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A MARKOV APPROACH TO LARGE MISSILE ATTACKS

Maurice M. Mizrahi

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ABSTRACT

An exact Markov approach is used to calculate the attrition suffered by a number of identical targets subjected to sequential missile attacks. Various missile-allocation situations are examined. An APL program is presented.

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A MARKOV APPROACH TO LARGE MISSILE ATTACKS
Maurice M. Mizrahi

INTRODUCTION

This paper exactly calculates the attrition suffered by a number of targets in a formation subjected to sequential missile attacks. The targets could be a raid of enemy bomber aircraft attempting to penetrate several layers of missile defenses, such as missile-firing fighter aircraft or surface-to-air missiles.

Specifically, we will calculate the distribution of the number of survivors of a set number of attacks with given parameters, for various missile-allocation situations, and the expected number of missiles fired. The emphasis will be on eliminating the complexity arising from a large number of missiles attacking simultaneously. APL programs for these calculations, written by Paul E. Klebe and the author, are presented in Appendix A.

ASSUMPTIONS AND METHODOLOGY

The targets are identical, and the missiles in a given attack are identical and fired simultaneously. Let

- n be the initial number of targets
- M_i be the maximum number of missiles fired in the i th attack
- p_i be the probability of kill of 1 missile against 1 target in the i th attack
- k be the total number of attacks.

Three cases will be examined, differing in the manner in which the missiles allocate themselves and the number of missiles fired. The first 2 cases reflect random targeting: the missiles allocate themselves randomly among all the available targets (Case I) or among a specific number b_i of targets -- say the closest b_i targets (Case II). Under those assumptions, if no target is present no missile is launched, but if at least 1 target is present all M_i missiles are launched. This corresponds to the situation where the defense sees only a blob

of targets and cannot tell them apart, or where the missiles cannot be precisely guided. The last case (Case III) reflects perfect targeting: the targets can be distinguished individually and only 1 missile is allocated per target, up to a total of M_i in the i th attack. Thus, no target can be hit by more than 1 missile in Case III.

It is assumed that the targets do not shoot back, or that, if they do, an adjustment is made to the M_i of the next attack. A target is either killed (it "disappears") or it is intact: no intermediate states are considered.

A simple Markov chain will be used in every case. The state of the system after the i th attack is therefore described by an $(n+1)$ - dimensional vector,

$$S_i = (P_0 \ P_1 \ \dots \ P_n) \ , \quad (1)$$

where P_r is the probability that r targets are present (have survived). The initial state is

$$S_0 = (0 \ 0 \ \dots \ 0 \ 1) \ . \quad (2)$$

The transition matrix T^i corresponding to the i th attack depends on p_i and M_i (and also on b_i in Case II). T^i is an $(n+1)$ by $(n+1)$, triangular, $(M_i + 1)$ - diagonal matrix. After the k th attack, the system is in the state

$$S_k = S_{k-1} T^k = S_0 T^1 T^2 \dots T^k \ . \quad (3)$$

The problem, therefore, reduces to calculating the transition matrices corresponding to each of the 3 cases. Once that is done and the final state S_k is obtained, one can extract the relevant statistics. Let $S_k(i)$ (for $i = 0, 1, \dots, n$) denote the i th element of S_k . Then:

- The expected number of targets surviving the k attacks is:

$$E = \sum_{j=0}^n j S_k(j) \quad . \quad (4)$$

- The standard deviation is:

$$\sigma = \left\{ \sum_{j=0}^n j^2 S_k(j) - \left[\sum_{j=0}^n j S_k(j) \right]^2 \right\}^{1/2} \quad . \quad (5)$$

- The expected number of missiles fired is:

$$E_k = \sum_{i=1}^k E(i) \quad , \quad (6)$$

where $E(i)$ is the expected number of missiles fired on the i th attack. In Cases I and II, we have:

$$E(i) = M_i \left[1 - S_{i-1}(0) \right] \quad . \quad (7)$$

The above reads: M_i missiles (the maximum for the i th attack) multiplied by the probability that there is at least one target present after the $(i-1)$ th attack.

In Case III, we have:

$$E(i) = \begin{cases} \sum_{j=0}^n j S_{i-1}(j) & \text{for } M_i \geq n , \\ \sum_{j=0}^{M_i} j S_{i-1}(j) \\ + M_i \left[S_{i-1}(M_i+1) + \dots + S_{i-1}(n) \right] & \text{for } M_i < n. \end{cases} \quad (8)$$

The above reads: if the number of targets present after the $(i-1)$ th attack is less than M_i , then only as many missiles as there are targets are fired, and if M_i targets or more are present after the $(i-1)$ th attack, then M_i missiles are fired.

CALCULATION OF THE TRANSITION MATRICES

The dependence of M_i , p_i , b_i , and T^i on i (the attack number) will be dropped in this section for simplicity. Let the elements of the transition matrix T be:

$$T_{ij} \equiv \text{probability that exactly } i \text{ targets survive given that } j \text{ are present } (i, j = 0, 1, \dots, n).$$

For purposes of use in Equation (3), it will be understood that i denotes the column and j the row.

Case 1: Random Targeting with Missiles Spread Among All Targets Present

This is the most difficult transition matrix to calculate. We have:

$$T_{ij} \equiv \begin{cases} 0 & \text{if } j-i < 0 \text{ or } j-i > M \\ P_{ij} & \text{otherwise,} \end{cases} \quad (9)$$

where

$$P_{ij} = j^{-M} \binom{j}{i} \sum \frac{M!}{k_1! k_2! \dots k_j!} s^{k_1 + \dots + k_i} (1-s)^{k_{i+1}} \dots (1-s)^{k_j}, \quad (10)$$

$s \equiv 1-p$ is the survival probability, and the sum extends over all k_r ranging from 0 to M but subject to the constraint that their sum be M ($\sum_{r=1}^j k_r = M$). We will first derive this result then simplify it.

Derivation of Transition Probability

We are looking for the probability that i targets survive if j are present. Label the j targets 1 to j . Assume that targets 1 to i survive after being targeted by k_1 to k_i missiles, respectively, (probability: $s^{k_1} \dots s^{k_i}$) and that targets $i+1$ to j are shot down after being targeted by k_{i+1} to k_j

missiles, respectively (probability: $(1-s^{k_{i+1}}) \dots (1-s^{k_j})$). The k 's must sum to M since there are only M missiles in the attack. There are $M!/(k_1!k_2! \dots k_j!)$ different ways of allocating k_1 missiles to target 1, k_2 missiles to target 2, etc., and k_j missiles to target j . Thus, the probability of a given combination is $[M!/(k_1!k_2! \dots k_j!)] / \sum_{k_r} [M!/(k_1!k_2! \dots k_j!)]$, which is $j^{-M} M!/(k_1!k_2! \dots k_j!)$ according to the multinomial expansion. Finally, there are $\binom{j}{i}$ different ways of choosing the i survivors. This leads to the sum in equation (10).

Simplification of Transition Probability

The sum [equation (10)] for the transition probability contains an exceedingly large number of terms and must be simplified for actual evaluation, even by computer. The basic approach in simplifying equation (10) is to get the summand to reproduce the well-known multinomial expansion:

$$(p_1 + p_2 + \dots + p_j)^M = \sum \frac{M!}{k_1!k_2! \dots k_j!} p_1^{k_1} p_2^{k_2} \dots p_j^{k_j}, \quad (11)$$

where $\sum$ means, as before, the sum from $k_r = 0$ to $k_r = M$, the sum of the k 's always being M .

The quantity we want to reduce is

$$A \equiv \sum \frac{M!}{k_1!k_2! \dots k_j!} s^{k_1 + \dots + k_i} (1-s^{k_{i+1}}) \dots (1-s^{k_j}) \quad (12)$$

Consider the auxiliary quantity

$$F(b) \equiv \sum \frac{M!}{k_1!k_2! \dots k_j!} s^{k_1 + \dots + k_i} (1-s^{b_{i+1}k_{i+1}}) \dots (1-s^{b_jk_j}). \quad (13)$$

We have, of course,

$$A = F(1) \quad (13a)$$

The integral

$$1 - s^x = x \int_s^1 u^{x-1} du \quad (14)$$

enables us to put $F(b)$ in the form

$$F(b) = \int_s^1 \dots \int_s^1 du_{i+1} \dots du_j \sum \frac{M!}{k_1! k_2! \dots k_j!} s^{k_1 + \dots + k_j} \\ \times b_{i+1}^{k_{i+1}} \dots b_j^{k_j} u_{i+1}^{b_{i+1} k_{i+1} - 1} \dots u_j^{b_j k_j - 1} \quad (15)$$

where sums and integrals have been interchanged. The identity

$$k u^{bk} = (\text{Log } u)^{-1} \frac{\partial u^{bk}}{\partial b} \quad (16)$$

puts $F(b)$ in the form

$$F(b) = \int_s^1 \dots \int_s^1 du_{i+1} \dots du_j b_{i+1} \dots b_j (u_{i+1} \dots u_j)^{-1} (\text{Log } u)^{i-j} \\ \times \frac{\partial^{j-i}}{\partial b_{i+1} \dots \partial b_j} \sum \frac{M!}{k_1! \dots k_j!} s^{k_1 + \dots + k_j} u_{i+1}^{b_{i+1} k_{i+1}} \dots u_j^{b_j k_j} \quad (17)$$

The sum in equation (17) is easily recognized as the multinomial expansion of $(is + u_{i+1}^{b_{i+1}} + \dots + u_j^{b_j})^M$. The derivatives can now be taken. The identity

$$\frac{\partial}{\partial b} (a+u^b)^M = M(a+u^b)^{M-1} u^b \text{Log } u \quad (18)$$

gives

$$\begin{aligned} & \frac{\partial^{j-i}}{\partial b_{i+1} \dots \partial b_j} (is + u_{i+1}^{b_{i+1}} + \dots + u_j^{b_j})^M \\ &= (\text{Log } u)^{j-i} M(M-1) \dots [M-(j-i)+1] (is + u_{i+1}^{b_{i+1}} + \dots + u_j^{b_j})^{M-(j-i)} \\ & \times u_{i+1}^{b_{i+1}-1} \dots u_j^{b_j-1}, \end{aligned} \quad (19)$$

which puts $F(b)$ in the form

$$\begin{aligned} F(b) &= M(M-1) \dots [M-(j-i)+1] \int_s^1 \dots \int_s^1 du_{i+1} \dots du_j u_{i+1}^{b_{i+1}-1} \dots u_j^{b_j-1} \\ & \times (is + u_{i+1}^{b_{i+1}} + \dots + u_j^{b_j})^{M-j+i}. \end{aligned} \quad (20)$$

The quantity we want is then

$$\begin{aligned} A = F(1) &= M(M-1) \dots [M-(j-i)+1] \int_s^1 \dots \int_s^1 (is + u_{i+1}^{b_{i+1}} + \dots \\ & + u_j^{b_j})^{M-j+i} du_{i+1} \dots du_j. \end{aligned} \quad (21)$$

The integral can now be evaluated by repeated use of

$$\int_s^1 dx (a+x)^n = \frac{(a+1)^{n+1} - (a+s)^{n+1}}{n+1} \quad (22)$$

The result, which can be established by induction, is

$$A = \sum_{d=0}^{j-i} (-1)^{j-i-d} \binom{j-i}{d} [d + (j-d)s]^M \quad (23)$$

The transition matrix is therefore:

$$T_{ij} = \begin{cases} 0 & \text{if } j-i < 0 \text{ or } j-i > M \\ j^{-M} \binom{j}{i} \sum_{d=0}^{j-i} (-1)^{j-i-d} \binom{j-i}{d} [d + (j-d)s]^M & \text{otherwise.} \end{cases} \quad (24)$$

This form for T_{ij} is far more tractable than equation (9). Indeed, equation (10) quickly becomes unwieldy as M increases, but that is not the case for equation (23), which is a simple sum of not more than $n+1$ terms. In other words, the number of missiles M is no longer a limit on the "tractability" of the problem. The number of targets n is still a limitation, however, but a mild one: the number of elements in the transition matrix grows as n^2 .

T_{00} is understood to be 1. Note that along the diagonal ($i=j \neq 0$) $T_{ij} = s^M$. Note also that $\sum_{i=0}^n T_{ij} = 1$ as expected. Detailed calculations for $n=10$ have verified that equations (9) and (24) give exactly the same T_{ij} .

Case II: Random Targeting, with Missiles Spread Among a Specific Number of Targets

Let us now assume that the M missiles allocate themselves randomly among only the closest b targets. Let T'_{ij} be the generic transition matrix element

(probability that i targets survive when j are present). If $b \geq j$, this restriction makes no difference and there is no change in the transition matrix T_{ij} of equation (24). But if $b < j$, T'_{ij} is the same as $T_{i-(j-b), j-(j-b)}$. Therefore, the full transition matrix in this case is:

$$T'_{ij} = \begin{cases} 0 & \text{if } j-i < 0 \text{ or } j-i > M \\ T_{ij} & \text{if } j \geq b \\ T_{i+b-j, b} & \text{otherwise,} \end{cases} \quad (25)$$

with T_{ij} in equation (24).

Case III: Perfect Targeting -- One Missile Per Target

In this case the distribution of survivors follows a simple binomial distribution, and the result is:

$$T_{ij} = \begin{cases} \binom{j}{i} p^{j-i} (1-p)^i & \text{for } i \leq j \leq M \\ \binom{M}{j-i} p^{j-i} (1-p)^{M-j+i} & \text{for } M < j \leq M+i \text{ and } j \geq i \\ 0 & \text{for } M+i < j \text{ or } j < i. \end{cases} \quad (26)$$

For example, if $n=10$ and $M=5$, the 11×11 transition matrix for arbitrary p is (notation: a, bc means $ap^b(1-p)^c$):

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p & 1-p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p^2 & 2,11 & (1-p)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p^3 & 3,21 & 3,12 & (1-p)^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p^4 & 4,31 & 6,22 & 4,13 & (1-p)^4 & 0 & 0 & 0 & 0 & 0 & 0 \\ p^5 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^5 & 0 & 0 & 0 & 0 & 0 \\ 0 & p^5 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^5 & 0 & 0 & 0 & 0 \\ 0 & 0 & p^5 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^5 & 0 & 0 & 0 \\ 0 & 0 & 0 & p^5 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & p^5 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^5 & 0 \\ 0 & 0 & 0 & 0 & 0 & p^5 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^5 \end{pmatrix}. \quad (27)$$

APL programs for the various calculations presented here are available.

ACKNOWLEDGEMENTS

I am grateful to A. Kaufman for a fruitful discussion and to P. Klebe for assistance with programming.

APPENDIX A

APL PROGRAMS FOR ATTRITION CALCULATIONS

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APL PROGRAMS FOR ATTRITION CALCULATIONS

This appendix contains APL programs, written by Paul E. Klebe and the author, which calculate the attrition based on the method described in the text.

The first listing is given in table A-1. Examples for each of the three targeting modes are given in tables A-2 to A-4. In the case of random allocation among only the closest b targets, only the case $b=M$ has been coded. Further, the probability of kill P and the number of missiles per attack M are the same for each attack. There are 6 inputs to the program:

- N = number of targets
- K = number of attacks
- M = number of missiles per attack
- P = probability a missile kills its target
- TGT $\left\{ \begin{array}{l} 0 \text{ for random allocation among the closest } M \text{ targets} \\ 1 \text{ for random allocation among all targets present} \\ 2 \text{ for perfect targeting} \end{array} \right.$
- TEST $\left\{ \begin{array}{l} 1 \text{ if } N, M, P \text{ and TGT have not changed from the previous run (so that} \\ \text{transition matrix does not have to be recalculated)} \\ 0 \text{ otherwise.} \end{array} \right.$

The program is initiated by keying in
(TGT, TEST) RUNNOW (N,K,M,P)

with the appropriate values substituted for the variables. If it is desired to make it interactive, keying in

GETDATA

will cause the program to ask for the required inputs by name. This is shown in table A-5.

The output returned by the program is as follows:

- A list of inputs
- The transpose of the transition matrix T (not T itself in order to save space)
- The number of targets surviving
- The standard deviation of targets killed
- The expected number of missiles fired
- The initial state vector
- The final state vector
- The sum of final state vector elements (as a check: it should be 1).

This program, written by P. Klebe, is an original improvement over an earlier version by the author in that, in order to save space, the transition matrix T is handled as a vector whose elements are only the nonzero elements of T. This enables the program to run for large values of N, for which the size of the T matrix is prohibitive. An example of this is provided in table A-6, where the case $n=100$ is treated. In cases when N is larger than 10, only the nonzero elements of T are printed in the output, as shown. For very large values of N, the number limit on the factorials may be reached and the program will not run. Roundoff errors may also be introduced for large N, due to the very small values of the elements of the final state vector. (This is particularly evident in standard deviation calculations¹).

Running time is of the order of seconds, at worst minutes, even for large N. Note that under perfect targeting and random targeting among closest M, no more than MK targets will ever get targeted. Thus, if $MK < N$, the input N can be chosen equal to MK and $N-MK$ added to the number of survivors given by the program. This reduces the run-time.

¹Example: $N=50$, $K=5$, $M=6$, $P=.1$, $TGT=1$. The program correctly calculates 47.3 survivors, but the final state vector elements sum to 1.01, which causes σ^2 in (5) to be -10.9. This is why a statement was introduced defining σ^2 to be the larger of the calculated value and 0.

The functions used in this program are:

- The overall control function RUNNOW
- The interactive function GETDATA
- TMR1, which calculates the transition matrix for the case TGT = 0
- TMR2, which calculates the transition matrix for the case TGT = 1
- TMR3, which calculates the transition matrix for the case TGT = 2
- SUM, which calculates the sum in equation (24) of the main text
- NOMISP, which calculates the successive state vectors and number of missiles fired under perfect targeting assumptions
- NOMISR, which calculates the successive state vectors and number of missiles fired under random targeting assumptions
- BIGDATA, which handles large transitions matrices (N larger than 10)
- OUTPUT, which organizes output presentation.

In certain cases, however, it is useful to be able to handle the actual transition matrix T, as opposed to a vector substitute for it. The following program, written by the author, performs the same functions as the previous one, but handles the T matrix as a matrix. Therefore, it cannot handle values of N that are larger than about 70 for an 85,000-byte workspace.

There are 4 inputs to the program:

- N = number of targets
- K = number of attacks
- M = number of missiles per attack
- P = probability a missile kills its target

The program is initiated by keying in:

PERTAR for perfect targeting,

RANTAR for random targeting among all targets present, and

RANTARM for random targeting among the closest M targets.

The output returned by the program is the same as before. A listing is given in Table A-7 and examples for the 3 targeting modes are given in table A-8. The functions used are:

- The 3 overall control functions PERTAR, RANTAR, RANTARM
- The functions calculating the state vectors:
NOMISP for perfect targeting and NOMISR for random targeting
- The 3 functions TM1, TM2, and TM3, calculating the transition matrices for the 3 targeting modes: random among closest M, random among all, and perfect, respectively.

TABLE A-1
LISTING OF FIRST APL PROGRAM

```

)LOAD KLEBE
SAVED 80/10/29 17.00.13
)FNS
BIGDATA GETDATA NOMISP NOMISR OUTPUT RUNNOW SUM THR1 THR2 THR3

    ▽BIGDATA[0]▽
    ▽ BIGDATA;KC;CT;VT
[1]  SPACE
[2]  'VALUES OF N,M,SIZE OF T'
[3]  N,M,pT
[4]  SPACE
[5]  →(TEST=1)p0
[6]  CA+KC+0
[7]  'THE TRANSPOSE OF THE TRANSITION MATRIX T IS: '
[8]  BERLIN:VT+1+MLN-CA
[9]  'I6;F7.4'p(CA;T(KC+1VT))
[10]  KC+KC+VT
[11]  →(N2CA+CA+1)pBERLIN
    ▽
    ▽GETDATA[0]▽
    ▽ GETDATA
[1]  SPACE+ ' '
[2]  DIO+0
[3]  'LIST THE NUMBER OF TARGETS, NUMBER OF SUCCESSIVE ATTACKS,'
[4]  '      THE NUMBER OF MISSILES IN EACH ATTACK, THE MISSILE PROBABILITY OF KILL'
[5]  PEST+0
[6]  SPACE
[7]  'TYPE 1 FOR PERFECT TARGETING, 2 FOR RANDOM TARGETING'
[8]  TGT+3-0
[9]  →(TGT=2)pBOSTON
[10]  SPACE
[11]  'TYPE 1 IF MISSILES SPREAD AMONG ALL AVAILABLE TARGETS'
[12]  '      2 IF MISSILES SPREAD AMONG A SUBGROUP OF M TARGETS'
[13]  TGT+2-0
[14]  BOSTON:SPACE
[15]  SPACE
[16]  'TYPE 1 IF THE T MATRIX IS SAME AS IN PREVIOUS RUN -- OTHERWISE TYPE 0'
[17]  TEST+0
[18]  (TGT,TEST)RUNNOW PEST
    ▽

    ▽NOMISP[0]▽
    ▽ NOMISP;AT;VT;JJ
[1]  EM+0
[2]  AT+1
[3]  ST+50
[4]  MN+MLN
[5]  GG:RT+1-1/ST(MN)
[6]  EM+EM+(MN×RT)+1/ST(MN)×1MN
[7]  JJ+KC+0
[8]  ROME:VT+1+MLN-JJ
[9]  ST(JJ)+1/ST(JJ+1VT)×T(KC+1VT)
[10]  KC+KC+VT
[11]  →(N2JJ+JJ+1)pROME
[12]  →(K2AT+AT+1)pGG
    ▽

```

▼NOMISRC[]▼

▼ NOMISR:AT:VT:JJ

```
[1] EM+0
[2] AT+1
[3] ST+SO
[4] GG:EM+EM+Mx1-ST[0]
[5] JJ+KC+0
[6] ROME:VT+1+MLN-JJ
[7] ST[JJ]++/ST[JJ+1VT]*TE[KC+1VT]
[8] KC+KC+VT
[9] →(N≥JJ+JJ+1)*ROME
[10] →(K≥AT+AT+1)*GG
```

▼

▼OUTPUT[]▼

▼ OUTPUT

```
[1] NAME+3 32p 'RANDOM TARGETING AMONG M CLOSESTRANDOM TARGETING AMONG ALL
[2] SPACE
[3] NAME[GT:]
[4] SPACE
[5] 'THE INITIAL NUMBER OF TARGETS IS 'IN;
[6] 'THE NUMBER OF SUCCESSIVE ATTACKS IS 'IK;
[7] 'THE NUMBER OF MISSILES IN EACH ATTACK IS 'IM;
[8] 'EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 'IP;
[9] SPACE
[10] →((NFM)>10)*PARIS
[11] KC+CA+0
[12] 'THE TRANSPOSE OF THE TRANSITION MATRIX T IS: '
[13] SPACE
[14] MADRID:VT+1+MLN-CA
[15] KT+N+1-CA+VT
[16] 9 5*(CA+0)*TE[KC+1VT]*(OKT)*0
[17] KC+KC+VT
[18] →(N≥CA+CA+1)*MADRID
[19] PARIS:SPACE
[20] 'THE NUMBER OF TARGETS SURVIVING IS 'IE;
[21] 'THE STANDARD DEVIATION OF TARGETS KILLED IS 'ISD;
[22] 'THE EXPECTED NUMBER OF MISSILES FIRED IS 'EM;
[23] SPACE
[24] 'THE INITIAL STATE IS '
[25] 10 7+SO
[26] SPACE
[27] 'THE FINAL STATE IS '
[28] 10 7+ST
[29] SPACE
[30] 'A1:F6.2'*( 'SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: '1+/ST)
[31] →((NFM)<11)*0
[32] BIGDATA
```

▼

PERFECT TARGETING

```

      ▽RUNNOW[0]▽
    ▽ BAKER RUNNOW ABLE;ST2
[1]  TGT←BAKER[0]
[2]  TEST←BAKER[1]
[3]  MAT1←3 4P'TMR1TMR2TMR3'
[4]  N←ABLE[0]
[5]  K←ABLE[1]
[6]  M←ABLE[2]
[7]  P←ABLE[3]
[8]  MAT2←3 6P'NOMISRNDMISRNDMISF'
[9]  SD←(N≠0),1
[10] →(TEST=1)P LONDON
[11] *MAT1[TGT;]
[12] LONDON:SPACE
[13] *MAT2[TGT;]
[14] I←1N+1
[15] E←+/I×ST
[16] ST2←(+/(I×2)×ST)-(+/I×ST)*2
[17] ST2←ST2Γ0
[18] SD←ST2×0.5
[19] OUTPUT
    ▽
      ▽SUM[0]▽
    ▽ SIG←I SUM J;VAR;C1;D1;D
[1]  D←11+C1+J-I
[2]  D1←D÷J
[3]  VAR←-1+2×2|C1+1
[4]  SIG←(I!J)×-/VAR×(D!C1)×(D1+S×1-D1)*M
    ▽
      ▽TMR1[0]▽
    ▽ TMR1;S;I;J;V
[1]  T←(L0.3+N+1+(M×0ΓN-M)+((1+S)×S+MLN)÷2)P0
[2]  V←I←0
[3]  SK←(S+1-P)*M
[4]  LOOP:J←I+1
[5]  T[V]←SK
[6]  V←V+1
[7]  →(I=N)P STEP
[8]  LOP:→(J>I+M)P STEP
[9]  T[V]←(I-0ΓJ-M)SUM JLM
[10] V←V+1
[11] →(N≥J+J+1)P LOP
[12] STEP:→(N≥I+I+1)P LOOP
[13] T[0]←1
    ▽

```

A-1 (con't)

```

      ▼TMR2[0]▼
      ▼ TMR2[S;I;J;V
[1]  T←(L0.3+N+1+(M×OΓN-M)+(1+S)×S+MLN)÷2)P0
[2]  V←I+0
[3]  SK←(S+1-P)×M
[4]  LOOP:J←I+1
[5]  TCVJ←SK
[6]  V←V+1
[7]  →(I=N)PSTEP
[8]  LOP:→(J>I+M)PSTEP
[9]  TCVJ←I SUM J
[10] V←V+1
[11] →(N≥J←J+1)PLOP
[12] STEP:→(N≥I←I+1)PLOOP
[13] T[0]←1

```

```

      ▼
      ▼TMR3[0]▼
      ▼ TMR3[S;I;J;V
[1]  T←(L0.3+N+1+(M×OΓN-M)+(1+S)×S+MLN)÷2)P0
[2]  V←I+0
[3]  SK←(S+1-P)×M
[4]  LOOP:J←I
[5]  LOP:→(J>I+M)PSTEP
[6]  TCVJ←(P×J-I)×((J-I)JLM)×S×I-OΓJ-M
[7]  V←V+1
[8]  →(N≥J←J+1)PLOP
[9]  STEP:→(N≥I←I+1)PLOOP
[10] T[0]←1

```

TABLE A-2

EXAMPLE OF FIRST PROGRAM FOR RANDOM
TARGETING AMONG M CLOSEST TARGETS

(0,0) RUNNOW (10,4,5,5) ← key in

RANDOM TARGETING AMONG M CLOSEST

THE INITIAL NUMBER OF TARGETS IS 10
THE NUMBER OF SUCCESSIVE ATTACKS IS 4
THE NUMBER OF MISSILES IN EACH ATTACK IS 5
EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 0.5

THE TRANSPOSE OF THE TRANSITION MATRIX T IS:

1.00000	0.96875	0.55664	0.15818	0.02197	0.00120	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.03125	0.41211	0.50926	0.22339	0.04200	0.00120	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.03125	0.30131	0.46692	0.25500	0.04200	0.00120	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.03125	0.25647	0.43800	0.25500	0.04200	0.00120	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500	0.04200	0.00120	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500	0.04200	0.00120
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500	0.04200
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125

THE NUMBER OF TARGETS SURVIVING IS 2.04
THE STANDARD DEVIATION OF TARGETS KILLED IS 1.49
THE EXPECTED NUMBER OF MISSILES FIRED IS 20

THE INITIAL STATE IS

0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 1.0000000

THE FINAL STATE IS

0.1654055 0.2340347 0.2455181 0.1863439 0.1065119 0.0453583 0.0136313 0.0027968 0.0003703 0.0000284 0.0000010

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1.00

TABLE A-3

EXAMPLE OF FIRST PROGRAM FOR RANDOM
TARGETING AMONG ALL TARGETS PRESENT

(1,0) RUNNOW (10,4,5,.5) ← key in

RANDOM TARGETING AMONG ALL

THE INITIAL NUMBER OF TARGETS IS 10
THE NUMBER OF SUCCESSIVE ATTACKS IS 4
THE NUMBER OF MISSILES IN EACH ATTACK IS 5
EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 0.5

THE TRANSPOSE OF THE TRANSITION MATRIX T IS:

1.00000	0.94875	0.55464	0.15818	0.02197	0.00120	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.03125	0.41211	0.50926	0.22339	0.04200	0.00289	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.03125	0.30131	0.46692	0.25500	0.05787	0.00449	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.03125	0.25647	0.43800	0.27247	0.07028	0.00641	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.41775	0.28309	0.08011	0.00800	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.21776	0.40296	0.28999	0.08802	0.00945
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.20774	0.39173	0.29473	0.09450
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.20051	0.38294	0.29813
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.19505	0.37589
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.19078
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125

THE NUMBER OF TARGETS SURVIVING IS 1.72
THE STANDARD DEVIATION OF TARGETS KILLED IS 1.44
THE EXPECTED NUMBER OF MISSILES FIRED IS 19.9

THE INITIAL STATE IS

0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 1.0000000

THE FINAL STATE IS

0.2350293 0.2578490 0.2312548 0.1553414 0.0790975 0.0305080 0.0087932 0.0018388 0.0002639 0.0000233 0.0000010

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1.00

TABLE A-4

EXAMPLE OF FIRST PROGRAM FOR PERFECT TARGETING

(2,0) RUNHOW (10,4,5,5)

PERFECT TARGETING

RA

THE INITIAL NUMBER OF TARGETS IS 10

THE NUMBER OF SUCCESSIVE ATTACKS IS 4

THE NUMBER OF MISSILES IN EACH ATTACK IS 5

EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 0.5

THE TRANSPOSE OF THE TRANSITION MATRIX T IS:

1.00000	0.50000	0.25000	0.12500	0.06250	0.03125	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.50000	0.50000	0.37500	0.25000	0.15625	0.03125	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.25000	0.37500	0.37500	0.31250	0.15625	0.03125	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.12500	0.25000	0.31250	0.31250	0.15625	0.03125	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.06250	0.15625	0.31250	0.31250	0.15625	0.03125	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250	0.31250	0.15625	0.03125
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250	0.31250	0.15625
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250	0.31250
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125

THE NUMBER OF TARGETS SURVIVING IS 1.44

THE STANDARD DEVIATION OF TARGETS KILLED IS 1.23

THE EXPECTED NUMBER OF MISSILES FIRED IS 17.1

THE INITIAL STATE IS

0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 1.0000000

THE FINAL STATE IS

0.2377138 0.3460693 0.2413464 0.1120377 0.0421381 0.0147858 0.0046206 0.0010872 0.0001812 0.0000191 0.0000010

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1.00

TABLE A-5

EXAMPLE OF INTERACTIVE VARIANT OF FIRST APL PROGRAM^a

► GETDATA
LIST THE NUMBER OF TARGETS, NUMBER OF SUCCESSIVE ATTACKS,
THE NUMBER OF MISSILES IN EACH ATTACK, THE MISSILE PROBABILITY OF KILL
Q:
► 10,4,5,.5

TYPE 1 FOR PERFECT TARGETING, 2 FOR RANDOM TARGETING
Q:
► 2

TYPE 1 IF MISSILES SPREAD AMONG ALL AVAILABLE TARGETS
2 IF MISSILES SPREAD AMONG A SUBGROUP OF M TARGETS
Q:
► 2

TYPE 1 IF THE T MATRIX IS SAME AS IN PREVIOUS RUN -- OTHERWISE TYPE 0
Q:
► 0

RANDOM TARGETING AMONG M CLOSEST, etc. (Output is as in Table A-2).

^a ► means "key in".

EXAMPLE OF FIRST PROGRAM FOR LARGE NUMBER
OF TARGETS ($N > 10$)^a

Key in

THE INITIAL NUMBER OF TARGETS IS 100
THE NUMBER OF SUCCESSIVE ATTACKS IS 20
THE NUMBER OF MISSILES IN EACH ATTACK IS 5
EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 0.1

[illegible][illegible]

^aNote: only the nonzero elements of the transition matrix are printed. To explain their location, it is best to give an example. Below is how the T matrix in table A-2 would be printed in this format (obtained by keying in BIGDATA after program execution).

VALUES OF N,M,SIZE OF T

0	1.0000	0.9488	0.5546	0.1582	0.0220	0.0012
1	0.0313	0.4121	0.5093	0.2234	0.0420	0.0012
2	0.0313	0.3013	0.4669	0.2550	0.0420	0.0012
3	0.0313	0.2565	0.4380	0.2550	0.0420	0.0012
4	0.0313	0.2326	0.4380	0.2550	0.0420	0.0012
5	0.0313	0.2326	0.4380	0.2550	0.0420	0.0012
6	0.0313	0.2326	0.4380	0.2550	0.0420	
7	0.0313	0.2326	0.4380	0.2550		
8	0.0313	0.2326	0.4380			
9	0.0313	0.2326				
10	0.0313					

[illegible]

... ..

TABLE A-7

LISTING OF SECOND APL PROGRAM

```

      VNMISPE[0]V
      V NMISP
[1]  AT←EM←0
[2]  ST←S0
[3]  GG:AT←AT+1
[4]  EM←EM+(M×+/(-O(N+1-M)†ST)†+/(O(M)-1)×M†ST
[5]  ST←ST+.xT
[6]  →(AT<K)pGG
[7]  EM
      V
      VNMISRE[0]V
      V NMISR
[1]  AT←EM←0
[2]  ST←S0
[3]  GG:AT←AT+1
[4]  EM←EM+M×1-ST[0]
[5]  ST←ST+.xT
[6]  →(AT<K)pGG
[7]  EM
      V
      VPERTARE[0]V
      V PERTAR
[1]  'PERFECT TARGETING RESULTS FOR 'N' TARGETS'
[2]  'SUBJECTED TO 'K' SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING'
[3]  'OF UP TO 'M' MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 'P'.'
[4]  '-----'
[5]  TM3
[6]  S0←(N<0),1
[7]  'THE EXPECTED NUMBER OF MISSILES FIRED IS:'
[8]  NMISP
[9]  'THE INITIAL STATE IS:'
[10] S0
[11] 'THE FINAL STATE IS:'
[12] ST
[13] 'THE SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: '†/ST
[14] I←(N+1)-1
[15] E←†/I×ST
[16] 'THE EXPECTED NUMBER OF TARGETS SURVIVING IS: '†E†
[17] SD2←(†/(1*2)×ST)-(†/I×ST)*2
[18] SD2←SD2†0
[19] SD←SD2*0.5
[20] 'THE STANDARD DEVIATION OF TARGETS KILLED IS: '†SD†
      V
      V RANTARE[0]V
      V RANTAR
[1]  'RANDOM TARGETING RESULTS FOR 'N' TARGETS'
[2]  'SUBJECTED TO 'K' SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING'
[3]  'OF 'M' MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 'P'.'
[4]  '-----'
[5]  TM2
[6]  S0←(N<0),1
[7]  'THE EXPECTED NUMBER OF MISSILES FIRED IS:'
[8]  NMISR
[9]  'THE INITIAL STATE IS:'
[10] S0
[11] 'THE FINAL STATE IS:'
[12] ST
[13] 'SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: '†/ST
[14] I←N+1
[15] E←†/I×ST
[16] 'THE EXPECTED NUMBER OF TARGETS SURVIVING IS: '†E†
[17] SD2←(†/(1*2)×ST)-(†/I×ST)*2
[18] SD2←SD2†0
[19] SD←SD2*0.5
[20] 'THE STANDARD DEVIATION OF TARGETS KILLED IS: '†SD†
      V

```

TABLE A-7 (con't)

```

▽RANTARM[0]▽
▽ RANTARM
[1] 'RANDOM TARGETING RESULTS FOR 'IN' TARGETS '
[2] '   SUBJECTED TO 'IK' SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING'
[3] '   OF 'IM' MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 'IP'.'
[4] '   (MISSILES SPREAD AMONG CLOSEST 'IM' TARGETS)'
[5] '-----'
[6] TM1
[7] SD←(N+0),1
[8] 'THE EXPECTED NUMBER OF MISSILES FIRED IS:'
[9] NOMISR
[10] 'THE INITIAL STATE IS:'
[11] SD
[12] 'THE FINAL STATE IS:'
[13] ST
[14] 'SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 'I+/ST
[15] I←IN+1
[16] E←+/IXST
[17] 'THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 'IE'
[18] SD2←(+/(I*2)*ST)-(+/IXST)*2
[19] SD2←SD2+0
[20] SN←SD2*0.5
[21] 'THE STANDARD DEVIATION OF TARGETS KILLED IS: 'ISD'

▽
▽SUM[0]▽
▽ SIG←SUMID
[1] D←-1
[2] SIG←0
[3] C1←J-I
[4] EE←D+D+1
[5] D1←D+J
[6] IG←(-1*(C1-D)*(D1C1))*(D1+(1-D1)*S)*M
[7] SIG←SIG+IG
[8] +(D<C1)*EE
[9] SIG←(I!J)*SIG

▽
▽TM1[0]▽
▽ TM1
[1] D10←0
[2] S←1-P
[3] T←((N+1),N+1)+0
[4] I←-1
[5] LOOP:I←I+1
[6] J←-1
[7] LOP:J←J+1
[8] +(((J-I)<0)^(J-I)>M)*BB
[9] +(I=J)*CC
[10] +(M<J)*YY
[11] T(I,J)←SIG+SUM
[12] BB←+(J<N)*LOP
[13] +(I<N)*LOOP
[14] T(I,0)←1
[15] 'THE TRANSITION MATRIX T IS:'
[16] ''
[17] T
[18] →0
[19] CC:T(I,J)*S*M
[20] →BB
[21] YY:I←I
[22] JJ←J
[23] I←I+M-J
[24] J←M
[25] T(I,J)←SIG+SUM
[26] I←II
[27] J←JJ
[28] →BB

```

TABLE A-7 (con't)

```

      ▼TM2[III]▼
      ▼ TM2
[11]  SIG←0
[2]   ←←←←←
[3]   T←((N+1)*N+1)*P0
[4]   I←1
[5]   LOOP:I←I+1
[6]   J←1
[7]   LOP:J←J+1
[8]   →(((J-I)≤0)∨(J-I)≥M)*BB
[9]   →(I=J)*CC
[10]  TEJ:II←SIG+SUM
[11]  BB:→(J≤N)*LOP
[12]  →(I≤N)*LOOP
[13]  TE0:OJ←1
[14]  'THE TRANSITION MATRIX T IS:'
[15]  ''
[16]  T
[17]  →0
[18]  CC:TEI:JJ←S*M
[19]  →BB
      ▼
      ▼TM3[III]▼
      ▼ TM3
[1]   OI0←1
[2]   T←((N+1)*N+1)*P0
[3]   J←0
[4]   LOOP:I←I+1
[5]   J←0
[6]   LOP:J←J+1
[7]   A←((I-1):J-1)*(P*J-I)*(1-P)*I-1
[8]   B←((J-I):M)*(P*(I-I))*(1-P)*M-I-I
[9]   T(I:J):←(A*(J<I)^(J-1):N)+B*(M-I-I)^(J-1)*M+1
[10]  →LOP*(J≤N+1)
[11]  →LOOP*(I≤N+1)
[12]  'THE TRANSITION MATRIX T IS:'
[13]  T
      ▼

```

TABLE A-8

EXAMPLE OF SECOND APL PROGRAM

$N=10$
 $K=4$
 $M=5$
 $P=.5$

← key in

PERTAR

PERFECT TARGETING RESULTS FOR 10 TARGETS
 SUBJECTED TO 4 SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING
 OF UP TO 5 MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 0.5.

THE TRANSITION MATRIX T IS:										
1	0	0	0	0	0	0	0	0	0	0
0.5	0.5	0	0	0	0	0	0	0	0	0
0.25	0.5	0.25	0	0	0	0	0	0	0	0
0.125	0.375	0.375	0.125	0	0	0	0	0	0	0
0.0625	0.25	0.375	0.25	0.0625	0	0	0	0	0	0
0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0	0
0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0
0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0
0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0
0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0
0	0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125
0	0	0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625
0	0	0	0	0	0	0	0.03125	0.15625	0.3125	0.3125

THE EXPECTED NUMBER OF MISSILES FIRED IS:
 17.11135864

THE INITIAL STATE IS:

0 0 0 0 0 0 0 0 0 0 1

THE FINAL STATE IS:

0.237713813R 0.3460693359 0.2413463593 0.1120376587 0.04213809967 0.0147857666 0.004620552063 0.001087188721
 0.0001811981201 0.0000190734863 9.536743164E-7

THE SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS! 1

THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 1.444320679

THE STANDARD DEVIATION OF TARGETS KILLED IS: 1.228997298

TABLE A-8
EXAMPLE OF SECOND APL PROGRAM

N=10
K=4
M=5
P=.5

← key In

PERTAR

PERFECT TARGETING RESULTS FOR 10 TARGETS
SUBJECTED TO 4 SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING
OF UP TO 5 MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 0.5.

THE TRANSITION MATRIX T IS:										
1	0	0	0	0	0	0	0	0	0	0
0.5	0.5	0	0	0	0	0	0	0	0	0
0.25	0.5	0.25	0	0	0	0	0	0	0	0
0.125	0.375	0.375	0.125	0	0	0	0	0	0	0
0.0625	0.25	0.375	0.25	0.0625	0	0	0	0	0	0
0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0	0
0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0
0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0
0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0
0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0
0	0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125
0	0	0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625
0	0	0	0	0	0	0	0.03125	0.15625	0.3125	0.3125

THE EXPECTED NUMBER OF MISSILES FIRED IS:

17.11135864

THE INITIAL STATE IS:

0 0 0 0 0 0 0 0 0 0 0 1

THE FINAL STATE IS:

0.0377138138 0.3460693359 0.2413463593 0.1120376587 0.04213809967 0.0147857666 0.004620552063 0.001087188721

0.0001811981201 0.0000190734863 9.536743164E-7

THE SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1

THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 1.444320679

THE STANDARD DEVIATION OF TARGETS KILLED IS: 1.228997298

TABLE A-8 (con't)

RANTAR ← key in

RANDOM TARGETING RESULTS FOR 10 TARGETS

SUBJECTED TO 4 SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING
OF 5 MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 0.5.-----
THE TRANSITION MATRIX T IS:

1	0	0	0	0	0	0	0	0	0	0
0.969	0.0313	0	0	0	0	0	0	0	0	0
0.557	0.412	0.0313	0	0	0	0	0	0	0	0
0.158	0.509	0.301	0.0313	0	0	0	0	0	0	0
0.022	0.223	0.467	0.256	0.0313	0	0	0	0	0	0
0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0	0	0
0	0.00289	0.0579	0.272	0.418	0.218	0.0313	0	0	0	0
0	0	0.00469	0.0703	0.283	0.403	0.208	0.0313	0	0	0
0	0	0	0.00641	0.0801	0.29	0.392	0.201	0.0313	0	0
0	0	0	0	0.008	0.088	0.295	0.383	0.195	0.0313	0
0	0	0	0	0	0.00945	0.0945	0.298	0.376	0.191	0.0313

THE EXPECTED NUMBER OF MISSILES FIRED IS:

19.9

THE INITIAL STATE IS:

0 0 0 0 0 0 0 0 0 0 1

THE FINAL STATE IS:

0.235 0.258 0.231 0.155 0.0791 0.0305 0.00879 0.00184 0.000264 0.0000233 9.54E-7

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1

THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 1.72

THE STANDARD DEVIATION OF TARGETS KILLED IS: 1.44

RANTARM ← key in

RANDOM TARGETING RESULTS FOR 10 TARGETS

SUBJECTED TO 4 SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING
OF 5 MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 0.5.
(MISSILES SPREAD AMONG CLOSEST 5 TARGETS)-----
THE TRANSITION MATRIX T IS:

1	0	0	0	0	0	0	0	0	0	0
0.969	0.0313	0	0	0	0	0	0	0	0	0
0.557	0.412	0.0313	0	0	0	0	0	0	0	0
0.158	0.509	0.301	0.0313	0	0	0	0	0	0	0
0.022	0.223	0.467	0.256	0.0313	0	0	0	0	0	0
0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0	0	0
0	0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0	0
0	0	0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0
0	0	0	0.0012	0.042	0.255	0.438	0.233	0.0313	0	0
0	0	0	0	0.0012	0.042	0.255	0.438	0.233	0.0313	0
0	0	0	0	0	0.0012	0.042	0.255	0.438	0.233	0.0313
0	0	0	0	0	0	0.0012	0.042	0.255	0.438	0.233

THE EXPECTED NUMBER OF MISSILES FIRED IS:

20

THE INITIAL STATE IS:

0 0 0 0 0 0 0 0 0 0 1

THE FINAL STATE IS:

0.165 0.234 0.246 0.186 0.107 0.0454 0.0136 0.0028 0.00037 0.0000284 9.54E-7

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1

THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 2.04

THE STANDARD DEVIATION OF TARGETS KILLED IS: 1.49

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