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ON STABILITY IN A COMPLEX ECONOMY

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ABSTRACT

In this paper we examine the local stability of a competitive equilibrium for a class of economies where strength of interaction among markets is explicitly parameterized. Within this class we establish necessary and sufficient conditions for an equilibrium to be locally stable in terms of the number of markets, the ratio of active links among markets to those combinatorially possible, and the average strength of an active link. In the Discussion we relate our result to the more standard sufficient conditions of weak gross substitutability over commodity pairs and negative diagonal dominance within the Jacobian of aggregate excess demands. Finally, we touch briefly upon the possibility that even a crude hierarchical organization of the markets may favorably alter the probability of stable equilibria.

ON STABILITY IN A COMPLEX ECONOMY*

1. Introduction

In Arrow and Hurwicz (1958) and Arrow, Block and Hurwicz (1959), the authors establish sufficient conditions for an equilibrium of the competitive pure exchange model to be locally stable under the Walrasian tâtonnement method of price adjustment. Foremost among these are: (1) that the Jacobian of the aggregate excess demand functions possess a dominant negative diagonal; (2) that each commodity be a weak gross substitute of every other commodity.[1] The crucial feature of these conditions for what follows is that each places a rather heavy restriction on the functional properties of aggregate excess demand to obtain stability. Much of the recent literature of equilibrium economics, however, has suggested that aggregate excess demand need possess very little structure (Sonnenschein, 1972, 1973; Debreu, 1974). Put briefly, the only properties which consumer rationality under the usual assumptions implies for aggregate excess demand are continuity and Walras' Identity. We are thus led to conclude that a competitive system need possess no properties which insure even the local stability of its equilibria under the usual price adjustment method.

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In this paper we present economically-meaningful necessary and sufficient conditions for a competitive equilibrium to be almost surely stable under the Walrasian tâtonnement. Our tool for doing so is the "Semi-Circle Theorem" for the distribution of the eigenvalues of a random matrix (Wigner, 1955, 1958, 1959, 1967: Mehta, 1967:43-46, 238-240). The first person to realize the potential utility of this result for the study of dynamics was Robert May (1972). From a technical perspective, our development differs from May's in two ways. First, our derivation of the condition equivalent to local stability is much simpler. Moreover, we derive the full distribution of the real parts of the eigenvalues of the random matrix and thus extend May's local result for the dominant real part. From an economic perspective our analysis indicates that conditions on the internal structure of the Jacobian of aggregate excess demands in equilibrium can be useful in making statements about stability. Whether such conditions on market interaction are stronger or weaker than the sufficiency conditions mentioned above is a matter which we leave for the Discussion. It may be noted at this point, however, that such conditions do not impose *functional* constraints on aggregate excess demand of the sort which Debreu and Sonnenschein have called into question. In this sense, at least, our interaction conditions seem more reasonable.

In Section 2 of what follows we present the main mathematical result, while in Section 3 we relate the construction to the more orthodox sufficiency conditions. Finally, we briefly discuss the possibility that introducing even a moderate degree of hierarchy among the sectors of the economy may radically alter the prospects for locally stable equilibria.

2. Formal Analysis

Consider an (n+1)-good pure exchange model in which the aggregate excess demand for the i^{th} good is given by $f_i(p)$, $i = 0,1,\ldots,n$, where $p \equiv (p_0,\ldots,p_n)$, and prices are quoted in terms of good 0 $(p_0 \equiv 1)$. Here, we assume that f_i is continuous, all i, and that

$$\sum_{i=0}^{n} p_i f_i(p) = 0 \quad ,$$

all p. Under the tâtonnement method of price adjustment we then have that

$$\frac{dp_i(t)}{dt} = H_i(f_i(p)) \quad , \qquad (i = 1,\ldots,n)$$

where H_i is continuous and preserves the sign of f_i. (It should be noted that the formal portion of our analysis does not require price adjustment of this form and that some other, less standard, adjustment process can be assumed, e.g., Arrow and Hahn (1971: 302).) For the sake of simplicity, let us assume that H_i is the identity, all i. If we assume non-satiation in at least one good, then an equilibrium price vector p* may be defined as a stationary point of this process, i.e., $f_i(p^*) = 0$, all i. We assume that at least one such p* exists. Now suppose that we introduce a C^1-perturbation $\varepsilon(t)$ on p* and write

$$p_i(t) = p_i^* + \varepsilon_i(t) \quad .$$

Taylor-expanding about p*, we have that

(1) $$\frac{dp_i(t)}{dt} = \sum_{j=1}^{n} a_{ij}\varepsilon_j(t) + O([\varepsilon(t)]^2) \quad , \qquad (i = 1,\ldots,n)$$

where $a_{ij} = [\partial f_i/\partial p_j](p^*)$ and $\varepsilon_j(t) = p_j(t) - p_j^* = dp_j$. Thus, neglecting terms of second order and higher, we have that equation (1) represents the total derivative of the i^{th} price path, and hence its best linear approximation, in the neighborhood of p*. Let us write (1) in matrix form as

$$\frac{dp(t)}{dt} = \frac{d\varepsilon(t)}{dt} \doteq A\varepsilon(t) \quad .$$

Now we note that generically A will possess distinct non-zero eigenvalues (Hirsch and Smale, 1974: 154-156). Hence we can write

$$\varepsilon_i(t) = \sum_{j=1}^{n} k_{ij}e^{\mu_j t} \quad ,$$

where $\mu_1,\ldots,\mu_n$ are the eigenvalues of A, and the k_{ij} are constants depending on $\varepsilon(0)$. Finally, since $\mu_1,\ldots,\mu_n$ will generically have non-zero real parts (Hirsch and Smale, 1974: 157), we may say that p* is locally stable if, and only if,

(2) $$\max_j \{\mathrm{Re}\ \mu_j\} < 0 \quad .$$

The matrix A is the Jacobian of aggregate excess demand evaluated at p*. In particular, a_{ij} measures the linear effect of a perturbation in p_j on the time path of p_i in a neighborhood of p*, and A thus reflects the type and magnitude of the interconnections among the n markets within this neighborhood. For example, if $a_{ij} \geq 0$, $i \neq j$, we have that goods i and j are weak gross substitutes near p*. Note, in particular, that the local stability properties of p* are completely determined by A according to (2). Hence, if we were to define a probability measure on the full linear group $G\ell(n,\mathbb{R})$, it might be feasible to determine conditions on the measure such that A satisfies (2) with some specified probability. To define such a measure, we suppose that if any market is individually perturbed from equilibrium, it returns with some characteristic damping time. For simplicity, we let $a_{ii} = -1$, all i. (This amounts to assuming that each market exhibits "Walrasian stability." For a further discussion, see Samuelson (1947:265).) The off-diagonal elements of A are chosen as follows. Let z denote a random variable with absolutely continuous distribution F(z), and assume that

(a) $E(z) = 0$;

(b) $E(z^2) = \sigma^2 < \infty$;

(c) $dF(z)/dz$ is symmetric.

Now for $c \in [0,1]$ let Prob $(a_{ij} = 0) = 1 - c$ and Prob $(a_{ij} = z) = c$, $i \neq j$. In words, the probability that a given off-diagonal entry a_{ij} is set equal to zero is $1 - c$, while with probability c, a_{ij} is a realization of a random variable with a symmetric density, mean of zero, and standard deviation σ. The economic meaning of this construction may be summarized in three points. First, for large n the choice of c provides a method for specifying the fraction of the $n(n-1)/2$ possible cross-market links that is active in the equilibrium. For example, a non-zero entry in the ij^{th} position indicates that a small change in the price prevailing on the j^{th} market affects, in some measure, demand for good i in the neighborhood of p*. Second, since $E(z) = 0$, σ^2 can be thought of as measuring the average strength of interaction among markets. Finally, the condition that $dF(z)/dz$ be symmetric says that two arbitrarily chosen goods are equally likely to be gross complements or gross substitutes, but that, on average, the interaction between their markets is nil. The economy we have in mind here consists of a large number of heterogeneous goods. If two of these are selected at random, then, we might expect them, on average, to be independent for purposes of price adjustment. Although it is not difficult to envision such a situation over pairs of goods, the question of whether this relation holds in an expected sense over *all* pairs of goods is less clear. Hicks (1946: 312), in fact, suggests that substitutability should be a more common relationship than complementarity, thus indicating that $E(z)$ should possibly be positive. Largely for technical reasons and because it does not seem too great a distortion of reality, however, we continue to assume that the mean of z is zero.

Having attempted to capture the richness of the economic structure under consideration through σ^2, c, and the number of goods n, we must now ask what sort of economy could give rise to a Jacobian lying in the class of those constructed. We note first that our construction says that $[\partial f_i/\partial p_j](p^*) = A$, $i,j = 1,\dots,n$, where A is a realization of our random assembly process. Further, we have assumed that

$$\sum_{i=0}^{n} p_i f_i(p) \equiv 0 \quad .$$

Differentiating with respect to p_j yields

$$\sum_{i=0}^{n} p_i \frac{\partial f_i}{\partial p_j} + f_j(p) \equiv 0$$

and

$$\sum_{i=0}^{n} p_i^* \frac{\partial f_i}{\partial p_j} (p^*) \equiv 0 \quad ,$$

since $f_j(p^*) = 0$, all j. Hence, in matrix form the Walras Identity with $p_0 \equiv 1$ implies that

$$\left[\begin{array}{c|ccc} a_{00} & a_{01} & \cdots & a_{0n} \\ \hline a_{10} & & & \\ \vdots & & A & \\ a_{n0} & & & \end{array}\right]^T (p^*)^T \equiv 0 \quad ,$$

where $a_{i0} = 0$, $i = 0,\ldots,n$, since $p_0 \equiv 1$. Therefore, if A is randomly assembled through the process described above, Walras' Identity insures that a_{0j}, $j = 1,\ldots,n$, will adjust so as to make this matrix identity valid. Recall, however, that the thrust of Debreu (1974) is that if f is continuous and satisfies Walras' Identity, then, apart from boundary considerations, there exist not more than n nonpathological consumers whose individual excess demands sum to $(f_0(p),\ldots,f_n(p))$. Thus, we have, in particular, that if f(p) satisfies the n^2 conditions imposed by A, as well as Walras' Identity, then there exists an underlying competitive exchange economy which could have given rise to it.

We are now in a position to ask how the local stability of p* depends on the complexity of the economy. In more precise terms, if Prob (n,c,σ) denotes the probability that the dominant real part in the spectrum of A is negative, we now wish to determine how this probability depends on n, c, and σ. This question was first answered for the case where A is a Wishart matrix by Eugene Wigner in an investigation of the energy levels of nuclear spectra (Wigner, 1958) and subsequently generalized by him to the case where A is constructed as above (Mehta, 1967: 238-240). Robert May (1972), however, was the first to apply these results to the analysis of linearized adjustment processes. To summarize this work briefly, let us first suppose that A is constrained to be symmetric. Further, we write A as B - I, where B is an nxn matrix consisting of the same off-diagonal elements as A and zeros along the principal diagonal, and I is the nxn unit matrix. Since A, and hence B, is symmetric, all its eigenvalues are real. Moreover, if $\mu_1,\ldots,\mu_n$ and $\lambda_1,\ldots,\lambda_n$ are the eigenvalue spectra of A and B, respectively, then $\mu_i = \lambda_i - 1$, all i. Now we assume that n is large enough that the probability of λ lying in the interval $\lambda + d\lambda$ is given by a continuous density function $\xi(\lambda)$. The general form of Wigner's result then says that

$$\xi(\lambda) = \begin{cases} \frac{1}{2\pi\sigma^2}(2nc\sigma^2 - \lambda^2)^{1/2} \;, & \lambda^2 \le 2nc\sigma^2 \\ 0 \;, & \lambda^2 > 2nc\sigma^2 \end{cases}$$

Geometrically, this means that ξ is a semi-circle centered at the origin with radius $\sigma(2nc)^{1/2}$, as shown in Figure 1. Hence, with

INSERT FIGURE 1 ABOUT HERE

probability one the dominant eigenvalue of B, say λ_1, is less than $\sigma(2nc)^{1/2}$. The dominant eigenvalue of A, μ_1, is thus almost surely less than $\sigma(2nc)^{1/2} - 1$. Therefore, when A is symmetric, p* will be locally stable with probability one if, and only if, $\sigma(2nc)^{1/2} < 1$, or

$$\sigma < (2nc)^{-1/2}$$

When A is regarded as the (large) finite-dimensional approximation of a quantum-mechanical Hamiltonian, then its self-adjointness follows from Stone's Theorem, since it is the infinitesimal generator of a strongly continuous one-parameter unitary group which describes quantum dynamics (Reed and Simon, 1972:266, 303).

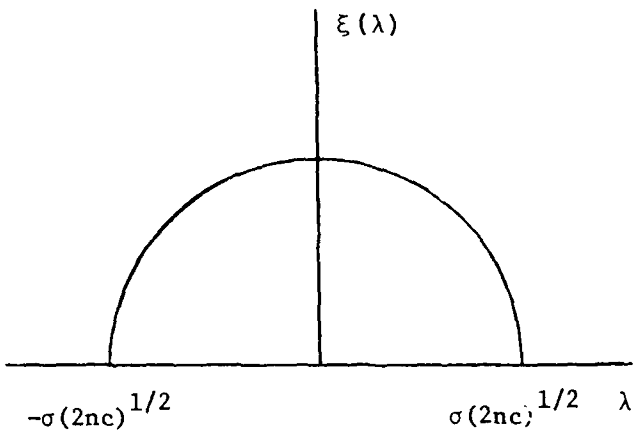


Figure 1

Unfortunately, in neither the general theory of linearized adjustment processes nor in our particular case of economic dynamics is there a similarly compelling reason for restricting attention to symmetric A. The primary difficulty encountered when this restriction is lifted is that A need no longer possess real eigenvalues, since the real field is not algebraically closed. Hence, a result as complete as the Semi-Circle Law is apparently not to be had. (See, e.g., Ginibre (1965).) Rather than seek a density for the eigenvalues of A, then, let us attempt to determine the density of their real parts and note that, by (2), this density may be used to probabilistically characterize the local stability of p*.

Let B be defined as previously, and note that, by the Cartesian Decomposition, it may be uniquely written as

$$B = X_{sym} + Y_{asym} \quad , \tag{3}$$

where $X_{sym} = \frac{B + B^T}{2}$ and $Y_{asym} = \frac{B - B^T}{2}$ are symmetric and antisymmetric, respectively. Now assume that B can be diagonalized over $\mathbb{C}^2$ by a sequence of similarity transformations $T_1,\ldots,T_m$, noting that this is always possible on some open dense subset of $G\ell(n,\mathbb{C}^2)$ (Hirsch and Smale, 1974: 157). Hence,

$$T_m \ldots T_1 B T_1^{-1} \ldots T_m^{-1} = \mathrm{diag}\ \{x_j + iy_j\} \equiv \mathrm{diag}\ \{\lambda_j\} \qquad (j=1,\ldots,n). \tag{4}$$

Further, substituting (3) into (4), it is clear that $T_1,\ldots,T_m$ will also diagonalize X_{sym} and Y_{asym} over $\mathbb{C}^2$. Note, however, that because X_{sym} is symmetric, its eigenvalues are purely real, while those of

Y_{asym} are purely imaginary by antisymmetry. If we denote these spectra by $\{a_1,\ldots,a_n\}$ and $\{ib_1,\ldots,ib_n\}$, respectively, we then have that

$$\text{diag}\ \{x_j + iy_j\} = \text{diag}\ \{a_j\} + \text{diag}\ \{ib_j\} \quad , \qquad (j=1,\ldots,n)$$

and hence that $a_j = \text{Re}\ \lambda_j$ and $b_j = \text{Im}\ \lambda_j$.

Now recall that the elements of B are independently identically distributed by construction. An easy computation then shows that the elements of X_{sym} are likewise independently identically distributed with mean zero and standard deviation $\sigma/\sqrt{2}$. Further, it can easily be shown (Feller, 1971: 149) that their common density is symmetric. Hence, the Semi-Cirle Law applies, and

$$\text{Re}\ \lambda_j \leq \sigma(nc)^{1/2} \quad , \qquad \text{all } j.$$

Moreover, since B = A - I, the dominant real part in the spectrum of A is no larger than $\sigma(nc)^{1/2} - 1$. We have thus proved:

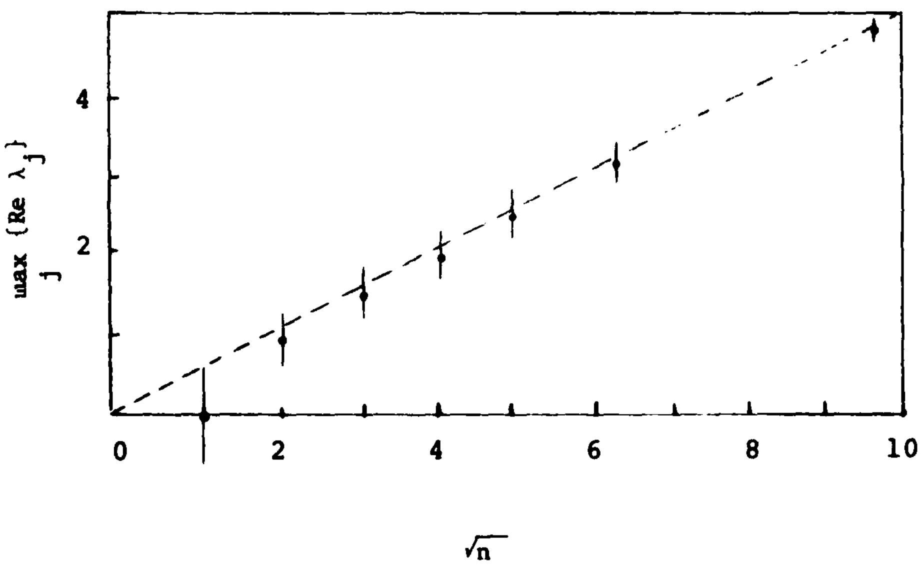
> Prob $(n,c,\sigma) \to 0$ *as* $n \to \infty$ *if, and only if,*
> $\sigma > (nc)^{-1/2}$.

Therefore, for any economy whose Jacobian of aggregate excess demands at equilibrium is generated in the manner described, we have that a high level of complexity (n,c, or σ large) is almost surely equivalent to local instability.

Two remarks, one substantive and another technical, are called for. First, that increasing n for fixed values of σ and c implies a greater likelihood of instability in this class of models is particularly interesting, since the usual way of introducing intertemporal considerations or uncertainty into the Arrow-Debreu model is through time-dated and state-contingent commodities. Thus, although the usual existence arguments remain unaltered by the introduction of such commodities, and the consequent increase in n, the stability of equilibrium can become less likely, unless c or σ decrease in an offsetting manner. Moreover, our proposition establishes the precise meaning of "offsetting" in this context. Second, strictly speaking, Wigner's derivation of the Semi-Circle Law applies only for n large. It is therefore interesting to ask how well the asymptotic result approximates cases in which n is relatively small. This is especially so since some recent studies (Ling, 1975, e.g.) have called into question technically similar results from the Erdös-Rényi theory of random graphs for all but large parameter values. Fortunately from our standpoint, several Monte Carlo studies (Gardner and Ashby, 1970; May, 1974: 65-66) indicate that even for values of n as small as 25; the discrepancy between the true real part of the dominant eigenvalue of B and that suggested by Wigner's analytic approximation is negligible, as suggested by Figure 2. (Cf., also, Bronk (1964) for

INSERT FIGURE 2 ABOUT HERE

an analytic result when $\sigma = c = 1$.) Thus, although the analytic result is of an asymptotic nature, convergence seems to be sufficiently rapid to insure its utility for models of reasonable size.



Parameters: $\sigma = 0.5$; $c = 1.0$. Bars correspond to one standard deviation.

Source: May (1974: 65)

Figure 2

3. Discussion

In this section we relate the Semi-Circle Condition to a portion of the previous literature on competitive stability. The first notion we examine is that of weak gross substitutability over pairs of commodities. In our notation this amounts to the assumption that $\partial f_i/\partial p_j \geq 0$, $i \neq j$, at all prices, and says that if the price of good j increases, then aggregate excess demand for good i must not decrease. That this condition is sufficient for local stability is shown in Arrow and Hurwicz (1958). In effect a Jacobian which displays this property over each pair of goods in equilibrium may be thought of as having been constructed from a probability density with support on the non-negative real line and, hence, having positive mean. We might thus conjecture that, since this condition insures local stability, the construction of A from such a distribution would yield a weaker criterion for local stability than does the Semi-Circle Condition. Unfortunately, all analytic eigenvalue-density results for random matrices to date rely crucially on the assumption that off-diagonal elements have a mean of zero. There thus appears to be no verification of our conjecture in the existing literature.

The assumption of negative diagonal dominance is more interesting from the standpoint of Section 2. Recall that A is said to have a dominant negative diagonal at p* if:

(a) $a_{ii} < 0$, all i;

(b) There is a vector $h(p^*) \gg 0$ such that

$$h_i(p^*)|a_{ii}| > \sum_{j \neq i} h_j(p^*)|a_{ij}|.$$

The interpretation of (a) is that, for each good i, the substitution term of the Slutsky equation dominates the income term; (b), on the other hand, says that there exist units for measuring the commodities in the economy such that the diagonal term dominates the sum of the off-diagonal terms in each row in absolute value. It is now an easy observation that if A possesses a negative dominant diagonal, then p* is locally stable. By Gerschgorin's Theorem (Wilkinson, 1965: 71), every eigenvalue of A lies in at least one of the circular disks with centers at a_{ii} and radii $\sum_{j \neq i} |a_{ij}|$. Now recall that $a_{ij} = -1$, all i. Hence, $\mu_1, \ldots, \mu_n$ lie in a disk centered at -1 with radius $\max_i \{ \sum_{j \neq i} |a_{ij}| \} < 1$, where the inequality follows from (b). It thus follows that Re $\mu_i < 0$, all i. Hence, a dominant negative diagonal a p* implies local stability, and local stability, in turn, is equivalent to the Semi-Circle Condition in economies whose equilibrium Jacobian can be realized through the assembly process of Section 2. Moreover, we can likewise show that if σ^2 is sufficiently small, then a dominant negative diagonal obtains with probability arbitrarily close to unity. (Note that the intuition behind this result is clear, since it certainly holds if $\sigma^2 = 0$. Thus, the result is essentially a continuity argument and is contained in a footnote.[2]) We can now know, however, when diagonal dominance is a strong sufficient condition for stability and when it is likely to be a relatively weak sufficient condition. Thus, if σ^2 or c is small, and average interaction or connectivity among markets is hence low, then assuming a dominant negative diagonal is a weak means of obtaining stability. Further, if the number of markets in the economy is held fixed, then the Semi-Circle Condition implies that the *only* way for diagonal dominance to be a reasonable stability criterion is for σ^2 or c to be low.

We close with several brief remarks on the relevance of our results for global stability theory. The question which the stability analysis of competitive equilibrium ultimately seeks to answer is whether a competitive model, beginning from an arbitrary price vector, ultimately reaches an equilibrium through some reasonable means of price adjustment. The question we have addressed, on the other hand, is more modest and amounts to asking whether an equilibrium price vector, if perturbed slightly, ultimately returns to that equilibrium or wanders away. Clearly, an affirmative stability result in our case is a necessary condition for an affirmative global result. It should, however, be noted that the Semi-Circle Criterion is likely to be a very stringent condition for local stability, particularly in the sort of model which equilibrium theorists usually envision. As already noted, a large number of markets in time-dated and state-contingent commodities will virtually preclude any market interaction if stability of either variety is to obtain. We are thus left with a conundrum of a most interesting sort: in the class of models investigated, global stability can obtain only when either thinness of markets prevents the standard existence and optimality results from carrying their usual force, or when topological constraints on the market network have been imposed. The latter certainly appears the more optimistic, and intriguing, possibility. Just how intriguing is suggested by the recent computer simulations of McMurtrie (1975). McMurtrie first generated 25 random matrices by sampling from a Gaussian distribution with mean of zero and variance of 0.5. The dimensions of each matrix were 24x24, and each had a connectivity of 0.083. Of these 25 matrices, 16 were found to have eigenvalues with negative real parts. Then, holding n, σ^2, and c constant, he generated another 25 random matrices, this time partitioning them such that rows i and $i+1$ were permitted to have non-zero entries only in columns $i+2$ and $i+3$ for $i = 1,\ldots,21$, and rows 23 and 24 non-zero entries only in columns 1 and 2. The results are striking: 23 of the 25 matrices generated possessed eigenvalues with negative real parts. This result was borne out in 11 other trials in which various configurations were specified, and randomly-generated matrices satisfying the configurational constraint were tested for stability. (For details of these runs, the interested reader may consult McMurtrie's above-mentioned article, especially Figure 3.) In each case, the same conclusion is, in some measure, obtained: weak hierarchy substantially increases the probability that the system will be stable. While these results are certainly not conclusive, they do suggest that a solution to the problem of obtaining stable equilibria within the Arrow-Debreu model may well lie in a more detailed specification of the trading structure or graph of the model. We might expect, in fact, that such a graph would take its place among the primitives of the model, such as preference orderings and price-adjustment mechanics. The problem then remaining would be to pair price-adjustment mechanisms with graphs such that stable equilibria are obtained.

FOOTNOTES

[1]When the equilibrium of the model is unique, (1) and (2) are, of course, sufficient for global stability as well.

[2]Here, we must determine a δ such that if $\sigma^2 < \varepsilon$, Prob $(\sum_{j\neq i} |a_{ij}| \geq 1) < \delta$. To accomplish this we use Markov's Inequality (Loève, 1977: 160), which says that if $X \geq 0$ is a random variable, then for any $t > 0$, Prob $(X \geq t) \leq \frac{1}{t} E(X)$. Let $X = \sum_{j\neq i} |a_{ij}| \geq 0$ and $t = 1$. We then have that

$$\text{Prob } (X \geq 1) \leq E(\sum_{j\neq i} |a_{ij}|) = \sum_{j\neq i} E(|a_{ij}|) \quad .$$

Now since $F(z)$ is absolutely continuous, it possesses a density function $f(z)$. Further

$$E(|z|) = \int_{\mathbb{R}} |z| f(z)dz \leq \int_{\mathbb{R}} (kz^2 + \frac{1}{4k}) f(z)dz \quad ,$$

all $k \geq 1$. Integrating this last expression gives

$$E(|z|) \leq k\sigma^2 + \frac{1}{4k} \quad .$$

Thus

$$\text{Prob } (\sum_{j\neq i} |a_{ij}| \geq 1) \leq (n-1)(k\sigma^2 + \frac{1}{4k}) \quad .$$

Now if $1 > \varepsilon > \sigma^2$, take $k = O(\varepsilon^{-1/2})$. Then

$$\text{Prob}\ \Big(\sum_{j\neq i} |a_{ij}| \geq 1\Big) < (n - 1)(\varepsilon^{1/2} + \frac{1}{4}\,\varepsilon^{1/2}) = \frac{5}{4}\,(n - 1)\varepsilon^{1/2} \quad ,$$

which can be made arbitrarily small.

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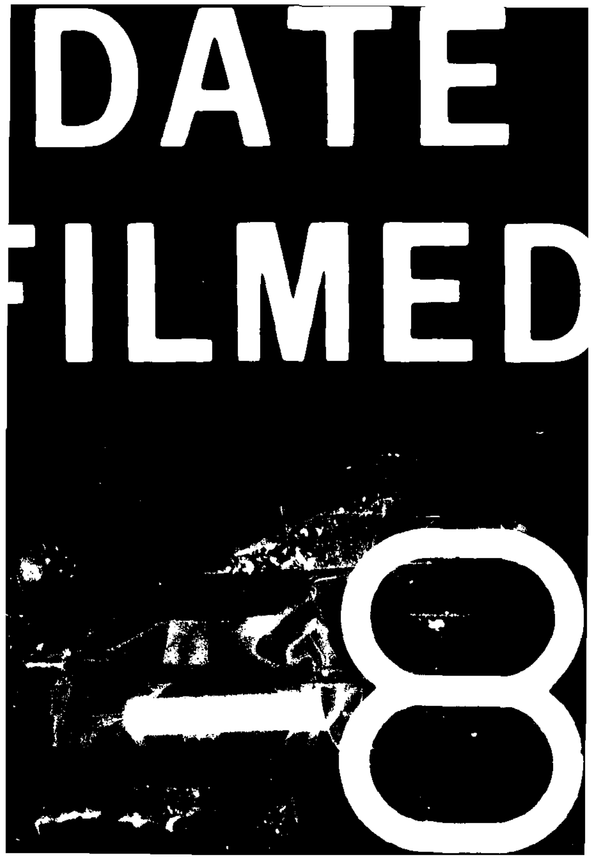
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