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6 COHERENT NOISE SYNTHESIZER

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11 November 1979

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CANADA

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ABSTRACT

A noise-generating algorithm and associated computer program for well-defined testing of beamformers are described. The algorithm is especially suitable for superdirective arrays of underwater hydrophones as it generates Gaussian noise of specified coherency. Statistical properties of the generator are confirmed to be those planned, and the ability of the generator to synthesize noise for isotropic or surface noise sources is verified for three-element arrays. Cumulative distributions for estimated coherency were obtained for the model.

INTRODUCTION

Computer programs for theoretical testing and comparison of beamforming algorithms require noise generating algorithms that synthesize noise of known coherency and statistical properties.

There is a significant advantage in using noise synthesizers to select suitable beamformers economically before field testing. The type of noise generated can be controlled and the beamformers tested for a set of defined and reproducible noise conditions. A considerable time-saving results since the testing of the beamformers for noise conditions that might be met in the field over several years can be done in the laboratory in a matter of days.

For arrays of widely spaced sensors, where the noise is uncorrelated from sensor to sensor, noise generators simply consist of uncorrelated noise sources, one noise source for each sensor. However, for arrays of closely spaced sensors, a model to generate noise correlated from sensor to sensor is required. This memorandum describes the simulator, verifies its statistical properties, and delineates those noise fields that can be represented by the simulator.

THEORY

A beamformer that explicitly includes a device to calculate Fourier transforms of the hydrophone outputs is shown in Figure 1. For computational efficiency, the noise generator described here produces the Fourier transforms of the noise directly, instead of generating the time series of the noise and subsequently calculating the transform. These transforms are arranged to be random variables with a Gaussian distribution that has been found to be characteristic of ambient noise over intervals of a few minutes[1].

To generate noise of specified coherencies between the n sensors, the Fourier transform $X_i(\omega)$, of the ith sensor at the frequency ω, is written as a linear combination of real and imaginary pairs of Gaussian distributed random variables $Z_i(\omega)$. Both the real and imaginary parts have a mean of 0 and a variance of 0.5. Dropping reference to frequency, these linear combinations are written:

$$
\begin{aligned}
X_1 &= a_{11} Z_1 + a_{12} Z_2 + \dots\dots + a_{1n} Z_n \\
X_2 &= a_{21} Z_1 + a_{22} Z_2 + \dots\dots + a_{2n} Z_n \\
X_i &= a_{i1} Z_1 + a_{i2} Z_2 + \dots\dots + a_{in} Z_n
\end{aligned}
\tag{1}
$$

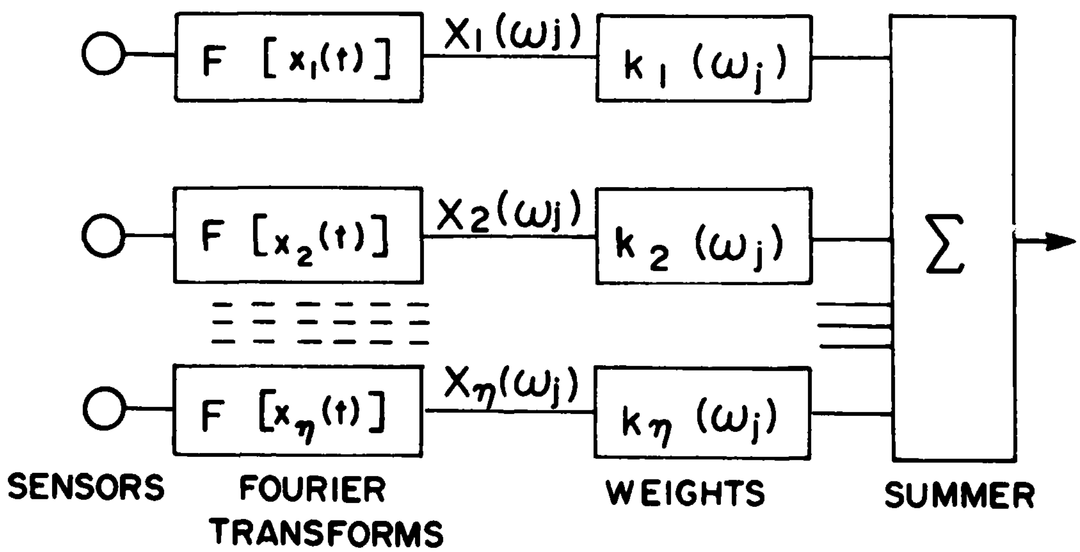

Figure 1. In the generalized beamformer shown the time series $x_i(t)$ is Fourier transformed to $X_i(\omega_j)$ and the transforms are multiplied by the weights $k_i(\omega_j)$.

The values of the a_{ij}, which are restricted to be real, are determined by the requirements that on the average the noise field power, q_{ij} $i=j$, be homogeneous (the same at all hydrophones and equal to unity) and that the average noise field coherency, q_{ij} $i\neq j$, between sensor pairs be as specified by the user (e.g. isotropic noise). These two conditions may be written

$$q_{ij} = \overline{X_i X_j^*} \qquad i,j=1,2,\ldots n. \qquad (2)$$

In addition, the simplifying assumption was made that

$$a_{ij} = 0 \qquad j>i. \qquad (3)$$

By combining (1), (2), and (3) and using the independence of the Z_i it can be shown that

$$q_{ij} = \overline{X_i X_j^*} = \sum_{k=1}^{i} a_{ik} a_{jk} \qquad j=1,..i;\ i+1,\ldots n. \qquad (4)$$

These equations are solved for a_{ij} and the Fourier transforms X_i are then calculated from Equation (1). A listing of the noise generating program is contained in Appendix A. The subroutine Gauss 4 called by the noise generator has been extensively tested and found to be faster computationally and better statistically than the random number generator 'Gauss' supplied with IBM systems software[2].

The noise generating algorithm cannot solve for a_{ij} for all arbitrary sets of coherency values. Firstly, the form of Equation (3) restricts noise fields modelled to those for which $q_{ij}=q_{ji}$. By doubling the number of random variables Z_i, complex q_{ij} could be accommodated. Secondly, even for a three-element array the requirement that a_{33} be real restricts permissible q_{ij}. To obtain some indication of whether this is a severe limitation, examples of noise fields that give real a_{33} for a three-element 'equispaced' horizontal line array were determined numerically and theoretically.

The condition on q_{ij} that must be satisfied for real a_{33} for any three-element array is,

$$q_{13}^2 \, q_{23}^2 + 2q_{13} \, q_{23} \, q_{12} + q_{12}^2 - 1 \leq 0 \tag{5}$$

This condition is a special case of the more general requirement that the cross spectral matrix be Hermitian positive semidefinite[3]. Equation (5), which is derived in Appendix B, was tested for isotropic noise, i.e. noise whose coherency is given by

$$q_{ij} = \frac{\sin(kd_{ij})}{kd_{ij}} \tag{6}$$

and for surface-generated noise for which the coherency can be expressed as

$$q_{ij} = \frac{2^m m! \, J_m(kd)}{(kd_{ij})^m} \tag{7}$$

where k is the wave number, d_{ij} is the sensor separation, and J_m is the Bessel function of the first kind of order m. The condition specified by Equation (5) is satisfied for three-element equispaced arrays for isotropic noise and for surface generated noise for m = 0, 1, 2 and $\frac{d}{\lambda}$ up to 0.95. This was shown theoretically for surface noise as outlined in Appendix C and numerically for isotropic noise. Beyond 0.95 of a wavelength the model approaches that of independent noise sources, one noise source for each hydrophone.

It might be thought that allowing a_{ij} to be complex would remove the restriction imposed by Equation (5) and allow modelling of a wider range of noise fields. However, even for complex a_{ij} the restriction on the noise coherency as defined by Equation (5) remains. Furthermore, allowing a_{ij} to be complex introduces a new difficulty. While for real a_{ij} all sensors will have a uniform distribution of the phase shift between the real and imaginary parts of the Fourier transform, complex a_{ij} introduces the situation where there are distinctly different distributions for different hydrophones; this is equivalent to saying that the noise field is not homogeneous in the phase shift distribution and is therefore rather unrealistic. The restriction to real a_{ij} is thus not purely arbitrary.

DISCUSSION OF RESULTS

Tests were carried out to determine whether the synthesizer produced noise with the desired statistical properties. Firstly, the Kolmogorov-Smirnov test was applied to test the hypothesis that the Fourier transform amplitudes are Gaussian distributed random variables. The test was applied to the cumulative distribution. Each cumulative distribution tested contained 500 samples of the transform and 100 cumulative distributions were tested. A significance level was calculated for each of the 100 cumulative distributions. The significance level indicates the probability that the cumulative distribution would have occurred by chance. Individual significance levels were consistent with the hypothesis that the sample came from a population of Gaussian distributions.

The 100 significance levels from the Kolmogorov-Smirnov test were also examined. They lie between 0 and 100% and should have an equal probability of occurrence, i.e. the significance levels should be uniformly distributed. The observed set of 100 significance levels obtained in the Kolmogorov-Smirnov test departed somewhat from a uniform distribution. It was necessary to know whether this departure from a uniform distribution was likely to occur by chance. Again the Kolmogorov-Smirnov test was used to investigate the hypothesis that the significance levels were uniformly distributed. This hypothesis of uniform distribution could not be rejected at the 27% level, i.e. there is approximately one chance in four of obtaining this particular distribution or one with a greater deviation from uniformity. Thus there is no reason to suspect the original hypothesis of the Fourier transform amplitudes being Gaussian distributed. Indeed confidence in the hypothesis is increased.

Secondly, the power from each sensor was tested to determine whether the power was chi-squared distributed with two degrees of freedom. Significance levels were calculated from the Kolmogorov-Smirnov test for cumulative distributions containing 100 samples of the power in 20 trials with 5 sensors. The calculated individual significance levels were consistent with the chi-squared hypothesis. Again to aid in the evaluation of the significance levels as a group, the hypothesis that the significance levels were uniformly distributed, as they should be, was tested with the Kolmogorov-Smirnov test. It was found that the hypothesis could not be rejected at the 77% level. These results are taken as confirmation that the power is indeed chi-squared distributed with two degrees of freedom as was intended.

Thirdly, the phase angle of the sensor outputs should be uniformly distributed. In the 20 trials with 5 sensors, significance levels were calculated using the Kolmogorov-Smirnov test for cumulative distributions containing 100 samples of the phase angle. Again the individual significance levels were consistent with the hypothesis under test. Since the significance levels should themselves be uniformly distributed, they were tested for a uniform distribution with the Kolmogorov-Smirnov test. The hypothesis of a uniform distribution of the significance levels could not be rejected at the 97% level so that the hypothesis that the phase of the sensor output is uniformly distributed gains further support.

Additional checks were made to verify that the algorithm produced noise whose coherencies converged to the specified coherence for the noise field. Hydrophone outputs were synthesized for isotropic noise and also for a surface noise field represented by $J_0(kd)$ as given by Equation (7) for m=0. This was carried out for up to five hydrophones for various sensor configurations and in all cases solutions were found for the a_{ij}. The calculated coherencies for estimates made from samples of 100 coherencies produced by the simulator showed a bias. That bias agreed well with the bias given by Benignus[5] for coherencies generated from two independent Gaussian noise sources.

Cumulative distributions for the coherencies were calculated for a sample size of 100 at 9 selected coherencies. These are plotted in Figure 2 to characterize the model and enable comparison of measured cumulative distributions of coherency with coherency calculated from the model. For sample sizes between 2 and 100 the 95% confidence limits are summarized in Figure 3.

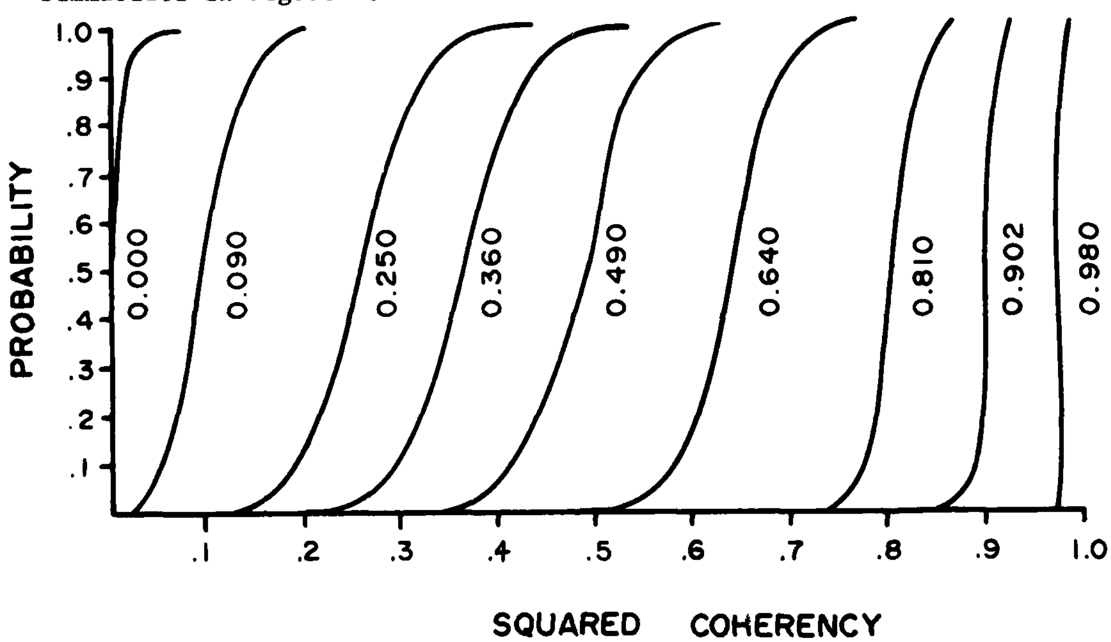

Figure 2. Cumulative frequency distributions for the calculated mean squared coherency. To obtain the curves plotted, 500 estimates of coherency were made with a sample size of 100. The true squared coherency is listed beside each curve.

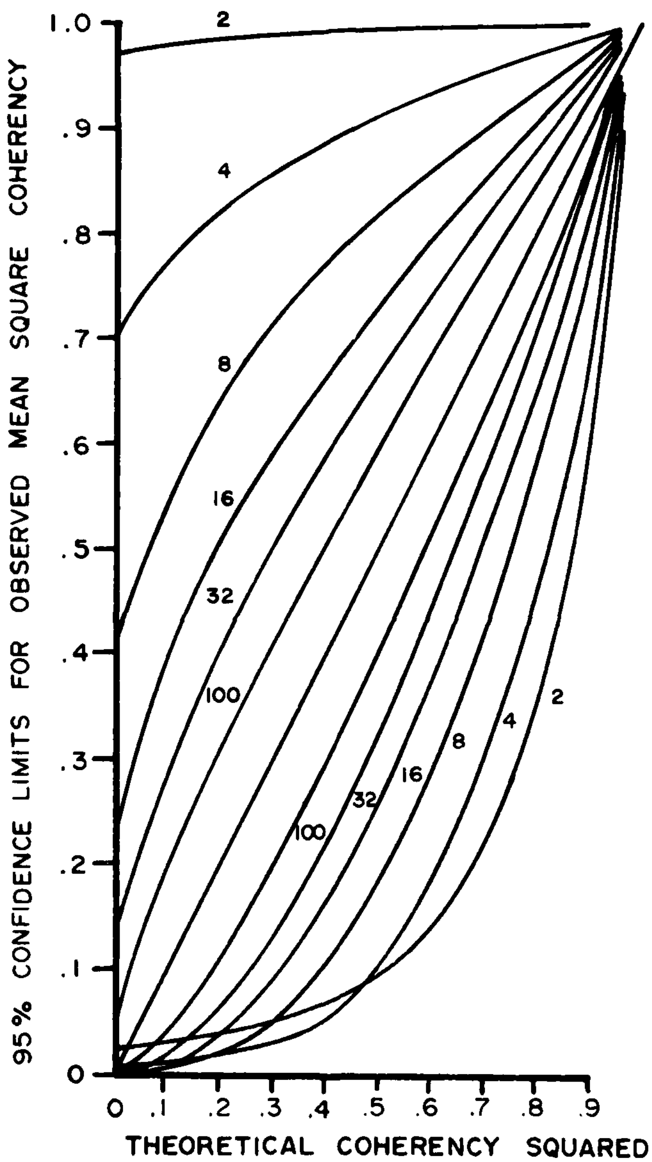

Figure 3. 95% confidence limits are shown for coherency squared for sample sized between 2 and 100 from 5000 estimates.

CONCLUSIONS

The algorithm meets the requirement of generating noise for testing beamformers for closely spaced arrays. This enables testing and comparison of beamformers in the laboratory for noise fields of defined and reproducible properties.

It was verified, for three-element equispaced arrays, that the algorithm is able to model noise fields with coherencies corresponding to isotropic noise and to surface noise fields. However, the algorithm does not generate noise for all arbitrary noise fields. An expression that must be satisfied by the coherencies for a three-element array was obtained.

The statistical properties of the synthesizer were confirmed to be those for Gaussian noise and cumulative distributions of the coherency were obtained.

REFERENCES

1. W.J. Jobst, and S.L. Adams, "Statistical Analysis of Ambient Noise", J. Acoust. Soc. Am., 62, 63-71, 1977.

2. IBM System 360 Scientific Subroutine Package (360A-CM-03X), Version III, 77, 1969.

3. G.M. Jenkins, and D.G. Watts, "Spectral Analysis and its Applications", Holden Day, 467, 1968.

4. B.F. Cron, and C.H. Sherman, "Spatial-Correlation Functions for Various Noise Models", J. Acoust. Soc. Am., 34, 1732-1736, 1962.

5. V.A. Benignus, "Estimation of the Coherence Spectrum and its Confidence Interval Using the fast Fourier Transform", IEEE Trans. Audio. Elect. Acoust., AU-17, 2, 145-150, 1969.

6. M. Abramowitz, and I.A. Stegun, "Handbook of Mathematical Functions", U.S. Dept. of Comm. National Bureau of Standards, Applied Maths. Series 55, 363, 1964.

APPENDIX A


```
C       SUBROUTINE COEFF (NUM,Q,A)
C
C
C       PURPOSE: THIS SUBROUTINE COMPUTES THE COEFFICIENTS FOR THE
C       GENERATION OF CORRELATED NOISE FOR NUM SENSORS FROM NUM GAUSSIAN
C       SOURCES.
C
C       PRAGRAMMER: N.J. SCHROEDER
C
C       LAST REVISION DATE: 10 AUGUST 1978
C
C       METHOD: THE SUBROUTINE ASSUMES THAT THE COEFFICIENTS FORM A
C       LOWER TRIANGULAR MATRIX, THAT IS THAT SENSOR (I) RECEIVES
C       NOISE COMPONENTS FROM A MAXIMUM OF (I) NOISE SOURCES. THE
C       SUBROUTINE ALSO ASSUMES THAT CERTAIN SIMPLIFYING ASSUMPTIONS
C       HAVE BEEN MADE: THAT THE AVERAGE POWER FROM ANY ONE (1) SENSOR
C       IS ONE (1); THAT THE NOISE SOURCES ARE TOTALLY UNCORRELATED;
C       AND THAT THE COHERENCE MATRIX IS KNOWN
C       THE ROUTINE CALCULATES THE COEFFICIENTS BY COLUMNS, FIRST
C       DETERMINING THE VALUE OF THE DIAGONAL ELEMENT AT THE TOP OF
C       THE NON-ZERO ELEMENTS OF EACH COLUMN, AND THEN THE ELEMENTS
C       BELOW.
C       THE METHOD FOLLOWS FROM THE FOLLOWING EQUATIONS:
C
C       THE EXAMPLE IS FOR A FOUR (4) SENSOR CASE.
C
C       Q(2,1)=A(1,1)*A(2,1)
C       Q(3,1)=A(1,1)*A(3,1)
C       Q(4,1)=A(1,1)*A(4,1)
C
C       Q(3,2)=A(2,1)*A(3,1)+A(2,2)*A(3,2)
C       Q(4,2)=A(2,1)*A(4,1)+A(2,2)*A(4,2)
C
C       Q(4,3)=A(3,1)*A(4,1)+A(3,2)*A(4,2)+A(3,3)*A(4,3)
C       FROM THE EQUATIONS IT IS CLEAR THAT FOR ANY NONDIAGONAL ELEMENT
C       A(I,J), I GREATER THAN J:
C
C       A(I,J)=(Q(I,J)-SUMATION(A(I,K)*A(J,K)),K=1,J-1)
C       -----------------------------------------------
C                                  A(J,J)
C
C       THE INPUT PARAMATERS ARE:
C
```

```
C         NUM***THE NUMBER OF SENSORS.
C         Q***THE COHERENCE MATRIX.
C
C         THE OUTPUT PARAMETERS ARE:
C
C         A***THE MATRIX WHICH CONTAINS THE COEFFICIENTS.
C
C         SUBROUTINES REQUIRED: NONE
C
C         PROGRAM OUTPUT: NONE
C
      SUBROUTINE COEFF (NUM,Q,A)
C
      REAL*8 B(10,10)
      REAL*8 SUM
      REAL*4 A(10,10)
      DIMENSION Q(10,10)
C   LOAD COEFFICIENT MATRIX WITH ZEROS
      DO 150 I150=1,NUM
      DO 151 I151=1,NUM
      B(I150,I151)=0.0
  151 CONTINUE
  150 CONTINUE
C   LOAD IN 1 FOR VALUE OF A(1,1)
      B(1,1)=1.0
      DO 100 I100=1,NUM-1
            DO 101 I101=I100+1,NUM
C   INITIALIZE SUM AS COHERENCE BETWEEN SENSORS I100 AND I101
                  SUM=DBLE (Q(I100,I101))
                  DO 102 I102=1,I100-1
C   SUBTRACT PRODUCTS FROM SUM
                        SUM=SUM-(B(I100,I102)*B(I101,I102))
  102             CONTINUE
C   DIVIDE SUM BY DIAGONAL ELEMENT
                  B(I101,I100)=SUM/B(I100,I100)
  101       CONTINUE
C   FIND DIAGONAL ELEMENT BY FINDING ROOT OF 1 MINUS THE SUM OF THE
C   SQUARES OF THE OTHER TERMS IN THE ROW
      SUM=1.0
            DO 103 I103=1,I100
            SUM=SUM-(B(I100+1,I103)*B(I100+1,I103))
  103       CONTINUE
      B(I100+1,I100+1)=DSQRT(SUM)
  100 CONTINUE
      DO 104 I104=1,NUM
C   CONVERT TO SINGLE PRECISION EQUIVALENT
      DO 105 I105=1,NUM
            A(I105,I104)=SNGL(B(I105,I104))
  105 CONTINUE
  104 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NOISE (NUM,ISEED,ISAM,A,X,S)
C
C
C     PURPOSE: TO GENERATE THE FOURIER COEFFICIENTS FOR SAMPLES OF CORREL-
C     ATED NOISE AT EACH OF NUM SENSORS FOR UP TO ONE HUNDRED (100) SAMPLES
C
C     PROGRAMMER: N.J.SCHROEDER
C
C     LAST REVISION DATE:24 JULY 1978
C
C     METHOD: THE FOURIER COEFFICIENTS ARE COMPUTED USING GAUSSIAN
C     DISTRIBUTED RANDOM VARIABLES GENERATED BY GAUSS4 WHICH ARE THEN
C     MULTIPLIED BY THE COEFFICIENTS WHICH ARE PART OF THE SUBROUTINE
C     INPUT.
C
C     THE INPUT PARAMETERS ARE:
C
C     NUM IS THE NUMBER OF SENSORS IN THE ARRAY
C     ISEED IS THE ODD INTEGER SEED FOR GAUSS4.IT MUST BE IN THE RANGE
C     2**31-1 TO -(2**31)
C     ISAM IS THE NUMBER OF SAMPLES DESIRED. THE RESPONSE AT EACH SENSOR I
C     COMPUTED FOR EACH SAMPLE. THE MAXIMUM NUMBER OF SAMPLES WHICH CAN BE
C     STORED IN THE ARRAY PROVIDE BY THE SUBROUTINE IS (100).
C     A IS THE NUM BY NUM MATRIX OF COEFFICIENTS. IT CAN BE PRODUCED BY
C     A SUBROUTINE SUCH AS COEFF. THE MAXIMUM NUMBER OF SENSORS IS TEN (10
C     THE PROGRAM ASSUMES THAT THE MATRIX IS LOWER TRIANGULAR.
C     S IS THE DESIRED STANDARD DEVIATION OF THE DATA.
C
C     THE OUTPUT PARAMETERS ARE:
C
C     X IS THE OUTPUT NOISE MATRIX. THE MAXIMUM SIZE IS TEN (10) SENSORS
C     BY ONE HUNDRED (100) SAMPLES. THE MATRIX IS COMPLEX.
C
C     SUBROUTINES REQUIRED: GAUSS4
C
C     SUBROUTINE OUTPUT: NONE
C
C
C
      COMPLEX C
      COMPLEX*8 X(10,100)
      REAL*4 A(10,10)
C     LOAD THE ARRAY WHICH WILL CONTAIN THE NOISE WITH ZEROS
      DO 98 I98=1,ISAM
           DO 99 I99=1,NUM
                X(I99,I98)=(0.0,0.0)
   99      CONTINUE
   98 CONTINUE
      DO 100 I100=1,ISAM
           DO 101 I101=1,NUM
C     FIND VALUE OF Z FOR GIVEN SENSOR AND SAMPLE
                CALL GAUSS4 (Z1,Z2,ISEED)
                C=CMPLX (Z1,Z2)
C     ADJUST VALUE OF Z FOR REQUIRED VARIANCE
                C=C*S
                DO 102 I102=I101,NUM
C     ADD COEFFICIENT TIMES Z TO VALUE AT EACH SENSOR
                     X(I102,I100)=X(I102,I100)+A(I102,I 101)*C
  102           CONTINUE
  101      CONTINUE
  100 CONTINUE
C     RETURN TO CALLING PROGRAM
      RETURN
      END
```

```
      SUBROUTINE GAUSS4(Z1,Z2,IX)
C
C     THIS SUBROUTINE GENERATES PAIRS OF INDEPENDENT NORMAL RANDOM
C     DEVIATES WITH MEAN ZERO AND STANDARD DEVIATION 1, USING THE
C     METHOD DESCRIBED IN THE REFERENCES.
C
C     Z1 AND Z2 ARE THE NORMAL RANDOM DEVIATES.
C
C     REFERENCES:
C
C     1)  JAMES R. BELL,  'ALGORITHM 334 (G5)  NORMAL RANDOM DEVIATES,'
C             COMM. ACM 11 (JULY 1968), 498.
C
C     2)  R. KNOP,  'REMARK ON ALGORITHM 334 (G5),'  COMM. ACM 12
C             (MAY 1969), 281.
C
C
C
C
C
C     GENERATE 3 UNIFORM RANDOM DEVIATES U(1),U(2), U(3)
 2101 CONTINUE
C     THE THREE RANDOM DEVIATES ARE DISTRIBUTED ON THE INTERVAL
C     -2**31-1 TO +2**31, MULTIPLICATION BY THE FACTOR .4656613E-9
C     CAUSES THE NEW RANDOM DEVIATES TO BE DISTRIBUTED UNIFORMLY ON
C     -1 TO 1 . IT IS POSSIBLE TO GENERATE BOTH POSITIVE AND NEGATIVE
C     RANDOM DEVIATES SINCE THE SIGN BIT IS NOT ZEROED.
      I1=IX*65539
      I2=I1*262147
      I3=I2*65539
      IX=I3
      X=I1*.4656613E-9
      Y=I2*.4656613E-9
C
C
C     GENERATE GAUSSIAN DEVIATES
C     THE FORMULA USED IN CALCULATING THE RANDOM DEVIATES IS:
C     Z1=SQRT(-2.0*ALOG(X1))* COS X2
C     Z2=SQRT(-2.0*ALOG(X1))* SIN X2
C     THE NEED TO CALCULATE SIN AND COS  IS ELIMINATED BY GENERATING
C     TWO RANDOM VARIABLES,X AND Y, WHICH CORRESPOND TO A POINT IN THE
C     UNIT DISC. S IS THE RADIUS SQUARED.
      S=X*X + Y*Y
C     THIS TEST DETERMINES IF THE POINT LIES OUTSIDE THE UNIT DISC.
C     IF IT DOES, THE POINT IS IGNORED AND A NEW POINT IS GENERATED.
      IF (S.GT.1.0) GO TO 2101
C
C     RHI IS A RANDOM DEVIATE WHICH IS UNIFORMLY DISTIBUTED ON 0 TO 1.
      RHI=(I3*.4656613E-9 + 1.)/2.
C     SINCE SIGN AND COS ARE SIMPLE THE RATIOS OF SIDES OF A RIGHT
C     TRIANGLE TO THE HYPOTENEUSE, IT IS NECESSARY ONLY TO CALCULATE THE
C     RATIO OF THE ABSCISSA AND ORDINATE OF THE RANDOM POINT TO THE
C     SQUARE ROOT OF S AND MULTIPLE THIS UNTO THE ROOT OF THE LOG
C     IN ORDER TO ARRIVE AT THE DESIRED RANDOM DEVIATE.
      R=SQRT(-2.*ALOG(RHI)/S)
      Z1=X*R
      Z2=Y*R
C
      RETURN
      END
```

APPENDIX B

In this appendix the condition on the noise coherencies q_{ij} for real a_{33} is derived for a three-element array. As previously the hydrophone output X_i is written

$$X_i = a_{i1} Z_1 + a_{i2} Z_2 + \ldots\ldots + a_{in} Z_n \tag{B1}$$

now

$$q_{ij} = \overline{X_i X_j^*} \text{ and } \overline{Z_i Z_j^*} = 1 \quad i = j \qquad = 0 \quad i \neq j \tag{B2}$$

so that

$$q_{ij} = \sum_{k=1}^{i} a_{ik}\, a_{jk} \tag{B3}$$

solving (B3) for a_{ij} we obtain:

$$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ q_{12} & \sqrt{1 - q^2_{12}} & 0 \\ q_{13} & \dfrac{q_{23} - q_{13}\, q_{12}}{\sqrt{1 - q^2_{12}}} & \sqrt{1 - q^2_{13} - \dfrac{(q_{23} - q_{13}\, q_{12})^2}{1 - q^2_{12}}} \end{bmatrix}$$

so that for a_{33} to be real

$$q^2_{13} + q^2_{12} + q^2_{23} - 2q_{23}\, q_{13}\, q_{12} - 1 \leq 0 \tag{B4}$$

APPENDIX C

In this appendix some noise fields that can be modelled by the algorithm are determined. The investigation is limited to three-element 'equispaced' horizontal arrays. For an equispaced array $q_{12} = q_{23}$ and (B4) becomes,

$$q_{13}^2 - 1 - 2q_{12}^2 (q_{13} - 1) \leq 0$$

for a_{33} real. This equation may be written

$$(q_{13} - 1)(q_{13} + 1 - 2 q_{12}^2) \leq 0$$

and since $(q_{13} - 1)$ is always negative we require

$$2 q_{12}^2 - q_{13} - 1 \leq 0 \tag{C1}$$

for real a_{33}.

<u>Case 1</u>

For surface noise whose coherency can be represented by $J_o(x)$ where x = kd, the left-hand side of (C1) becomes

$$2J_o{}^2(x) - J_o(2x) - 1 \tag{C2}$$

To evaluate this expression we have the addition theorems for Bessel functions[6]:

$$J_o{}^2(x) + 2 \sum_{k=1}^{\infty} J_k^2(x) = 1 \tag{C3}$$

and $$J_o(2x) = J_o{}^2(x) + 2 \sum_{k=1}^{\infty} (-1)^k J_k{}^2(x) \tag{C4}$$

Substituting (C4) in (C2) and splitting the sum into even and odd parts we obtain

$$J_o^2(x) - 2\sum_{k=1}^{\infty} J_{2k}^2(x) + 2\sum_{k=0}^{\infty} J_{2k+1}^2(x) - 1$$

$$= J_o^2(x) - 4\sum_{k=1}^{\infty} J_{2k}^2(x) + 2\sum_{k=1}^{\infty} J_k^2(x) - 1$$

and by applying (C3)

$$= -4\sum_{k=1}^{\infty} J_{2k}^2(x)$$

This verifies that the left-hand side of (C2) is certainly less than or equal to zero for all x. Thus the algorithm can find real a_{33} and synthesize acoustic noise for surface noise of the form $J_o(x)$ for all hydrophone separations with a three-element equispaced array.

<u>Case II</u>

For surface generated noise fields the noise coherency can be expressed by[4]:

$$q_{ij} = \frac{2^m m!\, J_m(x)}{x^m} = \sum_{k=0}^{\infty} \frac{(-1)^k\, x^{2k}\, n!}{2^{2k}\, k!\, (n+k)!} \qquad \text{(C5)}$$

To simplify substitution into (C1), the test for real a_{33}, we note that

$$q_{12}^2 = \left\{ \frac{J_m(x)}{x^m} - \frac{x^2\, m!}{2^2(m+1)!}\,\frac{J_m(x)}{x^m} + \ldots + \frac{(-1)^k\, x^{2k} m!\, J_m(x)}{2^{2k}\, k!\, (m+k)!} + \ldots \right\} \qquad \text{(C6)}$$

$$q_{13} = 1 - \frac{4x^2 m!}{2^{2(m+1)!}} + \ldots\ldots\ldots\ldots\ldots + \frac{4(-1)^k\, x^{2k}\, m!}{2^{2k}\, k!(m+k)!} + \ldots \qquad \text{(C7)}$$

Now substituting in (C1), grouping even and odd terms and using ℓ to denote the even terms, the left-hand side of (C1) becomes

$$\left(\frac{2\ J_m(x)}{x^m}\right) - \left(\frac{2x^2\ m!}{2^2(m+1)!}\ \frac{J_m(x)}{x^m} + \frac{4x^2\ m!}{2^2(m+1)!}\right) + \ldots\ldots$$

$$\ldots \left(\frac{(-1)^\ell\ x^{2\ell}\ m!}{2\ 2^{\ell}\ell!\ (m+\ell)!}\right)\left(\frac{J_m(x)}{x^m} - 2\right)\left(1 - \frac{x^2}{2^2(\ell+1)\ (m+\ell+1)}\right) + \ldots \quad (C8)$$

since $\dfrac{2^m\ m!\ J_m(kd)}{(kd)^m} \leq 1$, the first and second terms in the above expression are negative for all x. The third term is negative provided $x < 6$. This implies that a_{33} is known to be real under the following conditions,

1. the array consists of three equispaced sensors in a line;
2. the noise field is of the form (C5);
3. the largest hydrophone separations are $\leq$ 0.95 wavelengths.

It was also found from numerical evaluation of Equation (C1) that a_{33} is real out to hydrophone separations of 1.5 wavelengths for m = 1, 2, or 3 with surface noise fields of the form given by (C5).

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A noise-generating algorithm and associated computer program for well-defined testing of beamformers are described. The algorithm is especially suitable for superdirective arrays of underwater hydrophones as it generates Gaussian noise of specified coherency. Statistical properties of the generator are confirmed to be those planned, and the ability of the generator to synthesize noise for isotropic or surface noise sources is verified for three-element arrays. Cumulative distributions for estimated coherency were obtained for the model.

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