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**A FORTRAN Computer Program for Calculating  
The Propagation of Plane, Cylindrical, or Spherical  
Finite Amplitude Waves**

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| A numerical solution to the generalized Burgers' radial wave equation has been developed; it allows one to calculate stepwise the harmonic content of a finite amplitude wave in the frequency domain for the case of plane, cylindrical, or spherical geometry. The finite amplitude wave may have any initial harmonic content with arbitrary phase, and the absorption coefficient of each harmonic is independently adjustable. Remaining in the frequency domain allows a much larger step than conventional algorithms, which alternate between the time and frequency domains. A listing of the computer program is included. |                       |                                                             |  |

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# A FORTRAN COMPUTER PROGRAM FOR CALCULATING THE PROPAGATION OF PLANE, CYLINDRICAL, OR SPHERICAL FINITE AMPLITUDE WAVES

## INTRODUCTION

Several adequate theories [1,2,3], based on approximations to the nonlinear wave equation, have been developed to describe the behavior of a one-dimensional wave of moderate amplitude as it propagates through a nonlinear fluid. Most of these theories, however, are not conveniently applied to the problem of describing the propagation when the wave is of arbitrary initial waveform or when the linear absorption has an arbitrary frequency dependence. To handle these more general cases, investigators [4,5,6] have adapted the phenomenological model of Fox and Wallace [7] to use a high-speed computer to calculate the propagation stepwise. The distance of propagation in this model is divided into small intervals. The wave is first allowed to distort over one interval and is then corrected to account for absorption and geometrical spreading. The procedure is then repeated for the new waveform over the next interval. The use of small intervals preserves the interaction between the distortion, the absorption, and the geometrical spreading mechanisms. Since the distortion mechanism is applied in the particle velocity domain with absorption and geometrical spreading being applied in the frequency domain, it is necessary to switch back and forth between the two domains during each step. Even with the use of the Fast Fourier Transform (FFT), this procedure is a time-consuming process. In addition, one must take special care in applying the distortion mechanism when the waveform has a very steep shock-like portion.

We describe in this report a new procedure for calculating the propagation of plane, cylindrical, or spherical finite amplitude waves. This procedure performs the stepwise calculations entirely in the frequency domain, thus avoiding both the use of the FFT and the steep waveform problems. We also describe a FORTRAN computer program called FAW that implements the new procedure.

The theory behind the procedure is described in Sec. I. A discussion of the relative importance of linear absorption and nonlinear effects is presented in Sec. II. In Sec. III the numerical implementation of the procedure

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is developed. This is followed in Sec. IV by a description of the computer program FAW. Included are discussions of the significant FORTRAN variable names, the major computational blocks, parameter input, printed output, number of harmonics required in the calculation, and initial step size. Section V contains a comparison of results obtained using the new procedure with results obtained using the phenomenological model. The report concludes with appendices containing sample output and a program listing of FAW.

## I. THEORY

An approximate nonlinear wave equation valid for one-dimensional and progressive plane, cylindrical, and spherical waves in a lossless fluid is the generalized Burgers' equation [8]

$$\frac{\partial U}{\partial r} + (a/r)U - bU \frac{\partial U}{\partial \tau} = 0, \quad (1)$$

where  $U$  = particle velocity,

$r$  = spatial coordinate,

$\tau$  = retarded time  $(t - \frac{r}{c_0})$ ,

$c_0$  = small signal sound speed,

$b = \beta/c_0^2$

$\beta = (1 + \frac{B}{2A})$ , the nonlinearity parameter, and

$a = 0$  (plane waves),

$= 1/2$  (cylindrical waves), or

$= 1$  (spherical waves).

Equation (1) is modified to include linear absorption by noting that in the absence of nonlinearity the amplitude decays according to

$$U(r) = U_0 (r_0/r)^a \exp [-\alpha(\omega)(r-r_0)] \quad (2)$$

or

$$\frac{\partial U}{\partial r} = -(a/r) U - \alpha(\omega) U. \quad (3)$$

The first term on the right-hand side of Eq. (3) represents the loss due to geometrical spreading and is already included in Eq. (1). The second term represents the loss due to linear absorption. Adding this term to Eq. (1) results in the following nonlinear equation for a lossy fluid with arbitrary frequency-dependent absorption where dispersion has been neglected:

$$\frac{\partial U}{\partial r} + (a/r)U - bU (\partial U / \partial \tau) = -\alpha(\omega) U. \quad (4)$$

We now choose as a trial solution a Fourier series of linear damped waves of arbitrary phase with amplitudes that are a function of the spatial coordinate  $r$ :

$$U(r, \tau) = \sum_{k=1}^{\infty} (r_0/r)^a \left\{ G_k \sin(k\omega_0 \tau) + H_k \cos(k\omega_0 \tau) \right\} \exp[-\alpha_k(r-r_0)], \quad (5)$$

where  $\alpha_k$  is the absorption coefficient appropriate for the  $k^{\text{th}}$  harmonic. The fundamental frequency  $\omega_0$  is chosen less than or equal to  $1/\tau_0$ , where  $\tau_0$  is either the period of the initial waveform at  $r=r_0$  when the waveform is periodic or it is a time length sufficiently long to contain the resulting waveform at all desired distances when the waveform is transient. Substitution of Eq. (5) into Eq. (4) yields two coupled differential equations governing the behavior of the amplitude components  $G_k$  and  $H_k$  as a function of the spatial coordinate  $r$ :

$$\sum_{k=1}^{\infty} \frac{\partial G_k}{\partial r} \sin(k\omega_0 \tau) \exp[-\alpha_k(r-r_0)]$$

$$= \frac{b\omega_0}{2} (r_0/r)^a \sum_{\ell, m} m \exp[-(\alpha_\ell + \alpha_m)(r-r_0)]$$

$$\{ (G_\ell G_m - H_\ell H_m) \sin[(\ell+m)\omega_0 \tau]$$

$$+ (G_\ell G_m - H_\ell H_m) \sin[(\ell-m)\omega_0 \tau] \},$$

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and

$$\begin{aligned}
 & \sum_{k=1}^{\infty} \frac{\partial H_k}{\partial r} \cos(k\omega_0 \tau) \exp[-\alpha_k(r-r_0)] \\
 &= \frac{b\omega_0}{2} (r_0/r)^a \sum_{\ell, m} m \exp[-(\alpha_{\ell+m} + \alpha_m)(r-r_0)] \\
 & \quad \left\{ (H_{\ell} G_m + G_{\ell} H_m) \cos[(\ell+m)\omega_0 \tau] \right. \\
 & \quad \left. + (H_{\ell} G_m - G_{\ell} H_m) \cos[(\ell-m)\omega_0 \tau] \right\}.
 \end{aligned} \tag{7}$$

Factoring out terms of the same frequency (i.e., terms in which  $\ell+m = k$ ,  $\ell-m = k$ , and  $\ell-m = -k$ ) results in

$$\begin{aligned}
 \frac{\partial G_k}{\partial r} &= \frac{b\omega_0}{2} (r_0/r)^a \left\{ \sum_{m=1}^{k-1} m (G_{k-m} G_m - H_{k-m} H_m) \exp[-(\alpha_{k-m} + \alpha_m - \alpha_k)(r-r_0)] \right. \\
 &+ \sum_{m=1}^{\infty} m (G_{k+m} G_m + H_{k+m} H_m) \exp[-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_0)] \\
 &\left. - \sum_{m=k+1}^{\infty} m (G_{m-k} G_m + H_{m-k} H_m) \exp[-(\alpha_{m-k} + \alpha_m - \alpha_k)(r-r_0)] \right\}
 \end{aligned} \tag{8}$$

and

$$\begin{aligned}
 \frac{\partial H_k}{\partial r} &= \frac{b\omega_0}{2} (r_0/r)^a \left\{ \sum_{m=1}^{k-1} m (H_{k-m} G_m + G_{k-m} H_m) \exp[-(\alpha_{k-m} + \alpha_m - \alpha_k)(r-r_0)] \right. \\
 &+ \sum_{m=1}^{\infty} m (H_{k+m} G_m - G_{k+m} H_m) \exp[-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_0)] \\
 &\left. + \sum_{m=k+1}^{\infty} m (H_{m-k} G_m - G_{m-k} H_m) \exp[-(\alpha_{m-k} + \alpha_m - \alpha_k)(r-r_0)] \right\}.
 \end{aligned} \tag{9}$$

The first sum on the right-hand side of Eqs. (8) and (9) represents contributions to the  $k^{\text{th}}$  harmonic due to the interaction of lower harmonics producing a sum frequency component at the  $k^{\text{th}}$  harmonic. The second sum in the equations corresponds to the interaction of higher harmonics producing a difference frequency component at the  $k^{\text{th}}$  harmonic. The last sum in the equations represents the loss to the  $k^{\text{th}}$  harmonic due to its interaction with all of the harmonics.

Combining the last two sums in Eqs. (8) and (9) yields

$$\begin{aligned} \frac{\partial G_k}{\partial r} = \frac{b\omega_0}{2} (r_0/r)^a \left\{ \sum_{m=1}^{k-1} m (G_{k-m} G_m - H_{k-m} H_m) \exp [-(\alpha_{k-m} + \alpha_m - \alpha_k)(r-r_0)] \right. \\ \left. - k \sum_{m=1}^{\infty} (G_{k+m} G_m + H_{k+m} H_m) \exp [-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_0)] \right\} \end{aligned} \quad (10)$$

and

$$\begin{aligned} \frac{\partial H_k}{\partial r} = \frac{b\omega_0}{2} (r_0/r)^a \left\{ \sum_{m=1}^{k-1} m (H_{k-m} G_m + H_m G_{k-m}) \exp [-(\alpha_{k-m} + \alpha_m - \alpha_k)(r-r_0)] \right. \\ \left. + k \sum_{m=1}^{\infty} (H_m G_{k+m} - H_{k+m} G_m) \exp [-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_0)] \right\}. \end{aligned} \quad (11)$$

Equations (10) and (11) are the coupled nonlinear equations that are numerically integrated to obtain the harmonic amplitudes  $G_k$  and  $H_k$  as a function of the spatial coordinate  $r$ .

## II. RELATIVE IMPORTANCE OF LINEAR ABSORPTION AND NONLINEAR EFFECTS

The Goldberg number  $\Gamma$  is defined as

$$\Gamma = \frac{1}{\alpha \ell} \quad (12)$$



where  $\alpha$  is the absorption coefficient and  $\ell$  is the discontinuity distance, the point at which the waveform would shock if linear absorption were absent. The discontinuity distance is geometry dependent and for initially sinusoidal waves is given by

$$\ell_p = \frac{c_o^2}{\beta U_o \omega_o} \quad (\text{plane wave}), \quad (13)$$

$$\ell_c = r_o + \frac{c_o^2}{\beta U_o \omega_o} + \frac{c_o^4}{4r_o \beta^2 U_o^2 \omega_o^2} \quad (\text{cylindrical wave}), \quad (14)$$

and

$$\ell_s = r_o \exp(c_o^2 / \beta U_o \omega_o r_o) \quad (\text{spherical wave}). \quad (15)$$

If the Goldberg number is greater than unity, the nonlinear effect becomes important and shocks are likely. In this case, as the waveform approaches the discontinuity distance, nonlinear effects dominate the loss due to linear absorption. The amplitudes of all harmonics above the fundamental increase at the expense of the fundamental. After the discontinuity distance is reached, however, linear absorption plays an increasingly larger role and eventually the amplitudes of all the harmonics decrease with distance.

The use of the Goldberg number is important in deciding on the number of harmonics to retain in the calculation. If the Goldberg number is small compared to unity, then the nonlinear effect is small and the waveform is not going to shock. In this case, a small number of harmonics will adequately describe the waveform at any position. However, if the Goldberg number is large, then shocks are likely and a large number of harmonics must be retained in the calculation. A discussion of the relative error associated with the number of harmonics retained in the calculation is found in Section IV.E.

### III. NUMERICAL IMPLEMENTATION

#### A. Series Truncation

In order to numerically integrate Eqs. (10) and (11) the infinite series on the right-hand side of each equation must be truncated in such a manner that no instability is introduced into the algorithm. This problem can best be examined by assuming that the phase of the initial waveform is such that all the amplitude coefficients  $H_k$  are zero and that  $j$  harmonics are retained in the calculation. This results in Eq. (11) vanishing and Eq. (10) reducing to

$$\frac{\partial G_k}{\partial r} = -\frac{h\omega_0}{2} (r_0/r)^a \left\{ \sum_{m=1}^{k-1} G_{k-m} G_m \exp [-(\alpha_{k-m} + \alpha_m - \alpha_k)(r-r_0)] \right. \\ \left. - k \sum_{m=1}^{j-k} G_{k+m} G_m \exp [-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_0)] \right\}. \quad (16)$$

The simple truncation used in obtaining Eq. (16) is insufficient when an attempt is made to examine the propagation of the waveform beyond the discontinuity distance. In calculating the propagation, the flow of energy from lower to higher harmonics stops with the last harmonic retained in the series. This is obvious from the fact that the second series in Eq. (16) vanishes for the  $j^{\text{th}}$  harmonic. Thus the use of simple truncation eliminates the primary nonlinear energy-loss mechanism of the last ( $j^{\text{th}}$ ) harmonic. The harmonics preceding the last are affected in a similar, but less severe, manner. The calculated values for the last harmonics become abnormally large relative to the lower harmonics. Being "too large", these harmonics then cause an abnormal growth of the next lower harmonics so that eventually even the lowest harmonics are significantly in error. This instability is circumvented in the program by artificially increasing the loss of the last few (and least significant) harmonics by requiring that their amplitude never exceed the amplitude of the next lower harmonic.

## B. Integration Method

Equations (10) and (11) are stepwise numerically integrated by the first-order Runge-Kutta method to obtain the amplitudes of the harmonics at progressively increasing distances. This method, as applied to the numerical solution of the problem

$$G'_k = -\frac{\partial G_k}{\partial r} = R_k(r, G, H) \quad (17)$$

and

$$H'_k = -\frac{\partial H_k}{\partial r} = S_k(r, G, H) \quad (18)$$

yields the amplitudes at the  $(N+1)^{th}$  step as

$$G_k(N+1) = G_k(N) + \frac{h}{2} [G'_k(N) + G'_k(N+1)] \quad (19)$$

and

$$H_k(N+1) = H_k(N) + \frac{h}{2} [H'_k(N) + H'_k(N+1)], \quad (20)$$

where

$$G'_k(N) = R_k[r(N), G(N), H(N)], \quad (21)$$

$$H'_k(N) = S_k[r(N), G(N), H(N)], \quad (22)$$

$$G'_k(N+1) = R_k[r(N+1), G(N) + hG'_k(N), H(N) + hH'_k(N)], \quad (23)$$

$$H'_k(N+1) = S_k[r(N+1), G(N) + hG'_k(N), H(N) + hH'_k(N)], \quad (24)$$

and  $h$  is the incremental step size [9]. This procedure has the advantage of not requiring the calculation of any derivatives of  $R_k$  and  $S_k$  as would be necessary in a Taylor-series expansion. The procedure also allows the step size to be easily changed at any point in the calculation. The only disadvantage is that the right-hand sides of Eqs. (17) and (18) must be evaluated twice at each step.

### C. Variable Step Size

In calculating the propagation of a finite amplitude wave it is desirable to use the largest step size that will produce accurate results. During the initial portion of the propagation, when the waveform is undergoing its most rapid change due to nonlinear effects, a small step size is required. After the discontinuity distance is reached, however, the nonlinear effects become less pronounced and the step size can be increased. In order to minimize the running time of the computer program, both under these circumstances and on occasions when an overly conservative initial step size has been chosen, a variable step size feature is incorporated.

This feature doubles the step size whenever the average percentage change in the amplitude components  $|\Delta G_j/G_j|$  and  $|\Delta H_j/H_j|$ ,  $j=1, 2, \dots, k$ , over the previous step is below some arbitrary value  $\epsilon$ . The integer  $k$  is the number of harmonics printed in the output and is generally less than the number of harmonics retained in the calculation. There are two methods for controlling the doubling of the step size. Either the internal value  $\epsilon$  may be modified or the number of harmonics printed out may be changed.

The harmonics are printed out at fixed distance intervals (print out distance interval = specified integer  $\times$  initial step size). When the step size is doubled, it is unlikely that the harmonics will be calculated at positions coinciding with the print-out distance. The program circumvents this problem by linearly interpolating the output from the calculated values.

#### D. Renormalization

One of the standard programming problems associated with numerical solutions is the limited exponent range of computers. The Advanced Scientific Computer (ASC) at the Naval Research Laboratory (NRL), for which this program was written, has an exponent range of -76 to +76. The terms most likely to exceed this range are the exponentials in Eqs. (10) and (11).

To illustrate this problem the exponentials in Eqs. (10) and (11) are examined for the case of an omega-squared frequency dependence of the absorption coefficients (fresh water). The exponential in the second series on the right-hand sides of Eqs. (10) and (11) may then be written in the form

$$\exp [-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_0)] = \exp [-2m(k+m)\alpha_1(r-r_0)], \quad (25)$$

where  $\alpha_1$  is the absorption coefficient of the fundamental. Since  $m$  and  $k$  are positive integers, the exponential tends to zero with increasing  $r$ . This causes no problem if the computer is told to set underflow to zero (the error associated with setting numbers smaller than  $10^{-76}$  to zero is negligible).

The exponential in the first series on the right-hand sides of Eqs. (10) and (11) may be written as

$$\exp [-(\alpha_{k+m} + \alpha_m - \alpha_k)(r-r_0)] = \exp [+2m(k-m)\alpha_1(r-r_0)]. \quad (26)$$

Since  $m$  is always less than  $k$  in the first series, this exponential may exceed the upper bound of the exponent range. As an example, when 50 harmonics are retained in the calculation of a 100-kHz spherical wave in fresh water ( $\alpha_1 = 2.38 \times 10^{-4}$  N/m), the exponent range of +76 is exceeded at  $r = 255$  m. This distance is totally insufficient to examine the asymptotic decay of the spherical waves.

A simple method for circumventing this problem is to renormalize the waveform after each step. In this procedure the source position  $r_0$  is changed after each step and set equal to the current position  $r$ . This limits the size of the distance term in the exponentials to  $h$ , the step size, and merely requires that the amplitudes be transformed as

$$A_k \longrightarrow A_k [R_0 / (R_0 + h)] \exp (-\alpha_k h). \quad (27)$$

It has the additional advantage of not requiring the calculation of the exponentials at each step since they do not change and can be stored. If the step size doubles, the stored values are simply squared.

#### IV. DESCRIPTION OF COMPUTER PROGRAM FAW

The computer program FAW, which is listed in Appendix A, is written in universal FORTRAN and should run on any computer accepting this language. However, the program has been specifically written to take advantage of the vectorizing capability of the ASC at NRL.

While the use of the vectorizing option on the ASC greatly reduces the running time of the program, it does not allow the use of variable ranges on nested DO loops. This constraint results in FAW running inefficiently on computers that do not have vectorizing capability. Therefore, it may be necessary to modify FAW for use on a nonvectorizing computer.

The DO loops, which are modified in FAW for the vectorizing process, are the loops associated with the two series on the right-hand sides of Eqs. (10) and (11) (lines 243 to 256 and 320 to 333 in FAW). The first series on the right-hand sides of Eqs. (10) and (11) has an upper limit of  $k-1$  for the  $k^{\text{th}}$  harmonic. The second series is truncated to an upper limit of  $j-k$  for the  $k^{\text{th}}$  harmonic when  $j$  harmonics are retained in the calculation. Since both of these limits are a function of the harmonic increment being calculated, the range of the inner DO loop is not a constant. However, the ASC runs quicker

if the loops are vectorized, and to do this the ranges of the two inner loops have been set equal to J-1. This results in extraneous terms being calculated and increases the running time on nonvectorizing computers.

The extraneous terms do not contribute to the calculation and are all set equal to zero in the DO loops. This is accomplished by generating a matrix for each DO loop whose elements are zero for the extraneous terms and whose nonzero elements are the exponential terms in Eqs. (10) and (11). As explained in the section on renormalization, this matrix need only be calculated once and is then used at each step.

The remainder of this section describes FAW. Included are a listing of the significant FORTRAN variable names and descriptions of the major computation blocks in FAW followed by a discussion of the parameter input, printed output, number of harmonics to retain in calculation, and initial step size.

#### A. Significant FORTRAN Variable Names

The significant FORTRAN variable names in FAW are as follows:

|       |                                                                                                               |
|-------|---------------------------------------------------------------------------------------------------------------|
| A     | Geometrical spreading factor<br>= 0 (plane waves),<br>= 1/2 (cylindrical waves),<br>= 1 (spherical waves).    |
| ALPHA | Vector whose $I^{\text{th}}$ element ALPHA (I) is the absorption coefficient of the $I^{\text{th}}$ harmonic. |
| B     | The constant $b = \beta/c_0^2$ in Eqs. (10) and (11).                                                         |
| BETA  | Coefficient of nonlinearity.                                                                                  |

|      |                                                                                                                                                                                     |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| C    | Small signal sound speed $c_0$ .                                                                                                                                                    |
| DG   | Vector containing the differential change of the amplitude coefficients G. The element DG(I) contains the differential change $hG_I'(N)$ at the $N^{th}$ step as given by Eq. (21). |
| DG1  | Vector whose $I^{th}$ element DG1(I) contains the differential change $hG_I'(N+1)$ at the $N+1^{th}$ step as given by Eq. (23).                                                     |
| DH   | Vector containing the differential change of the amplitude coefficients H. The element DH(I) contains the differential change $hH_I'(N)$ at the $N^{th}$ step as given by Eq. (22). |
| DH1  | Vector whose $I^{th}$ element DH1(I) contains the differential change $hH_I'(N+1)$ at the $N+1^{th}$ step as given by Eq. (24).                                                     |
| DX   | Current step size.                                                                                                                                                                  |
| DXI  | Initial step size.                                                                                                                                                                  |
| E    | Normalization constant. The output is normalized to the constant E, which is an input parameter and generally set equal to the initial amplitude of the fundamental.                |
| F1   | Factor from Eqs. (10) and (11) equal to $(r_0/r)^a$ .                                                                                                                               |
| FREQ | Frequency of fundamental.                                                                                                                                                           |
| G    | Vector whose $I^{th}$ element contains the amplitude G(I) of the sine component of the $I^{th}$ harmonic.                                                                           |
| GH   | Vector whose $I^{th}$ element $GH(I) = \exp [-ALPHA (I) * DX]$ .                                                                                                                    |



GX            Array whose elements are all zero that is used as a buffer for the working array G2. In the DO loops which calculate DG, DG1, DH, and DH1 the ranges of the loops have been written to take advantage of vectorization. This results in negative indices for some of the G2 elements in the calculation. The GX array is placed before the G2 array to prevent incorrect results.

G2            Array whose 1<sup>th</sup> element first contains G(I) and in later calculations contains G(I) + DG(I) as required by the Runge-Kutta method.

H             Same as G for the cosine elements.

HX            Same as GX but placed in front of H2.

H2            Same as G2 for the cosine elements.

IAF           Input parameter equal to either zero, if only the absorption coefficient for the fundamental is input and an omega-squared dependence for the harmonic absorption coefficients is used, or one if the absorption coefficient is input for each harmonic retained in the calculations.

IP            Print-out interval = IP \* DX.

J             Number of harmonics retained in the calculations.

K             Number of input G coefficients.

KI            Number of input H coefficients.

L            Number of externally supplied absorption coefficients used to modify the omega-squared dependent coefficients. Used only if IAF = 0 and the omega-squared dependence is to be modified.

NI           Number of harmonics printed to output.

R            Spatial coordinate.

RMAX        Maximum distance that propagation is to be calculated.

RN           Source position. Modified after each step. Set equal to the present position R as described in section on renormalization.

RO           Initial source position.

X1           Doubly dimensioned array whose elements  $X1(M, N)$  are unity if  $X11(M, N)$  is nonzero and zero otherwise.

X11          Array whose elements  $X11(M, N) = \exp((\text{ALPHA}(M) - \text{ALPHA}(N) - \text{ALPHA}(M-N)) * DX)$  for  $M = 2$  to  $J$  and  $N = 1$  to  $M$ . All other elements are zero.

XZ           Doubly dimensioned array whose elements  $X2(M, N)$  are unity if  $X22(M, N)$  is nonzero and zero otherwise.

X22          Array whose elements  $X22(M, N) = \exp((\text{ALPHA}(M) - \text{ALPHA}(N) - \text{ALPHA}(M+N)) * DX)$  for  $M = 1$  to  $J-1$  and  $N = 1$  to  $J-M$ . All other elements are zero.

## B. Major Computation Blocks in FAW

Descriptions of the major computation blocks in FAW are as follows:

| <u>Computation Block</u>                                                                                                                | <u>Line Number</u> |           |
|-----------------------------------------------------------------------------------------------------------------------------------------|--------------------|-----------|
|                                                                                                                                         | <u>From</u>        | <u>To</u> |
| Initialize all arrays to zero.                                                                                                          | 29                 | 44        |
| Read first fourteen data cards and<br>print data to output.                                                                             | 51                 | 138       |
| Obtain absorption coefficients.                                                                                                         | 142                | 179       |
| Read input waveform and print waveform<br>to output.                                                                                    | 183                | 199       |
| Calculate matrix elements of $X_1$ ,<br>$X_{11}$ , $X_2$ , and $X_{22}$ and elements of<br>vector $GH$ .                                | 204                | 221       |
| Calculate $DG$ and $DH$ , first derivative<br>of Runge-Kutta method.                                                                    | 243                | 256       |
| Calculate new amplitudes for second<br>derivative.                                                                                      | 260                | 265       |
| Find last five non-zero harmonic<br>amplitudes and modify them, if<br>necessary, to insure that they form<br>a non-increasing sequence. | 289                | 308       |
| Calculate $DG_1$ and $DH_1$ , second<br>derivative of Runge-Kutta method.                                                               | 320                | 333       |

| <u>Computation Block</u>                                                                                                       | <u>Line Number</u> |           |
|--------------------------------------------------------------------------------------------------------------------------------|--------------------|-----------|
|                                                                                                                                | <u>From</u>        | <u>To</u> |
| Calculate new amplitudes for current position.                                                                                 | 337                | 340       |
| Find last five non-zero harmonic amplitudes and modify them, if necessary, to insure that they form a non-increasing sequence. | 344                | 382       |
| Check position for output.                                                                                                     | 386                | 386       |
| Interpolate output if output position does not coincide with position of calculated amplitudes.                                | 390                | 404       |
| Standard output (no interpolation necessary).                                                                                  | 405                | 408       |
| Print output.                                                                                                                  | 409                | 418       |
| Check step size (if incremental change is small, then double step size).                                                       | 422                | 431       |
| Modify waveform for renormalization.                                                                                           | 435                | 438       |
| Double step size.                                                                                                              | 443                | 454       |
| Next step.                                                                                                                     | 456                |           |

### C. Parameter Input

The input consists of a series of data cards:

Data Card 1: Format D17.10 - FREQ, the frequency of the fundamental.

Data Card 2: Format D17.10 - C, the small signal sound speed in meters per second.

Data Card 3: Format D17.10 - BETA, the coefficient of nonlinearity.

Data Card 4: Format D17.10 - E, the normalization constant.

Data Card 5: Format D17.10 - DXI, the initial step size in meters.

Data Card 6: Format D17.10 - RMAX, the maximum propagation distance in meters.

Data Card 7: Format D17.10 - R0, the source size, in meters, for cylindrical and spherical waves. If a plane wave is being calculated, R0 is not used but a value must be entered.

Data Card 8: Format F5.2 - A, the geometrical spreading factor.  
A = 0 (plane waves)  
= 1/2 (cylindrical waves)  
= 1 (spherical waves).

Data Card 9: Format I4 - IAF, the absorption flag. IAF = 0 if the omega-squared dependence for the absorption coefficients is being used. IAF = 1 if all absorption coefficients are entered on data cards.

Data Card 10: Format I4 - J, the number of harmonics retained in the calculation.

Data Card 11: Format I4 - K, the number of initial G coefficients entered.

Data Card 12: Format I4 - KI, the number of initial H coefficients entered.

Data Card 13: Format I4 - NI, the number of harmonics printed to output.

Data Card 14: Format I8 - IP, the integer multiplicative factor of  
DXI which gives the print-out interval.

If IAF = 0

Data Card 15: Format D17.10 - ALPHA (1), the absorption coefficient  
of the fundamental in nepers per meter.

Data Card 16: Format I4 - L, the number of harmonics being modified  
from the omega-squared dependence.

Next L Cards: Format I4, D17.10 - These cards contain an integer, right  
justified in the first four spaces on the card, specifying the  
number of the harmonic followed by the absorption coefficient  
for that harmonic.

If IAF = 1

Data Card 15 to 14+J: Format D17.10 - These cards contain the J absorption  
coefficients in order in nepers per meter.

Next K Cards: Format I4, D17.10 - These cards contain a right justified  
integer, in the first four spaces, specifying the number of  
the harmonic followed by the G amplitude coefficient of that  
harmonic in meters per second.

Next KI Cards: Format I4, D17.10 - These cards contain a right justified  
integer, in the first four spaces, specifying the number of  
the harmonic followed by the H amplitude coefficient of  
that harmonic in meters per second.

#### D. Printed Output

A sample output from FAW is shown in Appendix B. The first portion of the output contains a listing of all input parameters. This allows the input parameters to be checked for errors and is useful for future reference. The remaining portion contains the calculated output.

The distance in meters is printed at each output interval followed by two columns of numbers. The columns are labeled and contain the amplitudes of the sine and cosine components divided by the normalization constant  $E$ . The number of terms output at each interval is  $NI$ , which is a user-specified parameter. In addition, the step size  $DX$  in meters, is printed each time the step size is doubled. This gives the user useful information on the effects of varying the step size doubling parameter  $\epsilon$  and the number of harmonics printed out  $NI$ .

#### E. Number of Harmonics

A sufficient number of harmonics must be retained in the calculation to insure a negligibly small error in the highest harmonic of interest. In order to obtain some measure of the required number of harmonics, the algorithm was used to calculate the harmonic content of an initially pure sinusoidal plane wave with a variety of harmonics being retained in the calculation. The frequency of the fundamental was 2.5 MHz, and the initial pressure amplitude was 3 atmospheres, which gives a discontinuity distance of 21 cm. Table I lists the percentage deviations of the resulting amplitudes of the first five harmonics, at the discontinuity distance, from the values obtained when 40 harmonics were retained. As is obvious from the table, the required number of harmonics to retain depends on the harmonic of interest and the allowable error. If the fundamental is the only harmonic of interest, one need retain no more than seven harmonics in the calculation. On the other hand, an accurate value for the fifth harmonic may require twenty or more harmonics to be retained in the calculation.

Table 1 - percentage deviation in the first five harmonics  
for various numbers of retained harmonics N

| $\frac{N}{\text{HARMONIC}}$ | 7    | 8    | 9    | 10   | 15    | 20     | 25      | 30      |
|-----------------------------|------|------|------|------|-------|--------|---------|---------|
| 1st                         | 0.03 | 0.02 | 0.02 | 0.02 | 0.004 | 0.0003 | 0.00002 | 0.00000 |
| 2nd                         | 0.20 | 0.20 | 0.10 | 0.10 | 0.020 | 0.0020 | 0.00030 | 0.00003 |
| 3rd                         | 0.80 | 0.60 | 0.50 | 0.50 | 0.080 | 0.0090 | 0.00100 | 0.00010 |
| 4th                         | 2.40 | 1.80 | 1.60 | 1.50 | 0.200 | 0.0200 | 0.00300 | 0.00040 |
| 5th                         | 4.40 | 3.80 | 3.50 | 3.40 | 0.500 | 0.0500 | 0.00700 | 0.00080 |

#### F. Step Size

With the ability of the program to double the step size after each step, it is best to choose the initial step size conservatively and let the program find the best value. In order to determine a conservative initial value for the step size, the choice of step size was investigated for the same 2.5-MHz plane-wave case used in the previous section. The step size was not allowed to double and various step sizes from 1/200 to 1/10 of the discontinuity distance were used. Table II lists the percentage deviations of the first five harmonic amplitudes from the values obtained when the step size was 1/200 of the discontinuity distance. The amplitudes were those at the discontinuity distance, and forty harmonics were retained in the calculations. The table indicates that a step size of 1/10 of the discontinuity distance will yield results that are accurate to within the normal experimental error.



Table II - Percentage deviation in the first five harmonics  
for various step sizes

( $\sigma$  = discontinuity distance)

| STEP SIZE<br>HARMONIC | 1/100 $\sigma$ | 1/50 $\sigma$ | 1/20 $\sigma$ | 1/10 $\sigma$ |
|-----------------------|----------------|---------------|---------------|---------------|
| 1st                   | 0.0005         | 0.0030        | 0.0200        | 0.0600        |
| 2nd                   | 0.0001         | 0.0050        | 0.0300        | 0.1000        |
| 3rd                   | 0.0020         | 0.0100        | 0.0900        | 0.4000        |
| 4th                   | 0.0070         | 0.0400        | 0.2500        | 1.1000        |
| 5th                   | 0.0100         | 0.0700        | 0.5000        | 2.0000        |

#### V. COMPARISON WITH PHENOMENOLOGICAL MODEL

As a test of their validity, Eqs. (10) and (11) were used to compute the harmonic content of an initially pure sinusoidal 300-Hz plane wave. An omega-squared frequency dependence of the linear absorption terms was assumed with the absorption of the fundamental being  $1.12 \times 10^{-6}$  Np/m. The initial pressure was 1 atmosphere. This problem was also solved using a computer algorithm [6] based on the phenomenological model of Fox and Wallace [7]. Figure (1) illustrates the agreement between the results obtained using the algorithm presented in this paper, shown as solid curves, and the results obtained using the phenomenological model, shown as dots. Although only the first four harmonics are illustrated in this figure, the agreement was equally as good for higher harmonics.

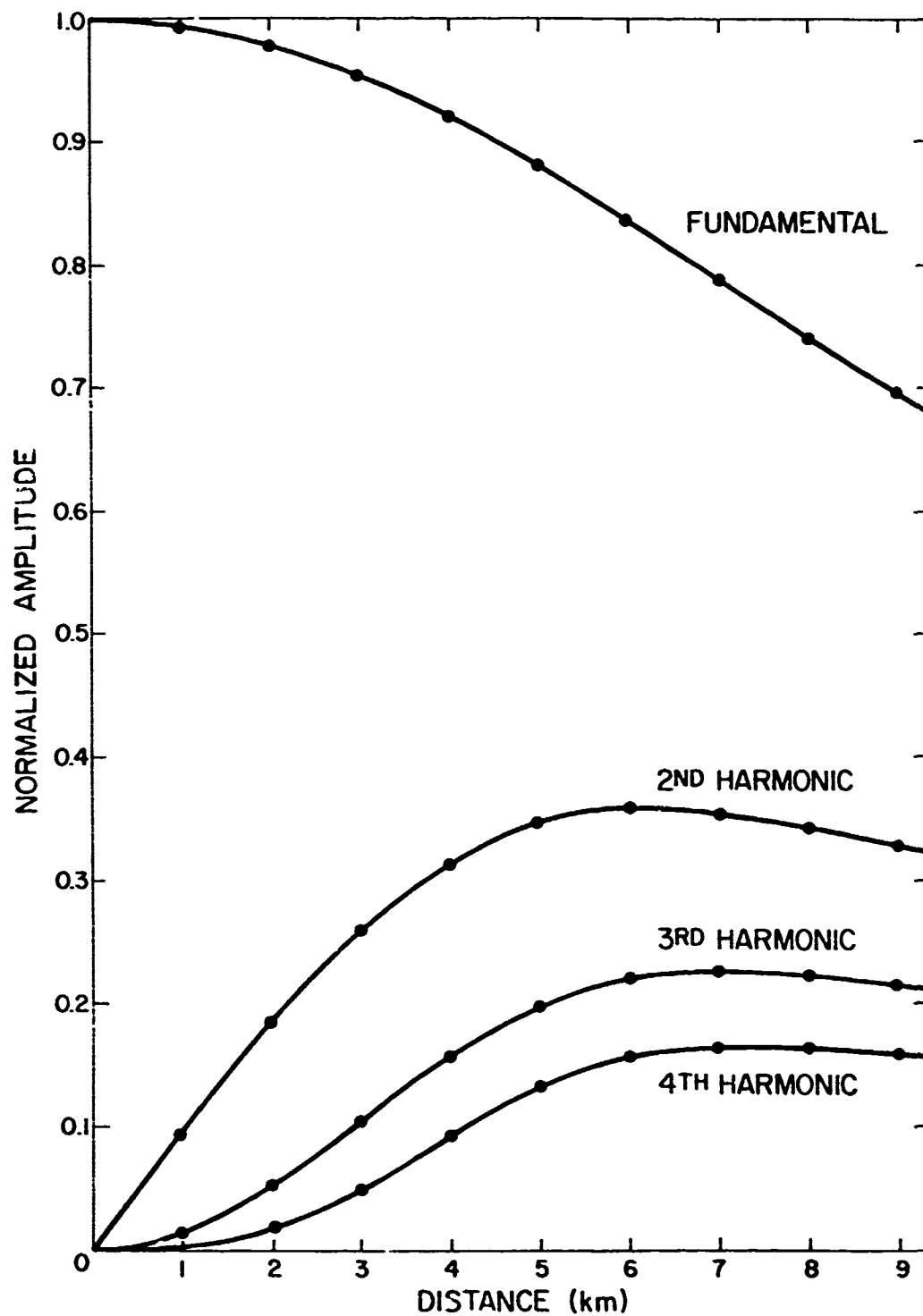


Fig. 1 - Calculated harmonic behavior of an initially pure<sub>0</sub> sinusoidal plane wave ( $\nu = 300$  Hz,  $\alpha = 1.12 \times 10^{-6}$  N/m, and  $P = 1$  atm.) by — frequency domain algorithm, • phenomenological model.

## REFERENCES

1. W. Keck and R. T. Beyer, *Physics of Fluids* 3, 342-352 (1960).
2. D. T. Blackstock, *J. Acous. Soc. Am.* 36, 534-542 (1964).
3. K. A. Naugol'nykh, S. I. Soluyan, and R. V. Khokhlov, *Sov. Phys. Acoust.* 9, 42-46 (1963).
4. B. D. Cook, *J. Acous. Soc. Am.* 34, 941-946 (1962).
5. A. L. Van Buren, *J. Acous. Soc. Am.* 44, 1021-1027 (1968).
6. A. L. Van Buren, *J. Sound Vib.* 42, 273-280 (1975).
7. F.E. Fox and W.A. Wallace, *J. Acous. Soc. Am.* 26, 994-1006 (1954).
8. D. T. Blackstock, *J. Acous. Soc. Am.* 36, 217-219 (1964).
9. F. B. Hildebrand, Introduction to Numerical Analysis (McGraw-Hill Book Company, Inc., New York, 1956).

# APPENDIX A

## COMPUTER PROGRAM FAW

```

1 C
2 C
3 C
4 C   THIS PROGRAM STEPWISE CALCULATES THE HARMONIC CONTENT
5 C   OF A FINITE AMPLITUDE WAVE AS A FUNCTION OF POSITION.
6 C   THE INITIAL WAVEFORM MAY HAVE ANY HARMONIC CONTENT
7 C   AND ARBITRARY PHASE. THE ABSORPTION COEFFICIENTS MAY
8 C   BE EXTERNALLY SUPPLIED OR AN INTERNAL OMEGA SQUARED
9 C   ALGORITHM IS SUPPLIED. THE INTERNAL ALGORITHM ALLOWS
10 C  MODIFICATION OF ANY OF THE OMEGA SQUARED COEFFICIENTS.
11 C  THE NUMERICAL INTEGRATION USES THE RUNGE-KUTTA METHOD
12 C  AND THE STEP SIZE WILL AUTOMATICALLY INCREASE WHEN THE
13 C  CHANGE OVER AN INTERVAL IS SMALL.
14 C
15 C
16 C
17   DIMENSION G(50),GX(50),G2(100),H(50),HX(50),H2(100)
18   DIMENSION X1(50,50),X11(50,50),X2(50,50)
19   DIMENSION X22(50,50),XL(50),XL1(50),GH(50)
20   DIMENSION ALPHA(50),DG(50),DG1(50),DH(50),DH1(50)
21   DIMENSION KG(5),KH(5)
22   DOUBLE PRECISION G,GX,G2,H,HX,H2,X1,X11,X2,X22,XL,XL1
23   DOUBLE PRECISION GH,ALPHA,DG,DG1,DH,DH1,R,C,FREQ,BETA
24   DOUBLE PRECISION E,B,X,F1,DX,RMAX,RN,Z,Z1,DXI,PD
25   DOUBLE PRECISION Y,RQ
26 C
27 C   INITIALIZE REGISTERS TO ZERO
28 C
29   DO 110 IB=1,100
30     G(IB)=0.D0
31     GX(IB)=0.D0
32     G2(IB)=0.D0
33     H(IB)=0.D0
34     HX(IB)=0.D0
35     H2(IB)=0.D0
36 110  CONTINUE
37   DO 130 IC=1,50
38     DO 120 ID=1,50
39       X1(IC,ID)=0.D0
40       X11(IC,ID)=0.D0
41       X2(IC,ID)=0.D0
42       X22(IC,ID)=0.D0
43 120  CONTINUE
44 130  CONTINUE
45 C
46 C   INPUT DATA
47 C
48 C
49 C   FREQ=FUNDAMENTAL FREQUENCY
50 C
51   READ(5,140)FREQ

```

```

52 140  FORMAT(D17.10)
53      PRINT 150,FREQ
54 150  FORMAT(5X,10HFREQUENCY=,D17.10)
55 C
56 C      C=SMALL SIGNAL SOUND SPEED
57 C
58      READ(5,140)C
59      PRINT 160,C
60 160  FORMAT(5X,12HSOUND SPEED=,D17.10)
61 C
62 C      BETA=COEFFICIENT OF NONLINEARITY(1+B/2A)
63 C
64      READ(5,140)BETA
65      PRINT 170,BETA
66 170  FORMAT(5X,5HBETA=,D17.10)
67 C
68 C      E=NORMALIZATION CONSTANT (OUTPUT IN DB RE(E))
69 C
70      READ(5,140)E
71      PRINT 180,E
72 180  FORMAT(5X,23HNORMALIZATION CONSTANT=,D17.10)
73 C
74 C      DXI=INITIAL STEP SIZE
75 C
76      READ(5,140)DXI
77      PRINT 190,DXI
78 190  FORMAT(5X,18HINITIAL STEP SIZE=,D17.10)
79 C
80 C      RMAX=MAXIMUM PROPAGATION DISTANCE
81 C
82      READ(5,140)RMAX
83      PRINT 200,RMAX
84 200  FORMAT(5X,17HMAXIMUM DISTANCE=,D17.10)
85 C
86 C      RO=SOURCE SIZE(INITIAL POSITION FOR GEOMETRICAL SPREADING)
87 C
88      READ(5,140)RO
89      PRINT 210,RO
90 210  FORMAT(5X,12HSOURCE SIZE=,D17.10)
91 C
92 C      A=SPREADING FACTOR
93 C          =0 (PLANE WAVES)
94 C          =1/2 (CYLINDRICAL WAVES)
95 C          =1 (SPHERICAL WAVES)
96 C
97      READ(5,220)A
98 220  FORMAT(F5.2)
99      PRINT 230,A
100 230  FORMAT(5X,17HSPREADING FACTOR=,F5.2)
101 C
102 C      IAF- FLAG(ABSORPTION COEFFICIENTS)

```

```

103 C      =0 (OMEGA SQUARED DEPENDENCE)
104 C      =1 (EXTERNALLY SUPPLIED)
105 C
106      READ(5,240)IAF
107 240      FORMAT(I4)
108      PRINT 250,IAF
109 250      FORMAT(5X,16HABSORPTION FLAG=,I4)
110 C
111 C      J=NUMBER OF HARMONICS RETAINED IN CALCULATION
112 C
113      READ(5,240)J
114      PRINT 260,J
115 260      FORMAT(5X,20HNUMBER OF HARMONICS=,I4)
116 C
117 C      K=NUMBER OF INITIAL G COEFFICIENTS
118 C      KI=NUMBER OF INITIAL H COEFFICIENTS
119 C
120      READ(5,240)K
121      READ(5,240)KI
122      PRINT 270,K
123 270      FORMAT(5X,23HINITIAL G COEFFICIENTS=,I4)
124      PRINT 280,KI
125 280      FORMAT(5X,23HINITIAL H COEFFICIENTS=,I4)
126 C
127 C      NI=NUMBER OF HARMONICS PRINTED OUT
128 C
129      READ(5,240)NI
130      PRINT 290,NI
131 290      FORMAT(5X,27HNUMBER OF HARMONICS OUTPUT=,I4)
132 C
133 C      IP-PRINT OUT INTERVAL=IP*DX
134 C
135      READ(5,300)IP
136 300      FORMAT(I8)
137      PRINT 310,IP
138 310      FORMAT(5X,19HPRINT OUT INTERVAL=,I8)
139 C
140 C      OBTAIN ABSORPTION COEFFICIENTS
141 C
142      IF(IAF)1110,320,400
143 C
144 C      OMEGA SQUARED DEPENDENCE
145 C
146 320      READ(5,140)ALPHA(1)
147      PRINT 330,ALPHA(1)
148 330      FORMAT(5X,9HALPHA(1)=,D17,10)
149      DO 340 IE=2,J
150      ALPHA(IE)=IE*IE*ALPHA(1)
151 340      CONTINUE
152 C
153 C      L=NUMBER OF ABSORPTION COEFFICIENTS BEING MODIFIED

```

```

154 C      FROM OMEGA SQUARED DEPENDENCE
155 C
156      READ(5,240)L
157      PRINT 350,L
158 350    FORMAT(5X,43HNUMBER OF MODIFIED ABSORPTION COEFFICIENTS=,I4)
159      IF(L)1110,440,360
160 360    PRINT 370
161 370    FORMAT(5X,33HMODIFIED ABSORPTION COEFFICIENTS.)
162      DO 390 IG=1,L
163      READ(5,380)N,X
164 380    FORMAT(I4,D17.10)
165      PRINT 380,N,X
166      ALPHA(N)=X
167 390    CONTINUE
168      GO TO 440
169 C
170 C      INPUT ABSORPTION COEFFICIENTS
171 C
172 400    PRINT 410
173 410    FORMAT(5X,23HABSORPTION COEFFICIENTS)
174      DO 430 IH=1,J
175      READ(5,140)X
176      ALPHA(IH)=X
177      PRINT 420,IH,ALPHA(IH)
178 420    FORMAT(1X,6HALPHA(,I3,2H)=,D17.10)
179 430    CONTINUE
180 C
181 C      READ INPUT WAVEFORM
182 C
183 440    CONTINUE
184      IF(K)1110,490,450
185 450    PRINT 460
186 460    FORMAT(5X,14HINPUT WAVEFORM)
187      DO 480 IJ=1,K
188      READ(5,380)N,X
189      G(N)=X
190      PRINT 470,N,G(N)
191 470    FORMAT(1X,2HG(,I3,2H)=,D17.10)
192 480    CONTINUE
193 490    IF(KI)1110,530,500
194 500    DO 520 IK=1,KI
195      READ(5,380)N,X
196      H(N)=X
197      PRINT 510,N,H(N)
198 510    FORMAT(1X,2HH(,I3,2H)=,D17.10)
199 520    CONTINUE
200 530    B=3.1415926536*BETA*FREQ/(C*C)
201 C
202 C      CALCULATE MATRIX ELEMENTS
203 C
204      DO 550 IL=2,J

```

```

205      IL1=IL-1
206      DO 540 IM=1,IL1
207          X11(IL,IM)=DEXP((ALPHA(IL)-ALPHA(IM)-ALPHA(IL-IM))*DXI)
208          X1(IL,IM)=1.D0
209 540      CONTINUE
210 550      CONTINUE
211      IFX=J-1
212      DO 570 IN=1,IFX
213          IN1=J-IN
214          DO 560 IP=1,IN1
215              X22(IN,IP)=DEXP((ALPHA(IN)-ALPHA(IP)-ALPHA(IN+IP))*DXI)
216              X2(IN,IP)=1.D0
217 560      CONTINUE
218 570      CONTINUE
219      DO 580 IQ=1,J
220          GH(IQ)=DEXP(-ALPHA(IQ)*DXI)
221 580      CONTINUE
222 C
223 C      SET COUNTER
224 C
225      RN=RO
226      R=RN
227      DX=DXI
228      PD=IP*DX
229      F1=1.D0
230 590      CB=DX*B
231 C
232 C      FILL ARRAYS
233 C
234      DO 600 IR=1,J
235          G2(IR)=G(IR)
236          H2(IR)=H(IR)
237          DG(IR)=0.D0
238          DH(IR)=0.D0
239 600      CONTINUE
240 C
241 C      ENTER LOOP FOR CALCULATING FIRST DERIVATIVE
242 C
243      DO 620 IS=2,J
244          DO 610 IT=1,IFX
245              CX=CB*IT*X1(IS,IT)
246              DG(IS)=CX*(G2(IS-IT)*G2(IT)-H2(IS-IT)*H2(IT))+DG(IS)
247              DH(IS)=CX*(H2(IS-IT)*G2(IT)+G2(IS-IT)*H2(IT))+DH(IS)
248 610      CONTINUE
249 620      CONTINUE
250      DO 640 IU=1,J
251          DO 630 IV=1,IFX
252              CX=CB*IU*X2(IU,IV)
253              DG(IU)=DG(IU)-CX*(G2(IU+IV)*G2(IV)+H2(IU+IV)*H2(IV))
254              DH(IU)=DH(IU)+CX*(G2(IU+IV)*H2(IV)-H2(IU+IV)*G2(IV))
255 630      CONTINUE

```



```

256 640    CONTINUE
257 C
258 C    CALCULATE NEW AMPLITUDES FOR SECOND DERIVATIVE
259 C
260        DO 650 IW=1,J
261        G2(IW)=G(IW)+DG(IW)
262        H2(IW)=H(IW)+DH(IW)
263        DG1(IW)=0.D0
264        DH1(IW)=0.D0
265 650    CONTINUE
266 C
267 C    FIND LAST FIVE NON-ZERO HARMONICS
268 C
269        DO 655 IW1=1,5
270        KG(IW1)=0
271        KH(IW1)=0
272 655    CONTINUE
273        DO 710 IX=1,J
274        IF(G2(IX))660,680,660
275 660        DO 670 IY=1,4
276        KG(IY)=KG(IY+1)
277 670        CONTINUE
278        KG(5)=IX
279 680        IF(H2(IX))690,710,690
280 690        DO 700 IZ=1,4
281        KH(IZ)=KH(IZ+1)
282 700        CONTINUE
283        KH(5)=IX
284 710    CONTINUE
285 C
286 C    INSURE THAT THE LAST FIVE HARMONICS ARE NOT
287 C    PROGRESSIVELY LARGER
288 C
289        DO 720 JA=1,4
290        JB=KG(JA)
291        IF(JB)712,720,712
292 712        Z=DABS(G2(JB))
293        JC=KG(JA+1)
294        IF(JC)714,720,714
295 714        Z1=DABS(G2(JC))
296        IF(Z1.LT.Z)GO TO 720
297        G2(JC)=0.95*G2(JC)*Z/Z1
298 720    CONTINUE
299        DO 730 JD=1,4
300        JE=KH(JD)
301        IF(JE)722,730,722
302 722        Z=DABS(H2(JE))
303        JF=KH(JD+1)
304        IF(JF)724,730,724
305 724        Z1=DABS(H2(JF))
306        IF(Z1.LT.Z)GO TO 730

```

```

307      H2(JF)=0.95*H2(JF)*Z/Z1
308 730    CONTINUE
309      R=R+DX
310      IF(A)1110,760,740
311 740    IF(A.LT.1.D0)GO TO 750
312      F1=RN/R
313      CB=DX*B*F1
314      GO TO 760
315 750    F1=(RN/R)**0.5
316      CB=DX*B*F1
317 C
318 C      ENTER LOOP FOR CALCULATING SECOND DERIVATIVE
319 C
320 760    DO 780 JG=2,J
321          DO 770 JH=1,IFX
322          CX=CB*JH*X11(JG,JH)
323          DG1(JG)=CX*(G2(JG-JH)*G2(JH)-H2(JG-JH)*H2(JH))+DG1(JG)
324          DH1(JG)=CX*(H2(JG-JH)*G2(JH)+G2(JG-JH)*H2(JH))+DH1(JG)
325 770    CONTINUE
326 780    CONTINUE
327      DO 800 JI=1,J
328          DO 790 JJ=1,IFX
329          CX=CB*JI*X22(JI,JJ)
330          DG1(JI)=DG1(JI)-CX*(G2(JI+JJ)*G2(JJ)+H2(JI+JJ)*H2(JJ))
331          DH1(JI)=DH1(JI)+CX*(G2(JI+JJ)*H2(JJ)-H2(JI+JJ)*G2(JJ))
332 790    CONTINUE
333 800    CONTINUE
334 C
335 C      CALCULATE NEW AMPLITUDES
336 C
337      DO 810 JK=1,J
338      G(JK)=G(JK)+0.5*(DG(JK)+DG1(JK))
339      H(JK)=H(JK)+0.5*(DH(JK)+DH1(JK))
340 810    CONTINUE
341 C
342 C      FIND LAST FIVE NON-ZERO HARMONICS
343 C
344      DO 820 JL=1,5
345      KG(JL)=0
346      KH(JL)=0
347 820    CONTINUE
348      DO 880 JM=1,J
349      IF(G(JM))830,850,830
350 830      DO 840 JN=1,4
351          KG(JN)=KG(JN+1)
352 840      CONTINUE
353      KG(5)=JM
354 850      IF(H(JM))860,880,860
355 860      DO 870 JO=1,4
356          KH(JO)=KH(JO+1)
357 870      CONTINUE

```

```

358      KH(5)=JM
359 880    CONTINUE
360 C
361 C      INSURE THAT THE LAST FIVE HARMONICS ARE NOT PROGRESSIVELY LARGER
362 C
363      DO 890 JF=1,4
364      JQ=KG(JF)
365      IF(JQ)882,890,882
366 882    Z=DABS(G(JQ))
367      JR=KG(JF+1)
368      IF(JR)884,890,884
369 884    Z1=DABS(G(JR))
370      IF(Z1.LT.Z)GO TO 890
371      G(JR)=0.95*G(JR)*Z/Z1
372 890    CONTINUE
373      DO 900 JS=1,4
374      JT=KH(JS)
375      IF(JT)892,900,892
376 892    Z=DABS(H(JT))
377      JU=KH(JS+1)
378      IF(JU)894,900,894
379 894    Z1=DABS(H(JU))
380      IF(Z1.LT.Z)GO TO 900
381      H(JU)=0.95*H(JU)*Z/Z1
382 900    CONTINUE
383 C
384 C      CHECK FOR OUTPUT DISTANCE
385 C
386 905    IF((R-RO)-PD)1010,950,910
387 C
388 C      INTERPOLATE OUTPUT
389 C
390 910    Y=DX-((R-RO)-PD)
391      DO 920 JV=1,NI
392      G2(JV)=G2(JV)-DG(JV)+0.5*(DG(JV)+DG1(JV))*Y/DX
393      H2(JV)=H2(JV)-DH(JV)+0.5*(DH(JV)+DH1(JV))*Y/DX
394 920    CONTINUE
395      X=F1
396      IF(A-0.5)940,925,930
397 925    X=(RN/(RN+Y))*0.5
398      GO TO 940
399 930    X=RN/(RN+Y)
400 940    DO 945 JV1=1,NI
401      XL(JV1)=X*G2(JV1)*DEXP(-ALPHA(JV1)*Y)/E
402      XL1(JV1)=X*H2(JV1)*DEXP(-ALPHA(JV1)*Y)/E
403 945    CONTINUE
404      GO TO 965
405 950    DO 960 JX=1,NI
406      XL(JX)=F1*G(JX)*GH(JX)/E
407      XL1(JX)=F1*H(JX)*GH(JX)/E
408 960    CONTINUE

```

```

409 965 PRINT 970,PD
410 970 FORMAT(5X,9HDISTANCE=,D17.10)
411 PRINT 980
412 980 FORMAT(5X,3HSIN,20X,3HCOS)
413 DO 1000 JY=1,NI
414 PRINT 990,JY,XL(JY),XL1(JY)
415 990 FORMAT(1X,I4,4X,D17.10,5X,D17.10)
416 1000 CONTINUE
417 PD=PD+IF*DXI
418 GO TO 905
419 C
420 C CHECK STEP SIZE
421 C
422 1010 N=0
423 Z=0.D0
424 DO 1050 J1=1,NI
425 IF(G(J1))1020,1030,1020
426 1020 Z=Z+DABS(0.5*((DG(J1)+DG1(J1))/G(J1)))
427 N=N+1
428 1030 IF(H(J1))1040,1050,1040
429 1040 Z=Z+DABS(0.5*(DH(J1)+DH1(J1))/H(J1))
430 N=N+1
431 1050 CONTINUE
432 C
433 C MODIFY WAVEFORM
434 C
435 DO 1060 J2=1,J
436 G(J2)=G(J2)*F1*GH(J2)
437 H(J2)=H(J2)*F1*GH(J2)
438 1060 CONTINUE
439 IF(Z.GT.(N*0.005))GO TO 1100
440 C
441 C DOUBLE STEP SIZE
442 C
443 DX=2.0*DX
444 DO 1070 J2=1,J
445 GH(J2)=GH(J2)*GH(J2)
446 1070 CONTINUE
447 DO 1090 J3=1,J
448 DO 1080 J4=1,J
449 X11(J3,J4)=X11(J3,J4)*X11(J3,J4)
450 X22(J3,J4)=X22(J3,J4)*X22(J3,J4)
451 1080 CONTINUE
452 1090 CONTINUE
453 PRINT 1106,DX
454 1106 FORMAT(1X,3HDX=,D17.10)
455 1100 RN=R
456 IF(RMAX-(R-R0))1110,590,590
457 1110 STOP
458 END

```

# APPENDIX B

## SAMPLE OUTPUT FROM FAW

```

FREQUENCY= 0.3000000000D 03
SOUND SPEED= 0.1500000000D 04
BETA= 0.3500000000D 01
NORMALIZATION CONSTANT= 0.6670000000D-01
INITIAL STEP SIZE= 0.1000000000D 02
MAXIMUM DISTANCE= 0.1000000000D 05
SOURCE SIZE= 0.0000000000D 00
SPREADING FACTOR= 0.00
ABSORPTION FLAG= 0
NUMBER OF HARMONICS= 40
INITIAL G COEFFICIENTS= 1
INITIAL H COEFFICIENTS= 0
NUMBER OF HARMONICS OUTPUT= 5
PRINT OUT INTERVAL= 100
ALPHA(1)= 0.1120000000D-05
NUMBER OF MODIFIED ABSORPTION COEFFICIENTS= 0
INPUT WAVEFORM
G( 1)= 0.6670000000D-01
DISTANCE= 0.1000000000D 04
SIN COS
1 0.9941231144D 00 0.0000000000D 00
2 0.9622622120D-01 0.0000000000D 00
3 0.1394984526D-01 0.0000000000D 00
4 0.2394449333D-02 0.0000000000D 00
5 0.4511965196D-03 0.0000000000D 00
DISTANCE= 0.2000000000D 04
SIN COS
1 0.9788882494D 00 0.0000000000D 00
2 0.1845999987D 00 0.0000000000D 00
3 0.5197039094D-01 0.0000000000D 00
4 0.1729678606D-01 0.0000000000D 00
5 0.6313700593D-02 0.0000000000D 00
DISTANCE= 0.3000000000D 04
SIN COS
1 0.9546672934D 00 0.0000000000D 00
2 0.2587699767D 00 0.0000000000D 00
3 0.1041698315D 00 0.0000000000D 00
4 0.4945203382D-01 0.0000000000D 00
5 0.2570994911D-01 0.0000000000D 00
DX= 0.2000000000D 02
DISTANCE= 0.4000000000D 04
SIN COS
1 0.9220214992D 00 0.0000000000D 00
2 0.3138414940D 00 0.0000000000D 00
3 0.1574155764D 00 0.0000000000D 00
4 0.9275848905D-01 0.0000000000D 00
5 0.5972644773D-01 0.0000000000D 00
DX= 0.4000000000D 02

```

|                            |                  |                  |
|----------------------------|------------------|------------------|
| DISTANCE= 0.5000000000D 04 |                  |                  |
| SIN                        |                  | COS              |
| 1                          | 0.8817469738D 00 | 0.0000000000D 00 |
| 2                          | 0.3469048248D 00 | 0.0000000000D 00 |
| 3                          | 0.1988133777D 00 | 0.0000000000D 00 |
| 4                          | 0.1330635772D 00 | 0.0000000000D 00 |
| 5                          | 0.9695954537D-01 | 0.0000000000D 00 |
| DX= 0.8000000000D 02       |                  |                  |
| DISTANCE= 0.6000000000D 04 |                  |                  |
| SIN                        |                  | COS              |
| 1                          | 0.8353073629D 00 | 0.0000000000D 00 |
| 2                          | 0.3581480161D 00 | 0.0000000000D 00 |
| 3                          | 0.2202056808D 00 | 0.0000000000D 00 |
| 4                          | 0.1567515441D 00 | 0.0000000000D 00 |
| 5                          | 0.1208128685D 00 | 0.0000000000D 00 |
| DX= 0.1600000000D 03       |                  |                  |
| DISTANCE= 0.7000000000D 04 |                  |                  |
| SIN                        |                  | COS              |
| 1                          | 0.7863635495D 00 | 0.0000000000D 00 |
| 2                          | 0.3538678457D 00 | 0.0000000000D 00 |
| 3                          | 0.2247481885D 00 | 0.0000000000D 00 |
| 4                          | 0.1638955398D 00 | 0.0000000000D 00 |
| 5                          | 0.1287869223D 00 | 0.0000000000D 00 |
| DX= 0.3200000000D 03       |                  |                  |
| DISTANCE= 0.8000000000D 04 |                  |                  |
| SIN                        |                  | COS              |
| 1                          | 0.7389351302D 00 | 0.0000000000D 00 |
| 2                          | 0.3423198787D 00 | 0.0000000000D 00 |
| 3                          | 0.2210334863D 00 | 0.0000000000D 00 |
| 4                          | 0.1629226548D 00 | 0.0000000000D 00 |
| 5                          | 0.1290571601D 00 | 0.0000000000D 00 |
| DISTANCE= 0.9000000000D 04 |                  |                  |
| SIN                        |                  | COS              |
| 1                          | 0.6947357383D 00 | 0.0000000000D 00 |
| 2                          | 0.3279410902D 00 | 0.0000000000D 00 |
| 3                          | 0.2137387066D 00 | 0.0000000000D 00 |
| 4                          | 0.1583862311D 00 | 0.0000000000D 00 |
| 5                          | 0.1259628330D 00 | 0.0000000000D 00 |
| DISTANCE= 0.1000000000D 05 |                  |                  |
| SIN                        |                  | COS              |
| 1                          | 0.6542737913D 00 | 0.0000000000D 00 |
| 2                          | 0.3128153020D 00 | 0.0000000000D 00 |
| 3                          | 0.2050573568D 00 | 0.0000000000D 00 |
| 4                          | 0.1523892258D 00 | 0.0000000000D 00 |
| 5                          | 0.1214645359D 00 | 0.0000000000D 00 |