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NONDISPERSIVE WAVE PROPAGATION

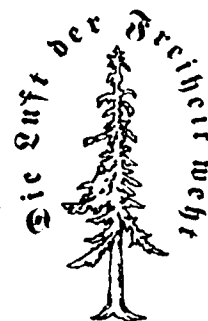IN A LAYERED COMPOSITE

by

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and

G. Herrmann

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

It is found that it is possible to propagate a horizontally polarized (SH) wave without dispersion through an elastic, periodically-layered composite. The nondispersive property of the wave is due to the fact that at each interface the angle of incidence is such that the wave is totally transmitted without reflection. In optics such an angle is referred to as the Brewster angle. It is determined that this particular case is contained as a special solution of the general dispersion equation for SH waves, which has not been noticed before

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ABSTRACT

It is found that it is possible to propagate a horizontally polarized (SH) wave without dispersion through an elastic, periodically-layered composite. The nondispersive property of the wave is due to the fact that at each interface the angle of incidence is such that the wave is totally transmitted without reflection. In optics such an angle is referred to as the Brewster angle. It is determined that this particular case is contained as a special solution of the general dispersion equation for SH waves, which has not been noticed before.

1.   INTRODUCTION

For horizontally polarized shear waves impinging upon an interface be-
tween two media with different properties, it is well known that in general
there will be both a transmitted and a reflected wave. For a special angle of
incidence the amplitude of the reflected wave is known to be zero and only
the transmitted wave will remain. In optics this angle is referred to as the
Brewster angle. Under these very special conditions it can be expected that
this same reflection-free phenomenon must exist in a bi-laminated composite.
This is a special case which has been overlooked in the general treatment of
wave propagation in anti-plane strain [1]. It is the aim of this paper to ex-
amine in detail this particular phenomenon of non .spersive wave propagation
in a layered medium.

2.   BREWSTER ANGLE

We begin by examining the case of an infinite SH wave train impinging at an angle θ upon the interface of two semi-infinite elastic homogeneous media of different properties. In general, a portion of the wave will be reflected and a portion will be transmitted (see Fig. 1a). For a particular angle of incidence the wave will be totally transmitted (Fig. 1b). The angle θ_B is called the Brewster angle and is given by an expression of the form [2]

$$\sin \theta_B = \left[\frac{\left(\frac{\mu \rho'}{\mu' \rho} \Big/ \left(\frac{\mu'}{\mu}\right)^2 \right) - 1}{\left(\frac{\mu}{\mu'}\right)^2 - 1} \right]^{\frac{1}{2}} \tag{1}$$

where μ, ρ and μ', ρ' are the shear moduli and density in the two media, respectively. The feature of no reflection can be generalized from the case of two semi-infinite media in contact to the case of an infinite laminated medium (Fig. 2). Here the condition of no reflection occurs at each interface. As a result we can state that the laminated composite is capable of supporting such reflection-free waves. An immediate observation is that these waves are independent of the frequency, i.e. "nondispersive". In frequency-wave number space, waves propagating in such a fashion are represented by a straight line. A discussion of this phenomenon for antiplane strain wave propagation in an infinite periodically layered composite has not been noted before. In the next section we will take a closer look at the dispersion relation for the infinite laminated composite and examine this noteworthy dispersion-free propagation phenomenon.

3. DISPERSION IN AN INFINITE LAMINATED MEDIUM

We consider horizontally polarized harmonic shear waves (SH waves) prop-agating through a periodically layered elastic body of unbounded extent. The union of any two contiguous layers in the body constitutes a *unit cell*, and this unit cell is invariant under a lattice translation along the positive and negative y-axes (Fig. 2). Each of the two layers in the unit cell are assumed to be homogeneous, isotropic, and perfectly bonded to the adjoining layers. The two lamellae of a typical unit cell have elastic constants $[(\lambda,\mu) ; (\lambda',\mu')]$, thicknesses $(2h;2h')$, and densities $(\rho;\rho')$, respectively.

Let u, v and w be the three Cartesian components of the displace-ment vector in the x, y and z-directions, respectively. For antiplane motion, we take

$$v \equiv w \equiv 0 \quad \text{and} \quad u = u(y,z;t) . \tag{2}$$

For the layers with unprimed constants in the Nth unit cell, the equation of motion is

$$\mu\left(\frac{\partial^2 u}{\partial y_N^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\frac{\partial^2 u}{\partial t^2} , \quad (-h \le y_N \le h) , \quad t > 0 \tag{3}$$

where y_N is a local coordinate with its origin at the midplane of the layer (Fig. 2). For plane waves traveling in the positive z-direction, we assume

$$u = f(y_N) e^{i(k_z z - \omega t)} \tag{4}$$

where k_z is the wave number in the z-direction and ω is the circular frequency in radians per unit of time. Substitution into (3) yields the ordinary differential equation

$$\frac{d^2 f}{dy_N^2} + \left(\frac{\rho\omega^2}{\mu} - k_z^2\right) f = 0 \ , \quad (-h \le y_N \le h) \tag{5}$$

whose solution may be written in the form

$$f(y_N) = C_1 \exp\left(\frac{i\pi\alpha y_N}{2h}\right) + C_2 \exp\left(-\frac{i\pi\alpha y_N}{2h}\right) \ . \tag{6}$$

Here $\alpha = \sqrt{\Omega^2 - \zeta^2}$, and Ω and ζ are, respectively, the nondimensional frequency and wave number defined by

$$\Omega = \frac{2h\omega}{\pi\sqrt{\mu/\rho}} \ ; \quad \zeta = \frac{2h}{\pi} k_z \tag{7}$$

The displacement component now takes the form

$$u(y_N, z; t) = \left[C_1 \exp\left(\frac{i\pi\alpha y_N}{2h}\right) + C_2 \exp\left(-\frac{i\pi\alpha y_N}{2h}\right)\right]$$

$$\times \exp\left[\frac{i\pi}{2h}\left(\zeta z - \sqrt{\frac{\mu}{\rho}}\,\Omega t\right)\right] \ , \quad (-h \le y_N \le h) \ , \ t > 0 \ . \tag{8}$$

In a similar manner, the displacement component of the layer with primed constants in the Nth unit cell can be written as

$$u'(y_N', z; t) = \left[C_1' \exp\left(\frac{i\pi\alpha' y_N'}{2h} \right) + C_2' \exp\left(-\frac{i\pi\alpha' y_N'}{2h} \right) \right]$$

$$\times \exp\left[\frac{i\pi}{2h}\left(\zeta z - \sqrt{\frac{\mu}{\rho}}\, \Omega t \right) \right] , \quad (-h' \leq y_N' \leq h') , \quad t > 0 \tag{9}$$

where $\alpha' = \sqrt{\sigma^2\Omega^2 - \zeta^2}$ and $\sigma^2 = (\mu\rho'/\mu'\rho)$.

The layers are considered to be perfectly bonded, so continuity of displacement and traction must be enforced at the layer interfaces. The continuity conditions between the two layers in the Nth unit cell and the adjoining layer in the (N + 1)th unit cell are

$$u(-h, z_N; t) = u'(h', z_N'; t) ,$$

$$\sigma_{xy}(-h, z_N; t) = \sigma_{xy}'(h', z_N'; t) ,$$

$$-\infty < z_N, z_{N+1} < \infty , \quad t > 0$$

$$u(h, z_N; t) = u'(-h', z_{N+1}'; t) ,$$

$$\sigma_{xy}(h, z_N; t) = \sigma_{xy}'(-h', z_{N+1}'; t) . \tag{10}$$

It may now be noted that the equation of motion for the laminated body B takes the form of a partial differential equation with coefficients which are periodic in the y-direction with period $d = 2(h + h')$. This follows from the fact that the coefficients of the equation of motion (the elastic moduli and mass density) are piecewise constant in each layer and have periodic

variation from cell to cell. It is well known from the one-dimensional theory of Floquet [4] or the three-dimensional theory of Bloch that differential equations with periodic coefficients admit solutions of the form

$$u(y,z;t) = g(y,z;t) \, e^{ik_y y} \tag{11}$$

where $g(y,z;t)$ has the same periodicity as the coefficients of the differential equation, i.e.,

$$g(y + d,z;t) = g(y,z;t) \, . \tag{12}$$

From (11) it is seen that k_y has the character of a wave number; it is said to be the wave number of the Floquet wave. Equations (11) and (12) lead immediately to the quasi-periodic recurrence relation

$$u(y + d,z;t) = u(y,z;t) \, e^{ik_y d} \, . \tag{13}$$

The Floquet solution (11) has several other interesting properties. In particular, it is not difficult to show that the wave number k_y is uniquely determined only to within an integer multiple of $2\pi/d$. This fact, its physical significance, and other properties of the Floquet solution are discussed in some detail by Lee [3].

Using (13), the last two equations of (10) may be written as

$$u(h,z_N;t) = u \, (-h',z_N';t) \, e^{ik_y d}$$
$$-\infty < z_N < \infty \quad , \quad t > 0$$
$$\sigma_{xy}(h,z_N;t) = \sigma'_{xy}(-h',z_N';t) \, e^{ik_y d} \tag{14}$$

which together with the first two equations of (10) now represent a set of displacement and traction continuity conditions written in terms of the displacements and stresses at the interfaces of the Nth unit cell. If these are satisfied, the quasi-periodicity condition (13) will insure continuity of displacement and traction across the interfaces of every unit cell of the laminated medium.

Substituting (8) and (9), and the isotropic stress-strain relation $\sigma_{xy} = \mu(\partial u/\partial y)$ into (14) and the first two equations of (10), we obtain a set of four homogeneous equations. For a nontrivial solution, the determinant of the matrix of coefficients must vanish, yielding the dispersion equation

$$
\begin{vmatrix}
e^{-\phi} & e^{\phi} & -e^{\epsilon\phi'} & -e^{-\epsilon\phi'} \\
\gamma\alpha e^{-\phi} & -\gamma\alpha e^{\phi} & -\alpha' e^{\epsilon\phi'} & \alpha' e^{-\epsilon\phi'} \\
e^{\phi} & e^{-\phi} & -e^{(\psi-\epsilon\phi')} & -e^{(\psi+\epsilon\phi')} \\
\gamma\alpha e^{\phi} & -\gamma\alpha e^{-\phi} & -\alpha' e^{(\psi-\epsilon\phi')} & \alpha' e^{(\psi+\epsilon\phi')}
\end{vmatrix} = 0 \qquad (15)
$$

where $\phi = i\pi\alpha/2$, $\phi' = i\pi\alpha'/2$, $\psi = i\pi(1+\epsilon)\eta$, $\epsilon = h'/h$, $\gamma = \mu/\mu'$, and $\eta = (2h/\pi)k_y$. This equation may be written in the convenient form

$$
H(\Omega,\eta,\zeta) = 4\gamma\alpha\alpha' \cos \pi\eta(1 + \epsilon) + (\gamma\alpha - \alpha')^2 \cos \pi(\alpha - \epsilon\alpha')
$$

$$
- (\gamma\alpha + \alpha')^2 \cos \pi(\alpha + \epsilon\alpha') = 0 \quad . \qquad (16)
$$

Since this equation remains unaffected by a change in sign in η or ζ , both these quantities may be assumed positive. However, if either one or both of

α and α´ is imaginary, (16) must be appropriately interpreted in terms of hyperbolic functions.

It may be noted that, since (16) relates a nondimensional frequency to two nondimensional wave numbers, the roots of this equation define a surface in frequency-wave number space, which is called the dispersion surface. A qualitative sketch of this surface, drawn on an extended zone scheme [3], is shown in Fig. 3. It can be seen that the surface is, in general, discontinuous at $\eta = n/(1 + \epsilon)$, $n = 1,2,\ldots$. These planes of discontinuity divide the surface into Brillouin zones, the first three of which are shown in the sketch. The geometric and material parameters used here and throughout the remainder of this paper are $\gamma = 0.02$, $\sigma^2 = 0.06$, and $\epsilon = 4$.

In the past the physical interpretation of the wave numbers η and ζ has been quite deficient. Until now no simple example was known which would give insight into the nature of these wave numbers.

A description of the nondispersive propagation in terms of the wave-numbers η and ζ illustrates a point which is, in general, not sufficiently appreciated. In some papers (e.g. [5]), the wave numbers η and ζ were treated as vector components of a single wave number measured normal to a plane wave front. It is clear from Fig. 5 that, in general, the wave forms of the type appropriate to layered composites are not plane waves, except in the case $\zeta = 0$. Intuitively, some average propagation direction should be described by the relation ζ/η . Our intuition fails us if we guess that this direction may be defined by connecting any two corresponding points in adjacent unit cells in the wave vector representation of Fig. 4a. Based on the previous numerical values chosen, we can calculate an effective propagation direction as defined by the line connecting two corresponding points (A,B) in adjacent

unit cells from the wave vector representation (see Fig. 4b). For the para-
meters chosen, an angle of ~6° is obtained for θ .

By careful inspection of the dispersion surface we find that the only
possible direction for a straight line in frequency-wave number space which
lies entirely on the surface is illustrated by the straight line OABC in
Fig. 2. It can be shown that the equation of this line satisfies the disper-
sion equation. Again, for the particular parameters chosen we find for a
propagation direction given by ζ/η a value of ~41°. Thus our initial guess
for the propagation direction based on the wave vector representation is not
correct.

A dual representation to the wave vector is by means of the wave front.
In an infinite homogeneous isotropic elastic space the wave vector is normal
to the wave front. For the laminated composite this is true within each layer,
but not in an average sense. Fig. 5a shows the wave front corresponding to
the wave vectors of Fig. 4a. Fig. 5b shows the "averaged" wave front. The
calculated angle θ for the assumed properties was found to be identical to
the value obtained from the dispersion surface. This can be shown mathemat-
ically by considering two such wave fronts as shown in the next section.

This shows quite clearly the manner in which η and ζ can be regarded
as vector components of some "equivalent" wave front. This illustrative ex-
ample derived from this nondispersive propagation direction appears to be the
only case where wave fronts can be drawn easily.

An unexpected sidelight of this analysis deals with the concept of what
is meant by a nondispersive wave. We previously stated that a straight line
in frequency-wave number space represents nondispersive wave propagation. This
could be interpreted as meaning the group velocity, as defined by

$$C_g = -\left(\frac{\partial H}{\partial \eta} e_\eta + \frac{\partial H}{\partial \zeta} e_\zeta\right) \Big/ \frac{\partial H}{\partial \Omega}$$

remains constant. This is in addition to the phase velocity

$$C_p = \frac{\Omega}{\left|\eta e_\eta + \zeta e_\zeta\right|}$$

where e_η and e_ζ are unit vectors perpendicular and parallel to the layers, respectively, remaining constant. For the present case the line OABC in Fig. 3 certainly satisfies the second criterion. But the group velocity is only constant if we restrict our attention to the line OABC . In an experiment in which the ratio of ζ/η is fixed and we move only along the line OABC , we would indeed find that the system is nondispersive. If, instead, one is permitted to sample in other propagation directions, we would find that the group velocity is not constant along the line OABC and the dispersive-free nature is lost.

4. SPECIAL SOLUTION OF THE GENERAL DISPERSION EQUATION

In this section we will show that there exists a straight line in frequency-wave number space which satisfies the dispersion equation identically. Also it will be shown that the ratio of the wave numbers for the effective plane wave is the same as the ratio of ζ to η found previously.

We first show that the equation of a line through two points $(\Omega_0, \zeta_0, \eta_0)$ and $(\Omega_1, \zeta_1, \eta_1)$ in frequency-wave number space can be written in the following form:

$$\frac{\eta - \eta_0}{\eta_1 - \eta_0} = \frac{\zeta - \zeta_0}{\zeta_1 - \zeta_0} = \frac{\Omega - \Omega_0}{\Omega_1 - \Omega_0} \qquad (17)$$

For convenience we choose $(\Omega_0, \zeta_0, \eta_0) = (0, 0, 0)$ and let $(\Omega_1, \zeta_1, \eta_1)$ be the coordinates of the point A in Fig. 3. In this case the equation of the line through these two points can be written as:

$$\frac{\eta}{\dfrac{1}{1 + \epsilon}} = \frac{\zeta}{\dfrac{1}{1 + \gamma\epsilon}\sqrt{\dfrac{\sigma^2 - \gamma^2}{1 - \sigma^2}}} = \frac{\Omega}{\dfrac{1}{1 + \gamma\epsilon}\sqrt{\dfrac{1 - \gamma^2}{1 - \sigma^2}}} . \qquad (18)$$

We can rewrite the dispersion equation in the following form:

$$4\gamma\zeta^2 \sqrt{\frac{\Omega^2}{\zeta^2} - 1} \sqrt{\frac{\sigma^2\Omega^2}{\zeta^2} - 1} \cos \pi\eta(1 + \epsilon) + \zeta^2 \left(\gamma \sqrt{\frac{\Omega^2}{\zeta^2} - 1} - \sqrt{\frac{\sigma^2\Omega^2}{\zeta^2} - 1} \right)^2 x$$

$$\cos \pi \left(\sqrt{\Omega^2 - \zeta^2} - \epsilon \sqrt{\sigma^2\Omega^2 - \zeta^2} \right)^2 - \zeta^2 \left(\gamma \sqrt{\frac{\Omega^2}{\zeta^2} - 1} + \sqrt{\frac{\sigma^2\Omega^2}{\zeta^2} - 1} \right)^2 x$$

$$\cos \pi\zeta \left(\sqrt{\frac{\Omega^2}{\zeta^2} - 1} + \epsilon \sqrt{\frac{\sigma^2\Omega^2}{\zeta^2} - 1} \right) = 0. \tag{19}$$

From (18) we find for Ω/ζ

$$\frac{\Omega}{\zeta} = \sqrt{\frac{1 - \gamma^2}{1 - \sigma^2}} . \tag{20}$$

Substituting into (19) and dividing by ζ^2 gives, after rearranging,

$$4\gamma^2 \frac{1 - \sigma^2}{\sigma^2 - \gamma^2} \cos \pi\eta(1 + \epsilon) - 4\gamma^2 \frac{1 - \sigma^2}{\sigma^2 - \gamma^2} \cos \left\{ \pi\zeta(1 + \gamma\epsilon)\sqrt{\frac{1 - \sigma^2}{\sigma^2 - \gamma^2}} \right\} = 0 . \tag{21}$$

But from (18) we see that

$$\eta(1 + \epsilon) = (1 + \gamma\epsilon)\zeta\sqrt{\frac{1 - \sigma^2}{\sigma^2 - \gamma^2}} . \tag{22}$$

In this case the dispersion equation is satisfied identically for the line defined by (18).

The only remaining question concerns the relationship between the wave number components of the effective plane wave and the wave numbers ζ and η .

To clarify this point we consider two consecutive wave fronts defined by BIF and EDH (see Fig. 6). The straight lines BF and EH are the effective plane wave fronts. Using some definitions from wave mechanics and trigonometry, we will now derive the relation between k_x and k_y (the effective wave numbers in the horizontal and vertical direction, respectively) and the wave number ζ and η . We note first that

$$\frac{k_x}{k_y} = \frac{EF}{FH} = \frac{2(h + h')}{2(h \cot \beta + h' \cot \beta')} \quad . \tag{23}$$

This can be rewritten as

$$\frac{k_x}{k_y} = \frac{1 + \epsilon}{\cot \beta' \left(\dfrac{z'}{z} \dfrac{c'_p}{c_p} + \epsilon \right)} \tag{24}$$

where $z = (\rho\mu)^{\frac{1}{2}}$, $z' = (\rho'\mu')^{\frac{1}{2}}$ are the impedances, and $c_p = (\mu/\rho)^{\frac{1}{2}}$ $c'_p = (\mu'/\rho')^{\frac{1}{2}}$ are the phase velocities. The expression in parentheses can be rewritten as

$$\frac{z'}{z} \frac{c'_p}{c_p} + \epsilon = \frac{1}{\gamma} + \epsilon \quad . \tag{25}$$

Substituting (25) into (24) gives

$$\frac{k_x}{k_y} = \frac{1 + \varepsilon}{\cot \beta' \left(\frac{1}{\gamma} + \varepsilon\right)} = \frac{\gamma(1 + \varepsilon) \tan \beta'}{(1 + \gamma\varepsilon)} \ . \tag{26}$$

Using Snell's Law we can obtain, after some manipulation

$$\tan \beta' = \left(\frac{\frac{z'^2}{z^2} - 1}{1 - \frac{c_p^2}{c_p'^2}} \right)^{\frac{1}{2}} \ . \tag{27}$$

This can be rewritten as

$$\tan \beta' = \left(\frac{\frac{\sigma^2}{\gamma^2} - 1}{1 - \sigma^2} \right)^{\frac{1}{2}} \ . \tag{28}$$

Thus we obtain after substituting into (26) and rearranging

$$\frac{k_x}{k_y} = \frac{(1 + \varepsilon) \left(\frac{\sigma^2 - \gamma^2}{1 - \sigma^2} \right)^{\frac{1}{2}}}{1 + \gamma\varepsilon} \ . \tag{29}$$

If we now return to equation (18), we note that the expression for ζ/η is identical to equation (29). Thus finally we have

$$\frac{k_x}{k_y} = \frac{\zeta}{\eta} \tag{30}$$

CONCLUSION

What we have shown here is that a singular propagation direction exists for SH waves in an infinite laminated composite along which no dispersion takes place. In addition, a simple wave front representation for this non-dispersive propagation provides a clear illustration of the manner in which the ratio, ζ/η , characterizes the propagation direction. This should clear up some confusion which has existed in the past concerning the sense in which ζ and η are related to vector components of plane waves. It should also be pointed out that this reflection-free phenomenon is possible only for wave systems which can be described by a single potential, e.g. SH waves. A similar situation is not possible for waves in plane strain.

ACKNOWLEDGMENT

REFERENCES

1. Delph, T.J., Herrmann, G. and Kaul, R.K., "Harmonic Wave Propagation in a Periodically Layered, Infinite Elastic Body: Antiplane Strain", Journal of Applied Mechanics, Vol. 45, 1978, pp. 343-349.

2. Eringen, A.C. and Suhubi, E.S., Elastodyanmics, Volume II, Linear Theory.

3. Lee, E.H., "Wave Propagation in Composites with Periodic Structure", Proceedings of the Fifth Canadian Congress of Applied Mechanics, University of New Brunswick, Fredericton, N.B. Canada 1975, pp. 49-59.

4. Ince, E.I., Ordinary Differential Equations, Dover Publications, New York 1956, pp. 381-384.

5. Sve, C., "Time Harmonic Waves Travelling Obliquely in a Periodically Laminated Medium", Journal of Applied Mechanics, Vol. 38, 1971, pp·477-482.

FIGURE CAPTIONS

Fig. 1a   SH wave impinging upon interface

Fig. 1b   SH wave impinging at Brewster angle

Fig. 2   Geometry of layered composite

Fig. 3   Dispersion surface for laminated medium

Fig. 4a   Wave vector representation

Fig. 4b   Effective wave vector

Fig. 5a   Wave front representation

Fig. 5b   Effective wave front
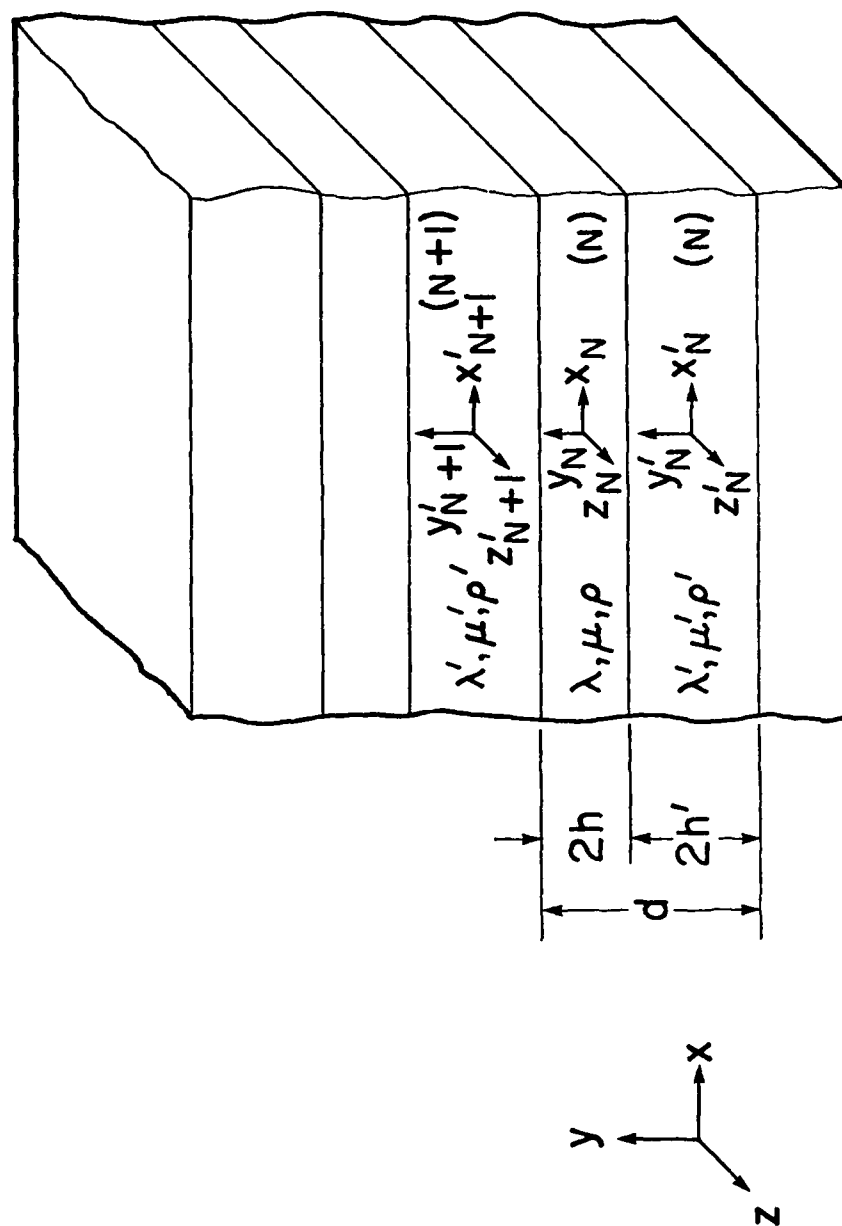
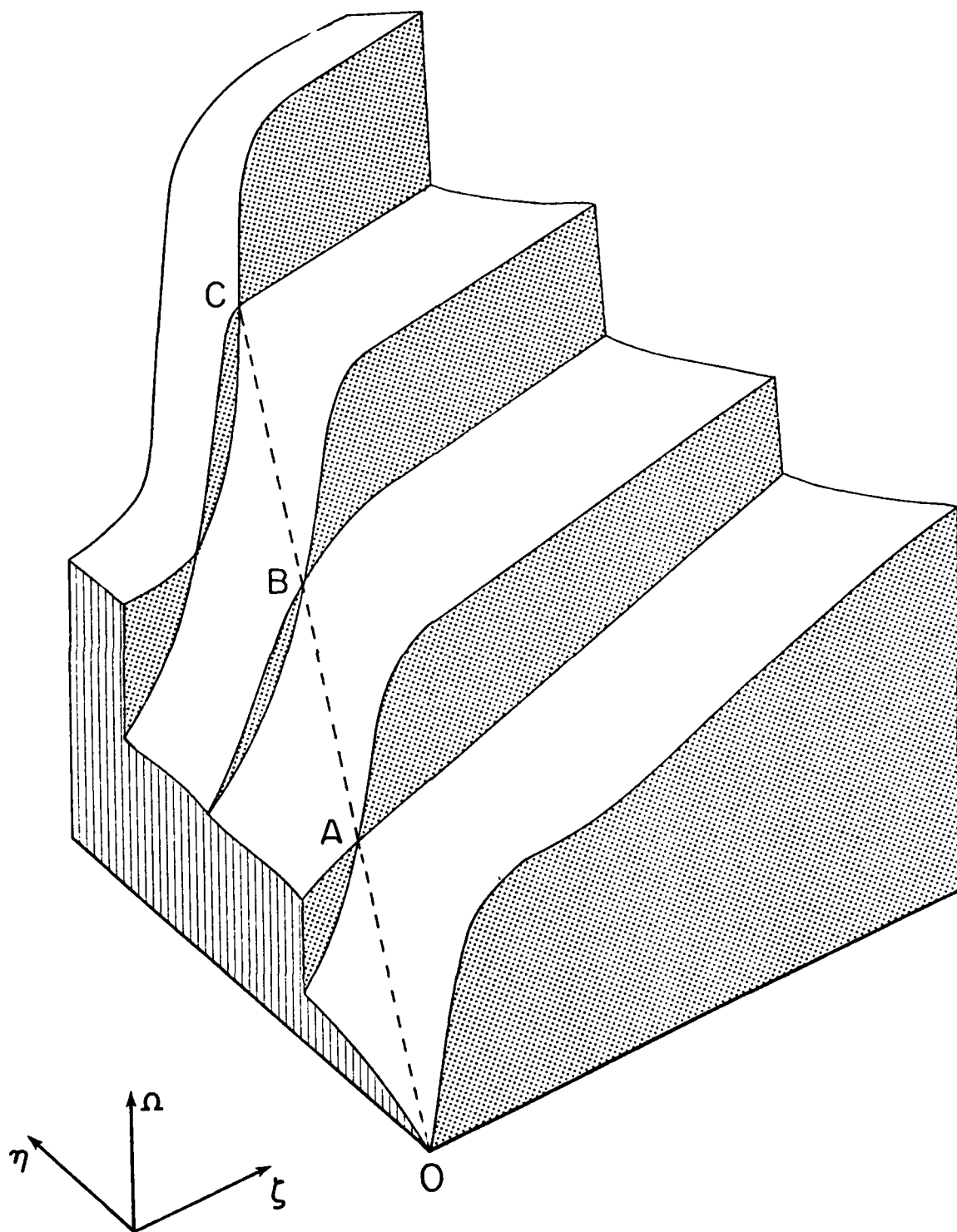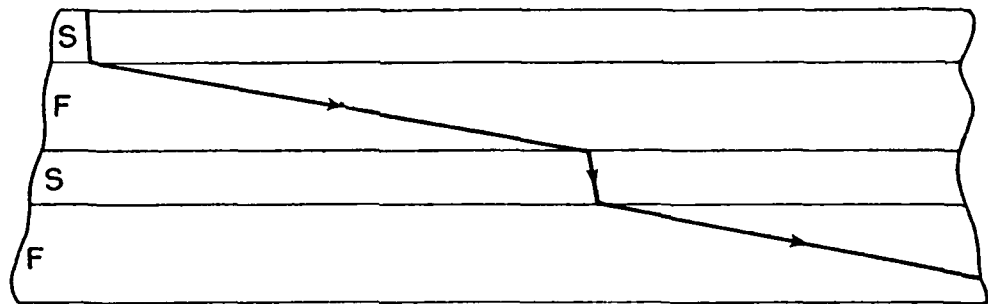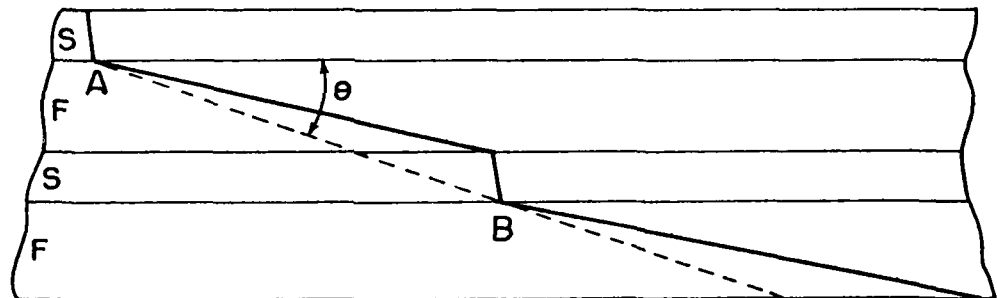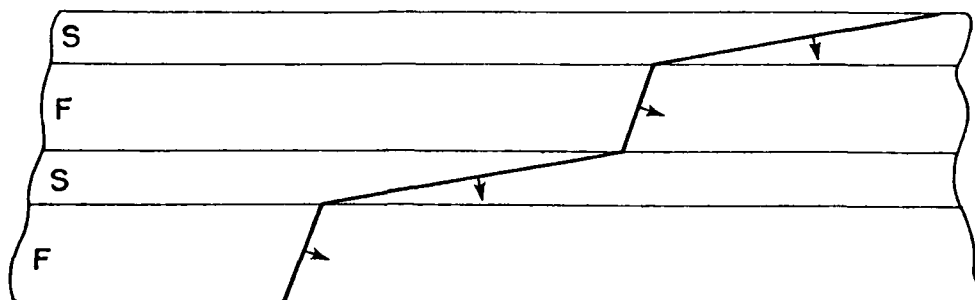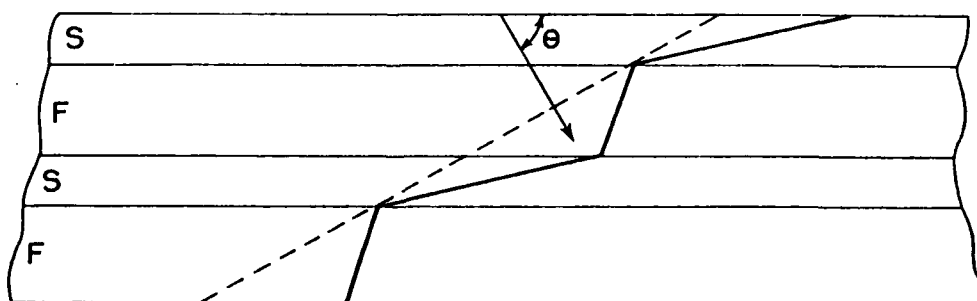Fig. 6   Detail of wave front
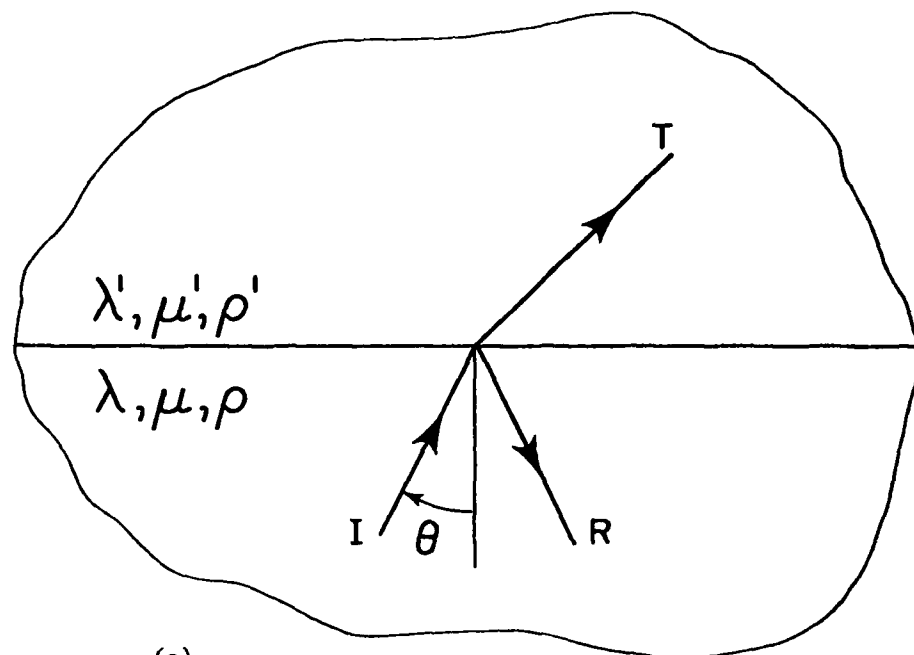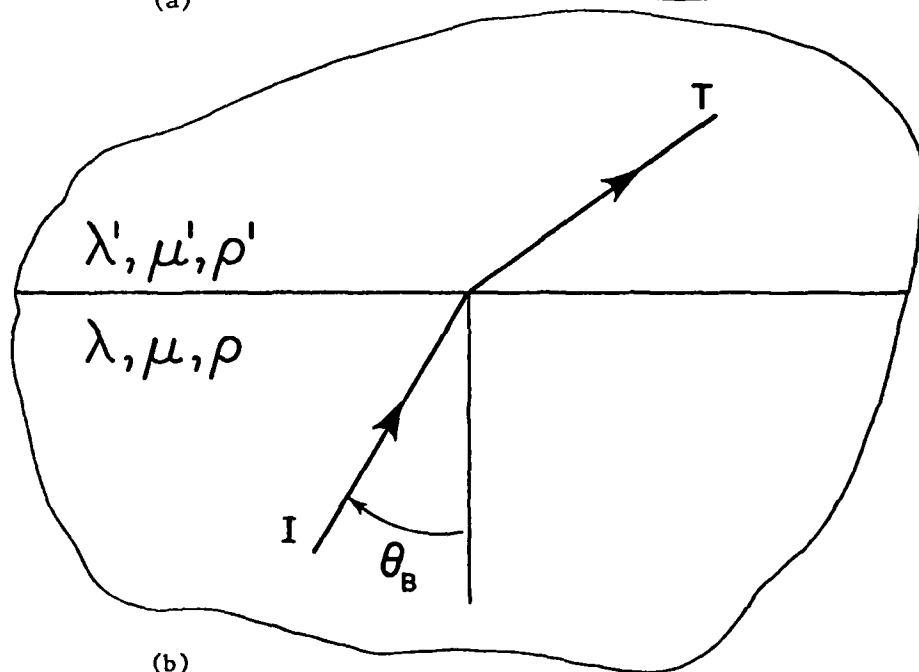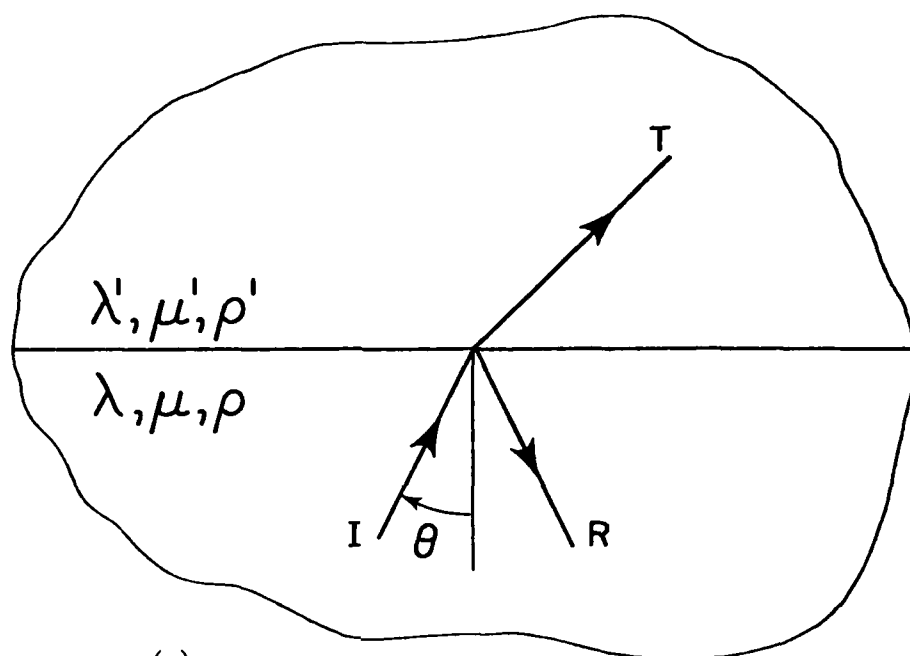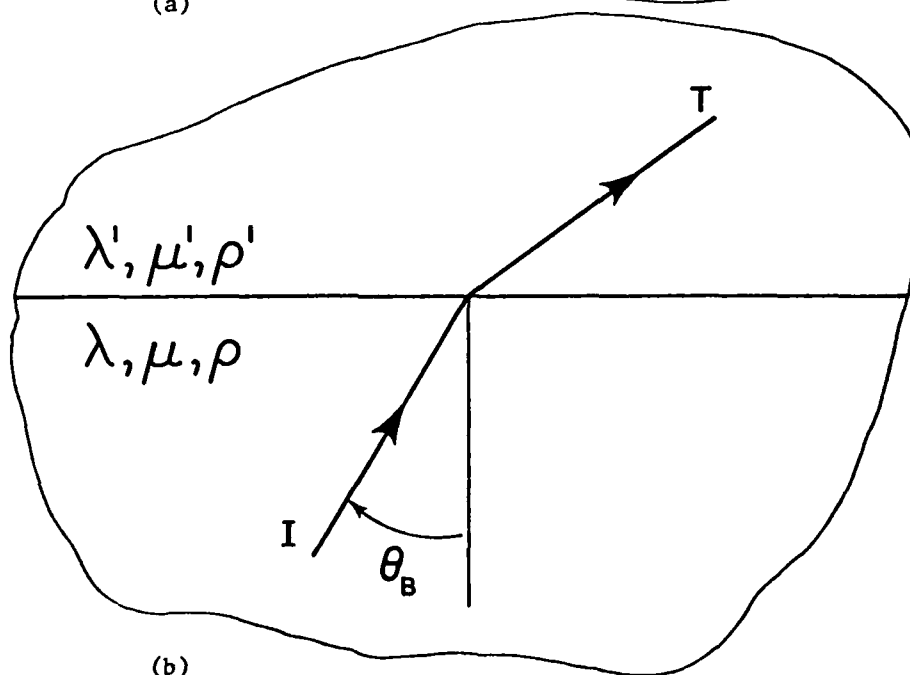
Figure 1

Figure 1

Figure 2

Figure 3

(a)

(b)

Figure 4

(a)

(b)

Figure 5

Figure 6

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