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A THEOREM ON TREND-FREE BLOCK DESIGNS[1]

by

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A THEOREM ON TREND-FREE BLOCK DESIGNS

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SUMMARY

An earlier, referenced report considered the existence of trend-free block designs and gave some general results for general block designs and some special results for complete block designs. This report presents a new existence theorem that is sufficiently general to cover broad classes of incomplete block designs and suggests a design construction method.

Suppose that a common trend affecting treatment responses exists over the plots within blocks of a block design with the usual design parameters, v, b, k, r. A trend-free block design, if it exists, is one for which treatments are assigned to plots within blocks so that treatment and block contrasts are orthogonal to specified trend components. We prove that, if a given block design is constructed and the common trend is linear, the necessary and sufficient condition for the existence of a trend-free block design with the same incidence matrix is that $\frac{1}{2}r(k + 1)$ be an integer.

Bradley and Yeh (1980) introduced the concept of trend-free block (TFB) designs and developed a necessary and sufficient condition for their existence. Yeh and Bradley (1979) established alternative existence conditions, in some cases for special trends and special block designs, and examined some design construction procedures. While some important results were obtained for complete block designs, extensions to incomplete block designs were limited. The purpose of this report is to present a new theorem that is very generally applicable to incomplete block designs.

In review, consider an experimental situation with v treatments applied to plots arranged in b blocks of size $k \leq v$. Each plot receives only one treatment and each treatment occurs at most once in a block. Plots in each block are arranged in the same m-dimensional array, indexed by vectors of positive integers, $\underset{\sim}{t} = (t_1, \ldots, t_m)$, $t_u = 1, \ldots, s_u$, $u = 1, \ldots, m$, where s_u is the number of plot positions in the u^{th} dimension, $\prod_u s_u = k$. A polynomial trend, common to all blocks, is assumed to exist over the plots in a block and to be a function of the plot position $\underset{\sim}{t}$.

The classical model for general block designs is extended through the addition of trend terms. The trend function is expressed as a linear combination of m-dimensional, orthogonal polynomials of the form,

$$(1) \qquad \phi_{\underset{\sim}{\alpha}}(\underset{\sim}{t}) \equiv \phi_{(\alpha_1, \ldots, \alpha_m)}(t_1, \ldots, t_m) = \prod_{u=1}^{m} \phi_{\alpha_u}(t_u),$$

where $\phi_{\alpha_u}(t_u)$ is a one-dimensional, orthogonal polynomial of degree α_u on the integers, $1, \ldots, s_u$. The model is

$$(2) \qquad y_{j\underset{\sim}{t}} = \mu + \sum_{i=1}^{v} \delta_{j\underset{\sim}{t}}^{i} \tau_i + \beta_j + \sum_{\underset{\sim}{\alpha} \in \Lambda} \theta_{\underset{\sim}{\alpha}} \phi_{\underset{\sim}{\alpha}}(\underset{\sim}{t}) + \epsilon_{j\underset{\sim}{t}},$$

$j = 1, \ldots, b$, $\underset{\sim}{t} = (t_1, \ldots, t_m)$, $t_u = 1, \ldots, s_u$, $u = 1, \ldots, m$, where $y_{j\underset{\sim}{t}}$ is the observation on plot position $\underset{\sim}{t}$ of block j, μ, τ_i and β_j are respectively the usual mean, treatment and block parameters, $\sum_{\underset{\sim}{\alpha} \in A} \theta_{\underset{\sim}{\alpha}} \phi_{\underset{\sim}{\alpha}}(\underset{\sim}{t})$ is the trend effect on plot $\underset{\sim}{t}$, not dependent on the particular block j, with $\theta_{\underset{\sim}{\alpha}}$ being the regression coefficient of $\phi_{\underset{\sim}{\alpha}}(\underset{\sim}{t})$ and A, an index set of p, m-dimensional, non-zero vectors of the form $\underset{\sim}{\alpha}$, $p < k$, and the $\epsilon_{j\underset{\sim}{t}}$ are random errors assumed to be i.i.d. with zero means. Designation of the treatment applied to plot $(j, \underset{\sim}{t})$ is effected through indicator variables, $\delta^i_{j\underset{\sim}{t}} = 1$ or 0 as treatment i is or is not on plot $(j, \underset{\sim}{t})$, $i = 1, \ldots, v$. A TFB design exists when it is possible to choose the $\delta^i_{j\underset{\sim}{t}}$ so that the estimators of treatment and block contrasts are unaffected by the presence or absence of the trend terms in (2). For a TFB design, the appropriate treatment and block sums of squares in analysis of variance have the same algebraic forms in the presence or absence of the trend terms.

Let $\underset{\sim}{\Delta}_j$, $j = 1, \ldots, b$, be the $k \times v$ matrix with $\delta^i_{j\underset{\sim}{t}}$ in row $\underset{\sim}{t}$ and column i and let $\underset{\sim}{\phi}$ be the $k \times p$ matrix with $\phi_{\underset{\sim}{\alpha}}(\underset{\sim}{t})$ in row $\underset{\sim}{t}$ and column $\underset{\sim}{\alpha}$. Further, let $\underset{\sim}{X}'_\tau = (\underset{\sim}{\Delta}'_1, \ldots, \underset{\sim}{\Delta}'_b)$ and $\underset{\sim}{X}_\theta = \underset{\sim}{1}_b \otimes \underset{\sim}{\phi}$, where $\underset{\sim}{1}_b$ is the b-dimensional column vector with unit elements and $\underset{\sim}{B} \otimes \underset{\sim}{C}$ is the Kronecker product of $\underset{\sim}{B}$ and $\underset{\sim}{C}$. Bradley and Yeh (1980, Theorem 3.1) show that a block design is trend-free (to the trend terms specified in the model) if and only if

$$(3) \qquad \underset{\sim}{X}'_\tau \underset{\sim}{X}_\theta = \underset{\sim}{0},$$

or equivalently, if and only if

$$(4) \qquad \underset{\sim}{\Delta}'_+ \underset{\sim}{\phi} = \underset{\sim}{0},$$

where $\underset{\sim}{\Delta}_+ = \sum_{j=1}^{b} \underset{\sim}{\Delta}_j$.

In this report, attention is limited to consideration of TFB designs when $r_1 = \dots = r_v = r$, $k \le v$, $r, k > 1$, $m = 1$. These designs will be designated as $TF_{\underset{\sim}{\phi}}B(v, b, k, r)$ designs, designs with treatments so arranged within blocks that they are trend-free to a common, one-dimensional trend over the k plot positions within the blocks. When $\underset{\sim}{\phi} = \underset{\sim}{\phi}_1$, the common trend is linear; when $\underset{\sim}{\phi} = (\underset{\sim}{\phi}_1, \underset{\sim}{\phi}_2)$, **the** common trend is quadratic.

Theorem 3.3 of Yeh and Bradley (1979) is the key to the new theorem of this report and is restated:

<u>Theorem 1</u>. If $\underset{\sim}{\phi} = (\underset{\sim}{\phi}_1, \dots, \underset{\sim}{\phi}_p)$ under model (2), then a $TF_{\underset{\sim}{\phi}}B(v, b, k, r)$ design exists if and only if there exists a $v \times b$ matrix $\underset{\sim}{W}$ with non-negative elements such that

(i) each column of $\underset{\sim}{W}$ has the integers $1, \dots, k$ as elements along with $(v - k)$ zero elements, and,

(ii) for any $\alpha = 1, \dots, p$, the sum of the α^{th} powers of elements for any single row of $\underset{\sim}{W}$ is $S_\alpha(k, r) = \frac{r}{k} \sum_{i=1}^{k} i^\alpha$.

The theorem to be proved is:

<u>Theorem 2</u>. If $\underset{\sim}{\phi} = \underset{\sim}{\phi}_1$ and a connected block design exists with parameters v, b, k and r, the design may be converted into a $TF_{\underset{\sim}{\phi}_1}B(v, b, k, r)$ design through exchanges of plot positions for treatments within blocks if and only if $\frac{1}{2}r(k + 1)$ is an integer.

Necessity follows at once from Theorem 1. Proof of sufficiency requires a series of lemmas given below. Theorem 2 implies Theorem 4.2 (Yeh and Bradley, 1979) when $k = v$ and application to various classes of incomplete block designs is obvious.

The key to proof of sufficiency for Theorem 2 is the matrix $\underset{\sim}{W}$ of Theorem 1. The typical element w_{ij} of $\underset{\sim}{W}$ is the plot position of treatment i in block j if $w_{ij} \neq 0$ and, if $w_{ij} = 0$, treatment i does not occur in block j. Each column of $\underset{\sim}{W}$ consists of the integers 1, ..., k along with v - k zeros and each row of $\underset{\sim}{W}$ contains r non-zero elements. The first and obvious lemma summarizes the situation.

Lemma 1. If $\underset{\sim}{W}$ represents any block design with parameters v, b, k and r with specified plot positions for treatments within each block, then

(i) (The minimum row sum of $\underset{\sim}{W}$) $\leq \frac{1}{2}r(k + 1)$, and

(ii) equality holds in (i) if and only if $\underset{\sim}{W}$ represents a $TF_{\underset{\sim}{\phi}_1}B(v, b, k, r)$ design.

Lemma 1 provides a basis for the desired proof of sufficiency. If exchanges of non-zero elements are made within columns of $\underset{\sim}{W}$, a new design with the designated parameters results but row sums in $\underset{\sim}{W}$ may change. If such exchanges result in equal row sums of $\frac{1}{2}r(k + 1)$, an integer by assumption in Theorem 2, then a $TF_{\underset{\sim}{\phi}_1}B(v, b, k, r)$ design is produced (Theorem 1). A matrix $\underset{\sim}{W}$ may have several rows with minimum row sum.

Definition 1. A matrix $\underset{\sim}{W}$, and its corresponding block design, *can be improved* if, through exchanges of non-zero elements in columns of $\underset{\sim}{W}$, the minimum row sum of the resultant matrix is not decreased from that of $\underset{\sim}{W}$ and the number of rows with the minimum row sum of $\underset{\sim}{W}$ is reduced.

Remarks 1. If $\frac{1}{2}r(k + 1)$ is an integer and if $\underset{\sim}{W}$, representing a block design of the class under consideration, is such that it can be improved through a sequence of improved matrices $\underset{\sim}{W}_1, \ldots, \underset{\sim}{W}_n$, $\underset{\sim}{W}_n$ having minimum row sum, $\frac{1}{2}r(k + 1)$, then n is finite and $\underset{\sim}{W}_n$ represents and specifies a $TF_{\underset{\sim}{\phi}_1}B(v, b, k, r)$ design.

Some exchange notation is needed. We write

(i) $w_{ij} \to w_{i'j}$ to mean that $w_{ij} + 1 = w_{i'j}$, $i \neq i'$, $w_{i'j} \neq 0$,

(ii) $w_{ij} \overset{E}{\to} w_{i'j}$ to mean that (i) holds and the exchange is made yielding a new $\underset{\sim}{W}$ matrix, and

(iii) $w_{i_1j_1} \overset{E}{\to} w_{i_2j_1} | w_{i_2j_2} \overset{E}{\to} w_{i_3j_2}$ to mean that the first exchange is made in $\underset{\sim}{W}$ followed by the second exchange in the matrix resulting from the first, $j_1 \neq j_2$.

More generally, we write

$$(5) \qquad w_{i_1j_1} \overset{E}{\to} w_{i_2j_1} | w_{i_2j_2} \overset{E}{\to} w_{i_3j_2} | \cdots | w_{i_nj_n} \overset{E}{\to} w_{i_{n+1}j_n}$$

to mean a sequence of exchanges, each based on the $\underset{\sim}{W}$ matrix resulting from the previous exchange, $j_t \neq j_{t+1}$, $t = 1, \ldots, n$.

Lemma 2. Given the sequence of exchanges (5) with $n > 1$, the resultant $\underset{\sim}{W}$ matrix is such that,

(i) If $i_1 \neq i_{n+1}$, the i_1-th row sum is increased by 1 (relative to the original matrix $\underset{\sim}{W}$), the i_{n+1}-th row sum is decreased by 1, and all of the remaining row sums are unchanged, or

(ii) If $i_1 = i_{n+1}$, all row sums are unchanged.

Definition 2. If the sequence of exchanges (5) is possible with $i_1 \neq i_{n+1}$, we say that row i_1 *can reach* row i_{n+1} in the initial matrix $\underset{\sim}{W}$. If $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are two matrices with rows consisting of two, non-empty, disjoint subsets of the rows of $\underset{\sim}{W}$, we say that $\underset{\sim}{A}$ *can reach* $\underset{\sim}{B}$ if a row in $\underset{\sim}{A}$ can reach a row in $\underset{\sim}{B}$.

Lemma 3. If $\underset{\sim}{A}$ and $\underset{\sim}{B}$ contain all row sums in $\underset{\sim}{W}$ and if $\underset{\sim}{W}$ represents a connected design with parameters v, b, k and r, then $\underset{\sim}{A}$ can reach $\underset{\sim}{B}$ through one exchange if the average row sum of $\underset{\sim}{A}$ does not exceed $\frac{1}{2}r(k + 1)$.

Proof. The proof is by contradiction. Suppose that $\underset{\sim}{A}$ cannot reach $\underset{\sim}{B}$ through one exchange.

Let n_j be the number of non-zero elements in the j-th column of $\underset{\sim}{A}$, $j = 1, \ldots, b$. If $n_j \neq 0$, the non-zero elements in column j of $\underset{\sim}{A}$ must be $k, (k - 1), \ldots, (k - n_j + 1)$, the largest n_j numbers in the set $\{1, \ldots, k\}$. If the number of rows in $\underset{\sim}{A}$ is $\geq k$, $0 < n_j < k$ for at least one j since the design is connected. It follows that the average row sum in $\underset{\sim}{A}$ is

$$\text{(6)} \qquad \frac{r}{2}\sum_{j=1}^{b} n_j(2k - n_j + 1)/\sum_{j=1}^{b} n_j = r(k + \tfrac{1}{2}) - \frac{r}{2}\Big(\sum_{j=1}^{b} n_j^2/\sum_{j=1}^{b} n_j\Big)$$

$$> r(k + \tfrac{1}{2}) - \frac{r}{2}\Big(\sum_{j=1}^{b} n_j k/\sum_{j=1}^{b} n_j\Big) = \frac{r}{2}(k + 1),$$

the inequality in (6) being strict because the design is connected. The condition of the lemma is violated and the proof is complete.

Lemma 4 follows from Lemma 2 and Lemma 5 presents a condition for continuation of the exchange process:

Lemma 4. If rows i and i' in $\underset{\sim}{W}$ have respectively the minimum row sum m and a row sum $\geq m + 2$, $\underset{\sim}{W}$ can be improved if row i can reach row i'.

Lemma 5. If $w_{i_1j_1} \overset{E}{\rightarrow} w_{i_2j_1}$ is possible in $\underset{\sim}{W}$ and if the resultant matrix does not represent a $TF_{\Phi_1}B$ design but does have row i_2 with minimum row sum, there exists a row i_3 and a column j_2, $i_3 \neq i_2$, $j_2 \neq j_1$, such that $w_{i_2j_2} \rightarrow w_{i_3j_2}$.

Proof. Assume that the claimed result is false. Let $\underset{\sim}{W}^*$ with typical element w^*_{ij} result after $w_{i_1j_1} \overset{E}{\to} w_{i_2j_1}$ in $\underset{\sim}{W}$. Then row i_2 of $\underset{\sim}{W}^*$ cannot reach any other row through a column different from j_1. This means that $w^*_{i_2j_1}$ is now at least 1 and all of the remaining $(r - 1)$ positive elements in row i_2 of $\underset{\sim}{W}^*$ must be k. Therefore, the i_2-th row sum of $\underset{\sim}{W}^* \geq (r - 1)k + 1 \geq \frac{1}{2}r(k + 1)$ since $r, k > 1$. A contradiction is established by Lemma 1 and the proof follows.

Notice that, if $i_3 = i_1$ in Lemma 5, each row sum in the matrix resulting from $w_{i_1j_1} \overset{E}{\to} w_{i_2j_1} | w_{i_2j_2} \overset{E}{\to} w_{i_3j_2}$ matches the corresponding row sum of $\underset{\sim}{W}$ by Lemma 2.

Proof of Sufficiency, Theorem 2. If the matrix $\underset{\sim}{W}$ corresponding to the block design of Theorem 2 does not represent a $TF_{\underset{\sim}{\phi}_1}B$ design, all rows of $\underset{\sim}{W}$ may be collected into three matrices, $\underset{\sim}{R}_0$, $\underset{\sim}{R}_1$, and $\underset{\sim}{R}_2$ such that $\underset{\sim}{R}_0$ consists of rows with minimum sum m, $\underset{\sim}{R}_1$ of rows with sums $(m + 1)$, and $\underset{\sim}{R}_2$ of rows with sums $\geq (m + 2)$. We show that $\underset{\sim}{R}_0$ can always reach $\underset{\sim}{R}_2$. Therefore, by Lemma 4, $\underset{\sim}{W}$ can be improved and a $TF_{\underset{\sim}{\phi}_1}B(v, b, k, r)$ design can be obtained by Remark 1.

It is easy to show that, when the row sums of $\underset{\sim}{W}$ are not all equal, $\underset{\sim}{R}_1$ may or may not exist, while $\underset{\sim}{R}_0$ and $\underset{\sim}{R}_2$ always exist. When $\underset{\sim}{R}_1$ does not exist, we may set $\underset{\sim}{A} = \underset{\sim}{R}_0$ and $\underset{\sim}{B} = \underset{\sim}{R}_2$ and use Lemma 3 to show that $\underset{\sim}{R}_0$ can reach $\underset{\sim}{R}_2$. When $\underset{\sim}{R}_1$ exists, we show that $\underset{\sim}{R}_0$ can reach $\underset{\sim}{R}_2$ by contradiction.

Assume that $\underset{\sim}{R}_0$ cannot reach $\underset{\sim}{R}_2$. Set $\underset{\sim}{A} = \underset{\sim}{R}_0$ and $\underset{\sim}{B} = \begin{bmatrix} \underset{\sim}{R}_1 \\ \underset{\sim}{R}_2 \end{bmatrix}$ and use Lemma 3 again: since $\underset{\sim}{R}_0$ cannot reach $\underset{\sim}{R}_2$, $\underset{\sim}{R}_0$ can reach $\underset{\sim}{R}_1$ through one exchange, say $w_{i_1j_1} \overset{E}{\to} w_{i_2j_1}$, yielding a new matrix $\underset{\sim}{W}_1$. Row i_2 in $\underset{\sim}{W}_1$

has sum m, the minimum row sum in $\underset{\sim}{W}_1$, and, by Lemma 5, a further exchange, $w_{i_2 j_2} \overset{E}{\rightarrow} w_{i_3 j_2}$, $j_2 \neq j_1$, may be made and yields a new matrix $\underset{\sim}{W}_2$. After the two exchanges, row i_3 of $\underset{\sim}{W}_2$ has sum m or (m - 1) depending on whether row i_3 was in $\underset{\sim}{R}_1$ or $\underset{\sim}{R}_0$; in either case, row i_3 has minimum row sum in $\underset{\sim}{W}_2$. Lemma 5 is applied again and an additional exchange may be made.

A successive exchange process has been established that continues without ending if $\underset{\sim}{R}_0$ cannot reach $\underset{\sim}{R}_2$. But the number of possible exchanges and orders of exchanges in columns of $\underset{\sim}{W}$ is finite and a closed loop of exchanges like (5) with $i_{n+1} = i_1$ must result. Rows of $\underset{\sim}{W}$ in the loop must come from $\begin{bmatrix} \underset{\sim}{R}_0 \\ \underset{\sim}{R}_1 \end{bmatrix}$ because otherwise $\underset{\sim}{R}_0$ would reach $\underset{\sim}{R}_2$. The loop is closed in the sense that rows in the loop cannot reach the remaining rows, if any, in $\begin{bmatrix} \underset{\sim}{R}_0 \\ \underset{\sim}{R}_1 \end{bmatrix}$ and cannot reach rows in $\underset{\sim}{R}_2$, again because then $\underset{\sim}{R}_0$ would reach $\underset{\sim}{R}_2$. A contradiction has resulted and is established through use of Lemma 3. Let $\underset{\sim}{A}$ be the subset of rows of $\begin{bmatrix} \underset{\sim}{R}_0 \\ \underset{\sim}{R}_1 \end{bmatrix}$ in the closed loop and let $\underset{\sim}{B}$ be the remaining rows of $\underset{\sim}{W}$. It is clear that the average row sum in $\underset{\sim}{A}$ does not exceed $\frac{1}{2}r(k + 1)$ and $\underset{\sim}{A}$ should be able to reach $\underset{\sim}{B}$. Theorem 2 has been established.

Remark 2. Theorem 2 has implications for rank-order statistics. If treatments are ranked within blocks of the connected block design and treatment rank sums are considered instead of position sums, the theorem establishes conditions under which the treatments may have equal rank sums.

Let a balanced incomplete block (BIB) design free of a trend described by $\underset{\sim}{\phi}$ be designated as a $TF_{\underset{\sim}{\phi}}BIB(v, b, k, r, \lambda)$ design. Many $TF_{\underset{\sim}{\phi}_1}BIB(v, b, k, r,$ designs exist. Raghavarao (1971, Table 5.10.1) gives solutions for BIB designs with $v, b \le 100$, $r, k \le 15$. Ninety-one designs are listed of which only fifteen do not yield $TF_{\underset{\sim}{\phi}_1}$ BIB designs, namely numbers 1, 6, 14, 15, 24, 27, 35, 36, 46, 48, 52, 53, 69, 72, and 88.

Symmetric balanced incomplete block designs, for which $v = b$ and hence $k = r$, free of a trend described by $\underset{\sim}{\phi}$, may be designated as $TF_{\underset{\sim}{\phi}}SBIB(v, k, \lambda)$ designs, where λ has the usual meaning. It is obvious that SBIB designs yield $TF_{\underset{\sim}{\phi}_1}SBIB(v, k, \lambda)$ designs by Theorem 2. A Youden Square design is a special case of an SBIB design in a rectangular array with the columns forming a CB design and the rows an SBIB design. Youden Squares are the incomplete block design counterpart to Latin Squares in complete block designs.

Theorems on trends in one dimension of higher order than the first are difficult to develop. An example of a $TF_{\underset{\sim}{\phi}}BIB(v, b, k, r, \lambda)$ design is given by Yeh and Bradley (1979) when $\underset{\sim}{\phi} = (\underset{\sim}{\phi}_1, \underset{\sim}{\phi}_2)$, a quadratic trend, and $v = 5$, $b = 10$, $k = 3$, $r = 6$, $\lambda = 3$.

The method of proof of Theorem 2 suggests a possible exchange method for derivation of $TF_{\underset{\sim}{\phi}_1}B(v, b, k, r)$ designs when they exist that should be feasible on a computer.

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ABSTRACT

An earlier, referenced report considered the existence of trend-free block designs and e some general results for general block designs and some special results for complete ck designs. This report presents a new existence theorem that is sufficiently general to er broad classes of incomplete block designs and suggests a design construction method.

Suppose that a common trend affecting treatment responses exists over the plots within cks of a block design with the usual design parameters, v, b, k, r. A trend-free block ign, if it exists, is one for which treatments are assigned to plots within blocks such t atment and block contrasts are orthogonal to specified trend components. We prove that, a given block design is constructed and the common trend is linear, the necessary and ficient condition for the existence of a trend-free block design with the same idence matrix is that $\frac{1}{2}r(k + 1)$ be an integer.