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WHOLESALE WAREHOUSE INVENTORY CONTROL WITH STATISTICAL DEMAND I--ETC(U)
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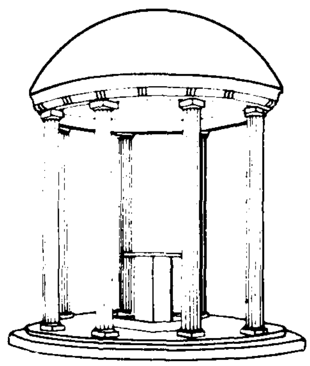

SCHOOL OF BUSINESS ADMINISTRATION
AND
CURRICULUM IN
OPERATIONS RESEARCH AND SYSTEMS ANALYSIS

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WHOLESALE WAREHOUSE INVENTORY CONTROL WITH STATISTICAL DEMAND INFORMATION.

Technical Report #15

Carl R. Schultz

December 1980

Work Sponsored By

Office of Naval Research (N00014-78-C0467)

Decision Control Models in Operations Research

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER Technical Report #15	2. GOVT ACCESSION NO. AD A095304	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) WHOLESALE WAREHOUSE INVENTORY CONTROL WITH STATISTICAL DEMAND INFORMATION		5. TYPE OF REPORT & PERIOD COVERED Technical 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Carl R. Schultz		8. CONTRACT OR GRANT NUMBER(s) N000-14-78-C-0467
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of North Carolina at Chapel Hill Chapel Hill, North Carolina 27514		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Mathematical and Information Sciences Division Office of Naval Research, Code 434 Arlington, Virginia 22217		12. REPORT DATE December 1980 13. NUMBER OF PAGES 74 and 75 (appendices)
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Inventory Control, (s,S) Policies, Multi-echelon Systems		

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Inventory managers often encounter erratic demand histories which are difficult to model. For example, periods of no demand are frequently observed, and when demand is positive, it tends to be quite large. Furthermore, periods of high demand are often followed by several periods of no demand. One possible explanation for this sporadic and correlated demand behavior is that demand originates from separate facilities which employ (s,S) replenishment policies. Each period's demand, therefore, ~~is the sum~~ cont.

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S/N 0102-014-6601

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) of the replenishment order quantities received from other inventory control facilities.

The management at a multi-item inventory system at the warehouse level, in which demand is comprised at the aggregated replenishment orders from lower-echelon inventory control facilities, was investigated by Schultz [1979]. Under the assumption that the warehouse observes only the aggregated replenishment orders, Schultz adapts an approximately optimal (s,S) policy rule (The Power Approximation of Ehrhardt [1976]), originally designed for independent and identically distributed demands, for use in a warehouse demand environment. The demand requirements of this new policy rule, referred to as the Correlation-Adjusted Power Approximation, are the mean, variance, and variance over one lead time of demand. For the situation in which these demand parameters are known exactly, Schultz empirically demonstrates that the operating characteristics of the policy rule are close to the operating characteristics of simulation-derived estimates of optimal (s,S) policies for a wide range of parameter settings.

In this investigation we evaluate, by means of computer simulation, the performance of the Correlation-Adjusted Power Approximation for the situation in which the decision-maker's knowledge of the warehouse demand process is limited to a sample of previously realized demands. In our simulation experiments, policy parameters are revised periodically using a fixed number of past demands to estimate the mean, variance, and variance over one lead time of the warehouse demand process. For our experimental design, we found the policy rule's performance to be quite good using a relatively small demand history. We also examine the accuracy of statistical forecasts that predict the future behavior of operating characteristics. We discuss forecasts of systemwide operating characteristics and of individual item characteristics. As a result, the manager of an inventory system is apprised of the extent of the bias in forecast estimates.

FOREWORD

As part of the on-going research program in "Decision Control Models in Operations Research," Mr. Carl R. Schultz investigates the behavior of multi-item inventory control systems at a warehouse level in which demand is comprised of the aggregated replenishment orders from lower-echelon inventory control facilities. In this environment, the warehouse demand probability distributions are sporadic and exhibit correlation from one time period to the next. This study builds on a previous one (Technical Report #14) by relaxing an assumed environment of perfect demand information to a setting where demand parameters are estimated from a limited sample of historical demands. Several sections of this report parallel similar findings in earlier reports. Other related reports dealing with the research program are listed on the following pages.

Harvey M. Wagner
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MacCormick, A. (1974), Statistical Problems in Inventory Control, ONR and ARO Technical Report 2, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A. S. and R. L. Kaufman (1975), Multi-Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 9), ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

Ehrhardt, R. (1975), Variance Reduction Techniques for an Inventory Simulation, ONR and ARO Technical Report 4, September 1975, School of Organization and Management, Yale University, 24 pp.

Kaufman, R. (1976), Computer Programs for (s,S) Policies Under Independent or Filtered Demands, ONR and ARO Technical Report 5, School of Organization and Management, Yale University, 65 pp.

Kaufman, R. and J. Klincewicz (1976), Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 6, School of Organization and Management, Yale University, 58 pp.

Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ONR and ARO Technical Report 7, June 1976, School of Organization and Management, Yale University, 106 pp.

Klincewicz, J. G. (1976), Biased Variance Estimators for Statistical Inventory Policies, ONR and ARO Technical Report 8, August 1976, School of Organization and Management, Yale University, 24 pp.

Klincewicz, J. G. (1976), Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 9, November 1976, School of Organization and Management, Yale University, 52 pp.

Klincewicz, J. G. (1976), The Power Approximation: Control of Multi-Item Inventory Systems with Constant Standard-Deviation-To-Mean Ratio for Demand, ONR and ARO Technical Report 10, November 1976, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Kaufman, R. L. (1977), (s,S) Inventory Policies in a Nonstationary Demand Environment, ONR and ARO Technical Report 11, April 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 155 pp.

Ehrhardt, R. (1977), Operating Characteristic Approximations for the Analysis of (s,S) Inventory Systems, ONR and ARO Technical Report 12, April 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 109 pp.

Schultz, C. R., R. Ehrhardt, and A. MacCormick (1977), Forecasting Operating Characteristics of (s,S) Inventory Systems, ONR and ARO Technical Report 13, December 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Schultz, C. R. (1979), (s,S) Inventory Policies for a Wholesale Warehouse Inventory System, ONR Technical Report 14, April 1979, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 75 pp.

ABSTRACT

Inventory managers often encounter erratic demand histories which are difficult to model. For example, periods of no demand are frequently observed, and when demand is positive, it tends to be quite large. Furthermore, periods of high demand are often followed by several periods of no demand. One possible explanation for this sporadic and correlated demand behavior is that demand originates from separate facilities which employ (s,S) replenishment policies. Each period's demand, therefore, is the sum of the replenishment order quantities received from other inventory control facilities.

The management at a multi-item inventory system at the warehouse level, in which demand is comprised at the aggregated replenishment orders from lower-echelon inventory control facilities, was investigated by Schultz (1979). Under the assumption that the warehouse observes only the aggregated replenishment orders, Schultz adapts an approximately optimal (s,S) policy rule (The Power Approximation of Ehrhardt [1976]), originally designed for independent and identically distributed demands, for use in a warehouse demand environment. The demand requirements of this new policy rule, referred to as the Correlation-Adjusted Power Approximation, are the mean, variance, and variance over one lead time of demand. For the situation in which these demand parameters are known exactly, Schultz empirically demonstrates that the operating characteristics of the policy rule are close to the operating characteristics of simulation-derived estimates of optimal (s,S) policies for a wide range of parameter settings.

In this investigation we evaluate, by means of computer simulation, the performance of the Correlation-Adjusted Power Approximation for the situation in which the decision-maker's knowledge of the warehouse demand process is limited to a sample of previously realized demands. In our simulation experiments, policy parameters are revised periodically using a fixed number of past demands to estimate the mean, variance, and variance over one lead time of the warehouse demand process. For our experimental design, we found the policy rule's performance to be quite good using a relatively small demand history. We also examine the accuracy of statistical forecasts that predict the future behavior of operating characteristics. We discuss forecasts of system-wide operating characteristics and of individual item characteristics. As a result, the manager of an inventory system is apprised of the extent of the bias in forecast estimates.

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1. (s,S) INVENTORY POLICIES FOR A WHOLESALE WAREHOUSE SYSTEM

In many inventory systems one often observes an erratic demand history whose underlying generating process is difficult to explain. For example, consider a situation in which periods of no demand are frequently observed, and when demand is positive it tends to be quite large. In addition, periods of positive demand are often followed by several periods of no demand. One possible explanation for this sporadic and correlated demand behavior is that the observed demand originates from other facilities which employ (s,S) replenishment policy rules. A natural setting for this type of demand behavior is a two-echelon inventory system, consisting of a number of lower-echelon facilities (stores) satisfying erogenous customer demand, and a single upper-echelon facility (warehouse) whose demand is comprised solely of replenishment orders placed by those stores.

The behavior of multi-item inventory control systems at a warehouse level, in which demand is comprised at the aggregated replenishment orders from lower-echelon inventory control facilities, was investigated by Schultz (1979). Under an assumption of limited warehouse demand information, Schultz adapts the Power Approximation of Ehrhardt (1976), which was originally designed for independent and identically distributed demands, for warehouse replenishment rules. The demand requirements of the modified policy rule are the mean, variance, and variance of demand over one leadtime. Schultz empirically investigates the performance of the policy rule when these demand parameters are known exactly. The rule performs well in the warehouse environment, yielding total costs that are typically within a few percent of those of a simulation-derived estimate of the minimum-cost stationary (s,S) policy.

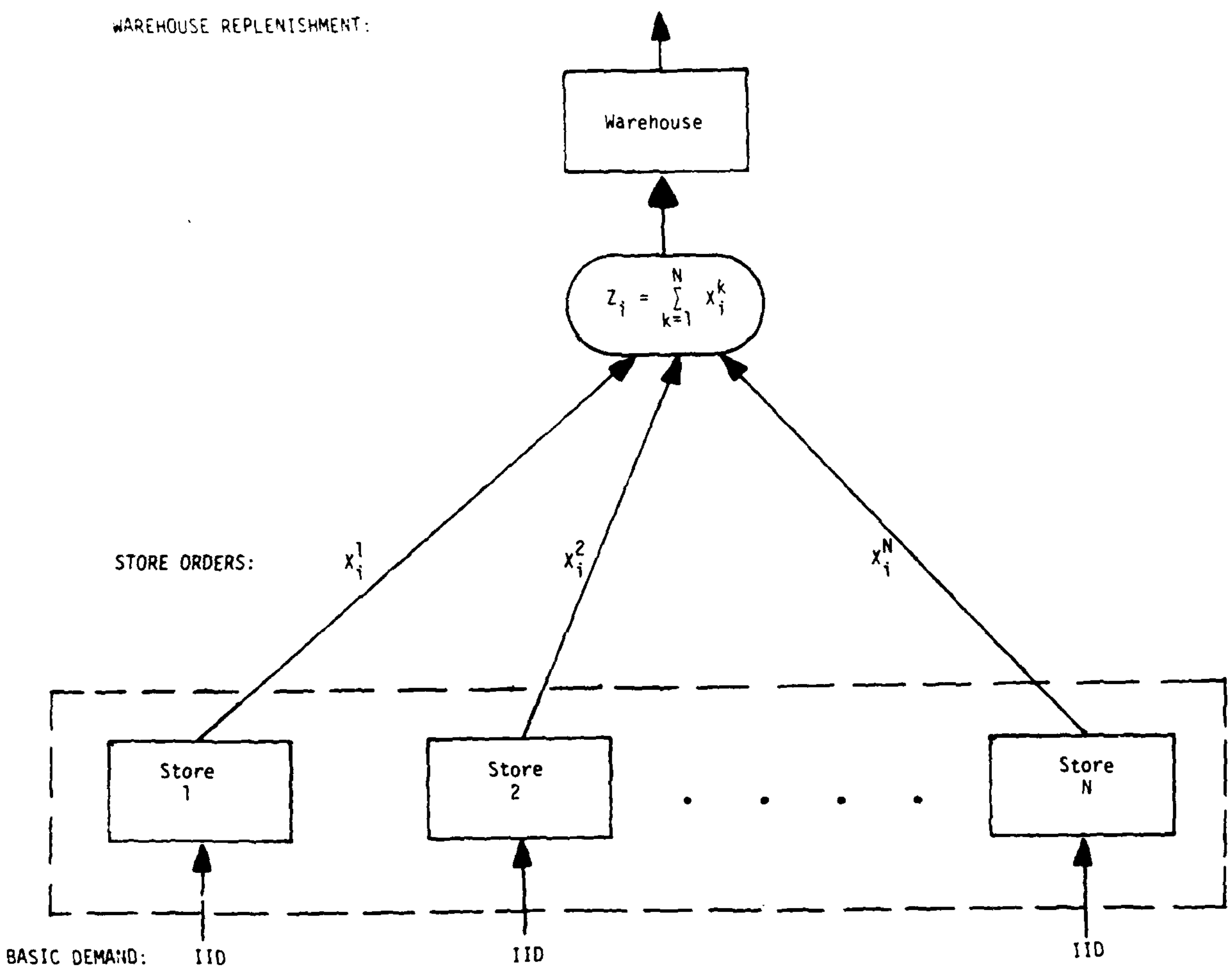
The primary goal of our study is to evaluate, by means of computer simulation, the performance of the policy rule for the situation in which the decision-makers knowledge of the warehouse demand process is limited to a sample of previously-realized demands.

1.1 A Wholesale Warehouse Inventory Model

We consider the management of a wholesale warehouse inventory system. We model the wholesale warehouse as the upper-echelon facility of the two-echelon inventory system depicted in Figure 1.1 . The system is designed to provide simple experimental designs for research purposes. We point out, however, that the policy rule developed for use at the warehouse is easily adopted to more complex systems with arbitrary autocorrelated demand processes.

We postulate a single-item, periodic review inventory model for each facility in the two-echelon system. We assume that demand at each facility is met as long as there is stock on hand, and when a stockout occurs, unfilled demand is backlogged until sufficient replenishments arrive. Items kept in inventory are assumed to be conserved, there being no losses by deterioration, obsolescence, or pilferage. Inventory on hand at the end of a given period is the inventory from the previous period plus any replenishment that arrives, less demand in the given period.

FIGURE 1.1 A TWO-ECHELON INVENTORY SYSTEM



If inventory on hand is negative, its absolute value is the amount of backlogged demand. The time sequence of events within any period is taken to be order, delivery, and demand.

Basic demand, which we assume is independent and identically distributed, enters the two-echelon inventory system at a lower-echelon comprised of N independently-operated stores in parallel. We make the additional assumption that control over replenishment at each store is exercised by an (s,S) policy: whenever inventory x on hand and on order is less than or equal to the value s, an order is placed for a replenishment of size $S - x$. The basic demand is then filtered through the (s,S) replenishment policy at each store and, disallowing transshipments of items between the stores, is passed on to the warehouse in the form of aggregated replenishment orders. Specifically, if X_i^k represents the order quantity received from store K in period i, then

$$Z_i = \sum_{k=1}^{N} X_i^k \tag{1.1}$$

is the entire demand realized at the warehouse in period i. Properties of the warehouse demand process $\{Z_i , i \geq 1\}$ were investigated by Schultz (1979). Simple expressions for the mean, variance, and autocorrelation function of the warehouse demand process were derived in terms of the stores' policy parameters and demand distributions. The warehouse demand properties are summarized in Section 1.5.1.

The dotted line in Figure 1.1 stresses that in our model the warehouse observes only the aggregate replenishment orders. Knowledge of the operations at individual lower-echelon facilities is assumed to be either unavailable or ignored in a deliberate attempt to decentralize inventory management. Thus, we desire cost effective easily computed warehouse replenishment rules which require, in addition to the warehouse economic parameters, only information directly obtainable from the observed warehouse demand process.

The cost structure at the warehouse is of a simple form. At the end of each period a unit holding cost h or a unit penalty cost p is incurred for each unit on hand or on backorder, respectively. The cost of a replenishment quantity is assumed to be linear with a fixed setup cost K, and warehouse replenishments are assumed to be delivered a fixed lead time L after being ordered. The criterion for optimal inventory control is minimization of the undiscounted expected cost per period over an infinite horizon.

Under the warehouse cost assumptions described above, a stationary (s,S) policy would be optimal if the warehouse demands were independent and identically distributed [Iglehart (1963 a,b)]. Unfortunately, warehouse demands in successive periods are dependent, and an optimal policy will not be of a stationary (s,S) form. Nevertheless, in this study we confine our attention to warehouse replenishment policies of the (s,S) form, since their simple form has led to their frequent use in applied situations.

1.2 Wholesale Warehouse Policy Rules

Recall that our criterion for optimal warehouse inventory control is minimization of the undiscounted expected cost per period over an infinite horizon. The complexity of the warehouse demand process, notably its autocorrelated behavior, makes the computation of a policy which meets this criterion prohibitive. Even the form of the optimal policy is very difficult to characterize. Nevertheless, as previously noted, we have confined our attention to the (s,S) form because of its popular use.

Even when we restrict the cost minimization to only those policies of a stationary (s,S) form, computational difficulties remain. Hence, we investigate approximately optimal (s,S) policy rules. Any investigation of approximately optimal (s,S) policy rules, however, would be fruitless unless we can establish a benchmark by which we can evaluate their performance. We obtain benchmark values by estimating an optimal stationary (s,S) policy via simulation. Specifically, we seek the stationary (s,S) policy that minimizes the total cost per period for a long history of generated warehouse demands; we refer to this policy as the <u>"very best" (s,S) policy</u>. The software package we use to estimate very best (s,S) policies is documented by Kaufman (1976). The methodology of his program is outlined in the following paragraphs.

A sequence of warehouse demands is generated, and a set of warehouse (s,S) policies is empirically tested through simulation on the generated demand sequence. The (s,S) policy with the smallest total cost per period is designated the very best (s,S) policy. In the software package a sequence of values for D is examined. For each D , a corresponding value for S is selected that minimizes the total per period using the generated demand sequence. The set of D values is determined by a Fibonacci search over a range of possible values for D [Wagner (1969)].

This technique would guarantee finding the very best (s,S) policyfor the generated demand sequence if the total cost per period was convex in D ; but such convexity is generally not present [Wagner, O'Hagan and Lundh (1965)]. Thus, the policy found may not be the very best (s,S) policy for the generated demand sequence. We refer to the policy resulting from the search technique described above as the "best (s,S) policy".

Once a best (s,S) policy has been found, we operate the policy on the same generated demand history and collect key operating characteristic values. Since the computed operating characteristic values are statistical estimates, confidence intervals are also computed. Kaufman's program also has an option that permits the computation of operating characteristic values and confidence intervals for an arbitrary (s,S) warehouse policy. By using this option, we evaluate the performance of other (s,S) policies by comparing their operating characteristic values with those of the best (s,S) policy on an identical warehouse demand sequence.

Although the simulation program discussed above computes nearly optimal (s,S) policies, its implementation in an actual wholesale warehouse inventory system is impractical. It requires knowledge of the warehouse demand distribution or, equivalently, the demand distribution and policy parameters at each store. This information usually is unavailable to the warehouse manager. In addition, deriving a policy rule by simulation for each item in a large multi-item system would be far too expensive. For these reasons we seek an easily-computed policy rule that requires only limited warehouse demand information, and that gives operating characteristic values close to those of best (s,S) policies.

1.2.1 The Correlation-Adjusted Power Approximation

The algorithm we use to determine values for the warehouse policy parameters s and S is an adaptation of the Power Approximation of Ehrhardt (1976), which was originally designed for independent and identically distributed demands, to a correlated demand environment. The Power Approximation, which is based on asymptotic renewal theory, computes values for the policy parameters s and S using only the mean μ and variance σ^2 of demand.

The Power Approximation is executed as follows.

Let

(1.2) $$D_p = (1.463)\mu^{.364}(K/h)^{.498}\sigma_L^{.1382}$$

and

(1.3) $$s_p = (L+1)\mu + \sigma_L^{.832}(\sigma^2/\mu)^{.187}(.220/z + 1.142 - 2.866z) ,$$

where

(1.4) $$z = \{D_p/[(1 + p/h)\sigma_L]\}^{.5} ,$$

and

(1.5) $$\sigma_L^2 = (L+1)\sigma^2 .$$

If D_p/μ is greater than 1.5, let $s = s_p$ and $S = s_p + D_p$. Otherwise, the empirical modification of Wagner (1969) is used.

The modification is based on the observation of Wagner, O'Hagan and Lundh (1965) that a μ grows large relative to K/h, the optimal policy converges to a single-critical-number. Therefore, when D_p/μ is sufficiently small, less than or equal to 1.5, S_p is compared with a single-critical-number which would be optimal if K were equal to zero. The smaller of these two numbers is then used as S in the policy, thereby reducing the separation between S and s. The single-critical-number used is one which would be optimal if demand followed a normal distribution and K were equal to zero. Define S_0 as

$$S_0 = (L + 1)\mu + v\sigma_L , \tag{1.6}$$

where v is the solution to

$$(2\pi)^{-\frac{1}{2}} \exp(-x^2/2)\, dx = p/(p + H) . \tag{1.7}$$

The policy parameters are given by

$$\begin{aligned} s &= \text{minimum } (s_p, S_0) \\ S &= \text{minimum } (S_p, S_0) . \end{aligned} \tag{1.8}$$

If demands are integer-valued, then s_p, D_p, and S_0 are rounded to the nearest integer.

Schultz [1979] adapted the Power Approximation to a correlated demand environment by replacing expression (1.5), which represents the variance of demand over $L+1$ periods when demands are independent and identically distributed, with the more general expression

$$(1.9) \qquad \sigma_L^2 = [(L+1) + 2\sum_{j=1}^{L} (L+1-j)\rho(j)]\sigma^2 ,$$

where $\rho(j)$ is the autocorrelation of the process at lag j. We refer to the Power Approximation with (1.9) replacing (1.5) as the Correlation-Adjusted Power Approximation. Notice that the Correlation-Adjusted Power Approximation does not rely on information about the structure of the multi-echelon replenishment system. It relies solely on information about the aggregate warehouse demand process. Accordingly, one must either be able to compute μ, σ^2 and σ_L^2 from known system parameters, or one must rely upon statistical estimates of these quantities based on historical records of demand. Schultz (1979) has demonstrated that when the former approach can be used the algorithm is an accurate approximation to best (s,S) policies found by simulation. In this study, we focus on the latter approach. That is, we extend the policy rule to an environment where demand parameters must be statistically estimated.

1.3 Evaluating the Performance of Warehouse Inventory Policies

The performance of warehouse inventory policies is empirically evaluated by comparing their key operating characteristics with those of best (s,S) policies in the same demand environment. The operating characteristics we examine are the expected values per period of period-end inventory, backlog quantity, frequency of period-end backlogs, replenishment quantity, frequency of replenishment and the total cost incurred.

We evaluate policies in a system of 72 independent inventory items with diverse parameter settings. The parameters are specified by using an analytical characterization of the warehouse demand process, which gives properties of warehouse demand as functions of store parameters. In the following subsections we present an analysis of the warehouse demand process, and specify parameter settings for our 72-item system.

1.3.1 The Warehouse Demand Process

We refer to the diagram of the warehouse replenishment system given in Figure 1.1 . Since warehouse demand is the sum of store replenishments, we first examine the replenishment process of a single following an (s,S) replenishment policy.

We assume the demand process at a store, denoted by q_1 , q_2 ,..., is a sequence of non-negative, integer-valued, independent and identically distributed random variables having cumulative distribution $\Phi(\cdot)$ and probability mass function $\phi(\cdot)$. Let $\Phi^n(\cdot)$ and $\phi^n(\cdot)$ be, for $n > 1$, the n-fold convolutions of $\Phi(\cdot)$ and $\phi(\cdot)$. Let $\phi^0(\cdot)$ represent the distribution whose full mass is located at zero.

We define the renewal functions

$$M(y) = \sum_{k=1}^{\infty} \Phi^k(y) \quad , \tag{1.10}$$

and

$$m(y) = \sum_{k=1}^{\infty} \phi^k(y) \quad . \tag{1.11}$$

Let μ_s and σ_s^2 denote, respectively, the mean and variance of the store's demand distribution. (We mention as a caution that the subscript s here denotes "store", and should not be confused with the (s,S) policy parameter.)

We assume that the store employs a stationary (s,S) replenishment policy. Let $D = S - s$ and note that the possible replenishment quantity values are 0 , $D+1$, $D+2$, ...

Using the store assumptions given above, we analyze the properties of the store's replenishment-quantity process. Several important results from this analysis are given by the following lemma.

<u>Lemma 1.1</u>:

If X_i represents the replenishment order quantity in period i , then

i) the stationary distribution of X_i is given by

$$\Pr[X_i = 0] = M(D)/[1 + M(D)] \; ,$$

and for $k = 0, 1, 2, \ldots$,

$$\Pr[X_i = D+k+1] = [\phi(D+k+1) + \sum_{j=0}^{D} \phi(j+k+1)m(D-j)]/[1 + M(D)] \; ,$$

ii) the mean of the stationary distribution, denoted by μ_r , satisfies

$$\mu_r = \mu_s \; ,$$

iii) the variance of the stationary distribution, denoted by σ_r^2 , satisfies

$$\sigma_r^2 = \sigma_s^2 + [2\mu_s \sum_{k=0}^{D} km(k)]/[1 + M(D)] \quad ,$$

and

iv) the correlation between replenishment order quantities separated by j periods, denoted by $\rho_r(j)$, satisfies the recursive relationship

$$\rho_r(j) = \begin{cases} -[\mu_r \sum_{k=0}^{D} k\phi(k)]/\sigma_r^2 & , \; j = 1 \\ -[\mu_r \sum_{k=0}^{D} k\phi^j(k)]/\sigma_r^2 - \sum_{\ell=1}^{j-1} \phi^{j-\ell}(D)\rho_r(\ell), & j > 1 \end{cases} \quad .$$

A proof of Lemma 1.1 is given in Schultz (1979).

We now proceed to examine properties of the warehouse demand process as functions of the store parameters. The properties of our wholesale warehouse inventory model, described in Section 1.1 , enable us to derive simple expressions for the mean, variance, and autocorrelation function of the warehouse demand process in terms of their counterparts for the single-store replenishment processes.

Let $\mu_{r,k}$, $\sigma_{r,k}^2$, and $\gamma_{r,k}(\cdot)$ be, respectively, the mean, variance, and autocovariance function of the replenishment process at store k . Let μ_w , σ_w^2 , and $\rho_w(\cdot)$ be, respectively, the mean, variance, and autocorrelation function of the demand process at the warehouse. Several important properties of the warehouse demand process are given in the following lemma.

Lemma 1.2:

A. If all stores operate independently, and have independent demand distributions, then

i) $\mu_w = \sum_{k=1}^{N} \mu_{r,k}$

ii) $\sigma_w^2 = \sum_{k=1}^{N} \sigma_{r,k}^2$

iii) $\rho_w(j) = \sum_{k=1}^{N} \gamma_{r,k}(j) / \sum_{k=1}^{N} \sigma_{r,k}^2$ for $j = 1,2,3, \ldots$;

B. Furthermore, if each store has the same demand distribution and replenishment policy, then

iv) $\mu_w = N\mu_r$

v) $\sigma_w^2 = N\sigma_r^2$

vi) $\rho_w(j) = \rho_r(j)$ for $j = 1,2,3,\ldots$.

A proof of Lemma 1.2 is given in Schultz (1979).

We use Lemmas 1.1 and 1.2 to set store parameters (μ_s , σ_s^2 , and D) so that the warehouse demand process has the desired properties.

1.3.2 Parameter Settings

We consider a set of 72 independent inventory items to be stocked at the warehouse; Table 1.1 lists the warehouse parameter settings. The four values for mean warehouse demand are 4, 8, 12, and 16. The variance-to-mean ratio of the warehouse demand process is 9 . Three values, L = 0, 2, and 4, are assigned to lead time. Since the cost function is linear in the parameters K , h , and p , the value of unit holding cost is normalized at unity. The penalty cost values are p = 4 , 9 , and 99 . The setup cost values are K = 32 and 64 .

Table 1.1

Warehouse Parameters

PARAMETER	PARAMETER SETTINGS	NUMBER OF SETTINGS
Mean Demand, μ_W	4, 8, 12, 16	4
Demand Variance/ Mean, σ_W^2/μ_W	9	1
Delivery Leadtime, L	0, 2, 4	3
Unit Holding Cost, h	1	1
Unit Penalty Cost, p	4, 9, 99	3
Ordering Setup Cost, K	32, 64	2

We use Lemmas 1.1 and 1.2 to set store parameters so that desired levels of correlation are achieved in the warehouse demand process, while still maintaining the specified values for warehouse demand mean and variance. We present the resulting experimental designs in the following paragraphs. See Schultz (1979) for details of the analysis.

To facilitate our research analysis, we specify two warehouse demand environments. In each environment we assign the same auto-correlation function to all of the 72 items listed in Table 1.1. The store parameters corresponding to these demand environments are given in Table 1.2 . Both demand environments assume a value of $D = 8$ and a negative binomial demand distribution at each store. The demand environments are characterized by the number of stores N in each, with one having four times as many stores as the other. In the few-stores environment each store has a mean of 4 , a variance of 12.80 , and a replenishment variance of 36 , while for the many-stores environment, the values are, respectively, 1 , 1.70 , and 9 . Thus, warehouse mean demand values of 4 , 8 , 12 , and 16 correspond to $N = 1$, 2 , 3 , and 4 for the few-stores environment and correspond to $N = 4$, 8 , 12 , and 16 in the many-stores environment.

Table 1.2

Store Parameters
($D = 8$, negative binomial demand distribution at each store)

DEMAND ENVIRONMENT	N	μ_s	σ_s^2	σ_r^2
Few Stores	$\mu_w/4$	4	12.80	36
Many Stores	μ_w	1	1.70	9

The warehouse autocorrelation functions for the demand environments are listed in Table 1.3 up to lag four. For the few-stores environment the autocorrelation function has a value of -0.30 at lag one and rapidly approaches zero for lags greater than one. For the many-stores environment the autocorrelation function has a value of -0.11 at lag one and slowly goes to zero as the lag number increases. The difference between the autocorrelation functions arises from the different values for the mean and variance of the store's demand distribution.

Table 1.3
Warehouse Autocorrelation Functions

DEMAND ENVIRONMENT	$\rho_W(1)$	$\rho_W(2)$	$\rho_W(3)$	$\rho_W(4)$
Few Stores	-0.30	-0.05	+0.03	+0.01
Many Stores	-0.11	-0.10	-0.09	-0.07

Figures 1.2 and 1.3 are plots of the warehouse demand distribution for each of the four values of mean demand in the few-stores environment. The distributions are quite sporadic, i.e., demand is most likely to be zero but when it is positive it tends to be rather large.

Figure 1.2

Probability Mass Functions for the Few-Stores Environment

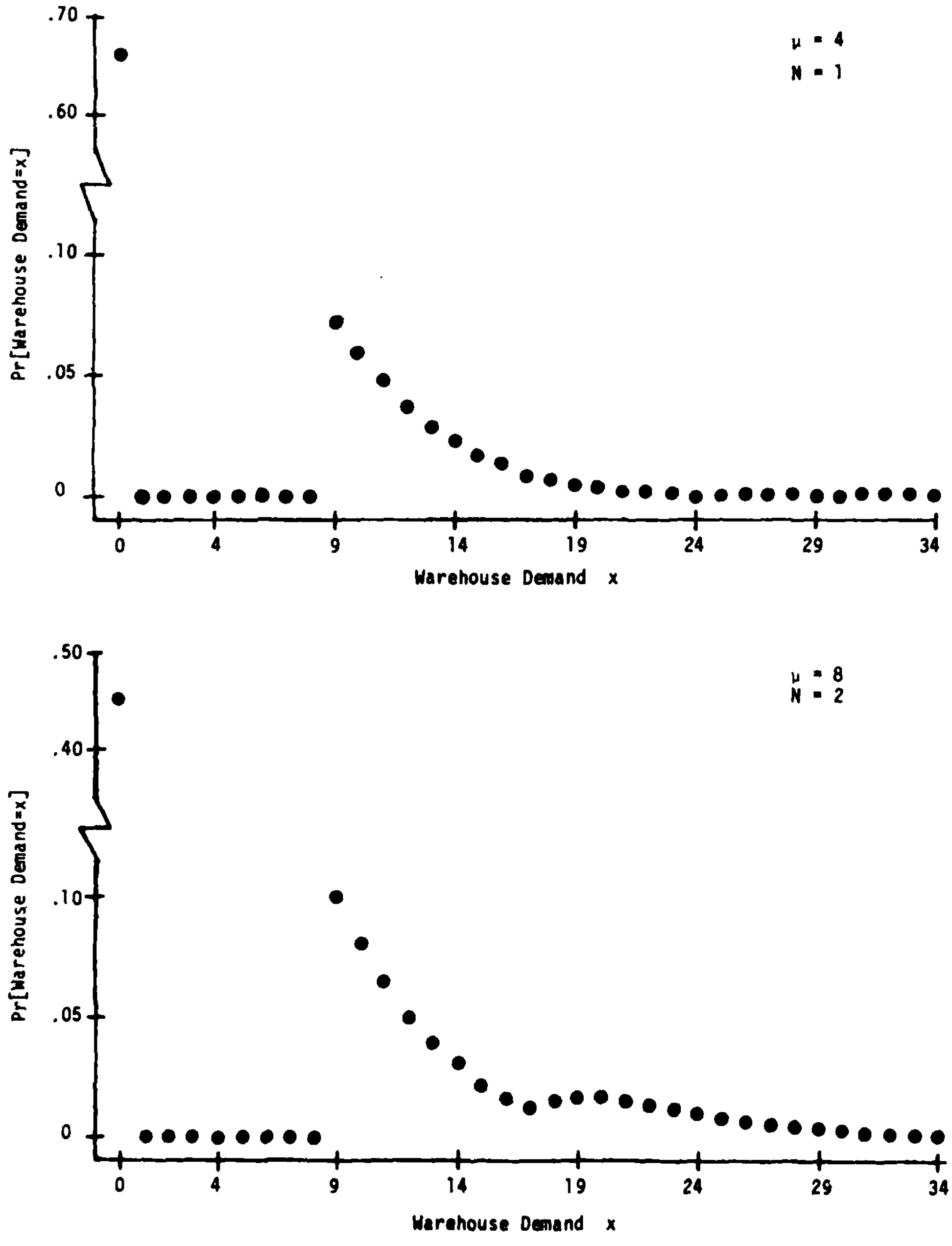
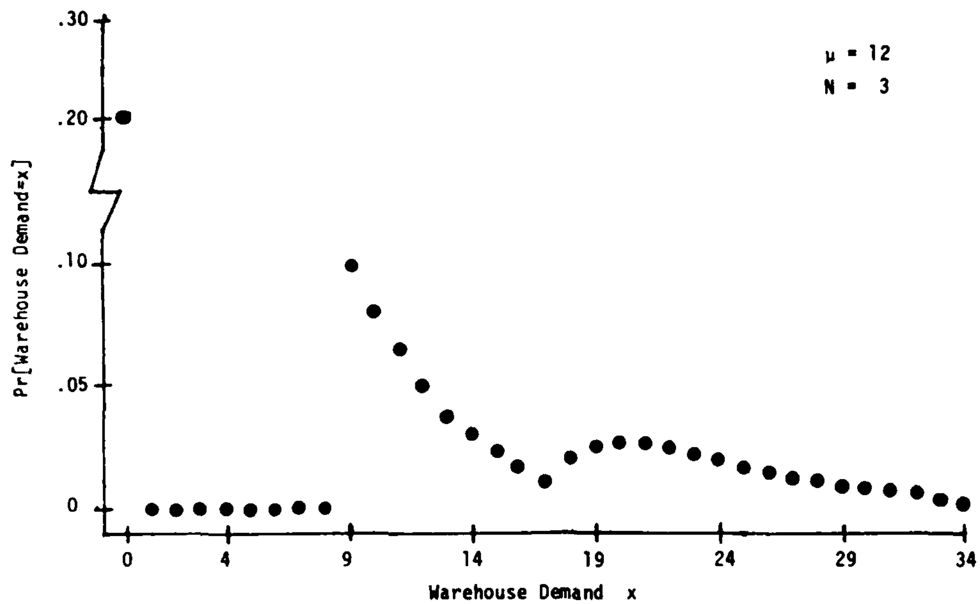
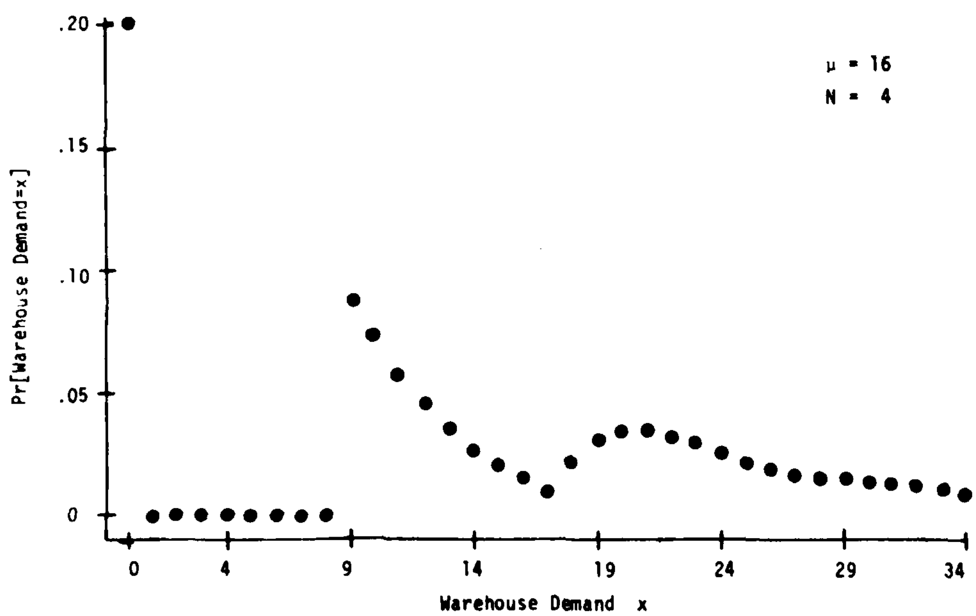


Figure 1.3

Probability Mass Functions for the Few-Stores Environment





Note for $N = 1$ the high probability of zero demand and the monotonically decreasing curve for demand values beyond $D + 1$. When $N = 2$ notice a slight increase in the probability mass begins at $2(D + 1)$. This is the effect of the second store. The effect of still more stores is to shift more mass into the tail of the distribution.

In Section 2 we empirically evaluate the performance of the Statistical Correlation-Adjusted Power Approximation policy rule for the experimental design given above. In Section 3 we examine the accuracy of statistical forecasts that estimate the future behavior of warehouse operating characteristics. In Section 4 we examine the robustness of the Correlation-Adjusted Power Approximation by evaluating its performance in a replenishment system with non-identical stores. Finally, in Section 5 we summarize our findings and suggest several ideas for improving the Correlation-Adjusted Power Approximation.

2. WHOLESALE WAREHOUSE INVENTORY CONTROL WITH STATISTICAL DEMAND INFORMATION

In this section we investigate the behavior of multi-item inventory control systems at the warehouse level when the only demand information available to the decision-maker is that provided by a finite sample from the realized demand history. Specifically, we assume that T periods of demand data are utilized to construct an inventory control policy for use during the next T weeks of operation. A simulation experiment is used to assess policy performance. In the experiment 200 replications of policy construction and operation are simulated. Policy performance is then evaluated by comparing average operating characteristic values with estimates of the expected operating characteristic values for the same systems controlled with full information and with best policies.

2.1 The Statistical Correlation-Adjusted Power Approximation

Recall that the demand information requirements of the Correlation-Adjusted Power Approximation are the mean, variance, and variance over one lead time of demand. When demand information is limited to a time series of previous warehouse demands, we obtain values for (s,S) by substituting statistical estimates for demand parameters in equations (1.2) through (1.6). We refer to policy parameters obtained in this manner as Statistical Correlation-Adjusted Power Approximation policies.

In this study, we assume that a warehouse demand history of T periods, with equal weight being given to each observation in this history, is used in constructing the policy parameters s and S for use during the next T periods of operation. This is not optimal if the demand process is known to be stationary, for then the entire history should be accumulated to give better knowledge and performance. Even when demand is known to be nonstationary but varying in a regular manner, such as by a trend or periodic cycle, or both, an optimal decision rule would generally utilize the entire history.

The decision-maker usually is not in a position to know, however, that conditions observed will continue to prevail. This provides justification for making frequent revisions and placing greater weight on demands from the immedicate past and less on earlier demands.

2.2 Estimating the Mean, Variance, and Lead Time Variance of Demand

The statistics required by our decision rule are the sample estimators of the mean, variance, and variance of demand over one lead time, denoted $\hat{\mu}$, $\hat{\sigma}^2$, and $\hat{\sigma}_L^2$, respectively. *Let* T *be the number* of observations of past demands, and let $t(\geq T)$ be the period in which a policy revision is to be made. We estimate the mean μ of the warehouse demand process $\{Z_i, i \geq 1\}$ by

$$\hat{\mu} = T^{-1} \sum_{\tau=1}^{T} Z_{t-\tau} , \tag{2.1}$$

the variance σ^2 by

$$\hat{\sigma}^2 = (T-1)^{-1} \sum_{\tau=1}^{T} (Z_{t-\tau} - \hat{\mu})^2 , \tag{2.2}$$

and the variance of demand over one lead time σ_L^2 by

$$\hat{\sigma}_L^2 = [(L+1) + 2 \sum_{j=1}^{L} (L+1-j)\hat{\rho}(j)]\hat{\sigma}^2 , \tag{2.3}$$

where

$$(2.4) \qquad \hat{\rho}(j) = \left[\sum_{\tau=1}^{T-j} (Z_{t-\tau}-\hat{\mu})(Z_{t-\tau+j}-\hat{\mu})\right] / [(T-j-1)\hat{\sigma}^2]$$

is the estimate of the correlation between two warehouse demands separated by j periods.

In some applied settings, especially in those where the number of inventoried items is quite large, the computational burden and data storage requirements of estimating L autocorrelation coefficients for each inventoried item can be large. In addition, if T is small, estimates of higher-order autocorrelation coefficients may be unstable. For these reasons we also consider an alternative estimate of σ_L^2 which uses only the first-order autocorrelation coefficient estimate $\hat{\rho}(1)$ and sets all the estimates of higher-order autocorrelation coefficients equal to zero. This alternate estimate for σ_L^2 is given by

$$(2.5) \qquad \hat{\sigma}_L^2 = [L(1+2\hat{\rho}(1)) + 1]\hat{\sigma}^2 \quad ,$$

and is termed the truncated lead time demand variance estimator.

2.3 The Performance of the Statistical Correlation-Adjusted Power Approximation

In this subsection, we present the results from the simulation of multi-item warehouse inventory systems controlled by the Correlation-Adjusted Power Approximation with statistical demand information. In the simulation experiments, average operating characteristic values are collected for 200 replications using a demand history length of 26 periods. The parameter settings for the multi-item systems studied are given in Section 1 . Comparisons are made with estimates of the operating characteristic values for the same systems controlled with full information and with best policies. In addition, the sensitivity of the operating characteristics to values of the input parameters is investigated.

We first examine the policy rule's performance when lead time demand variance is estimated using (2.3), which requires estimation of the first L autocorrelation coefficients. We then investigate the policy rule's performance using the truncated lead time demand variance estimator given by (2.5) which only requires estimation of the first-order autocorrelation coefficient $\rho(1)$.

2.3.1 Performance Using the First L Sample Autocorrelation Coefficients to Estimate Lead Time Demand Variance

Estimated expected total cost per period, and its components, under full and statistical demand information are shown for the few-stores environment in Table 2.1, and for the many-stores environment in Table 2.2. In addition, Tables 2.1 and 2.2 display estimates of absolute and percentage differences in expected costs per period when the Correlation-Adjusted Power Approximation, under full and statistical demand information, is compared with best (s,S) policy control. Total costs under statistical demand information are 7.8% and 10.9% above best policy's total cost for the few-stores and many-stores environments, respectively. Notice that under full information total costs are, respectively, 2.6% and 3.4% above best policy costs. From a practitioner's viewpoint, the results under statistical demand information are encouraging in that total costs are within 11% of best values, and are within 7.5% of performance

Table 2.1

Summary of the Performance of Correlation-Adjusted Power Approximation Policies for Few-Stores Warehouse Inventory Systems Under Full and Statistical Information

72-Item System

COST COMPONENT	FULL INFORMATION		STATISTICAL INFORMATION 26-REVISION INTERVAL	
	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES
INVENTORY	1930 (61.1)	183 [10.5]	2042 (61.5)	295 [16.9]
BACKLOG	461 (14.6)	- 5 [- 1.0]	501 (15.1)	35 [7.5]
REPLENISHMENT	768 (24.3)	-100 [-11.5]	777 (23.4)	- 91 [-10.5]
TOTAL	3160 (100.0)	79 [2.6]	3320 (100.0)	239 [7.8]

NOTE: Numbers in parentheses are percentages of total cost.

Numbers in brackets are percentage differences in cost components over best values.

Table 2.2

Summary of the Performance of Correlation-Adjusted Power Approximation Policies for Many-Stores Warehouse Inventory Systems Under Full and Statistical Information

72-Item System

COST COMPONENT	FULL INFORMATION		STATISTICAL INFORMATION 26-REVISION INTERVAL	
	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES
INVENTORY	2080 (63.1)	225 [13.3]	2227 (63.0)	392 [21.4]
BACKLOG	454 (13.8)	- 27 [- 5.5]	546 (15.4)	65 [13.7]
REPLENISHMENT	765 (23.2)	-109 [-12.5]	764 (21.6)	-110 [-12.6]
TOTAL	3299 (100.0)	109 [3.4]	3537 (100.0)	347 [10.9]

NOTE: Numbers in parentheses are percentages of total cost.

Numbers in brackets are percentage differences in cost components over best values.

levels with known demand parameters using a relatively small demand history of 26 periods. We also note, that our results are comparable to those obtained in a previous study (Ehrhardt [1976]) in which demand was independent and identically distributed. In that study, total costs were found to be 11.5% above optimal costs for a similar 72-item inventory system with a demand variance-to-mean ratio of 9 and a demand history of 26 periods controlled by the Statistical Power Approximation.

We next examine the individual components of total cost given in Tables 2.1 and 2.2. For both the few-stores and many-stores environment controlled with statistical demand information, inventory and backlog costs are above best values while replenishment costs are below best values. The distributions of costs are similar to those when the system is controlled with full information. The most notable difference is that with full information backlog costs are slightly below best values.

The cost component differences mentioned above indicate that one shortcoming of the Correlation-Adjusted Power Approximation is that, in general, it holds too much inventory and orders too infrequently. The major cause of this shortcoming can be attributed to the setting of $D \equiv S - s$. Detailed comparisons of best policy parameters with Correlation-Adjusted Power Approximation policy parameters, given in Table A9 of Appendices I, II, and III, demonstrates that the latter policy rule sets D too large for almost every item. The result is less frequent ordering and higher average inventory levels. At present, we are not aware of a good way to correct this discrepancy.

When the items in each system controlled with the Statistical Correlation-Adjusted Power Approximation are divided into subsystems according to values taken by the input parameters, the resulting percentages above best costs are shown in Tables 2.3 and 2.4 . Total cost data reveal that performance degrades with increasing penalty costs, increasing lead times, and decreasing setup costs. This behavior is similar for both demand environments, with the many-stores demand environment exhibiting slightly more pronounced trends. Notice that inventory costs are above best values for all parameter settings, especially for a high penalty cost or a low setup cost. Backlog costs are above best values for nearly all of the parameter settings. Note, however, that backlog costs are considerably less than best values when the penalty cost parameter is 99 . Furthermore, percentages above best backlog cost values tend to increase with lead time. This trend is opposite to what was observed by Schultz [1979] under full information. Finally, we note that replenishment costs are below best values for all parameter settings.

The percentage apportionment of aggregate costs per period for various parameter classifications is shown in Tables 2.5 and 2.6 for best policies and for Statistical Correlation-Adjusted Power Approximation policies. The distributions of costs are similar for both policy rules. The most notable difference is that the statistical policies yield a larger proportion in the inventory cost component and a smaller proportion in the replenishment component. Notice also that trends in percentage apportionment of aggregate costs, as functions of the input parameters, are similar for both policy rules.

Table 2.3

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System Controlled Under Statistical Demand Information

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
STAT. CORR.-ADJUSTED P.A.													
INVENTORY	16.9	1.4	9.5	27.8	22.4	12.1	16.5	17.6	16.6	19.0	20.3	13.4	16.5
BACKLOG	7.5	21.9	16.8	-25.5	- 4.0	18.7	3.4	1.7	15.3	10.8	8.2	5.5	6.9
REPLENISHMENT	-10.5	- 4.2	- 8.7	-17.5	-12.3	- 9.3	- 9.9	-11.3	-10.3	-11.5	-14.0	- 7.3	-10.1
TOTAL	7.8	4.0	5.3	11.9	9.4	6.4	6.3	7.0	9.6	9.6	8.4	6.5	7.4

Table 2.4

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System Controlled Under Statistical Demand Information

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
<u>STAT. CORR.-ADJUSTED P.A.</u>													
INVENTORY	21.4	5.7	13.2	32.7	27.6	15.8	18.0	20.7	24.3	26.9	23.2	18.3	19.9
BACKLOG	13.7	26.5	25.7	-22.1	1.9	25.2	3.4	13.8	22.1	13.1	29.4	3.2	12.4
REPLENISHMENT	-12.6	- 7.7	-12.7	-16.9	-15.3	-10.7	-14.3	-11.9	-11.4	-16.2	-20.4	- 4.1	-11.8
TOTAL	10.9	5.9	7.7	16.4	12.9	9.2	5.3	11.2	15.3	13.2	11.4	10.3	10.0

Table 2.5

Percentage Apportionment of Aggregate Costs Per Period for a 72-Item Warehouse Inventory System

Under Best and Statistical Correlation-Adjusted Power Approximation Policy Control

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
INVENTORY	56.7	11.9	16.5	28.2	26.5	30.2	16.3	18.9	21.4	9.4	12.8	16.3	18.3
BACKLOG	15.1	5.8	5.3	4.0	7.5	7.6	4.3	5.0	5.9	2.5	3.5	4.3	4.8
REPLENISHMENT	28.2	8.8	9.2	10.2	11.3	16.9	9.5	9.4	9.2	4.4	6.7	7.8	9.3
TOTAL	100.0	26.5	31.0	42.5	45.3	54.7	30.1	33.3	36.5	16.3	23.0	28.3	32.4
STAT. CORR.-ADJUSTED P.A.													
INVENTORY	61.5	11.3	16.3	33.5	30.1	31.4	17.7	20.7	23.2	10.4	14.3	17.1	19.7
BACKLOG	15.1	6.6	5.8	2.8	6.7	8.4	4.1	4.7	6.3	2.6	3.5	4.2	4.8
REPLENISHMENT	23.4	7.8	7.8	7.8	9.2	14.2	8.0	7.8	7.7	3.6	5.4	6.6	7.8
TOTAL	100.0	25.6	30.3	44.1	46.0	54.0	29.7	33.1	37.2	16.5	23.2	28.0	32.3

Table 2.6

Percentage Apportionment of Aggregate Costs Per Period for a 72-Item Warehouse Inventory System

Under Best and Statistical Correlation-Adjusted Power Approximation Policy Control

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
INVENTORY	57.5	12.1	16.7	28.8	27.0	30.5	15.5	20.2	21.7	9.5	12.9	16.6	18.5
BACKLOG	15.1	5.9	5.3	3.9	7.5	7.6	4.5	5.1	5.4	2.5	3.4	4.3	4.9
REPLENISHMENT	27.4	8.6	9.2	9.6	11.2	16.2	9.6	9.1	8.8	4.4	6.7	7.3	9.0
TOTAL	100.0	26.6	31.2	42.3	45.7	54.3	29.7	34.4	36.0	16.4	23.0	28.2	32.4
STAT. CORR.-ADJUSTED P.A.													
INVENTORY	63.0	11.5	17.0	34.5	31.1	31.9	16.5	22.1	24.4	10.9	14.4	17.7	20.0
BACKLOG	15.4	6.7	6.0	2.7	6.0	8.5	4.2	5.2	5.9	2.5	3.9	4.0	5.0
REPLENISHMENT	21.6	7.2	7.2	7.2	8.6	13.0	7.5	7.1	7.0	3.3	4.8	6.3	7.1
TOTAL	100.0	25.4	30.3	44.3	46.5	53.5	28.2	34.4	37.4	16.7	23.1	28.0	32.1

Tables 2.7 and 2.8 show the values of other operating characteristics of the systems under best policies and Statistical Correlation-Adjusted Power Approximation policies. The statistical policies display higher backlog frequencies and lower replenishment frequencies. Note that for best policies replenishment frequency increases with unit penalty cost while it remains nearly constant for the statistical policies. This lack of dependence on penalty cost is due to the fact that the Correlation-Adjusted Power Approximation expression for D is independent of p . Finally, percentages above best values for these same operating characteristics values of the statistical policies are given in Tables 2.9 and 2.10 . The results are consistent with trends noted for the cost components.

2.3.2 *Performance Using Only the First-Order Sample Autocorrelation Coefficient to Estimate Lead Time Demand Variance*

As we pointed out earlier, in some inventory systems the number of inventoried items is so large that the computional and data storage requirements, necessary to estimate the first L autocorrelation coefficients for each item in the system, are excessive. In addition, if the revision history is small, higher-order sample autocorrelation coefficient estimates may be quite unstable. For these reasons, we have considered an alternative lead time demand variance estimator, given by 2.5 , which only requires estimation of the first-order autocorrelation coefficient.

Tables 2.11 and 2.12 , corresponding to the few-stores and many-stores environments, display estimates of total cost per period and its components along with estimates of absolute and percentage

Table 2.7

Operating Characteristics of a 72-Item Warehouse Inventory System Under
Best and Statistical Correlation-Adjusted Power Approximation Policy Control

Few-Stores Environment

DECISION RULE AND OPERATING CHARACTERISTIC	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
Period-End Inventory	1747	369	508	870	816	931	503	583	660	290	395	501	563
Backlog Frequency	.097	.191	.093	.009	.098	.097	.096	.097	.099	.093	.098	.099	.099
Weighted Proportion of Demand Backlogged	.017	.186	.076	.005	.017	.018	.015	.017	.020	.029	.020	.017	.014
Replenishment Frequency	.264	.246	.258	.287	.303	.225	.260	.267	.256	.161	.253	.291	.351
STAT. CORR.-ADJUSTED P.A.													
Period-End Inventory	2042	374	557	1113	999	1043	586	686	770	345	474	568	655
Backlog Frequency	.107	.214	.100	.006	.100	.113	.104	.105	.111	.109	.107	.107	.105
Weighted Proportion of Demand Backlogged	.019	.227	.088	.004	.017	.021	.015	.017	.023	.032	.022	.017	.015
Replenishment Frequency	.235	.235	.235	.235	.266	.204	.241	.234	.230	.143	.217	.267	.313

Table 2.8

Operating Characteristics of a 72-Item Warehouse Inventory System Under Best and Statistical Correlation-Adjusted Power Approximation Policy Control

Many-Stores Environment

DECISION RULE AND OPERATING CHARACTERISTIC	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
Period-End Inventory	1835	385	532	918	861	974	494	646	694	303	413	528	589
Backlog Frequency	.097	.188	.095	.009	.097	.097	.095	.098	.099	.093	.097	.099	.100
Weighted Proportion of Demand Backlogged	.018	.195	.079	.005	.018	.018	.016	.018	.019	.029	.020	.017	.015
Replenishment Frequency	.267	.251	.268	.283	.310	.224	.283	.264	.254	.169	.258	.286	.355
STAT. CORR.-ADJUSTED P.A.													
Period-End Inventory	2227	408	601	1218	1100	1127	583	782	853	385	510	626	706
Backlog Frequency	.106	.213	.101	.006	.098	.115	.106	.106	.108	.103	.114	.102	.107
Weighted Proportion of Demand Backlogged	.020	.246	.099	.004	.018	.023	.017	.021	.024	.033	.026	.018	.016
Replenishment Frequency	.231	.231	.231	.231	.263	.200	.240	.229	.226	.145	.207	.269	.305

Table 2.9

Percentage Above Best (s,S) Operating Characteristic Values for a 72-Item Warehouse Inventory System
Under Statistical Correlation-Adjusted Power Approximation Policy Control

Few-Stores Environment

DECISION RULE AND OPERATING CHARACTERISTIC	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
STAT. CORR.-ADJUSTED P.A.													
Period-End Inventory	16.9	1.4	9.5	27.8	22.4	12.1	16.5	17.6	16.6	19.0	20.3	13.4	16.5
Backlog Frequency	9.4	12.1	7.9	-32.2	2.1	16.8	7.7	8.5	12.0	14.4	8.9	8.6	6.1
Weighted Proportion of Demand Backlogged	7.5	21.9	16.8	-25.5	- 4.0	18.7	3.4	1.7	15.3	10.8	8.2	5.5	6.9
Replenishment Frequency	-11.0	- 4.5	- 9.1	-18.3	-12.3	- 9.3	-10.8	-12.2	- 9.9	-11.1	-14.3	- 8.2	-10.8

Table 2.10

Percentage Above Best (s,S) Operating Characteristic Values for a 72-Item Warehouse Inventory System
Under Statistical Correlation-Adjusted Power Approximation Policy Control

Many-Stores Environment

DECISION RULE AND OPERATING CHARACTERISTIC	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
STAT. CORR.-ADJUSTED P.A.													
Period-End Inventory	21.4	5.7	13.2	32.7	27.6	15.8	18.0	20.7	24.3	26.9	23.2	18.3	19.9
Backlog Frequency	9.5	13.2	6.4	-35.3	0.4	18.6	11.2	8.3	9.0	11.4	16.7	3.1	7.0
Weighted Proportion of Demand Backlogged	13.7	26.5	25.7	-22.1	1.9	25.2	3.4	13.8	22.1	13.1	29.4	3.2	12.4
Replenishment Frequency	-13.4	- 7.8	-13.4	-18.3	-15.3	-10.7	-15.4	-13.2	-11.3	-14.4	-20.0	- 5.8	-14.1

differences over best values using both methods to estimate the lead time demand variance. For the few-stores environment, estimating only $\rho(1)$ results in only a 0.6% degradation in performance when compared with estimating the first L autocorrelation coefficients. The excellent results for this environment are not surprising since the theoretical higher-order autocorrelation coefficients are close to zero. For the many-stores environment, however, the theoretical higher-order autocorrelation coefficients are substantially non-zero, and yet the degradation in performance is less than 2% . Examining the individual components of total cost, we find that using only the first-order sample autocorrelation coefficient results in higher inventory costs and lower backlog and replenishment costs than using the first L sample autocorrelation coefficients to estimate σ_L^2 .

Percentages above best costs, when aggregated by the individual input parameters, are given in Tables 2.13 and 2.14 . Comparing with Tables 2.3 and 2.4 , we find that ignoring the higher-order autocorrelation coefficients results in higher total costs mainly for those items with higher penalty costs and, as one would expect, those items with long lead times. Notice, however, that the degradation is not severe even for these items.

In conclusion, the results in this subsection suggest that if one estimates only the first-order autocorrelation coefficient, it may be possible to attain performance levels which are near those obtained by estimating the first L autocorrelation coefficients. This limited estimation procedure would not be justified, of course, if the degradation in cost performance outweighs the additional cost of estimating the higher-order autocorrelation coefficients.

Table 2.11

Summary of the Performance of Correlation-Adjusted Power Approximation Policies for a 72-Item Warehouse Inventory System Under Statistical Information Using (2.3) and (2.5) to Estimate Lead Time Demand Variance

Few-Stores Environment

COST COMPONENT	PERFORMANCE USING FIRST L SAMPLE AUTOCORRELATION COEFFICIENTS		PERFORMANCE USING ONLY THE FIRST SAMPLE AUTOCORRELATION COEFFICIENTS	
	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES
INVENTORY	2042 (61.5)	295 [16.9]	2107 (63.1)	360 [20.6]
BACKLOG	501 (15.1)	35 [7.5]	459 (13.7)	- 7 [- 1.6]
REPLENISHMENT	777 (23.4)	- 91 [-10.5]	774 (23.2)	- 94 [-10.9]
TOTAL	3320 (100.0)	239 [7.8]	3339 (100.0)	258 [8.4]

NOTE: Numbers in parentheses are percentages of total cost.

Numbers in brackets are percentage differences in cost components over best values.

Table 2.12

Summary of the Performance of Correlation-Adjusted Power Approximation Policies for a 72-Item Warehouse Inventory System Under Statistical Information Using (2.3) and (2.5) to Estimate Lead Time Demand Variance

Many-Stores Environment

COST COMPONENT	PERFORMANCE USING FIRST L SAMPLE AUTOCORRELATION COEFFICIENTS		PERFORMANCE USING ONLY THE FIRST SAMPLE AUTOCORRELATION COEFFICIENTS	
	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES
INVENTORY	2227 (63.0)	392 [21.4]	2352 (65.5)	517 [28.2]
BACKLOG	546 (15.4)	65 [13.7]	481 (13.4)	0 [- 0.0]
REPLENISHMENT	764 (21.6)	-110 [-12.6]	759 (21.1)	-115 [-13.1]
TOTAL	3537 (100.0)	347 [10.9]	3592 (100.0)	402 [12.6]

NOTE: Numbers in parentheses are percentages of total cost.

Numbers in brackets are percentage differences in cost components over best values.

Table 2.13

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System Under Statistical Information Using (2.5) to Estimate Lead Time Demand Variance

Few-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
CORR. - ADJUSTED P.A.													
INVENTORY	20.6	4.1	12.6	32.3	26.4	15.6	16.5	21.0	23.4	23.6	24.1	17.1	19.8
BACKLOG	- 1.6	16.6	8.3	-40.8	-12.6	9.1	3.4	- 4.5	- 2.8	- 0.4	0.9	- 5.3	- 0.9
REPLENISHMENT	-10.9	- 4.7	- 8.9	-17.9	-12.5	- 9.7	- 9.9	-11.7	-11.0	-11.9	-14.5	- 7.7	-10.4
TOTAL	8.4	3.9	5.5	13.3	10.2	6.9	6.3	7.9	10.6	10.4	9.3	6.9	8.1

Table 2.14

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System Under Statistical Information Using (2.5) to Estimate Lead Time Demand Variance

Many-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
CORR. - ADJUSTED P.A.													
INVENTORY	28.2	10.3	18.9	41.1	35.3	21.8	18.0	23.9	39.3	33.9	30.1	24.8	26.8
BACKLOG	- 0.0	17.6	11.4	-42.1	-11.2	11.1	3.4	6.0	- 8.6	0.8	13.4	-10.7	- 0.2
REPLENISHMENT	-13.1	- 8.2	-13.3	-17.4	-15.8	-11.3	-14.3	-12.3	-12.6	-16.5	-20.7	- 5.0	-12.3
TOTAL	12.6	5.9	8.1	20.1	15.2	10.4	5.3	11.8	19.4	15.3	12.8	11.7	11.9

3. FORECASTING WAREHOUSE OPERATING CHARACTERISTICS WITH STATISTICAL DEMAND INFORMATION

The manager of an inventory system operating with statistical demand information will need to forecast system behavior. Forecasts may be needed to aid or justify the installation of scientific control. It may also be a routine requirement because of periodic revision of the control parameters or even as part of a regular budgeting procedure. In this section we are interested in examining the accuracy of statistical forecasts that predict the future behavior of warehouse operating characteristics.

Consider an inventoried item whose control parameters for the next T periods of operation are constructed using the previous T periods of demand data. A forecast f is to be made of the average value of each operating characteristic of interest over the next T periods of operation. Let a be the actual realization of the average value of the operating characteristic over the same interval. The major issues to be resolved here are the extent of the bias B of the forecast, defined as

$$B = E(a - f) \ , \tag{3.1}$$

and the level of dispersion D of the forecast, defined as

$$D = [\mathrm{Var}(a-f)]^{\frac{1}{2}} \ . \tag{3.2}$$

Note that our definition of bias is the negative of the definition usually given for bias, since our biases tend to run in this direction.

We estimate B and D with the aid of a simulation experiment. In the experiment 200 replications of policy construction, performance forecasting, and operation are simulated. Let f_t and a_t be, respectively, the forecast and actual average operating characteristic values for replication t . Consider the time series of differences between the forecasts and the subsequently realized values, $\{b_t = a_t - f_t ; t = 1, 2, \cdots 200\}$, resulting from the simulation experiment. We estimate the bias B by the mean

$$\bar{b} = (200)^{-1} \sum_{t=1}^{200} b_t \tag{3.3}$$

of the time series, and the level of dispersion D by the variance

$$\hat{\sigma}_B^2 = (200)^{-1} \sum_{t=1}^{200} (b_t - \bar{b})^2 \tag{3.4}$$

of the time series.

Previous forecasting studies [MacCormick (1974), Estey and Kaufman (1975), Ehrhardt (1976), and Kaufman (1977)] have utilized a method known as "retrospective simulation" to forecast system behavior. Briefly, this method entails using the same sample of previously realized demands both to set the policy parameters s and S , and to forecast the system's performance. The forecast is constructed by simulating how the policy would have performed in response to these demands. In this report we have also adopted this forecasting method.

The forecasting studies mentioned above have all demonstrated that the double use of the demand history, for control and forecasting, results in forecasts which are biased in the positive direction; that is, the forecasts tend to underestimate the realized operating characteristic values. In our study of statistical forecasts in a warehouse demand environment, we also observe this phenomenon. We note, however, that in our demand environment the forecast bias observed is substantially less than that found in previous studies where demand was independent and identically distributed. We believe that the decrease in forecast bias can, for the most part, be attributed to the negatively correlated nature of the warehouse demand process, which results in a lower variance of demand over a given revision interval.

3.1 Properties of Multi-Item Forecasts of Warehouse Operating Characteristics

In our study, forecasts of costs per period, for the few-stores and many-stores multi-item inventory systems, have been computed for every item in the systems. Little consideration has been given to an important sampling problem that would be present in a large system. For such a system it is prohibitively expensive to gather a history of demands for every item. Therefore, a representative sample of items must be selected to make an overall system forecast. The accuracy of such a system forecast is likely to be critically dependent on the number of items in the sample and its composition. The process of choosing a representative sample from a large system is presently an open topic for research, and beyond the scope of this study.

We estimate the bias of forecasts of expected costs per period for a multi-item system by the sum of the corresponding estimated biases for each item in the system. In turn, we estimate the dispersion of system forecasts by the square root of the sum of the squared dispersion levels for each item in the system.

The accuracy of the statistical forecasts of several key system operating characteristics is presented in Tables 3.1 and 3.2 .

Table 3.1

Accuracy of Statistical Forecasts of System Operating Characteristics for the 72-Item Few-Stores Warehouse Inventory System Controlled with Statistical Demand Information

(Revision Interval of 26 Periods; Revision History of 26 Periods)

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS	PERCENTAGE UNDERESTIMATE	LEVEL OF DISPERSION
HOLDING QUANTITY	2042.0000	2030.0000	0012.000	0000.6000	0001.6100
BACKLOG QUANTITY	0076.5000	0065.1000	0011.4000	0014.9000	0000.5800
BACKLOG FREQUENCY	0000.1065	0000.0992	0000.0073	0006.8000	0000.0464
REPLENISHMENT FREQUENCY	0000.2350	0000.2341	0000.0009	0000.4000	0000.0182
TOTAL COST	3320.0000	3160.0000	0160.0000	0004.8000	0006.6300

Table 3.2

Accuracy of Statistical Forecasts of System Operating Characteristics for the 72-Item Many-Stores Warehouse Inventory System Controlled with Statistical Demand Information

(Revision Interval of 26 Periods; Revision History of 26 Periods)

OPERATING CHARACTERISTIC	AVERAGE ACTUAL VALUE	AVERAGE FORECASTED VALUE	BIAS	PERCENTAGE UNDERESTIMATE	LEVEL OF DISPERSION
HOLDING QUANTITY	2227.0000	2221.0000	0006.0000	0000.3000	0001.6800
BACKLOG QUANTITY	0083.8000	0074.1000	0009.7000	0011.5000	0000.5230
BACKLOG FREQUENCY	0000.1064	0000.1011	0000.0053	0005.0000	0000.0410
REPLENISHMENT FREQUENCY	0000.2314	0000.2312	0000.0002	0000.1000	0000.0174
TOTAL COST	3537.0000	3397.0000	0140.0000	0004.0000	0006.9700

Notice that in both the few-stores and many-stores environments, the forecasts underestimate each average operating characteristic value. The forecasts of holding quantity and replenishment frequency incur small percentage errors. Backlog quantity, backlog frequency, and total cost, however, have percentage errors of 14.7% , 6.8% , and 4.8% , respectively, for the few-stores environment, and 11.5% , 5.0% , and 4.0% , respectively, for the many-stores environment. Most of the bias in forecasting total cost is due to the large underestimation of backlog quantities. Notice that for backlog quantity, backlog frequency, and total cost the bias of the forecasts is significant in comparison with the dispersion level of the forecasts. Thus, it may be worthwhile to investigate making a correction to each forecast of these system operating characteristics if the sample of items used in making the forecasts is sufficiently large and representative of the overall structure of the multi-item system.

We pointed out earlier that in our demand environment the forecast bias observed is substantially less than that found in previous studies where demand was assumed to be independent and identically distributed. To illustrate, Ehrhardt (1976) evaluated the behavior of statistical forecasts for a similar 72-item system with independent and identically distributed negative binomial demands having a variance-to-mean ratio of 9 . Using the same simulation experiment to evaluate forecasting accuracy, Ehrhardt found that retrospective forecasts of backlog frequency, backlog cost, and total cost had percentage errors of 16.3% , 59.3% , and 17.1% , respectively. Note that the percentage errors of backlog and total cost are, respectively, about two and four times greater than those observed in our demand environment.

To check sensitivity of the bias of the forecasts to individual system parameters, Tables 3.3 and 3.4 were constructed showing percentage values for the biases. Total cost data in Table 3.3 reveal that forecasting accuracy diminishes with increasing penalty cost and lead time, and with decreasing setup cost. Holding and replenishment costs are accurately predicted for all parameter settings. Backlog costs, however, are severely underestimated; especially, for those items with a unit penalty cost of 99 . Notice that the trends in the bias of the total cost forecasts as functions of the input parameters are directly attributable to similar trends found in the bias of backlog cost forecasts. Finally, Table 3.4 presents biases of forecasts of operating characteristics other than costs. Notice that while replenishment frequency is accurately forecasted for all parameter settings, backlog frequency is accurately forecasted only for those items with zero lead time and a penalty cost of four.

3.2 Properties of Single-Item Forecasts of Warehouse Operating Characteristics

Table A7 of Appendices I and II displays the estimated percentage bias of total cost per period for each item in the few-stores and many-store multi-item systems, respectively. The table has been constructed by the analysis of the time series of differences between the forecasts and the values subsequently realized for the operating characteristics.

The estimators of the forecast bias B and its dispersion level D are given by equations (3.3) and (3.4) , respectively. Recall that our definition of bias implies that the bias is positive if the

Table 3.3

Forecasting for a Multi-Item System Under Statistical Control:

Estimated Percentage Difference Between Forecasts and Average Values of Operating Characteristics

(Rev sion Interval 26 Periods, Revision History 26 Periods, Forecasting History 26 Periods)

COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
72-ITEM SYSTEM WITH WAREHOUSE DEMANDS: <u>FEW-STORES</u>													
INVENTORY	0.6	1.5	0.9	0.1	0.6	0.5	0.2	0.3	1.1	0.4	0.6	0.6	0.6
BACKLOG	28.9	10.9	22.3	85.0	31.4	27.0	11.2	25.8	42.8	28.1	27.5	28.7	30.7
REPLENISHMENT	0.4	0.5	0.4	0.3	0.3	0.4	0.6	0.4	0.2	0.8	0.3	0.7	0.0
TOTAL	4.8	3.6	4.8	5.5	5.1	4.6	1.8	3.9	8.0	4.8	4.6	4.9	4.9
72-ITEM SYSTEM WITH WAREHOUSE DEMANDS: <u>MANY-STORES</u>													
INVENTORY	0.3	0.9	0.3	0.0	0.4	0.2	- 0.2	0.4	0.5	0.1	0.4	0.0	0.5
BACKLOG	24.4	7.7	18.3	79.0	27.9	21.7	11.9	23.2	34.5	26.1	24.6	24.9	23.1
REPLENISHMENT	0.1	0.0	0.2	0.0	0.1	0.0	0.2	- 0.0	0.0	0.3	0.5	- 0.3	0.1
TOTAL	4.0	2.5	3.9	4.9	4.4	3.6	1.8	3.8	5.8	4.1	4.5	3.5	3.9

Table 3.4

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Average Values of Operating Characteristics

(Revision Interval 26 Periods, Revision History 26 Periods, Forecasting History 26 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS												
		PENALTY			SETUP		LEADTIME			MEAN				
		4	9	99	32	64	0	2	4	4	8	12	16
72-ITEM SYSTEM WITH WAREHOUSE DEMANDS: <u>FEW-STORES</u>													
Period-End Inventory	0.6	1.5	0.9	0.1	0.6	0.5	0.2	0.3	1.1	0.4	0.6	0.6	0.6
Backlog Frequency	6.8	3.0	11.0	74.4	8.5	5.4	0.5	6.2	13.3	7.8	6.5	6.6	6.5
Weighted Proportion of Demand Backlogged	28.9	10.9	22.3	85.0	31.4	27.0	11.2	25.8	42.8	28.1	27.5	26.7	30.7
Replenishment Frequency	0.4	0.4	0.4	0.3	0.3	0.4	0.6	0.4	0.2	0.8	0.3	0.6	0.1
72-ITEM SYSTEM WITH WAREHOUSE DEMANDS: <u>MANY-STORES</u>													
Period-End Inventory	0.3	0.9	0.3	0.0	0.4	0.2	- 0.2	0.4	0.5	0.1	0.4	0.0	0.5
Backlog Frequency	5.0	1.8	8.2	65.9	6.4	3.9	1.8	4.8	8.5	6.2	5.4	5.1	3.4
Weighted Proportion of Demand Backlogged	24.4	7.7	18.3	79.0	27.9	21.7	11.9	23.2	34.5	26.1	24.6	24.9	23.1
Replenishment Frequency	0.1	- 0.0	0.3	0.1	0.1	0.0	0.2	0.1	0.1	0.2	0.5	- 0.2	0.1

realized value of an operating characteristic exceeds its forecasted value. To test whether bias is significantly positive or negative, a two-tailed large sample test has been applied at the 0.05 level of significance, assuming that the sample mean differences for the 200 observations are normally distributed. The standard deviation used to construct the unit normal test statistic is the estimated forecast dispersion level. Results of the tests are shown in column (2) of Table A7 of Appendices I, II, and III. Note that the forecast bias of total cost is significantly positive for 47 of the 48 items which have a replenishment lead time of four, and is significantly positive for 41 of the 48 items which have a replenishment lead time of two. The forecast bias of backlog quantity follows a similar pattern. The forecast bias of backlog quantity is significantly positive for 46 and 38 items when the replenishment lead time is, respectively, four and two. A tendency for backlog frequency to be significantly underestimated also shows up for longer replenishment lead times.

4. A WAREHOUSE INVENTORY SYSTEM WITH HETEROGENEOUS STORE PARAMETERS

In Section 2 we empirically evaluated the performance of the Statistical Correlation-Adjusted Power Approximation in two multi-item inventory systems. The multi-item systems were characterized by the number of identical stores N generating warehouse demand through their replenishment orders, with one having four times as many stores as the other. In the few-stores environment, each store had a mean demand of 4 and a variance of 12.8 ; in the many-stores environment, these values were 1 and 1.7 , respectively . In both environments, the difference in the store's parameters s and S was 8 . The results of Section 2 clearly demonstrate that, for both multi-item systems, the policy rule provides a good approximation to best (s,S) policies found by simulation.

The assumption of identical stores, while convenient for research purposes, is admittedly an artificial and unrealistic assumption. A natural question arises as to whether or not the assumption is a significant factor influencing system performance. In this section we drop the assumption of identical stores and test the Correlation-Adjusted Power Approximation in a more realistic demand environment.

We begin by presenting an experimental design in which the store parameters are heterogeneous. We then evaluate the performance of the policy rule in this heterogeneous-stores environment under full and statistical demand information. The results of our experiments indicate that performance is as good or better than that observed under the assumption of identical stores.

4.1 An Experiment Design with Heterogeneous Store Parameters

As before, we consider a full-factorial 72-item warehouse inventory system. The warehouse parameter values are the same as those used in the previous design, and are given in Table 1.1 . The four values for mean warehouse demand are 4 , 8 , 12 , and 16 . The variance-to-mean ratio of the warehouse demand process is 9 . Three values, $L = 0$, 2 , and 4 , are assigned to the replenishment lead time. The penalty cost values are $p = 4$, 9 , and 99 , and the setup cost values are $K = 32$ and 64 .

The store parameters are chosen so that stores have different (s,S) policies and demand distributions. Our design consists of 12 different lower-echelon configurations, three for each of the 4 mean warehouse demand values, with six warehouse items assigned to each configuration.

At each store we assume a negative binomial demand distribution. From the results given in Section 1 , it is easy to show that if a store has a negative binomial demand distribution and uses an (s,S) policy, then the stationary replenishment process at the store is completely characterized by the mean μ_s and variance σ_s^2 of the store's demand distribution and the difference D in its policy parameters s and S . In order to obtain a broad range of D values, and, at the same time guarantee a warehouse demand variance-to-mean ratio of 9 . we make the additional assumption that σ_r^2/μ_s is 9 for each store, where σ_r^2 is the variance of the store replenishment process. We also inject additional realism into our design by setting D at each store approximately equal to $4\sqrt{\mu_s}$. This corresponds roughly to a Wilson lot-size model at each store with $K/h = 8$.

A complete description of the 12 lower-echelon configurations is given in Table 4.1 . The entries in the table are the number of stores in a given design with a given vector of store parameters (μ_s, σ_s^2, D) . For example, design V has one store with a mean of 1 , a variance of 6.2 , and a D value of 4 , two stores each with a mean of 2 , a variance of 9.2 , and a D value of 6 , and another store with a mean of 3 , a variance of 11.7 , and a D value of 7. Notice the broad range of parameter settings. Store mean demand values range from 0.33 to 6 , and D values range from 2 to 10 . Within a given design, the number of stores ranges from 1 to 12 , and the number of different types of stores ranges from 1 to 4 .

Table 4.2 lists the autocorrelation functions of the warehouse demand process up to lag 4 for each of the 12 designs of Table 4.1. The diversity of the autocorrelation functions is evidenced by the first-order autocorrelation coefficients which range from -.09 in design VI to -.43 in design IV . Design X has a first-order autocorrelation coefficient of -.38 and rapidly approaches zero for higher lags. Design XII , on the other hand, has a first-order autocorrelation coefficient of -.10 and slowly goes to zero for higher lags.

4.2 The Performance of the Correlation-Adjusted Power Approximation with Full Demand Information

A summary of the performance of the Correlation-Adjusted Power Approximation in the heterogeneous-stores environment, described above is given in Table 4.3 for the case of known mean, variance, and

Table 4.1

Heterogeneous Stores Environment

[Entries in table are the number of stores in each design with a given vector (μ_s , σ_s^2 , D) of store parameters]

WAREHOUSE MEAN μ_w	DESIGN	μ_s / σ_s^2 / D	0.33 2.60 2.00	1.00 6.20 4.00	1.33 7.20 5.00	2.00 9.20 6.00	2.33 11.00 7.00	3.00 11.70 7.00	4.00 12.80 8.00	5.00 12.50 9.00	6.00 10.50 10.00
4	I								1		
	II			1				1			
	III		1		1		1				
8	IV					1					1
	V			1		2		1			
	VI		1		4		1				
12	VII					1			1		1
	VIII			2		2		2			
	IX		2		5		2				
16	X					1		1		1	1
	XI			2		4		2			
	XII		3		6		3				

Table 4.2

Warehouse Autocorrelation Coefficients

DESIGN	ρ(1)	ρ(2)	ρ(3)	ρ(4)
I	-.30	-.05	+.02	+.01
II	-.17	-.07	-.01	+.00
III	-.11	-.06	-.03	-.01
IV	-.43	+.09	+.00	-.03
V	-.15	-.06	-.02	-.00
VI	-.09	-.05	-.03	-.01
VII	-.39	+.04	+.01	-.02
VIII	-.16	-.07	-.02	-.00
IX	-.10	-.05	-.03	-.01
X	-.38	+.03	+.02	-.02
XI	-.15	-.07	-.02	-.00
XII	-.10	-.06	-.03	-.01

lead time variance of demand. Estimated average total cost per period, and its components, are shown together with estimates of absolute and percentage increases over the corresponding best values. Notice that the estimated total cost per period is within 2% of the corresponding best value. Examination of the individual cost components reveals a familiar characteristic of the Correlation-Adjusted Power Approximation, higher-than-best inventory costs being offset somewhat by lower-than-best backlog and replenishment costs. Comparison with the policy rule's performance in the homogeneous store environments, given in Table 2.1 and 2.2 , demonstrates that not only has a high overall performance level been maintained, but, in fact, we find a modest reduction in the absolute difference with best values, especially for the holding cost component.

The percentage above best values of total cost and its components is broken down by individual parameter settings in Table 4.4 . Observe that, with the exception of those items with a high unit penalty cost, the Correlation-Adjusted Power Approximation total cost performance is close to best performance. Inventory costs are higher than the corresponding best values for all parameter settings except a unit penalty cost of 4 . In addition, inventory cost performance degrades with increasing penalty cost and replenishment lead time, and with decreasing setup costs and mean demand. Since holding costs account for over 60% of total costs, these trends appear in the total cost component as well. Backlog costs fluctuate above and below best values. They tend to go from above to below best values with increasing penalty cost and replenishment lead time and with decreasing setup cost.

Table 4.3

Summary of the Performance of the Correlation-Adjusted Power Approximation for a 72-Item Warehouse Inventory System with Heterogeneous Store Parameters Under Full Demand Information

(26 Period Revision Interval, and a 26 Period Revision History)

COST COMPONENT	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUE
INVENTORY	2031 (61.1)	162 [8.7]
BACKLOG	530 (16.0)	- 6 [- 1.0]
REPLENISHMENT	761 (22.9)	- 95 [-11.1]
TOTAL	3322 (100.0)	62 [1.9]

NOTE: Numbers in parentheses are percentages of total cost.

Numbers in brackets are percentage differences in cost components over best values.

Table 4.4

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System Controlled Under Full Demand Information

Heterogeneous-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
CORR. - ADJUSTED P.A.													
INVENTORY	8.7	- 2.6	1.8	17.2	11.2	6.4	6.7	8.6	10.2	10.4	9.2	8.3	7.8
BACKLOG	- 1.0	16.8	9.9	-38.3	-11.3	9.1	8.9	- 3.5	- 5.8	- 0.1	2.3	- 5.9	- 0.0
REPLENISHMENT	-11.1	- 6.5	- 7.7	-18.0	-11.1	-11.0	-11.3	- 9.2	-12.6	-13.9	-14.0	- 8.4	- 9.8
TOTAL	1.9	0.6	0.7	3.5	2.1	1.7	1.4	2.1	2.1	2.5	1.9	1.7	1.8

Finally, observe that replenishment costs are below best values for all parameter settings.

The percentage apportionment of aggregate costs per period for various parameter settings is shown in Table 4.5 for best policies and for Correlation-Adjusted Power Approximation policies. The distribution of costs is similar for both policy rules. Total cost figures are nearly identical for the two policy rules. The only noticeable difference exists in replenishment costs. The Correlation-Adjusted Power Approximation again lacks the slight dependence on unit penalty cost that is evidenced by the best policies.

Table 4.6 shows the values of operating characteristics other than costs for the multi-item system under best and Correlation-Adjusted Power Approximation Control. Backlog frequency under the latter policy is higher, and displays a slight dependence on replenishment lead time which is absent under best policy control. Replenishment frequency is lower under Correlation-Adjusted Power Approximation control and, as with replenishment costs, lacks the dependence on p which is present under best policy control. Finally, Table 4.7 displays the percentages above best values for these same operating characteristics of the Correlation-Adjusted Power Approximation policies. Note that replenishment frequencies are below best values for all parameter settings, and that backlog frequencies are above best values for all parameter settings except for a unit penalty cost of 99 , which is a substantial 43% below the best value.

Table 4.5

Percentage Apportionment of Aggregate Costs Per Period for a 72-Item Warehouse Inventory System

Under Best and Correlation-Adjusted Power Approximation Policy Control

Heterogeneous-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
<u>BEST POLICIES</u>													
INVENTORY	57.3	11.8	16.5	29.0	27.2	30.1	15.7	19.5	22.1	9.5	13.3	16.1	18.4
BACKLOG	16.4	6.0	5.8	4.6	8.2	8.2	4.4	5.7	6.3	2.9	4.0	4.3	5.2
REPLENISHMENT	26.2	8.3	8.4	9.5	10.3	15.9	9.1	8.5	8.6	4.1	6.2	7.2	8.6
TOTAL	100.0	26.2	30.7	43.1	45.8	54.2	29.2	33.7	37.1	16.5	23.6	27.7	32.2
<u>CORR. - ADJUSTED P.A.</u>													
INVENTORY	61.1	11.2	16.5	33.3	29.7	31.5	16.4	20.8	23.9	10.3	14.2	17.1	19.4
BACKLOG	16.0	6.9	6.3	2.8	7.1	8.9	4.8	5.4	5.8	2.8	4.0	4.0	5.1
REPLENISHMENT	22.9	7.6	7.6	7.6	9.0	13.9	7.9	7.6	7.4	3.5	5.3	6.5	7.6
TOTAL	100.0	25.8	30.4	43.8	45.9	54.1	29.1	33.8	37.1	16.6	23.6	27.6	32.2

Table 4.6

Operating Characteristics of a 72-Item Warehouse Inventory System Under

Best and Correlation-Adjusted Power Approximation Policy Control

Heterogeneous-Stores Environment

DECISION RULE AND OPERATING CHARACTERISTIC	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
Period-End Inventory	1869	385	538	946	888	981	512	635	921	310	434	525	600
Backlog Frequency	.099	.191	.096	.009	.098	.100	.098	.100	.099	.097	.099	.099	.100
Weighted Proportion of Demand Backlogged	.020	.205	.087	.006	.020	.020	.016	.021	.023	.035	.024	.018	.016
Replenishment Frequency	.259	.246	.251	.281	.293	.225	.266	.255	.256	.162	.243	.286	.345
CORR. - ADJUSTED P.A.													
Period-End Inventory	2031	374	548	1107	987	1044	546	691	794	343	473	569	646
Backlog Frequency	.108	.216	.102	.005	.101	.114	.111	.108	.104	.104	.112	.106	.108
Weighted Proportion of Demand Backlogged	.020	.240	.096	.004	.018	.022	.018	.020	.022	.035	.025	.017	.016
Replenishment Frequency	.230	.231	.230	.231	.261	.200	.238	.229	.224	.139	.212	.262	.308

Table 4.7

Percentage Above Best (s,S) Operating Characteristic Values for a 72-Item Warehouse Inventory System
Under Correlation-Adjusted Power Approximation Policy Control

Heterogeneous-Stores Environment

DECISION RULE AND OPERATING CHARACTERISTIC	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
CORR. - ADJUSTED P.A.													
Period-End Inventory	8.7	- 2.6	1.8	17.2	11.2	6.4	6.7	8.6	10.2	10.4	9.2	8.3	7.8
Backlog Frequency	8.9	12.6	6.3	-43.4	3.5	14.1	13.5	8.8	4.4	7.3	12.1	6.7	8.3
Weighted Proportion of Demand Backlogged	- 1.0	16.8	9.9	-38.3	-11.3	9.1	8.9	- 3.5	- 5.8	- 0.1	2.3	- 5.9	- 0.0
Replenishment Frequency	-11.1	- 6.2	- 8.4	-17.7	-11.1	-11.0	-10.5	-10.2	-12.5	-14.0	-12.7	- 8.4	-10.7

4.3 The Performance of the Correlation-Adjusted Power Approximation with Statistical Demand Information

The methodology we use to evaluate the Correlation-Adjusted Power Approximation in the heterogeneous-stores environment, when demand information is limited to a sample of previously-realized demands, is identical to that employed in Section 2 . As before, we simulate 200 replications of policy construction and operation. We again use the previous 26 demand observations to compute the policy parameters to be employed during the next 26 periods of operation. In this section we estimate the variance of demand over one lead time L by expression (2.3), which requires the estimation of the first L autocorrelation coefficients of demand. We again use a revision interval and revision history of 26 periods.

Table 4.8 summarizes the performance of the Statistical Correlation-Adjusted Power Approximation in the heterogeneous-stores environment. Estimated average costs per period and estimated absolute and percentage increases over the corresponding best costs are shown. Total cost is 7.2% above the corresponding best value, and when compared with total cost performance in the identical stores environments, this represents a slightly better performance level. Inventory costs and backlog costs are, on the average, 12.8% and 14.5% above best values, respectively, while replenishment costs are 9.7% below best values. The greatest discrepancy between performance under full and statistical demand information exists in the backlog cost component. Backlog costs under full information are 1% below best values while under statistical information they are 14.5% above best values.

Table 4.8

Summary of the Performance of the Correlation-Adjusted Power Approximation for a 72-Item Warehouse Inventory System with Heterogeneous Store Parameters Under Statistical Demand Information

(26 Period Revision Interval, and a 26 Period Revision History)

COST COMPONENT	AVERAGE COSTS PER PERIOD	INCREASE OVER BEST VALUES
INVENTORY	2108 (60.3)	239 [12.8]
BACKLOG	613 (17.5)	77 [14.5]
REPLENISHMENT	773 (22.1)	- 83 [- 9.7]
TOTAL	3494 (100.0)	234 [7.2]

NOTE: Numbers in parentheses are percentages of total cost.

Numbers in brackets are percentage differences in cost components over best values.

The percentage increases in Table 4.8 are divided into subsystems according to values taken by the input parameters in Table 4.9 . As in the identical stores environments of Section 2 , inventory costs are above best values and replenishment costs are below best values for all parameter settings. For a penalty cost of 99 , backlog costs are slightly above best values whereas they are substantially below best values in the identical stores environments. Notice also that percentage increases of backlog costs over best values increase with lead time. This is opposite to what was observed in the case of full information.

The percentage apportionment of aggregate costs per period for various parameter settings is shown in Table 4.10 for best policies and Statistical Correlation-Adjusted Power Approximation policies. The distributions of costs are similar for both policy rules. Note, however, that backlog costs under the latter policy rule exhibit a dependence on the setup cost parameter which is absent under best policy control. This same phenomenon is present under full information and in the identical stores environments.

The values of operating characteristics other than costs are given in Table 4.11 , and the percentage increases over best values are given in Table 4.12 . Trends are consistent with those noted earlier when the system is controlled under full demand information.

Table 4.9

Percentage Above Best Costs Per Period for a 72-Item Warehouse Inventory System Controlled Under Statistical Demand Information

Heterogeneous Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
STAT. CORR.-ADJUSTED P.A.													
INVENTORY	12.8	1.3	6.3	21.1	15.8	10.1	12.5	13.6	12.3	14.9	14.1	11.7	11.7
BACKLOG	14.5	20.0	19.6	0.8	4.2	24.6	5.3	6.5	28.3	18.6	11.8	21.8	10.9
REPLENISHMENT	- 9.7	- 5.1	- 5.8	-17.2	- 9.9	- 9.6	-10.2	- 7.9	-11.0	-11.2	-12.9	- 7.0	- 9.1
TOTAL	7.2	3.6	5.5	10.5	7.9	6.5	4.4	7.0	9.6	8.2	6.6	8.4	6.0

Table 4.10

Percentage Apportionment of Aggregate Costs Per Period for a 72-Item Warehouse Inventory System

Under Best and Statistical Correlation-Adjusted Power Approximation Policy Control

Heterogeneous-Stores Environment

DECISION RULE AND COST COMPONENT	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
INVENTORY	57.3	11.8	16.5	29.0	27.2	30.1	15.7	19.5	22.1	9.5	13.3	16.1	18.4
BACKLOG	16.4	6.0	5.8	4.6	8.2	8.2	4.4	5.7	6.3	2.9	4.0	4.3	5.2
REPLENISHMENT	26.2	8.3	8.4	9.5	10.3	15.9	9.1	8.5	8.6	4.1	6.2	7.2	8.6
TOTAL	100.0	26.2	30.7	43.1	45.8	54.2	29.2	33.7	37.1	16.5	23.6	27.7	32.2
STAT. CORR.-ADJUSTED P.A.													
INVENTORY	60.3	11.2	16.4	32.7	29.4	30.9	16.5	20.6	23.2	10.2	14.2	16.8	19.2
BACKLOG	17.5	6.8	6.4	4.3	7.9	9.6	4.4	5.7	7.5	3.0	4.2	4.9	5.3
REPLENISHMENT	22.1	7.4	7.4	7.3	8.7	13.3	7.6	7.3	7.2	3.4	5.1	6.3	7.3
TOTAL	100.0	25.3	30.3	44.4	46.1	53.9	28.5	33.6	37.9	16.7	23.4	28.0	31.9

Table 4.11

Operating Characteristics of a 72-Item Warehouse Inventory System Under
Best and Statistical Correlation-Adjusted Power Approximation Policy Control

Heterogeneous-Stores Environment

DECISION RULE AND OPERATING CHARACTERISTIC	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
BEST POLICIES													
Period-End Inventory	1869	385	538	946	888	981	512	635	921	310	434	525	600
Backlog Frequency	.099	.191	.096	.009	.098	.100	.098	.100	.099	.097	.099	.099	.100
Weighted Proportion of Demand Backlogged	.020	.205	.087	.006	.020	.020	.016	.021	.023	.035	.024	.018	.016
Replenishment Frequency	.259	.246	.251	.281	.293	.225	.266	.255	.256	.162	.243	.286	.345
STAT. CORR.-ADJUSTED P.A.													
Period-End Inventory	2108	390	573	1145	1029	1079	575	721	809	356	495	586	670
Backlog Frequency	.110	.216	.106	.008	.104	.116	.105	.109	.116	.108	.110	.112	.110
Weighted Proportion of Demand Backlogged	.023	.246	.105	.006	.021	.025	.017	.022	.029	.040	.027	.021	.017
Replenishment Frequency	.234	.234	.234	.233	.264	.203	.241	.232	.228	.144	.215	.266	.310

Table 4.12

Percentage Above Best (s,S) Operating Characteristic Values for a 72-Item Warehouse Inventory System
Under Statistical Correlation-Adjusted Power Approximation Policy Control

Heterogeneous-Stores Environment

DECISION RULE AND OPFRATING CHARACTERISTIC	TOTAL	INPUT PARAMETERS											
		PENALTY			SETUP		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	4	8	12	16
STAT. CORR.-ADJUSTED P.A.													
Period-End Inventory	12.8	1.3	6,3	21,1	15.8	10.1	12.5	13.6	12.3	14.9	14.1	11.7	11.7
Backlog Frequency	11,2	12.8	9.9	-10.4	5.9	16.3	7.5	9.7	16.3	11.6	10.2	13.0	9.9
Weighted Proportion of Demand Backlogged	14,5	20.0	19.6	0.8	4.2	24.6	5.3	6,5	28,3	13.6	11.8	21.8	10.9
Replenishment Frequency	- 9.8	- 4.9	- 6.6	-16.8	- 9.9	- 9.6	- 9.4	- 9.0	-10.9	-11.5	-11.5	- 7.1	-10.0

5. CONCLUSIONS AND TOPICS FOR FUTURE RESEARCH

In this section we summarize the principal results of this report and outline several topics to be investigated further.

Our main intent in this report has been to evaluate the performance of the Correlation-Adjusted Power Approximation in a warehouse inventory system under statistical demand information. We tested the policy rule on three multi-item inventory systems using a demand history of 26 periods to estimate the required demand parameters. The first two multi-item systems studied were characterized by the number of identical stores generating warehouse demand through their replenishment orders. In the few-stores and many-stores multi-item systems, average total costs were 7.8% and 10.9% , respectively, above the corresponding best values. The other multi-item system considered was designed to test the policy rule in a more realistic, heterogeneous-stores environment. Under full and statistical demand information total costs were 1.9% and 7.2% , respectively, above the corresponding best values. To summarize then we have found that, with: a limited history of previously-realized demands, the policy rule performs quite well in several diverse warehouse demand environments. The effectiveness of the rule is underscored when we note that most of the total cost increases are attributable to items with unit penalty costs of 99 , a parameter setting considerably larger than those typically found in practice.

We have noted throughout this report that one of the shortcomings of the Correlation-Adjusted Power Approximation is that, in general, the policy rule holds too much inventory, especially for those items with a high penalty cost. One cause for excessive inventory levels is setting the value of s too high. Recall that the approximation for s is given by

$$(5.1) \quad s = (L + 1)\mu + \sigma_L^{.832}(\sigma^2/\mu)^{.187}(.220/z + 1.142 - 2.866z) \ ,$$

Notice the term σ^2/μ in the expression for s. In an independent and identically distributed demand environment, for which the expression for s was derived, this term is equivalent to the variance-to-mean ratio of demand over one lead time σ_L^2/μ_L; that is,

$$(5.2) \quad \sigma_L^2/\mu_L = \sigma_L^2/[\mu(L + 1)] = \sigma^2(L + 1)/[\mu(L + 1)] = \sigma^2/\mu \ .$$

In a correlated demand environment, however, the terms σ_L^2/μ_L and σ^2/μ are not equal. In fact, in a negatively-correlated demand environment, σ_L^2/μ_L will be less than σ^2/μ. Thus, if the quantity $(.220/z + 1.142 - 2.866z)$ is positive, we will obtain a smaller value for s by using σ_L^2/μ_L in (5.1) instead of σ^2/μ. In this case, the result will be lower inventory levels. The benefits, if any, to be derived from this substitution need to be investigated.

Finally, the robustness of the Correlation-Adjusted Power Approximation, under full and statistical demand information, in other types of correlated demand environments needs to be studied. For example, its performance in an environment where demand follows a Markov process could be investigated.

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APPENDIX I

Single-Item Data

Few-Stores Environment

Summary of Data for 72 Items in a Few-Stores Environment

Controlled with:

Best (s,S) Policies (SB)

Power Approximation (PA)

Statistical Power Approximation

			page
Table	A1	Average Total Cost	I.1 to I.3
	A3	Period-End Inventory	I.4 to I.6
	A4	Period-End Backlog as Proportion of Mean Demand	I.7 to I.9
	A5	Frequency of Periods with Backlog	I.10 to I.12
	A6	Replenishment Frequency	I.13 to I.15
	A7	Estimated Bias of Forecast of Total Cost	I.16 to I.18
	A9	Values for (s,S)	I.19 to I.21
	A10	Standard Deviations of (s,S) Values	I.22 to I.24

TABLE A1 AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL SE ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SE	PA	(26,26)
4	4	32	18.1	0.6	2.7
8	4	32	25.5	0.7	4.3
12	4	32	31.0	0.2	4.2
16	4	32	36.0	0.6	2.0
4	9	32	21.5	0.8	5.5
8	9	32	29.6	2.5	5.3
12	9	32	36.3	0.1	3.2
16	9	32	41.7	1.6	5.3
4	99	32	27.5	12.3	17.3
8	99	32	40.5	4.9	11.9
12	99	32	50.4	2.6	8.3
16	99	32	55.9	5.8	11.3
4	4	64	23.2	2.4	3.2
8	4	64	32.6	1.3	2.4
12	4	64	40.0	1.0	2.3
16	4	64	45.7	0.9	3.1
4	9	64	27.1	1.7	3.5
8	9	64	37.3	2.8	2.8
12	9	64	45.9	0.7	2.5
16	9	64	52.4	0.5	4.2
4	99	64	33.2	9.0	14.0
8	99	64	49.2	3.1	7.7
12	99	64	60.3	0.5	6.1
16	99	64	67.5	5.1	10.8

TABLE A1 AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL SB ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	19.1	1.3	6.4
8	4	32	28.0	1.5	5.7
12	4	32	34.2	0.5	2.7
16	4	32	39.3	-0.7	2.6
4	9	32	23.2	2.3	7.6
8	9	32	33.3	0.0	5.8
12	9	32	40.5	1.2	3.1
16	9	32	46.5	-0.6	5.8
4	99	32	34.3	6.5	11.4
8	99	32	46.1	7.4	16.9
12	99	32	56.4	9.0	14.1
16	99	32	64.0	5.8	14.2
4	4	64	24.4	3.2	4.8
8	4	64	34.8	0.7	2.5
12	4	64	42.8	1.0	3.0
16	4	64	49.2	0.8	2.0
4	9	64	28.4	1.2	4.9
8	9	64	40.4	1.3	2.8
12	9	64	49.6	0.6	2.3
16	9	64	56.7	0.5	3.2
4	99	64	38.6	3.0	10.1
8	99	64	54.6	2.0	12.4
12	99	64	66.1	4.5	8.2
16	99	64	76.1	1.8	6.9

TABLE A1 AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL SB ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	21.0	0.7	6.7
8	4	32	30.5	0.3	6.9
12	4	32	37.1	0.0	6.3
16	4	32	42.9	-0.0	6.0
4	9	32	25.2	2.5	12.9
8	9	32	36.2	1.2	10.6
12	9	32	44.3	0.4	9.1
16	9	32	51.4	-0.3	8.8
4	99	32	37.0	11.1	20.3
8	99	32	51.4	9.0	14.5
12	99	32	65.2	6.3	13.1
16	99	32	73.9	4.0	16.3
4	4	64	26.0	2.4	5.7
8	4	64	36.9	2.3	6.4
12	4	64	45.2	1.3	3.5
16	4	64	52.7	0.3	4.3
4	9	64	30.2	1.5	10.4
8	9	64	43.1	2.0	6.7
12	9	64	53.1	0.6	4.9
16	9	64	61.7	-0.3	3.6
4	99	64	42.7	3.6	11.8
8	99	64	59.0	4.7	13.7
12	99	64	73.9	2.3	10.3
16	99	64	84.8	3.0	11.4

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SR	PA	(26,26)
4	4	32	5.8	7.6	8.2
8	4	32	10.9	10.5	12.0
12	4	32	14.0	13.7	14.4
16	4	32	15.4	15.8	16.8
4	9	32	13.1	11.8	12.8
8	9	32	14.6	16.1	18.0
12	9	32	19.6	20.5	21.1
16	9	32	21.7	23.3	25.0
4	99	32	19.8	24.3	26.2
8	99	32	26.4	32.8	35.7
12	99	32	31.3	39.9	42.7
16	99	32	36.4	45.5	48.1
4	4	64	10.5	8.9	9.2
8	4	64	13.1	12.2	12.8
12	4	64	17.9	15.1	15.8
16	4	64	20.2	17.5	18.6
4	9	64	12.7	13.1	14.2
8	9	64	18.2	18.5	19.4
12	9	64	22.4	22.5	23.7
16	9	64	27.1	25.8	27.9
4	99	64	19.8	26.3	27.8
8	99	64	30.6	36.1	38.0
12	99	64	39.8	42.5	45.0
16	99	64	41.1	48.3	52.0

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	8.9	9.3	9.7
8	4	32	12.8	12.8	14.2
12	4	32	15.5	16.6	16.9
16	4	32	18.8	18.4	19.7
4	9	32	12.3	14.5	15.2
8	9	32	17.2	19.1	20.6
12	9	32	22.4	24.3	25.1
16	9	32	25.7	26.9	29.6
4	99	32	24.0	30.1	31.3
8	99	32	31.8	39.9	43.3
12	99	32	38.0	48.6	51.4
16	99	32	43.7	55.0	58.6
4	4	64	11.0	9.5	10.5
8	4	64	14.8	14.0	14.7
12	4	64	19.3	17.0	18.3
16	4	64	21.0	20.2	21.0
4	9	64	14.4	15.2	16.1
8	9	64	19.9	21.0	22.2
12	9	64	27.0	25.8	26.9
16	9	64	28.7	29.5	30.9
4	99	64	28.3	30.6	32.4
8	99	64	35.7	41.7	44.4
12	99	64	44.8	50.2	52.7
16	99	64	47.1	57.2	60.0

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SP	PA	(26,26)
4	4	32	10.1	10.4	11.1
8	4	32	15.7	14.9	15.8
12	4	32	18.3	18.3	19.4
16	4	32	20.9	21.5	22.2
4	9	32	14.3	16.0	17.1
8	9	32	20.9	21.7	23.6
12	9	32	25.5	26.8	28.3
16	9	32	29.9	30.9	32.9
4	99	32	26.8	34.5	36.0
8	99	32	34.7	45.9	49.0
12	99	32	46.7	56.0	58.7
16	99	32	51.2	63.9	66.8
4	4	64	12.7	10.6	11.4
8	4	64	16.1	15.7	16.4
12	4	64	20.6	19.5	19.8
16	4	64	23.1	22.6	23.5
4	9	64	17.8	16.3	17.9
8	9	64	22.5	23.1	24.5
12	9	64	27.8	29.0	29.2
16	9	64	32.9	33.1	34.5
4	99	64	26.8	34.6	37.0
8	99	64	37.9	47.1	49.4
12	99	64	50.2	56.9	58.7
16	99	64	57.5	64.8	67.3

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(CUT) /C(IN)	C(FIX) /C(IN)	SE	PA	(26,26)
4	4	32	0.3953	0.3204	0.3240
8	4	32	0.1963	0.2253	0.2001
12	4	32	0.1408	0.1551	0.1683
16	4	32	0.1274	0.1432	0.1286
4	9	32	0.0768	0.1224	0.1289
8	9	32	0.0711	0.0924	0.0711
12	9	32	0.0620	0.0539	0.0596
16	9	32	0.0442	0.0537	0.0506
4	99	32	0.0051	0.0026	0.0022
8	99	32	0.0050	0.0022	0.0019
12	99	32	0.0041	0.0016	0.0017
16	99	32	0.0035	0.0015	0.0016
4	4	64	0.2762	0.4241	0.3985
8	4	64	0.1974	0.2714	0.2630
12	4	64	0.1550	0.2172	0.2096
16	4	64	0.1272	0.1703	0.1684
4	9	64	0.0950	0.1732	0.1529
8	9	64	0.0714	0.1102	0.0894
12	9	64	0.0666	0.0812	0.0754
16	9	64	0.0555	0.0637	0.0614
4	99	64	0.0051	0.0048	0.0046
8	99	64	0.0058	0.0024	0.0036
12	99	64	0.0040	0.0027	0.0030
16	99	64	0.0033	0.0029	0.0031

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-s=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.2841	0.2996	0.3412
8	4	32	0.2036	0.2559	0.2378
12	4	32	0.1642	0.1682	0.1819
16	4	32	0.1350	0.1491	0.1442
4	9	32	0.1442	0.1087	0.1273
8	9	32	0.0800	0.0908	0.0940
12	9	32	0.0674	0.0642	0.0656
16	9	32	0.0619	0.0568	0.0574
4	99	32	0.0114	0.0028	0.0047
8	99	32	0.0048	0.0028	0.0035
12	99	32	0.0055	0.0027	0.0028
16	99	32	0.0034	0.0010	0.0020
4	4	64	0.2830	0.4917	0.4452
8	4	64	0.2026	0.2891	0.2868
12	4	64	0.1824	0.2393	0.2278
16	4	64	0.1474	0.1918	0.1836
4	9	64	0.1409	0.1597	0.1564
8	9	64	0.0962	0.1172	0.1027
12	9	64	0.0731	0.0895	0.0832
16	9	64	0.0646	0.0707	0.0722
4	99	64	0.0068	0.0037	0.0057
8	99	64	0.0062	0.0027	0.0061
12	99	64	0.0043	0.0035	0.0033
16	99	64	0.0044	0.0020	0.0026

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.3261	0.3361	0.3877
8	4	32	0.2459	0.2576	0.2834
12	4	32	0.1946	0.1946	0.2191
16	4	32	0.1630	0.1644	0.1889
4	9	32	0.1437	0.1313	0.1720
8	9	32	0.0918	0.1041	0.1216
12	9	32	0.0885	0.0760	0.0985
16	9	32	0.0666	0.0653	0.0831
4	99	32	0.0115	0.0040	0.0088
8	99	32	0.0080	0.0033	0.0026
12	99	32	0.0070	0.0032	0.0046
16	99	32	0.0052	0.0013	0.0051
4	4	64	0.3550	0.5160	0.5101
8	4	64	0.2495	0.3283	0.3487
12	4	64	0.1950	0.2522	0.2552
16	4	64	0.1867	0.2060	0.2266
4	9	64	0.1330	0.1841	0.2098
8	9	64	0.1039	0.1299	0.1375
12	9	64	0.0833	0.0928	0.1109
16	9	64	0.0696	0.0807	0.0850
4	99	64	0.0115	0.0049	0.0075
8	99	64	0.0066	0.0040	0.0076
12	99	64	0.0053	0.0036	0.0069
16	99	64	0.0052	0.0036	0.0063

TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-s=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SP	PA	(26,26)
4	4	32	0.1864	0.1886	0.1637
8	4	32	0.1952	0.2111	0.1929
12	4	32	0.1989	0.2113	0.2127
16	4	32	0.1986	0.2207	0.1927
4	9	32	0.0808	0.1094	0.1037
8	9	32	0.0900	0.1154	0.0867
12	9	32	0.0968	0.0859	0.0912
16	9	32	0.0943	0.1015	0.0921
4	99	32	0.0080	0.0025	0.0027
8	99	32	0.0090	0.0045	0.0038
12	99	32	0.0096	0.0041	0.0035
16	99	32	0.0090	0.0037	0.0046
4	4	64	0.1647	0.2524	0.2498
8	4	64	0.1911	0.2414	0.2169
12	4	64	0.1892	0.2332	0.2213
16	4	64	0.1886	0.2336	0.2221
4	9	64	0.0898	0.1103	0.0992
8	9	64	0.0914	0.1236	0.1033
12	9	64	0.0927	0.1125	0.1038
16	9	64	0.0951	0.1041	0.1025
4	99	64	0.0080	0.0047	0.0046
8	99	64	0.0088	0.0039	0.0063
12	99	64	0.0092	0.0065	0.0067
16	99	64	0.0094	0.0070	0.0063

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WHOLESALE WAREHOUSE INVENTORY CONTROL WITH STATISTICAL DEMAND I--ETC(U)
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TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SP	PA	(26,26)
4	4	32	0.1739	0.1901	0.2079
8	4	32	0.1939	0.2199	0.2021
12	4	32	0.1884	0.1872	0.1988
16	4	32	0.1880	0.2068	0.1946
4	9	32	0.0923	0.0765	0.0856
8	9	32	0.0937	0.0894	0.0904
12	9	32	0.0941	0.0851	0.0887
16	9	32	0.0955	0.0939	0.0935
4	99	32	0.0082	0.0029	0.0040
8	99	32	0.0076	0.0031	0.0048
12	99	32	0.0094	0.0039	0.0062
16	99	32	0.0088	0.0031	0.0050
4	4	64	0.1829	0.2801	0.2419
8	4	64	0.1825	0.2377	0.2292
12	4	64	0.1970	0.2445	0.2217
16	4	64	0.1939	0.2320	0.2161
4	9	64	0.0908	0.0964	0.0977
8	9	64	0.0933	0.1090	0.0960
12	9	64	0.0904	0.1086	0.1040
16	9	64	0.0968	0.1052	0.1004
4	99	64	0.0082	0.0037	0.0050
8	99	64	0.0086	0.0045	0.0063
12	99	64	0.0084	0.0051	0.0069
16	99	64	0.0090	0.0041	0.0054

TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SP	PA	(26,26)
4	4	32	0.1972	0.1943	0.1971
8	4	32	0.1948	0.2048	0.2115
12	4	32	0.1948	0.1948	0.2025
16	4	32	0.1939	0.1982	0.2081
4	9	32	0.0894	0.0822	0.1000
8	9	32	0.0923	0.0941	0.1058
12	9	32	0.0992	0.0878	0.1077
16	9	32	0.0921	0.0896	0.1035
4	99	32	0.0078	0.0029	0.0063
8	99	32	0.0090	0.0043	0.0050
12	99	32	0.0096	0.0041	0.0065
16	99	32	0.0096	0.0031	0.0096
4	4	64	0.1950	0.2414	0.2367
8	4	64	0.1962	0.2310	0.2365
12	4	64	0.1894	0.2257	0.2261
16	4	64	0.1997	0.2185	0.2240
4	9	64	0.0886	0.1129	0.1115
8	9	64	0.0970	0.1172	0.1162
12	9	64	0.0935	0.0966	0.1133
16	9	64	0.0929	0.1035	0.1025
4	99	64	0.0078	0.0031	0.0048
8	99	64	0.0092	0.0049	0.0071
12	99	64	0.0092	0.0061	0.0110
16	99	64	0.0088	0.0061	0.0098

TABLE A6   REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.186	0.171	0.162
8	4	32	0.259	0.247	0.254
12	4	32	0.319	0.311	0.307
16	4	32	0.389	0.352	0.365
4	9	32	0.178	0.171	0.165
8	9	32	0.309	0.239	0.252
12	9	32	0.311	0.311	0.309
16	9	32	0.425	0.352	0.361
4	99	32	0.178	0.171	0.163
8	99	32	0.317	0.247	0.255
12	99	32	0.444	0.311	0.306
16	99	32	0.437	0.352	0.362
4	4	64	0.128	0.125	0.129
8	4	64	0.206	0.190	0.191
12	4	64	0.230	0.233	0.237
16	4	64	0.271	0.276	0.277
4	9	64	0.171	0.128	0.130
8	9	64	0.218	0.185	0.194
12	9	64	0.254	0.233	0.237
16	9	64	0.271	0.276	0.280
4	99	64	0.178	0.125	0.128
8	99	64	0.218	0.200	0.190
12	99	64	0.245	0.233	0.241
16	99	64	0.332	0.281	0.281

TABLE A6 REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-s=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SP	PA	(26,26)
4	4	32	0.177	0.165	0.161
8	4	32	0.271	0.232	0.245
12	4	32	0.339	0.305	0.297
16	4	32	0.371	0.346	0.357
4	9	32	0.178	0.166	0.163
8	9	32	0.322	0.239	0.244
12	9	32	0.339	0.305	0.300
16	9	32	0.371	0.345	0.352
4	99	32	0.181	0.166	0.159
8	99	32	0.326	0.232	0.244
12	99	32	0.372	0.305	0.302
16	99	32	0.468	0.345	0.352
4	4	64	0.139	0.121	0.124
8	4	64	0.212	0.185	0.185
12	4	64	0.230	0.230	0.232
16	4	64	0.292	0.267	0.271
4	9	64	0.139	0.121	0.126
8	9	64	0.212	0.180	0.187
12	9	64	0.230	0.226	0.232
16	9	64	0.292	0.271	0.268
4	99	64	0.119	0.121	0.123
8	99	64	0.218	0.185	0.188
12	99	64	0.254	0.230	0.232
16	99	64	0.345	0.267	0.271

TABLE A6 REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	RA	(26,26)
4	4	32	0.177	0.165	0.158
8	4	32	0.218	0.232	0.243
12	4	32	0.296	0.296	0.296
16	4	32	0.361	0.340	0.350
4	9	32	0.178	0.159	0.160
8	9	32	0.270	0.232	0.239
12	9	32	0.289	0.295	0.295
16	9	32	0.371	0.340	0.344
4	99	32	0.178	0.159	0.158
8	99	32	0.322	0.232	0.241
12	99	32	0.319	0.295	0.297
16	99	32	0.451	0.340	0.342
4	4	64	0.119	0.121	0.123
8	4	64	0.200	0.180	0.183
12	4	64	0.240	0.222	0.230
16	4	64	0.276	0.267	0.265
4	9	64	0.119	0.121	0.124
8	9	64	0.206	0.180	0.182
12	9	64	0.254	0.226	0.226
16	9	64	0.292	0.263	0.268
4	99	64	0.178	0.121	0.121
8	99	64	0.247	0.180	0.183
12	99	64	0.272	0.226	0.229
16	99	64	0.298	0.263	0.267

TABLE I.7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

			(1)	(2)
MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(26,26,26)	(26,26,26)
4	4	32	0.2	+ + + - + +
8	4	32	0.3	+ + - + + +
12	4	32	0.3	+ + - + + +
16	4	32	0.1	+ + - + + +
4	9	32	0.3	+ + + + # +
8	9	32	0.0	+ - - + + +
12	9	32	0.7	+ # + + - #
16	9	32	1.0	+ # + - - #
4	99	32	0.8	- # # + + #
8	99	32	1.0	- # # - + #
12	99	32	1.4	# + + + + #
16	99	32	2.2	+ # # + + #
4	4	64	0.0	- - + - + +
8	4	64	0.3	+ + - + + +
12	4	64	0.3	- # + + - +
16	4	64	0.1	+ - - - + +
4	9	64	0.2	- + + - + +
8	9	64	0.1	+ - - + + +
12	9	64	-0.0	- - + - + -
16	9	64	0.8	+ + - - + +
4	99	64	1.4	+ # + - + #
8	99	64	1.7	- # # - - #
12	99	64	2.1	+ + + + # #
16	99	64	3.0	+ # + - + #

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; * : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

			(1)	(2)
MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(26,26,26)	(26,26,26)
4	4	32	0.6	+ + + - + +
8	4	32	1.2	* * + + + *
12	4	32	1.3	+ * + - - *
16	4	32	1.2	+ * + - + *
4	9	32	1.5	* * + - + *
8	9	32	1.5	+ * * - - *
12	9	32	2.2	+ * * + + *
16	9	32	2.0	+ * * - - *
4	99	32	2.0	+ * * - - *
8	99	32	2.5	- * * + + *
12	99	32	2.9	+ * * + + *
16	99	32	2.6	- * * * + *
4	4	64	0.6	- + + + + *
8	4	64	0.7	- * + + + +
12	4	64	1.0	* + - + * C
16	4	64	1.4	+ * + - - *
4	9	64	1.6	- * + + + *
8	9	64	1.0	+ + + - - +
12	9	64	1.5	+ + + + + *
16	9	64	1.2	+ + + - - +
4	99	64	1.9	- * * - - *
8	99	64	4.4	+ * * + + *
12	99	64	3.8	+ * * + + *
16	99	64	2.8	- * + + + *

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS R=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(1) (26,26,26)	(2) (26,26,26)
4	4	32	1.5	# # + - + #
8	4	32	2.7	# # # + + #
12	4	32	3.2	# # + - + #
16	4	32	3.9	# # # - + #
4	9	32	3.0	+ # # + + #
8	9	32	3.6	# # # - - #
12	9	32	5.1	+ # # - + #
16	9	32	5.9	# # # - + #
4	99	32	3.5	- # # + + #
8	99	32	2.2	+ # # + + #
12	99	32	5.3	+ # # - + #
16	99	32	7.7	- # # - - #
4	4	64	1.7	+ # # - - #
8	4	64	2.5	# # + - - #
12	4	64	2.4	# # + + # #
16	4	64	3.4	# # # + - #
4	9	64	2.9	+ # # - + #
8	9	64	3.7	+ # # - - #
12	9	64	4.2	+ # # + + #
16	9	64	4.5	# # + + + #
4	99	64	2.7	- # # - + #
8	99	64	5.7	+ # # - + #
12	99	64	8.0	+ # # - - #
16	99	64	9.1	+ # # - - #

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S			LITTLE S			BIG S - LITTLE S		
			SB	PA	(26,26)	SB	PA	(26,26)	SB	PA	(26,26)
4	4	32	13	16	16.4	0	-1	-1.0	13	17	17.4
8	4	32	25	25	25.7	3	2	2.0	22	23	23.7
12	4	32	33	33	33.5	6	5	5.3	27	28	28.2
16	4	32	38	40	40.9	10	8	8.8	28	32	32.1
4	9	32	22	21	21.9	7	4	4.4	15	17	17.5
8	9	32	27	32	32.6	9	8	9.0	18	24	23.6
12	9	32	40	41	41.2	12	13	13.1	28	28	28.0
16	9	32	44	49	49.8	20	17	17.9	24	32	31.9
4	99	32	29	34	35.7	14	17	18.3	15	17	17.5
8	99	32	39	49	51.0	22	26	27.3	17	23	23.7
12	99	32	47	61	63.5	32	33	35.3	15	28	28.2
16	99	32	59	72	73.7	36	40	41.9	23	32	31.8
4	4	64	23	21	20.8	-1	-4	-4.0	24	25	24.8
8	4	64	31	31	31.2	1	-2	-2.3	30	33	33.5
12	4	64	44	40	40.2	3	0	0.3	41	40	39.9
16	4	64	52	48	48.1	6	3	3.1	46	45	45.1
4	9	64	22	26	26.6	5	2	1.8	17	24	24.7
8	9	64	36	39	38.9	8	5	5.6	28	34	33.3
12	9	64	47	49	49.3	11	9	9.5	36	40	39.7
16	9	64	60	58	59.2	14	13	13.8	46	45	45.4
4	99	64	29	40	40.7	14	15	16.1	15	25	24.6
8	99	64	49	56	58.1	21	25	24.7	28	31	33.4
12	99	64	66	70	71.5	28	30	31.7	38	40	39.8
16	99	64	69	81	83.9	34	37	38.7	35	44	45.2

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S			LITTLE S			BIG S - LITTLE S		
			SB	PA	(26,26)	SB	PA	(26,26)	SB	PA	(26,26)
4	4	32	25	26	26.2	10	8	8.1	15	18	18.1
8	4	32	42	44	44.6	21	19	19.9	21	25	24.7
12	4	32	57	60	60.0	33	31	30.8	24	29	29.1
16	4	32	74	75	75.4	44	42	42.5	30	33	32.9
4	9	32	29	32	32.6	14	14	14.4	15	18	18.2
8	9	32	45	51	52.4	29	27	27.8	16	24	24.6
12	9	32	65	69	69.7	41	40	40.5	24	29	29.2
16	9	32	82	85	87.4	52	52	54.1	30	33	33.2
4	99	32	41	48	48.6	27	30	30.7	14	18	17.9
8	99	32	60	73	75.1	45	48	50.6	15	25	24.5
12	99	32	80	94	96.8	59	65	67.6	21	29	29.3
16	99	32	97	114	117.2	76	81	84.1	21	33	33.1
4	4	64	30	30	30.0	8	4	4.5	22	26	25.5
8	4	64	48	49	49.0	19	15	14.6	29	34	34.5
12	4	64	69	66	66.5	28	25	25.4	41	41	41.2
16	4	64	82	83	82.3	40	36	35.9	42	47	46.3
4	9	64	34	37	36.9	12	11	11.4	22	26	25.6
8	9	64	54	58	57.8	25	23	23.4	29	35	34.4
12	9	64	78	77	77.1	37	35	35.9	41	42	41.2
16	9	64	91	94	94.4	49	48	47.9	42	46	46.6
4	99	64	51	53	53.0	24	27	27.8	27	26	25.2
8	99	64	70	79	81.2	42	45	46.6	28	34	34.6
12	99	64	94	102	103.5	58	61	62.4	36	41	41.0
16	99	64	106	123	124.8	73	76	78.1	33	47	46.7

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF PEW-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S			LITTLE S			BIG S - LITTLE S		
			SB	PA	(26,26)	SB	PA	(26,26)	SB	PA	(26,26)
4	4	32	34	35	35.0	19	17	16.7	15	18	18.3
8	4	32	64	62	62.1	36	37	37.1	28	25	25.0
12	4	32	86	86	86.1	56	56	56.4	30	30	29.7
16	4	32	108	110	109.5	77	76	75.9	31	34	33.6
4	9	32	39	42	42.1	24	23	23.7	15	19	18.4
8	9	32	67	70	70.7	46	45	45.7	21	25	25.0
12	9	32	95	96	96.0	64	66	66.4	31	30	29.6
16	9	32	118	121	121.9	88	87	88.2	30	34	33.8
4	99	32	52	61	61.3	37	42	42.8	15	19	18.5
8	99	32	79	95	96.9	63	70	71.8	16	25	25.0
12	99	32	115	126	127.8	88	96	98.0	27	30	29.8
16	99	32	137	155	157.8	115	121	123.9	22	34	33.8
4	4	64	42	39	38.9	15	13	13.0	27	26	26.0
8	4	64	66	67	67.4	35	32	32.0	31	35	35.4
12	4	64	93	93	92.1	54	50	50.2	39	43	41.9
16	4	64	117	117	116.9	72	70	69.4	45	47	47.5
4	9	64	48	46	47.0	21	20	20.9	27	26	26.1
8	9	64	73	76	76.0	43	41	41.0	30	35	35.1
12	9	64	100	104	102.9	64	62	61.1	36	42	41.8
16	9	64	127	130	130.0	85	82	82.4	42	48	47.6
4	99	64	52	65	65.9	37	39	39.8	15	26	26.1
8	99	64	86	101	102.3	63	66	67.1	23	35	35.3
12	99	64	122	133	134.2	89	91	92.2	33	42	42.0
16	99	64	152	163	165.1	111	115	117.3	41	48	47.8

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S (26,26)	LITTLE S (26,26)
4	4	32	1.93	0.55
8	4	32	2.19	0.86
12	4	32	2.60	1.19
16	4	32	2.83	1.44
4	9	32	2.36	1.00
8	9	32	2.76	1.49
12	9	32	3.11	1.86
16	9	32	3.85	2.48
4	99	32	3.68	2.47
8	99	32	5.01	4.08
12	99	32	6.61	5.57
16	99	32	7.00	5.93
4	4	64	2.07	0.23
8	4	64	2.39	0.68
12	4	64	2.80	0.95
16	4	64	2.86	1.09
4	9	64	2.82	0.81
8	9	64	3.09	1.25
12	9	64	3.70	1.65
16	9	64	4.00	1.97
4	99	64	4.05	2.19
8	99	64	5.39	3.81
12	99	64	6.13	4.63
16	99	64	7.74	5.98

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

			BIG S	LITTLE S
MEAN	C(CUT) /C(IN)	C(FIX) /C(IN)	(26,26)	(26,26)
4	4	32	3.64	2.18
8	4	32	4.74	3.28
12	4	32	5.18	3.79
16	4	32	5.56	4.15
4	9	32	4.25	2.85
8	9	32	5.32	3.94
12	9	32	6.08	4.62
16	9	32	6.76	5.32
4	99	32	6.79	5.50
8	99	32	8.91	7.57
12	99	32	10.30	8.93
16	99	32	11.00	9.66
4	4	64	3.92	1.84
8	4	64	4.36	2.53
12	4	64	5.12	3.19
16	4	64	5.77	3.90
4	9	64	4.64	2.51
8	9	64	5.74	3.64
12	9	64	6.51	4.37
16	9	64	6.57	4.61
4	99	64	6.13	4.40
8	99	64	8.42	6.53
12	99	64	9.97	8.11
16	99	64	10.02	8.17

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF FEW-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

			BIG S	LITTLE S
MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(26,26)	(26,26)
4	4	32	5.07	3.58
8	4	32	6.96	5.41
12	4	32	8.17	6.59
16	4	32	9.45	7.92
4	9	32	6.12	4.57
8	9	32	7.81	6.28
12	9	32	9.29	7.71
16	9	32	10.66	9.03
4	99	32	9.96	8.46
8	99	32	12.00	10.54
12	99	32	15.41	13.81
16	99	32	17.05	15.34
4	4	64	5.23	3.20
8	4	64	7.32	5.09
12	4	64	7.81	5.77
16	4	64	9.37	7.26
4	9	64	6.90	4.55
8	9	64	8.44	6.09
12	9	64	9.12	6.95
16	9	64	10.08	7.98
4	99	64	9.73	7.61
8	99	64	11.94	9.88
12	99	64	13.93	11.72
16	99	64	15.36	13.15

APPENDIX II

Single-Item Data

Many-Stores Environment

Summary of Data for 72 Items in a Many-Stores Environment

Controlled with:

Best (s,S) Policies (SB)

Power Approximation (PA)

Statistical Power Approximation

			page
Table	A1	Average Total Cost	II.1 to II.3
	A3	Period-End Inventory	II.4 to II.6
	A4	Period-End Backlog as Proportion of Mean Demand	II.7 to II.9
	A5	Frequency of Periods with Backlog	II.10 to II.12
	A6	Replenishment Frequency	II.13 to II.15
	A7	Estimated Bias of Forecast of Total Cost	II.16 to II.18
	A9	Values for (s,S)	II.19 to II.21
	A10	Standard Deviations of (s,S) Values	II.22 to II.24

TABLE A1 AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL SB ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	18.4	1.3	1.9
8	4	32	25.7	0.2	4.3
12	4	32	31.1	0.4	0.6
16	4	32	35.8	-0.0	3.3
4	9	32	21.2	1.6	7.2
8	9	32	29.8	0.2	5.8
12	9	32	36.9	0.8	4.3
16	9	32	42.2	0.4	3.4
4	99	32	30.9	5.9	13.0
8	99	32	42.0	2.7	7.5
12	99	32	51.9	1.7	10.8
16	99	32	59.5	3.7	6.9
4	4	64	22.5	1.6	4.4
8	4	64	32.7	0.6	2.9
12	4	64	39.6	2.1	2.2
16	4	64	45.6	0.2	3.5
4	9	64	27.2	1.0	4.4
8	9	64	37.6	1.6	3.9
12	9	64	45.7	1.0	3.4
16	9	64	52.3	0.0	3.7
4	99	64	36.9	-0.6	9.4
8	99	64	49.9	5.4	8.2
12	99	64	61.5	0.6	4.6
16	99	64	70.2	0.6	4.5

TABLE A1 AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL SB ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	22.0	0.1	5.4
8	4	32	30.0	0.2	11.8
12	4	32	36.8	0.9	8.1
16	4	32	41.9	0.7	7.8
4	9	32	25.6	4.2	10.3
8	9	32	36.4	-1.5	8.1
12	9	32	43.5	1.2	8.2
16	9	32	50.5	0.1	8.0
4	99	32	36.0	11.7	20.7
8	99	32	49.3	11.1	21.5
12	99	32	61.1	7.1	17.1
16	99	32	70.3	9.6	19.1
4	4	64	26.1	0.4	6.4
8	4	64	37.5	0.6	4.1
12	4	64	44.7	0.9	5.6
16	4	64	51.5	0.2	6.2
4	9	64	30.6	0.5	8.5
8	9	64	43.5	-1.2	5.7
12	9	64	52.2	0.8	7.3
16	9	64	59.6	0.8	7.4
4	99	64	40.7	9.5	17.0
8	99	64	57.3	5.7	15.0
12	99	64	69.6	6.7	13.1
16	99	64	79.4	6.0	12.1

TABLE A1   AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL SB ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	22.7	2.1	8.1
8	4	32	31.9	0.3	9.9
12	4	32	39.2	0.3	8.6
16	4	32	45.2	0.4	8.0
4	9	32	26.7	3.0	12.2
8	9	32	37.5	0.8	13.1
12	9	32	46.5	-0.0	10.7
16	9	32	54.1	0.0	13.3
4	99	32	35.5	20.0	38.1
8	99	32	51.2	16.1	30.8
12	99	32	63.7	9.6	27.8
16	99	32	75.2	9.2	22.3
4	4	64	27.0	1.2	7.2
8	4	64	38.5	0.2	7.8
12	4	64	47.2	1.5	4.7
16	4	64	54.4	1.3	7.2
4	9	64	31.6	2.0	9.2
8	9	64	45.2	-0.6	7.7
12	9	64	54.9	1.5	11.3
16	9	64	63.1	1.2	7.5
4	99	64	41.8	13.2	27.3
8	99	64	59.1	9.0	18.7
12	99	64	72.6	6.1	17.4
16	99	64	82.1	6.9	18.8

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	5.5	8.7	8.7
8	4	32	9.4	10.5	11.3
12	4	32	12.5	13.4	14.2
16	4	32	15.1	15.2	16.7
4	9	32	11.1	13.0	13.4
8	9	32	15.4	16.3	17.3
12	9	32	19.5	20.1	21.7
16	9	32	19.8	22.7	24.8
4	99	32	21.8	25.7	27.4
8	99	32	27.1	32.7	34.9
12	99	32	34.6	39.3	43.9
16	99	32	33.3	44.8	47.9
4	4	64	8.8	9.2	8.6
8	4	64	12.2	11.9	12.7
12	4	64	15.2	14.8	15.9
16	4	64	19.3	17.7	18.3
4	9	64	11.1	13.5	14.0
8	9	64	15.4	18.5	19.3
12	9	64	22.3	22.2	24.2
16	9	64	26.3	26.1	26.6
4	99	64	21.7	26.9	27.6
8	99	64	30.6	35.4	37.8
12	99	64	40.8	42.2	45.3
16	99	64	45.5	48.5	51.0

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	11.9	10.6	12.2
8	4	32	15.4	14.6	16.2
12	4	32	17.8	17.5	20.1
16	4	32	20.7	21.1	22.5
4	9	32	14.6	15.7	18.4
8	9	32	19.8	22.3	23.3
12	9	32	24.6	25.8	28.8
16	9	32	27.1	30.6	32.1
4	99	32	26.1	34.2	36.8
8	99	32	33.2	46.7	48.6
12	99	32	43.8	54.4	59.1
16	99	32	50.0	62.6	66.1
4	4	64	11.7	11.6	11.9
8	4	64	15.6	15.7	16.3
12	4	64	20.3	18.6	20.8
16	4	64	23.2	22.3	23.7
4	9	64	17.7	17.5	18.0
8	9	64	23.3	23.5	24.9
12	9	64	28.0	28.3	30.4
16	9	64	31.7	32.7	34.4
4	99	64	28.7	36.0	37.0
8	99	64	37.8	47.7	50.8
12	99	64	50.5	56.5	60.2
16	99	64	53.2	64.3	68.1

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	12.4	11.7	13.5
8	4	32	16.7	16.7	17.6
12	4	32	20.0	19.6	22.0
16	4	32	23.1	23.5	24.9
4	9	32	16.3	17.7	20.2
8	9	32	22.6	24.4	25.7
12	9	32	27.6	28.5	32.4
16	9	32	30.3	33.8	36.6
4	99	32	26.1	37.0	42.3
8	99	32	39.0	51.7	54.6
12	99	32	44.9	60.0	68.0
16	99	32	52.6	69.7	75.5
4	4	64	12.7	12.6	13.0
8	4	64	18.1	16.7	17.8
12	4	64	23.2	20.7	22.0
16	4	64	24.7	24.2	26.4
4	9	64	18.8	18.9	19.5
8	9	64	22.6	25.1	26.7
12	9	64	31.7	30.6	32.8
16	9	64	34.9	35.0	37.0
4	99	64	26.1	39.0	42.2
8	99	64	39.0	51.8	53.3
12	99	64	52.1	61.4	64.8
16	99	64	58.8	70.0	74.3

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.4213	0.2435	0.2836
8	4	32	0.1682	0.2319	0.2477
12	4	32	0.1594	0.1659	0.1475
16	4	32	0.1167	0.1488	0.1364
4	9	32	0.1124	0.0710	0.1062
8	9	32	0.0932	0.0815	0.0923
12	9	32	0.0586	0.0666	0.0621
16	9	32	0.0598	0.0596	0.0499
4	99	32	0.0075	0.0025	0.0053
8	99	32	0.0050	0.0034	0.0033
12	99	32	0.0047	0.0030	0.0030
16	99	32	0.0048	0.0037	0.0025
4	4	64	0.3053	0.3382	0.4348
8	4	64	0.2031	0.2837	0.2761
12	4	64	0.1507	0.2266	0.1904
16	4	64	0.1300	0.1564	0.1804
4	9	64	0.1124	0.1589	0.1793
8	9	64	0.0932	0.1098	0.1060
12	9	64	0.0581	0.0862	0.0698
16	9	64	0.0547	0.0571	0.0688
4	99	64	0.0081	0.0041	0.0121
8	99	64	0.0049	0.0062	0.0051
12	99	64	0.0052	0.0042	0.0029
16	99	64	0.0041	0.0025	0.0029

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIC IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATICN CF MANY-STCRES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.2550	0.3800	0.3542
8	4	32	0.2128	0.2395	0.3101
12	4	32	0.1677	0.2203	0.2038
16	4	32	0.1555	0.1575	0.1843
4	9	32	0.1325	0.1729	0.1233
8	9	32	0.0791	0.0801	0.1204
12	9	32	0.0734	0.0852	0.0775
16	9	32	0.0670	0.0628	0.0809
4	99	32	0.0093	0.0017	0.0031
8	99	32	0.0065	0.0004	0.0049
12	99	32	0.0054	0.0015	0.0023
16	99	32	0.0041	0.0022	0.0043
4	4	64	0.3834	0.4054	0.5140
8	4	64	0.2474	0.3291	0.3546
12	4	64	0.2056	0.2561	0.2458
16	4	64	0.1596	0.1891	0.2212
4	9	64	0.1258	0.1391	0.2059
8	9	64	0.0864	0.1067	0.1339
12	9	64	0.0901	0.0939	0.1023
16	9	64	0.0680	0.0747	0.0882
4	99	64	0.0082	0.0012	0.0077
8	99	64	0.0050	0.0014	0.0048
12	99	64	0.0045	0.0029	0.0032
16	99	64	0.0049	0.0017	0.0026

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.3441	0.3855	0.3560
8	4	32	0.2307	0.2350	0.3156
12	4	32	0.2150	0.2208	0.2275
16	4	32	0.1699	0.1702	0.2056
4	9	32	0.1198	0.1221	0.1207
8	9	32	0.0984	0.0782	0.1304
12	9	32	0.0742	0.0854	0.0884
16	9	32	0.0700	0.0655	0.0972
4	99	32	0.0080	0.0021	0.0039
8	99	32	0.0054	0.0	0.0063
12	99	32	0.0060	0.0007	0.0034
16	99	32	0.0055	0.0009	0.0037
4	4	64	0.3729	0.4059	0.5178
8	4	64	0.2447	0.3241	0.3907
12	4	64	0.2138	0.2735	0.2697
16	4	64	0.1823	0.2238	0.2413
4	9	64	0.1253	0.1494	0.2005
8	9	64	0.0984	0.1166	0.1491
12	9	64	0.0874	0.1018	0.1292
16	9	64	0.0704	0.0853	0.0988
4	99	64	0.0080	0.0002	0.0090
8	99	64	0.0054	0.0010	0.0071
12	99	64	0.0050	0.0012	0.0050
16	99	64	0.0033	0.0010	0.0041

TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.1909	0.1876	0.1698
8	4	32	0.1682	0.2394	0.2085
12	4	32	0.1870	0.1984	0.1811
16	4	32	0.1950	0.1997	0.1944
4	9	32	0.0990	0.0481	0.0790
8	9	32	0.0937	0.0804	0.1013
12	9	32	0.0945	0.0857	0.0894
16	9	32	0.0947	0.0992	0.0900
4	99	32	0.0090	0.0020	0.0046
8	99	32	0.0092	0.0053	0.0056
12	99	32	0.0088	0.0053	0.0046
16	99	32	0.0088	0.0055	0.0058
4	4	64	0.1377	0.1692	0.2669
8	4	64	0.1884	0.2160	0.2360
12	4	64	0.1986	0.2302	0.2206
16	4	64	0.1827	0.2218	0.2315
4	9	64	0.0990	0.1268	0.1075
8	9	64	0.0937	0.1084	0.1154
12	9	64	0.0937	0.1084	0.0929
16	9	64	0.0964	0.0980	0.1075
4	99	64	0.0090	0.0043	0.0094
8	99	64	0.0086	0.0100	0.0060
12	99	64	0.0090	0.0078	0.0060
16	99	64	0.0094	0.0068	0.0062

TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.1802	0.2306	0.1787
8	4	32	0.1988	0.2138	0.2133
12	4	32	0.1796	0.2132	0.1929
16	4	32	0.1972	0.1921	0.2015
4	9	32	0.0843	0.1090	0.0794
8	9	32	0.0974	0.0865	0.1054
12	9	32	0.0959	0.1009	0.0890
16	9	32	0.0931	0.0849	0.0992
4	99	32	0.0074	0.0018	0.0035
8	99	32	0.0096	0.0010	0.0062
12	99	32	0.0098	0.0027	0.0033
16	99	32	0.0092	0.0031	0.0073
4	4	64	0.1778	0.1856	0.2333
8	4	64	0.1952	0.2279	0.2386
12	4	64	0.1921	0.2318	0.2063
16	4	64	0.1972	0.2132	0.2185
4	9	64	0.0931	0.0947	0.1094
8	9	64	0.0886	0.0986	0.1167
12	9	64	0.0998	0.1023	0.1050
16	9	64	0.0988	0.1041	0.1044
4	99	64	0.0082	0.0020	0.0052
8	99	64	0.0080	0.0023	0.0063
12	99	64	0.0092	0.0051	0.0062
16	99	64	0.0094	0.0033	0.0046

TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.1911	0.2320	0.1852
8	4	32	0.1941	0.1954	0.2110
12	4	32	0.1988	0.2054	0.1960
16	4	32	0.1929	0.1907	0.2079
4	9	32	0.0816	0.0747	0.0783
8	9	32	0.0947	0.0769	0.1067
12	9	32	0.0964	0.0982	0.0950
16	9	32	0.0964	0.0890	0.1035
4	99	32	0.0098	0.0016	0.0038
8	99	32	0.0088	0.0	0.0062
12	99	32	0.0094	0.0008	0.0050
16	99	32	0.0094	0.0018	0.0067
4	4	64	0.1817	0.1919	0.2308
8	4	64	0.1911	0.2273	0.2444
12	4	64	0.1997	0.2398	0.2246
16	4	64	0.1991	0.2277	0.2185
4	9	64	0.0951	0.1013	0.1035
8	9	64	0.0947	0.1033	0.1096
12	9	64	0.0957	0.1078	0.1173
16	9	64	0.0992	0.1125	0.1085
4	99	64	0.0098	0.0008	0.0067
8	99	64	0.0088	0.0018	0.0073
12	99	64	0.0082	0.0033	0.0063
16	99	64	0.0086	0.0025	0.0067

TABLE A6 REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	FA	(26,26)
4	4	32	0.191	0.188	0.171
8	4	32	0.340	0.244	0.236
12	4	32	0.342	0.310	0.314
16	4	32	0.414	0.347	0.361
4	9	32	0.188	0.188	0.172
8	9	32	0.243	0.243	0.240
12	9	32	0.346	0.308	0.313
16	9	32	0.429	0.346	0.364
4	99	32	0.190	0.188	0.171
8	99	32	0.340	0.243	0.239
12	99	32	0.365	0.308	0.311
16	99	32	0.581	0.346	0.365
4	4	64	0.138	0.128	0.123
8	4	64	0.218	0.185	0.189
12	4	64	0.268	0.230	0.241
16	4	64	0.281	0.281	0.272
4	9	64	0.188	0.128	0.124
8	9	64	0.243	0.184	0.189
12	9	64	0.269	0.230	0.243
16	9	64	0.283	0.281	0.277
4	99	64	0.188	0.128	0.125
8	99	64	0.242	0.192	0.190
12	99	64	0.226	0.230	0.243
16	99	64	0.283	0.283	0.277

TABLE A6 REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.189	0.166	0.166
8	4	32	0.243	0.243	0.231
12	4	32	0.342	0.283	0.307
16	4	32	0.351	0.342	0.339
4	9	32	0.195	0.149	0.169
8	9	32	0.340	0.242	0.229
12	9	32	0.343	0.282	0.309
16	9	32	0.429	0.341	0.337
4	99	32	0.195	0.167	0.169
8	99	32	0.340	0.242	0.230
12	99	32	0.343	0.292	0.306
16	99	32	0.428	0.341	0.340
4	4	64	0.129	0.128	0.120
8	4	64	0.218	0.180	0.178
12	4	64	0.226	0.222	0.229
16	4	64	0.283	0.269	0.263
4	9	64	0.130	0.128	0.120
8	9	64	0.218	0.184	0.179
12	9	64	0.226	0.220	0.227
16	9	64	0.283	0.260	0.263
4	99	64	0.138	0.128	0.118
8	99	64	0.243	0.184	0.177
12	99	64	0.214	0.222	0.231
16	99	64	0.289	0.269	0.263

TABLE A6 REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.149	0.166	0.166
8	4	32	0.244	0.243	0.228
12	4	32	0.276	0.283	0.301
16	4	32	0.351	0.342	0.334
4	9	32	0.188	0.167	0.167
8	9	32	0.243	0.242	0.226
12	9	32	0.340	0.275	0.299
16	9	32	0.429	0.341	0.335
4	99	32	0.195	0.149	0.165
8	99	32	0.246	0.242	0.230
12	99	32	0.365	0.282	0.295
16	99	32	0.436	0.341	0.329
4	4	64	0.129	0.128	0.120
8	4	64	0.197	0.180	0.176
12	4	64	0.214	0.220	0.226
16	4	64	0.283	0.260	0.258
4	9	64	0.130	0.124	0.122
8	9	64	0.243	0.180	0.177
12	9	64	0.214	0.220	0.224
16	9	64	0.283	0.260	0.259
4	99	64	0.195	0.128	0.117
8	99	64	0.246	0.184	0.174
12	99	64	0.226	0.220	0.225
16	99	64	0.283	0.253	0.261

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

			(1)	(2)
MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(26,26,26)	(26,26,26)
4	4	32	0.1	+ + + - + +
8	4	32	0.3	+ + + + + +
12	4	32	0.2	+ + + - - +
16	4	32	0.2	- + + + - +
4	9	32	0.4	- # + + + +
8	9	32	0.4	- + + + + +
12	9	32	0.6	+ + + - - +
16	9	32	0.5	+ + + - - +
4	99	32	1.2	- # + + + #
8	99	32	1.2	+ + + - + +
12	99	32	2.7	+ # # - = #
16	99	32	2.6	- # # - - #
4	4	64	-0.2	- + + - + -
8	4	64	0.0	- - + + + +
12	4	64	-0.3	- + - - - -
16	4	64	-0.1	+ - - + + -
4	9	64	-0.1	- - - - + -
8	9	64	-0.1	- + + + + -
12	9	64	-0.4	- - - + + -
16	9	64	0.3	+ + + + + +
4	99	64	2.9	- # # - - #
8	99	64	2.3	+ + + + + +
12	99	64	1.2	+ + + - - +
16	99	64	1.5	+ + + - + +

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

			(1)	(2)
MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(26,26,26)	(26,26,26)
4	4	32	0.7	- # + - - #
8	4	32	1.2	# # + - + #
12	4	32	1.4	+ # + + + #
16	4	32	0.9	+ # + - + #
4	9	32	1.3	# # # + + #
8	9	32	2.2	+ # # + + #
12	9	32	1.7	+ # + + + #
16	9	32	1.8	+ # + + + #
4	99	32	1.1	- # # - - #
8	99	32	3.4	- # # - + #
12	99	32	2.7	+ # # + + #
16	99	32	6.6	+ # # - + #
4	4	64	0.4	- + - - + +
8	4	64	0.8	+ + + + + +
12	4	64	0.9	# + - - - #
16	4	64	1.2	+ # + - - #
4	9	64	1.8	+ # # + + #
8	9	64	1.3	+ # + - - #
12	9	64	1.5	- # + - = +
16	9	64	1.4	- # + + - #
4	99	64	3.3	+ # # - - #
8	99	64	3.5	+ # # - - #
12	99	64	2.5	+ # # - - #
16	99	64	2.6	+ + + - - #

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS C=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(1) (26,26,26)	(2) (26,26,26)
4	4	32	1.3	+ # + - - #
8	4	32	2.0	+ # + - - #
12	4	32	2.3	+ # # + - #
16	4	32	2.3	# # + + + #
4	9	32	1.8	- # # + + #
8	9	32	3.1	+ # # + + #
12	9	32	3.2	+ # # + - #
16	9	32	5.4	# # + - + #
4	99	32	1.5	- # # - + #
8	99	32	4.8	+ # # + + #
12	99	32	3.8	- # # + - #
16	99	32	5.6	+ # # + + #
4	4	64	0.8	+ + + - - #
8	4	64	2.5	+ # + # + #
12	4	64	1.1	+ + + - - +
16	4	64	2.3	# # + + - #
4	9	64	2.1	- # + + + #
8	9	64	2.8	+ # + + + #
12	9	64	4.9	- # # - - #
16	9	64	3.6	# # + = - #
4	99	64	3.8	+ # # + + #
8	99	64	5.5	+ # # - - #
12	99	64	4.9	- # # - - #
16	99	64	5.6	- # # - + #

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S SB	BIG S PA	BIG S (26,26)	LITTLE S SB	LITTLE S PA	LITTLE S (26,26)	BIG S - LITTLE S SB	BIG S - LITTLE S PA	BIG S - LITTLE S (26,26)
4	4	32	12	16	16.6	0	-1	-0.9	12	17	17.5
8	4	32	20	25	25.6	3	2	1.9	17	23	23.7
12	4	32	30	33	33.3	7	5	5.1	23	28	28.1
16	4	32	37	40	40.6	10	8	8.6	27	32	31.9
4	9	32	19	21	21.9	2	4	4.4	17	17	17.5
8	9	32	31	32	32.9	7	8	9.2	24	24	23.8
12	9	32	38	41	41.6	16	13	13.5	22	28	28.2
16	9	32	42	49	49.9	17	17	17.9	25	32	32.0
4	99	32	30	34	36.1	18	17	18.6	12	17	17.5
8	99	32	39	49	51.1	22	26	27.5	17	23	23.6
12	99	32	53	61	64.8	33	33	36.3	20	28	28.4
16	99	32	52	72	74.3	35	40	42.3	17	32	32.0
4	4	64	20	21	20.7	0	-4	-4.0	20	25	24.7
8	4	64	29	31	31.3	1	-2	-2.2	28	33	33.5
12	4	64	38	40	39.9	4	0	0.2	34	40	39.7
16	4	64	50	48	48.1	5	3	3.0	45	45	45.1
4	9	64	19	26	26.7	2	2	2.0	17	24	24.8
8	9	64	31	39	39.2	7	5	5.7	24	34	33.5
12	9	64	46	49	49.7	13	9	9.7	33	40	40.0
16	9	64	58	58	58.7	14	13	13.5	44	45	45.2
4	99	64	30	40	40.8	13	15	16.1	17	25	24.7
8	99	64	47	56	58.5	21	25	24.9	26	31	33.6
12	99	64	69	70	71.6	28	30	31.8	41	40	39.8
16	99	64	78	81	83.8	34	37	38.6	44	44	45.2

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S SB	BIG S PA	BIG S (26,26)	LITTLE S SB	LITTLE S PA	LITTLE S (26,26)	BIG S - LITTLE S SB	BIG S - LITTLE S PA	BIG S - LITTLE S (26,26)
4	4	32	27	27	28.0	14	9	9.4	13	18	18.6
8	4	32	46	45	47.1	21	20	21.7	25	25	25.4
12	4	32	59	62	63.0	36	32	33.1	23	30	29.9
16	4	32	77	78	79.0	46	44	45.1	31	34	33.9
4	9	32	30	34	35.0	19	15	16.4	11	19	18.6
8	9	32	47	54	55.1	30	29	30.0	17	25	25.0
12	9	32	67	72	73.5	44	42	43.5	23	30	30.0
16	9	32	81	89	90.8	56	55	56.8	25	34	34.0
4	99	32	42	52	53.7	31	34	35.2	11	18	18.5
8	99	32	61	79	81.4	44	54	56.3	17	25	25.1
12	99	32	87	101	104.1	64	72	74.3	23	29	29.8
16	99	32	105	122	125.7	79	88	91.8	26	34	33.9
4	4	64	31	31	31.6	8	5	5.3	23	26	26.3
8	4	64	48	51	50.9	20	15	15.6	28	36	35.3
12	4	64	70	68	69.0	29	26	26.8	41	42	42.2
16	4	64	85	85	86.3	41	38	38.3	44	47	47.9
4	9	64	38	38	39.3	16	12	13.0	22	26	26.3
8	9	64	57	60	61.4	29	25	25.9	28	35	35.5
12	9	64	79	80	81.4	38	37	38.9	41	43	42.4
16	9	64	95	98	99.0	51	50	51.2	44	48	47.9
4	99	64	49	57	58.2	29	31	32.1	20	26	26.1
8	99	64	70	85	88.1	47	50	52.6	23	35	35.5
12	99	64	104	109	111.4	58	67	69.3	46	42	42.1
16	99	64	117	130	133.8	76	83	86.0	41	47	47.8

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S			LITTLE S			BIG S - LITTLE S		
			SB	PA	(26,26)	SB	PA	(26,26)	SB	PA	(26,26)
4	4	32	38	36	37.3	19	18	18.4	19	18	18.9
8	4	32	63	63	64.3	40	38	38.7	23	25	25.6
12	4	32	89	88	88.3	58	58	58.0	31	30	30.3
16	4	32	111	112	113.5	80	78	79.0	31	34	34.5
4	9	32	40	43	45.6	26	25	26.4	14	18	19.1
8	9	32	70	72	73.5	47	47	48.0	23	25	25.5
12	9	32	94	99	100.3	69	68	70.0	25	31	30.4
16	9	32	116	124	127.4	91	90	92.7	25	34	34.6
4	99	32	50	64	67.0	39	45	48.1	11	19	18.9
8	99	32	87	100	103.7	65	74	78.2	22	26	25.6
12	99	32	111	131	137.0	91	101	106.6	20	30	30.4
16	99	32	139	161	167.5	117	127	132.9	22	34	34.6
4	4	64	40	40	40.7	17	14	14.0	23	26	26.6
9	4	64	68	68	68.8	38	32	32.8	30	36	35.9
12	4	64	98	94	94.9	52	51	52.0	46	43	42.9
16	4	64	118	119	120.8	74	71	71.9	44	48	48.9
4	9	64	47	48	49.0	25	21	22.3	22	27	26.8
8	9	64	70	78	79.4	47	42	43.4	23	36	36.0
12	9	64	108	106	107.6	62	63	64.6	46	43	42.9
16	9	64	130	132	133.1	86	84	84.6	44	48	48.4
4	99	64	50	68	70.8	39	42	44.2	11	26	26.6
8	99	64	87	105	106.8	65	70	71.1	22	35	35.7
12	99	64	128	138	141.0	87	95	98.1	41	43	42.9
16	99	64	155	169	172.3	111	120	123.7	44	49	48.5

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S (26,26)	LITTLE S (26,26)
4	4	32	1.60	0.48
8	4	32	1.77	0.72
12	4	32	1.94	0.92
16	4	32	2.68	1.38
4	9	32	2.39	1.14
8	9	32	2.95	1.76
12	9	32	3.30	2.09
16	9	32	3.66	2.42
4	99	32	4.32	3.30
8	99	32	5.42	4.51
12	99	32	6.76	5.82
16	99	32	6.92	6.09
4	4	64	1.76	0.23
8	4	64	2.19	0.61
12	4	64	2.34	0.82
16	4	64	2.66	1.01
4	9	64	2.41	0.84
8	9	64	2.91	1.26
12	9	64	3.05	1.45
16	9	64	3.59	1.77
4	99	64	4.12	2.74
8	99	64	5.40	4.01
12	99	64	6.15	4.74
16	99	64	6.66	5.34

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

			BIG S	LITTLE S
MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(26,26)	(26,26)
4	4	32	3.29	2.04
8	4	32	4.70	3.28
12	4	32	5.15	3.82
16	4	32	5.63	4.29
4	9	32	4.15	2.95
8	9	32	5.45	4.19
12	9	32	6.44	5.06
16	9	32	6.27	5.03
4	99	32	7.00	6.12
8	99	32	10.48	9.36
12	99	32	11.21	10.02
16	99	32	14.32	13.06
4	4	64	3.45	1.66
8	4	64	4.48	2.60
12	4	64	5.29	3.34
16	4	64	6.27	4.16
4	9	64	4.25	2.55
8	9	64	5.32	3.49
12	9	64	6.41	4.42
16	9	64	6.82	4.91
4	99	64	7.09	5.67
8	99	64	10.03	8.36
12	99	64	10.88	9.34
16	99	64	11.83	10.26

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS D=S-S=8 AND A NEGATIVE BINOMIAL DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF MANY-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S (26,26)	LITTLE S (26,26)
4	4	32	4.88	3.53
8	4	32	6.40	4.98
12	4	32	7.94	6.39
16	4	32	8.05	6.63
4	9	32	5.36	4.17
8	9	32	6.99	5.76
12	9	32	8.51	7.13
16	9	32	10.50	8.92
4	99	32	9.88	8.73
8	99	32	13.57	12.43
12	99	32	17.26	15.90
16	99	32	17.31	16.04
4	4	64	4.85	2.99
8	4	64	6.59	4.54
12	4	64	7.53	5.55
16	4	64	7.97	6.05
4	9	64	5.57	3.80
8	9	64	6.78	5.02
12	9	64	8.96	6.92
16	9	64	9.33	7.31
4	99	64	10.14	8.50
8	99	64	12.99	11.20
12	99	64	13.10	11.33
16	99	64	18.71	16.48

APPENDIX III

Single-Item Data

Heterogeneous-Stores Environment

Summary of Data for 72 Items in a Heterogeneous-Stores Environment

Controlled with:

Best (s,S) Policies (SB)

Power Approximation (PA)

Statistical Power Approximation

			page
Table	A1	Average Total Cost	III.1 to III.3
	A3	Period-End Inventory	III.4 to III.6
	A4	Period-End Backlog as Proportion of Mean Demand	III.7 to III.9
	A5	Frequency of Periods with Backlog	III.10 to III.12
	A6	Replenishment Frequency	III.13 to III.15
	A7	Estimated Bias of Forecost of Total Cost	III.16 to III.18
	A9	Values for (s,S)	III.19 to III.21
	A10	Standard Deviations of (s,S) Values	III.22 to III.24

TABLE A1 AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIC IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION CF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE CPTIMAL SE ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	17.9	1.1	3.9
8	4	32	25.5	1.5	3.6
12	4	32	30.9	0.1	2.4
16	4	32	36.1	0.6	1.2
4	9	32	22.0	0.9	4.1
8	9	32	30.7	0.0	3.9
12	9	32	36.2	1.6	3.5
16	9	32	41.9	1.1	4.9
4	99	32	33.5	-1.0	6.9
8	99	32	40.6	3.7	8.8
12	99	32	52.3	1.2	7.4
16	99	32	60.1	1.9	6.2
4	4	64	22.9	1.4	3.9
8	4	64	32.8	1.0	1.6
12	4	64	39.9	1.4	1.6
16	4	64	46.5	0.9	1.2
4	9	64	27.1	0.6	2.2
8	9	64	38.5	0.3	1.1
12	9	64	45.5	0.6	4.2
16	9	64	53.0	1.4	2.8
4	99	64	38.5	3.3	1.8
8	99	64	47.4	4.2	9.1
12	99	64	62.0	2.2	6.0
16	99	64	71.2	1.7	5.7

TABLE A1 AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL SB ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	21.3	0.2	3.9
8	4	32	32.1	0.9	2.3
12	4	32	33.5	0.3	3.4
16	4	32	42.7	-0.4	3.8
4	9	32	27.0	1.2	2.3
8	9	32	32.1	0.8	6.0
12	9	32	43.7	0.1	7.1
16	9	32	53.7	-0.3	2.5
4	99	32	33.2	11.2	14.1
8	99	32	56.2	5.8	7.7
12	99	32	65.7	3.0	15.8
16	99	32	64.4	5.0	10.7
4	4	64	25.7	1.5	3.3
8	4	64	38.8	-0.7	1.3
12	4	64	41.7	1.3	4.3
16	4	64	52.1	0.7	2.4
4	9	64	31.8	0.6	3.7
8	9	64	39.7	0.4	3.9
12	9	64	52.2	1.0	4.7
16	9	64	62.3	1.2	6.4
4	99	64	39.1	2.8	13.9
8	99	64	64.6	1.0	2.2
12	99	64	72.0	2.6	16.4
16	99	64	73.5	4.9	9.0

TABLE A1 AVERAGE TOTAL COST
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL SB ARE % EXCESS OVER SB VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	24.0	1.0	6.3
8	4	32	28.8	0.7	8.1
12	4	32	39.7	0.1	8.6
16	4	32	49.8	-1.0	2.0
4	9	32	25.3	2.4	14.8
8	9	32	41.6	0.2	7.5
12	9	32	50.2	0.4	11.8
16	9	32	50.5	0.7	3.9
4	99	32	42.8	6.1	18.3
8	99	32	63.0	5.8	16.1
12	99	32	61.2	6.0	10.3
16	99	32	82.1	3.2	13.7
4	4	64	28.9	-0.2	5.6
8	4	64	35.5	0.5	4.2
12	4	64	47.6	1.2	7.4
16	4	64	58.6	1.0	2.8
4	9	64	30.2	0.4	7.1
8	9	64	48.2	2.5	6.9
12	9	64	58.4	-0.2	10.8
16	9	64	60.3	0.6	4.4
4	99	64	46.7	5.2	16.3
8	99	64	72.0	0.6	11.9
12	99	64	70.8	3.2	11.0
16	99	64	92.2	4.1	11.1

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NC. CF PERICDS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	6.6	7.8	8.2
8	4	32	11.1	10.7	11.6
12	4	32	13.9	13.1	14.0
16	4	32	16.4	16.0	16.8
4	9	32	11.5	11.7	12.7
8	9	32	16.8	15.9	17.5
12	9	32	19.7	20.2	21.2
16	9	32	21.7	22.9	24.2
4	99	32	21.6	23.9	26.2
8	99	32	28.0	33.1	35.3
12	99	32	36.5	39.4	41.5
16	99	32	40.4	45.0	47.0
4	4	64	8.9	8.9	9.3
8	4	64	13.1	12.0	12.8
12	4	64	16.0	15.1	15.6
16	4	64	18.0	17.9	18.7
4	9	64	13.1	13.4	14.0
8	9	64	16.8	18.3	19.2
12	9	64	21.7	22.4	23.8
16	9	64	25.2	25.8	27.3
4	99	64	23.9	26.4	27.0
8	99	64	31.4	35.2	37.8
12	99	64	36.5	42.9	44.3
16	99	64	43.6	48.5	50.2

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	10.6	10.2	10.7
8	4	32	15.1	15.0	15.6
12	4	32	15.5	15.3	16.4
16	4	32	20.5	20.2	21.7
4	9	32	15.3	15.7	16.3
8	9	32	16.7	18.3	20.0
12	9	32	23.6	26.2	27.1
16	9	32	30.1	31.2	31.4
4	99	32	23.1	30.3	31.4
8	99	32	39.2	44.7	47.1
12	99	32	47.5	56.1	58.6
16	99	32	42.2	53.2	57.7
4	4	64	11.9	10.7	11.2
8	4	64	16.6	15.7	16.7
12	4	64	18.6	16.9	17.6
16	4	64	23.5	21.5	22.8
4	9	64	17.2	17.2	17.4
8	9	64	19.6	20.6	21.5
12	9	64	29.9	28.2	28.9
16	9	64	34.7	33.0	34.0
4	99	64	24.0	31.0	32.8
8	99	64	39.8	45.9	47.9
12	99	64	51.6	57.5	58.6
16	99	64	48.9	55.5	58.3

TABLE A3 PERIOD-END INVENTORY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	12.3	12.4	12.8
8	4	32	14.6	13.5	15.1
12	4	32	20.5	21.3	21.0
16	4	32	24.2	24.9	25.0
4	9	32	14.1	16.3	17.1
8	9	32	24.5	23.9	25.4
12	9	32	29.7	31.2	31.7
16	9	32	27.6	29.6	31.1
4	99	32	30.7	38.3	39.0
8	99	32	44.5	55.0	54.3
12	99	32	43.1	53.2	54.2
16	99	32	59.0	71.3	71.5
4	4	64	11.9	12.6	13.3
8	4	64	15.3	14.7	15.5
12	4	64	22.7	22.1	21.9
16	4	64	27.3	26.1	26.0
4	9	64	17.7	16.8	17.8
8	9	64	25.5	25.5	27.0
12	9	64	33.2	32.7	33.3
16	9	64	33.1	31.6	33.2
4	99	64	36.1	39.0	39.7
8	99	64	45.0	55.3	54.2
12	99	64	45.3	54.9	57.4
16	99	64	63.4	71.9	72.9

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.3504	0.3021	0.3240
8	4	32	0.1753	0.2201	0.2101
12	4	32	0.1492	0.1666	0.1616
16	4	32	0.1123	0.1380	0.1269
4	9	32	0.1314	0.1459	0.1307
8	9	32	0.0850	0.0976	0.0893
12	9	32	0.0540	0.0635	0.0591
16	9	32	0.0539	0.0561	0.0580
4	99	32	0.0105	0.0099	0.0108
8	99	32	0.0045	0.0012	0.0011
12	99	32	0.0051	0.0031	0.0040
16	99	32	0.0043	0.0031	0.0033
4	4	64	0.2906	0.4139	0.3938
8	4	64	0.2213	0.2788	0.2578
12	4	64	0.1552	0.2120	0.1989
16	4	64	0.1354	0.1760	0.1623
4	9	64	0.1291	0.1579	0.1499
8	9	64	0.0850	0.1163	0.1001
12	9	64	0.0661	0.0773	0.0770
16	9	64	0.0572	0.0703	0.0647
4	99	64	0.0099	0.0135	0.0100
8	99	64	0.0040	0.0025	0.0020
12	99	64	0.0051	0.0047	0.0052
16	99	64	0.0044	0.0037	0.0046

TABLE A4 PERIOD-END BACKLCG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NC. CF PERICDS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.3422	0.3817	0.3917
8	4	32	0.2551	0.3078	0.2991
12	4	32	0.1603	0.1834	0.1786
16	4	32	0.1615	0.1746	0.1812
4	9	32	0.1845	0.1882	0.1705
8	9	32	0.0884	0.0888	0.0842
12	9	32	0.0800	0.0743	0.0944
16	9	32	0.0810	0.0784	0.0877
4	99	32	0.0096	0.0033	0.0033
8	99	32	0.0102	0.0090	0.0073
12	99	32	0.0064	0.0019	0.0067
16	99	32	0.0048	0.0020	0.0014
4	4	64	0.3717	0.4853	0.4593
8	4	64	0.2815	0.3496	0.3409
12	4	64	0.1855	0.2225	0.2331
16	4	64	0.1806	0.2205	0.2093
4	9	64	0.1830	0.2023	0.2119
8	9	64	0.0948	0.1028	0.1060
12	9	64	0.0841	0.0945	0.1026
16	9	64	0.0824	0.0918	0.1060
4	99	64	0.0105	0.0040	0.0095
8	99	64	0.0108	0.0098	0.0082
12	99	64	0.0050	0.0016	0.0090
16	99	64	0.0035	0.0027	0.0028

TABLE A4 PERIOD-END BACKLOG AS PROPORTION OF MEAN DEMAND
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.4157	0.4491	0.4810
8	4	32	0.2259	0.2414	0.2563
12	4	32	0.2121	0.1964	0.2650
16	4	32	0.2147	0.2133	0.2347
4	9	32	0.1513	0.1279	0.1919
8	9	32	0.1319	0.1443	0.1620
12	9	32	0.0952	0.0908	0.1394
16	9	32	0.0655	0.0707	0.0707
4	99	32	0.0140	0.0059	0.0168
8	99	32	0.0117	0.0055	0.0143
12	99	32	0.0054	0.0018	0.0031
16	99	32	0.0061	0.0017	0.0070
4	4	64	0.4588	0.5473	0.5936
8	4	64	0.2305	0.2902	0.3054
12	4	64	0.2154	0.2523	0.3090
16	4	64	0.2124	0.2576	0.2738
4	9	64	0.1438	0.1620	0.1855
8	9	64	0.1346	0.1738	0.1803
12	9	64	0.0963	0.1072	0.1579
16	9	64	0.0735	0.0847	0.0862
4	99	64	0.0086	0.0066	0.0177
8	99	64	0.0116	0.0072	0.0189
12	99	64	0.0055	0.0032	0.0055
16	99	64	0.0047	0.0045	0.0081

TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.1795	0.1837	0.1637
8	4	32	0.1907	0.2141	0.2023
12	4	32	0.1877	0.2083	0.2000
16	4	32	0.1839	0.2143	0.1973
4	9	32	0.0966	0.1040	0.0973
8	9	32	0.0899	0.1012	0.0913
12	9	32	0.0901	0.0964	0.0908
16	9	32	0.0964	0.1000	0.0981
4	99	32	0.0094	0.0074	0.0106
8	99	32	0.0098	0.0028	0.0033
12	99	32	0.0088	0.0054	0.0077
16	99	32	0.0092	0.0064	0.0075
4	4	64	0.1903	0.2398	0.2396
8	4	64	0.1979	0.2436	0.2210
12	4	64	0.1917	0.2364	0.2250
16	4	64	0.1993	0.2342	0.2235
4	9	64	0.0929	0.1032	0.0983
8	9	64	0.0899	0.1164	0.1052
12	9	64	0.0988	0.1116	0.1071
16	9	64	0.0980	0.1046	0.1006
4	99	64	0.0074	0.0090	0.0094
8	99	64	0.0080	0.0042	0.0044
12	99	64	0.0088	0.0070	0.0094
16	99	64	0.0090	0.0072	0.0069

TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(PIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.1779	0.1975	0.1973
8	4	32	0.1899	0.2111	0.2031
12	4	32	0.1903	0.2099	0.2023
16	4	32	0.1955	0.2049	0.2058
4	9	32	0.0994	0.1008	0.0933
8	9	32	0.0976	0.0984	0.0900
12	9	32	0.0992	0.0901	0.0981
16	9	32	0.0976	0.0978	0.1069
4	99	32	0.0086	0.0028	0.0040
8	99	32	0.0096	0.0064	0.0085
12	99	32	0.0094	0.0038	0.0098
16	99	32	0.0096	0.0044	0.0031
4	4	64	0.1897	0.2270	0.2346
8	4	64	0.1917	0.2382	0.2206
12	4	64	0.1995	0.2358	0.2377
16	4	64	0.1965	0.2336	0.2288
4	9	64	0.0962	0.1004	0.1110
8	9	64	0.0992	0.1030	0.1054
12	9	64	0.0990	0.1038	0.1073
16	9	64	0.0960	0.1088	0.1196
4	99	64	0.0076	0.0034	0.0075
8	99	64	0.0094	0.0084	0.0083
12	99	64	0.0098	0.0036	0.0117
16	99	64	0.0098	0.0052	0.0056

TABLE A5 FREQUENCY OF PERIODS WITH BACKLOG
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.1857	0.1889	0.2104
8	4	32	0.1947	0.2129	0.2086
12	4	32	0.1887	0.1781	0.2142
16	4	32	0.1961	0.1977	0.2186
4	9	32	0.0966	0.0829	0.1121
8	9	32	0.0954	0.1034	0.1196
12	9	32	0.0972	0.0970	0.1238
16	9	32	0.0927	0.0976	0.0935
4	99	32	0.0096	0.0048	0.0113
8	99	32	0.0094	0.0038	0.0115
12	99	32	0.0096	0.0032	0.0046
16	99	32	0.0098	0.0034	0.0102
4	4	64	0.1993	0.2143	0.2333
8	4	64	0.1963	0.2306	0.2283
12	4	64	0.1885	0.2077	0.2319
16	4	64	0.1931	0.2125	0.2367
4	9	64	0.0907	0.1004	0.1065
8	9	64	0.0996	0.1188	0.1265
12	9	64	0.0976	0.0996	0.1273
16	9	64	0.0992	0.1112	0.1048
4	99	64	0.0090	0.0040	0.0090
8	99	64	0.0094	0.0056	0.0135
12	99	64	0.0088	0.0052	0.0065
16	99	64	0.0088	0.0064	0.0115

TABLE A6 REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIC IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.179	0.170	0.162
8	4	32	0.273	0.255	0.253
12	4	32	0.309	0.309	0.310
16	4	32	0.392	0.360	0.363
4	9	32	0.179	0.164	0.169
8	9	32	0.244	0.244	0.247
12	9	32	0.332	0.304	0.308
16	9	32	0.390	0.356	0.357
4	99	32	0.243	0.166	0.168
8	99	32	0.283	0.250	0.251
12	99	32	0.305	0.305	0.310
16	99	32	0.400	0.354	0.361
4	4	64	0.147	0.122	0.129
8	4	64	0.198	0.192	0.193
12	4	64	0.256	0.236	0.240
16	4	64	0.309	0.277	0.281
4	9	64	0.145	0.127	0.129
8	9	64	0.244	0.186	0.195
12	9	64	0.261	0.234	0.239
16	9	64	0.305	0.277	0.278
4	99	64	0.166	0.125	0.127
8	99	64	0.200	0.190	0.192
12	99	64	0.305	0.232	0.238
16	99	64	0.321	0.281	0.279

TABLE A6 REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.164	0.158	0.162
8	4	32	0.277	0.236	0.239
12	4	32	0.321	0.298	0.302
16	4	32	0.373	0.349	0.345
4	9	32	0.159	0.152	0.163
8	9	32	0.283	0.241	0.248
12	9	32	0.358	0.296	0.295
16	9	32	0.371	0.342	0.346
4	99	32	0.194	0.164	0.160
8	99	32	0.279	0.238	0.238
12	99	32	0.332	0.292	0.298
16	99	32	0.455	0.352	0.352
4	4	64	0.123	0.119	0.126
8	4	64	0.206	0.182	0.183
12	4	64	0.222	0.229	0.230
16	4	64	0.267	0.263	0.269
4	9	64	0.125	0.117	0.123
8	9	64	0.206	0.185	0.188
12	9	64	0.206	0.224	0.230
16	9	64	0.246	0.262	0.266
4	99	64	0.172	0.119	0.125
8	99	64	0.255	0.182	0.181
12	99	64	0.226	0.226	0.226
16	99	64	0.297	0.270	0.271

TABLE A6 REPLENISHMENT FREQUENCY
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	SB	PA	(26,26)
4	4	32	0.159	0.146	0.159
8	4	32	0.216	0.241	0.243
12	4	32	0.281	0.281	0.292
16	4	32	0.371	0.336	0.338
4	9	32	0.179	0.155	0.158
8	9	32	0.238	0.230	0.240
12	9	32	0.317	0.292	0.292
16	9	32	0.420	0.343	0.348
4	99	32	0.204	0.150	0.158
8	99	32	0.288	0.228	0.237
12	99	32	0.366	0.298	0.300
16	99	32	0.420	0.333	0.337
4	4	64	0.152	0.117	0.121
8	4	64	0.200	0.182	0.184
12	4	64	0.228	0.218	0.226
16	4	64	0.277	0.259	0.261
4	9	64	0.113	0.119	0.122
8	9	64	0.203	0.179	0.181
12	9	64	0.231	0.218	0.223
16	9	64	0.259	0.262	0.270
4	99	64	0.112	0.119	0.120
8	99	64	0.277	0.178	0.177
12	99	64	0.298	0.225	0.230
16	99	64	0.333	0.263	0.262

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(1) (26,26,26)	(2) (26,26,26)
4	4	32	0.2	+ + + - + +
8	4	32	0.2	- + + + + +
12	4	32	0.5	+ + + + + +
16	4	32	0.4	+ + + - - +
4	9	32	0.5	- # + - - #
8	9	32	0.4	- + + + + +
12	9	32	0.4	- + + + + +
16	9	32	0.8	+ + + + - +
4	99	32	3.0	+ # # + + #
8	99	32	0.9	+ # # - - #
12	99	32	3.6	- # # + + #
16	99	32	3.3	+ # # - - #
4	4	64	0.0	- + - + + +
8	4	64	0.5	+ - + - + +
12	4	64	0.3	+ + + - - +
16	4	64	0.3	+ + + + - +
4	9	64	0.4	+ + + - + +
8	9	64	0.5	- # + - - +
12	9	64	0.3	- + + = - +
16	9	64	0.4	+ + - + + +
4	99	64	2.8	+ # # - - #
8	99	64	1.0	+ + + + + +
12	99	64	3.7	- # # - - #
16	99	64	3.9	+ + + + + #

TABLE A7   ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1)   % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2)   + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(1) (26,26,26)	(2) (26,26,26)
4	4	32	0.7	- # + - - #
8	4	32	2.0	# # - - - #
12	4	32	0.9	+ # + + + #
16	4	32	2.6	# # # - - #
4	9	32	2.4	# # # - + #
8	9	32	1.2	+ # + + - #
12	9	32	3.4	+ # # - - #
16	9	32	4.4	+ # # - + #
4	99	32	1.4	+ # # - + #
8	99	32	5.7	- # # - - #
12	99	32	7.8	+ # # - + #
16	99	32	1.9	+ # + - + #
4	4	64	1.1	+ + + + # #
8	4	64	2.0	# + - + + #
12	4	64	1.1	+ # + - - #
16	4	64	1.8	+ # + + + #
4	9	64	2.2	+ # # = - #
8	9	64	1.5	+ + + + # #
12	9	64	2.4	# # + - + #
16	9	64	4.8	+ # # + + #
4	99	64	3.1	- # # - - #
8	99	64	5.5	- # # - - #
12	99	64	10.0	+ # # + + #
16	99	64	4.0	+ # # - + #

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMN (1) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMN (2) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(1) (26,26,26)	(2) (26,26,26)
4	4	32	2.7	# # + + + #
8	4	32	2.1	# # # + - #
12	4	32	4.4	# # # + - #
16	4	32	4.4	# # # + + #
4	9	32	4.1	# # # - + #
8	9	32	6.2	# # # = = #
12	9	32	8.8	# # # - + #
16	9	32	4.5	+ # # - - #
4	99	32	7.1	# # # - + #
8	99	32	11.4	+ # # + + #
12	99	32	3.5	+ + # + - +
16	99	32	10.8	+ # # - - #
4	4	64	2.8	# # + + + #
8	4	64	1.9	# # + - + #
12	4	64	3.7	# # # - - #
16	4	64	5.5	# # # - + #
4	9	64	1.9	+ # # - - #
8	9	64	6.1	# # # - - #
12	9	64	8.1	# # # - + #
16	9	64	4.8	+ # + + + #
4	99	64	7.0	+ # # + + #
8	99	64	14.5	+ # # + + #
12	99	64	6.7	# # # + + #
16	99	64	12.3	+ # # + + #

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S SB	BIG S PA	BIG S (26,26)	LITTLE S SB	LITTLE S PA	LITTLE S (26,26)	BIG S - LITTLE S SB	BIG S - LITTLE S PA	BIG S - LITTLE S (26,26)
4	4	32	14	16	16.4	0	-1	-1.0	14	17	17.4
8	4	32	25	25	25.9	4	2	2.1	21	23	23.8
12	4	32	34	33	32.8	6	5	5.0	28	28	27.8
16	4	32	39	40	40.4	11	8	8.5	28	32	31.8
4	9	32	20	21	21.8	5	4	4.3	15	17	17.5
8	9	32	33	32	32.5	9	8	8.9	24	24	23.6
12	9	32	39	41	41.4	14	13	13.3	25	28	28.1
16	9	32	46	49	49.6	18	17	17.7	28	32	31.9
4	99	32	28	34	35.4	19	17	18.0	9	17	17.4
8	99	32	42	49	50.6	23	26	27.0	19	23	23.6
12	99	32	58	61	62.6	30	33	34.5	28	28	28.1
16	99	32	65	72	72.9	38	40	41.2	27	32	31.7
4	4	64	19	21	20.7	-1	-4	-4.0	20	25	24.7
8	4	64	32	31	30.9	0	-2	-2.3	32	33	33.1
12	4	64	40	40	39.9	4	0	0.3	36	40	39.6
16	4	64	46	48	48.4	7	3	3.2	39	45	45.2
4	9	64	24	26	26.2	4	2	1.8	20	24	24.5
8	9	64	33	39	39.2	9	5	5.7	24	34	33.5
12	9	64	46	49	49.6	11	9	9.7	35	40	39.9
16	9	64	55	58	58.6	15	13	13.5	40	45	45.1
4	99	64	34	40	39.7	17	15	15.4	17	25	24.3
8	99	64	51	56	58.0	20	23	24.6	31	33	33.4
12	99	64	58	70	71.0	30	30	31.3	28	40	39.7
16	99	64	73	81	82.3	36	37	37.5	37	44	44.8

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S			LITTLE S			BIG S - LITTLE S		
			SB	PA	(26,26)	SB	PA	(26,26)	SB	PA	(26,26)
4	4	32	27	27	27.2	10	9	8.8	17	18	18.4
8	4	32	44	46	45.3	24	21	20.5	20	25	24.8
12	4	32	58	59	59.3	32	30	30.4	26	29	28.9
16	4	32	76	77	77.1	46	44	43.7	30	33	33.4
4	9	32	33	34	33.3	15	15	15.1	18	19	18.2
8	9	32	46	50	51.5	27	26	27.1	19	24	24.4
12	9	32	65	71	71.4	43	42	41.9	22	29	29.5
16	9	32	87	90	88.9	57	56	55.4	30	34	33.5
4	99	32	39	48	49.0	27	30	31.0	12	18	18.1
8	99	32	70	78	78.7	50	53	53.9	20	25	24.7
12	99	32	92	103	104.4	67	73	74.5	25	30	29.9
16	99	32	96	112	116.3	74	79	83.2	22	33	33.1
4	4	64	32	31	30.5	7	5	4.8	25	26	25.7
8	4	64	50	51	50.8	20	16	15.6	30	35	35.2
12	4	64	69	66	66.5	26	25	25.4	43	41	41.1
16	4	64	87	85	85.5	40	37	37.9	47	48	47.6
4	9	64	38	39	38.0	13	12	12.2	25	27	25.8
8	9	64	54	57	57.2	24	23	23.0	30	34	34.2
12	9	64	83	79	78.9	37	37	37.2	46	42	41.7
16	9	64	102	98	98.3	50	50	50.7	52	48	47.6
4	99	64	41	53	54.2	25	27	28.7	16	26	25.6
8	99	64	72	84	84.6	49	49	49.6	23	35	34.9
12	99	64	104	110	109.8	62	68	68.0	42	42	41.9
16	99	64	112	121	123.0	71	75	76.7	41	46	46.3

TABLE A9 VALUES FOR (S,S)

WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S SB	BIG S PA	BIG S (26,26)	LITTLE S SB	LITTLE S PA	LITTLE S (26,26)	BIG S - LITTLE S SB	BIG S - LITTLE S PA	BIG S - LITTLE S (26,26)
4	4	32	37	38	36.5	19	18	17.8	18	20	18.6
8	4	32	63	60	61.7	35	36	36.8	28	24	24.9
12	4	32	88	89	86.9	57	58	56.9	31	31	29.9
16	4	32	111	114	112.0	81	79	77.8	30	35	34.2
4	9	32	38	42	42.0	24	23	23.6	14	19	18.4
8	9	32	73	73	73.3	48	47	47.8	25	26	25.4
12	9	32	98	101	99.5	71	71	69.3	27	30	30.2
16	9	32	114	120	120.5	89	86	87.0	25	34	33.4
4	99	32	54	65	63.9	42	46	45.2	12	19	18.6
8	99	32	91	105	102.2	72	79	76.8	19	26	25.4
12	99	32	109	123	123.8	87	94	94.3	22	29	29.5
16	99	32	147	164	162.0	122	129	127.9	25	35	34.1
4	4	64	37	41	40.0	18	14	13.8	19	27	26.2
8	4	64	65	66	65.4	34	31	30.7	31	35	34.7
12	4	64	95	95	94.1	54	52	51.5	41	43	42.6
16	4	64	121	121	118.3	76	72	70.2	45	49	48.1
4	9	64	48	46	46.3	20	20	20.4	28	26	26.0
8	9	64	77	79	78.9	46	43	43.1	31	36	35.8
12	9	64	108	109	107.2	67	65	64.4	41	44	42.8
16	9	64	131	129	128.8	82	81	81.5	49	48	47.2
4	99	64	67	69	68.6	39	43	42.2	28	26	26.4
8	99	64	92	110	106.6	72	74	71.0	20	36	35.6
12	99	64	115	131	132.6	86	89	90.8	29	42	41.8
16	99	64	156	171	170.3	121	123	122.0	35	48	48.3

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 0

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S (26,26)	LITTLE S (26,26)
4	4	32	1.93	0.55
8	4	32	2.72	1.11
12	4	32	3.15	1.42
16	4	32	2.78	1.38
4	9	32	2.52	1.11
8	9	32	3.47	1.88
12	9	32	3.22	1.99
16	9	32	4.14	2.59
4	99	32	5.54	3.90
8	99	32	4.12	3.38
12	99	32	7.36	5.93
16	99	32	7.87	6.47
4	4	64	2.12	0.24
8	4	64	2.93	0.78
12	4	64	3.36	1.10
16	4	64	2.59	0.97
4	9	64	3.21	0.97
8	9	64	4.01	1.56
12	9	64	3.35	1.52
16	9	64	4.49	2.16
4	99	64	5.13	3.13
8	99	64	4.59	3.22
12	99	64	7.30	5.32
16	99	64	7.43	5.58

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 2

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	BIG S (26,26)	LITTLE S (26,26)
4	4	32	3.97	2.39
8	4	32	6.39	4.40
12	4	32	5.00	3.62
16	4	32	7.37	5.53
4	9	32	5.65	3.84
8	9	32	4.82	3.50
12	9	32	7.54	5.80
16	9	32	9.65	7.62
4	99	32	6.38	5.15
8	99	32	11.80	10.07
12	99	32	14.65	12.90
16	99	32	10.64	9.34
4	4	64	4.92	2.31
8	4	64	6.39	3.68
12	4	64	4.97	3.20
16	4	64	7.18	4.79
4	9	64	6.30	3.52
8	9	64	5.00	3.11
12	9	64	8.04	5.48
16	9	64	10.10	7.18
4	99	64	7.19	5.26
8	99	64	10.32	8.22
12	99	64	14.13	11.67
16	99	64	9.29	7.60

TABLE A10 STANDARD DEVIATIONS OF (S,S) VALUES
WHOLESALE WAREHOUSE INVENTORY SYSTEM
EACH STORE HAS A NEGATIVE BINOMIAL DEMAND DISTRIBUTION
WAREHOUSE DEMAND VARIANCE/MEAN RATIO IS 9
REPLENISHMENT LEADTIME = 4

STATISTICAL SIMULATION OF HETEROGENEOUS-STORES ENVIRONMENT
(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

			BIG S	LITTLE S
MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	(26,26)	(26,26)
4	4	32	6.76	4.81
8	4	32	6.09	4.72
12	4	32	9.89	8.02
16	4	32	12.57	10.46
4	9	32	6.36	4.76
8	9	32	9.68	7.85
12	9	32	12.67	10.58
16	9	32	8.41	7.09
4	99	32	11.55	9.86
8	99	32	16.67	14.91
12	99	32	12.27	10.96
16	99	32	18.51	16.84
4	4	64	7.06	4.34
8	4	64	6.30	4.39
12	4	64	9.79	7.27
16	4	64	13.09	9.97
4	9	64	6.32	4.13
8	9	64	10.87	8.02
12	9	64	12.25	9.41
16	9	64	9.66	7.55
4	99	64	12.36	9.71
8	99	64	16.04	13.47
12	99	64	12.97	10.90
16	99	64	18.60	16.03

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