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COMPLETE INDEPENDENCE
IN
THE MULTIVARIATE NORMAL DISTRIBUTION*

by

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ABSTRACT

Testing complete independence is one of the simplest problems oncerning the covariance structure of a set of measurements. A stepwise procedure proposed by S. N. Roy and R. E. Bargmann (1958) and a trace criterion due to H. Nagao (1973) are two well known copetitors of the likelihood ratio test of the hypothesis derived assuming the multivariate normality. We consider some modifications of the Roy-Bargmann procedure based on combinations of independent tests and find them to be asymptotically equivalent to the likelihood ratio test, which is optimal in terms of the exact slopes. The operating characteristics of various tests with samples of moderate size are examined empirically.

Key Words: Combination of tests, Exact slopes, Stepdown procedure.

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1. INTRODUCTION AND SUMMARY

Let $\mathbf{X}_1, \mathbf{X}_2, \ldots\ldots, \mathbf{X}_N$ be a random sample from a p-variate normal population with the covariance matrix $\mathbf{\Sigma}$. One of the simplest problems concerning the covariance structure of the multivariate normal distribution is of testing the complete independence of the p measurements comprising the vectors $\mathbf{X}_i$. The likelihood ratio test for the complete independence which depends upon the determinant $|\mathbf{R}|$ of the correlation matrix was derived by Wilks (1935). The exact distribution of the likelohood ratio statistic is discussed and tabulated by Mathai and Katiyar (1979). An alternative solution termed a step-down procedure, which consists of p-1 independent tests was proposed by Roy and Bargmann (1958). This procedure, unlike the likelihood ratio test, permits post-hoc analysis of the nature of dependence in case of a rejection of the null hypothesis and depends upon only the well tabulated F-distribution for its implementation.

In this paper we introduce a class of tests asymptotically equivalent, in terms of the exact Bahadur slopes, to the likelihood ratio test which is optimal in this sense. The presently available methods of testing complete independence are summarized in section 2. The new tests are introduced and shown to be Bahadur-optimal in section 3. Section 4 contains a Monte Carlo comparison of these tests with the likelihood ratio test and the step-down procedure when the samples are of moderate size. The empirical study also includes a test proposed by Nagao (1973).

2. SOME TESTS OF COMPLETE INDEPENDENCE

Let $\mathbf{R}$ be the correlation matrix of a sample of size N from the

$N_p(\underset{\sim}{\mu},\underset{\sim}{\Sigma})$ population. The likelihood ratio test for Ho: $\underset{\sim}{\Sigma} = \text{diag}\,(\sigma_1^2, \sigma_2^2, \ldots, \sigma_p^2) = \Sigma_d$ rejects it if

$$\Lambda = |\underset{\sim}{R}|^{-N/2} < c \,, \tag{2.1}$$

where the critical constant c may be obtained from Mathai and Katiyar (1979) or obtained by using approximations such as Box's, and Bartlett's discussed by Mudholkar, Trivedi, and Lin (1980). Nagao (1973) noted that asymptotically $-2 \log \Lambda$ is a χ^2-variable when the null hypothesis is true but it is an asymptotic normal variable for any fixed alternative. He suggested regarding $\tau^2 = \text{tr}(\Sigma\, \Sigma_d^{-1} - I)^2$ which is proportional to the variance of this normal distribution as a noncentrality parameter, i.e., a measure of departure from the null hypothesis, and proposed a consistent estimator

$$T = \frac{N-1}{2}\, \text{tr}\, (S\, S_D^{-1} - I)^2 = \frac{N-1}{2}\, \text{tr}\, (R^2 - I) \tag{2.2}$$

of a multiple of τ^2 as a test statistics for H_0. He obtained the asymptotic expansion for T in the form

$$\Pr(T \le x) = P_f + \frac{1}{N-1}\,[a_6\, P_{f+6} + a_4\, P_{f+4} + a_2\, P_{f+2} + a_0\, P_f] + O(n^{-2}) \,, \tag{2.3}$$

where $f = p(p-1)/2$ and $P_f = P[\chi^2_f \le x]$ and $a_6 = (p^3 - 3p^2 + 2p)/12$, $a_4 = (-2p^3 + 3p^2 - p)/8$, and $a_2 = (p^3 - p)/4$. He showed that it is satisfactory for $n = 100$.

Roy and Bargmann (1958) consider the null hypothesis of complete independence in the form $H_0 = \bigcap_{i=2}^{p} \{H_{0i} : \rho^2_{i\,\cdot\,12\ldots(i-1)} = 0\}$, where $\rho^2_{i\,\cdot\,12\ldots(i-1)}$ is the multiple correlation between X_i and $(X_1, X_2, \ldots, X_{i-1})$, and note that the sample step-down multiple correlation coefficients

$R^2_{i \cdot 12....(i-1)}$ are independently distributed when Σ is diagonal. They propose rejecting the null hypothesis when at least one of the component hypotheses H_{oi} is rejected by the usual test for it,i.e. when $R^2_{i \cdot 12....(i-1)}$ constant. This procedure is simple to implement as independently distributed

$$F_i = (N-i)/(i-1)\, R^2_{i\cdot 12...(i-1)}/\{1-R^2_{i\cdot 12...(i-1)}\} \tag{2.4}$$

have variance ratio distributions with $(i-1, N-i)$ d.f., $i = 2, 3, ..., p$.

However, the procedure does require an a priori ordering among the measured variables and a decision regarding the levels α_i of the component tests which, because of the independence, are related to allowable overall type I error α by $(1-\alpha) = \prod_{i=2}^{p} (1-\alpha_i)$. It is common to take $\alpha_i = 1-(1-\alpha)^{1/(p-1)}$ $i = 2,...,p$. Roy and Bargmann gave the confidence bounds associated with this step-down procedure which can be used to gain an understanding of the nature of dependence in case H_o is rejected.

3. A CLASS OF B-OPTIMAL TESTS

The problem of the allocation of the overall type I error among the component tests of the step-down procedure may be avoided by considering, instead of the variance ratio statistics F_{i+1}, the P-values P_i associated with the individual tests $i = 1,...,k$, where $k = p-1$. Since the statistics F_{i+1} are independent under H_o, the P-values P_i have independent uniform null distributions. These can therefore be combined variously to construct an overall test for H_o. The problem of combining independent tests of significance is classical and the literature on the subject is extensive. It is well reviewed in Liptak (1958), Oosterhoff (1969), George (1977) and Mudholkar and George (1979).

A combination procedure for the P-values $P_1, P_2, \ldots, P_k$ associated with k independent tests of significance for hypotheses $H_{oi}: \theta_i = \theta_{oi}$ vs. $H_{1i}: \theta_i > \theta_{oi}$, $i = 1,2,\ldots,k$ is based upon a combination statistic $\Psi(P_1,\ldots,P_k)$ which is used for testing the overall hypothesis $H_o = \bigcap H_{oi}$ vs. the alternative $H_1 = \bigcup H_{1i}$. The overall null hypothesis Ho is rejected when $\Psi(P_1,\ldots,P_k)$ is large. The following are some of the well known combination statistics: (i) The earliest proposed $\Psi_T = \min\{-2 \log P_i\}$ due to Tippett, (ii) $\Psi_F = \Sigma -2 \log P_i$ due to Fisher, (iii) $\Psi_N = \Phi^{-1}(1-P_i)$, Φ being the c.d.f. of standard normal, considered by Lipkak (1958), and (iv) $\Psi_L = \sum_i \log [P_i/(1-P_i)]$ introduced by George (1977). These statistics have simple null distributions. Ψ_T is distributed as the smallest order statistic of a sample from the exponential population, Ψ_F is a χ^2_{2k}-variable, Ψ_N has N(0, k) distribution, and Ψ_L a k-fold convolution of logistic distribution is approximately a t variable with 5k + 4 degrees of freedom. It is easily seen that the stepdown procedure with equal α_i's is equivalent to the Tippett combination of its P-values. An account of various studies of the operating characteristics of combination methods in the Neyman-Pearson and decision theoretic framework may be found in Oosterhoff (1969). However, none of the methods can be preferred on the basis of these works. Littel and Folks (1971) examined Bahadur ARE's of various methods and found that among all monotone combination procedures Fisher's is optimal according to this criterian. Mudholkar and George (1979) showed that Ψ_L has the same exact slope as Ψ_F and is consequently optimal. For a recent account of this aspect see Berk and Cohen (1979). These studies of asymptotic relative efficiencies concern combinations of independent tests; but can be extended to the methods of combining tests which are independent under the null hypothesis only.

The exact slope used in defining Bahadur ARE of a test at an alternative is the rate at which -2 log (P-value of the test) increases with respect to the sample size n, when the alternative is true. Specifically, let large values of a statistic T_n be significant in testing H_o: $\theta \in \hat{H}_o$ vs. H_1: $\theta \notin \hat{H}_o$, $F_{\theta,n}(t)$ denote the distribution function of T_n and $P_n(T_n) = 1 - F_{o,n}(T_n)$ be the associated P-value. Then $c(\theta) = \lim_{n\to\infty} -2n^{-1} \log P_n(T_n)$, when it exists, is the exact slope of T_n. $c(\theta)$ is often obtained using the following result due to Bahadur (1971, p. 27).

<u>Proposition</u>. Suppose that $\lim Tn/\sqrt{n} = b(\theta)$ a.s. for each $\theta \in \hat{H} - H_o$. Let $\rho(t) = -n^{-1}\log[1-F_{o,n}(\sqrt{n}\, t)]$ and suppose that $\lim_{n\to\infty} \rho_n(t) = \rho(t)$ exists and is continuous on an open interval containing the range of $b(\theta)$. Then the exact slope of T_n is $c(\theta) = 2 \cdot \rho(b(\theta))$.

<u>Remark</u>. $\rho(t)$ is sometimes referred as the index of the sequence $\{T_n\}$ or of the sequence of distributions $\{F_{o,n}\}$ of $\{T_n\}$.

Now consider the present problem of testing the null hypothesis H_o that the covariance matrix Σ is diagonal. The step-down procedure which involves testing the component hypothesis H_{oi}: $\rho^2_{i+1\cdot 12\ldots i} = 0$ with

$$F_{i+1} = [N-i-1)/i] \cdot [R^2_{i+1\cdot 12\ldots i}/(1-R^2_{i+1\cdot 12\ldots i})], \tag{3.1}$$

$i = 1,2,\ldots,k$, may be modified by combining these independent (only under H_o) tests using a combination statistic $\Psi(P_1,\ldots,P_k)$, where P_i's are the P-values associated with F_{i+1}'s. We are interested in the statistics of the form $\Psi(P_1,\ldots,P_k) = \sum_i \phi_i(P_i)$, where $\phi_i(P_i) = \phi_i(P_{i,n})$ are monotone decreasing with index $\rho(t) = t$, $i - 1, 2, \ldots, k$. Let L denote the family of these tests. Note that if $\phi = G_i^{-1}(1-P_i)$ then under H_o, $\psi(P_1,\ldots,P_k)$ is distributed as the convolution of $G_1, G_2, \ldots, G_k$. In fact L includes in this manner the combinations of the step-down tests based on Fisher's method with $G^{-1}(1-t) = -2 \log t$ and on Logit method with $G^{-1}(t) - \log[t/(1-t)]$.

The tests in the family L are asymptotically optimal and equivalent to the likelihood ratio test for H_o. In order to demonstrate this, i.e. to obtain the exact slope of $\psi(P_1,\ldots,P_k) = \Sigma\phi(P_i)$, we examine $\phi_i(P_i)$ which has the same slope as F_{i+1} given in (3.1). Since F_{i+1} is a variance-ratio with (i, N-i-1) d.f., for an alternative $\rho^2_{i+1\cdot 12\ldots i}$, $\{F_{i+1}/(N-i-1)\}^{1/2}$ converges in probability to $\{[\rho^2_{i+1\cdot 12\ldots i}/(1-\rho^2_{i+1\cdot 12\ldots i})]/i\}^{1/2}$ Moreover, it can be shown (eg., see Bahadur (1971, p. 13)) that $-n^{-1} \log(1-F_{o,n}(\sqrt{nt})) \to \log(1 + i\, t^2)$. Hence by the above proposition, the exact slope of F_{i+1} or its monotone function $\phi_i(P_i)$ is

$$c_i(\rho^2_{i+1\cdot 12\ldots i}) = -\log(1-\rho^2_{i+1\cdot 12\ldots i}) . \tag{3.2}$$

Now, in view of the results by Berk and Cohen (1979) it follows that the index of $\Sigma\phi_i(P_i)$ is the same as the index $\rho_i(t) = t$ of each $\phi_i(P_i)$ and consequently the exact slope of $\psi(P_i,\ldots,P_k)$ is

$$\begin{aligned} c(\mathcal{P}) &= \sum_i -\log(1-\rho^2_{i+1\cdot 12\ldots i}) \\ &= -\log|\mathcal{P}| , \end{aligned} \tag{3.3}$$

where $|\mathcal{P}|$ denotes the determinant of the population correlation matrix $\mathcal{P} = (\rho_{ij})$. By particularizing the result in section 3.4 of Hsieh (1979) it is seen that the likelihood ratio test for complete independence is asymptotically optimal with the exact slope -log | |, the same as (3.3).

4. AN EMPIRICAL EVALUATION

In this section we present a Monte Carlo study of the operating characteristics of some of the asymptotic Bahadur equivalents, of the likelihood ratio test for complete independence, described in section 3, when the samples are of moderate size. The study also includes Nagao's test given in section 2. The finite sample behavior of the tests are investigated in terms of the power function as well as in terms of the means and s.d.'s of the P-values of the tests at various alternatives.

The Monte Carlo Experiment. The simulation study was conducted on IBM 3032 at the University of Rochester, generating the random samples from IMSL routine GGNRM. 3000 samples of size n=20 and n=30 were drawn from $N_p(\underset{\sim}{0}, \Sigma)$ with $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_p^2 = 1$ and various configurations of correlations ρ_{ij} from values 0, .2, .4, .6 and .8 for p = 3, 4, 5. For each sample drawn, the following test statistics were obtained from the sample correlation matrix $\underset{\sim}{R} = (r_{ij})$.

(i) Likelihood ratio based statistic $\ell = 1 - \{N-1-(2p+5)/6\} \log |R|$,

(ii) Nagao's test statistic $T = (N-1) \sum_{i<j} r_{ij}^2$,

(iii) Step-down statistics $F_i = [N-i)/(i-1)].[R^2_{i\cdot 12\ldots(i-1)}/(1-R^2_{i\cdot 12\ldots i-1})]$, for i = 2, 3, ..p

(iv) Combination statistic based on Logit method $\psi_L = -\sum_{i=2}^{p} \log(P_i/(1-P_i))$,

(v) Combination statistic based on Fisher's method $\psi_F = -2\sum_{i=2}^{p} \log P_i$, where P_i are the P-values corresponding to the step down statistic.

These test statistics were compared with their critical constants determined using the following facts regarding the null distributions:

(i) ℓ is well approximated with χ^2 distribution having d.f. p(p-1)/2;

(ii) the critical constant T_α for T may be approximated by

$$T_\alpha \doteq u + \frac{1}{n}\left[\frac{2a_6 u}{f(f+2)(f+4)}\{u^2 + (f+4)u + (f+2)(f+4)\} + \frac{2a_4 . u}{f(f+2)}(u+f+2) + \frac{2a_2 u}{f}\right],$$

where u is the upper 100α percentage point of the χ^2 distribution with d.f. $f = p(p-1)/2$, and a_2, a_4, a_6 as given in (2.3) ;

(iii) F_i is distributed as a variance ratio F with (i-1,N-i) d.f., for i = 2, 3;

(iv) ψ_L is approximated with $a \cdot t_\nu$, a constant times student's t, where $a = \pi\{k(5k+2)/(3(5k+4))\}^{1/2}$, k = p-1, and d.f. $\nu = 5k+4$;

(v) ψ_F is distributed as a χ^2 with 2(p-1) d.f.

The pow of each test was estimated by the proportion of times the null hypothesis was rejected by the corresponding test. The s.d. of any of these estimates $\leqq \{(3000)x4\}^{-1/2}$. The P-values corresponding to the tests were obtained using the equation (2.3) and the results on the null distributions as mentioned above. The P-values in each case were averaged and their standard deviation was computed.

The estimated power functions and the means of the P-values of the five tests at various alternatives are given in Table 1 and Table 2 respectively. The Monte Carlo experiment with 3000 simulation was first conducted with n=20 and p=3 for the correlation configurations appearing in the tables. After an examination of the results it was performed with p=4 and 5 for two special configurations, namely (i) the extreme configuration in which only the first correlation configuration ρ_{21} is nonzero, and (ii) the symmetrical configuration where all correlation coefficients are equal. As a confirmation of the findings, the procedure was repeated with n=30. The s.d.'s of the P-values and the results for n=30 which are not included in this paper are available from the authors.

Conclusions. Two features of the comparative behavior of the five tests clearly emerge from the two tables: (i) In case of the extreme configuration, with $\rho_{21} \neq 0$, $\rho_{ij} = 0$ otherwise, the step down procedure is preferable. Its superiority over the other four tests increases as p increases. Nagao's test is the poorest in this case. (ii) Nagao's test dominates others if H_o is violated in a symmetric manner, i.e., when ρ_{ij}'s are nonzero and equal. The stepdown test is the weakest in this case. It is also observed that the likelihood ratio test, and the two combinations of the P-values of the stepdown components are generally comparable and are preferable except against the two special alternatives.

Table 1. The Empirical Power Functions for Samples of Size n=20 with Monte Carlo of Size 3000

p	Nonzero Corr.	L.R. Test	Nagao Test	Step Dn Test	Logit Comb.	Fisher Comb.
3	H_o	.0490	.0480	.0527	.0503	.0473
	ρ_{21} = .2	.0880	.0900	.0993	.1003	.1027
	= .4	.2673	.2650	.3300	.2957	.3227
	= .6	.6503	.6357	.7540	.6437	.7217
	= .8	.9823	.9780	.9930	.9717	.9877
	ρ_{31} = .2	.0963	.0957	.0867	.0893	.0883
	= .4	.2710	.2663	.2420	.2197	.2347
	= .6	.6733	.6493	.6470	.5347	.6107
	= .8	.9800	.9770	.9757	.9390	.9717
	ρ_{32} = .2	.0990	.0967	.0930	.0877	.0890
	= .4	.2713	.2660	.2373	.2280	.2350
	= .6	.6570	.6430	.6273	.5393	.5953
	= .8	.9717	.9650	.9707	.9210	.9587
	$\rho_{21} = \rho_{31}$ = .2	.1393	.1417	.1320	.1420	.1403
	= .4	.5983	.5400	.5270	.6200	.6170
	= .6	.9990	.9930	.9933	.9977	.9990
	$\rho_{31} = \rho_{32}$ = .2	.1387	.1357	.1187	.1173	.1220
	= .4	.5967	.5403	.5663	.4767	.5407
	= .6	.9980	.9947	.9977	.9817	.9973
	$\rho_{21} = \rho_{32}$ = .2	.1583	.1550	.1543	.1660	.1673
	= .4	.5843	.5410	.5203	.6050	.5987
	= .6	.9987	.9940	.9923	.9980	.9987
	All ρ_{ij}'s = .2	.1890	.2150	.1693	.1933	.1937
	= .4	.6340	.6847	.5353	.6453	.6353
	= .6	.9490	.9650	.9073	.9567	.9527
	= .8	1.000	.9997	.9993	.9997	.9997
4	H_o	.0473	.0457	.0457	.0480	.0463
	ρ_{21} = .2	.0763	.0803	.0960	.0953	.1020
	= .4	.2013	.1937	.2817	.2350	.2707
	= .6	.5227	.4837	.7150	.5530	.6643
	= .8	.9400	.8993	.9840	.9167	.9727

Continued

Table 1 Continued

p	Nonzero Corr.	L.R. Test	Nagao Test	Step Dn Test	Logit Comb.	Fisher Comb.
4	All ρ_{ij}'s = .2	.2253	.2860	.1680	.2283	.2257
	= .4	.7270	.8153	.5823	.7440	.7343
	= .6	.9833	.9923	.9427	.9860	.9837
	= .8	1.000	1.000	.9997	1.000	1.000
5	H_o	.0470	.0463	.0447	.0527	.0473
	ρ_{21} = .2	.0670	.0677	.0897	.0817	.0857
	= .4	.1520	.1493	.2517	.2043	.2313
	= .6	.3923	.3607	.6760	.4573	.5823
	= .8	.8727	.7377	.9840	.8673	.9597
	All ρ_{ij}'s = .2	.2773	.3820	.1887	.2867	.2763
	= .4	.8223	.9080	.6150	.8347	.8220
	= .6	.9897	.9963	.9537	.9903	.9897
	= .8	1.000	1.000	1.000	1.000	1.000

Table 2. Estimated Means of the P-values at Various Alternatives for Samples of Size 20 with Monte Carlo of Size 3000

p	Nonzero Corr.	L.R. Test	Nagao Test	Step Dn Test	Logit Comb.	Fisher Comb.
3	H_o	.4969	.4975	.4978	.4956	.4971
	ρ_{21} = .2	.4345	.4340	.4250	.4278	.4239
	= .4	.2482	.2485	.2201	.2418	.2213
	= .6	.0775	.0805	.0555	.0863	.0610
	= .8	.0048	.0062	.0023	.0069	.0033
	ρ_{31} = .2	.4229	.4215	.4314	.4339	.4302
	= .4	.2544	.2553	.2750	.2926	.2749
	= .6	.0793	.0823	.0900	.1234	.0968
	= .8	.0048	.0061	.0052	.0155	.0070
	ρ_{32} = .2	.4221	.4216	.4334	.4330	.4310
	= .4	.2531	.2543	.2739	.2916	.2736
	= .6	.0834	.0832	.0919	.1248	.0975
	= .8	.0059	.0076	.0065	.0170	.0087
	$\rho_{21} = \rho_{31}$ = .2	.3659	.3658	.3656	.3708	.3610
	= .4	.0984	.1081	.1117	.1000	.0950
	= .6	.0007	.0035	.0028	.0008	.0007
	$\rho_{31} = \rho_{32}$ = .2	.3627	.3641	.3791	.3864	.3773
	= .4	.1009	.1102	.1141	.1459	.1190
	= .6	.0007	.0034	.0009	.0069	.0012
	$\rho_{21} = \rho_{32}$ = .2	.3606	.3613	.3607	.3637	.3562
	= .4	.1011	.1101	.1134	.1018	.0976
	= .6	.0007	.0034	.0027	.0008	.0006
	All ρ_{ij}'s = .2	.3217	.3152	.3340	.3250	.3237
	= .4	.0985	.0871	.1183	.0970	.0978
	= .6	.0115	.0084	.0192	.0112	.0114
	= .8	.0001	.0000	.0003	.0001	.0001
4	H_o	.5030	.5017	.5037	.5064	.5047
	ρ_{21} = .2	.4509	.4491	.4291	.4405	.4312
	= .4	.3090	.3105	.2523	.2929	.2586
	= .6	.1202	.1280	.0689	.1191	.0812
	= .8	.0124	.0204	.0031	.0177	.0057

Continued

Table 2 Continued

p	Nonzero Corr.	L.R. Test	Nagao Test	Step Dn Test	Logit Comb.	Fisher Comb.
4	All ρ_{ij}'s = .2	.2876	.2705	.3145	.2910	.2904
	= .4	.0660	.0484	.0981	.0643	.0653
	= .6	.0040	.0018	.0125	.0034	.0039
	= .8	.0000	.0000	.0002	.0000	.0000
5	H_o	.5025	.5047	.5012	.5035	.5029
	ρ_{21} = .2	.4640	.4643	.4400	.4483	.4397
	= .4	.3353	.3346	.2626	.3056	.2724
	= .6	.1658	.1769	.0838	.1534	.1008
	= .8	.0251	.0451	.0041	.0288	.0089
	All ρ_{ij}'s = .2	.2639	.2334	.3042	.2674	.2668
	= .4	.0407	.0236	.0832	.0393	.0414
	= .6	.0023	.0008	.0099	.0018	.0020
	= .8	.0000	.0000	.0002	.0000	.0000

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Testing complete independece is one of the simplest problems concerning the covariance structure of a set of measurements. A stepwise procedure proposed by S. N. Roy and R. E. Bargmann (1958) and a trace criterion due to H. Nagao (1973) are two well known copetitors of the likelihood ratio test of the hypothesis derived assuming the multivariate normality. We consider some modifications of the Roy-Bargmann procedure based on combinations of independent tests and find them to be asymptotically equivalent to the likelihood ratio test, which is optimal in terms of the exact slopes. The operating

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characteristics of various tests with samples of moderate size are examined empirically.

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