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MODEL PROPELLERS INTENDED FOR CAVITATION TESTING.

(10) B. R./Parkin

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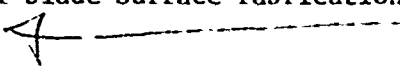
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NOMENCLATURE

c	profile chord length
C_p	pressure coefficient, $(p - p_o) / \frac{1}{2} \rho v_o^2$
$C_p _{\min}$	minimum pressure coefficient
C_{p_s}	pressure coefficient on blade surface
C_l	section lift coefficient
C_{l_i}	section lift coefficient at ideal attack angle (design lift coefficient)
E	relative error in cavitation number caused by blade surface asperity, $(\sigma_{cR} - \sigma_{cs}) / \sigma_{cs}$
f_M	maximum camber of blade profile
h	maximum camber of profile or maximum height of surface asperity
k_c	camber correction factor
Re	Reynolds number based c blade section chord
v_o	free stream velocity
v_*	skin friction velocity, $\sqrt{\tau_o / \rho}$
x	distance along chord line
y	distance normal to chord line
α	angle of attack measured with respect to the chord line
α_i	ideal or "design" angle of attack
$\bar{\beta}$	mean relative inflow angle measured with respect to propeller plane
$\pm \Delta \beta$	variations in propeller inflow angle caused by hull-induced inflow distortions
δ	boundary layer thickness
δ_1	displacement thickness

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δ_2	momentum thickness
δ_ℓ	viscous sublayer thickness
ϵ	dimensionless bump height h/ℓ
$\eta(x)$	ordinate of bump
ν	kinematic viscosity
ρ	fluid density
ℓ	length of limb
σ	cavitation number, $(p_o - p_v) / \frac{1}{2} \rho v_c^2$
σ_{cs}	inception cavitation number on surface of a smooth blade
σ_{cR}	incipient cavitation number on roughened body or on body with asperity
σ_{fp}	cavitation number of an asperity on a flat plate

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INTRODUCTION

Experience with model propeller tests in cavitation tunnels and towing tanks seems to suggest that quality control requirements with respect to blade-surface contours and roughness may be more severe for cavitation onset measurements than is the case for overall propulsive performance evaluation. Therefore, the 16th ITTC Cavitation Committee has been assigned the task of determining whether or not sufficient basic data are now available to enable the formulation of quality control recommendations which member organizations can use when fabricating propeller models for cavitation testing. This report documents one aspect of a more comprehensive review of the subject carried out by the committee upon the effects of blade surface roughness and contour irregularities on cavitation inception.

The committee had agreed that the present study should be carried out for a propeller such as that of the "Sydney Express" for which observations of cavitation have been made on the full scale propeller at sea and at model scale in various laboratories. However it was found that the propeller fitted to the "Sydney Express" has blades with sharp leading edges. This geometry can produce laminar separation or laminar bubbles on the blade leading edges. Since analysis methods are not generally available for such noncavitating flows it was decided to use a different example for the present calculations. Accordingly, the skewed propeller fitted to one of the San Clemente OBO class merchant ships was selected.

The design parameters for this propeller are documented in DTNSRDC report P-500-H-04 [1, 2]*. Only a few of the characteristics given in this

*Numbers in brackets indicate references cited below.

source are required for the present calculations. Moreover, Reference [1] summarizes towing tank results on the inflow velocities in the plane of the propeller at the stern of the hull. Therefore estimates for the relative inflow velocity and blade element attack angles are also available. These data provide the basic ingredients needed for the present work in which a blade element at the .7 radius point is taken to be representative of the hydrodynamics of the propeller. Only blade surface cavitation is considered.

PRELIMINARY CALCULATIONS

The full scale OBO propeller has a radius of 13 ft. The blade chord at .7 radius is 6.557 ft. The towing tank model has a radius of 4.333 in. and at .7 radius the blade chord is 2.186 in. The factor between full scale and model is 36.

Reference [1] gives the average relative inflow angle $\bar{\beta}_1$ as 11.72° at the .7 radius station. If we assume a ship speed of 17 knots (28.39 fps) full scale and 3 knots (≈ 5 fps) for towing tank tests we can use this information to estimate the Reynolds number range for the present calculations. If we base the Reynolds number on blade element chord length and relative inflow speed we find for the model that $Re \approx 3 \times 10^5$. For the full scale propeller we find $Re \approx 9 \times 10^7$. For the full scale ship the speed of 17 knots is close to its maximum speed. Therefore we should consider a range of Reynolds numbers in order to allow for lesser speeds. In the case of the model we need to consider a range of Reynolds numbers which will permit one to consider towing tank tests and at higher speeds cavitation tunnel tests. As a result of these considerations we have taken the following range of Reynolds numbers for the present work.

Re for model tests

$$3 \times 10^5$$

$$10^6$$

$$3 \times 10^6$$

Re for full scale

$$10^7$$

$$3 \times 10^7$$

$$10^8$$

Turning now to blade element geometry at the .7 radius point, we note that a NACA series 6 camber line with $a = 0.8$ was combined with a modified thickness distribution [3] in order to obtain a typical blade section. The modified profile thickness distribution is given in terms of percent chord as follows.

<u>x/c</u>	<u>y/c</u>	<u>x/c</u>	<u>y/c</u>	<u>x/c</u>	<u>y/c</u>
0	0	20	3.9957	65	4.3992
.5	.6648	25	4.3629	70	4.0545
.75	.8120	30	4.6382	75	3.6303
1.25	1.0436	35	4.8325	80	3.1265
2.5	1.4663	40	4.9527	85	2.5437
5.	2.0655	45	5.000	90	1.8828
7.5	2.5253	50	4.9635	95	1.1453
10.	2.9073	55	4.8525	97.5	0.7485
15.	3.5213	60	4.6649	100.	0.3333

According to Abbott and von Doenhoff [4], the equation of the mean-line for series 6 profiles is

$$\frac{y}{c} = \frac{C_{li}}{2\pi(1+a)} \left\{ \frac{1}{1-a} \left[\frac{1}{2} \left(a - \frac{x}{c} \right)^2 \ln \left| a - \frac{x}{c} \right| - \frac{1}{2} \left(1 - \frac{x}{c} \right)^2 \ln \left(1 - \frac{x}{c} \right) \right. \right. \\ \left. \left. + \frac{1}{4} \left(1 - \frac{x}{c} \right)^2 - \frac{1}{4} \left(a - \frac{x}{c} \right)^2 \right] - \frac{x}{c} \ln \frac{x}{c} + g - h \frac{x}{c} \right\} ,$$

where

$$g = -\frac{1}{1-a} \left[a^2 \left(\frac{1}{2} \ln a - \frac{1}{4} \right) + \frac{1}{4} \right] ,$$

$$h = \frac{1}{1-a} \left[\frac{1}{2}(1-a)^2 \ln (1-a) - \frac{1}{4}(1-a)^2 \right] + g .$$

The ideal attack angle is

$$\alpha_i = \frac{C_{\ell_i} h}{2\pi(a+1)} ,$$

where h is the maximum camber of the blade section as a fraction of its chord. In the present calculations we put $a = .8$ and eventually combined the mean line with the above thickness distribution in order to obtain the profile ordinates. However before this could be done it was necessary to convert the .7 radius profile for the OBO propeller into an equivalent two-dimensional section by removing the induction effects which were necessary because of lifting surface considerations in the propeller design.

On page 402, Abbott and von Doenhoff, the data listed for the mean line having $a = 0.8$ show that if $C_{\ell_i} = 1.0$, $\alpha_i = 1.54^\circ$ and the camber is

$$\frac{f_M}{c} \Big|_{2-D} = .0679 .$$

From Table 10 on page 38 of Reference [1] we find at .7 radius that

$$\frac{f_M|_{\text{due to loading}}}{f_M|_{2-D}} = k_c = 2.025 .$$

at Table 7 of the same report shows that $\left. \frac{f_M}{c} \right|$ due to loading is .0323 at .7 radius. Therefore,

$$\left. \frac{f_M}{c} \right|_{2D} = \frac{.0323}{2.025} = .01595$$

moreover, $\left. \frac{f_M}{c} \right|_{2-D} = .0679 C_{\ell_i}$ from which it follows that

$$C_{\ell_i} = \frac{.01595}{.0679} = .2349$$

and

$$\alpha_i = 1.54 \times .2349 = .362^\circ$$

Continuing with data from Reference [1] we find from Table 16 that at the .7 radius station the variations of inflow angle from towing tank tests are

$$+\Delta\beta = 5.08^\circ$$

and

$$-\Delta\beta = -5.69^\circ$$

These values of $\Delta\beta$ provide for the distorted inflow caused by the ship's wake. These values can be summed with $\bar{\beta}$ in order to obtain the maximum and minimum values of attack angle for the two dimensional profile. As a result, in the following calculations we consider three attack angles:

$$\alpha|_{\max} = 6.05^\circ, \alpha_i = 0.362^\circ, \text{ and } \alpha|_{\min} = -4.72^\circ.$$

Having determined C_{ℓ_i} as noted above, we can then determine the profile shape by adding the thickness distribution to the camber line. A plot of the resulting profile is shown in Figure 1. Once the profile ordinates have been defined, one can find the pressure distribution around the profile in accordance with potential theory. Three pressure distributions were obtained using a Douglas-Neumann program for the three values of α determined above. The resulting pressure distributions are plotted in Figure 2 below.

Once the pressure distributions are determined one can carry out boundary layer calculations. In the course of this study we considered a number of possibilities for executing such calculations. We have chosen Truckenbrodt's revised method [5] because of its simplicity. Basic steps in this calculation and the final results are given in Appendix A of this report. We need only mention here that such calculations were carried out for upper and lower surfaces of the profile of Figure 1 for the three attack angles called out in Figure 2 and for all six of the Reynolds numbers listed above. In all, thirty-six separate calculations were carried out. For those Reynolds numbers appropriate to model tests both laminar and turbulent boundary layers are expected to be present and a very simple criterion* was employed to fix the transition point. For the higher Reynolds numbers of the full scale propeller the boundary layer is predominantly turbulent. It should be noted that high precision is not the aim of these calculations. Our goal is to provide a straightforward engineering analysis which might lead to useful quality control requirements.

* Appendix A

EFFECTS OF ROUGHNESS

Having dispensed with the necessary preliminaries, we are free to consider the matter of limits on roughness. The basic reference for all that follows is contained in the J.S.R. (1979) paper by Arndt, Holl, Bohn, and Bechtel [6]. Figure 12 of this paper shows that an isolated two-dimensional triangular roughness element will produce a much stronger effect on cavitation onset than do isolated cones or triangular irregularities. Moreover, from the data of Figure 7 of the paper it appears that the triangular 2-D element produces a stronger effect than the circular-arc 2-D element. Therefore we are justified in considering the triangular element as the dominant case to consider because criteria based upon it will certainly contain other forms of roughness. We shall investigate the maximum roughness height which will permit no more than a relative effect, E , in increasing the onset of cavitation. In order to do this we can proceed as follows.

In this study we will neglect all cavitation scale effects and put the critical cavitation number for cavitation onset on a smooth body equal to the magnitude of $C_{p|_{\min}}$. Thus

$$\sigma_{cs} = -C_{p|_{\min}}.$$

From the plot of Figure 2, page 25, below we see that at the three values of α we have

α	σ_{cs}	x/c
6.05°	2.77	0.01
.362°	0.35	0.45
-4.72°	2.83	0.01

The critical case is the third of these because the boundary layer will be thin and the critical cavitation number is large. However we will consider all three cases in order to compare the outcome at these three attack angles.

Next we can turn to the results summarized in Figure 7 of Arndt et al. [6]. For the triangular 2-D element we find

$$\sigma_{fp} = .152 \left(\frac{h}{\delta} \right)^{.361} \left(\frac{U\delta}{\nu} \right)^{.196},$$

where σ_{fp} is the onset cavitation number for a protuberance on a flat plate having no pressure gradient, h is the height of the two-dimensional element, δ is the boundary layer thickness, U is the velocity at the edge of the boundary layer and ν is the kinematic viscosity. Suppose the flat plate has the same length as the profile chord c . Then the preceding equation can be expressed as

$$\sigma_{fp} = .152 (Re)^{.196} \left(\frac{h}{c} \right)^{.361} \left(\frac{\delta}{c} \right)^{.165}.$$

Next we can introduce the superposition equation,

$$\sigma_{cR} = -C_{p_s} + (1 - C_{p_s}) \sigma_{fp},$$

from Equation (6) of Arndt et.al [6]. Now let us define the relative effect of roughness by

$$E = \frac{\sigma_{cR} - \sigma_{cs}}{\sigma_{cs}},$$

where σ_{cR} is the onset cavitation number on the roughened body, σ_{cs} is the same quantity on the smooth body, and C_{ps} is the pressure coefficient on the smooth body. Recalling our definition for E given above we can write

$$E = .152 \frac{(1-C_{ps})(Re)^{.196}(h/c)^{.361}}{(-C_{ps})(\delta/c)^{.165}}$$

Solving for h/c we get

$$\frac{h}{c} = \frac{E^{2.77}(\delta/c)^{.457}}{.0054(Re)^{.543}} \left(\frac{1-C_{ps}}{-C_{ps}} \right)^{2.77} \quad (A)$$

In the calculations to follow we will consider two values of E: 5 and 10%. For these two values we have $E^{2.77}/.0054 = .046$ and $.313$ respectively. The ratio of permissible h/c values in these cases is 6.8. The following table gives calculated values of h/c for these two values of E.

E	α°	x/c	-Re-					
			3×10^5	10^6	3×10^6	10^7	3×10^7	10^8
5%	6.05	0.01	4.36×10^{-6}	1.72×10^{-6}	7.42×10^{-7}	2.91×10^{-7}	1.25×10^{-7}	4.94×10^{-8}
	0.362	0.45	1.69×10^{-5}	9.84×10^{-6}	6.53×10^{-6}	3.22×10^{-6}	1.61×10^{-6}	8.10×10^{-7}
	-4.72	0.01	7.01×10^{-6}	5.42×10^{-6}	5.69×10^{-7}	1.75×10^{-7}	7.48×10^{-8}	3.0×10^{-8}
10%	6.05	0.01	3.19×10^{-5}	1.17×10^{-5}	5.04×10^{-6}	1.98×10^{-6}	8.48×10^{-7}	3.36×10^{-7}
	0.362	0.45	1.15×10^{-4}	6.69×10^{-5}	4.44×10^{-5}	2.19×10^{-5}	1.09×10^{-5}	5.51×10^{-6}
	-4.72	0.01	4.81×10^{-5}	3.68×10^{-5}	3.87×10^{-6}	1.18×10^{-6}	5.08×10^{-7}	2.04×10^{-7}
			model			full scale		

The values tabulated above must be supplemented by additional calculations for the minimum value of C_p on the lower surface at $\alpha = 0.362$, not because it is the minimum C_p at this value of α but because the value of x/c is 3.0% and although $C_p = -.159$ at this point the boundary layer will be very thin at this position.

E	-Re-					
	3×10^5	10^6	3×10^6	10^7	3×10^7	10^8
5%	1.33×10^{-5}	6.97×10^{-6}	2.99×10^{-6}	1.18×10^{-6}	1.11×10^{-6}	5.37×10^{-7}
10%	9.07×10^{-4}	4.74×10^{-5}	2.03×10^{-5}	8.02×10^{-6}	7.55×10^{-6}	3.65×10^{-6}
	model			full scale		

These new values can be compared with appropriate entries in the table above and we find that they are all larger than those associated with the minimum pressure coefficient. Therefore, it appears for the present three cases at least that all critical roughnesses are those calculated first.

On the otherhand it must be recognized that the data which underlie Equation (A) are obtained from roughness elements which were not smaller than 40 microns and for values of h/δ no smaller than .0136. This value of h/δ can be compared with the displacement and momentum thicknesses. For a laminar boundary layer on a flat plate we have

$$\frac{\delta_1}{\delta} = .344 \text{ and } \frac{\delta_2}{\delta} = .133$$

Data in Table I of Reference [6] for triangular elements show height ratios as small as

$$\frac{h}{\delta} = .0136 \quad .$$

Since the experiments were carried out with turbulent boundary layers, it is of interest to compare this smallest height with the viscous sublayer height, δ_ℓ . We can estimate this quantity by using (see Schlichting, op cit.)

$$\delta_\ell = \frac{5\nu}{v_*} \quad ,$$

where $v_* = \sqrt{\tau_o/\rho}$. Assuming a 1/7 power law profile and allowing for variations in pressure coefficient along a body, we would have for the shearing stress at the wall

$$\frac{\tau_o}{\rho} = U_o^2 (1 - C_p) (.0296) (Re \frac{x}{c})^{-1/5} \quad .$$

After some manipulation we get

$$\frac{\delta_\ell}{c} = 29 \frac{(x/c)^{1/10}}{Re^{9/10} \sqrt{1-C_p}} \quad .$$

If we put $C_p = 0$ for a flat plate and change 29 to 100, neglect $(x/c)^{1/10}$ and replace the $Re^{9/10}$ with $Re = U_o c/\nu$ we get the admissible roughness formula in Schlichting (page 660-661). It appears therefore that admissible roughness can be as large as four times the viscous sublayer height.

For the 1/7 power law profile we have

$$\frac{\delta}{c} = .37 \frac{(x/c)^{4/5}}{Re^{1/5}},$$

and it follows that

$$\frac{\delta_l}{\delta} = \frac{78}{(x/c Re)^{7/10}},$$

where we have retained the flat plate values. For a Reynolds number of 10^6 and $x/c = 1$ we get

$$\frac{\delta_l}{\delta} = .005$$

This value is probably the largest of the viscous sublayer heights encountered by Arndt et al., but even so it is significantly smaller than their smallest h/δ noted above. If we were to multiply the ratio δ_l/δ by 4 we find that h/δ is in the range of admissible roughness, as can be seen from data in Table 12.3 of Schlichting. Since the present analysis appears to show a possible effect on cavitation of roughness elements which are smaller than those of the experiments it would be of great interest to extend these experiments to even smaller roughness sizes. When one couples this finding with the fact in many situations encountered in the towing tank or water tunnel that laminar boundary layers are present, the extension of flat plate roughness data to include cavitation in laminar boundary layers would also be beneficial.

At this time it appears that our attempt, using 2-D triangular roughness, to define criteria for the onset of cavitation on marine propellers is hampered by a lack of data although the methods by which such rational standards might be developed seem straightforward.

As we have noted, the preceding calculations assume an isolated two-dimensional roughness of triangular shape because of its strong effect on cavitation inception. On the other hand if we were to go to the other extreme and consider distributed roughness only, we may be justified in using the usual criterion based on skin friction which defines admissible roughness. We base this assertion upon the correlation given in Figure 6 of Reference [6]. In this case we would have

$$\frac{h}{c} \sim \frac{100}{Re\sqrt{1-C_p(x/c)}} \quad (B)$$

for the admissible height of distributed roughness. In this case there would be no measurable effect on cavitation inception; that is, in terms of the above terminology $E = 0$. Clearly, the critical value of h would be found at $C_p|_{\min}$.

EFFECTS OF A CONTOUR IRREGULARITY

The formulation of a method by which one can examine the effects of variations in profile contour seem somewhat less demanding than the preceding considerations. One way is to use potential theory to calculate the pressure distribution of the deformed profile from which the cavitation performance can be reassessed. The chief problem with this approach is

that not all organizations have a computer program possessing the "numerical flexibility" which would enable them to handle small bumps or waves without difficulty. For those who do not have such a "flexible" program, we would propose the following approximate procedure.

The basic idea here is to use the superposition equation from Reference [6] as quoted above. For σ_{fp} one would simply use the magnitude of $C_p|_{min}$ of one of the bumps given in Appendix B of this report. See Figure 3 below. The conditions under which this procedure is valid are shown on Figure 2 of Reference [6]. The curves and the data given there show that if

$$h/\delta \geq 4 \quad ,$$

then the proposed procedure should work satisfactorily. Here h is the maximum height of the bump. Otherwise the methods which apply to smaller roughness elements as discussed above should be used.

In terms of the bumps and pressure distributions of Appendix B we may note that the parameter ϵ is given by

$$\epsilon = h/\ell \quad ,$$

where ℓ is the length of the bump (taken as unity in those calculations) and h is the actual height of the bump. Therefore we can write the criterion of applicability as

$$\frac{\ell}{\delta} \epsilon \geq 4 \quad .$$

In those cases where the observed bump is not well represented by either of those given in Appendix B one can develop the measured contour deviation about

a straight line. Then he can superpose bumps having various values of ϵ along this line until the observed deviation from the line is fairly well represented. After that he can superpose the pressure distributions from all bumps. Values of x (or x/l), η/ϵ and C_p/ϵ tabulated in Appendix E could be useful in this phase of the calculations. The σ_{fp} will be equal to the magnitude of $C_p|_{\min}$ obtained from the composite pressure distribution.

As a simple illustration, let us suppose that only one bump is needed to characterize at least approximately a typical deformation. If we settle on Case #2 from Appendix B we see that

$$C_p|_{\min} = -.68\epsilon$$

Then since

$$E = \left(\frac{1-C_{ps}}{-C_{ps}} \right) \sigma_{fp}$$

we have

$$E = .68 \left(\frac{1-C_{ps}}{-C_{ps}} \right) \epsilon$$

Now if we put $E = .1$ we have

$$\epsilon = .147 \left(\frac{-C_{ps}}{1-C_{ps}} \right)$$

Finally we can use the "criterion of applicability" from above and write

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$$\frac{\delta}{\ell} \leq .0368 \left[\frac{-C_{p_s}}{1-C_{p_s}} \right] .$$

and we also see that it leads to the alternate expression

$$\epsilon \leq .147 \left[\frac{-C_{p_s}}{1-C_{p_s}} \right] .$$

As an application of these results suppose that we consider the critical case from the plot of Figure 2 which is found at $\alpha = -4.72$.

$C_p|_{\min}$ for this case is -2.84 and we find

$$\epsilon \leq .109$$

$$\frac{\delta}{\ell} \leq .0272$$

On the other hand we find from Appendix A that

$$\frac{\delta}{c} = .00015 \text{ when } Re = 10^6 .$$

Hence it follows that

$$\frac{\ell}{c} > .005 .$$

These data, together with the fact that the bump is given by

$$\eta(x) = \epsilon(1-x^2)^2 ,$$

where x is measured from the center of the bump and is an actual distance normalized by l and η is the normalized ordinate of the bump; serves to characterize a permissible deformation which will produce a relative error in cavitation number of about 10%. The effect of more complex bumps can be explored by use of the composite pressure distribution discussed above.

CONCLUSIONS

An investigation which makes use of the most complete information available on the effects of surface asperities with respect to cavitation inception, has sought to determine how these data might be used to formulate quality control criteria for cavitation inception on model propeller blades. The OBO propeller was used as an example. It was found that the dominant effect is caused by the two-dimensional isolated triangular element. In this case the data are not complete enough to permit the formulation of a roughness criterion. On the other hand, if one considers the effects of distributed roughness or single bumps which protrude well outside the boundary layer one can formulate methods for obtaining simple quality control criteria for these two cases.

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REFERENCES

1. Morgan, W. B., Valentine, D. T., Denny, S. B., Weaver, A. H., "Highly-Skewed Propeller Design for the San Clemente Class Ore-Bulk-Oil Carrier (OBO)," David Taylor Naval Ship Research and Development Center, Ship Performance Department Evaluation Report P-500-H-04, Bethesda, MD 20034, April 1973. See also:
2. Valentine, D. T., and Dashnaw, F. S., "Highly-Skewed Propeller for San Clemente Class Ore/Bulk/Oil Carrier Design Considerations, Model and Full Scale Evaluations," Proc. First Ship Technology and Research (STAR) Symposium, Washington, DC, Society of Naval Architects and Marine Engineers, August 26-29, 1975.
3. Brockett, T., Private Communication, 1980.
4. Abbott, I. H., and von Doenhoff, A. E., Theory of Wing Sections, Dover Publications, Inc., New York, 1959.
5. Schlichting, H., Boundary Layer Theory, 7th Edition, McGraw-Hill Book Company, New York, 1979, page 672.
6. Arndt, R. E. A., Holl, J. W., Bohn, J. C., and Bechtel, W. T., "Influence of Surface Irregularities on Cavitation Performance," Journal of Ship Research, Vol. 23, No. 3, September 1979, pp. 157-170.

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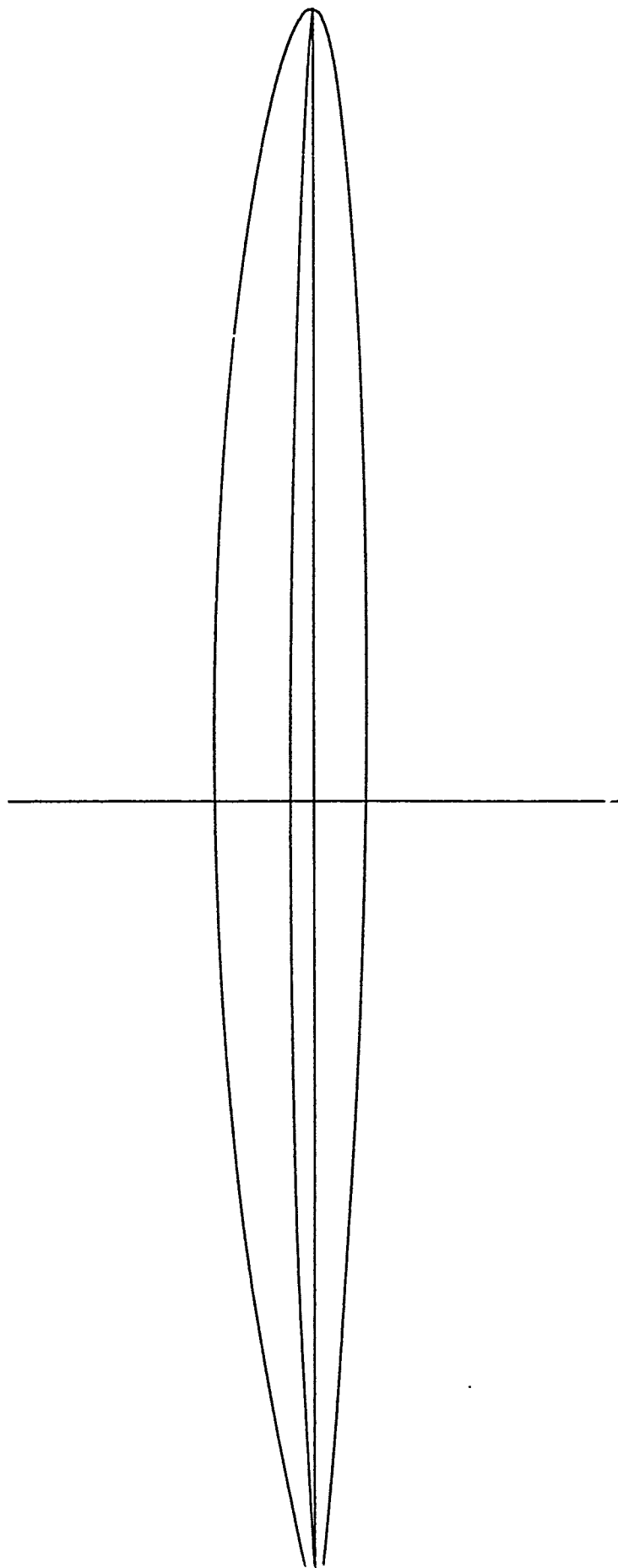


Figure 1. Modified NACA66 Blade Section

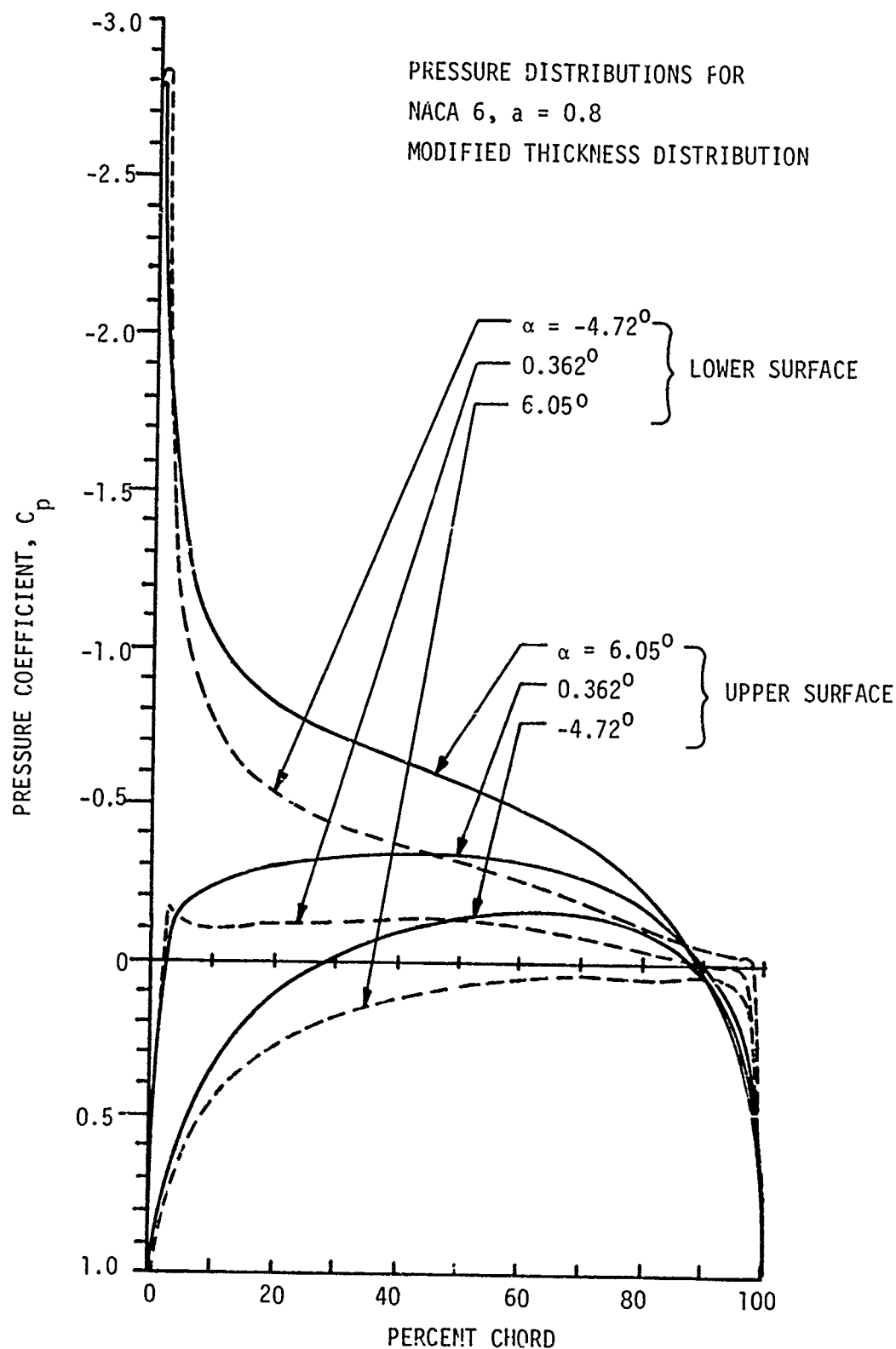


Figure 2. Calculated Pressure Distribution on Modified NACA 6 ($a = 0.8$) Profile of Figure 1

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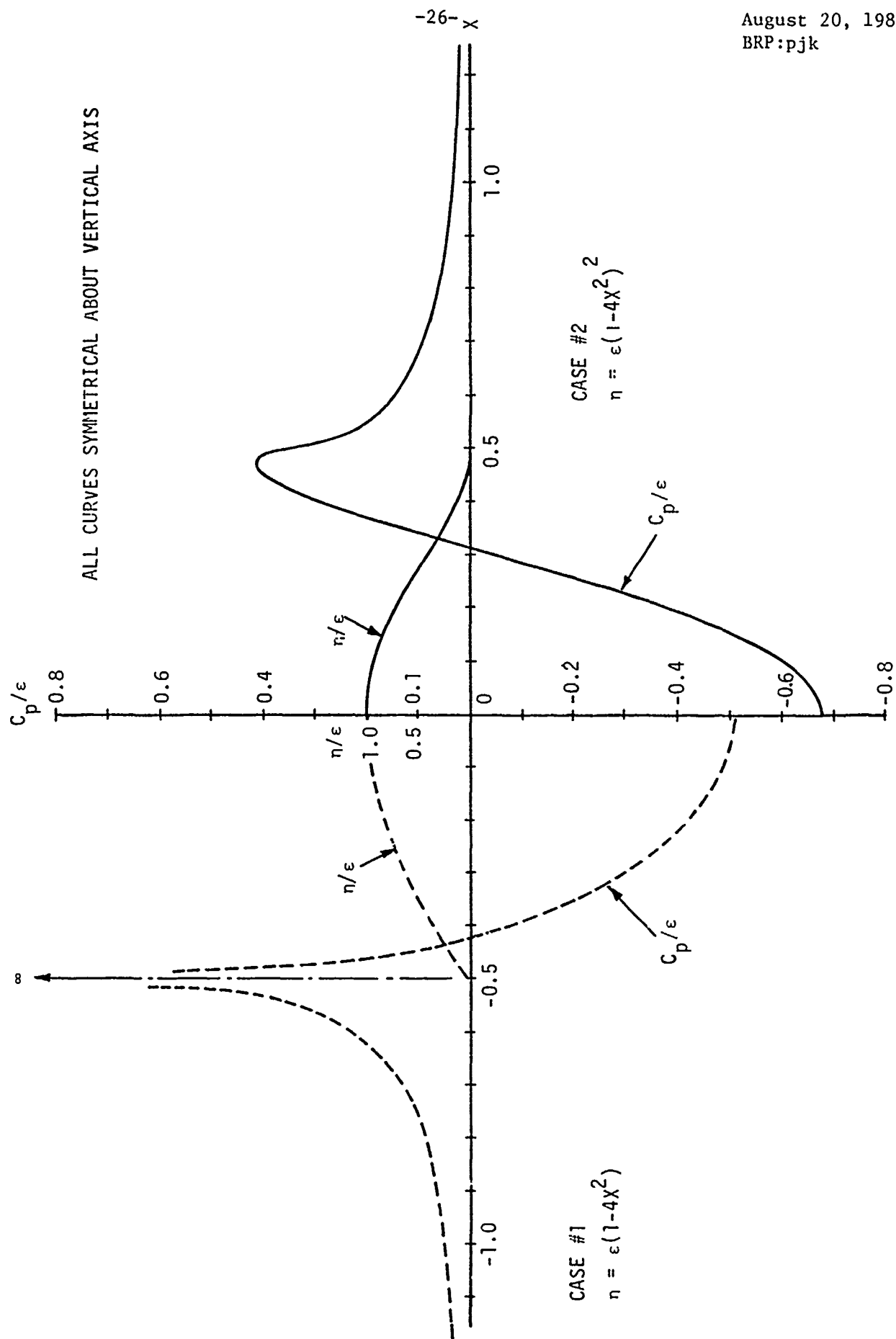


Figure 3. Pressure Distributions from Appendix B of Two Bumps on a Flat Plate.
Case #1: Linearized "Circular" Bump (Parabolic Contour). Case #2:
Bump with Cusp.

APPENDICIES

A: Boundary Layer Thickness

Truckenbrodt's Revised Integral Method.

See: Boundary-Layer Theory by H. Schlichting, 7th Edition, McGraw-Hill Book Company, New York, 1980, Chapter XXII.

Basic Equations Used: Let x be arc length. Then

$$R_3(x) = \frac{\delta_3 U(x)}{\nu} = \frac{\delta_3}{c} \frac{c U_o}{\nu} \cdot \frac{U(x)}{U_o} = \left(\frac{\delta_3}{c} \right) Re \sqrt{1-C_p(x)} \quad ,$$

where

$$\delta_3(x) = \text{energy thickness} = \int_0^{\delta} [1 - (u/U(x))^2] (u/U) dy \quad ,$$

ν = kinematic viscosity,

c = profile chord length,

$U(x)$ = velocity at edge of boundary layer,

$u(y)$ = velocity profile at any arc length, x ,

U_o = free stream velocity

Re = Reynolds number based on free stream velocity and chord length

$\delta(x)$ = boundary layer thickness at the arc length x

Laminar Boundary Layer:

If we assume that $u(y)$ can be approximated by the Blasius profile we get

$$\frac{\delta}{c} = 4.788 \frac{\delta_3}{c} \quad ,$$

as can be found from a simple Simpson's rule integration as shown below.

Calculations for the ratio δ/δ_s for the Laminar Boundary Layer.

Blasius Flat Plate Solution (see Schlichting p. 139, 7th Edition).

$$\delta_3 = \frac{\delta}{5} \int_0^{\xi} \overbrace{f' (1 - [f']^2)}^{f(\xi)} d\xi = \frac{\delta}{5} I$$

ξ	F	ξ	F
1. 0	0	22. 4.2	.06284
2. .2	.06612	23. 4.4	.04653
3. .4	.13043	24. 4.6	.03373
4. .6	.19107	25. 4.8	.02397
5. .8	.24616	26. 5.0	.01669
6. 1.0	.29392	27. 5.2	.01140
7. 1.2	.33272	28. 5.4	.00764
8. 1.4	.36128	29. 5.6	.00502
9. 1.6	.37876	30. 5.8	.00323
10. 1.8	.38489	31. 6.0	.00204
11. 2.0	.38000	32. 6.2	.00126
12. 2.2	.36505	33. 6.4	.00078
13. 2.4	.34158	34. 6.6	.00046
14. 2.6	.31154	35. 6.8	.00026
15. 2.8	.27708	36. 7.0	.00016
16. 3.0	.24045	37. 7.2	.00008
17. 3.2	.20366	38. 7.4	.00004
18. 3.4	.16846	39. 7.6	.00002
19. 3.6	.13616	40. 7.8	0
20. 3.8	.10756	41. 8.0	0
21. 4.0	.08311		

n = 40

h = 0.2

Using a TI 59 with NL 10 for Simpson's Rule we get $I = 1.04434$. Therefore

$$\delta = \frac{5\delta_3}{1.04434} = 4.78771\delta_3$$

The calculator results are shown on the next page.

-29-
ML 10 - Simpson's Rule

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40. INTERVALS
0.2 h

1.044338667 $\leftarrow I$

List Data registers.

0.	00
6.	01
0.	02
0.2	03
1.044338667	04
40.	05
0.	06
0.06612	07
0.13043	08
0.19107	09
0.24616	10
0.29392	11
0.33272	12
0.36128	13
0.37876	14
0.38489	15
0.38	16
0.36505	17
0.34158	18
0.31154	19
0.27708	20
0.24045	21
0.20366	22
0.16846	23
0.13516	24
0.10756	25
0.08311	26
0.06284	27
0.04653	28
0.03373	29
0.02397	30
0.01669	31
0.0114	32
0.00764	33
0.00502	34
0.00323	35
0.00204	36
0.00126	37
0.00078	38
0.00046	39
0.00026	40
0.00016	41
0.00008	42
0.00004	43
0.00002	44
0.	45
	46

Calculator results for the
determination of δ/δ_3

Then, following Schlichting, page 682, we can put $v'_\ell = .917v$ and

$$\frac{\delta}{c} = \frac{4.788}{\sqrt{Re}} \left[\frac{\int_0^{\xi_1} [1-C_p(\xi)]^{2.5} d\xi}{.917 [1-C_p(\xi_1)]^3} \right]^{1/2} = \frac{5}{\sqrt{Re}} \left[\frac{\int_0^{\xi_1} [1-C_p(\xi)]^{2.5} d\xi}{[1-C_p(\xi_1)]^3} \right]^{1/2} \quad (A-1)$$

It is worth observing that when $C_p = 0$ in the laminar boundary Equation (A-1) that we get

$$\frac{\delta}{c} = \frac{5}{\sqrt{Re}} \left(\frac{x}{c} \right)^{1/2}$$

which is the well known Blasius result. Moreover in Equation (A-1) the quantity ξ is given by

$$\xi = x/c \quad ,$$

and as is customary, at the forward stagnation point one puts $\delta = 0$.

Equation (A-1) is used to calculate the laminar boundary layer thickness from the nose of the profile to the transition point. For many purposes it is permissible to use a simple criterion in order to estimate the location of transition. One can suppose that transition may occur at an arc-length Reynolds number of

$$Re_t = 3 \times 10^5$$

or in the neighborhood of $C_{p_{min}}$, whichever occurs first. The criterion is probably most appropriate if the outer flow is highly turbulent as in the wake of a ship although its chief virtue is simplicity rather than accuracy.

Once the transition point x_1 has been selected one can calculate the laminar energy-layer Reynolds number,

$$R_3(x_1) = \frac{\delta(x_1)}{c} \frac{Re \sqrt{1-C_p(x_1)}}{4.788} , \quad (A-2)$$

for the transition point only. One assumes that the energy layer δ_3 is continuous at transition.

Turbulent Boundary Layer

Once the laminar boundary layer thickness distribution and the location of transition have been determined, calculations for the turbulent boundary layer can be made. The formula for the energy thickness Reynolds number is

$$R_3(x) = \left\{ [R_3(x_1)]^{1.152} + \frac{Re}{v'} \frac{\int_{\xi_1}^{\xi} [1-C_p(\xi)]^{1.65} d\xi}{[1-C_p(x)]^{1.15}} \right\}^{\frac{1}{1.152}} , \quad (A-3)$$

where $R_3(x_1)$ is given by Equation (A-2), $\xi = x/c$ as before and $\xi_1 = x_1/c$. Schlichting suggests that for v' one should use $v' = 80v$. The boundary layer thickness can be determined from the energy thickness if it is assumed that the velocity profile can be approximated by a one-seventh power law profile which is known to give rather good results for flat-plate skin friction at Reynolds numbers less than 10^7 . As a result one finds that

$$\delta = 40 \delta_3(x)/7 ,$$

from which it follows that

$$\frac{\delta(x)}{c} = \frac{5.714 R_3(x)}{Re \sqrt{1-C_p}(x)} \quad (A-4)$$

In the course of preliminary calculations based upon Equations (A-1) through (A-4) the recommended value of $v' = 80 v$ was used. Surprisingly, when laminar and turbulent boundary layer calculations were started from the nose of a profile it was found that the laminar boundary layer thickness was larger than the turbulent boundary layer thickness for a considerable range of arc lengths starting from the nose.

Therefore we conducted a short investigation to see if a better choice for the value of v' could be made. We considered a flat plate of length c at $Re = 10^6$. The laminar boundary layer thickness for this case can be found from the formula at the top of page 30. The corresponding turbulent boundary layer thickness for a one-seventh power law profile is known to be

$$\frac{\delta(x)}{c} = \frac{.37(x/c)^{4/5}}{Re^{1/5}},$$

and if one puts $C_p(\xi) = R_3(x_1) = 0$ in Equations (A-3) and (A-4) it follows that

$$\frac{\delta(x)}{c} = \frac{5.714 \left(\frac{v}{v'} \frac{x}{c} \right)^{.868}}{(Re)^{.132}}.$$

If one equates these two values of turbulent boundary layer thickness he finds that the ratio v/v' is

$$\frac{v}{v'} = \frac{.0427}{[(x/c)Re]^{.0452}} \quad (A-5)$$

Equation (A-5) can be used to explore various possible values for v' . For example we have used $Re = 10^6$ as noted above and we considered a few values of x/c as tabulated below.

<u>$Re = 10^6$</u>		Table showing the effect upon possible values of v' for three transition point locations on a flat plate.
$\frac{x}{c}$	$\frac{v'}{v}$	
.5	65	
.1	58	
.05	38	

Next, two likely values of v' were used to compare 1/7 power and Truckenbrodt boundary layer thicknesses directly at $Re = 10^6$ and $x/c = .08$ in which case $\delta/c|_{1/7} = .00309$.

$$\text{For } v' = 50v, \quad \left. \frac{\delta}{c} \right|_T = .00345$$

$$\text{For } v' = 55v, \quad \left. \frac{\delta}{c} \right|_T = .0032$$

As a result of these comparisons it seemed best to take

$$v' = 55v$$

for the present calculations. Nevertheless as a final illustration of the effect of changing from $v' = 55v$ to $v' = 80v$ on a typical profile is illustrated by the data on page 39 below, which are given for $Re = 3 \times 10^5$

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and $Re = 10^8$ when $v' = 80v$. These data can be compared with the corresponding results for $v' = 55v$ on pages 36 and 38 respectively.

Numerical Results and Calculator Program

Equations (A-1) through (A-4) were coded for execution on a TI-59 programmable calculator. For the record, we give a description of the procedure and other details of this phase of the calculations.

Data Registers

R_{00} - for DSZ index	R_{61} - blank
R_{01} - used, R_{02}^j and R_3 laminar	R_{62} - $[1-C_{p_j}], 1 \leq j \leq 51$
R_{03} - h in Simpson's rule	R_{63} - j
R_{04} - I , Simpson's rule result	R_{64} - k
R_{05} - $j_{\max}$ = total no of C_p entries	R_{65} - Re
R_{06} - $f_0, k = 6$ } R_{07} - $f_1, k = 7$ } R_{08} - $f_2, k = 8$ }	R_{66} - ΣI from R_{04}
	R_{67} - δ/c laminar
R_{09} - $j(x_1)$ = an odd integer denoting location of x_1 (transition)	R_{68} - $R_3(x)$ turbulent
	R_{69} - $f_k, 6 \leq k \leq 8$ where $f_k = [1-C_{p_k}]^{1.65}$
$R_{10} = C_p(0) - R_{60} = C_p(x_{\max})$, a total of 51 $C_p(x)$ entries can be used	
$\therefore 1 \leq j \leq 51$	

Preliminaries

- Partition memory $\boxed{7}$ $\boxed{2nd}$ $\boxed{Op}$ $\boxed{17}$, display 399.69. To check,
 $\boxed{2nd}$ $\boxed{Op}$ $\boxed{16}$.
- Enter program from magnetic card $\boxed{0}$ $\boxed{INV}$ $\boxed{2nd}$ $\boxed{WRITE}$ for both 1 and 2.

Procedure

- (a) enter data: $\overline{h}$ $\overline{ST0}$ $\overline{03}$ $\overline{Jmax}$ $\overline{ST0}$ $\overline{05}$ $\overline{j(x_1)}$ $\overline{ST0}$ $\overline{09}$
 $\overline{RE}$ $\overline{ST0}$ $\overline{65}$
- (b) enter Cp_j : $\overline{2nd}$ $\overline{Pgm}$ $\overline{10}$ $\overline{0}$ $\overline{C}$ $\overline{R/S}$ $\overline{0}$ $\overline{R/S}$ $\overline{0}$ $\overline{R/S}$ $\overline{0}$
 $\overline{R/S}$ + this moves pointer to R_{10} . $\overline{Cp_1}$ $\overline{R/S}$...
 $\overline{Cp_{jmax}}$ $\overline{R/S}$
- (c) Run program: $\overline{CLR}$ $\overline{RST}$ $\overline{R/S}$

—//—

Program Listing and Data Register contents given on pages 55 - 57. Note value of v' in Program Memory addresses 306 and 307. Specific boundary layer calculation results for OBO propeller are given on pages 36 - 54.

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300000. Re
0.04 x/c
.0009376418 s/c
0.08
.0019023488
0.12
.0024701031
0.16
.0029294425
0.2 *laminar boundary layer*
.0033101646
0.24
.0036683173
0.28
.004005352
0.32
.0043112649
0.36
.0046009608
0.4
.0048823734
0.44
.0051374603
0.48 x/c
.0054083593 s/c
394.0277908 Re
.0064542164 s/c
0.52
.0074032394
0.56
.0093435161
0.6 *turbulent boundary layer*
.0107853203
0.64
.0122513752
0.68
.0137749593

0.72
.0153751967
0.76
.0170827223
0.8
.01900231
0.84
.0215967958
0.88
.0243971632
0.92
.0293914488
0.96
.0360203913
1000000.
0.04
.0005135673
0.08
.0010419593
0.12
.0013529339
0.16
.0016045217
0.2
.0018130518
0.24
.0020092101
0.28
.0021940965
0.32
.0023613758
569.9173607
.002618071
0.36
.0041381976

0.4
.0053035476
0.44
.0065919314
0.48
.0077832053
0.52
.0089988211
0.56
.010215503
0.6
.0114606336
0.64
.0127500398
0.68
.0141143953
0.72
.015574235
0.76
.017157801
0.8
.0189917176
0.84
.0215633229
0.88
.0249739471
0.92
.028601172
0.96
.0369520417

Upper Surface
Design $d_s = 0.362$

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3000000.

0.04
.0003965052

0.08
.0006015755
413.6391705
.0007179303

0.12
.0013875458

0.16
.0031015664

0.2
.0041323305

0.24
.0051403602

0.28
.0061329708

0.32
.0070821016

0.36
.0080333702

0.4
.0089842869

0.44
.0089942447

0.48
.0108467404

0.52
0.011870836

0.56
0.012913222

0.6
.0140066658

0.64
.0151650424

0.72
.0177997613

0.76
.0193257107

0.8
0.021120936

0.84
0.013926819

0.88
.0274130114

0.92
.0324843673

0.96
.0405975963

10000000.

0.04
.0001624042
353.5666038
.0001938133

0.08
.0012434488

0.12
.0022990653

0.16
.0031877932

0.2
.0040229164

0.24
0048550071

0.28
.0058624513

0.32
.00684841817

0.36
.0078763417

0.4
.0080758409

0.44
.0083425472

0.48
.0096489643

0.52
.0105221947

0.56
.0114124465

0.6
.0123489776

0.64
.0133440408

0.68
.0144273451

0.72
.0156160003

0.76
.0159343275

0.8
.0165023544

0.84
.0204578199

0.88
.0239263079

0.92
.0264460041

0.96
.0355060476

Upper Surface
Design $\alpha_i = 2.342^\circ$

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300000000.

0.72
0.013646715

0.44
.0066354555

0.04
.0004724448

0.76
.0147935809

0.48
.0072290981

0.08
.0019375542

0.8
0.016158505

0.52
.0078734211

0.12
.0021766187

0.84
.0182126616

0.56
.0085307066

0.16
.0019285496

0.98
.0209524713

0.6
.0092228613

0.2
.0036380299

0.92
.0248369054

0.64
.0099589716

0.24
.0043494203

0.96
.0310581546

0.68
.0107612023

0.28
.0050593536

100000000.

0.72
.0116422569

0.32
.0057478721

0.04
.0004030512

0.76
.0126206687

0.36
.0064313292

0.08
.0011837473

0.8
.0137851099

0.4
.0071174302

0.12
.0018569124

0.84
.0155375476

0.44
.0077778881

0.16
.0024983981

0.88
.0178749283

0.48
.0084737386

0.2
.0031036684

0.92
.0211883086

0.52
.0092289953

0.24
.0037105683

0.96
.0264962676

0.56
.0099994463

0.28
.0043152251

0.6
.0108107699

0.32
.0049036126

0.64
0.011673617

0.36
0.005486682

0.68
.0126139685

0.4
.0060720071

Upper Surface
Design $\alpha_s = 0.342^\circ$

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300000.
0.04
.0009376413
0.08
.0019023488
0.12
.0024701081
0.16
.0029294425
0.2
.0033101646
0.24
.0036683173
0.28
0.004005852
0.32
.0043112699
0.36
.0046009608
0.4
.0048822734
0.44
.0051374603
0.48
.0054082563
394.0277938
.0064542164
0.52
.0074635519
0.56
.0084683804
0.6
.0094854955
0.64
.0105271771

$v' = 80v$, DESIGN $d_x = 0.362^\circ$ Upper Surface.

0.68
.0116146284
0.72
.0127634949
0.76
.0139946322
0.8
0.015385856
0.84
.0172822115
0.88
.0197017415
0.92
.0230016833
0.96
.0280911424
1000000000.
0.04
.0002911413
0.08
0.000855072
0.12
.0013413283
0.16
.0018047012
0.2
.0022419142
0.24
.0026803043
0.28
.0031177964
0.32
.0035420918

0.4
.0043860738
0.44
0.004793077
0.48
.0052218908
0.52
.0056873132
0.56
0.006162099
0.6
.0066620723
0.64
.0071937967
0.68
.0077732827
0.72
.0084097066
0.76
.0091164558
0.8
.0099575821
0.84
.0112234439
0.88
.0129118352
0.92
.0153055945
0.96
.0191394021

$v' = 80v$ DESIGN $d_x = 0.362^\circ$ Upper Surface.

Illustrating the effect of changing $v' = 55v$

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300000.

0.68
.0242723047

0.4
.0119499067

0.04
.0012546472
84.6460003
.0014972961

0.72
.0260029472

0.44
.0130074652

0.08
0.003280661

0.7:
0.02760954:

0.48
.0141089661

0.12
.0048554775

0.6
0.029829419

0.52
.0152955959

0.16
.0063365071

0.84
.0314327358

0.56
.0164902529

0.2
.0077284185

0.88
.0328029596

0.6
.0177329005

0.24
.0091087414

0.92
.0346754874

0.64
.0190270274

0.28
.0104725978

0.96
.0370996329

0.68
.0204073319

1000000.

0.32
.0117913058

0.04
.0006871986
154.5417459
.0008201029

0.72
.0218844464

0.36
.0130887217

0.76
0.023426065

0.4
.0143804516

0.08
.0023911334

0.8
.0251492058

0.44
.0156147706

0.12
.0037602971

0.84
.0265175783

0.48
0.016901508

0.16
.0050404595

0.88
.0276877322

0.52
.0182890569

0.2
.0062405811

0.92
.0292852685

0.56
.0196864444

0.24
.0074277516

0.96
.0313522677

0.6
0.021140644

0.28
.0085989369

Lower Surface
Design $d_i = 0.362^\circ$

0.64
.0226556808

0.32
.0097305293

0.36

3000000.

0.04
.0003967543
267.6741558
.0004734866

0.08
.0018673537

0.12
.0030659324

0.16
.0041821153

0.2
.0052267881

0.24
.0062586347

0.28
.0072756406

0.32
.0082578309

0.36
.0092230411

0.4
0.01018297

0.44
.0111000692

0.48
.0120547969

0.52
.0130827365

0.56
.0141174371

0.6
.0151934176

0.64
0.016313723

-41-

0.72
.0187864753

0.76
.0201202685

0.8
.0216107965

0.84
.0227948835

0.88
.0238077233

0.92
.0251897227

0.96
.0269772887

10000000.

0.04
.0002173113
488.7039106
.0002593393

0.08
.0014758664

0.12
.0025072945

0.16
.0034648479

0.2
.0043599536

0.24
.0052431286

0.28
.0061130317

0.32
.0069528908

0.36
0.007777982

August 20, 1980

BRP:pjk

0.4
.0085983302

0.44
.0093820344

0.48
0.010197616

0.52
.0110754067

0.56
.0119588578

0.6
0.012877389

0.64
.0138336139

0.68
.0148530959

0.72
.0159437009

0.76
.0170817043

0.8
.0183532519

0.84
.0193636298

0.88
.0202280417

0.92
.0214070732

0.96
.0229317861

Lower Surface
Design $\alpha_x = 0.0362^\circ$

August 20, 1980

BRP:pjk

300000000.
0.04
0.000702417
0.08
.0017112778
0.12
.0025723731
0.16
.0033798262
0.2
.0041364489
0.24
.0048884047
0.28
.0056329345
0.32
.0063503765
0.36
.0070571528
0.4
.0077611864
0.44
0.008433027
0.48
.0091347991
0.52
.0098935034
0.56
.0106577044
0.6
.0114536817
0.64
.0122835724
0.68
.0131700395

0.72
0.014119868
0.76
.0151117488
0.8
0.016222192
0.84
.0171004331
0.88
.0178490865
0.92
.0186766001
0.96
.0202100413
1000000000.
0.04
.0005992445
0.08
.0014599217
0.12
.0021945367
0.16
.0028633902
0.2
.0035288786
0.24
.0041703855
0.28
.0048047894
0.32
.0054176199
0.36
.0060205873
0.4
0.006631207

0.44
.0071943663
0.48
.0077930607
0.52
.0084403249
0.56
.0090922704
0.6
.0097713409
0.64
.0104793355
0.68
.0112355967
0.72
.0120459121
0.76
0128921004
0.8
.0138394424
0.84
.0145886856
0.88
.0153273752
0.92
.0161039655
0.96
0.0173415-8

Lower Surface

Design $\alpha_s = 0.362$

August 20, 1980

BRP:pjk

300000.

0.04
.0015603912
149.7837723
.0018627678

0.08
0.003732662

0.12
.0055490341

0.16
0.007275482

0.2
.0089630575

0.24
.0106526303

0.28
.0122929082

0.32
.0139136793

0.36
.0155342076

0.4
.0171270695

0.44
.0167232899

0.48
.0204788167

0.52
0.022301576

0.56
0.0242097796

0.6
.0262603733

0.64
.0284823377

0.68
.0309664627

0.72
.0337351769

0.76
.0369307918

0.8
.0413830652

0.84
.0478748142

0.88
0.056648394

0.92
.0710190852

0.96
.0769179697

1000000.

0.04
.0008549353
273.4665028
.0010202799

0.08
.0026435913

0.12
0.004207965

0.16
0.005691551

0.2
.0071393511

0.24
.0085864248

0.28
.0095907159

0.32
.0113776528

0.36
0.012763578

0.4
.0141255594

0.44
.0154899326

0.48
.0169888489

0.52
.0185445153

0.56
.0201725053

0.6
.0219211818

0.64
.0238153107

0.68
.0259320936

0.72
.0281907468

0.76
.0310123659

0.8
.0345025492

0.84
.0403275103

0.88
.0477958678

0.92
.0506297449

0.96
.0550525446

Upper surface
 $d = 6.05^\circ$

August 20, 1980
BRP:pjk

3000000.

0.04
0004935971
473.6578769
.0005890589

0.08
.0020190109

0.12
.0033819654

0.16
.0046716732

0.2
.0059284952

0.24
.0071835703

0.28
.0084010807

0.32
.0096030363

0.36
.0108037606

0.4
.0119835495

0.44
.0131651701

0.48
.0144625057

0.52
.0158086272

0.56
.0172170208

0.6
.0187294401

0.64
.0203673108

0.72
.0242360799

0.76
.0265882463

0.8
.029863088

0.84
.0346365544

0.88
.0410886754

0.92
.0516593351

0.96
.0559996618

10000000.

0.04
.0002703543
864.7770125
.0003226408

0.08
.0015658644

0.12
.0027353681

0.16
.0038397064

0.2
.0049147341

0.24
.005987436

0.28
.0070277329

0.32
.008054507

0.36
.0090799587

0.4
.0100874214

0.44
.0110962962

0.48
.0122034976

0.52
.0133521385

0.56
.0145537331

0.6
.0158438556

0.64
.0172407859

0.68
.0188013351

0.72
.0205397464

0.76
.0225451698

0.8
.0253367686

0.84
.0294055319

0.88
.0349052954

0.92
.0439161426

0.96
.0476162741

Upper Surface
 $d = 6.05^\circ$

30000000.

0.04
.0001560891
1437.837723
.0001862768

0.08
.0012779455

0.12
.0022933232

0.16
.0032507615

0.2
.0041821632

0.24
.0051111183

0.28
.0060118789

0.32
.0069007563

0.36
.0077883808

0.4
0.008660375

0.44
.0095335167

0.48
.0104915258

0.52
.0114852936

0.56
.0125247955

0.6
.0136407529

0.64
.0148490099

45-

0.68
.0161986702

0.72
.0177020666

0.76
.0194362776

0.8
.0218501064

0.84
.0253681122

0.88
.0301234795

0.92
.0379149576

0.96
.0411145013

100000000.

0.04
.0000854935
2734.665028
0.000102028

0.08
.0010458544

0.12
.0019143769

0.16
.0027324596

0.2
.0035279102

0.24
.0043210133

0.28
.0050899522

0.32
.0058489643

August 20, 1980

BRP:pjk

0.36
.0066062326

0.4
.0073504264

0.44
.0080955552

0.48
.0089129753

0.52
.0097608504

0.56
.0106476846

0.6
.0115997001

0.64
.0126303885

0.68
.0137816316

0.72
.0150639576

0.76
.0165430945

0.8
.0186017645

0.84
.0216020505

0.88
0.025657646

0.92
.0322027066

0.96
.0350315532

Upper Surface
d = 6.05"

August 20, 1980

BRP:pjk

<u>300000.</u>	0.32 .0037119391	0.68 .0061098284
0.04 .0010477876	0.36 .0049118113	0.72 .0063874935
0.08 .0017955816	0.4 .0060310817	0.76 .0066682963
0.12 .0022658061	0.44 .0070826136	0.8 0.006987488
0.16 .0026793723	0.48 .0081041583	0.84 .0072071126
0.2 .0030243667	0.52 .0091492846	0.88 .0073230886 450.0784852 .0087513092
0.24 .0033451912	0.56 .0101763168	0.92 0.010162042
0.28 .0036515341	0.6 .0112106699	0.96 0.01169864
0.32 .0039243218	0.64 .0122493383	1. .0172139093
0.36 .0041827611	0.68 .0133295067	<u>1000000.</u>
0.4 .0044260237	0.72 .0144464778	0.04 .0005738969
0.44 .0046472245	0.76 .0155821524	0.08 .0009834806
0.48 0.004867167	0.8 .0168163146	0.12 0.001240224
0.52 .0051143566	0.84 .0178374157	0.16 .0014675536
0.56 .0053547413	0.88 .0188429144	0.2 .0016565118
0.6 .0055996502	0.92 .0198953464	0.24 .0018333367
0.64 .0058455233	0.96 .0213090116	0.28 .0020000276 372.904255 .0022566072
Lower Surface	1. .0253800968	

August 20, 1980

BRP:pjk

<u>3000000.</u>	0.68	0.36
	.0132490079	.0059033218
0.04	0.72	0.4
(.0003313395	.0141979837	.0064477353
0.08	0.76	0.44
.0005678128	0.015170199	.0070719675
244.1671758		
.0006776278	0.8	0.48
	.0162462101	.0077049049
0.12	0.84	0.52
.0018231789	.0171076354	.0083939787
0.16	0.88	0.56
.0027486256	.0177551207	.0095834267
0.2	0.92	0.6
.0035894509	.0186512793	.0097940436
0.24	0.96	0.64
.0044090508	.0201047459	0.010513957
0.28	1.	0.68
.0052222702	.0306670575	.0113901332
(0.32	<u>10000000.</u>	0.72
0.006005714		.0120995158
0.36	0.04	0.76
.0067816882	.0003397118	.0131296657
0.4	0.08	0.8
.0075469519	.0011110725	.0133494413
0.44	0.12	0.84
.0082879265	.0018015232	.0143347514
0.48	0.16	0.88
.0090354237	.0024903148	.0151340117
0.52	0.2	0.92
.0098470344	.0031791101	.0159397771
0.56	0.24	0.96
.0106583167	.003981348	.0167411127
0.6	0.28	1.
(.0114934446	.0044994736	.0175417971
0.64	0.32	
.0123449531		

August 20, 1980

BRP:pjk

<u>30000000.</u>	0.72 .0104671632	0.4 .0047584198
0.04 (.0002938713	0.76 .0111851339	0.44 .0052191024
0.08 .0003611502	0.8 .0119805112	0.48 .0056862093
0.12 .0015585131	0.84 .0126160832	0.52 .0061947448
0.16 .0021547056	0.88 .0130921054	0.56 .0067035554
0.2 .0027302263	0.92 .0137542897	0.6 .0072278845
0.24 .0033075417	0.96 .0146298736	0.64 .0077629758
0.28 .0038914572	1. .0249206332	0.68 .0083320659
0.32 (.0044569339	<u>100000000.</u>	0.72 .0089297243
0.36 .0050202247	0.04 .0002507068	0.76 .009542238
0.4 .0055776313	0.08 .0008199744	0.8 .0102208738
0.44 0.00611788	0.12 .0013295954	0.84 .0107530107
0.48 .0066652092	0.16 .0018382179	0.88 .0111691054
0.52 .0072612997	0.2 .0023292049	0.92 .0117340117
0.56 0.007857714	0.24 .0028321723	0.96 .0122519316
0.6 0.008472445	0.28 .0033199718	1. .0212902425
0.64 (.00919916	0.32 .0038022952	
0.68 .0097000000	0.36 .0042923406	

Lower Surface
d = 6.05°

300000.
0.04
(.0014273925
0.08
- .0019080852
0.12
- .0020151029
0.16
- .0026733149
0.2
- .0027866957
0.24
- .0032356392
0.28
- .0035669673
0.32
- .0038230785
0.36
(0.004066352
0.4
- .0042994953
0.44
- .0045111377
0.48
- .0047498256
0.52
- .0049869618
0.56
- .0052444108
0.6
- .0055021377
0.64
(0.005774617
0.68
- .00604241
0.72
- .0063091479

0.68
.0083474128
0.72
.0098021416
0.76
.0112837588
0.8
.0124327642
0.84
.0149683226
0.88
.0173821669
0.92
.0205865816
0.96
.0260815243
1000000.
0.04
.0007818183
0.08
.0010423627
0.12
.0012680341
0.16
.0014645115
0.2
.0016343415
0.24
.0018012819
0.28
.0019537185
0.32
408.11-11-10
0.36
0.0021131576
0.4
.0022644108

0.36
.0046423827
0.4
.0059801342
0.44
.0070141121
0.48
.0083771942
0.52
.0091406713
0.56
.0113014029
0.6
.0113813241
0.64
.01233563-8
0.68
.0135746298
0.72
.0148203049
0.76
.0161899499
0.8
.0173441104
0.84
.0201250218
0.88
.0217381176
0.92
.0274017113
0.96
.0351714398
.....

August 20, 1980

BRP:pjk

3000000.
0.04
0.000451383
0.08
.0006018084
303.5945488
.0007181982
0.12
.0018794981
0.16
0.002817789
0.2
.0036677105
0.24
.0045046659
0.28
.0053144198
0.32
.0061022753
0.36
.0068841857
0.4
.0076540251
0.44
0.008400148
0.48
0.009206628
0.52
.0100433745
0.56
.0108962655
0.6
.0117851952
0.64
0.01272319

0.68
.0137361872
0.72
.0148257537
0.76
.0160253321
0.8
.0175628167
0.84
.0198249493
0.88
.0226560227
0.92
.0268154896
0.96
.0345514573
10000000.
0.04
.0007563246
0.08
.0014063404
0.12
.0020730203
0.16
.0027359162
0.2
.0036214319
0.24
0.004049649
0.28
.0047975158
0.32
.0057552134
0.36
.0067777718

0.4
.0066457334
0.44
0.007263556
0.48
.0079492533
0.52
.0086574431
0.56
.0093807799
0.6
.0101363094
0.64
.0109348346
0.68
0.011800753
0.72
.0127327223
0.76
.0137604964
0.8
.0150620764
0.84
.01703630323
0.88
.01947752-8
0.92
.022017439.9
0.96
.024777-02103

Upper Surface
 $\alpha = -4.72^\circ$

August 20, 1980

BRP:pjk

30000000.
0.04
(.0006542666
0.08
0.012165696
0.12
0.001792433
0.16
(.0023667331
0.2
(.0029260516
0.24
(.0035031915
0.28
(.0040724322
0.32
(.0046326141
0.36
(.0051936261
0.4
(.0057489617
0.44
(.006299606
0.48
(.0068766718
0.52
(.0074892104
0.56
(.0081149425
0.6
(.0087686942
0.64
(.0094592969
0.68
(.0101745104

0.72
.0110145756
0.76
.0119036624
0.8
.0130469091
0.84
0.014734656
0.88
.0168492381
0.92
.0199497363
0.96
.0257563369
100000000.
0.04
.0005581665
0.08
.0010378773
0.12
.0015291481
0.16
.0020191024
0.2
0.002496267
0.24
0.002986354
0.28
.0034742587
0.32
0.003952156
0.36
.0044307754
0.4

0.44
.0053649234
0.48
.0058666118
0.52
.0063891812
0.56
.00692330027
0.6
0.00748073
0.64
0.008069893
0.68
.0087089395
0.72
.0093967316
0.76
.0101552275
0.8
.0111305517
0.84
.0125703988
0.88
.0143743866
0.92
.0170255615
0.96
.0219731921

Upper Surface
 $\alpha = -4.72^\circ$

-52-
300000.

August 20, 1980
BRP:pjk

0.72
.0340767333

B

0.04
.0005327188
65.36899749
.0006357467

A

0.4
.0158073999

C

0.76
.0365411337

0.08
.0033923696

0.44
.0172106471

0.8
.0393495444

0.12
.0005772202

0.48
.0188013074

0.84
.0420313604

0.16
.0079154906

0.52
.0201395943

0.88
.0441123411

0.2
.009870125

0.56
.021708189

0.92
.0466144009

0.24
.0117313082

0.6
.0233391023

0.96
.0499865583

0.28
.0135837779

0.64
.0250526844

1000000.

0.04
.0002917821
119.3468967
.0003482128

0.32
.0153464849

0.68
.0268796339

0.08
.0026846164

0.36
.0170621393

0.72
.0289705027

0.12
.004714378

0.4
.018766551

0.76
.0309721176

0.16
.0065439078

0.44
.0204096396

0.8
.0333668146

0.2
.008213802

0.48
.0330787773

0.84
.0356537153

0.24
.0029010252

0.52
.0234415033

0.88
.0374227447

0.28
.0113834584

0.56
.0254739115

0.92
.0397835614

0.32
.0128835596

0.6
.02940928

0.96
.0424378102

0.36
.0143530431

0.64
.02940928

Lower Surface

$\alpha = -4.72^\circ$

(Read A → B → C)

August 20, 1980

RRP:pjk

3000000.
0.04
.0001684605
206.7148888
.0003010408
0.08
.0022201189
0.12
.0039761353
0.16
0.005559316
0.2
.0070038553
0.24
.0082793557
0.28
.0097475758
0.32
.0110503709
0.36
.0123179352
0.4
.0135770023
0.44
0.014790525
0.48
0.015994235
0.52
.0171251219
0.56
0.018622183
0.6
.0200930257
0.64
.0215755082

0.72
.0248774638
0.76
.0266951013
0.8
.0287560793
0.84
.0307439038
0.88
.0322792745
0.92
.0341248052
0.96
.0366113247
10000000.
0.04
.0000922626
377.4080352
.0001101146
0.08
0.001824906
0.12
.0033332369
0.16
.0046842826
0.2
.0059171531
0.24
.0070910576
0.28
.0081567563
0.32
.0091703345
0.36
.0104520716

0.4
.0115264672
0.44
.0125622633
0.48
.0135891503
0.52
.0147246596
0.56
.0156824415
0.6
.0170860901
0.64
0.016350914
0.68
.0196986435
0.72
.0211874651
0.76
.0227179037
0.8
.0244842604
0.84
.0261714015
0.88
.0274311759
0.92
.0290354401
0.96
0.0291174307

Lower Surface
 $\alpha = -4.72^\circ$

300000000.

0.04
.0000532719
653 6898749
.0000635747

0.08
.0015586066

0.12
.0028549159

0.16
.0040238703

0.2
.0050906206

0.24
.0061063472

0.38
.0071168942

0.32
.0080784205

0.36
.0090142965

0.4
.0099438414

0.44
0.010839396

0.48
0.011728431

0.52
0.012710767

0.56
.0137123576

0.6
.0147535892

0.64
.0158474604

0.68
.0170135461

-54-

0.72
.0182840755

0.76
.0196251926

0.8
.0211531246

0.84
0.022612384

0.88
.0237453808

0.92
.0251071091

0.96
.0269415679

1000000000.

0.04
.0000291782
1193.468967
.0000348213

0.08
.0013131389

0.12
.0024191562

0.16
.0034165422

0.2
.0043267488

0.24
.0051934116

0.28
.0060555453

0.32
.0069710125

0.36
.0078745123

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0.4
.0084675982

0.44
.0092321979

0.48
.0099902046

0.52
.0100282832

0.56
.0116827723

0.6
0.012571076

0.64
.0135042881

0.68
.0144990487

0.72
.0155829113

0.76
.0167269315

0.8
.0180303959

0.84
.0192752348

0.88
.0202417983

0.92
.0214034574

0.96
0.022369306

Lower Surface

$\alpha = -4.72^\circ$

057 16 A'
090 17 B'
119 12 B
136 18 C'
184 13 D'
253 10 E'
285 11 A
368 14 D

399.69

000 43 RCL
001 65 65
002 99 PRT
003 98 ADV
004 43 RCL
005 05 05
006 75 -
007 43 RCL
008 09 09
009 95 =
010 55 ÷
011 02 2
012 95 =
013 42 STD
014 01 01
015 42 STD
016 00 00
017 00 0
018 42 STD
019 04 04
020 42 STD
021 66 66
022 42 STD
023 02 02
024 43 RCL
025 09 09
026 85 +
027 09 9
028 95 =
029 32 XIT
030 01 1
031 00 0
032 67 EQ
033 14 D
034 01 1
035 75 -
036 43 RCL
037 10 10
038 95 =
039 45 YX
040 02 2
041 93 .
042 05 5
043 95 =
044 42 STD
045 06 06
046 22 INV
047 86 STF

-55-

049 07 7
050 42 STD
051 64 64
052 01 1
053 01 1
054 42 STD
055 63 63
056 76 LBL
057 16 A'
058 01 1
059 75 -
060 73 PC#
061 63 63
062 95 =
063 42 STD
064 62 62
065 45 YX
066 02 2
067 93 .
068 05 5
069 95 =
070 72 ST#
071 64 64
072 01 1
073 44 SUM
074 63 63
075 87 IFF
076 01 01
077 12 B
078 01 1
079 44 SUM
080 64 64
081 86 STF
082 01 01
083 43 RCL
084 63 63
085 42 STD
086 02 02
087 61 GTD
088 16 A'
089 76 LBL
090 17 B'
091 53 (
092 53 (
093 43 RCL
094 06 06
095 85 +
096 04 4
097 65 X
098 43 RCL
099 07 07
100 85 +
101 43 RCL
102 08 08
103 54)
104 65 X
105 43 RCL
106 03 03
107 55 ÷
108 03 3
109 54)
110 42 STD

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111 04 04
112 44 SUM
113 66 66
114 07 7
115 42 STD
116 64 64
117 92 RTN
118 76 LBL
119 12 B
120 71 SBR
121 17 B'
122 22 INV
123 86 STF
124 01 01
125 43 RCL
126 02 02
127 22 INV
128 77 GE
129 18 C'
130 22 INV
131 67 EQ
132 18 C'
133 96 STF
134 02 02
135 76 LBL
136 18 C'
137 42 RCL
138 02 02
139 75 -
140 01 1
141 00 0
142 95 =
143 65 X
144 43 RCL
145 03 03
146 95 =
147 99 PRT
148 53 (
149 43 RCL
150 66 66
151 55 ÷
152 53 (
153 43 RCL
154 62 62
155 45 YX
156 03 3
157 54)
158 54)
159 34 FX
160 65 X
161 05 5
162 95 =
163 55 ÷
164 53 (
165 43 RCL
166 55 65
167 34 FX
168 54)
169 95 =
170 42 STD
171 67 67
172 99 PRT

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173	87	IFF
174	02	02
175	19	D'
176	98	ADV
177	43	RCL
178	08	08
179	42	STD
180	06	06
181	61	GTD
182	16	A'
183	76	LBL
184	19	D'
185	43	RCL
186	62	62
187	34	FX
188	65	X
189	43	RCL
190	65	65
191	65	X
192	43	RCL
193	67	67
194	55	÷
195	04	4
196	93	.
197	07	7
198	08	8
199	08	8
200	95	=
201	42	STD
202	02	02
203	99	PRT
204	65	X
205	05	5
206	93	.
207	07	7
208	01	1
209	04	4
210	55	÷
211	53	(
212	43	RCL
213	65	65
214	65	X
215	43	RCL
216	62	62
217	34	FX
218	54	)
219	95	=
220	99	PRT
221	98	ADV
222	43	RCL
223	02	02
224	45	YX
225	01	1
226	93	.
227	01	1
228	05	5
229	02	2
230	95	=
231	42	STD
232	02	02

233	22	INV
234	86	STF
235	02	02
236	43	RCL
237	62	62
238	45	YX
239	01	1
240	93	.
241	06	6
242	05	5
243	95	=
244	42	STD
245	06	06
246	00	0
247	42	STD
248	66	66
249	07	7
250	42	STD
251	64	64
252	76	LBL
253	10	E'
254	01	1
255	75	-
256	73	RC#
257	63	63
258	95	=
259	42	STD
260	62	62
261	45	YX
262	01	1
263	93	.
264	06	6
265	05	5
266	95	=
267	42	STD
268	69	69
269	72	ST*
270	64	64
271	01	1
272	44	SUM
273	63	63
274	87	IFF
275	01	01
276	11	A
277	86	STF
278	01	01
279	01	1
280	44	SUM
281	64	64
282	61	GTD
283	10	E'
284	76	LBL
285	11	A
286	43	RCL
287	03	03
288	65	X
289	53	(
290	43	RCL
291	63	63
292	75	-

295	54	)
296	95	=
297	99	PRT
298	71	SBR
299	17	B'
300	43	RCL
301	66	66
302	65	X
303	43	RCL
304	65	65
305	55	÷
306	08	.85
307	00	.05
308	95	=
309	55	÷
310	53	(
311	43	RCL
312	62	62
313	45	YX
314	01	1
315	93	.
316	01	1
317	05	5
318	54	)
319	95	=
320	85	+
321	43	RCL
322	02	02
323	95	=
324	22	INV
325	45	YX
326	01	1
327	93	.
328	01	1
329	05	5
330	02	2
331	95	=
332	65	X
333	05	5
334	93	.
335	07	7
336	01	1
337	04	4
338	55	÷
339	53	(
340	43	RCL
341	65	65
342	65	X
343	50	.
344	43	RCL
345	62	62
346	34	FX
347	54	)
348	54	)
349	95	=
350	99	PRT
351	98	ADV
352	22	INV
353	86	STF

355	43	RCL
356	69	69
357	42	STD
358	06	06
359	97	DSZ
360	00	00
361	10	E'
362	43	RCL
363	01	01
364	42	STD
365	00	00
366	91	R/S
367	76	LBL
368	14	J
369	00	0
370	42	STD
371	06	06
372	07	7
373	42	STD
374	64	64
375	01	1
376	01	1
377	42	STD
378	63	63
379	61	GTD
380	10	E'
381	00	0
382	00	0
383	00	0
384	00	0
385	00	0

12.	00
-57- 12.	01
977.3428482	02
0.02	03
.0335920914	04
49.	05
.7456336761	06
.8404774784	07
.7456336761	08
25.	09
1.	10
0.4290993	11
-0.1488981	12
-0.1738043	13
-0.2042837	14
-0.2297392	15
-0.2494459	16
-0.2646265	17
-0.277503	18
-0.2902193	19
-0.3021669	20
-0.3112488	21
-0.3171539	22
-0.3218098	23
-0.3263865	24
-0.3312559	25
-0.3353662	26
-0.3389635	27
-0.3420992	28
-0.344265	29
-0.3462343	30
-0.3490858	31
-0.351841	32
-0.3531389	33
-0.352087	34
-0.3489647	35
-0.3453159	36
-0.3415661	37
-0.3369379	38
-0.331439	39
-0.3251038	40
-0.3177958	41
-0.309659	42
-0.3003378	43
-0.289526	44
-0.2774239	45
-0.2647638	46
-0.2506704	47
-0.235302	48
-0.2181101	49
-0.1971979	50
-0.1685343	51
-0.1328144	52
-0.09420395	53
-0.009378433	54
-0.9378433	55
0.04023716	56
0.09995742	57

0.	61
0.8370033	62
59.	63
7.	64
300000.	65
.7160031549	66
.0054082583	67
977.3428482	68
.7456336761	69

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B: Perturbed Pressure Distributions Caused by Surface Irregularities

In this discussion we include two bumps because they lead to the simplest computations. The bumps presented are:

1st case: $\eta(x) = \epsilon(1 - 4x^2)$, $|x| \leq 1/2$; $= 0$, $|x| > 1/2$.

2nd case: $\eta(x) = \epsilon(1 - 4x^2)^2$, $|x| \leq 1/2$; $= 0$, $|x| > 1/2$.

For Case 1: we have

$$\eta'(x) = -8\epsilon x \quad , \quad |x| \leq 1/2 \quad ; \quad \eta'(x) = 0 \quad \text{elsewhere} \quad .$$

$$\eta''(x) = -8\epsilon \quad , \quad |x| \leq 1/2 \quad ; \quad \eta''(x) = 0 \quad \text{elsewhere} \quad .$$

For Case 2: we have

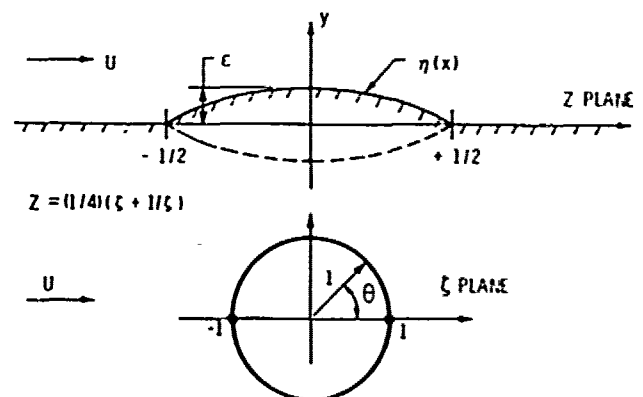
$$\eta'(x) = -16\epsilon x(1 - 4x^2) \quad , \quad |x| \leq 1/2 \quad ; \quad \eta'(x) = 0 \quad \text{elsewhere}$$

$$\eta''(x) = -16\epsilon(1 - 12x^2) \quad , \quad |x| \leq 1/2 \quad ; \quad \eta''(x) = 0 \quad \text{elsewhere}$$

Note that both bumps have discontinuous curvature at $|x| = 1/2$ but that in Case 2 the slope is continuous. We have considered bumps having a continuous slope and curvature at $|x| = 1/2$ but the series convergence is not too good.

Case 1

Consider the bump for which $\eta'(x) = -8\epsilon x$ as noted above.



In accordance with the ideas of thin airfoil theory we consider the complex velocity, $W(z) = U(1 + u - iv)$, to be invariant at corresponding points in the z and ζ planes. Next we consider the mapping,

$$z = (x + iy) = \frac{1}{4} \left(\zeta + \frac{1}{\zeta} \right) .$$

on the unit circle, $\zeta = e^{i\theta}$, and $2x = \cos\theta$. Accordingly, $\eta'(x) = -4\epsilon \cos\theta$. But in accordance with the scheme of linearization the boundary condition, $v(x)/U = \eta'(x)$, is to be evaluated at $y = 0$ on the real axis. Thus we have

$$\text{Im} \left[\frac{W(x)}{U} \right] = \begin{cases} \eta'(x) & , \quad |x| \leq \frac{1}{2} \quad \text{on the bump} , \\ 0 & , \quad |x| \leq \frac{1}{2} \quad \text{off the bump} . \end{cases}$$

But we know that for flows without circulation

$I(z) = U[1 + w(z)]$ can be written with

$$w(z) = \sum_{i=1}^{\infty} \frac{a_i + ib_i}{\zeta^i} \quad (\text{where } w = u - iv \text{ is the perturbation velocity})$$

in order that $w(z) \rightarrow 0$ as $\zeta \rightarrow \infty$ and that the flow direction at ∞ will be invariant under the mapping. This latter aspect is accounted for in the Joukowski transformation.

In general terms, $\zeta = re^{i\theta}$. But on the unit circle

$$\zeta = e^{i\theta} .$$

Therefore, the real axis in the interval, $(y = 0, |x| \leq 1/2)$ is on the unit circle with $0 \leq \theta \leq \pi$. The point $(y = 0, x = 0)$ and at the center of the bump, where $\eta'(0) = 0$ and $\eta(0) = \epsilon$, maps into the point $\theta = \pi/2, r = 1$. Accordingly, we have

$$w = u - iv = \sum_{i=1}^{\infty} (a_i + ib_i) e^{-i\theta} = \sum_{i=1}^{\infty} (a_i \cos i\theta + b_i \sin i\theta) - i \sum_{i=1}^{\infty} (a_i \sin i\theta - b_i \cos i\theta) .$$

Applying the boundary conditions on the bump we have

$$-iv = -i \sum_{i=1}^{\infty} (a_i \sin i\theta - b_i \cos i\theta) = +i8\epsilon x, \quad |x| \leq \frac{1}{2} = 0, \quad |x| > \frac{1}{2}$$

Therefore in the ζ plane we have

$$v = \sum_{i=1}^{\infty} (a_i \sin i\theta - b_i \cos i\theta) = -4\epsilon \cos\theta \text{ on } \zeta = e^{i\theta} .$$

But if $v = 0$ at points on the real axis outside the unit circle and on $|\zeta| = 1$ if $v(-\theta) = -v(\theta)$, in order that $v(\theta)$ is odd with respect to θ so that it will represent a thickness distribution and not a camber function, we must consider the expansion of $\cos\theta$ as a sine series. In this case $b_i = 0$ and

$$a_i = \frac{-8\epsilon}{\pi} \int_0^{\pi} (\cos\theta) \sin i\theta \, d\theta = \begin{cases} \frac{-16\epsilon i}{\pi(i^2-1)} & , \quad i = 2, 4, 6, \dots, \\ 0 & , \quad \text{otherwise} . \end{cases}$$

As a result of this finding it turns out that the u component is given by

$$u = -\frac{16\epsilon}{\pi} \sum_{i=2}^{\infty} \frac{i \cos i\theta}{(i^2-1)}, \quad (i \text{ is even only})$$

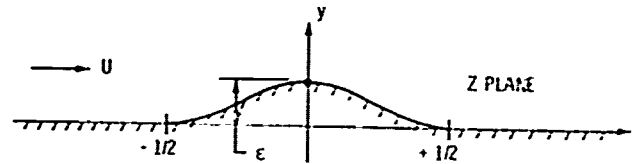
and since the linearized bernoulli equation is $C_p = -2u$, we have

$$C_p = \frac{32\epsilon}{\pi} \sum_{i=2, \dots}^{\infty} \frac{i \cos i\theta}{i^2-1} \quad ; \quad \text{on } \zeta = e^{i\theta}, \text{ and } i \text{ even}.$$

At this point we need to pause and consider the convergence of $C_p(\theta)$. In accordance with the Laurent expansion for w given on page 59 it is clear that for points outside and on the circle we must have $\zeta = re^{i\theta}$ and

$$C_p(\zeta) = \frac{32\epsilon}{\pi} \sum_{i=2}^{\infty} \frac{i \cos i\theta}{r^i (i^2-1)}, \quad \text{with } i \text{ being even}.$$

But, on the circle $r = 1$ and on the real axis $r \geq 1$ and $\theta = 0, \pi$. In particular, if $r > 1$ the series converges for all θ . When $r = 1$ the series diverges at $\theta = 0, \pi$. This can be verified by employing the usual tests for convergence of power series. It can also be seen for sufficiently large values of i when $\theta = 0$ (or π) that the series becomes equivalent to the harmonic series which is known to be divergent. Therefore we see that in terms of thin airfoil theory we can consider the corner between the wall and the bump as a linearized stagnation point. Since the series at other points is convergent, we can compute values of C_p with it, but we should not expect rapid convergence. Indeed, as we shall see below even though we take many terms (as high as 40) the accuracy is not great.



Case 2: $\eta'(x) = -16 \epsilon x (1 - 4x^2) = -8 \epsilon \cos \theta \sin^2 \theta$
 $= -4 \epsilon \cos \theta \sin^2 \theta = -2 \epsilon (\cos \theta - \cos 3\theta)$

Note that this case contains the first case as part of the solution.

Thus we see that the term $-2\epsilon \cos \theta$ will lead to a contribution for $u(\theta)$

which is

$$-\frac{8\epsilon}{\pi} \sum_{i=2}^{\infty} \frac{i \cos i\theta}{(i^2-1)}, \quad i = 2, 4, \dots$$

It remains for us to evaluate the term $2\epsilon \cos 3\theta$. In this instance we have to consider

$$+\frac{4\epsilon}{\pi} \int_0^{\pi} \cos 3\theta \sin i\theta \, d\theta = \begin{cases} \frac{8\epsilon i}{\pi(i^2-9)} & , \quad i = 2, 4, 6, \dots \\ 0 & , \quad \text{otherwise} \end{cases}$$

If we add these results, term by term, we obtain

$$\frac{8\epsilon}{\pi} \left[\frac{i}{i^2-9} - \frac{i}{i^2-1} \right] = \frac{8\epsilon}{\pi} \left[\frac{i^3-i-i^3+9i}{(i^2-1)(i^2-9)} \right] = \frac{64\epsilon}{\pi} \frac{i}{(i^2-1)(i^2-9)}$$

Therefore we have

$$C_p(x) = -\frac{128\epsilon}{\pi} \sum_{i=2}^{\infty} \frac{i \cos i\theta}{r^i(i^2-1)(i^2-9)} = \frac{32\epsilon}{\pi} \sum_{i=2}^{\infty} \left\{ \frac{i \cos i\theta}{r^i(i^2-1)} \left(\frac{4}{9-i^2} \right) \right\},$$

where $i = 2, 4, 6 \dots$, as before.

Note that we have written this expression so that each term of the sum for C_p can be obtained from the corresponding term of the first case by multiplying it by the factor $4/(9-i^2)$.

The convergence of this series is no problem. As $i \rightarrow \infty$ each term approaches $1/i^2$, and it is known that this will produce a uniformly and absolutely convergent series both on and off the circle $r = 1$.

Similarly, it is worth noting that the formulae for $\eta(x)$ in the two cases are very simply related. In fact, if we want to get case 2 from case 1 we need only take $\eta(x)/\epsilon$ for case 1 and square it. As a result it is fairly easy to program a routine which will do both cases in sequence and this has been done below.

It should be noted that the 2nd case was computed on the basis that the multiplicative factor $4/(9-i^2)$ was $2/(9-i^2)$, and this is incorrect. Therefore we should multiply all results for $C_p(x)/\epsilon$ which are tabulated below by a factor of 2. This error was corrected in the case $N = 40$.

Notes on T.I. - 59 program for finding $C_p(x)/\epsilon$ for the two bumps:

$$\text{Case 1: } \eta(x) = \epsilon(1 - 4x^2),$$

$$\text{Case 2: } \eta(x) = \epsilon(1 - 4x^2)^2.$$

Procedure: Enter program from keyboard. See below.

Enter Input Data:

Data Register Address

Total number of terms in sum = N

$R_{01} \rightarrow \boxed{\text{ST0}} \ 01$

Increment between successive $x = \Delta x$

$R_{07} \rightarrow \boxed{\text{ST0}} \ 07$

Maximum value of $x = x_{\max}$

$R_{08} \rightarrow \boxed{\text{ST0}} \ 08$

In order to execute program after input data are entered:

Press: $\boxed{\text{CLR}}$

$\boxed{\text{RST}}$

$\boxed{\text{R/S}}$

If it is desired to run Case 2 only, enter input data as before. Then

Press: $\boxed{\text{CLR}}$

$\boxed{\text{2nd}} \ \boxed{\text{St flg}}$

$\boxed{\text{GTO}} \ \boxed{03}$

$\boxed{\text{R/S}}$

Storage Map:

R_{00} for DSZ(n)

R_{04} for $r(x)$

* R_{08} for $x|_{\max}$

* R_{01} for N

R_{05} for i

R_{09} for $C_p(x)\epsilon$

R_{02} for x

R_{06} for $\eta(x)/\epsilon$

R_{10} for $(x-1/2)$

R_{03} for θ

* R_{07} for Δx

R_{11} used in
summation of
 $C_p(x)$.

* Input quantities.

—//—

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Data calculated for two cases, $N = 20$ and $N = 40$ are given below. These are followed by the program listing and other information. A plot of Case 1 and Case 2 is given in Figure 3, page 26 above.

$N = 40$
 Case 1.
 $\eta = \epsilon(1 - 4x^2)$

0.	
1.	
-5.030082152	
0.05	
0.99	
-5.058153468	
0.1 x	
0.96 η/ϵ	
-4.939254581 C_p/ϵ	
0.15	
0.91	
-4.561662501	
0.2	
0.84	
-4.253528991	
0.25	
0.75	
-3.693644385	
0.3	
0.64	
-2.997821768	
0.35	
0.51	
-1.912952636	
0.4	
0.36	
-1.5158775378	
0.45	
0.19	
1.521340967	
0.5	
0.	
23.7574337	
0.55	
0.	
3.435135819	
0.6	
0.	
2.234470026	
0.65	
0.	
1.649982227	

-66-

1.4
0.
0.2347862
1.45
0.
.2176112402
1.5
0.
.2022962247
1.55
0.
.1885765801
1.6
0.
.1762339944
1.65
0.
.1650868378
1.7
0.
.1549831282
1.75
0.
.1457944004
1.8
0.
.1374118072
1.85
0.
0.129742402
1.9
0.
0.122706431
1.95
0.
0.116235122
2.
0.
.1102688982

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0.7
0.
1.294791052
0.75
0.
1.054641806
0.8
0.
.8814365151
0.85
0.
0.750894088
0.9
0.
.6492842871
0.95
0.
.5682095794
1.
0.
.5022282621
1.05
0.
.4476533426
1.1
0.
0.401896424
1.15
0.
.36308624.4
1.2
0.
.3292394931
1.25
0.
.30110755.1
1.3
0.
.2760856015
1.35
0.
.25111755.1

N = 40

Case 2

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$$\eta = \epsilon(1-4x^2)^2$$

0.
1.
-6.790649262

0.7
0.
.9093066337

1.4
0.
0.183430237

0.05 x
0.9801 η/ϵ
-6.58756297 C_p/ϵ

0.75
0.
.7587009382

1.45
0.
.1703068729

0.1
0.9216
-5.986658923

0.8
0.
.6452235254

1.5
0.
.1585658571

0.15
0.8281
-5.013216552

0.85
0.
0.5569258

1.55
0.
.1480167431

0.2
0.7056
-3.710924596

0.9
0.
.4865118466

1.6
0.
.1385012363

0.25
0.5625
-2.145937846

0.95
0.
.4292514483

1.65
0.
.1298868128

0.3
0.4096
-.4124959147

1.
0.
.3819358799

1.7
0.
.1220617842

0.35
0.2601
1.354261938

1.05
0.
.3423096564

1.75
0.
.1149314434

0.4
0.1296
2.951140955

1.1
0.
.3087409166

1.8
0.
.1084150255

0.45
0.0361
4.024363908

1.15
0.
.2800203531

1.85
0.
.1024432906

0.5
0.
3.396858894

1.2
0.
0.255233431

1.9
0.
.0969565865

0.55
0.
1.952548409

1.25
0.
.2306763757

1.95
0.
.0819032844

0.6
0.
1.42897183

1.3
0.
.2147993238

2.
0.
.0872385066

0.65

1.35
0.

No. of terms in sum = $N \approx 20$

$0. \times$
 $1. \frac{1}{e}$
 -4.968739687 $\frac{C_p/E}{C_{\text{ave}}}$
 0.05
 0.99
 -5.112666205 $\frac{1}{e(1-4x^2)}$
 0.1
 0.96
 -4.936602309
 0.15
 0.91
 -4.490128291
 0.2
 0.84
 -4.28217104
 0.25
 0.75
 -3.819338671
 0.3
 0.64
 -2.92673393
 0.35
 0.51
 -1.840841503
 0.4
 0.36
 -1.4227985398
 0.45
 0.19
 1.542741737
 0.5
 $0.$
 20.28900423
 0.55
 $0.$
 3.435135815
 0.6
 $0.$
 2.234470026

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0.65
 $0.$
 1.649982227
 0.7
 $0.$
 1.294791052
 0.75
 $0.$
 1.054641806
 0.8
 $0.$
 $.8814365151$
 0.85
 $0.$
 0.750894088
 0.9
 $0.$
 $.6492842871$
 0.95
 $0.$
 $.5682095794$
 $1.$
 $0.$
 $.5022282621$
 1.05
 $0.$
 $.4476533426$
 1.1
 $0.$
 0.401896424
 1.15
 $0.$
 $.3630868414$
 1.2
 $0.$
 $.3298394981$
 1.25
 $0.$
 $.3011075311$
 1.3
 $0.$
 $.2762250015$

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BRP:pjk

1.35
 $0.$
 $.2541445596$
 1.4
 $0.$
 0.2347862
 1.45
 $0.$
 $.2176112402$
 1.5
 $0.$
 $.2022962247$
 1.55
 $0.$
 $.1885765801$
 1.6
 $0.$
 $.1762339944$
 1.65
 $0.$
 $.1650868878$
 1.7
 $0.$
 $.1549831282$
 1.75
 $0.$
 $.1457944004$
 1.8
 $0.$
 $.1374118072$
 1.85
 $0.$
 0.129742402
 1.9
 $0.$
 0.122706431
 1.95
 $0.$
 0.116235122
 $2.$
 $0.$
 0.116235122

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ERP:pjk

1.35

0.

.0990834462

1.4

0.

.0917151185

1.45

0.

.0851534364

1.5

0.

.0792829285

1.55

0.

.0740083715

1.6

0.

.0692506182

1.65

0.

.0649434064

1.7

0.

.0610308921

1.75

0.

.0574657217

1.8

0.

.0542075128

1.85

0.

.0512216453

1.9

0.

.0464782933

1.95

0.

.0459516422

2.

0.

No of terms in sum
0. N=20
1. Case 2
 $\eta = \epsilon(1-4x^2)^2$

0.05

0.9801

-3.293702658

0.1 x

0.9216 η/ϵ

-2.993284339 ϵ/ϵ

0.15

0.8281

-2.506745124

0.2

0.7056

-1.855410596

0.25

0.5625

-1.072817063

0.3

0.4096

-2.2063038399

0.35

0.2601

.6769714676

0.4

0.1296

1.475377376

0.45

0.0361

2.012301881

0.5

0.

1.700689672

0.55

0.

.9762742044

0.6

0.

.7144859148

0.65

0.7

0.

.4546533169

0.75

0.

.3793504691

0.8

0.

.3226117627

0.85

0.

0.2784629

0.9

0.

.2432559233

0.95

0.

.2146257241

1.

0.

0.19096794

1.05

0.

.1711548282

1.1

0.

.1543704583

1.15

0.

.1400101765

1.2

0.

.1276167155

1.25

0.

.1168381879

1.3

0.

.1073996619

All values of ϵ/ϵ on this page should be multiplied by 2 in order to correct an error in the program.

DATA REGISTERS

17.	for DSZ	00
40.	N	01
0.35	X	02
.7953988302	θ	03
1.	n	04
2.	i	05
0.2601	$\eta(x)/\epsilon$	06
0.05	ΔX	07
2.	X_{max}	08
1664847757	$C_p(x)/\epsilon$	09
-0.15	$(x - 1/2)$	10
-.0133333333	USED	11
0.	contents	12
0.	variable name	13
0.	not used	14
0.		15
0.		16
0.		17
0.		18
0.	address	19
0.		20
0.		21
0.		22
0.		23

Labels used OP 08

041	13	C
068	15	E
071	16	A'
101	48	EXC
107	17	B'
129	18	C'
148	19	D'
154	14	D
195	11	A
221	12	B
242	10	E'
265	66	PAU

Program Listing

000	22	INW	Flag = 1
001	86	STF	
002	01	01	
003	00	0	First
004	42	STD	x
005	02	02	
006	43	RCL	for
007	01	01	DSZ
008	42	STD	
009	00	00	

010	00	0	for
011	42	STD	C_p
012	09	09	Sum
013	02	2	first
014	42	STD	in
015	05	05	Sum
016	43	RCL	
017	02	02	
018	75	-	
019	93	$(x - \frac{1}{2})$	
020	05	5	for
021	95	=	test.
022	42	STD	
023	10	10	
024	43	RCL	
025	08	08	Test
026	75	-	on
027	43	RCL	$(X_{max} - x)$
028	02	02	
029	95	=	$x_{max} \geq x?$
030	77	GE	
031	13	C	yes
032	87	IFF	no
033	01	01	
034	15	E	exit.
035	86	STF	set
036	01	01	Flag
037	61	GTO	start
038	00	00	next
039	03	03	case
040	76	LBL	
041	13	C	
042	43	RCL	Print
043	02	02	X
044	99	PRT	
045	71	SBR	$\sqrt{\epsilon}$
046	16	A'	$\& n(x)$
047	43	RCL	Print
048	06	06	$\eta(x)/\epsilon$
049	99	PRT	
050	71	SBR	
051	18	C'	C_p/ϵ
052	43	RCL	Print
053	09	09	
054	99	PRT	$(C_p(x)/\epsilon)$
055	98	ADV	E
056	43	RCL	
057	02	02	
058	85	+	
059	43	RCL	Δx
060	07	07	
061	95	=	$x + \Delta x$
062	42	STD	new
063	02	02	X
064	61	GTO	start
065	00	00	next
066	06	06	calc.
067	76	LBL	end
068	15	E	calc.

Subroutine	070	76	LBL
A'	071	16	A'
	072	70	RAD
set $n=1$	073	01	1
	074	42	STD
	075	04	04
set $\theta=0$	076	00	0
	077	42	STD
	078	03	03
	079	43	RCL
	080	10	10 is
	081	77	GE $x \geq \frac{1}{2}?$
	082	17	B' yes
	083	53	(no
	084	01	1
	085	75	-
	086	04	4
	087	65	x
	088	53	(
	089	43	RCL
	090	02	02
	091	33	x^2
	092	54	$(1-4x^2)$
	093	54	)
	094	42	STD First
	095	06	06 $\eta(x)/\epsilon$
	096	87	IFF in flag?
	097	01	01 set?
	098	48	EXC yes
	099	92	RTH no
	100	76	LBL flag
	101	48	EXC set
	102	33	x^2 2nd
	103	42	STD $\eta(x)/\epsilon$
	104	06	06 $(1-4x^2)^2$
	105	92	RTH
	106	76	LBL $x \geq \frac{1}{2}$
	107	17	B'
	108	53	(
	109	43	RCL
	110	02	02 x
	111	65	x
	112	02	2 2x
	113	85	+
	114	53	(
	115	24	CE
	116	33	$x^2 (2x)^2$
	117	75	-
	118	01	1
	119	54	)
	120	34	$\sqrt{x} \sqrt{2x} = 1$
	121	54	)
	122	42	STD $n=2x$
	123	04	04 $4 + \sqrt{x}$
	124	00	0
	125	42	STD $\eta(x)/\epsilon$
	126	06	06 $=0$.
	127	92	RTH

first 2nd 2nd
when we calculate $\eta(x)$, $x \geq \frac{1}{2}$,
we must have $n=1$, θ is calculated before
in Subroutine C'.

Tests for Program Direction
is flag 1 set?
yes
no

Main Program

calculate

calculate

exit

Initialize main
variables.

Subtractive
 $\downarrow$ calculate θ if $x < \frac{1}{2}$
 $\downarrow$ calculate term in sum for $C_p(x)$.
 in for first $y(x)$.

128	76	LBL
129	18	C'
130	70	RAD
131	43	RCL
132	10	10 $\times \frac{1}{2}$
133	77	GE
134	19	D' $\frac{1}{2}$
135	53	(
136	53	(
137	43	RCL
138	02	02 $\times$
139	65	$\times$
140	02	2
141	54	) $2x$
142	22	INV
143	39	COS
144	54	)
145	42	STD θ
146	03	03
147	76	LBL $\times \frac{1}{2}$
148	19	D' $\frac{1}{2}$
149	43	RCL $\frac{1}{2}$
150	10	10 $\times \frac{1}{2}$
151	67	EQ ?
152	66	PAU $\frac{1}{2}$
153	76	LBL $\frac{1}{2}$
154	14	D
155	53	(
156	53	(
157	43	RCL $\frac{1}{2}$
158	05	05
159	65	$\times$
160	53	(
161	53	(
162	24	CE e
163	65	$\times$
164	43	RCL $\frac{1}{2}$
165	03	03
166	54	)
167	39	COS
168	54	) $\cos i$
169	54	) $\sin i$
170	55	+
171	53	(
172	53	(
173	43	RCL $\frac{1}{2}$
174	04	04 $\frac{1}{2}$
175	45	$\times$
176	43	RCL $\frac{1}{2}$
177	05	05 $\frac{1}{2}$
178	54	) $\frac{1}{2}$
179	65	$\times$
180	53	(
181	43	RCL
182	05	05 $\frac{1}{2}$
183	33	$\times \frac{1}{2}$
184	75	-
185	01	1
186	54	) $\frac{1}{2}$
187	54	) $\frac{1}{2}$

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For case 1 or case 2
 $\downarrow$ accumulate sum
 of terms
 Det
 $\downarrow$ sum is complete.
 $\downarrow$ calculate $C_p(x)/e$
 $\downarrow$ calculate term in sum for $C_p(x)/e$.

Sum is not complete calculate
 next value of i for calculation
 of next term in $C_p(x)/e$.

191	87	IFF $\frac{1}{2}$
192	01	01 $\frac{1}{2}$
193	10	E' $\frac{1}{2}$
194	76	LBL $\frac{1}{2}$
195	11	A
196	53	(
197	43	RCL $\frac{1}{2}$
198	11	11 $\frac{1}{2}$
199	85	+
200	43	RCL $\frac{1}{2}$
201	09	09 $\frac{1}{2}$
202	54	)
203	42	STD $\frac{1}{2}$
204	09	09 $\frac{1}{2}$
205	97	DSZ $\frac{1}{2}$
206	00	00 $\frac{1}{2}$
207	12	B $\frac{1}{2}$
208	53	($\frac{1}{2}$
209	43	RCL $\frac{1}{2}$
210	09	09 $\frac{1}{2}$
211	65	$\times$
212	03	3
213	02	2
214	55	+
215	89	$\frac{1}{2}$
216	54	)
217	42	STD $\frac{1}{2}$
218	09	09 $\frac{1}{2}$
219	92	RTH $\frac{1}{2}$
220	76	LBL $\frac{1}{2}$
221	12	B $\frac{1}{2}$
222	53	($\frac{1}{2}$
223	53	($\frac{1}{2}$
224	43	RCL $\frac{1}{2}$
225	01	01 $\frac{1}{2}$
226	75	- $\frac{1}{2}$
227	53	($\frac{1}{2}$
228	43	RCL $\frac{1}{2}$
229	00	00 $\frac{1}{2}$
230	75	- $\frac{1}{2}$
231	01	1 $\frac{1}{2}$
232	54	) $\frac{1}{2}$
233	54	) $\frac{1}{2}$
234	65	$\times$ $\frac{1}{2}$
235	02	2 $\frac{1}{2}$
236	54	) $\frac{1}{2}$
237	42	STD $\frac{1}{2}$
238	05	05 $\frac{1}{2}$
239	61	GTD $\frac{1}{2}$
240	18	C' $\frac{1}{2}$

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 $\downarrow$ calculate term in sum for 2nd $y(x)$
 $\downarrow$ if $x = \frac{1}{2}$ make
 $\theta = 0$.

241	76	LBL
242	10	E'
243	53	(
244	43	RCL
245	11	11
246	65	$\times$
247	53	(
248	02	2 4
249	94	+/-
250	55	+
251	53	(
252	43	RCL
253	05	05
254	33	$\times \frac{1}{2}$
255	75	-
256	09	9
257	54	)
258	54	)
259	54	)
260	42	STD
261	11	11
262	61	GTD
263	11	A
264	76	LBL
265	66	PAU
266	00	0 $\frac{1}{2}$
267	42	STD $\frac{1}{2}$
268	03	03 $\frac{1}{2}$
269	61	GTD $\frac{1}{2}$
270	14	D $\frac{1}{2}$
271	00	0 $\frac{1}{2}$
272	00	0 $\frac{1}{2}$
273	00	0 $\frac{1}{2}$
274	00	0 $\frac{1}{2}$

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