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HALL EFFECT NEAR THE MOBILITY EDGE: A SCALING ARGUMENT.(U)
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Hall Effect Near the Mobility Edge: A Scaling Argument*

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ABSTRACT

Critical behaviour of the zero temperature Hall constant in a disordered electronic system is considered. It is shown, by means of a scaling argument, that near (above) the mobility edge E_c the Hall constant R diverges according to $R(E) \sim (E-E_c)^{-t}$, where t is the conductivity exponent.

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Abrahams et al.[1] have developed a single parameter scaling theory of electron localization in disordered systems. The scaling parameter in their theory is the dimensionless conductance $g = G/(e^2/\hbar)$. which characterizes the degree of localization in a finite sample.[2] The theory predicts $d = 2$ as the lower critical dimensionality for the Anderson transition. A mobility edge E_c exists, and a true metallic conduction is possible, only for $d > 2$. The zero temperature conductivity near (above) E_c is proportional to $(E-E_c)^t$. The conductivity exponent t is related to the correlation (localization) length exponent ν by

$$t = (d-2)\nu \qquad (1)$$

a result first obtained by Wegner.[3]

The purpose of this note is to investigate the critical (i.e. near E_c) behaviour of the Hall constant $R(E)$ in the framework of a single parameter scaling theory for noninteracting electrons.

I first derive Eq. (1) using a scaling argument which can then be easily extended to obtain the Hall constant behaviour near E_c . It will be more convenient to use the parameter

$$\Delta(L) = (g(L) - g_c)/g_c \qquad (2)$$

(rather than $g(L)$ itself as in Ref. 1) as the basic scaling parameter. The length L in Eq. (2) represents some arbitrary scale, and g_c is conductance at the mobility edge. In the critical region, i.e. for $\Delta \ll 1$, the correlation length $\xi = L\Delta^{-\nu}(L)$, and hence the parameter $\Delta(L)$ scales as

$$\Delta(L) = \Delta_0(L/L_0)^{1/\nu} \qquad (3)$$

where $\Delta_0 \sim (E-E_c)$ is the initial value of the parameter at some, e.g. microscopic, scale L_0 .

I consider now a sample of length $\mathcal{L}$ and divide it into blocks of size L^d . According to the scaling hypothesis[1] the conductance G of the sample must be some function of $\mathcal{L}/L$ and $\Delta(L)$ only. For a large ($\mathcal{L} >> \xi$) sample, and in the metallic region

$$G = (\mathcal{L}/L)^{d-2} f(\Delta(L)) \qquad (4)$$

Let us choose $L << \xi$, so that $\Delta(L) << 1$. Then $f(\Delta)$ is proportional to Δ^t which, via Eq. (3), leads to

$$G \sim (\mathcal{L}/L)^{d-2} \Delta_0^t (L/L_0)^{t/\nu} \qquad (5)$$

Since G cannot depend on L , Eq. (5) immediately gives the relation (Eq. (1)) between the exponents. Thus there is a connection between the conductivity behaviour near E_c (the exponent t) and the behaviour of the conductance as a function of $\mathcal{L}$ for a large (metallic) sample (the exponent (d-2) in Eqs.(4,5)).

In the presence of an external magnetic field B the sample is characterized by the Hall conductance G_H , in addition to the usual ohmic conductance G . The relation between G_H and G is given by

$$G_H = GU_H/U \qquad (6)$$

where U_H and U are the Hall-voltage and the external voltage (in current direction) respectively. The Hall conductivity is defined as

$$\sigma_H = \lim_{\mathcal{L}\to\infty} G_H \mathcal{L}^{2-d} \qquad (7)$$

Below the mobility edge $\sigma_H = 0$, since there can be no Hall current without an ohmic current. When the mobility edge is approached from above, σ_H presumably approaches zero according to

$$\sigma_H(E) \sim (E-E_c)^{t_H} \tag{8}$$

which defines the Hall conductivity exponent t_H .

I now make the basic assumption that the one-parameter scaling hypothesis of Abrahams et al.[1] is valid also for the Hall conductance G_H , at least in the critical region.[4] Then, for large $\mathcal{L}$ and near (above) the mobility edge , G_H can be written as (compare to Eq. (5))

$$G_H \sim h(\mathcal{L}/L)^{d-2} \Delta_0^{t_H}(L/L_0)^{t_H/\nu} \tag{9}$$

since for a large (metallic) sample G_H , as well as G , must be proportional to $\mathcal{L}^{d-2}$. The dimensionless parameter h is proportional to the magnetic field B , which is assumed to be weak (i.e. $h \ll 1$) . This parameter will generally depend on energy E , i.e. on Δ_0 . For instance, in the weak scattering limit $h = \omega_c\tau$, where ω_c is the cyclotron frequency and τ is the relaxation time. Eq. (9) , in complete analogy with Eq. (5), leads to

$$t_H = t = (d-2)\nu \tag{10}$$

The Hall constant R is proportional to σ_H/σ^2 and therefore diverges, for $d > 2$, near the mobility edge as

$$R(E) \sim (E-E_c)^{-t} \tag{11}$$

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References

1. E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979); E. Abrahams and T.V. Ramakrishnan, J. Non-Crystalline Sol. 35, 15 (1980)
2. D.J. Thouless, Proc. Les Houches Summer School, Session XXXI, July 3 - August 18, 1978, eds. R. Balian, R. Maynard and G. Toulouse (Amsterdam: North-Holland) (1979), p. 41
3. F.J. Wegner, Z. Phys. B25, 327 (1976)
4. This assumption, of course, can be true only if the magnetic field does not represent the second relevant scaling variable, as it does for instance in the theory of magnetic phase transitions.

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