boundary Layers" in Studies in Convection, vol. 1, ed. by Launder, B. E., Academic Press, New York.
33. RODI, W. and SPALDING, D. B., 1970, "A Two-Parameter Model of Turbulence and its Applications to Free Jets" WARME-UND Stoffubertragung Bd. 3, pp. 85-95.
34. SCHLICHTING, H., 1966, "Boundary-Layer Theory," McGraw-Hill, New York.
35. TOLLMIEEN, W., 1926, "Berechnung Turbulenter Ausbreitungsvorgaenge," Z. A. M. M. vol. 6, pp. 468-478.
36. ZIJNEN, B. G., Van der Hegge, 1958, "Measurements of the Velocity Distribution in a Plane Turbulent Jet of Air", Appl. Sci., Res., Sect. A, vol. 7, pp. 256-276.

Appendix A Structure and Listings of Computer Program

A.1 Sturcture of the Steady Jet Program

The structure of the input data is as follows:

CARD 1	NRG, (RG(I), I = 1, NRG), DEGR
FORMAT	I5, 6F10.6
CARD 2	PROD , CONST 1, CONST 2
FORMAT	3F10.6
CARD 3	CO, XO, AA, BB, NPG, INC, IFREQ, NXI
FORMAT	4F10.6, 4I4
CARD 4	(NXTT(I), I = 2, NXII)
FORMAT	10I4
CARD 5	(DELX(I), I=1,NXI), X(1)
FORMAT	8F10.6
CARD 6	(DETAC(I), I = 1,8)
FORMAT	8F10.6
CARD 7	(VC(I), I = 1,7), ETAE
FORMAT	8F10.6
CARD 8	E1, E2, E3
FORMAT	3F10.6
CARD 9	OUTDSK, LN
FORMAT	L1, I3
CARD 10	ETAG
FORMAT	F10.6

The symbols used have the following meaning:

NRG	No. of times grid points have to be rearranged
RG(I)	Upper η value above which grid points will not be dropped
DEGR	Lower η value below which grid points will not be dropped.
PROD	V defined in section 2.1 (Jet Exit Reynolds Number)
CONST 1, CONST 2	c_1 and c_2 defined in Eq. (12)
CO	$\zeta_0^{1/2}/(ab)$
XO	ζ_0
AA	a defined in Eq. (15a)
BB	b defined in Eq. (15b)
NPG	n_g , No. of grid points to be added if the jet spreads
INC	interval, in terms of the number of transverse grid points, at which a value of the velocity profile is required
IFREQ	interval, in terms of the number of streamwise stations, at which velocity profile is required
NXI	No. of different grid sizes in streamwise direction
NXTT(I)	Streamwise station number at which a different grid size is used.
DELX (I)	Streamwise grid size $\Delta\zeta_i$
X(1)	Value of ζ at initial streamwise station (zero at nozzle exit)
DETAC(I)	Transverse grid size $\Delta\eta$
VC(I)	Values of η for each sub-region
ETAE	$(\eta_\infty)_i$
E1	Convergence limit defined in Eq. (36)
E2, E3	Jet edge definition defined in Eq. (37)

OUTDSK Assumes logical value TRUE if solutions are
 written to disk

LN Device number of writing to disk

ETAG η_a defined in Eq. (28)

~~A.2. Listing of Steady Jet Programs~~

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
84

47
46
45
44
43
42
41
40
39
38
37
36
35
34
33
32
31
30
29
28
27
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

[illegible]

105 100 95 90 85 80 75 70 65 60 55 50 45 40 35 30 25 20 15 10 5 0

A.3 Structure of the Unsteady Jet Program

The structure of the input data is as follows:

CARD 1	NRG, (RG(I), I = 1, NRG), DEGR
FORMAT	I5, 6F10.6
CARD 2	(NXG(I), I = 1, NRG)
FORMAT	5I5
CARD 3	NPG, NPG2, START, LN, NEW, NTRANS, NPRINT
FORMAT	7I5
CARD 4	CONST 1, CONST 2
FORMAT	2F10.4
CARD 5	CO, XO, EPS, OMG, NT, INC
FORMAT	4F10.4, 2I5
CARD 6	IFREQ, IFR, IFA, NXI
FORMAT	4I5
CARD 7	(NXTT(I), I=2, NXII)
FORMAT	10I5
CARD 8	(DELX(I), I = 1, NXI), X(1)
FORMAT	8F10.4
CARD 9	(DETAC(I), I = 1,8)
FORMAT	8F10.4
CARD 10	(VC(I), I = 1,7), GTAG
FORMAT	8F10.4
CARD 11	E1, E2, E3, E4, E5, AA, BB, PROD
FORMAT	8F10.4
CARD 12	ETAG
FORMAT	F10.4

Symbols which appear also in the steady jet program have the same meanings here. All other symbols are defined as follows: -

NXG(I)	Streamwise station at which grid has to be rearranged.
NPG 2	Second choice of number of transverse grid points to be added if jet spreads as discussed in section 4.2.4
START	Streamwise station number at which computations start
NEW	See below
NTRANS	Streamwise station number at and beyond which constant c_2 is used.
NPRINT	Streamwise station number at and beyond which velocity profiles are required
EPS	ϵ , amplitude of pulsation
OMG	ω , angular frequency of pulsation
NT	Number of time intervals in a period + 1
IFR	interval, in terms of the number of streamwise stations, at which instantaneous velocity profiles are required.
IFA	interval, in terms of the number of streamwise stations, at which mean quantities are required
E4	ϵ_4 , ϵ_5 and ϵ_7 defined in Eq. (40) and (41)
E5	ϵ_6 defined in Eq. (40)

Because of the initially steep velocity gradient, the velocity gradient curve behaves erratically at the region around the initial jet edge which constitutes about 3% of the total jet-width. To eliminate such numerical erratic behavior, a first-order smoothing function can be applied and is supplied through the Subroutine SMOOTH. The use of the

smoothing function does not affect the overall results and is optional. The parameter NEW specifies the streamwise station number where smoothing function starts to be used. If smoothing function is not required, NEW can assume the value of the final streamwise station number.

A.4 Listing of Unsteady Jet Program

FAC = 0.000

[illegible]

```

C**** STATION SOLUTION
C1400 IF (INT,1).GT.53
C1410 INT=INT+1
C1420 GO TO 100
C1430 IF (INT,1).GT.53
C1440 INT=INT+1
C1450 GO TO 100
C1460 IF (INT,1).GT.53
C1470 INT=INT+1
C1480 GO TO 100
C1490 IF (INT,1).GT.53
C1500 INT=INT+1
C1510 GO TO 100
C1520 IF (INT,1).GT.53
C1530 INT=INT+1
C1540 GO TO 100
C1550 IF (INT,1).GT.53
C1560 INT=INT+1
C1570 GO TO 100
C1580 IF (INT,1).GT.53
C1590 INT=INT+1
C1600 GO TO 100
C1610 IF (INT,1).GT.53
C1620 INT=INT+1
C1630 GO TO 100
C1640 IF (INT,1).GT.53
C1650 INT=INT+1
C1660 GO TO 100
C1670 IF (INT,1).GT.53
C1680 INT=INT+1
C1690 GO TO 100
C1700 IF (INT,1).GT.53
C1710 INT=INT+1
C1720 GO TO 100
C1730 IF (INT,1).GT.53
C1740 INT=INT+1
C1750 GO TO 100
C1760 IF (INT,1).GT.53
C1770 INT=INT+1
C1780 GO TO 100
C1790 IF (INT,1).GT.53
C1800 INT=INT+1
C1810 GO TO 100
C1820 IF (INT,1).GT.53
C1830 INT=INT+1
C1840 GO TO 100
C1850 IF (INT,1).GT.53
C1860 INT=INT+1
C1870 GO TO 100
C1880 IF (INT,1).GT.53
C1890 INT=INT+1
C1900 GO TO 100
C1910 IF (INT,1).GT.53
C1920 INT=INT+1
C1930 GO TO 100
C1940 IF (INT,1).GT.53
C1950 INT=INT+1
C1960 GO TO 100
C1970 IF (INT,1).GT.53
C1980 INT=INT+1
C1990 GO TO 100
C2000 IF (INT,1).GT.53
C2010 INT=INT+1
C2020 GO TO 100
C2030 IF (INT,1).GT.53
C2040 INT=INT+1
C2050 GO TO 100
C2060 IF (INT,1).GT.53
C2070 INT=INT+1
C2080 GO TO 100
C2090 IF (INT,1).GT.53
C2100 INT=INT+1
C2110 GO TO 100
C2120 IF (INT,1).GT.53
C2130 INT=INT+1
C2140 GO TO 100
C2150 IF (INT,1).GT.53
C2160 INT=INT+1
C2170 GO TO 100
C2180 IF (INT,1).GT.53
C2190 INT=INT+1
C2200 GO TO 100
C2210 IF (INT,1).GT.53
C2220 INT=INT+1
C2230 GO TO 100
C2240 IF (INT,1).GT.53
C2250 INT=INT+1
C2260 GO TO 100
C2270 IF (INT,1).GT.53
C2280 INT=INT+1
C2290 GO TO 100
C2300 IF (INT,1).GT.53
C2310 INT=INT+1
C2320 GO TO 100
C2330 IF (INT,1).GT.53
C2340 INT=INT+1
C2350 GO TO 100
C2360 IF (INT,1).GT.53
C2370 INT=INT+1
C2380 GO TO 100
C2390 IF (INT,1).GT.53
C2400 INT=INT+1
C2410 GO TO 100
C2420 IF (INT,1).GT.53
C2430 INT=INT+1
C2440 GO TO 100
C2450 IF (INT,1).GT.53
C2460 INT=INT+1
C2470 GO TO 100
C2480 IF (INT,1).GT.53
C2490 INT=INT+1
C2500 GO TO 100
C2510 IF (INT,1).GT.53
C2520 INT=INT+1
C2530 GO TO 100
C2540 IF (INT,1).GT.53
C2550 INT=INT+1
C2560 GO TO 100
C2570 IF (INT,1).GT.53
C2580 INT=INT+1
C2590 GO TO 100
C2600 IF (INT,1).GT.53
C2610 INT=INT+1
C2620 GO TO 100
C2630 IF (INT,1).GT.53
C2640 INT=INT+1
C2650 GO TO 100
C2660 IF (INT,1).GT.53
C2670 INT=INT+1
C2680 GO TO 100
C2690 IF (INT,1).GT.53
C2700 INT=INT+1
C2710 GO TO 100
C2720 IF (INT,1).GT.53
C2730 INT=INT+1
C2740 GO TO 100
C2750 IF (INT,1).GT.53
C2760 INT=INT+1
C2770 GO TO 100
C2780 IF (INT,1).GT.53
C2790 INT=INT+1
C2800 GO TO 100
C2810 IF (INT,1).GT.53
C2820 INT=INT+1
C2830 GO TO 100
C2840 IF (INT,1).GT.53
C2850 INT=INT+1
C2860 GO TO 100
C2870 IF (INT,1).GT.53
C2880 INT=INT+1
C2890 GO TO 100
C2900 IF (INT,1).GT.53
C2910 INT=INT+1
C2920 GO TO 100
C2930 IF (INT,1).GT.53
C2940 INT=INT+1
C2950 GO TO 100
C2960 IF (INT,1).GT.53
C2970 INT=INT+1
C2980 GO TO 100
C2990 IF (INT,1).GT.53
C3000 INT=INT+1
C3010 GO TO 100
C3020 IF (INT,1).GT.53
C3030 INT=INT+1
C3040 GO TO 100
C3050 IF (INT,1).GT.53
C3060 INT=INT+1
C3070 GO TO 100
C3080 IF (INT,1).GT.53
C3090 INT=INT+1
C3100 GO TO 100
C3110 IF (INT,1).GT.53
C3120 INT=INT+1
C3130 GO TO 100
C3140 IF (INT,1).GT.53
C3150 INT=INT+1
C3160 GO TO 100
C3170 IF (INT,1).GT.53
C3180 INT=INT+1
C3190 GO TO 100
C3200 IF (INT,1).GT.53
C3210 INT=INT+1
C3220 GO TO 100
C3230 IF (INT,1).GT.53
C3240 INT=INT+1
C3250 GO TO 100
C3260 IF (INT,1).GT.53
C3270 INT=INT+1
C3280 GO TO 100
C3290 IF (INT,1).GT.53
C3300 INT=INT+1
C3310 GO TO 100
C3320 IF (INT,1).GT.53
C3330 INT=INT+1
C3340 GO TO 100
C3350 IF (INT,1).GT.53
C3360 INT=INT+1
C3370 GO TO 100
C3380 IF (INT,1).GT.53
C3390 INT=INT+1
C3400 GO TO 100
C3410 IF (INT,1).GT.53
C3420 INT=INT+1
C3430 GO TO 100
C3440 IF (INT,1).GT.53
C3450 INT=INT+1
C3460 GO TO 100
C3470 IF (INT,1).GT.53
C3480 INT=INT+1
C3490 GO TO 100
C3500 IF (INT,1).GT.53
C3510 INT=INT+1
C3520 GO TO 100
C3530 IF (INT,1).GT.53
C3540 INT=INT+1
C3550 GO TO 100
C3560 IF (INT,1).GT.53
C3570 INT=INT+1
C3580 GO TO 100
C3590 IF (INT,1).GT.53
C3600 INT=INT+1
C3610 GO TO 100
C3620 IF (INT,1).GT.53
C3630 INT=INT+1
C3640 GO TO 100
C3650 IF (INT,1).GT.53
C3660 INT=INT+1
C3670 GO TO 100
C3680 IF (INT,1).GT.53
C3690 INT=INT+1
C3700 GO TO 100
C3710 IF (INT,1).GT.53
C3720 INT=INT+1
C3730 GO TO 100
C3740 IF (INT,1).GT.53
C3750 INT=INT+1
C3760 GO TO 100
C3770 IF (INT,1).GT.53
C3780 INT=INT+1
C3790 GO TO 100
C3800 IF (INT,1).GT.53
C3810 INT=INT+1
C3820 GO TO 100
C3830 IF (INT,1).GT.53
C3840 INT=INT+1
C3850 GO TO 100
C3860 IF (INT,1).GT.53
C3870 INT=INT+1
C3880 GO TO 100
C3890 IF (INT,1).GT.53
C3900 INT=INT+1
C3910 GO TO 100
C3920 IF (INT,1).GT.53
C3930 INT=INT+1
C3940 GO TO 100
C3950 IF (INT,1).GT.53
C3960 INT=INT+1
C3970 GO TO 100
C3980 IF (INT,1).GT.53
C3990 INT=INT+1
C4000 GO TO 100

```

PAGE 0012

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
84

FAC: 1934

[illegible]

[illegible]

444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517

[illegible]

[illegible]

36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
841
842
843
844
845
846
847
848
849
850
851
852
853
854
855
856
857
858
859
860
861
862
863
864
865
866
867

FAO 1994

BAC - CCI

```

COMMON/ALSCC/NTW
LOGICAL LOGFAC
C----- THIS SUBROUTINE SETS V-LIMITS GRADIENT PROFILE
C
      IF (XV.LT.ATFICC) GO TO 3120
      ICS1=1
      ICS2=1
      N=NSZ-1
      DO 3110 J=1,NSZ
        L=0
        C=C(I,J,2).LJ.C.C.CC(I,J+2,2).LJ
        IF (C(I,J,2).GT.C(I,J+1,2).AND.C(I,J+2,2).GT.C(I,J+1,2).AND.
        C(I,J+1,2).LT.-1.0) L=1
        GO TO 3115
      3120 J=J+1
      3115 C=C(I,J,2)
      3110 CONTINUE
      J=J+1
      ICS1=J
      ICS2=J
      IF (C(I,J,2).GT.C(I,J-1,2).AND.C(I,J,2).LT.0.1) GO TO 3170
      ICS1=J-1
      Y=J
      CCOMP=C(I,ICS1,2)
      3170 CONTINUE
      IF (Y-1.GT.1) GO TO 3470
      3180 J=J-1
      IF (C(I,J,2).LT.CCOMP) CCOMP=C(I,J,2)
      GO TO 3225
      3200 CONTINUE
      IF (Y-2.GT.1) GO TO 3460
      3210 J=J-1
      C=C(I,ICS2)-I*(C(I,ICS1)
      C=C(I,ICS1,2)-C(I,ICS2,2)
      F=C/2
      J=ICS1+1
      N=NSZ-1
      DO 3440 J=1,NSZ
        F=I*(J)-I*(J-1)
        C(I,J,2)=C(I,ICS1,2)+F*F
        WRITE (LP,4) C(I,J,2)
      3450 CONTINUE
      3460 C=CCOMP
      3470 WRITE (LP,4) C(I,J,2)
      3480 RETURN
C-----
      3470 F=I*(J-1)+I*(J-2)+I*(J-3)+I*(J-4)+I*(J-5)
      3480 F=I*(J-1)+I*(J-2)+I*(J-3)+I*(J-4)+I*(J-5)
      3490 F=I*(J-1)+I*(J-2)+I*(J-3)+I*(J-4)+I*(J-5)

```

Appendix B. Figures

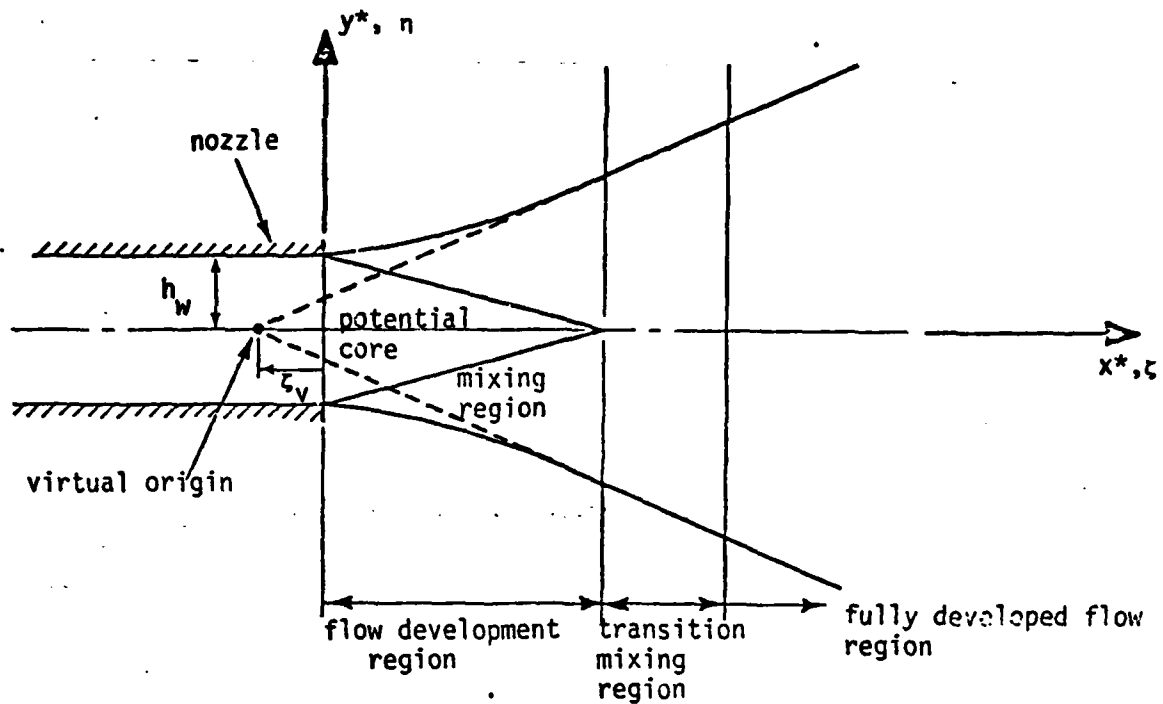


Figure 1. Configuration of the Plane Jet

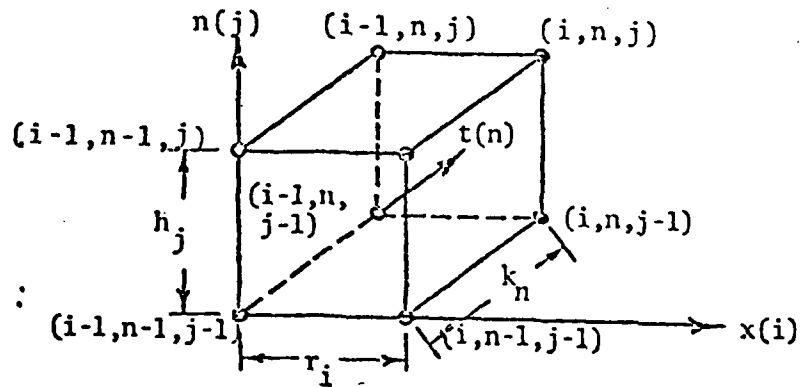


FIGURE 2. Net cube for the difference equations for Eq (21).

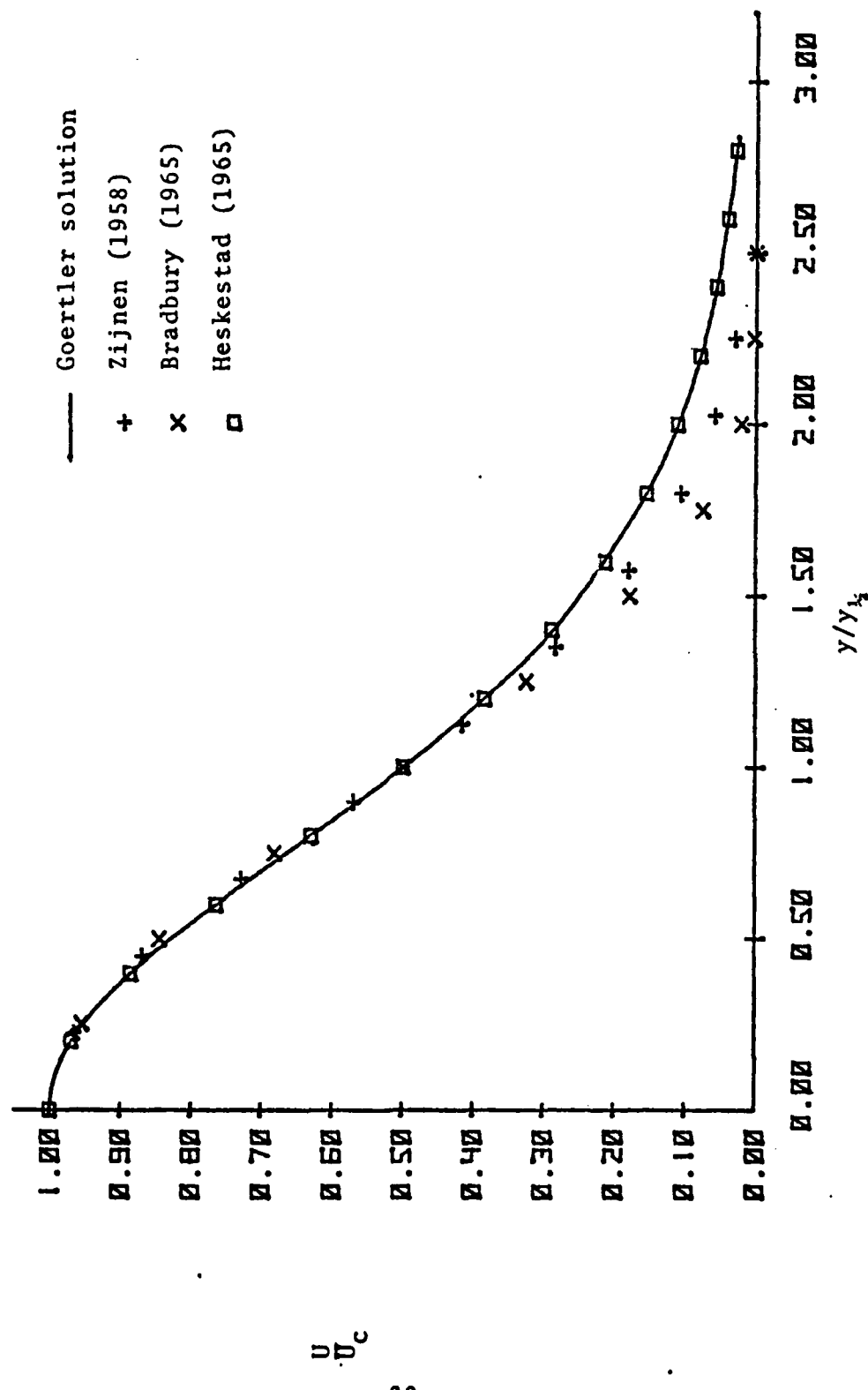


Figure 3. Non-Dimensional Self-Preserved Velocity Profile of the Steady Jet

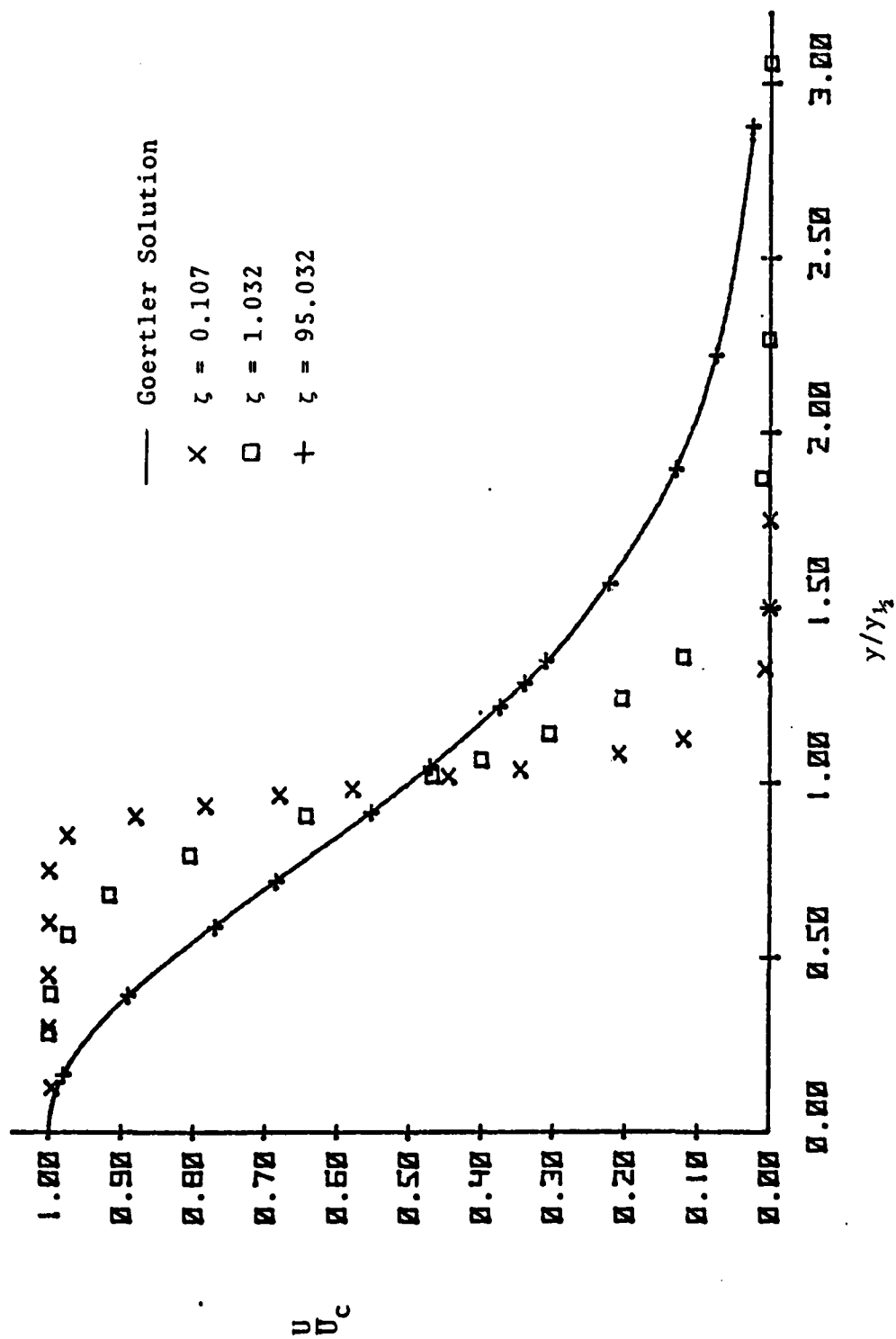


Figure 4(a) Non-Dimensional Velocity Distribution of the Steady Jet

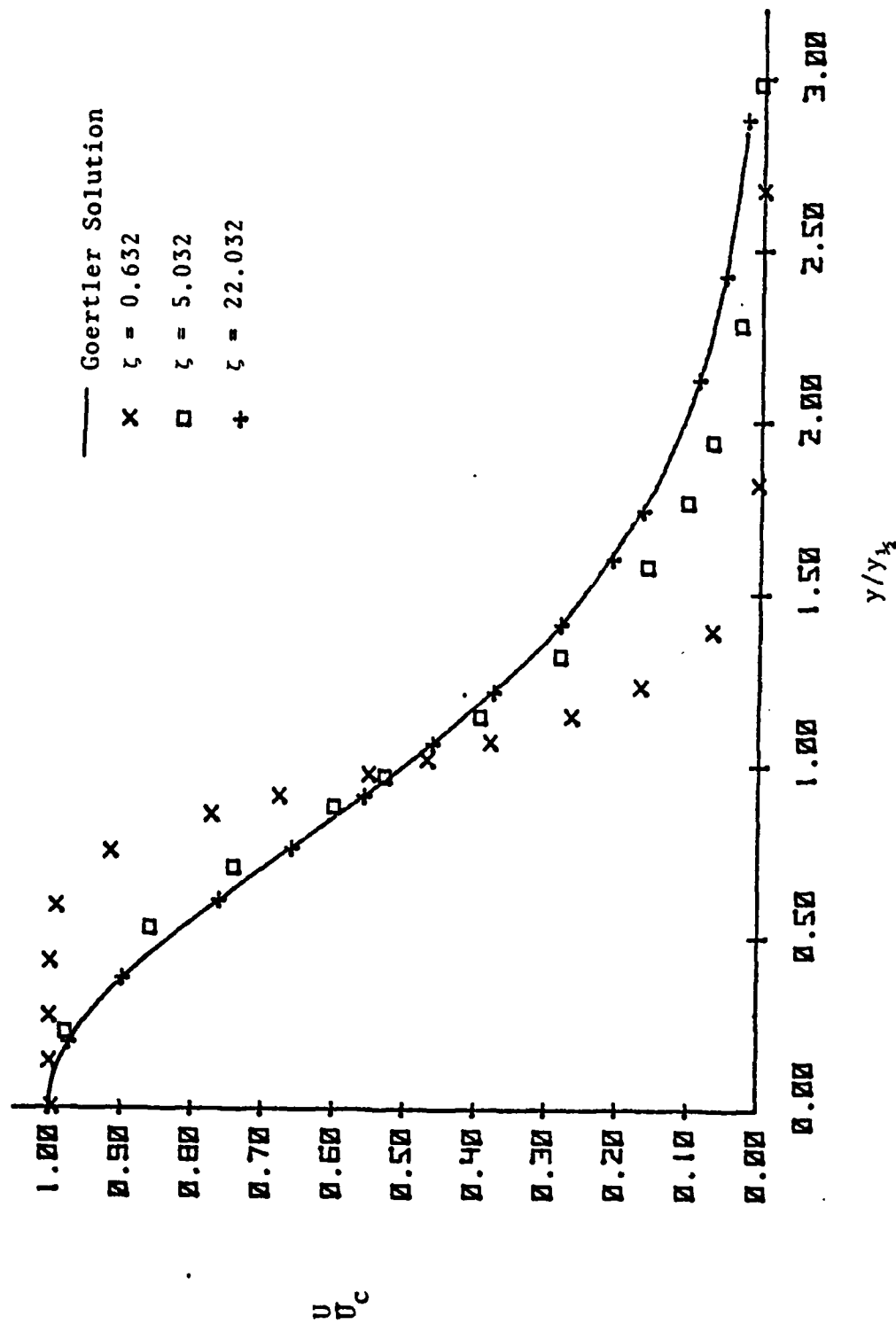


Figure 4(b) Non-Dimensional Velocity Distribution of the Steady Jet

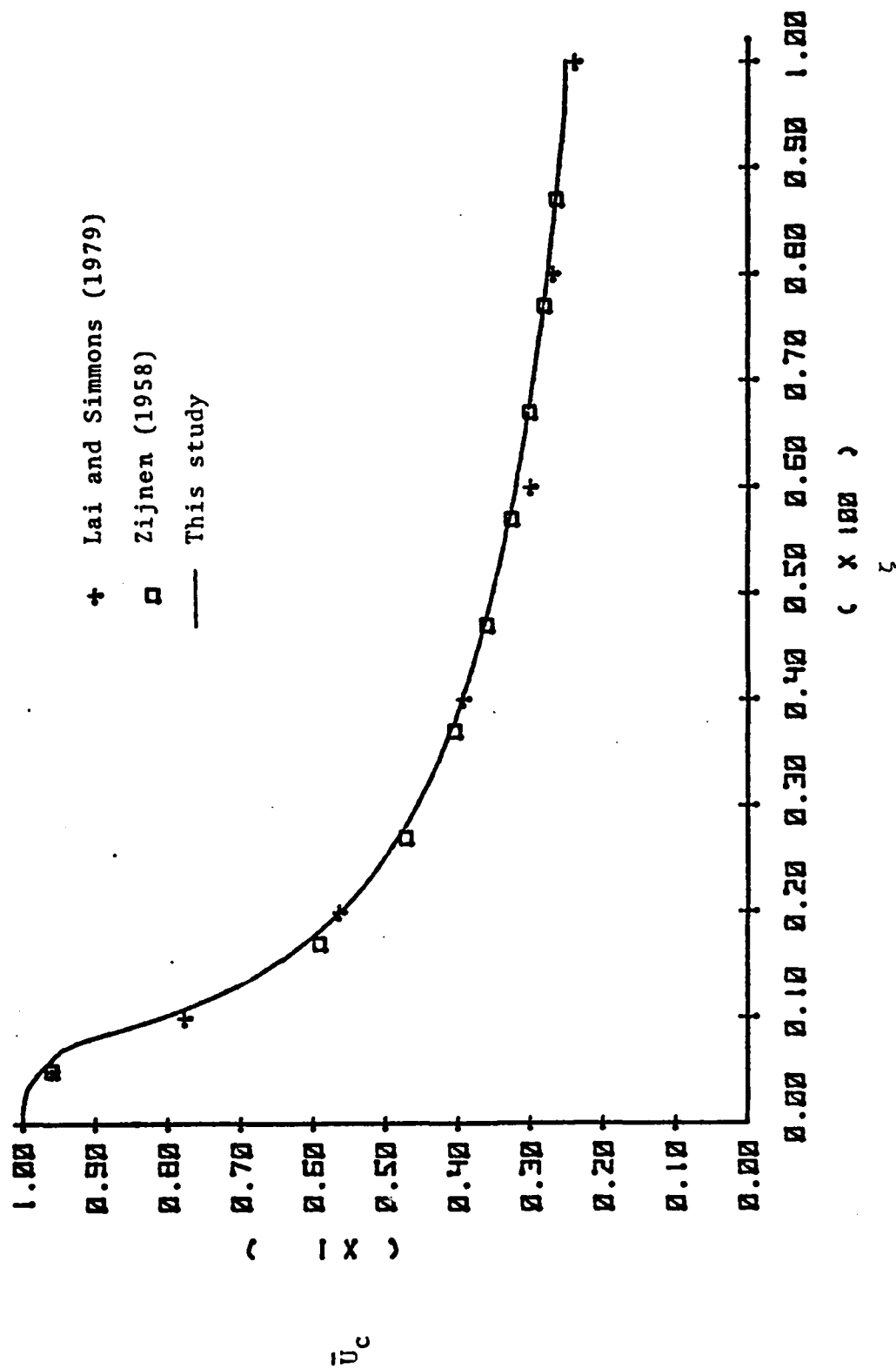
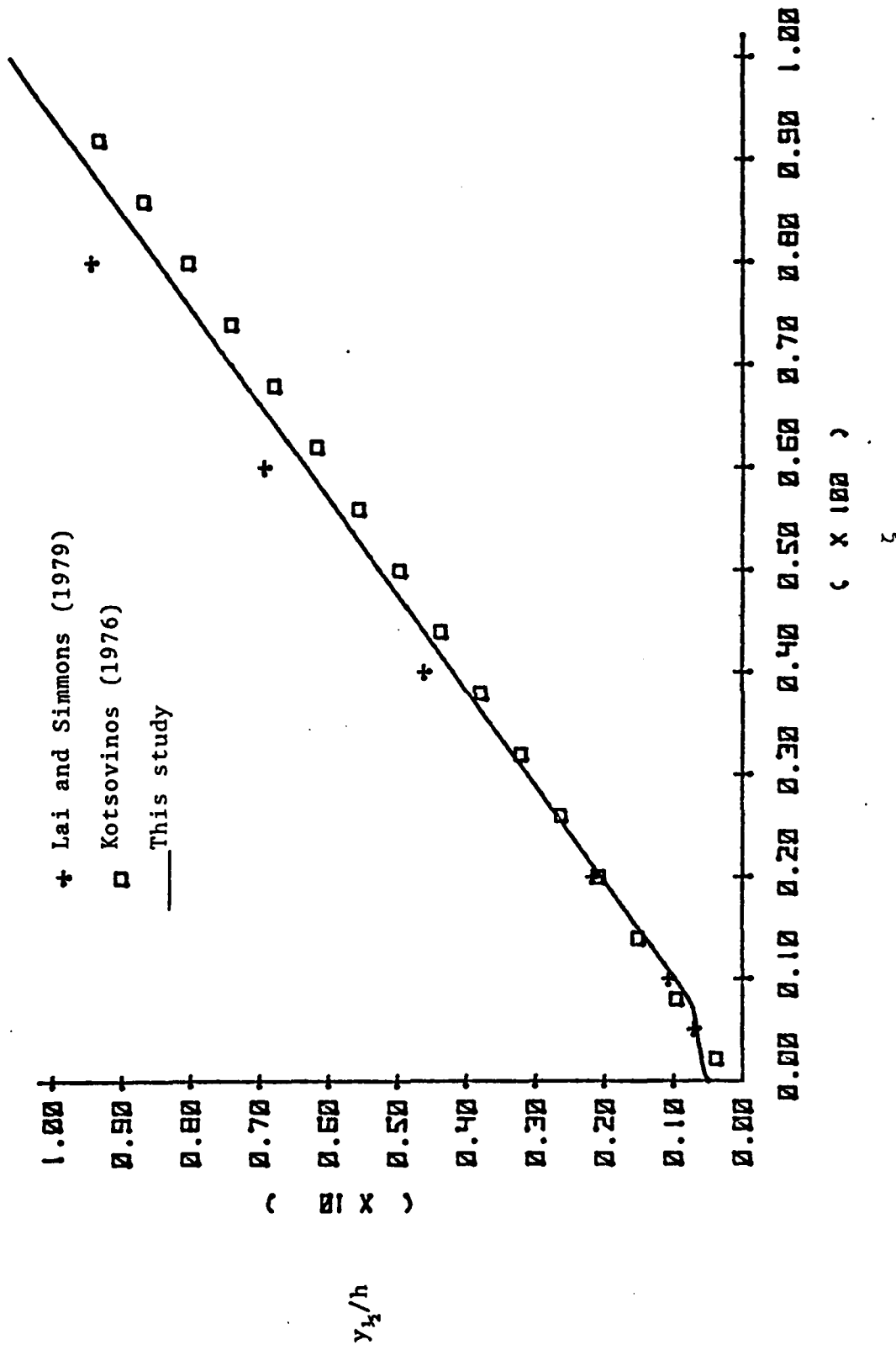


Figure 5 Variation of Centre-Line Velocity of the Steady Jet with Streamwise Distance



• Figure 6 Variation of the Half-Width of the Steady Jet with Streamwise Distance

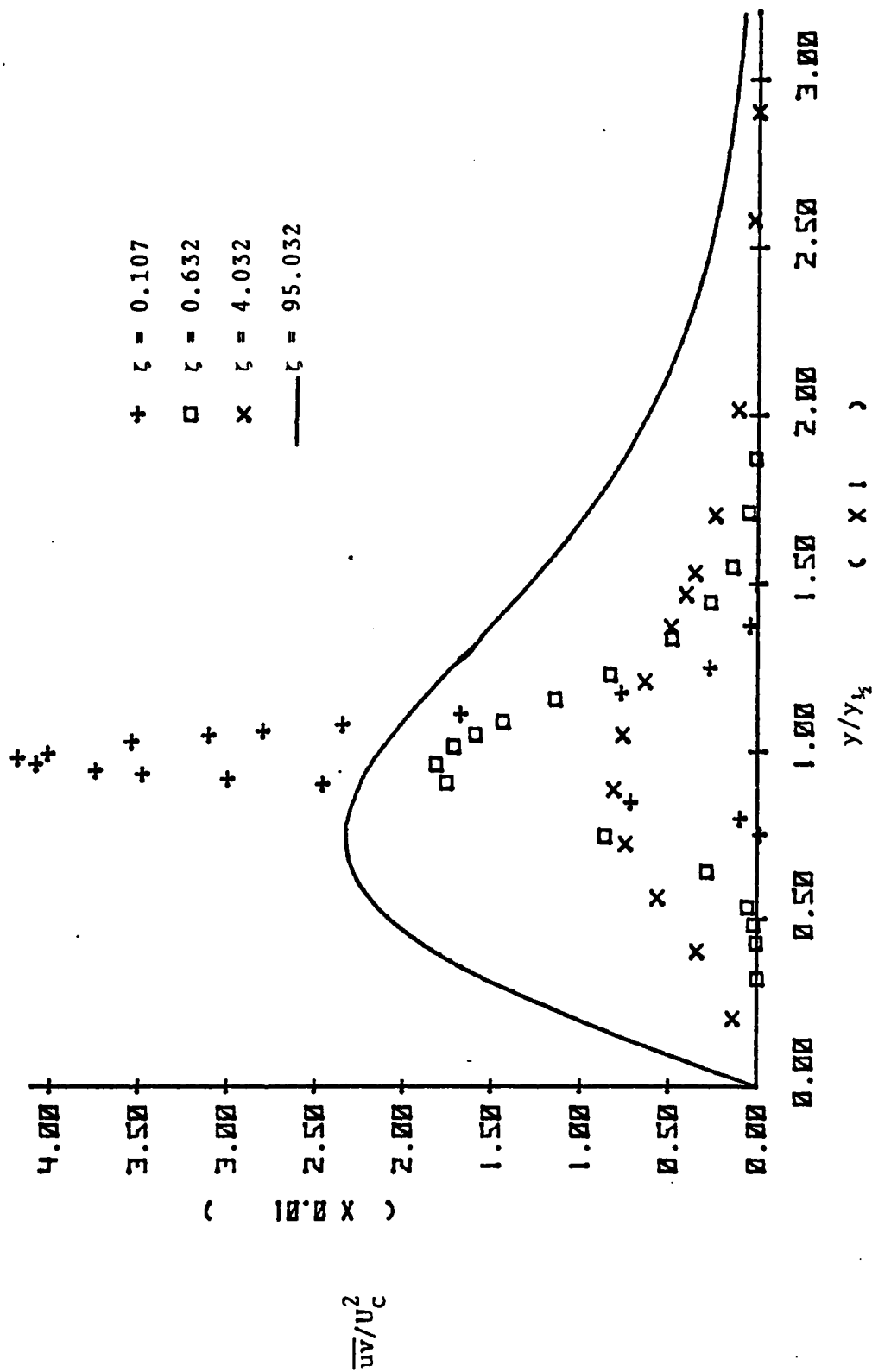


Figure 7(a) Nondimensional Shear Stress Profile for the Steady Jet

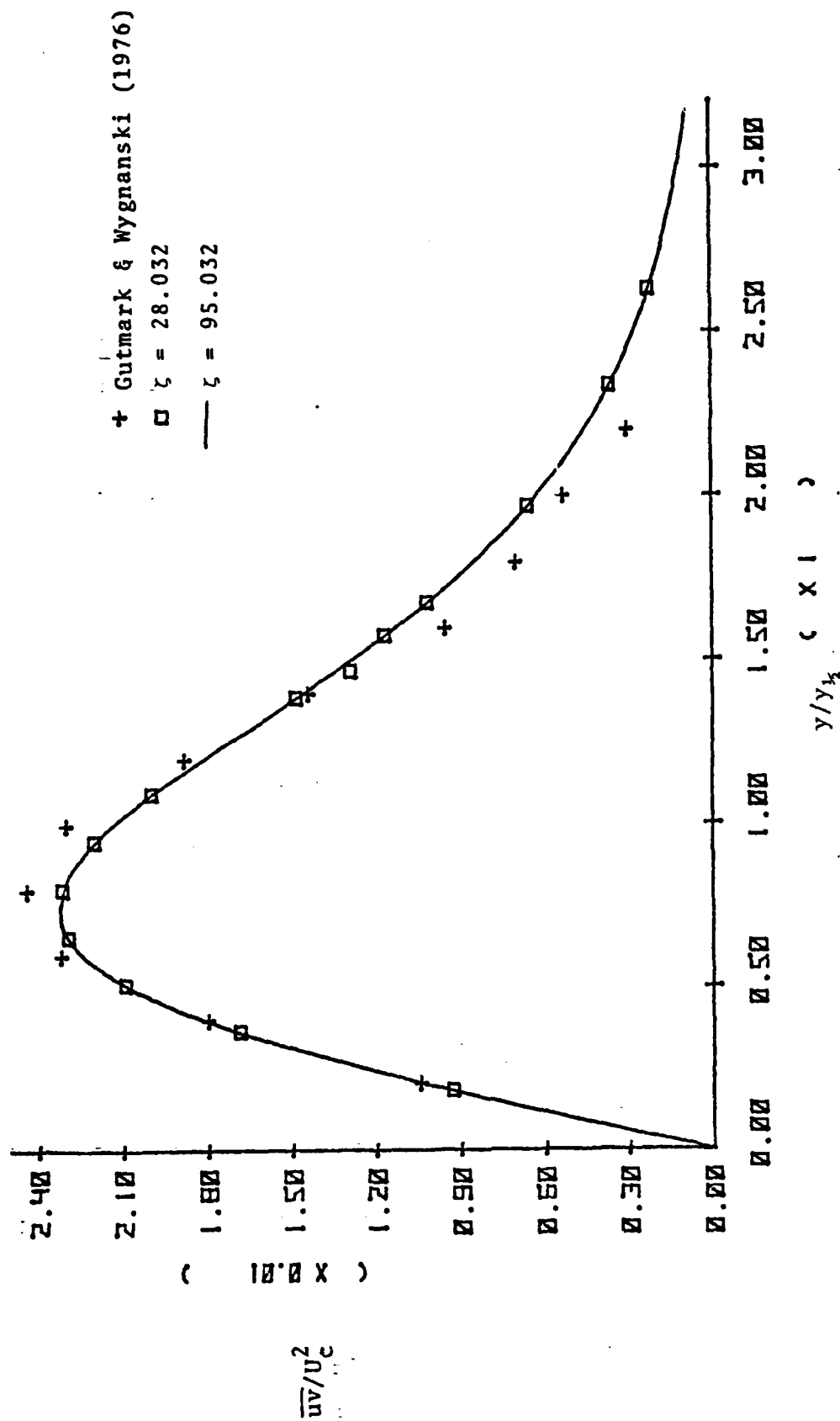


Figure 7(b) Nondimensional Shear Stress Profile for the Steady Jet

AD-A094 614

NAVAL POSTGRADUATE SCHOOL MONTEREY CA F/G 20/4
NUMERICAL SOLUTION OF STEADY AND PERIODICALLY PULSED TWO-DIMENS--ETC(U)
APR 80 J C LAI, J M SIMMONS

UNCLASSIFIED

NL

20/4
20/4-1/12

END

DATE

FILED

8h

DTIC

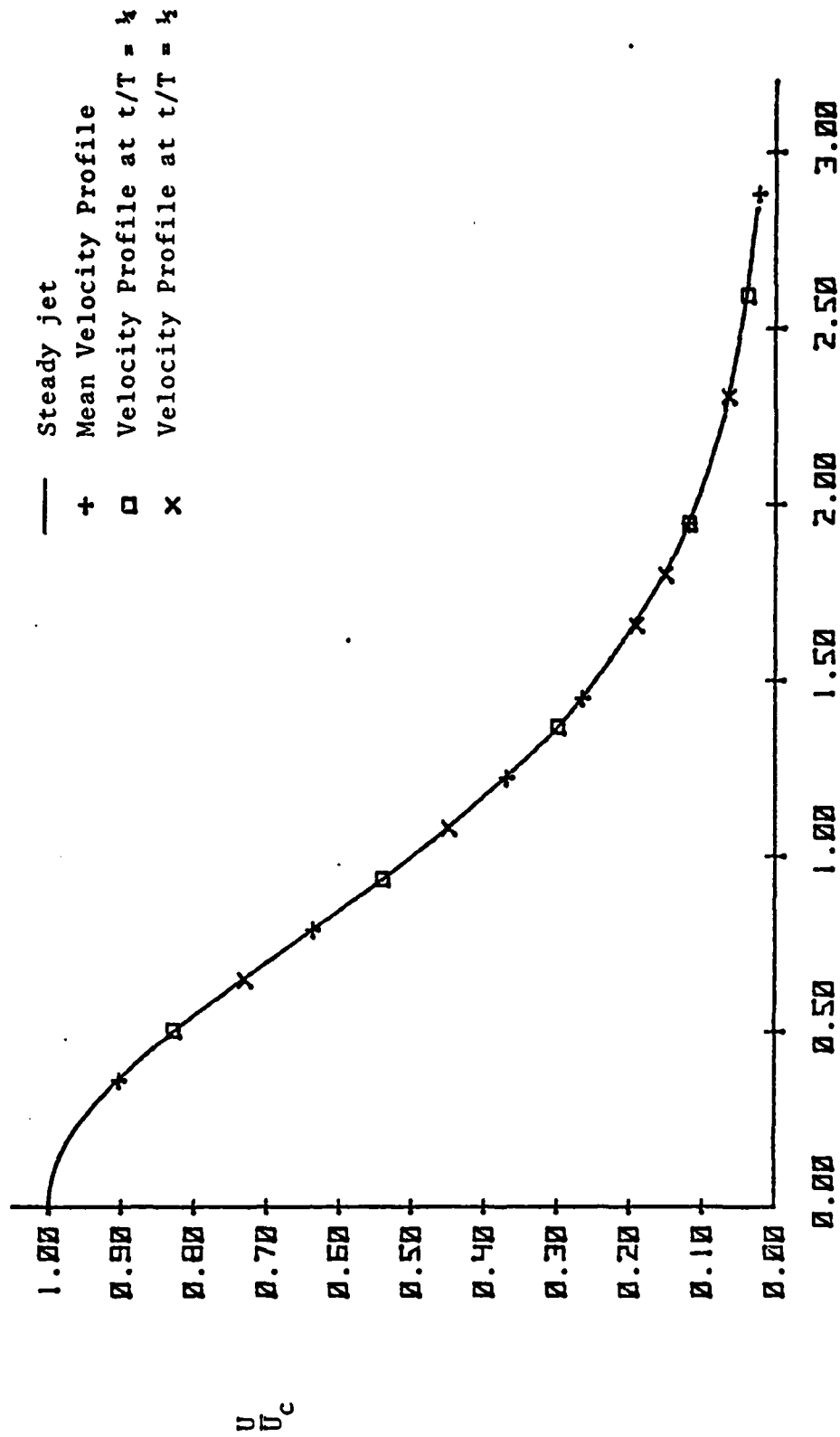


Figure 8(a) Non-dimensional Velocity Distribution for $\zeta = 40.032$,
 $\omega = 0.000871$,
 $\epsilon = 0.1$

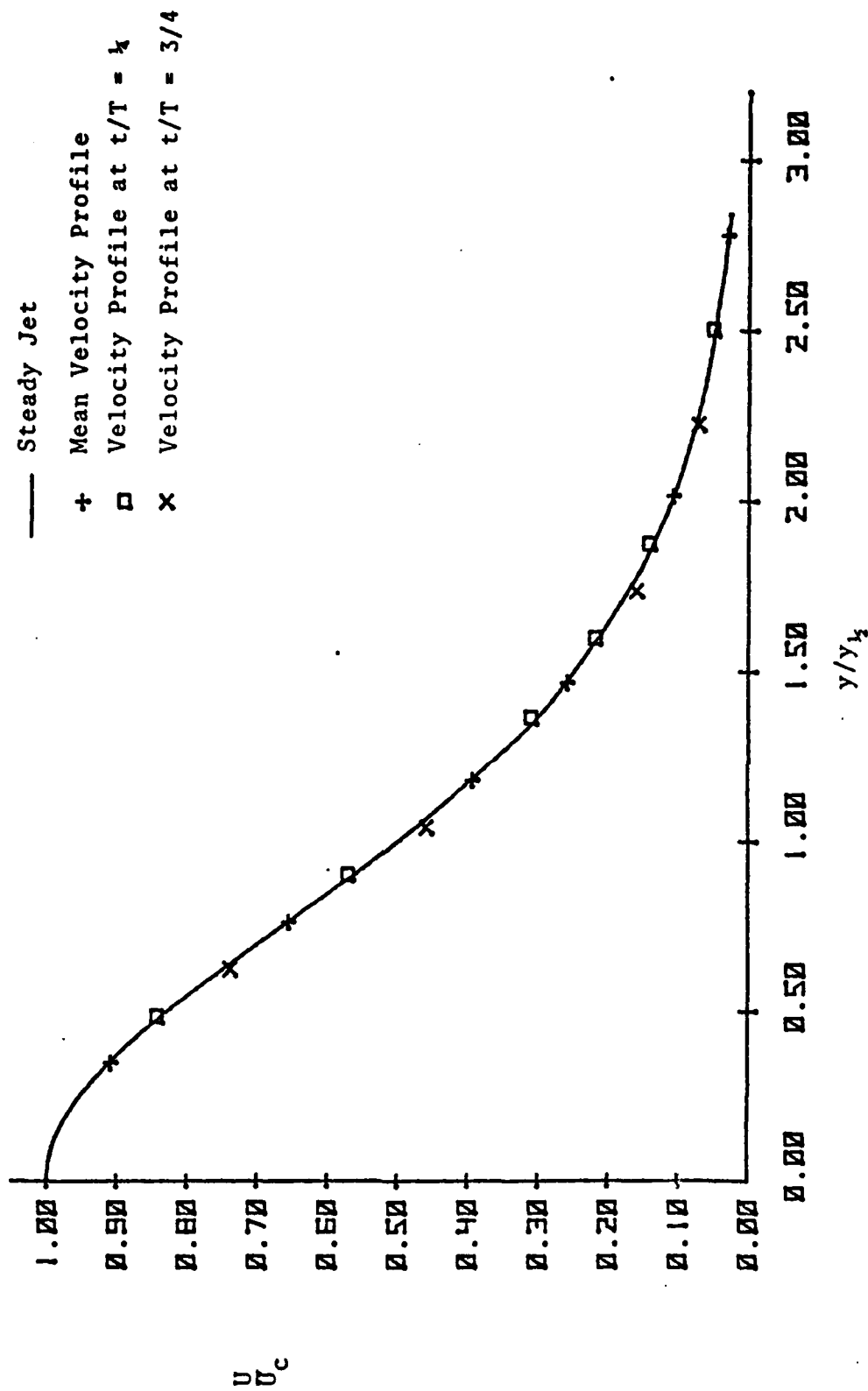


Figure 8(b) Non-dimensional Velocity Distribution for $\zeta = 40.032$,
 $\omega = 0.00871$
 $\epsilon = 0.1$

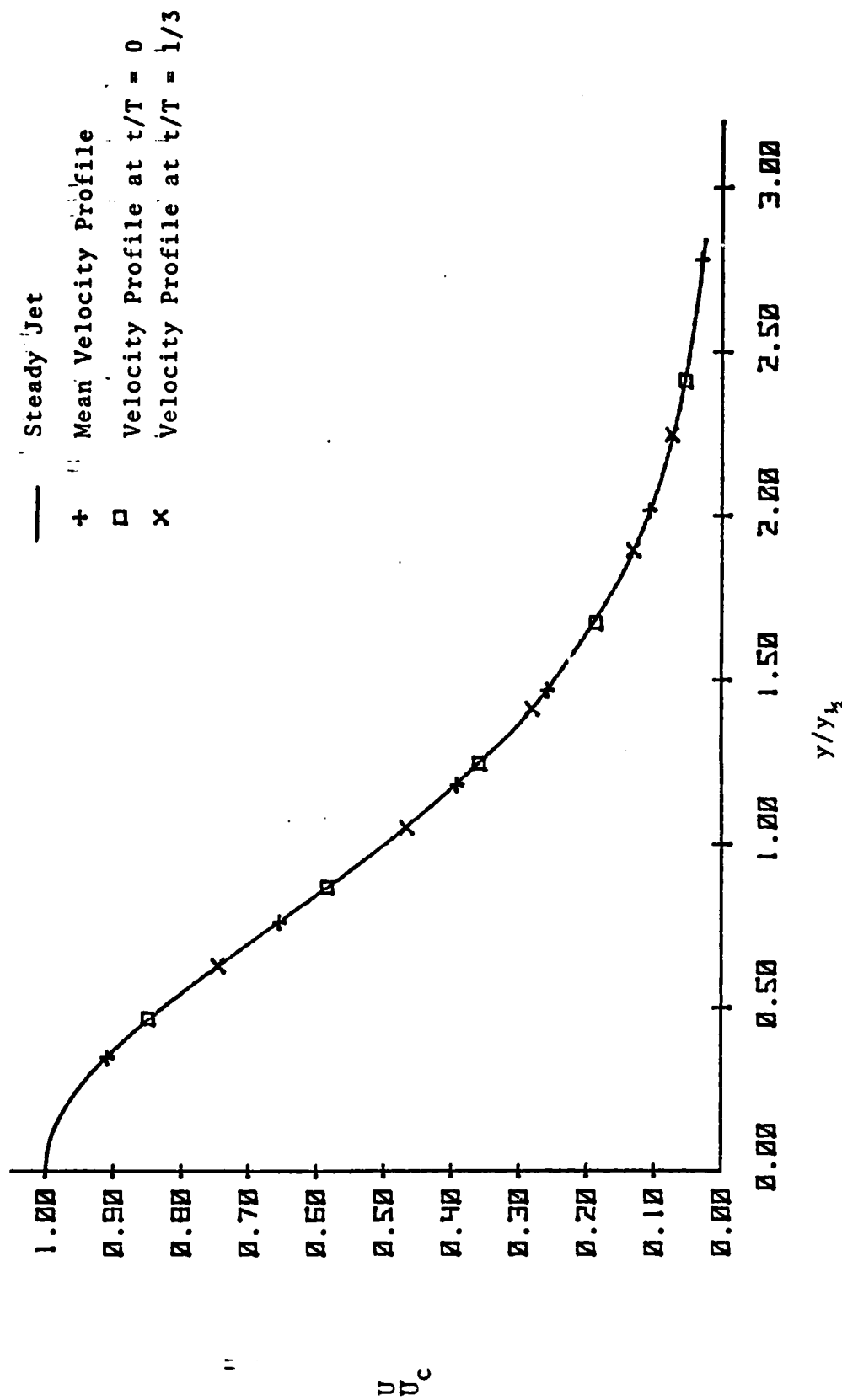


Figure 8(c) Non-dimensional Velocity Distribution for $\zeta = 40.032$
 $\omega = 0.0871$
 $\epsilon = 0.1$

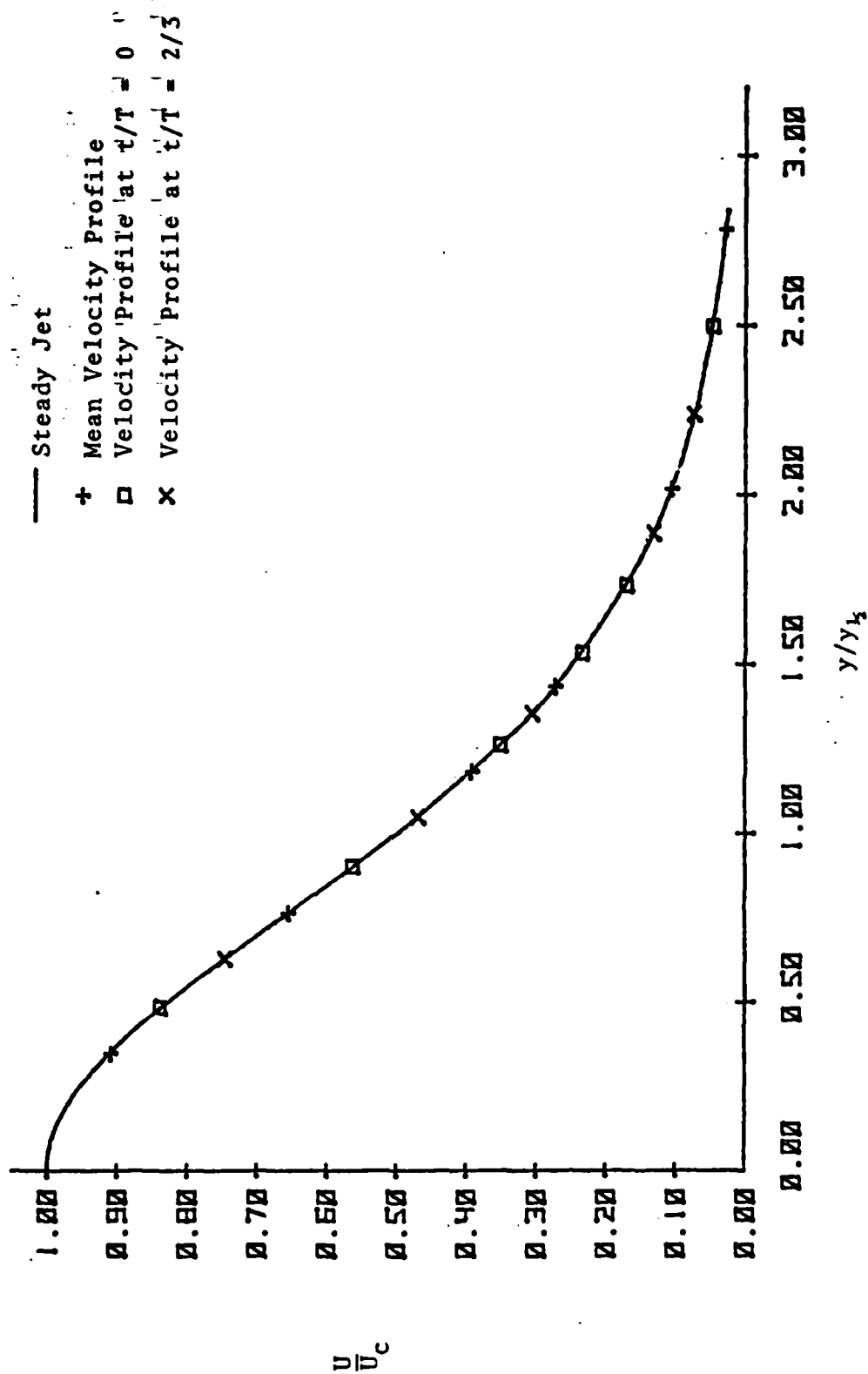


Figure 8(d) Non-dimensional Velocity distribution for $\zeta = 40.032$
 $\omega = 0.000871$
 $\epsilon = 0.15$

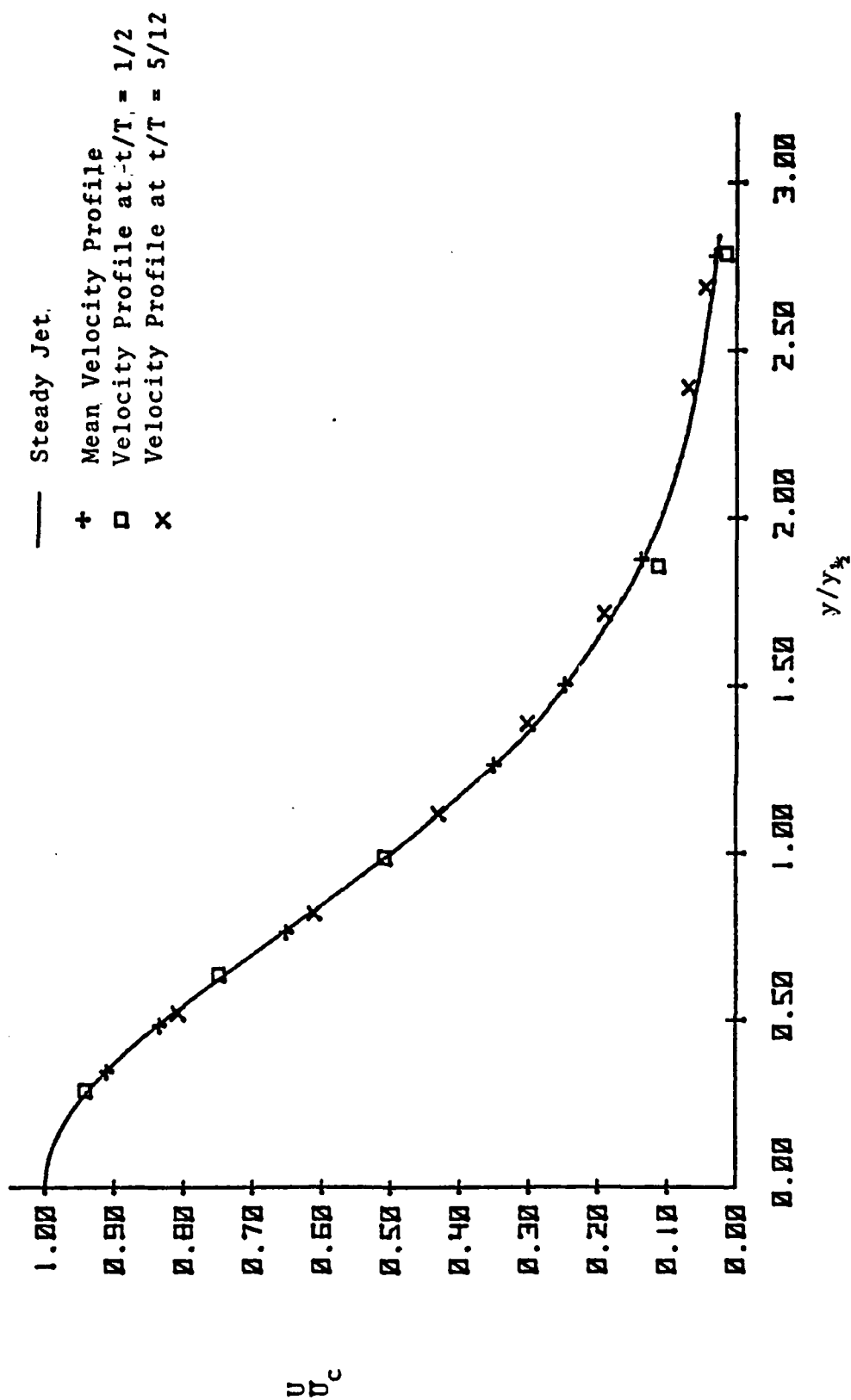


Figure 8(e) Non-dimensional Velocity Distribution for $\zeta = 40.032$
 $\omega = 0.00871$
 $\epsilon = 0.15$

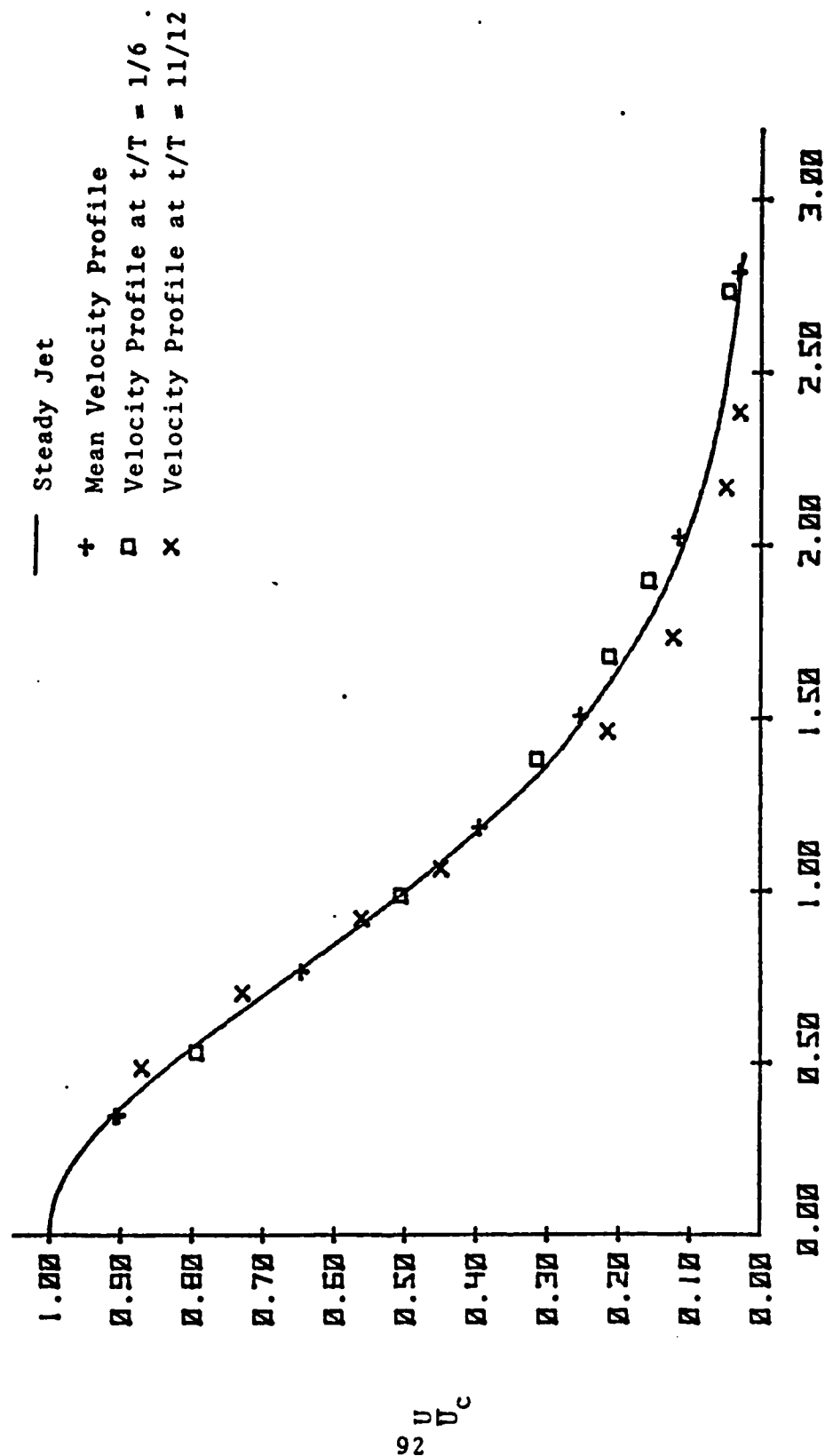


Figure 8 (f) Non-dimensional Velocity Distribution for $\zeta = 40.032$
 $\omega = 0.0871$
 $\epsilon = 0.15$

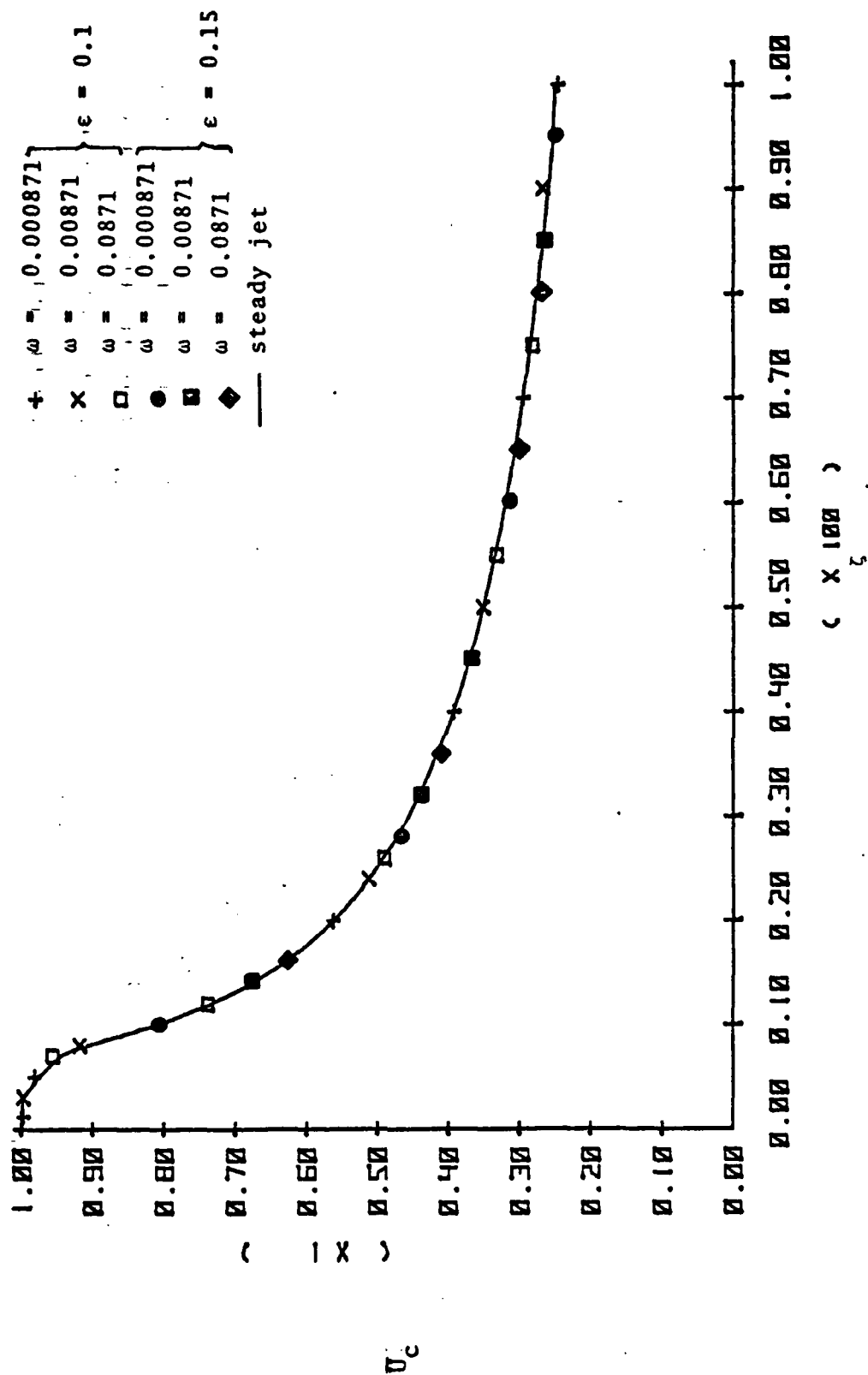


Figure 9 Variation of Mean Centre-Line Velocity with Streamwise Distance

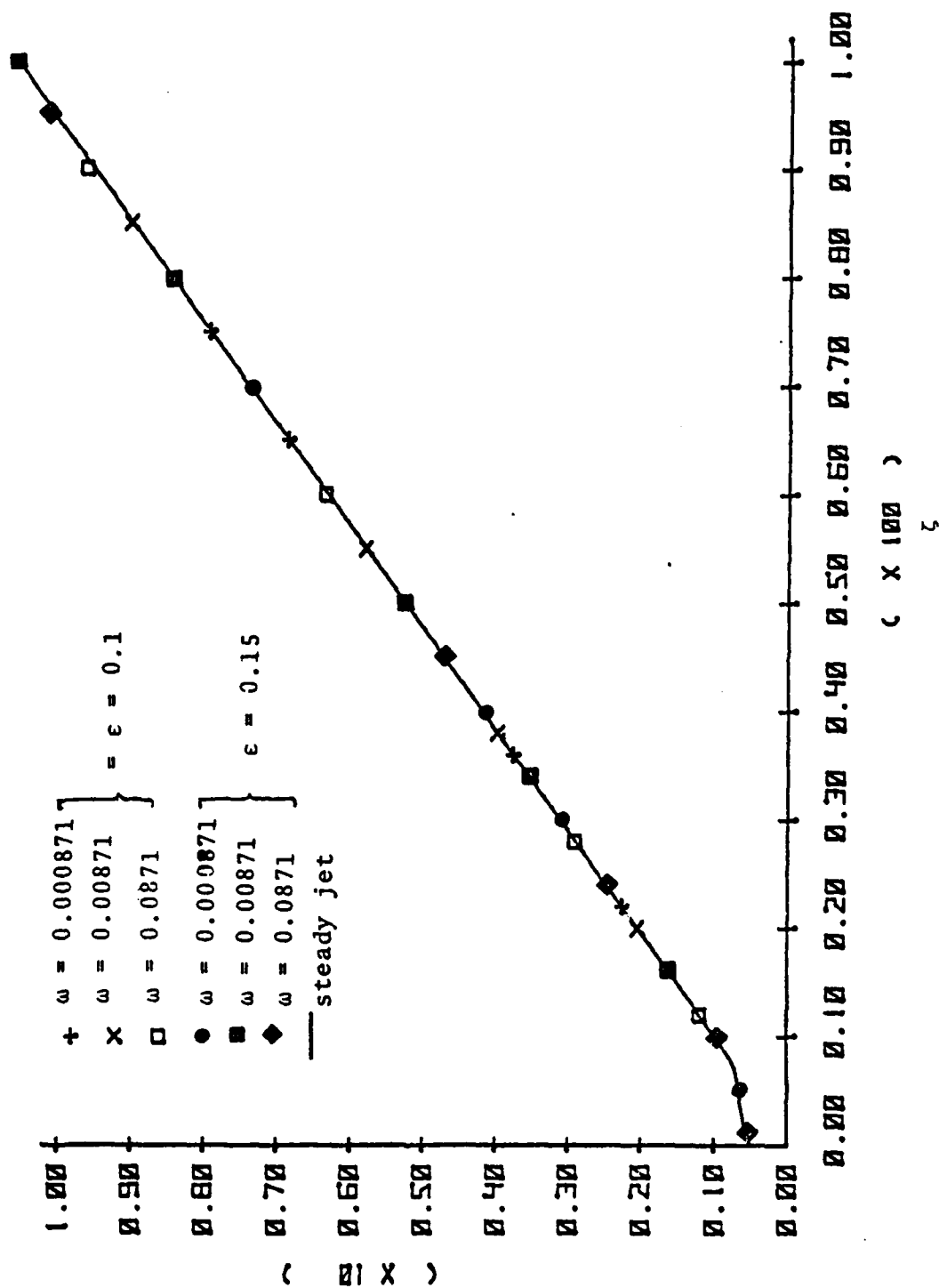


Figure 10 Variation of Mean Jet Half-width Streamwise Distance

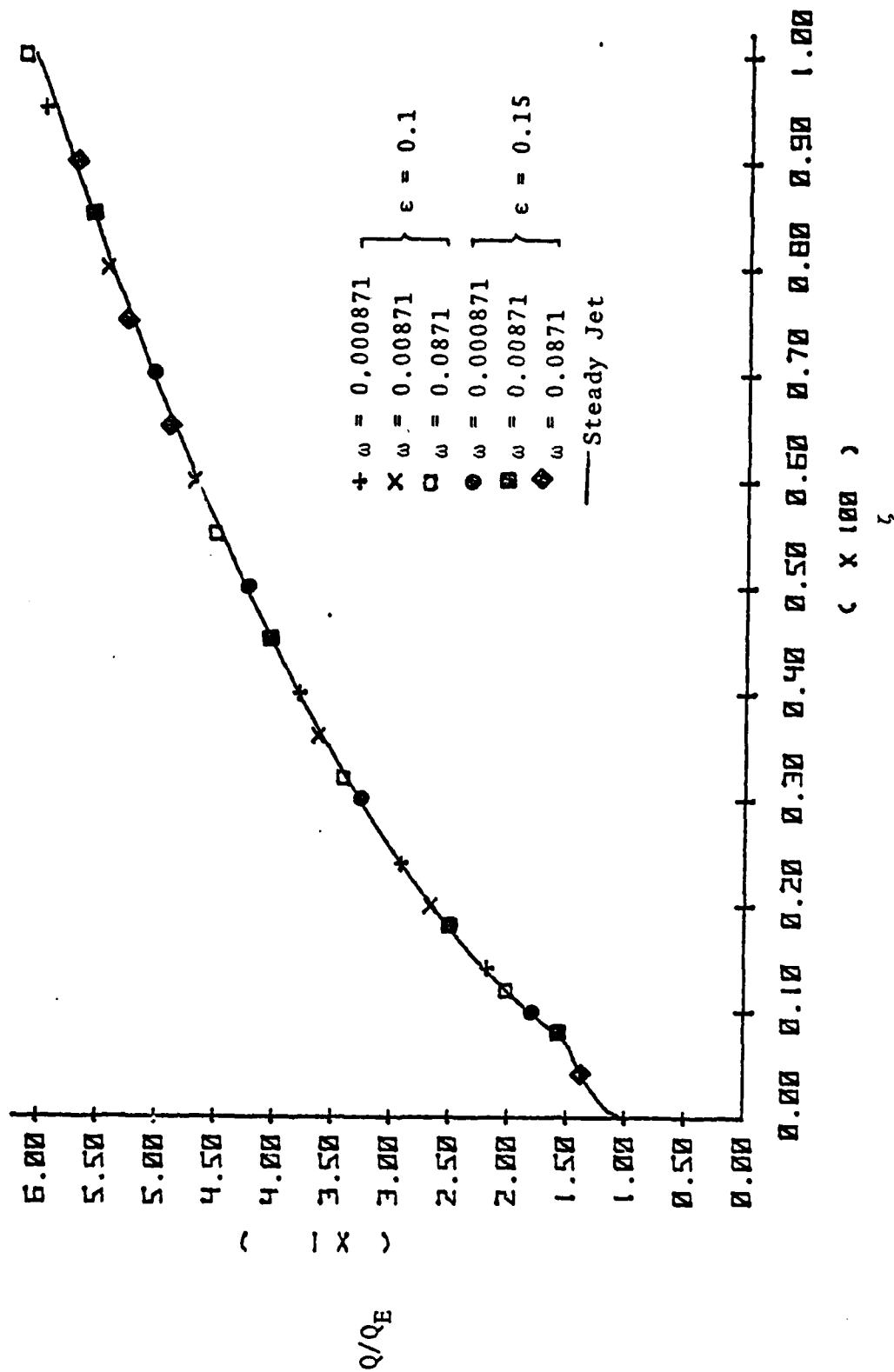


Figure 11 Variation of Non-Dimensional Mass Flow with Streamwise Distance

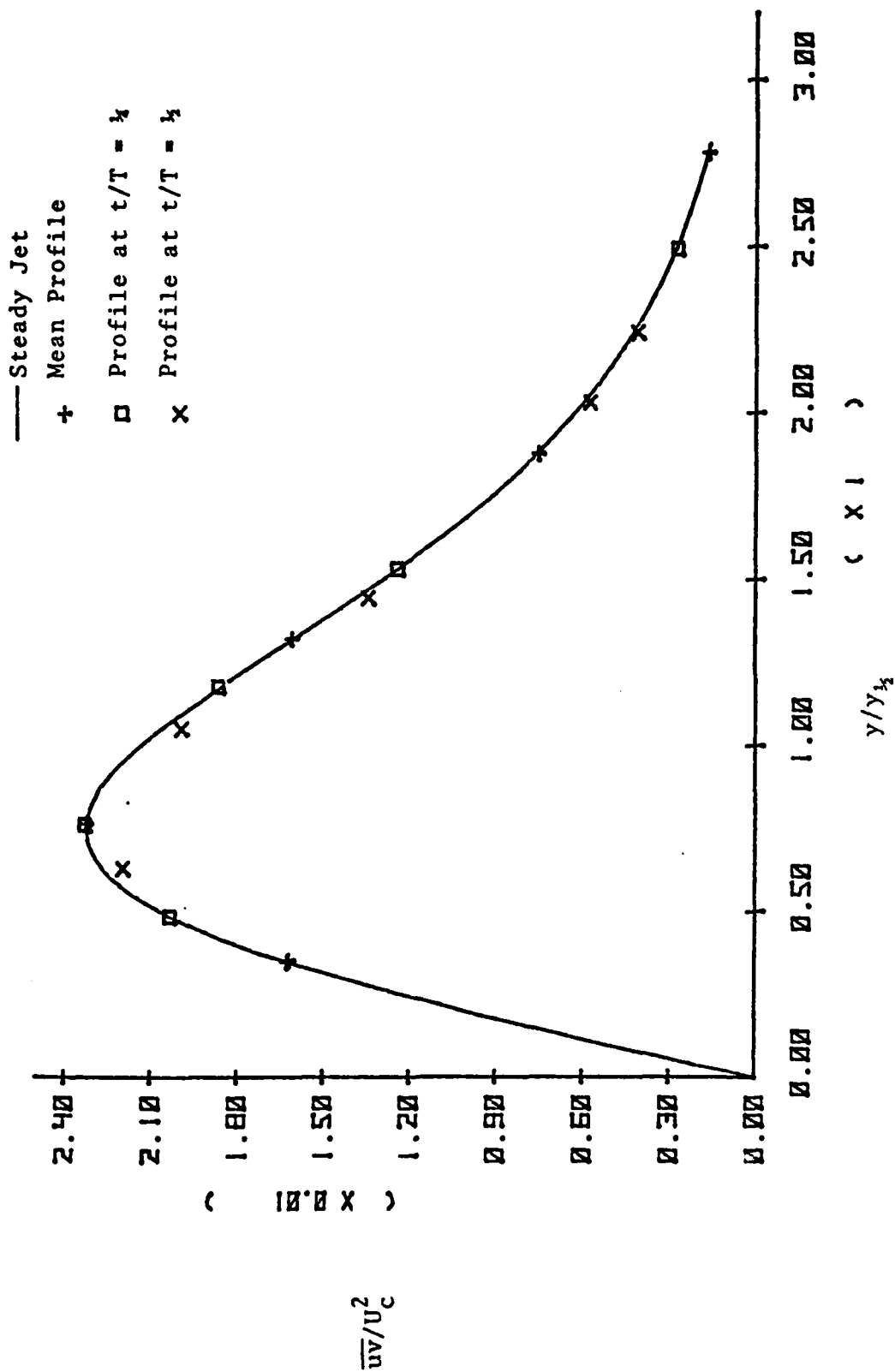


Figure 12(a) Non-Dimensional Shear Stress Profile for $\zeta = 40.032$, $\omega = 0.000871$, $\epsilon = 0.1$

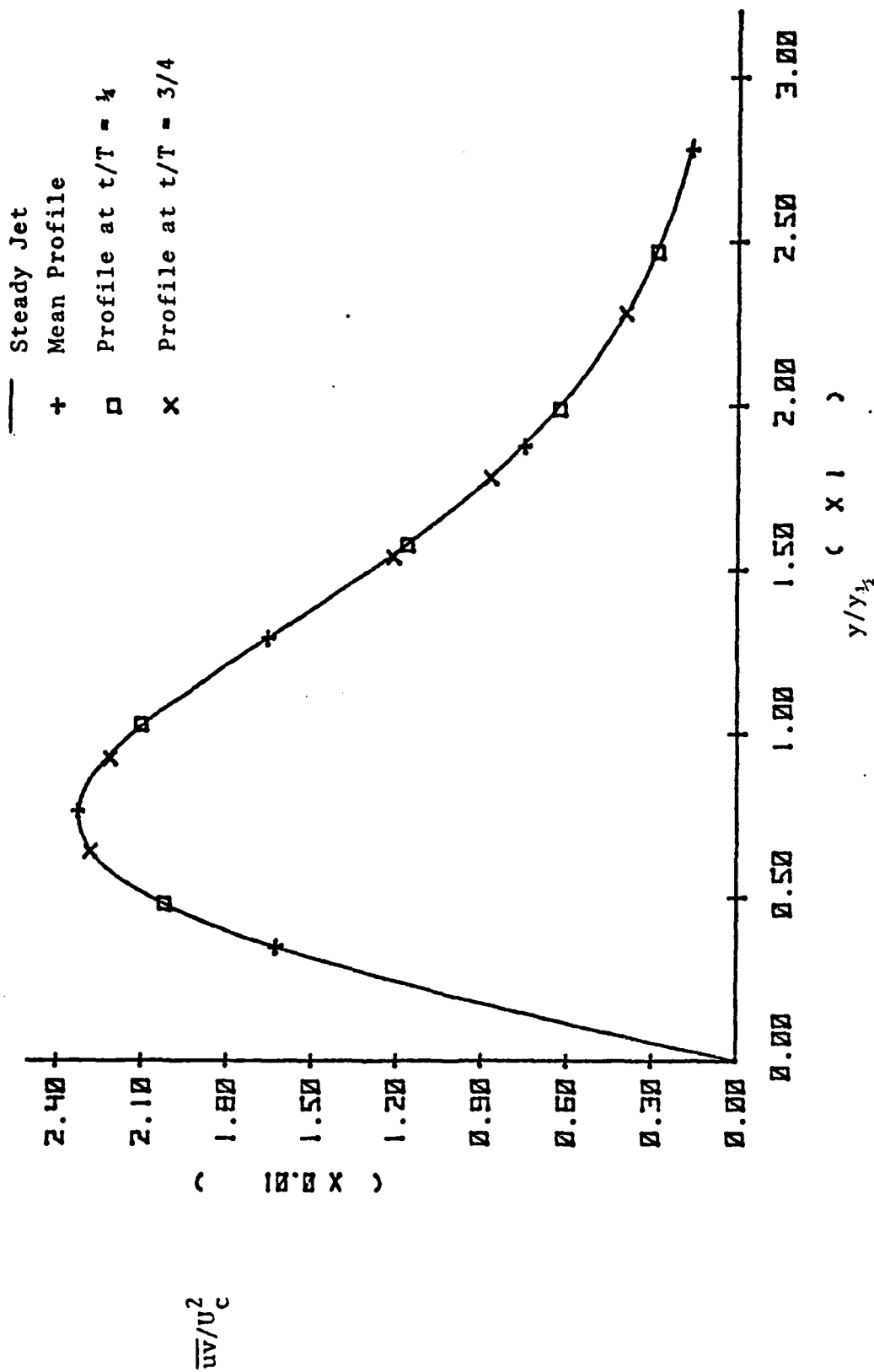


Figure 12(b) Non-dimensional Shear Stress Profile for $\zeta = 40.032$, $\omega = 0.00871$, $\epsilon = 0.1$

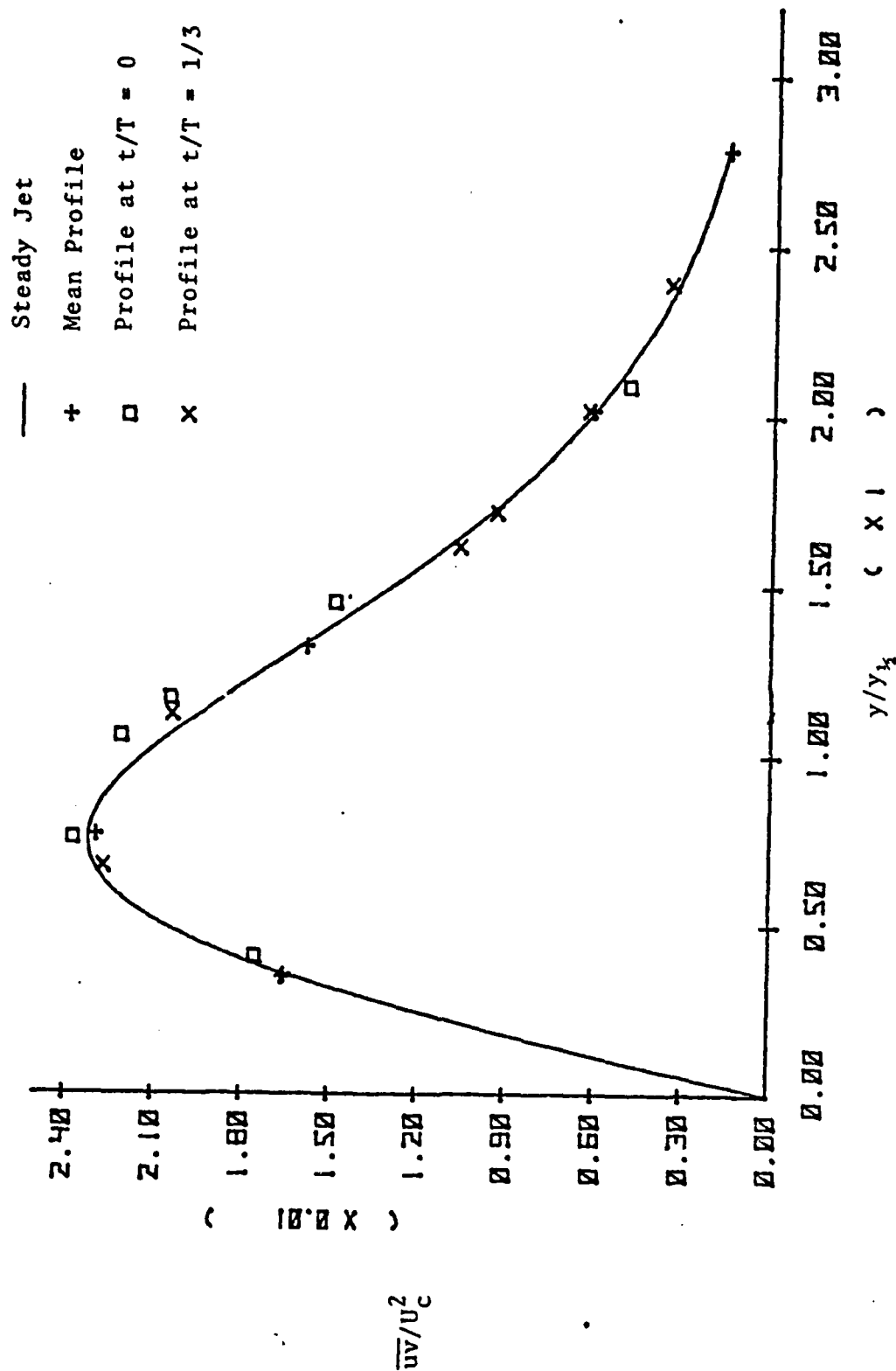


Figure 12(c) Non-dimensional Shear Stress Profile for $\zeta = 40.032$, $\omega = 0.0871$, $\epsilon = 0.1$

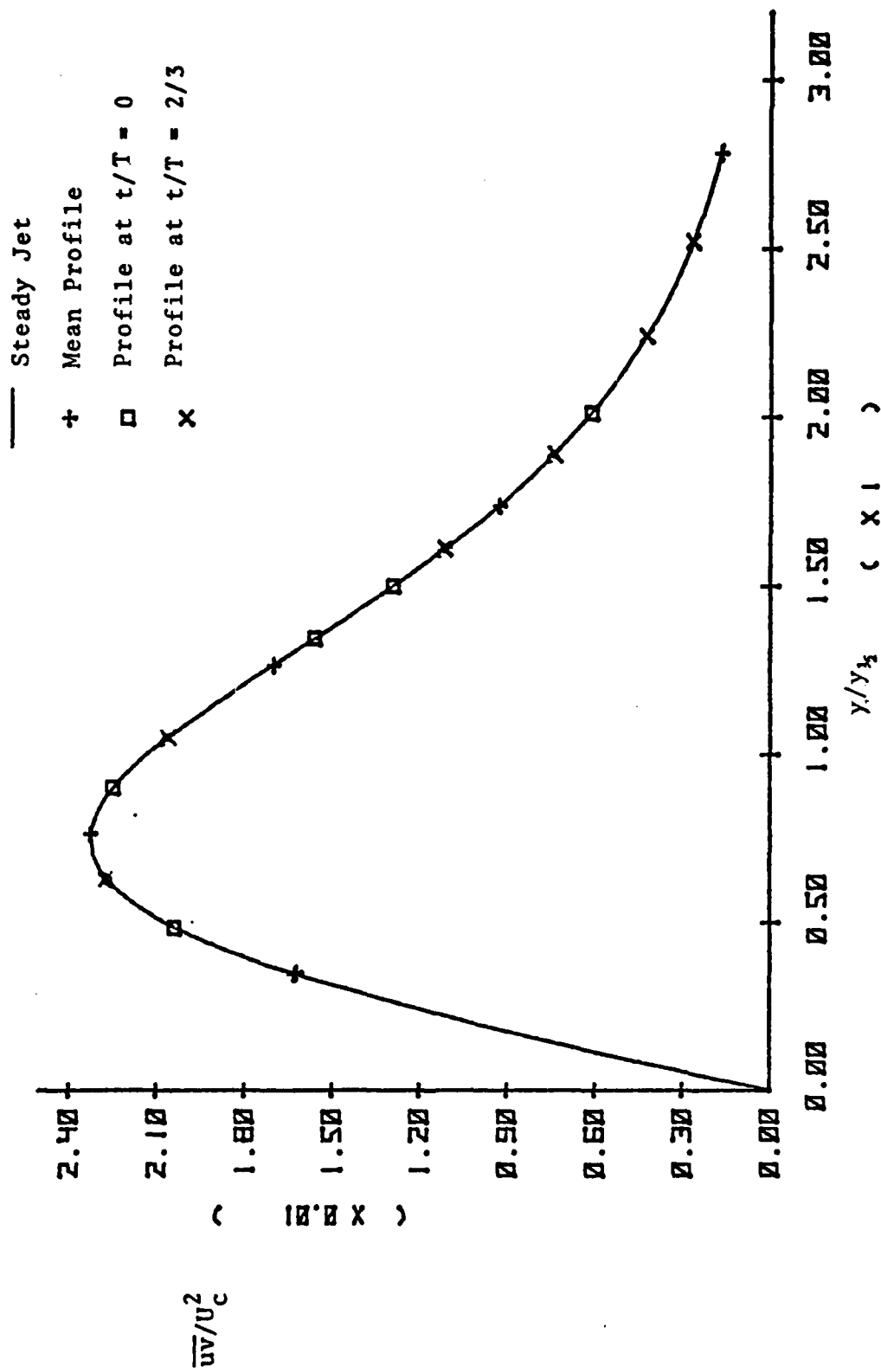


Figure 12(d) Non-Dimensional Shear Stress Profile for $\zeta = 40.032$, $\omega = 0.000871$, $\epsilon = 0.15$

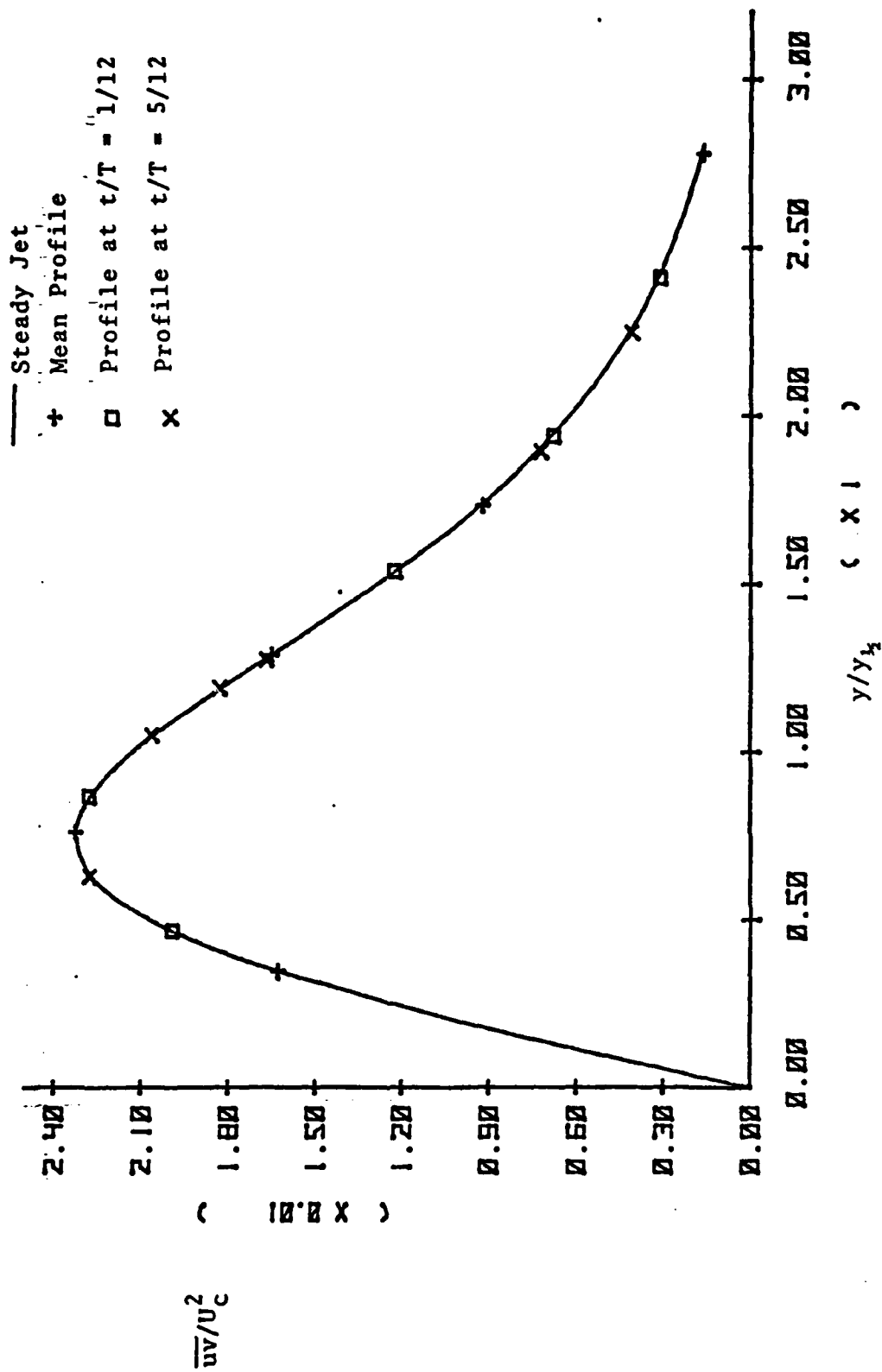


Figure 12(e) Non-Dimensional Shear Stress Profile for $\zeta = 40.032$, $\omega = 0.00671$, $\epsilon = 0.15$

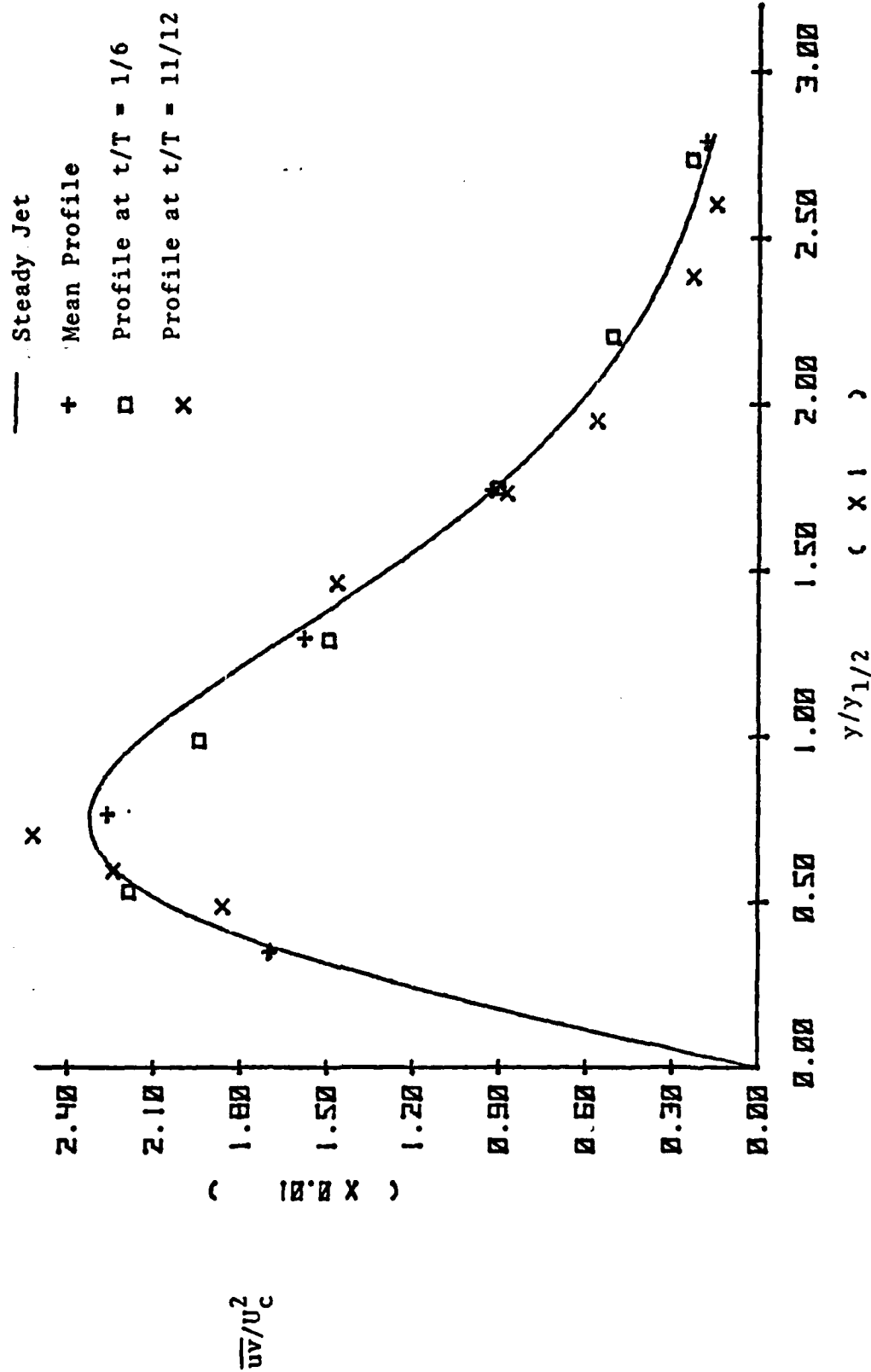


Figure 12(f) Non-Dimensional Shear Stress Profile for $\zeta = 40.032$, $\omega = 0.0871$, $\epsilon = 0.15$

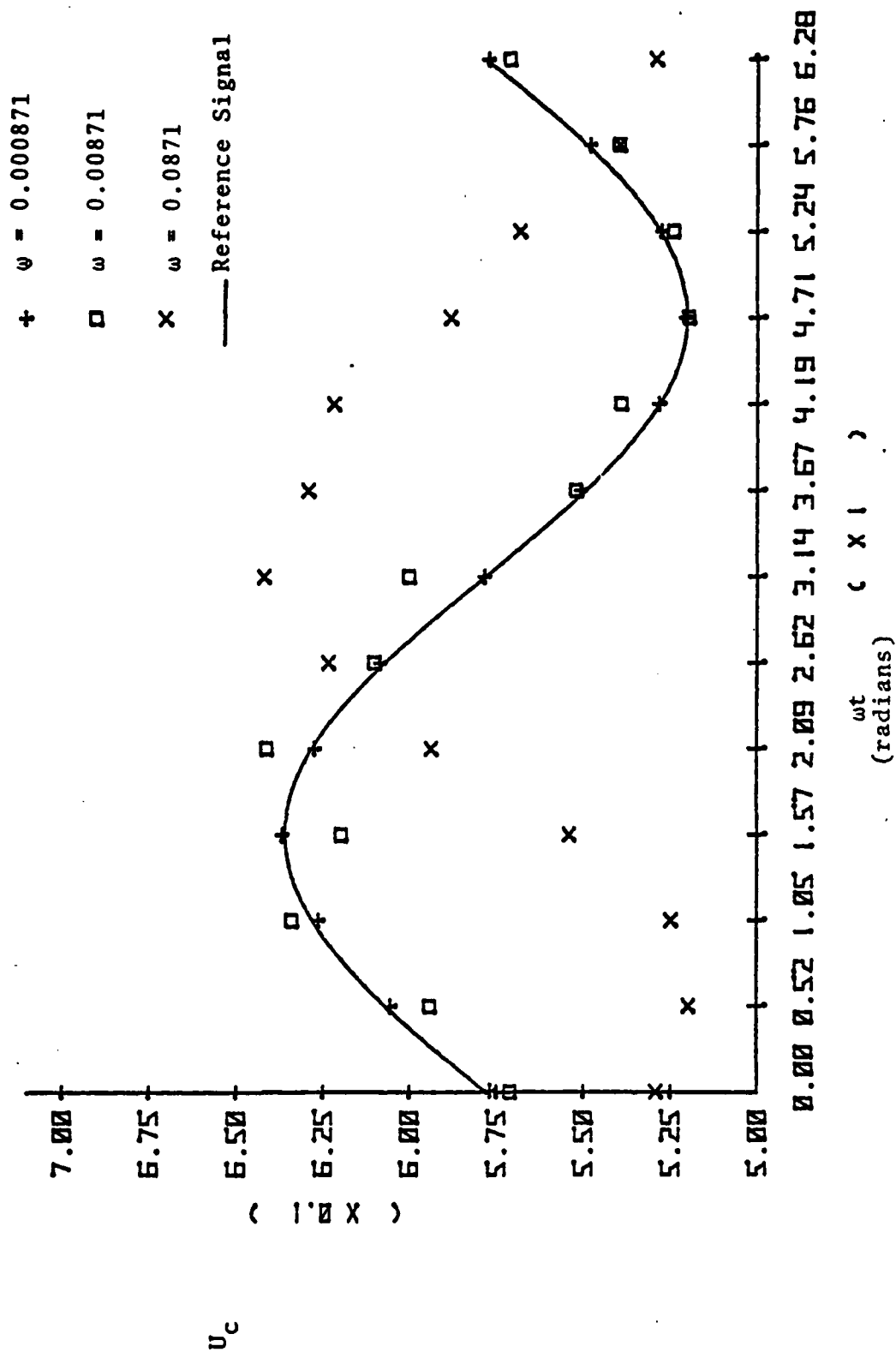


Figure 13(a) Variation of Instantaneous Centre-Line Velocity with Time for $\zeta = 19.032$
 $\zeta = 0.1$

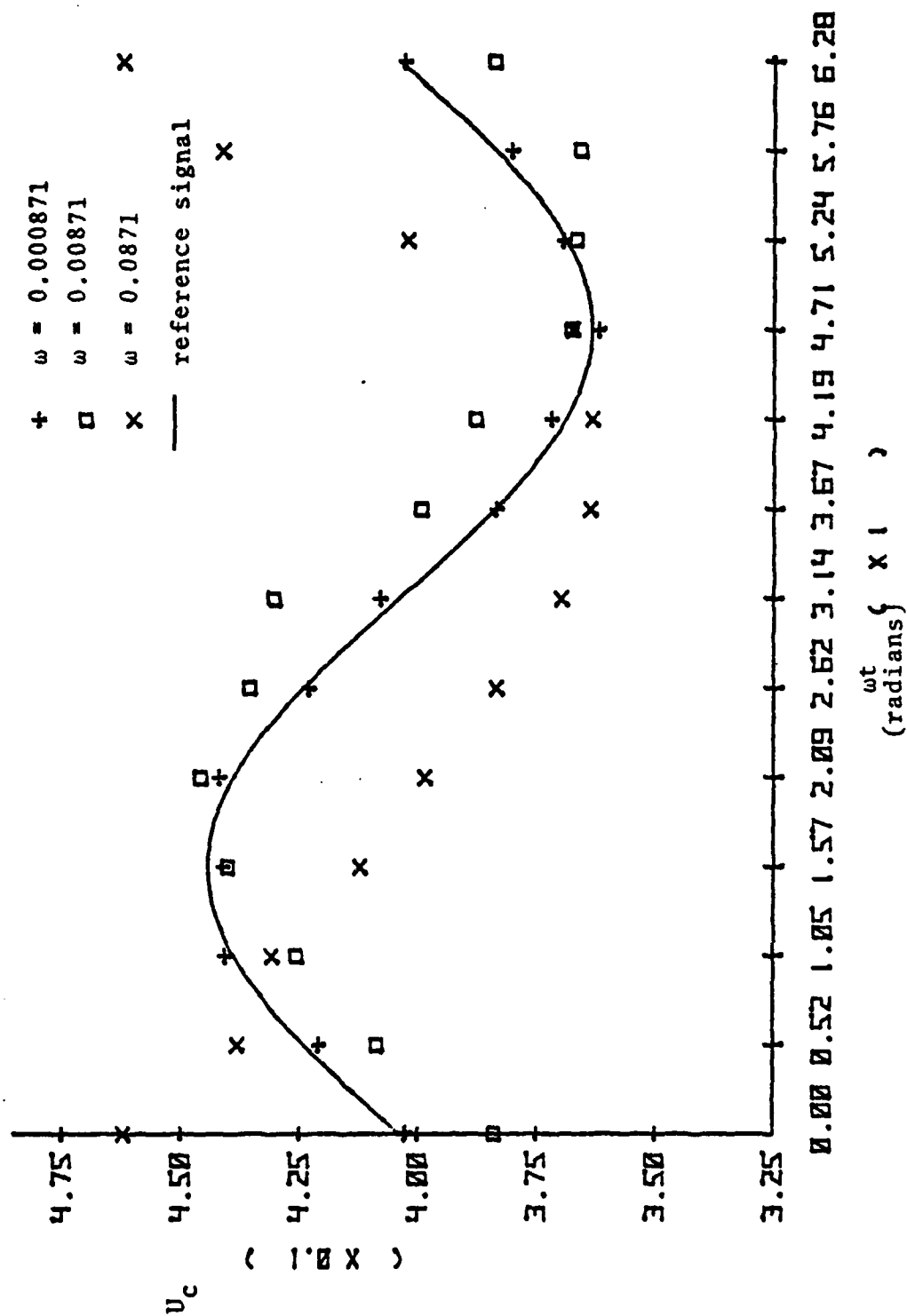


Figure 13(b) Variation of Instantaneous Centre-Line Velocity with Time for $z = 38.032$
 $\epsilon = 0.1$

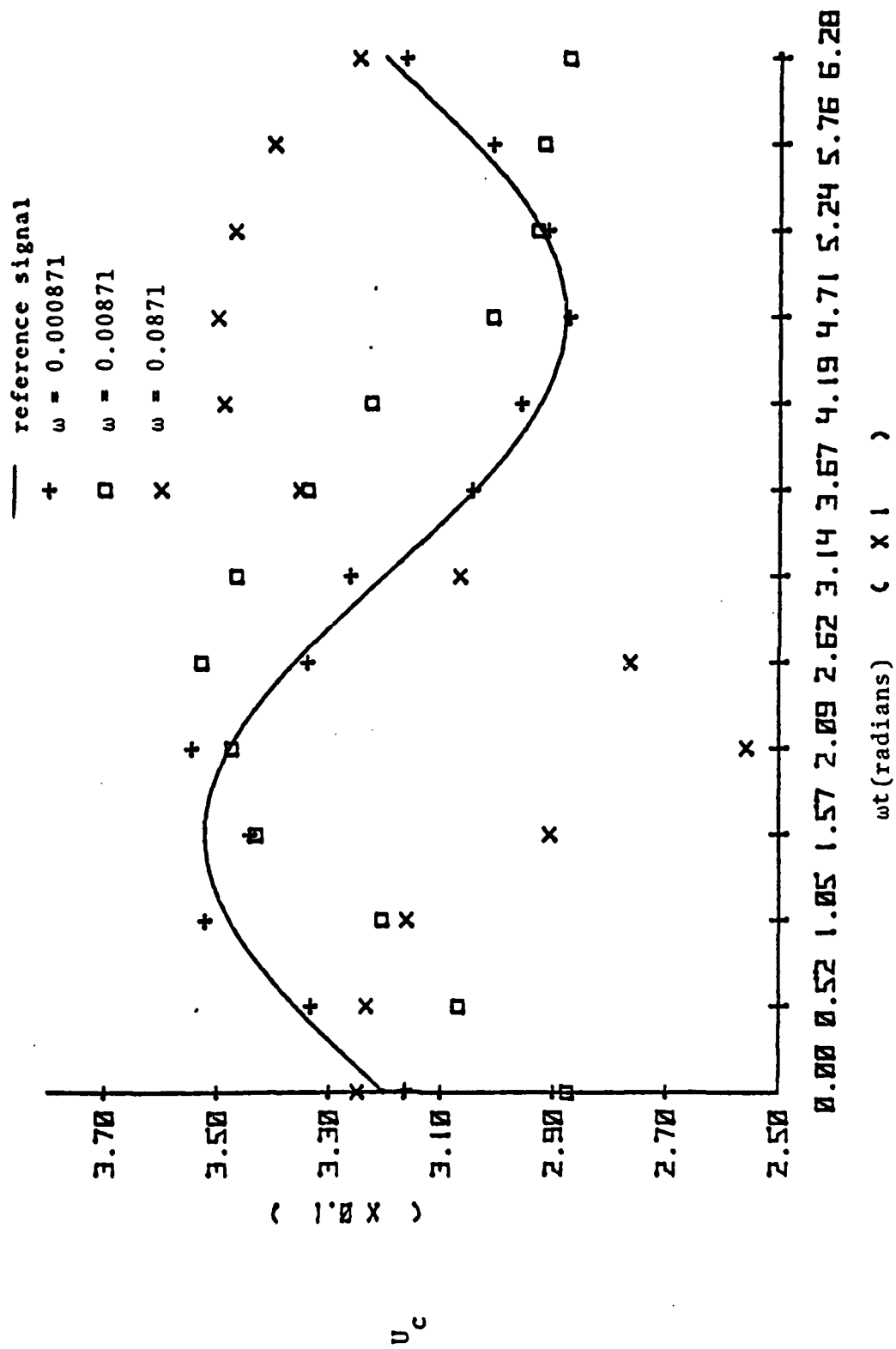


Figure 13(c) Variation of Instantaneous Centre-Line Velocity with Time for $\zeta = 60.032$
 $\epsilon = 0.1$

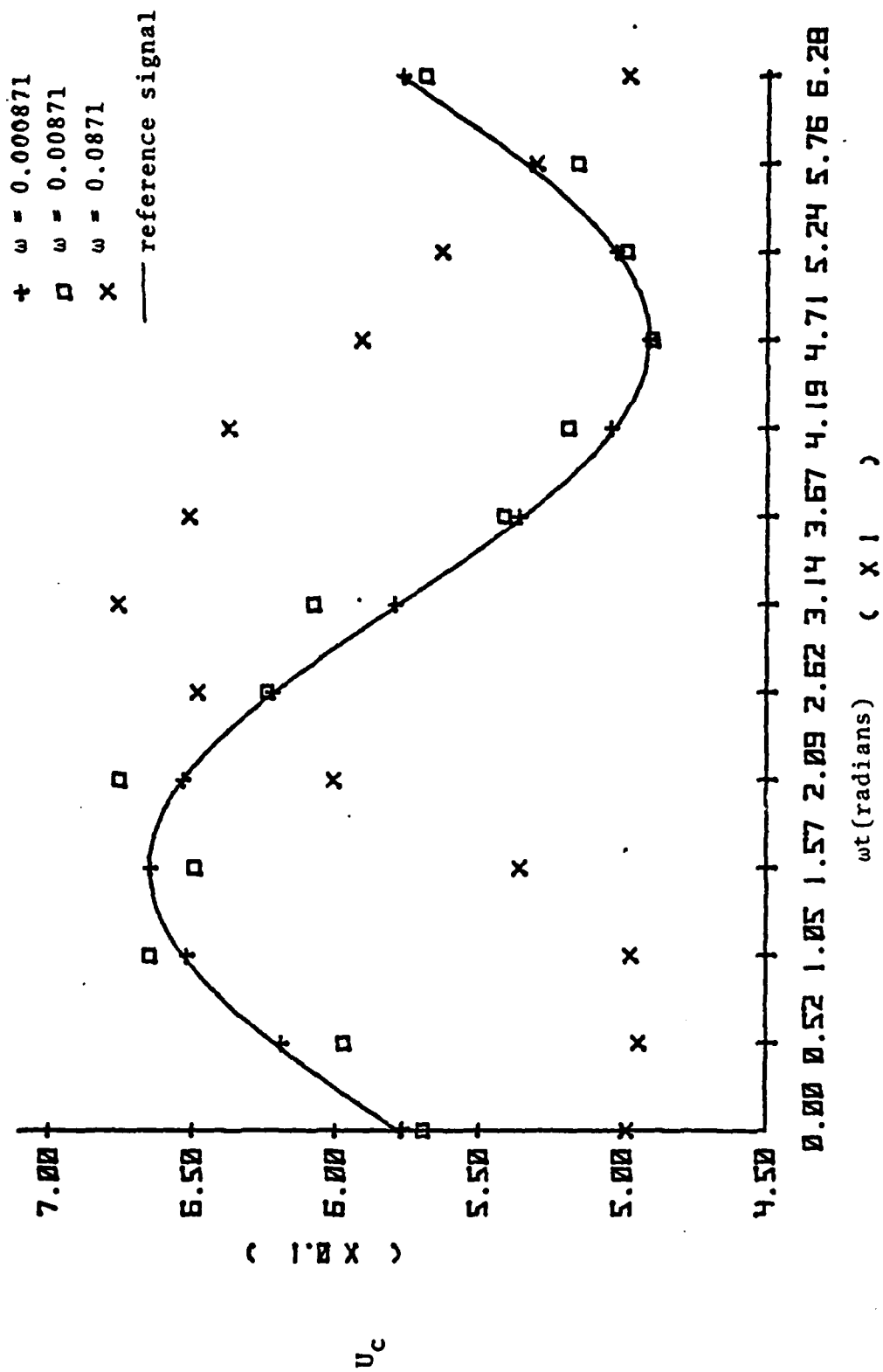


Figure 13(d) Variation of Instantaneous Centre-line Velocity with Time for $\zeta = 19.032$
 $\epsilon = 0.15$

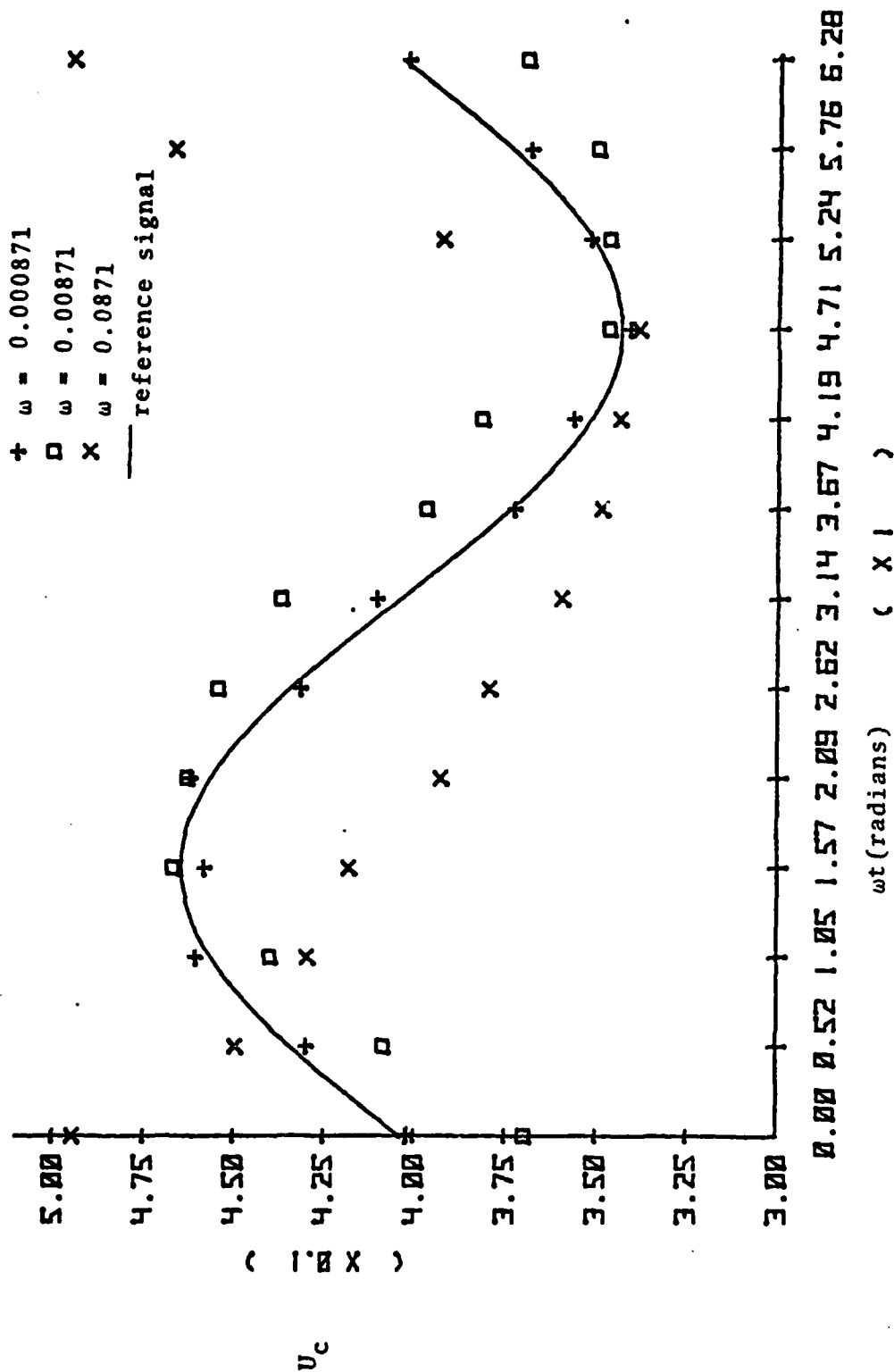


Figure 13(e) Variation of Instantaneous Centre-Line Velocity with Time for $\zeta = 38.032$
 $\epsilon = 0.15$

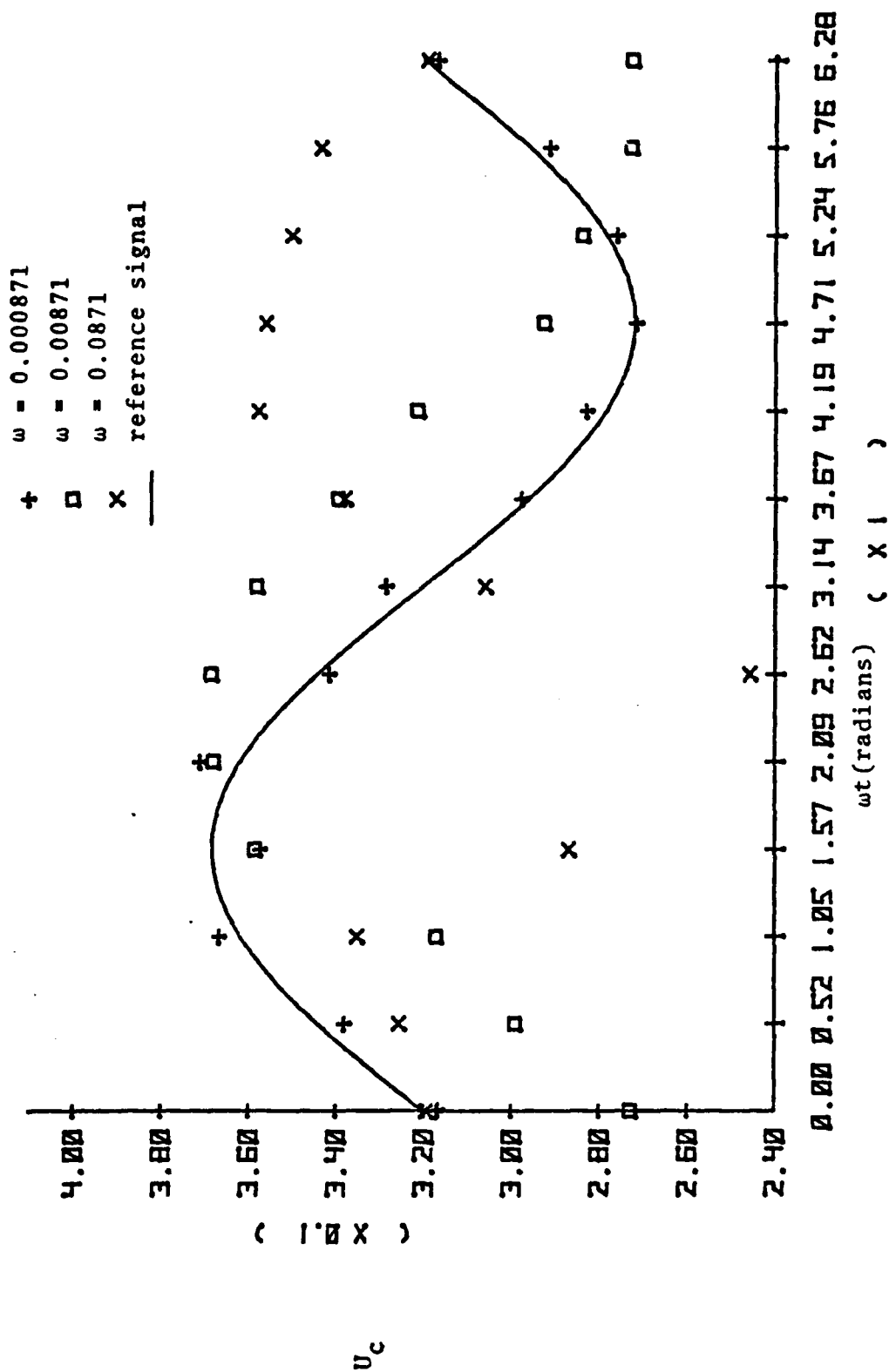


Figure 13(f) Variation of Instantaneous Centre-Line Velocity with time For $\zeta = 60.032$
 $\epsilon = 0.15$

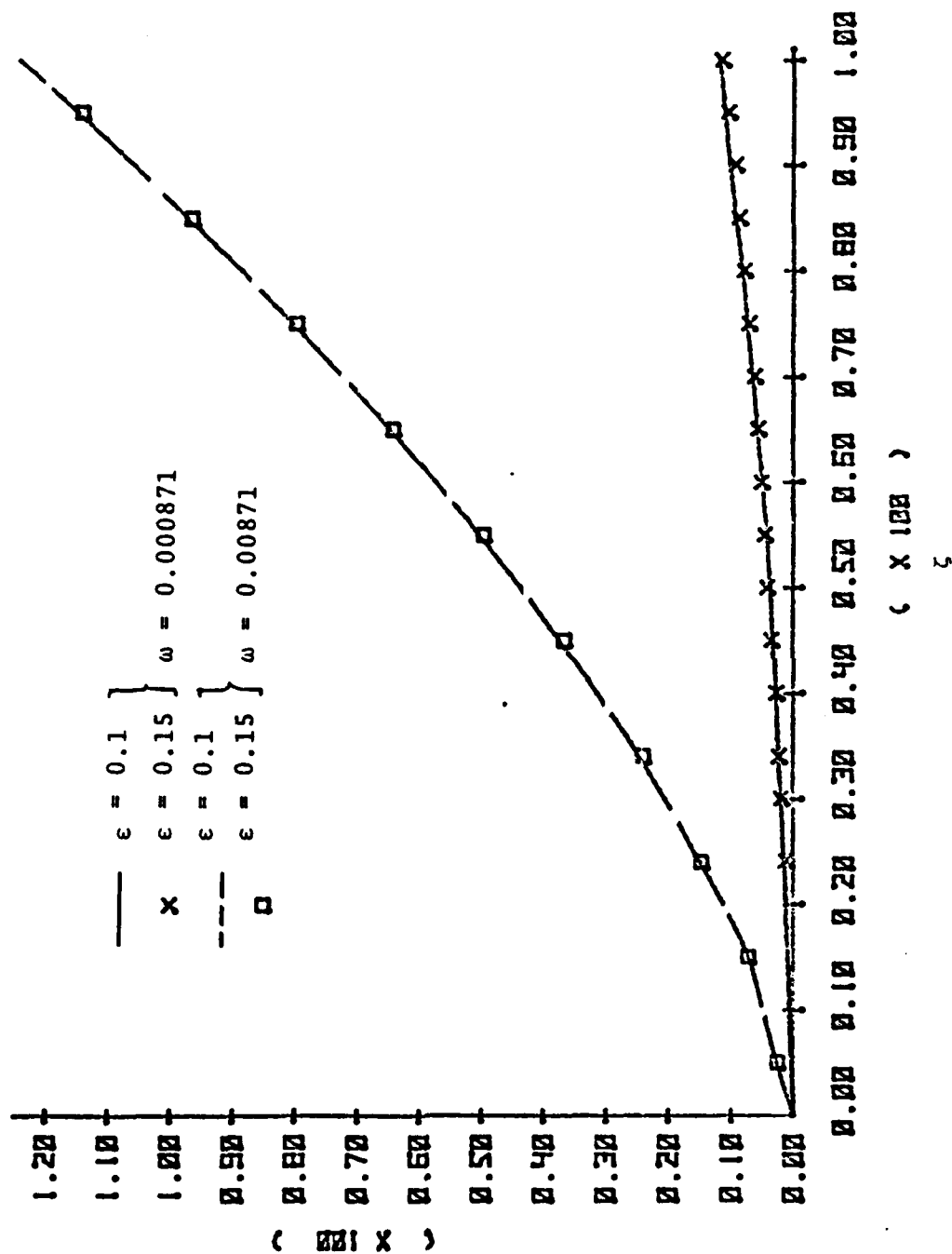


Figure 14(a) Variation of Phase Angle with Streamwise Distance

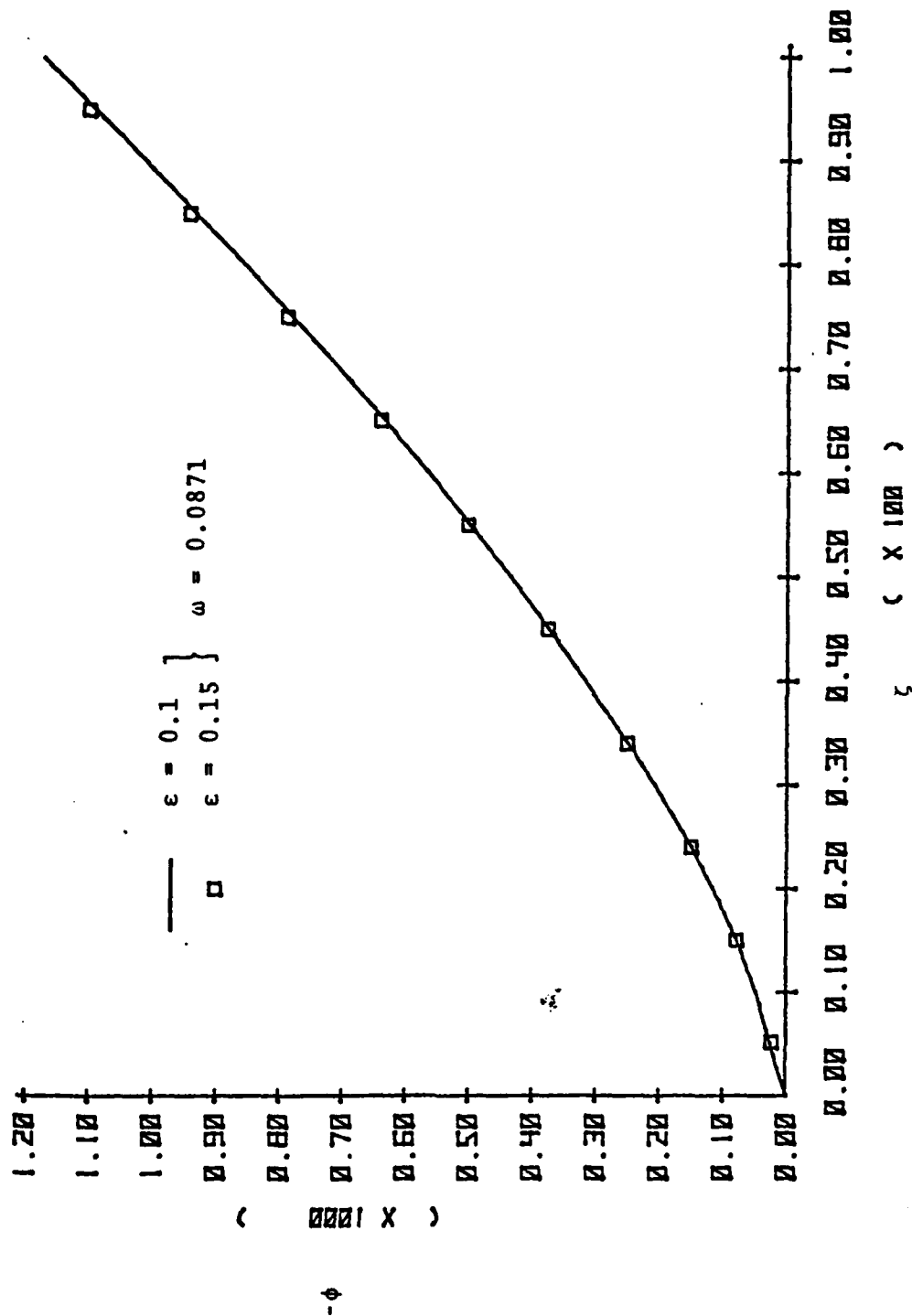


Figure 14(b) Variation of Phase Angle with Streamwise Distance

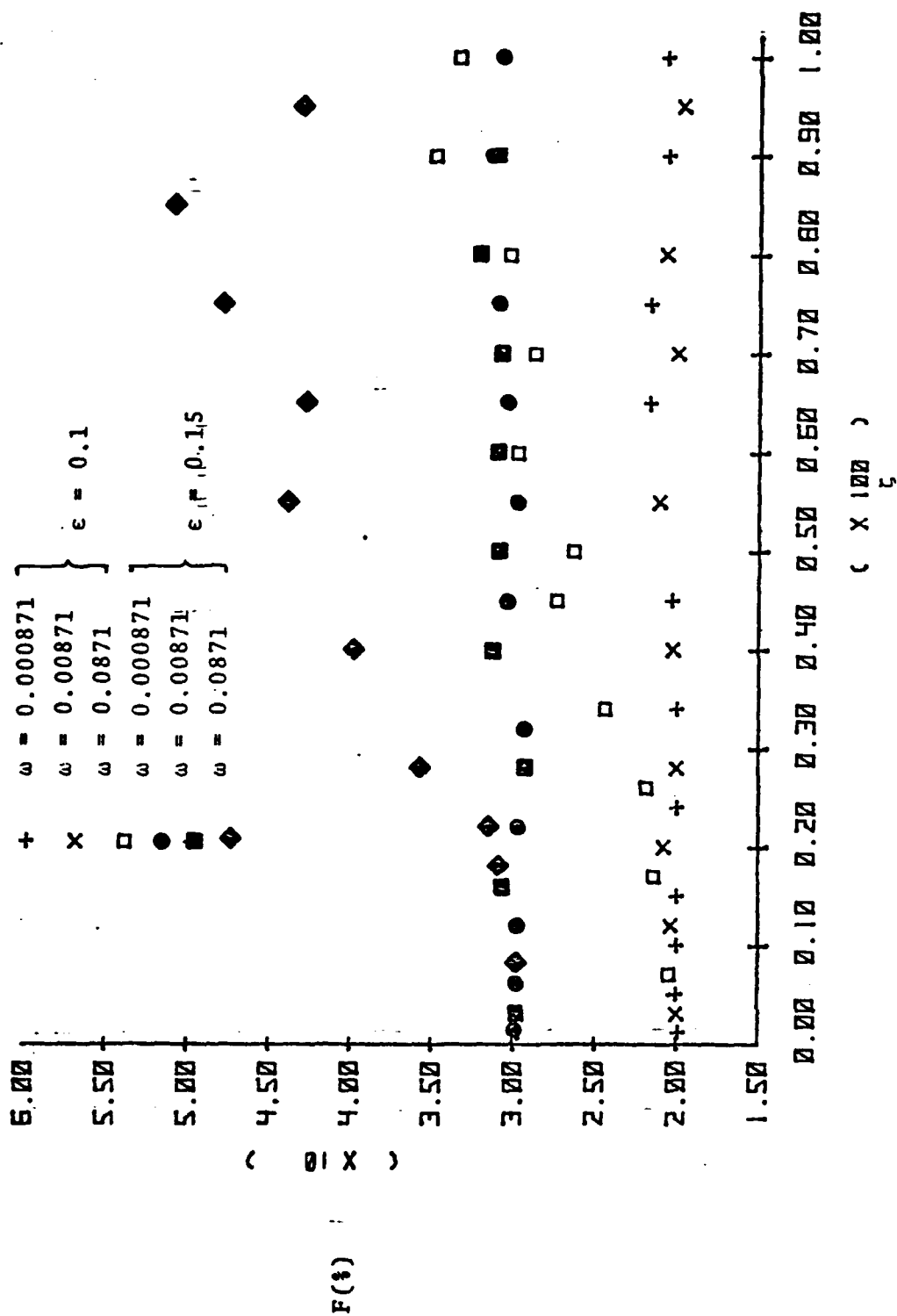


Figure 15 Variation of Peak-to-Peak Oscillation of Centre-Line Velocity with Streamwise Distance

DISTRIBUTION LIST

	No. of Copies
1. Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, CA 93940	2
3. Professor K. J. Bullock Department of Mechanical Engineering University of Queensland St. Lucia, Brisbane Australia 4067	1
4. Dr. J. M. Simmons Department of Mechanical Engineering University of Queensland St. Lucia, Brisbane Australia 4067	5
5. Mr. J. C. S. Lai Department of Mechanical Engineering University of Queensland St. Lucia, Brisbane Australia 4067	5
6. DR. T. Cebeci Distinguished Professor Mechanical Engineering Department Cal State University Long Beach, CA	1
7. Professor H. H. Korst Mechanical Engineering Department University of Illinois Urbana, Illinois	1
8. Professor M. F. Platzler Chairman, Department of Aeronautics Naval Postgraduate School Monterey, CA 93940	5
9. Professor R. P. Shreeve Department of Aeronautics Naval Postgraduate School Monterey, CA 93940	1

