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6 Mathematical Modeling of

Multi-Element Monopole Antennas

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10 [redacted] N.N. Puri, G. Goubau

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This research document presents a new theory for the analysis of multi-element antennas which consist of interconnected conductive structure elements of electrically small dimensions. The theory is based on the retarded electromagnet potentials which permit a diakoptic approach to the problem. The antenna is broken up into its individual structure elements. Each element is assumed to be excited, a) by currents which are impressed at its terminals, i.e. junctions with adjacent elements (current coupling), and b) by the electric fields of the currents and charges on all the other elements (field coupling). Both excitations are

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20. Abstract (continued)

treated independently. Each impressed current produces a "dominant" current distribution, a characteristic of the element, which can be readily computed. Current coupling is formulated by "intrinsic" impedance matrices which relate the scalar potentials at the terminals of an element, caused by its dominant current distributions, to the impressed currents of the element. Field coupling produces "scatter" currents on all the elements, and is formulated by a "field coupling" matrix which relates the scalar potentials at the terminals, caused by field coupling, to the impressed currents at all the terminals. Intrinsic and "field coupling" are combined to form the "complete" impedance matrix of the diakopted antenna. Enforcing continuity of the currents and equality of the scalar potentials at all the interconnections between the elements yields a system of linear equations for the junction currents and the input impedance of the antenna. Current coupling dominates over field coupling. Field coupling due to the dominant current distributions of the elements is of primary importance while field coupling due to the scatter currents is, in general, negligible. This theory is applied to several multi-element antennas and the results are compared with other methods to highlight the numerical advantages.

MATHEMATICAL MODELING OF MULTI-ELEMENT MONOPOLE ANTENNAS

FINAL REPORT

by

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Summary

This research document presents a new theory for the analysis of multi-element antennas which consist of interconnected conductive structure elements of electrically small dimensions. The theory is based on the retarded electromagnetic potentials which permit a diakoptic approach to the problem. The antenna is broken up into its individual structure elements. Each element is assumed to be excited, a) by currents which are impressed at its terminals, i.e. junctions with adjacent elements (current coupling), and b) by the electric fields of the currents and charges on all the other elements (field coupling). Both excitations are treated independently. Each impressed current produces a "dominant" current distribution, a characteristic of the element, which can be readily computed. Current coupling is formulated by "intrinsic" impedance matrices which relate the scalar potentials at the terminals of an element, caused by its dominant current distributions, to the impressed currents of the element. Field coupling produces "scatter" currents on all the elements, and is formulated by a "field coupling" matrix which relates the scalar potentials at the terminals, caused by field coupling, to the impressed currents at all the terminals. Intrinsic and "field coupling" are combined to form the "complete" impedance matrix of the diakopted antenna. Enforcing continuity of the currents and equality of the scalar potentials at all the interconnections between the elements yields a system of linear equations for the junction currents and the input impedance of the antenna. Current coupling dominates over field coupling. Field coupling due to the dominant current distributions of the elements is of primary importance while field coupling due to the scatter currents is, in general, negligible. This theory is applied to several multi-element antennas and the results are compared with other methods to highlight the numerical advantages.

This research document is dedicated to Dr. Goubau who expounded most of the ideas developed here and whose untimely death is a irreparable loss to the Scientific Community.

I. Introduction

Improved tactical communication systems require antennas which are electrically small (i.e. small compared with the wavelength), have very large bandwidths and reasonably high efficiency. It is well known to antenna experts that these requirements work against each other. The problem therefore, is to find sophisticated antenna structures which provide the best compromise between these contradicting requirements.

Experimental investigations of empirically designed multielement antennas, i.e., antennas which comprise a number of interconnected and closely spaced conductive elements, have shown promising results. An example of such a broadband multielement monopole antenna is shown in Figure 1. This antenna consists of four vertical conductors. The two thicker ones are grounded, while the other two are interconnected near the ground plane and connected to the input terminal. Each vertical conductor has a top capacitor in the form of a metal plate, and there are inductive interconnections between the plates in the form of wire loops. But antennas like the one mentioned, whose functioning is not quite understood, are not amenable to conventional computer analysis.

An analytical treatment of such a composite structure appears to be a rather hopeless undertaking. Commonly used numerical techniques are impracticable because they would require computers with enormous storage. Moreover, these techniques do not always yield reliable results [2].

This research offers a new approach to problems of this kind. According to this approach the composite structure is diakopted into its individual structure elements. As a simple example, Figure 2 shows a diakopted dipole with end capacitor plates. Each structure element is characterized by electrical quantities which depend only on size and shape of the element, and

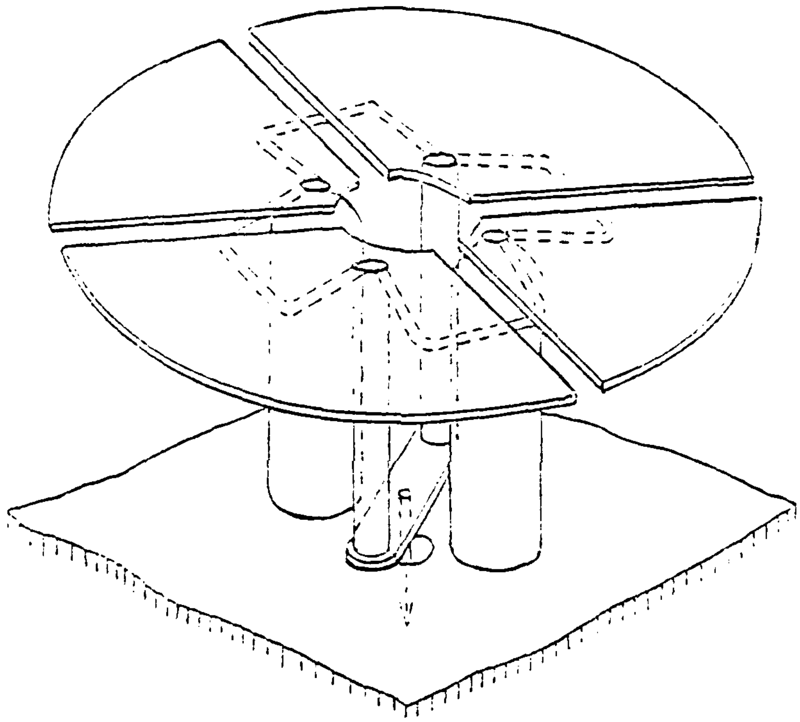


FIGURE 1

Broad-band Multi-element Monopole Antenna

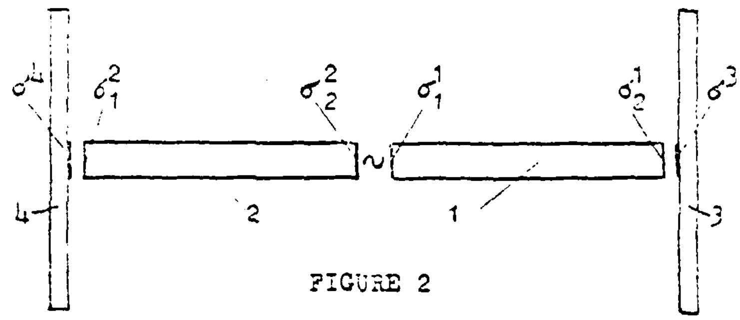


FIGURE 2

Diakopted Capacitively Loaded Dipole

the assembly is treated similarly to the interconnection of n-port networks.

The excitation of each element is ascribed to two causes, a) the currents entering the element at its "terminals," i.e. junctions with adjacent elements or the source, and b) the fields of the currents and charges on all the other elements. The first is referred to as "current coupling" and the second as "field coupling." Both excitations are treated separately. Current coupling implies hypothetical sources with a single terminal and the capability of impressing a current onto a conductor. Although such sources violate the continuity condition, their assumption is permissible if the electro-magnetic fields are expressed by the retarded electromagnetic potentials. Although the continuity condition is violated in the treatment of individual structure elements, it is restored when the elements are interconnected. Thus, current coupling is computed by impressing a current at a terminal of a structure element. This current spreads over the surface of the element and produces a current distribution which is uniquely determined by the geometry of the element and the location of the terminal and is called the dominant current distribution associated with a given terminal. There are as many dominant current distributions as there are terminals. The relationship between the scalar potentials at the terminals (produced by the dominant current distributions) and the impressed currents is formulated by the "intrinsic impedance matrix" of the element.

Field coupling,on the other hand,excites scatter currents which are superimposed on the dominant current distributions. The scalar potentials at the terminals due to field coupling depend on all the impressed currents. Their relationship with the impressed currents is forumlated by a "field coupling" matrix. The intrinsic impedance matrix and "field coupling" matrix combined together form the "complete impedance matrix" of the diakopted antenna. This matrix relates the total scalar potentials at the terminals of all the elements to all the impressed currents.

Interconnection of the structure elements, which requires equal scalar potentials at the interconnected terminals and continuity of the junction currents, it formulated by an interconnection matrix. In this manner a system of linear equations is obtained which yields the junction currents and the input impedance of the antenna.

A most simple antenna to which the theory applies is a simple monopole antenna with a top capacitor. In this case, there are two structure elements, the vertical conductor and the top capacitor. The ground plane can be replaced by the antenna image. No systematic way of computing the impedance characteristics of this antenna has been reported in the literature.

II. Diakoptic Theory of Multi-Element Antennas

In this section we shall develop the essential theoretical results required to implement the diakoptic theory.

Consider a multi-element radiating structure such as shown in Figure 1. Various elements are interconnected to each other via terminals of junctions. Let each radiating element be disconnected or (diakopted) from all other elements and be suspended in space. The assemblage of these disconnected elements is called the diakopted (or primitive) system. Each element has many terminals on each of which certain impressed current and potential is assumed. The essential requirement for this diakopted system with impressed currents along the junctions is that it be performancewise identical to the assembled antenna. Thus,

a) The sum of the impressed currents is zero at every junction between the structure elements and the continuity condition is satisfied at every input terminal. This requirement assures that the field of assembled antenna is Maxwellian.

b) The scalar potentials at the interconnected terminals are equal.

c) The potential difference between the input in terminals is equated with the driving voltage of the antenna source.

Let the potential-current relationship at every terminal be written in matrix form:

$$[\hat{\phi}] = [Z][I] \quad \text{[diakopted antenna]} \qquad \text{II.1}$$

$$[\hat{\phi}]' = [Z]'[I]' \quad \text{[assembled actual antenna]} \qquad \text{II.2}$$

Requirements (a), (b) and (c) represent Kirchoff's laws for interconnected structures and can be written as

$$[I] = [C][I]' \qquad \text{II.3}$$

$$[\hat{\phi}]' = [C]_t[\hat{\phi}] \qquad \text{II.4}$$

$$[\hat{\phi}]'_t[I]' = [\hat{\phi}]_t[\bar{I}] \qquad \text{II.5}$$

$[C]_t$ represents the transpose $[C]$.

Matrix $[Z]$ represents the impedance of the diakopted antenna and primed quantities refer to the actual assembled antenna. $[C]$ may be a rectangular matrix with $\{C_{ij}\}$ as 0 or 1.

From II.3, II.4 and II.5, the impedance matrix of the actual structure can be written as

$$[Z]' = [C]_t[Z][C]$$

An example at the end of this section shows how $[C]$ and $[Z]'$ are obtained.

The essential results of this section show that in order to obtain $[Z]'$, we have to only compute the impedance matrix $[Z]$ of the so called diakopted structure.

The most important point here to remember is that the elements of the impedance matrix $[Z]'$ depend upon simultaneously knowing current distribution on all the radiating structure elements. Thus, without diakopting the structure, we have to simultaneously solve as many integral equals as there are radiating elements. On the other hand, the elements of the impedance matrix $[Z]$ of the diakopted structure can be found by computing the current distribution on individual elements separately and hence involves solving as many integral equations as there are radiating structures, but only individually. This results in a tremendous savings of numerical computation. *In what follows* we shall show how the so called total, primitive (or diakopted) impedance matrix $[Z]$ can be computed.

III. Impedance Matrix of a Diakopted Antenna

Consider each element of a diakopted antenna.

The excitation of each element is ascribed to two causes. a) the currents entering the element at its "terminals," i.e., junctions with adjacent elements or the source, and b) the fields of the currents and charges on all the other elements. The first is referred to as "current coupling" and the second as "field coupling." Both excitations can be treated separately and the resulting compling can be superimposed due to linearity. Current coupling implies hypothetical sources with a single terminal and the capability of impressing a current onto a conductor. Although such sources violate the continuity condition, their assumption is permissible if the electro-magnetic fields are expressed by the retarded electromagnetic potentials. Although the continuity condition is violated in the treatment of individual structure elements, it is restored when the elements are interconnected. If a current is impressed at a terminal of a structure element,the current spreads over the surface of the element and produces a current distribution which is uniquely determined by the geometry of the element and the location of the terminal. This is called dominant current distribution of a particular element. There are as many dominant current distributions as there are terminals. The relationship between the scalar potentials at the terminals, produced by the dominant current distributions, and the impressed currents is formulated by the "intrinsic impedance matrix" of the element and is referred to as [Z(I)].

Field coupling excites scatter currents which are superimposed on the dominant current distributions. The scalar potentials at the terminals due to field coupling depend on all the impressed currents. Their relationship with the impressed currents is formulated by a "field coupling" matrix [Z(F)].

$$[Z] = [Z(I)] + [Z(F)]$$

This matrix [Z] is called the total impedance matrix of the diakopted antenna and relates the total scalar potentials at all the terminals of all the elements of the diakopted structure to all the impressed currents.

III.1 Current Coupling Between Structure Elements and Intrinsic Impedance Matrix [Z(I)].

A. Structure elements with one terminal

Consider one of the capacitor plates of the dipole in Fig. 2 separated from the other elements and suspended in space, with a current I impressed at the terminal, i.e., contact area in the center of the plate (Fig. 3). The contact area σ is considered very small compared with the surface area of the element. Excitation by an impressed current cannot be treated with Maxwell's equations, because Maxwell's equations imply sources which separate positive and negative charges. In contrast, impressed currents require sources which produce charges. The retarded electromagnetic potentials do not impose any conditions on the source, and can therefore be used for our problem.

If $\bar{i}(\bar{r})$ is the surface current density, and $q(\bar{r})$ the surface charge density due to the impressed current I, the retarded potentials are

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi}\int_S \bar{i}(\bar{r}')G(\bar{r},\bar{r}')dS \quad \text{(vector potential)} \qquad \text{III.1}$$

$$\phi(\bar{r}) = \frac{1}{4\pi\varepsilon}\int_S q(r')G(\bar{r},\bar{r}')dS \quad \text{(scalar potential)} \qquad \text{III.2}$$

with

$$G(\bar{r},\bar{r}') = \frac{\exp(-jk|\bar{r} - \bar{r}'|)}{|\bar{r} - \bar{r}'|} \qquad k = 2\pi/\lambda$$

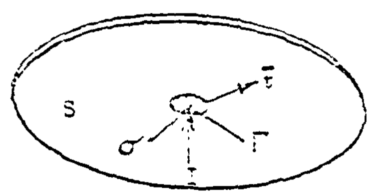


FIGURE 3

Excitation of Single Terminal
Terminal Structure Element

where $\bar{r}'$ is the position vector of the charges and currents on the surface elements dS, and $\bar{r}$ that of the point of observation. The quantities $\bar{i}(\bar{r})$ and $q(\bar{r})$ must satisfy the following two equations on the surface of the element outside the contact area σ:

$$\bar{E}(\bar{r}) \times d\bar{S} = -\,[j\omega\bar{A}(\bar{r}) + \bar{\nabla}\phi(\bar{r})] \times d\bar{S} = 0 \qquad \text{(Boundary condition)} \qquad \text{III.3}$$

$$\bar{\nabla}\cdot\bar{i}(\bar{r}) + j\omega q(\bar{r}) = 0 \qquad \text{(Continuity condition)} \qquad \text{III.4}$$

The condition that the current flux through the boundary curve Γ of the contact area σ is the continuation of the impressed current I, is given as:

$$\oint_{\Gamma} \bar{i}(\bar{r})\cdot\bar{t}(\bar{r})d\Gamma = I \qquad \text{III.5}$$

where $\bar{t}(\bar{r})$ is a unit vector tangential to the surface S and normal Γ. The current and charge distributions $\bar{i}(\bar{r})$ and $q(\bar{r})$ due to the impressed current I are termed as "dominant" distributions since the currents due to field coupling between the elements are, in general, relatively small. From the boundary condition III.3

$$\int_S \bar{E}(\bar{r})\cdot\bar{i}(\bar{r})dS = -\int_S [j\omega\bar{A}(\bar{r}) + \bar{\nabla}\phi(\bar{r})]\cdot\bar{i}(\bar{r})dS = 0 \qquad \text{III.6}$$

The surface of integration S is the surface of the element with the exclusion of the contact area. Using the relations

$$\bar{\nabla}\phi\cdot\bar{i}(\bar{r}) = \bar{\nabla}\cdot(\phi(\bar{r})\bar{i}(r)) - \phi(\bar{r})\bar{\nabla}\cdot\bar{i}(\bar{r}) = \bar{\nabla}\cdot(\phi(\bar{r})\bar{i}(\bar{r})) + j\omega\phi(\bar{r})q(\bar{r}) \qquad \text{III.7}$$

and applying Gauss' theorem, one obtains from III.6

$$j\omega\int_S [\bar{A}(\bar{r})\cdot\bar{i}(\bar{r}) + \phi(\bar{r})q(\bar{r})]dS = -\int_S \bar{\nabla}\cdot(\phi(\bar{r})\bar{i}(\bar{r}))dS = \oint_\Gamma \phi(\bar{r})\bar{i}(\bar{r})\cdot\bar{t}(\bar{r})d\Gamma \qquad \text{III.8}$$

If the contact area σ is sufficiently small, ϕ can be considered constant within the contact area. Thus, with (5), equation (8) reduces to

$$j\omega\int_S [\bar{A}(\bar{r})\cdot\bar{i}(\bar{r}) + \phi(\bar{r})q(\bar{r})]dS = \phi I \qquad \text{III.9}$$

where ϕ is the scalar potential at the contact area.

The ratio between ϕ and I can be used to define an impedance which shall be termed "intrinsic impedance." If $\bar{A}$ and ϕ are expressed by the current and charge distribution, the intrinsic impedance of the element is

$$\bar{Z}(I) = \frac{\phi}{I} = \frac{j\omega}{I^2}\int_S [\bar{A}(\bar{r})\cdot\bar{i}(\bar{r}) + \phi(\bar{r})q(\bar{r})]dS$$

$$= \frac{j\omega\mu}{4\pi}\int_S\int_{S'} G(\bar{r},\bar{r}')\left[\frac{\bar{i}(\bar{r})\cdot\bar{i}(\bar{r}')}{I^2} - \frac{1}{k^2}\frac{q(\bar{r})q(\bar{r}')}{Q^2}\right]dS\,dS \qquad \text{III.10}$$

where $Q = I/j\omega$ is the total charge on the element. The current and charge distribution functions $\bar{i}/I$ and q/Q are solely determined by the geometry of the element and the location of the coupling area.

When the intrinsic impedance is computed with III.10 for a conductor of any shape, for extremely low frequencies, it takes the form

$$Z(I) = \frac{1}{j\omega C} - \frac{1}{4\pi}\sqrt{\frac{\mu}{\varepsilon}} \qquad (\omega\to 0) \qquad \text{III.11}$$

where C is the static capacity of the element. The first term $1/j\omega C$ is the one which is to be expected. The second term represents a negative resistance of -30 ohms and is not quite obvious. It is brought about by the fact that an impressed current produces a charge on the element without a countercharge, in contrast to a Maxwell source. If the scalar potential is expanded in a power series in ω, one obtains

$$\phi(\bar{r}) = \frac{1}{4\pi\varepsilon}\int_{S'} G(\bar{r},\bar{r}')q(\bar{r}')dS' = \frac{1}{4\pi\varepsilon}\left\{\int_{S'}\frac{q(\bar{r}')}{|\bar{r}-\bar{r}'|}dS' - jk\int_{S'} q(\bar{r}')dS' + \ldots\right\}$$

The first term of this expansion is the static potential of the charges. The second term which is independent of $\bar{r}$ represents a potential, termed "background" potential ϕ_0, which is uniform in space and has no gradient. This means it does not produce a field. It is this background potential which produces the -30Ω term in III.11. When the element which we assumed to be suspended in space is within the antenna structure the background potential is compensated because the combined charges on all the other elements are negatively equal to the charges of the considered element. The background potential can be avoided if the retarded scalar potential is redefined as modified scalar potential

$$\hat{\phi} = \phi - \phi_0 = \frac{1}{4\pi\varepsilon}\int_{S'} \hat{G}(\bar{r},\bar{r}')q(\bar{r}')dS' \qquad \text{III.12}$$

where

$$\hat{G}(\bar{r},\bar{r}') = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|} + jk \qquad \text{III.13}$$

This modified scalar potential which will be used throughout the paper is legitimate as it is not conflicting with Maxwell's theory. Since $\nabla\hat{\phi} = \nabla\phi$, the boundary condition III.3 and the dominant current distribution derived therefrom remain unchanged if the conventional potential ϕ is substituted by the modified potential $\hat{\phi}$. For a Maxwell system ϕ and $\hat{\phi}$ are identical, since $\int q dS$ extended over the entire surface of the system is zero. The intrinsic impedance of a structure element with one connection becomes

$$Z = \frac{\hat{\phi}}{I} = \frac{j\omega}{I^2} \int_S [\bar{A}(\bar{r}) \cdot \bar{i}(\bar{r}) + \hat{\phi}(\bar{r}) q(\bar{r})] dS$$

$$= \frac{j\omega\mu}{4\pi} \int_S \int_S \left[G(\bar{r},\bar{r}') \frac{\bar{i}(\bar{r}) \cdot \bar{i}(\bar{r}')}{I^2} - \frac{1}{k^2} \hat{G}(\bar{r},\bar{r}') \frac{q(\bar{r}) q(\bar{r}')}{Q^2} \right] dS' dS \qquad \text{III.14}$$

III.14 represents a stationary formulation of the intrinsic impedance. This means, small errors in the dominant current distribution have only a second order effect on the intrinsic impedance. (See Appendix 4)

Excitation by an impressed current I at the terminal can be considered equivalent with the excitation by an oscillating charge

$$Q = \frac{I}{j\omega} \qquad \text{III.15}$$

which is placed above the contact area at a distance d→0 as shown in Fig. 4. The charge on the contact area σ consists essentially of the image charge -Q, with the charge +Q distributed over the surface areas of the structure element, because the net charge on the element must be zero. The equivalence between charge and current excitation is shown in Appendix 1.

The intrinsic impedance Z(I) of an element with one terminal can be represented by a lumped element circuit as shown in Fig. 5. For low frequencies, i.e. when the dimensions of the element are small compared with the wave length, C and L can be considered constant, while R increases proportionally with ω^2:

$$Z = \frac{1}{j\omega C} + j\omega L + R(\omega^2) \qquad \text{III.16}$$

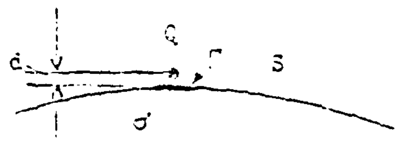


FIGURE 4

Excitation of Structure Element
by Oscillating Charge

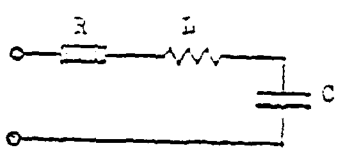


FIGURE 5

Low Frequency Equivalent Circuit
for a Single Terminal Structure
Element

In case φ instead of $\hat{\varphi}$ is used, the individual elements will show an additional -30Ω. fictitious resistance in the intrinsic impedance. However, the impedance of the totally assembled antenna is the same as the conventional impedance due to automatic compensation of -30Ω.

3. Structure elements with two or more terminals

A structure element with two terminals such as the cylindrical conductors of Fig. 2 has two dominant current distributions, one associated with each of the impressed currents (Fig. 6). Each dominant current distribution produces a scalar potential at both contact areas. If $\hat{\Phi}_{11}$ and $\hat{\Phi}_{21}$ are the potentials at the terminals 1 and 2 due to I_1, and $\hat{\Phi}_{12}$, $\hat{\Phi}_{22}$ those due to I_2, then, the relationship between the total potentials $\hat{\Phi}_1$ and $\hat{\Phi}_2$ at the terminals and the impressed currents can be written as

$$\begin{aligned} \hat{\Phi}_1 &= \hat{\Phi}_{11} + \hat{\Phi}_{12} = Z_{11}(I)I_1 + Z_{12}(I)I_2 \\ \hat{\Phi}_2 &= \hat{\Phi}_{21} + \hat{\Phi}_{22} = Z_{12}(I)I_1 + Z_{22}(I)I_2 \end{aligned} \qquad \text{III.17}$$

For a structure element with M terminals the relationship between the terminal potentials and the impressed currents is formulated by an M x M intrinsic impedance matrix.

$$[\hat{\Phi}] = [Z][I] \qquad \text{III.18}$$

where

$$Z_{jk}(I) = \frac{j\omega}{I_j I_k} \int_S [\bar{A}_k(\bar{r}) \cdot \bar{i}_j(\bar{r}) + \hat{\phi}_k(\bar{r}) q_j(\bar{r})] dS$$

$$= \frac{j\omega\mu}{4\pi} \int_S \int_{S'} \left[G(\bar{r},\bar{r}') \frac{\bar{i}_j(\bar{r}) i_k(\bar{r}')}{I_j I_k} - \frac{1}{k^2} \hat{G}(\bar{r},\bar{r}') \frac{q_j(\bar{r}) q_k(\bar{r}')}{Q_j Q_k} \right] dS' dS \qquad \text{III.19}$$

The quantities $\bar{i}_j, q_j$ and $\bar{i}_k, q_k$ are the dominant current and charge distributions generated by the impressed currents $I_j = j\omega Q_j$ and $I_k = j\omega Q_k$, and $\bar{A}_k, \Phi_k$ are the retarded potentials associated with $\bar{i}_k, q_k$. (III.19) is derived in Appendix 2.

The symmetry of the intrinsic impedance matrix, $Z_{jk}(I) = Z_{kj}(I)$, is evident from the second formulation of III.19. In Appendix 4 it is shown that III.19 is a stationary representation of the matrix elements.

A lumped element equivalent circuit for a structure element with two terminals is shown in Fig. 7. For sufficiently low frequencies the capacitors and inductors can be considered constant, while the resistors increase with ω^2. The resistor which is in series with the capacitor is negative, but smaller than the resistors associated with the inductors.

III.2 Field Coupling Between Structural Elements.

Field Coupling Impedance Matrix

We now consider a diakopted structure and arbitrary currents impressed at the terminals. The capacitively loaded dipole of Figure 2 may serve as an example. The terminals are identified by a superscript i and a subscript k, the superscript referring to the number of the element, and the subscript referring to the number of the terminal on the element. If there were no field coupling between the elements, the current distributions on all the elements would be the dominant distributions associated with the impressed currents.

The field of a dominant current distribution is non-Maxwellian since the associated net charge is nonzero. If a current I_k^i is impressed at the terminal $\binom{i}{k}$, the non-Maxwellian field of the dominant current and charge distribution $\bar{i}_k^i$, q_k^i induces currents on all the other elements. The scatter fields excited by these induced currents are Maxwellian, since induced current distributions have no net charge. These "first order" scatter fields excite second order

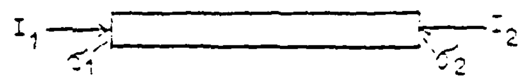


FIGURE 6

Structure Element with Two Terminals

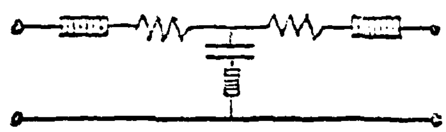


FIGURE 7

Equivalent Circuit for Two Terminal Structure Element scatter fields and so on, each higher order having a greatly reduced amplitude. All these scatter fields, when summed up, form a multiple scatter field which is Maxwellian. The currents and charges associated with the multiple scatter field are distributed over all the surfaces S^n (including S^i) and shall be denoted δi_k^{in}, q_k^{in}, the super and subscripts indicating that they are produced by the impressed current I_k^i and located on the element n.

The total field generated by I_k^i satisfies on every element the boundary conditions

$$(j\omega(\bar{A}_k^i + \delta\bar{A}_k^i) + \bar{\nabla}(\hat{\phi}_k^i + \delta\hat{\phi}_k^i)) \times d\bar{S}^n = 0 \ (n = 1, \ldots, i, \ldots N) \qquad \text{III.20}$$

where $\bar{A}_k^i$, $\hat{\phi}_k^i$ are the retarded potentials of the dominant current and charge distribution $\bar{i}_k^i$, q_k^i, and $\delta\bar{A}_k^i$, $\delta\hat{\phi}_k^i$ those of the scatter current and charge distributions $\delta\bar{i}_k^{in}$, δq_k^{in} combined. N is the number of elements.

Since the electric field of $\bar{i}_k^i$, q_k^i satisfies the boundary condition on S^i, it follows from (20) for n = i that

$$(j\omega\delta\bar{A}_k^i + \bar{\nabla}\delta\hat{\phi}_k^i) \times d\bar{S}^i = 0$$

Thus

$$\int_{S^i} \left[(j\omega\delta\bar{A}_k^i + \bar{\nabla}\delta\hat{\phi}_k^i)\cdot\bar{i}_k^i\right] dS^i = 0, \quad \begin{matrix} i = 1, \ldots, N \\ k = 1, \ldots, M_i \end{matrix} \qquad \text{III.21}$$

Using the relations III.7 and Gauss' theorem one obtains the "backscatter" potential due to the field interaction of the excited element with the other elements:

$$\hat{\Phi}(F)^{i,i}_{k,k}\, I^i_k = j\omega \int_{S^i} (\delta\bar{A}^i_k \cdot \bar{I}^i_k + \delta\hat{\phi}^i_k q^i_k) dS^i \qquad \text{III.22}$$

The letter F indicates field coupling, the first pair of indices $\binom{i}{k}$ refers to the terminal at which $\hat{\Phi}$ is determined, and the second pair to the terminal of the impressed current which produces this potential.

As shown in Appendix 3

$$\int_{S^i} (\delta\bar{A}^i_k \cdot \bar{I}^i_k + \delta\hat{\phi}^i_k\, q^i_k) dS^i = \sum_{n=1}^{N} \int_{S^n} (\bar{A}^i_k \cdot \delta\bar{I}^{i,n}_k + \hat{\phi}^i_k \delta q^{i,n}_k) dS^n \qquad \text{III.23}$$

Furthermore, from the boundary conditions III.20 using the relations III.7 and Gauss' theorem, follows

$$\int_{S^n} \left[(\bar{A}^i_k + \delta\bar{A}^i_k) \cdot \delta\bar{I}^{i,n}_k + (\hat{\phi}^i_k + \delta\hat{\phi}^i_k)\, \delta q^{i,n}_k \right] dS^n = 0, \text{ for every } n \text{ including } i \qquad \text{III.24}$$

The right-hand side of III.24 is zero since for scatter currents the rim integral of Gauss' theorem in III.5 is zero (see Appendix 5).

From the last three equations, one obtains the "back scatter" impedances

$$Z(F)^{i,i}_{k,k} = \frac{\hat{\Phi}(F)^{i,i}_{k,k}}{I^i_k} = -j\omega \left(\frac{1}{I^i_k} \right)^2 \sum_{n=1}^{N} \int_{S^n} (\delta\bar{A}^i_k \cdot \delta\bar{I}^{i,n}_k + \delta\hat{\phi}^i_k \delta q^{i,n}_k) dS^n \qquad \text{III.25}$$

which has to be added to the diagonal terms of the intrinsic impedance matrix $Z^{i,i}_{k,k}$, using the notation of this section. Generalization of III.25 to obtain the scatter field contributions to the off-diagonal terms is straightforward. One obtains

$$\overset{i,i}{Z(F)}_{k,j} = \frac{\overset{i,i}{\hat{\Phi}(F)}_{k,j}}{I_j^i} = -j\omega \frac{1}{I_k^i I_j^i} \sum_{n=1}^{N} \int_{S^n} (\delta\bar{A}_k^i \cdot \delta\bar{i}_j^{i,n} + \delta\hat{\phi}_k^i \delta q_j^{i,n}) dS^n \qquad \text{III.26}$$

For $k = j$ equation (26) transforms into III.25

Let us now determine the potential $\overset{i,\ell}{\hat{\Phi}(F)}_{k,m}$ produced by the impressed current I_m^ℓ at the terminal $\binom{i}{k}$. Because of the boundary condition III.20

$$\int_{S^i} [j\omega(\bar{A}_m^\ell + \delta\bar{A}_m^\ell) + \bar{\nabla}(\hat{\phi}_m^\ell + \delta\hat{\phi}_m^\ell)] . \bar{i}_k^i d\bar{S}^i = 0, \qquad \text{III.27}$$

and

$$\int_{S^n} [j\omega(\bar{A}_k^i + \delta\bar{A}_k^i) + \bar{\nabla}(\hat{\phi}_k^i + \delta\hat{\phi}_k^i)] \cdot \delta\bar{i}_m^{i,n} dS^n = 0 \qquad \text{III.28}$$

where the second equation holds for every n including i. The potentials $\delta\bar{A}_k^i$ and $\delta\hat{\phi}_k^i$ characterize the scatter field which would be excited by I_k^i.

As before, we apply III.7 and Gauss' theorem to the above two equations to obtain

$$\overset{i,\ell}{\hat{\Phi}(F)}_{k,m} I_k^i = j\omega \left[\int_{S^i} (\bar{A}_m \bar{i}_k^i + \hat{\phi}_m^\ell q_k^i) dS^i + \int_{S^i} (\delta\bar{A}_m^\ell \cdot \bar{i}_k^i + \delta\hat{\phi}_m^\ell q_k^i) dS^i \right] \qquad \text{III.29}$$

$$0 = \int_{S^n} (\bar{A}_k^i \cdot \bar{i}_m^{\ell n} + \hat{\phi}_k^i q_m^{\ell n}) dS^n + \int_{S^n} (\delta\bar{A}_k^i \cdot \delta\bar{i}_m^{\ell n} + \delta\hat{\phi}_k^i \delta q_m^{\ell,n}) dS^n \qquad \text{III.30}$$

The first term in III.29 represents the contribution to the terminal potential

$\hat{\phi}(F)^{i,\ell}_{k,m}$ from the non-Maxwellian field of the dominant current and charge distribution $\bar{i}^{\ell}_{m}$, q^{ℓ}_{m}, and the second term that from the scatter current and charge distributions $\delta\bar{i}^{\ell,n}_{m}$, $\delta q^{\ell,n}_{m}$

As shown in Appendix 3

$$\int_{S^i}(\delta\bar{A}^{\ell}_{m}\cdot\bar{i}^{i}_{k} + \delta\hat{\phi}^{\ell}_{m}q^{i}_{k})dS^{i} = \sum_{n=1}^{N}\int_{S^n}(\bar{A}^{i}_{k}\cdot\delta\bar{i}^{\ell,n}_{m} + \hat{\phi}^{i}_{k}\delta q^{\ell,n}_{m})dS^{n} \qquad \text{III.31}$$

Expressing $\hat{\phi}(F)^{i,\ell}_{k,m}$ in terms of an impedance

$$\hat{\phi}(F)^{i,\ell}_{k,m} = Z(F)^{i,\ell}_{k,m}\, I^{\ell}_{m}\ , \qquad \text{III.32}$$

the field coupling impedance between the terminals $\binom{i}{k}$ and $\binom{\ell}{m}$ becomes

$$Z(F)^{i,\ell}_{k,m} = \frac{1}{I^{i}_{k}I^{\ell}_{m}}\, j\omega\left[\int_{S^i}(\bar{A}^{\ell}_{m}\cdot\bar{i}^{i}_{k} + \hat{\phi}_{m}q^{i}_{k})dS^{i} - \sum_{n=1}^{N}\int_{S^n}(\delta\bar{A}^{i}_{k}\cdot\delta\bar{i}^{\ell,n}_{m} + \delta\hat{\phi}^{i}_{k}\delta q^{\ell,n}_{m})dS^{n}\right],\ i \neq \ell \qquad \text{III.33}$$

Equations III.26 and III.27 formulate the elements of the field coupling impedance matrix [Z(F)] which relates the scalar potentials $\hat{\phi}(F)^{i}_{k}$ at the terminals, caused by field coupling, to the impressed currents:

$$[\hat{\phi}(F)] = [Z(F)][I],\ \hat{\phi}(F)^{i}_{k} = \sum_{\ell=1}^{N}\sum_{m=1}^{M_\ell}\hat{\phi}(F)^{i,\ell}_{k,m} = \sum_{\ell=1}^{N}\sum_{m=1}^{M_\ell}Z(F)^{i,\ell}_{k,m}\, I^{\ell}_{m} \qquad \text{III.34}$$

IV. Complete Impedance Matrix $[\bar{Z}]$ of the Diakopted Antenna

The intrinsic impedance matrices of the individual structure elements can be combined into diagonal block impedance matrix $[\bar{Z}(I)]$ by writing the matrix elements $\bar{Z}_{k,j}$ (Eq. III.19) in the form $\bar{Z}^{i,i}_{k,j}(I)$. The superscript i identifies the terminals k and j as belonging to the element. The block matrix $[Z(I)]$ whose elements $Z^{i,\ell}_{k,j}(I)$ are zero for $i \neq \ell$ is the "current coupling matrix" of the diakopted system, and relates the terminal potentials

$$\hat{\Phi}^{i}_{k}(I) = \sum_{i=1}^{N} \sum_{j=1}^{M_i} Z^{i,i}_{k,j}(I)\; I^{i}_{j}$$

due to current coupling to the M_i impressed currents of the element i.

The sum of the matrices $[Z(I)]$ and $[Z(F)]$, i.e.

$$[\bar{Z}] = [Z(I)] + [Z(F)] \qquad \text{IV.1}$$

forms the "complete impedance matrix" of the diakopted antenna, which formulates the relationship between the total terminal potentials

$$\hat{\Phi}^{i}_{k} = \sum_{\ell=1}^{M} \sum_{m=1}^{M_\ell} \hat{\Phi}^{i,\ell}_{k,m}$$

produced by current and field coupling, to all impressed currents. In matrix form

$$[\bar{\hat{\Phi}}] = [\bar{Z}][I] \qquad \text{IV.2}$$

If the matrix elements $Z^{i,i}_{k,j}(I)$ (eq. III.19) and $Z^{i,i}_{k,j}(F)$ (eq. III.26) are added, the resulting elements $\bar{Z}^{i,i}_{k,j}$ have the same formulation as those which pertain to field coupling between two different elements (eq. III.33). In other words, if the condition $i \neq \ell$ is dropped, equation III.33 can be used as the general formulation for all the elements of the complete impendance matrix of the diakopted system.

Calculation of the impedances according to eq. III.33 requires, in principle, computation of the scatter current and charge distributions. However, numerical results obtained with this theory indicate that coupling by the scatter currents is a negligible effect. It has been found that coupling by the junction currents prevails over field coupling, and field coupling by the non-Maxwellian fields dominates over that by the (Maxwellian) scatter fields. In principle, the field coupling effect by the scatter currents can be obtained with an iterative procedure which is not discussed here.

If coupling by the scatter fields is neglected, the formula for the elements of the complete impedance matrix for the diakopted system reduces to

$$\bar{Z}^{i,\ell}_{k,m} = \frac{j\omega}{K^i_k I^\ell_m} \int_{S^i} (\bar{A}^\ell_m \cdot \bar{I}^i_k + \phi^\ell_m q^i_k) dS^i \qquad \begin{array}{l} i,\ell = 1,2,\ldots, N \\ k = 1,\ldots, M_i \\ m = 1,\ldots, M_\ell \end{array} \qquad \text{IV.3}$$

Thus all the matrix elements can be computed from the dominant current distributions.

The symmetry of the $[\bar{Z}]$, i.e.

$$\bar{Z}^{i,\ell}_{k,m} = \bar{Z}^{\ell,i}_{m,k} \qquad \text{IV.4}$$

can be easily verified, by expressing in (III.33) the vector and scalar potentials by the current and charge distributions according to (III.1) and (III.12).

Equation (III.37) represents a stationary formulation of the matrix elements of $[\bar{Z}]$. This means first order errors in the current and charge distributions lead to second order errors in the impedances (Appendix 4).

V. Interconnection of Diakopted Elements to Obtain Impedance of Assembled Multi Element Antenna

The requirement for the diakopted structure with impressed currents to be performancewise identical with the assembled antenna are that:

a) The sum of the impressed currents is zero at every junction between the structure elements and the continuity condition is satisfied at every input terminal. This requirement assures that the field of assembled antenna is Maxwellian.

b) The scalar potentials at the interconnected terminals are equal.

c) The potential difference between the input in terminals is equated with the driving voltage of the antenna source.

Imposing these junction conditions; the matrix equation (IV.2) yields a system of linear equations for the unknown junction currents and the input impedance of the antenna. Using network theory concepts the reduction of (IV.2) to this linear system of equations by enforcing the junction conditions can be formulated with a connection matrix [C] which reduces the number of potentials and currents of the diakopted structure to those of the actual structure [3]. As discussed in Section II, the impedance of the actual assembled antenna can be written as:

$$[\bar{Z}]' = [C]_t[\bar{Z}][C]$$

where $[\bar{Z}]$ and $[\bar{Z}]'$ refers to the actual and diakopted structure respectively. The following example shows how [C] and $[\bar{Z}]'$ are obtained.

<u>Example</u>

As an example we apply the diakoptic theory to an ordinary thin-wire dipole antenna and compare the results with the exact data available in the literature. To obtain a multielement structure we cut each wire in halves, as shown in Figure 8, and consider each half as a structure element. The diakopted dipole comprises two structure elements with one terminal and two structure elements with two terminals, so that the total number of terminals is six. The complete impedance matrix of the diakopted structure [Z] is therefore a 6x6 matrix. However there are only 8 different impedances because the four structure elements have been assumed to be alike.

Using the enumerations of Figure 8 the matrix equation (1.2) has the form

$$\begin{bmatrix} \hat{\phi}_1^3 \\ \hat{\phi}_2^1 \\ \hat{\phi}_1^1 \\ \hat{\phi}_2^2 \\ \hat{\phi}_1^2 \\ \hat{\phi}_2^4 \end{bmatrix} = \begin{bmatrix} Z_0 & Z_1 & Z_3 & Z_4 & Z_6 & Z_7 \\ Z_1 & Z_0 & Z_2 & Z_3 & Z_5 & Z_6 \\ Z_3 & Z_2 & Z_0 & Z_1 & Z_3 & Z_4 \\ Z_4 & Z_3 & Z_1 & Z_0 & Z_2 & Z_3 \\ Z_6 & Z_5 & Z_3 & Z_2 & Z_0 & Z_1 \\ Z_7 & Z_6 & Z_4 & Z_3 & Z_1 & Z_0 \end{bmatrix} \begin{bmatrix} I_1^3 \\ I_2^1 \\ I_1^1 \\ I_2^2 \\ I_1^2 \\ I_2^4 \end{bmatrix}$$

with

$$Z_0 = Z_{11}^{33} = Z_{22}^{11} = Z_{11}^{11} = Z_{22}^{22} = Z_{11}^{22} = Z_{22}^{44}$$

$$Z_1 = Z_{12}^{31} = Z_{21}^{13} = Z_{12}^{12} = Z_{21}^{21} = Z_{12}^{24} = Z_{21}^{42}$$

$$Z_2 = Z_{21}^{11} = Z_{12}^{11} = Z_{21}^{22} = Z_{12}^{22}$$

$$Z_3 = Z_{11}^{31} = Z_{11}^{13} = Z_{11}^{12} = Z_{11}^{21} = Z_{22}^{12} = Z_{22}^{21} = Z_{22}^{24} = Z_{22}^{42}$$

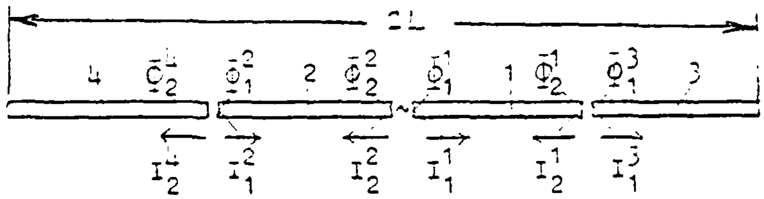


FIGURE 8

Thin Wire Dipole Treated as a Diakopted Four Element System

$$Z_4 = Z^{32}_{12} = Z^{23}_{21} = Z^{14}_{12} = Z^{41}_{21}$$

$$Z_5 = Z^{12}_{21} = Z^{21}_{12}$$

$$Z_6 = Z^{32}_{11} = Z^{23}_{11} = Z^{14}_{22} = Z^{41}_{22}$$

$$Z_7 = Z^{34}_{12} = Z^{43}_{21}$$

If coupling by the scatter currents is neglected, the darkened portion of the impedance matrix Z is the current coupling matrix Z(I).

The interconnection conditions require

$$I^3_1 = - I^1_2 = I_1 \qquad \hat{\Phi}^3_1 = \hat{\Phi}^1_2 = \Phi_1$$

$$I^2_2 = - I^1_1 = -I_0 \qquad \hat{\Phi}^1_1 - \hat{\Phi}^2_2 = V_0$$

$$I^4_2 = - I^2_1 = I_2 \qquad \hat{\Phi}^4_2 = \hat{\Phi}^2_1 = \Phi_3$$

where I_0, V_0 are input current and driving voltage of the antenna.

Because of the symmetry of the antenna

$$I_3 = I_2 \; ; \; \Phi_1 = - \Phi_3 \; ; \; \hat{\Phi}^2_2 = - \Phi^1_1$$

Current and voltage matrix of the interconnected antenna are

$$[I]' = \begin{bmatrix} I_0 \\ I_1 \end{bmatrix}, [\hat{\Phi}]' = \begin{bmatrix} V_0 \\ 0 \end{bmatrix}$$

Thus, the interconnection matrix becomes

$$[C]_t = \begin{array}{c} \\ \binom{1}{1} \\ \binom{3}{1} \end{array} \begin{array}{cccccc} \binom{3}{1} & \binom{1}{2} & \binom{1}{1} & \binom{2}{2} & \binom{2}{1} & \binom{4}{2} \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & -1 \end{array}$$

and the impedance matrix of the assembled antenna

$$[Z'] = 2 \begin{bmatrix} (Z_0 - Z_1) & (2Z_3 - Z_2 - Z_4) \\ (2Z_3 - Z_2 - Z_4) & (2Z_0 - 2Z_1 - Z_5 + 2Z_6 - Z_7) \end{bmatrix}$$

For the numerical calculation of the impedances, the following simplifying assumptions have been made:

a) coupling by the scatter currents is negligible

b) the dominant current distributions which, in this example, are the same for all the elements, can be approximated by linear current distributions (uniform charge distribution).

Although the latter approximation is rather crude, one should expect reasonable results if the wire sections are short compared with the wave length, because all the impedance formulas are stationary expressions. Linear current distribution permits analytic formulations of all the impedances Z_0, Z_1, Z_2 etc., and numerical calculations with a pocket calculator (such as HP 25). The results obtained are presented in Figure 9. The curves are plots (from a table by King [4]) of the real and the imaginary part of the input impedance of a dipole for $\ln \frac{2L}{a} = 5$ as a function of kL; 2L is the total length of the dipole, and a the wire radius. The crosses mark the values of the input impedance from (45) with the above assumptions. For $kL < 0.8$ the deviation of the real part of the input impedance from the exact value is less than 10% and for the imaginary part less than 1%. From this one can conclude that the linear approximation for the dominant current distribution is adequate if the length of a wire section is $<1/15\lambda$. This has been born out by computer results which were obtained when each dipole wire was diakopted into 4 equal sections. These results are marked in Figure 9 by dots and are in good agreement with the exact curves even beyond the resonance of the antenna.

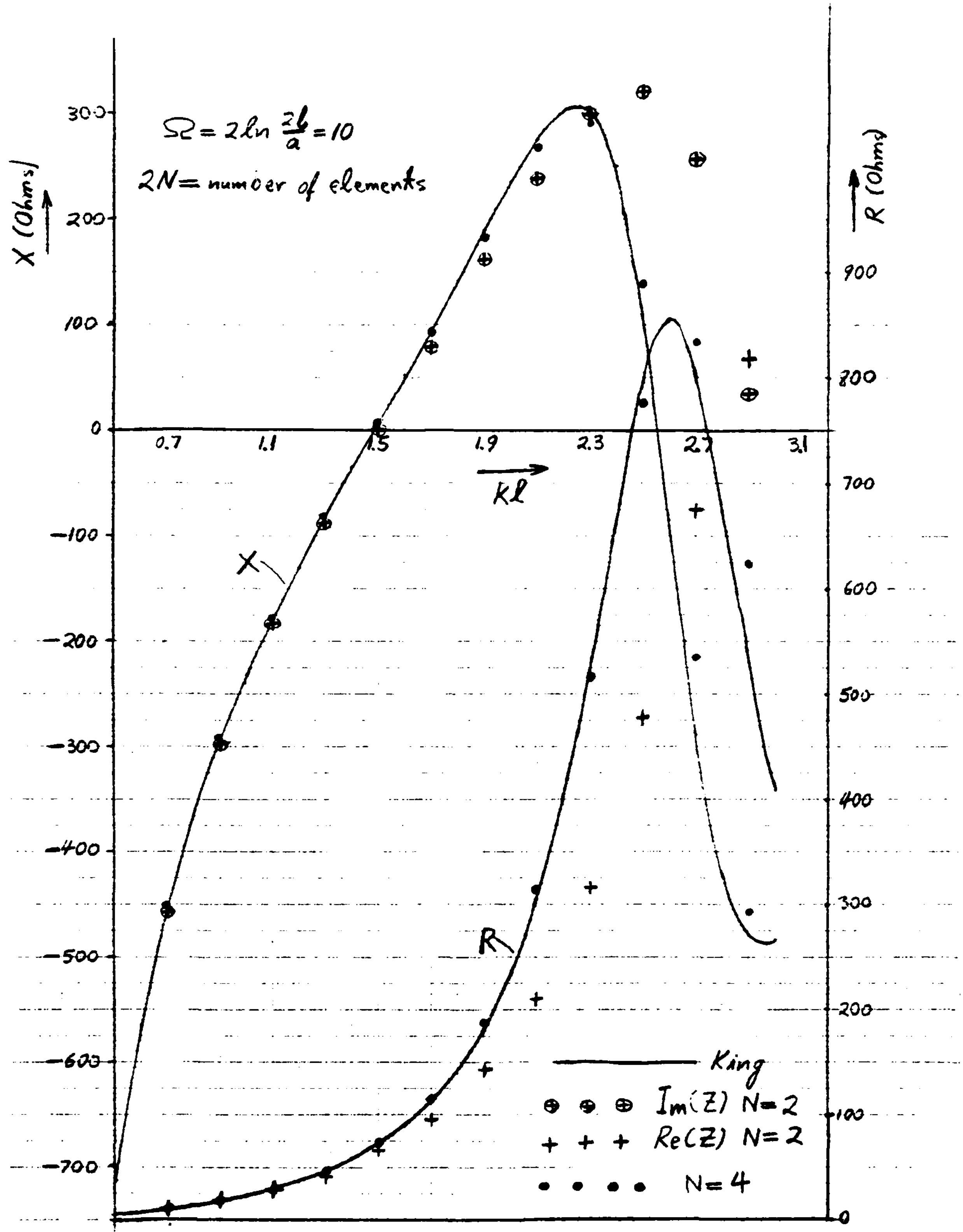


Fig. 9. : Comparison of Dipole Impedance Calculated with Diakoptic Theory vs. King

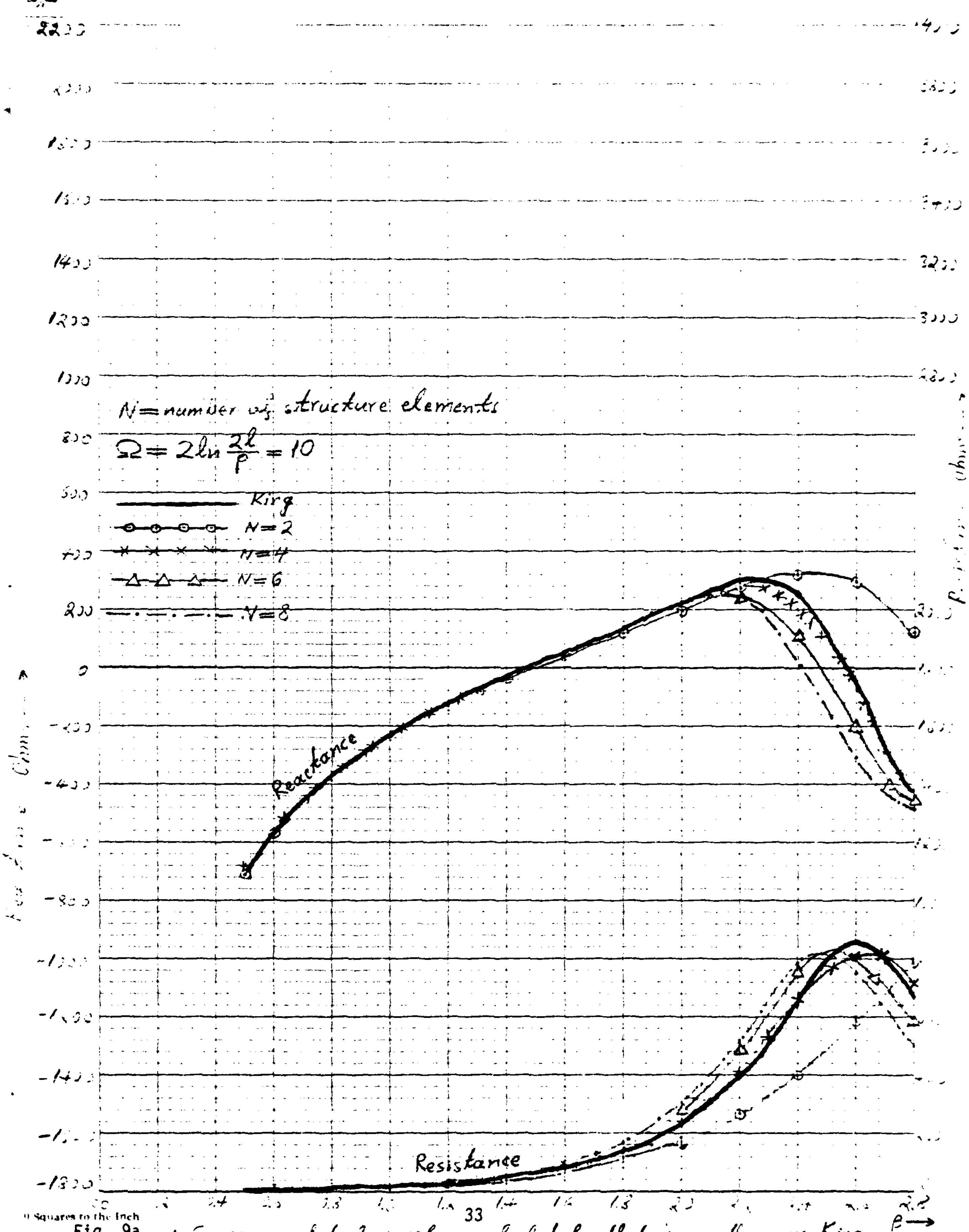


Fig. 9a. : Comparison of dipole impedance calculated with diakoptic theory vs. King

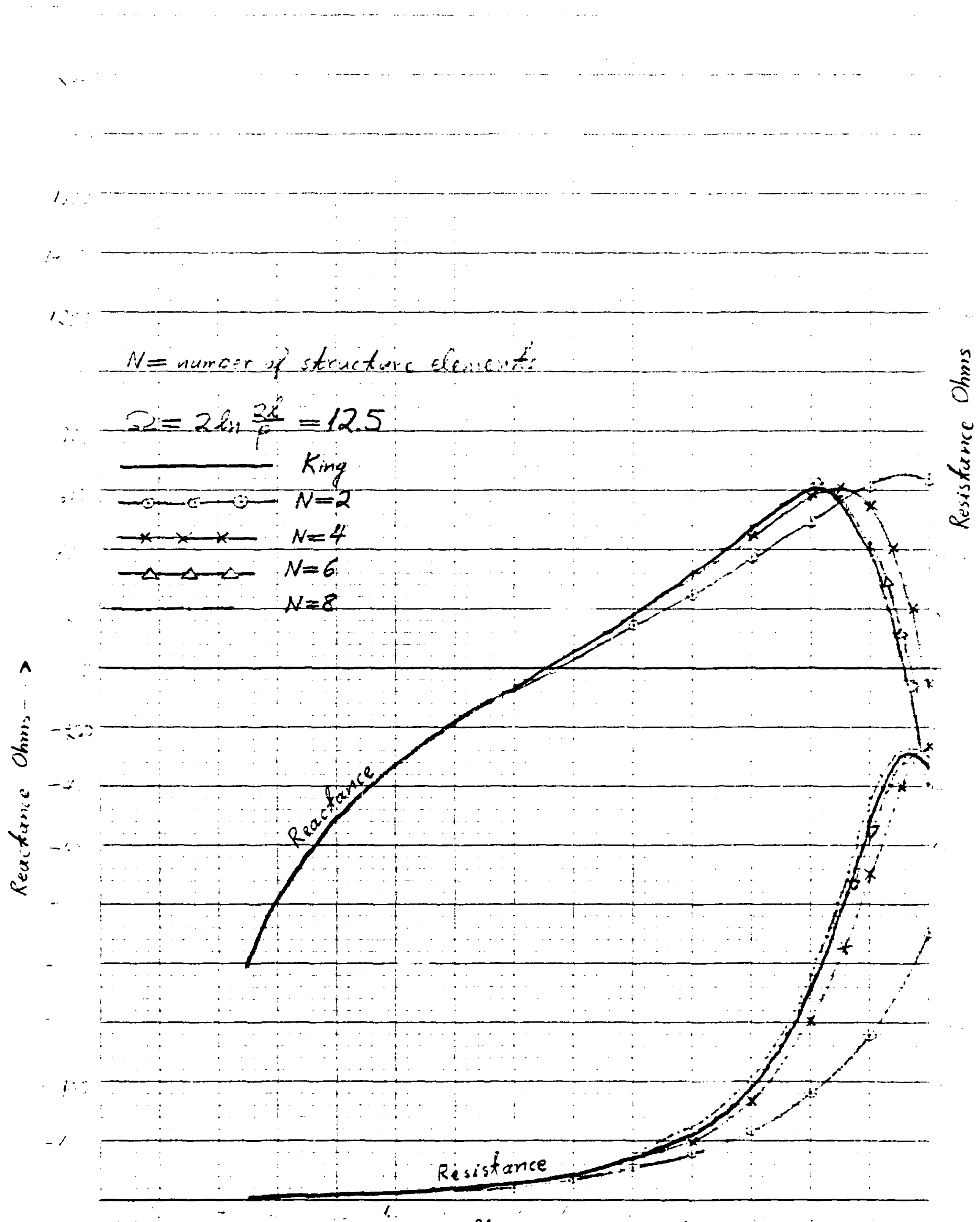


Fig. 9b. : Comparison of dipole impedance calculated with diakoptic theory vs King

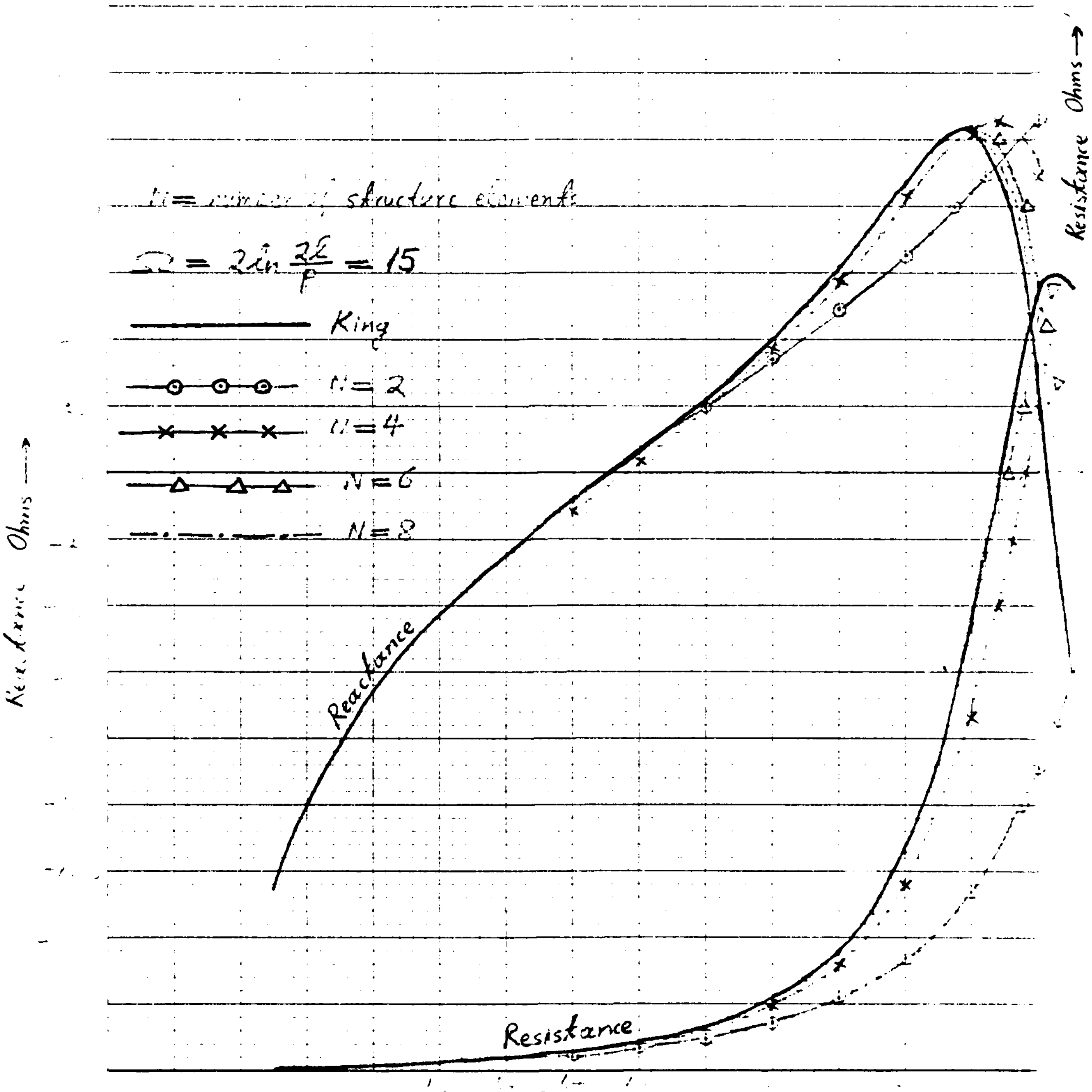


Fig. 9c. : Comparison of dipole impedance calculated with structure theory and King

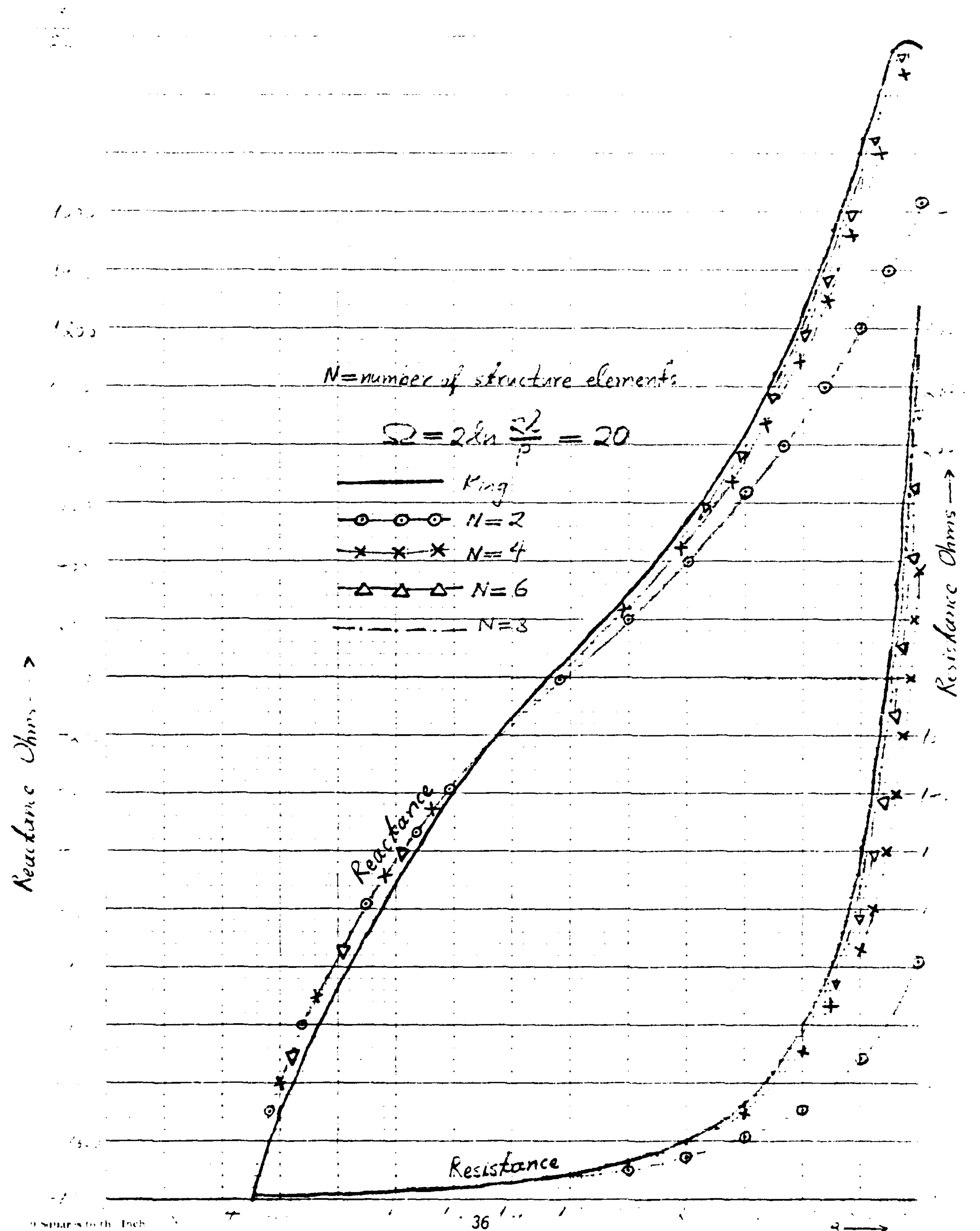

N=number of structure elements
$\Omega = 2\ln\frac{2l}{\rho} = 20$
King
N=2
N=4
N=6
N=3
Reactance
Resistance
Reactance Ohms
Resistance Ohms

Fig. 9d.

VI. Receiving Antennas

In the case of a receiving antenna the excitation is produced by an external field E(e). The source element is replaced by the input impedance of the receiver. All the quantities associated with the external field should be characterized by the subscript e.

The boundary conditions on the element i yields

$$\int_{S^i}\left\{\left[j\omega\bar{A}(e) + \bar{\nabla}\hat{\phi}(e) - \bar{E}(e)\right]\cdot\bar{i}(e)\right\}dS^i = 0$$

or

$$I_k^i\, \Phi_k^i(e) = j\omega\int_{S^i}(\bar{A}(e)\cdot\bar{i}_k^i + \hat{\Phi}(e)q_k^i)dS^i - \int_{S^i}\bar{E}(e)\ \bar{i}(e)dS^i \qquad \text{VI.1}$$

where $\bar{i}_k^i$, q_k^i are the dominant current and charge distributions which the impressed current I_k^i would produce on this element. $\Phi_k^i(e)$ the potential at the junction i,k caused by the external field. From the boundary condition for the dominant current distribution one obtains

$$\int_{S^i}\left[(j\omega\bar{A}_k^i + \text{grad}\ \hat{\Phi}_k^i)\cdot\bar{i}(e)\right]dS^i = j\omega\int_{S^i}(\bar{A}_k^i\cdot\bar{i}(e) + \hat{\Phi}_k^i q(e))dS^i = 0 \qquad \text{VI.2}$$

since $\bar{i}(e)$ is zero at the junction. As shown in Appendix 2

$$\int_{S^i}(\bar{A}_k^i\cdot\bar{i}(e) + \hat{\Phi}_k^i q(e))dS^i = \int_{S^i}(\bar{A}(e)\cdot\bar{i}_k^i + \hat{\Phi}(e)q_k^i)dS^i \qquad \text{VI.3}$$

Thus Equation (VI.1) reduces to

$$I_k^i\, \hat{\Phi}_k^i(e) = -\int \bar{E}(e)\bar{i}_k^i\ dS^i \qquad \text{VI.4}$$

The potential $\hat{\Phi}_k^i(e)$ produced by an external field at the junction i,k is given by the scalar product between the external field $\bar{E}(e)$ and the dominant current distribution function i_k^i/I_k^i, integrated over the surface S^i of the element. In the network presentation excitation by an external field is equivalent to voltage sources V(e) in series with the terminal voltages of the intrinsic impedance networks.

VII Numerical Results and Computer Programs

A. Cylindrical Wire

A.1. Dominant Current Distribution, Dominant Charge Distribution and Intrinsic Impedance Calculations

$i(x)$ = Current density

x = Source point

x' = Observation point

c = Velocity of light

λ = wave length

i_o = $i(o) = I_0/2\pi\rho$

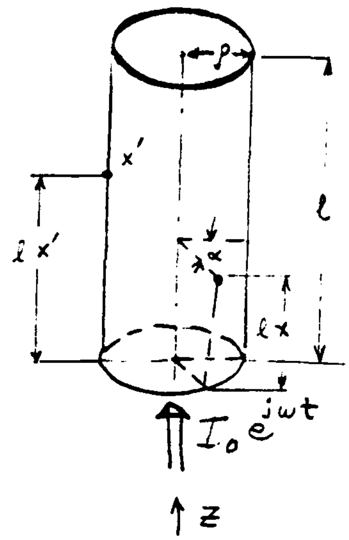
$i(x)$ = current distribution at x

ℓ = wire length

ρ = wire radius

$\omega = \frac{2\pi c}{\lambda}$, $k = \frac{2\pi}{\lambda}$, $k\ell = \beta$

$d = \sqrt{(x-x')^2 + 4\left(\frac{\rho}{\ell}\right)^2 \sin^2(\alpha/2)}$



The vector and scalar potentials at an observation point x are:

$$A_z(x) = \frac{\mu}{4\pi} \int_0^1 \int_0^{2\pi} i(x') \frac{e^{-j\beta d}}{d} \rho d\alpha dx'$$

$$\hat{\Phi}(x) = \frac{1}{4\pi\varepsilon\ell} \int_0^1 \int_0^{2\pi} \frac{1}{-j\omega} \frac{d}{dx'} i(x') \frac{e^{-j\beta d}}{d} \rho d\alpha dx'$$

Component of electric current field intensity parallel to the surface of the wiere is zero, and can be written as:

$$-\left(j\omega A_z(x) + \frac{1}{\ell} \frac{d\hat{\Phi}(x)}{dx}\right) = 0$$

or

$$\int_0^1 \int_0^{2\pi} \left[\frac{d}{dx'}\, i(x')\, \frac{d}{dx'} \left(\frac{e^{-j\beta d}}{d}\right) + \beta^2 i(x')\, \frac{e^{-j\beta d}}{d}\right] d\alpha dx' = 0 \qquad \text{VII.1}$$

Let

$$\frac{d}{dx}\, i(x) = a_j \cos\beta(1-x_j) \qquad x_{j-1}<x<x_j \qquad \text{VII.2}$$

$$x_j = \frac{1}{N}\, j \qquad j = 1,\ldots, N-1$$

Integrating:

$$i(x_{j-1}) = i(x_j) + \int_{x_j}^{x_{j-1}} a_j \cos\beta(1-x)dx$$

$$= i(x_j) + \frac{a_j}{\beta} (-\sin\beta(1-x_{j-1}) + \mathrm{Sin}\beta(1-x_j))$$

$$i(x_N) = 0$$

Thus

$$i(x) = \sum_{\lambda=j+1}^{N} \left[\frac{a_\lambda}{\beta}(-\sin\beta(1-x_{\lambda-1}) + \sin\beta(1-x_\lambda)) + \right.$$

$$\left. \frac{a_{j+1}}{\beta} (\sin\beta(1-x_j) - \sin\beta(1-x))\right] \qquad \text{VII.3}$$

Substituting VII.2 and VII.3 into VII.1 and choosing

$$x = \frac{(i-\frac{1}{2})}{N}, \quad i = 1,2,\ldots,N-1, \qquad \text{we obtain}$$

$$\sum_{j=1}^{N} a_j(g_{ji} + k_{ji} + \sum_{\lambda=1}^{j} \ell_{\lambda ji}) = 0 \qquad i = 1,\ldots,N-1$$

$$\text{VII.4}$$

$$\sum_{j=1}^{N} a_j(\frac{1}{\beta} \sin\beta(1-x_j) - \frac{1}{\beta} \sin\,(1-x_{j-1})) = i_o$$

where

$$g_{ji} = \left[\int_{x_{j-1}}^{x_j} \int_0^{\pi} \cos\beta(1-x') \frac{d}{dx'} \left(\frac{e^{-j\beta d}}{d}\right) \rho d\alpha dx' \right]_{x=x_i} \qquad \text{VII.5}$$

$$k_{ji} = \left[\int_{x_{j-1}}^{x_j} \int_0^{\pi} \beta \frac{e^{-j\beta d}}{d} \left(\sin\beta(1-x_{j-1}) - \sin\beta(1-x)\right) \rho d\alpha dx' \right]_{x=x_i} \qquad \text{VII.6}$$

$$\ell_{\lambda ji} = \left[\int_{x_{j-1}}^{x_j} \int_0^{\pi} \beta \frac{e^{-j\beta d}}{d} \left(\sin\beta(1-x_{\lambda}) - \sin\beta(1-x_{\lambda-1})\right) \rho d\alpha dx' \right]_{x=x_j} \qquad \text{VII.7}$$

Equation VII.4 is simultaneously solved to obtain $a_1,\ldots,a_N$ and hence the current and charge densities.

Quantities VII.5 to VII.7 are computed by quadrature integration formula. These expressions can be considerably simplified when $i \neq j$, resulting in computation saving.

A.2 Impedance Calculations

$$Z = \frac{\hat{\Phi}(x)}{I_o} \Big|_{x=0} = \frac{j\omega}{I_o^2} 2\pi\rho\ell \int_0^1 i(x') A_z(x') + \frac{j}{\omega} \frac{d}{dx'} i(x') \hat{\Phi}(x') dx' \qquad \text{VII.8}$$

$$Z = \frac{j}{4\pi^2\beta i_o^2} \left(\frac{\mu}{\varepsilon}\right)^{1/2} \int_0^1 \int_0^1 \int_0^{\pi} \Big[\beta^2 i(x) i(x') \frac{e^{-j\beta d}}{d}$$

$$- \frac{d}{dx} i(x) \frac{d}{dx'} i(x') \left(\frac{e^{-j\beta d}}{d} + j\beta\right)\Big] d\alpha dx dx'$$

or

$$Z = \frac{j}{4\pi\beta i_o^2} \left(\frac{\mu}{\varepsilon}\right)^2 \sum_{i=1}^{N} \sum_{j=1}^{N} \left[(\beta^2 i_i i_j - q_i q_j)\, d_{ij} - \frac{j\beta q_i q_j}{N^2}\right] \qquad \text{VII.8}$$

where

$$i_i = i(x) \Big]_{x = i - \frac{1}{2}} \quad , \quad q_i = \frac{d}{dx} i(x) \Big]_{x = i - \frac{1}{2}}$$

$$i_j = i(x') \Big]_{x' = j - \frac{1}{2}} \quad , \quad q_j = \frac{d}{dx} i(x') \Big]_{x' = j - \frac{1}{2}}$$

$$d_{ij} = \int_{x_{j-1}}^{x_j} \int_{x_{i-1}}^{x_i} \frac{e^{-j\beta d}}{d} \, dx dx'$$

```
'SOLUTION OF THE INTEGRAL EQUATION FOR THE CURRENT IN A DIPOLE OR MON
OPOLE
SUBROUTINE INTEG(A,B,C,D,LX,MY,X1,BETA,S,FUNCTI,RHO,DELTA,THETA)
REAL*8 Z,WEIGHT,X1,BETA,XI,YJ,RHO,DELTA,THETA
COMPLEX*16 FUNC,S,FUNCTI
DIMENSION Z(24),WEIGHT(24)
DATA Z/0.577350269,0.0,0.774596669,%
0.339981044,0.861136312,0.0,0.538469310,%
0.906179846,0.238619186,0.661209387,0.932469514,%
0.148874339,0.433395394,0.679409568,0.865063367,%
0.973906529,0.0,0.201194094,0.394151347,%
0.570972173,0.724417731,0.848206583,0.937273392,%
0.987992518/
DATA WEIGHT/1.0,0.888888889,0.555555556,%
0.652145155,0.347854845,0.568888889,0.478628671,%
0.236926885,0.467913935,0.360761573,0.171324493,0.295524225,0.269266
719,%
0.219086363,0.149451349,%
0.066671344,0.1012891,0.198431485,0.186161000,%
0.166269206,0.139570678,0.107159221,0.070366047,%
0.030753242/
S=(0.0,0.0)
DO 10 I=17,24
DO 10 I1=1,LX
DO 10 I2=1,2
DO 10 J=17,24
DO 10 J1=1,MY
DO 10 J2=1,2
30 STEPY=(D-C)/MY
D1=C+STEPY*J1
C1=D1-STEPY
STEPX=(B-A)/LX
B1=A+STEPX*I1
A1=B1-STEPX
XI=((-1)**I2*Z(I)*(B1-A1)+B1+A1)/2
YJ=((-1)**J2*Z(J)*(D1-C1)+D1+C1)/2
FUNC=FUNCTI(X1,XI,YJ,BETA,RHO,DELTA,THETA)
10 S=S+(B1-A1)*(D1-C1)/4*WEIGHT(I)*WEIGHT(J)*FUNC
RETURN
END
SUBROUTINE SIMQ (A,B,N,KS)
REAL*8 A,B,BIGA,TOL,SAVE
DIMENSION A(N,N),B(N)
'FORWARD SOLUTION'
TOL=0.0
KS=0
DO 65 J=1,N
JY=J+1
BIGA=0
DO 30 I=J,N
'SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN'
IF(DABS(BIGA).GE.DABS(A(I,J)))GO TO 30
BIGA=A(I,J)
```

```
520 IMAX=I
530 30 CONTINUE
540 'TEST FOR PIVOT LESS THAN TOLERANCE   SINQULAR MATRIX'
550 IF(DABS(BIGA)>TOL)GO TO 40
560 KS=1
570 RETURN
580 'INTERCHANGE ROWS IF NECESSARY'
590 40 I1=J+N*(J-2)
600 DO 50 K=J,N
610 SAVE=A(J,K)
620 A(J,K)=A(IMAX,K)
630 A(IMAX,K)=SAVE
640 'DIVIDE EQUATION BY LEADING COEFFICIENT'
650 50 A(J,K)=A(J,K)/BIGA
660 SAVE=B(IMAX)
670 B(IMAX)=B(J)
680 B(J)=SAVE/BIGA
690 'ELIMINATE NEXT VARIABLE'
700 IF(J=N)GO TO 70
710 DO 65 IX=JY,N
720 DO 60 JX=JY,N
730 60 A(IX,JX)=A(IX,JX)-A(IX,J)*A(J,JX)
740 65 B(IX)=B(IX)-(B(J)*A(IX,J))
750 'BACK SOLUTION'
760 70 NY=N-1
770 IT=N*N
780 DO 80 J=1,NY
790 IB=N-J
800 IC=N
810 DO 80 K=1,J
820 B(IB)=B(IB)-A(IB,IC)*B(IC)
830 80 IC=IC-1
840 RETURN
850 END
860 SUBROUTINE INTEGR(INITIA,FINAL,NUMINT,KERNEL,RESUL,X1,X2,BETA,RHO,DE
LTA,THETA)
870 REAL*8 Z(7),WEIGHT(7),INITIA,FINAL,INTERV,A,B,BETA,X1,RHO,X2,DELTA,T
HETA
880 COMPLEX*16 S,RESUL,KERNEL
890 DATA Z /0.201194094,0.394151347,%
900          0.570972173,0.724417731,0.848206583,0.937273392,%
910          0.987992518/
920 '
930 '
940 DATA WEIGHT /0.198431485,0.186161000,%
950                0.166269206,0.139570678,0.107159221,0.070366047,%
960                0.030753242/
970 '
980 INTERV=(FINAL-INITIA)/NUMINT
990 A=INITIA
1000 RESUL=(0.0D00,0.0D00)
1010 DO 20 J=1,NUMINT
1020 B=A+INTERV
```

```
*
S=(0.0D00,0.0D00)
DO 10 I=1,7
10 S=S+WEIGHT(I)*(KERNEL(X1,X2    ,(Z(I)*(B-A)+B+A)/2,    BETA,RHO,DE
LTA,THETA)+KERNEL(X1,X2    ,(-Z(I)*(B-A)+B+A)/2,    BETA,RHO,DELTA,THETA))
S=S+0.202578242*KERNEL(X1,X2    ,(B+A)/2,    BETA,RHO,DELTA,THETA)
S=(B-A)/2.0*S
A=A+INTERV
20 RESUL=RESUL+S
RETURN
END
*
*
FUNCTION G(X1,X2,ALPHA,BETA,RHO,DELTA,THETA)
COMPLEX*16 G,E,E1
REAL*8 X1,X2,BETA,RHO,ALPHA,D,D1,DELTA,THETA
D=DSQRT((X1-X2)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
D1=DSQRT((X1+X2+DELTA)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
E=(1.0D00,0.0D00)*DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
E1=(1.0D00,0.0D00)*DCOS(BETA*D1)-(0.0D00,1.0D00)*DSIN(BETA*D1)
G=(-E/D**2-(0.0D00,1.0D00)*BETA*E/D)*(X1-X2)/D-THETA*(-E1/D1**2-(0.
0D00,1.0D00)*BETA*E1/D1)*(X1+X2)/D1
RETURN
END
*
FUNCTION H(X1,X2,ALPHA,BETA,RHO,DELTA,THETA)
COMPLEX*16 H,E,E1
REAL*8 X1,X2,BETA,RHO,ALPHA,D,D1,DELTA,THETA
D=DSQRT((X1-X2)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
D1=DSQRT((X1+X2+DELTA)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
E=(1.0D00,0.0D00)*DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
E1=(1.0D00,0.0D00)*DCOS(BETA*D1)-(0.0D00,1.0D00)*DSIN(BETA*D1)
H=E/D+THETA*E1/D1
RETURN
END
FUNCTION HPR(ALPHA,X1,X2,BETA,RHO,DELTA,THETA)
COMPLEX*16 HPR,E,E1
REAL*8 X1,X2,BETA,RHO,ALPHA,D,D1,DELTA,THETA
D=DSQRT((X1-X2)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
D1=DSQRT((X1+X2+DELTA)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
E=(1.0D00,0.0D00)*DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
E1=(1.0D00,0.0D00)*DCOS(BETA*D1)-(0.0D00,1.0D00)*DSIN(BETA*D1)
HPR=E/D+THETA*E1/D1
RETURN
END
FUNCTION H1(ALPHA,X1,X2,BETA,RHO,DELTA,THETA)
COMPLEX*16 H1,E,E1
REAL*8 X1,X2,BETA,RHO,ALPHA,D,D1,DELTA,THETA
D=DSQRT((X1-X2)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
D1=DSQRT((X1+X2+DELTA)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
E=(1.0D00,0.0D00)*DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
E1=(1.0D00,0.0D00)*DCOS(BETA*D1)-(0.0D00,1.0D00)*DSIN(BETA*D1)
H1=E/D-THETA*E1/D1+(1.0-THETA)*(0.0D00,1.0D00)*BETA
RETURN
```

```
END
FUNCTION IMPKER(X1,X2,ALPHA,BETA,RHO,DELTA,THETA)
COMPLEX*16 IMPKER,E,E1
REAL*8 X1,X2,ALPHA,BETA,RHO,D,DELTA,THETA,D1
D=DSQRT((X1-X2)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
D1=DSQRT((X1+X2+DELTA)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
E=(1.0D00,0.0D00)*DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
E1=(1.0D00,0.0D00)*DCOS(BETA*D1)-(0.0D00,1.0D00)*DSIN(BETA*D1)
IMPKER=DCOS(BETA*(1.0-X2))*(E/D-THETA*E1/D1+(1.0-THETA)*(0.0D00,1.0
D00)*BETA)
RETURN
END
EXTERNAL G,H,IMPKER,H1,HPR
COMPLEX*16 RESG,RESH,G,H,A(40,40),AB(40),RESUL,CHAR(30),CURR,IMPKER
,Z,SUMH,ZPR,RESU1,CUR(30),IMP,H1,RESH1,CHARGE,HPR
REAL*8 X1,X2,STEP,BETA,RHO,X,REALPA,IMAGPA,REAATB,IMAATB,RA(1600),R
B(50),ATA(1600),ATB(50),MAGCHA,MAGCUR,IDCHA,IDCUR,DELTA,THETA,ALPHA
WRITE (6,2200)
READ (5,*) N,BETA,RHO,DELTA,THETA
IF (THETA-0.5) 171,171,172
171 WRITE (6,1171)
GO TO 173
172 WRITE (6,1172)
173 WRITE (6,999) BETA,RHO,DELTA
STEP=1.0/N
NTIM2=2*N
NMIN1=N-1
X1=STEP
DO 10 I=1,NMIN1
X2=STEP/2.0
SUMH=(0.0,0.0)
DO 20 J=1,N
CALL INTEGR(0.0,3.1415,1,G,RESG,X1,X2,BETA,RHO,DELTA,THETA)
CALL INTEGR(0.0,3.1415,1,H,RESH,X1,X2,BETA,RHO,DELTA,THETA)
SUMH=SUMH+RESH
A(I,J)=(DSIN(BETA*(1.0-(X2-STEP/2.0)))-DSIN(BETA*(1.0-(X2+STEP/2.0)
)))/BETA*RESG+(DSIN(BETA*(1.0-(X2-STEP/2.0)))-DSIN(BETA*(1.0-X2)))*BETA/
N*RESH+(DSIN(BETA*(1.0-(X2+STEP/2.0)))-DSIN(BETA*(1.0-(X2-STEP/2.0))))*B
ETA/N*SUMH
20 X2=X2+STEP
10 X1=X1+STEP
X=STEP
DO 30 J=1,N
A(N,J)=(DSIN(BETA*(1.0-X))-DSIN(BETA*(1.0-(X-STEP))))/BETA
30 X=X+STEP
DO 40 I=1,N
40 AB(I)=I/N*1.0
DO 50 I=1,N
DO 50 J=1,N
REALPA=(A(I,J)+DCONJG(A(I,J)))/2.0D00
IMAGPA=(A(I,J)-DCONJG(A(I,J)))/2.0D00/(0.0D00,1.0D00)
REAATB=(AB(I)+DCONJG(AB(I)))/2.0D00
IMAATB=(AB(I)-DCONJG(AB(I)))/2.0D00/(0.0D00,1.0D00)
RA(I+(J-1)*NTIM2)=REALPA
```

```
RA(I+(J+N-1)*NTIM2)=-IMAGPA
RA(I+N+(J-1)*NTIM2)=IMAGPA
RA(I+N+(J+N-1)*NTIM2)=REALPA
RB(I)=REAATB
RB(I+N)=IMAATB
50 CONTINUE
DO 410 I=1,NTIM2
DO 410 J=1,NTIM2
ATA((J-1)*NTIM2+I)=0.0D00
DO 410 K=1,NTIM2
410 ATA((J-1)*NTIM2+I)=ATA((J-1)*NTIM2+I)+RA((I-1)*N*2+K)*RA((J-1)*
N*2+K)
DO 420 I=1,NTIM2
ATB(I)=0.0D00
DO 420 K=1,NTIM2
420 ATB(I)=ATB(I)+RA((I-1)*N*2+K)*RB(K)
DO 430 I=1,NTIM2
430 RB(I)=ATB(I)
CALL SIMQ(ATA,RB,NTIM2,KS)
WRITE (6,3000)
WRITE (6,100)
DO 110 I=1,N
110 WRITE (6,*) RB(I),RB(I+N)
WRITE (6,100)
WRITE (6,2000)
WRITE (6,6000)
X=STEP/2.0
DO 150 I=1,N
CHAR(I)=(RB(I)*(1.0D00,0.0D00)+RB(I+N)*(0.0D00,1.0D00))*DCOS(BETA*(
1.0-X))
CHARGE=CHAR(I)/BETA
MAGCHA=CDABS(CHAR(I))
MAGCHA=MAGCHA/BETA
WRITE (6,5000) X,CHARGE,MAGCHA
150 X=X+STEP
WRITE (6,100)
WRITE (6,2100)
WRITE (6,6000)
CURR=(0.0D00,0.0D00)
X=1.0
DO 160 I=1,N
CURR=CURR+(DSIN(BETA*(1.0-X))-DSIN(BETA*(1.0-(X-STEP))))/BETA*(RB(N
-I+1)*(1.0D00,0.0D00)+RB(N-I+1+N)*(0.0D00,1.0D00))
X=X-STEP
MAGCUR=CDABS(CURR)
CUR(N-I+1)=CURR
160 WRITE (6,5000) X,CURR,MAGCUR
WRITE (6,100)
WRITE (6,2300)
'ZPR IS THE POTENTIAL AT THE END
Z=(0.0D00,0.0D00)
ZPR=(0.0D00,0.0D00)
X2=0.0D00
DO 170 I=1,N
```

```
CALL INTEG(X2,X2+STEP,0.0,3.1415,1,1.0.0,BETA,RESUL,IMPKER,RHO,DELT
A.THETA)
CALL INTEG(X2,X2+STEP,0.0,3.1415,1,1,1.0,BETA,RESU1,IMPKER,RHO,DELT
A.THETA)
Z=Z+RESUL*(RB(I)*(1.0D00,0.0D00)+RB(I+N)*(0.0D00,1.0D00))
ZPR=ZPR+RESU1*(RB(I)*(1.0D00,0.0D00)+RB(I+N)*(0.0D00,1.0D00))
170 X2=X2+STEP
Z=Z*377.0*(0.0D00,1.0D00)/4.0/3.1415**2/BETA
ZPR=ZPR*377.0*(0.0D00,1.0D00)/4.0/3.1415**2/BETA
ALPHA=1.0471
IMP=(0.0D00,0.0D00)
X1=STEP/2.0
DO 66 J=1,N
X2=STEP/2.0
DO 55 I=1,N
D=DSQRT((X1-X2)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
D1=DSQRT((X1+X2+DELTA)**2+(2.0*RHO*DSIN(ALPHA/2.0))**2)
E=(1.0D00,0.0D00)*DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
E1=(1.0D00,0.0D00)*DCOS(BETA*D1)-(0.0D00,1.0D00)*DSIN(BETA*D1)
CALL INTEG(X1-STEP/2.0,X1+STEP/2.0,X2-STEP/2.0,X2+STEP/2.0,1,1,0.78
54,BETA,RESH,HPR,RHO,DELTA,THETA)
CALL INTEG(X1-STEP/2.0,X1+STEP/2.0,X2-STEP/2.0,X2+STEP/2.0,1,1,0.78
54,BETA,RESH1,H1,RHO,DELTA,THETA)
IMP=IMP+BETA**2*CUR(I)*CUR(J)*RESH-((1.0D00,0.0D00)*RB(I)+(0.0D00,1
.0D00)*RB(I+N))*((1.0D00,0.0D00)*RB(J)+(0.0D00,1.0D00)*RB(J+N))*DCOS(BET
A*(1.0D00-X1))*DCOS(BETA*(1.0D00-X2))*RESH1
55 X2=X2+STEP
66 X1=X1+STEP
IMP=IMP*(0.0D00,1.0D00)*377.0/4.0/3.1415/BETA*(1.0D00+THETA)
WRITE (6,*) IMP
WRITE (6,2400)
WRITE (6,*) ZPR
100 FORMAT('      ')
2000 FORMAT(' DISTANCE            CHARGE')
2100 FORMAT(' DISTANCE           CURRENT')
6000 FORMAT(' DISTANCE  REAL   IMAG   MAGN')
2200 FORMAT(' N,BETA,RHO,DELTA,THETA')
2300 FORMAT(' IMPEDANCE')
5000 FORMAT(F6.3,3F8.3)
3000 FORMAT('  ')
2400 FORMAT(' POTENTIAL AT THE END')
1171 FORMAT(' MONOPOLE')
1172 FORMAT(' DIPOLE')
999 FORMAT(' BETA=',F8.4,'RADIUS=',F8.4,'GAP=',F8.4)
STOP
END
```

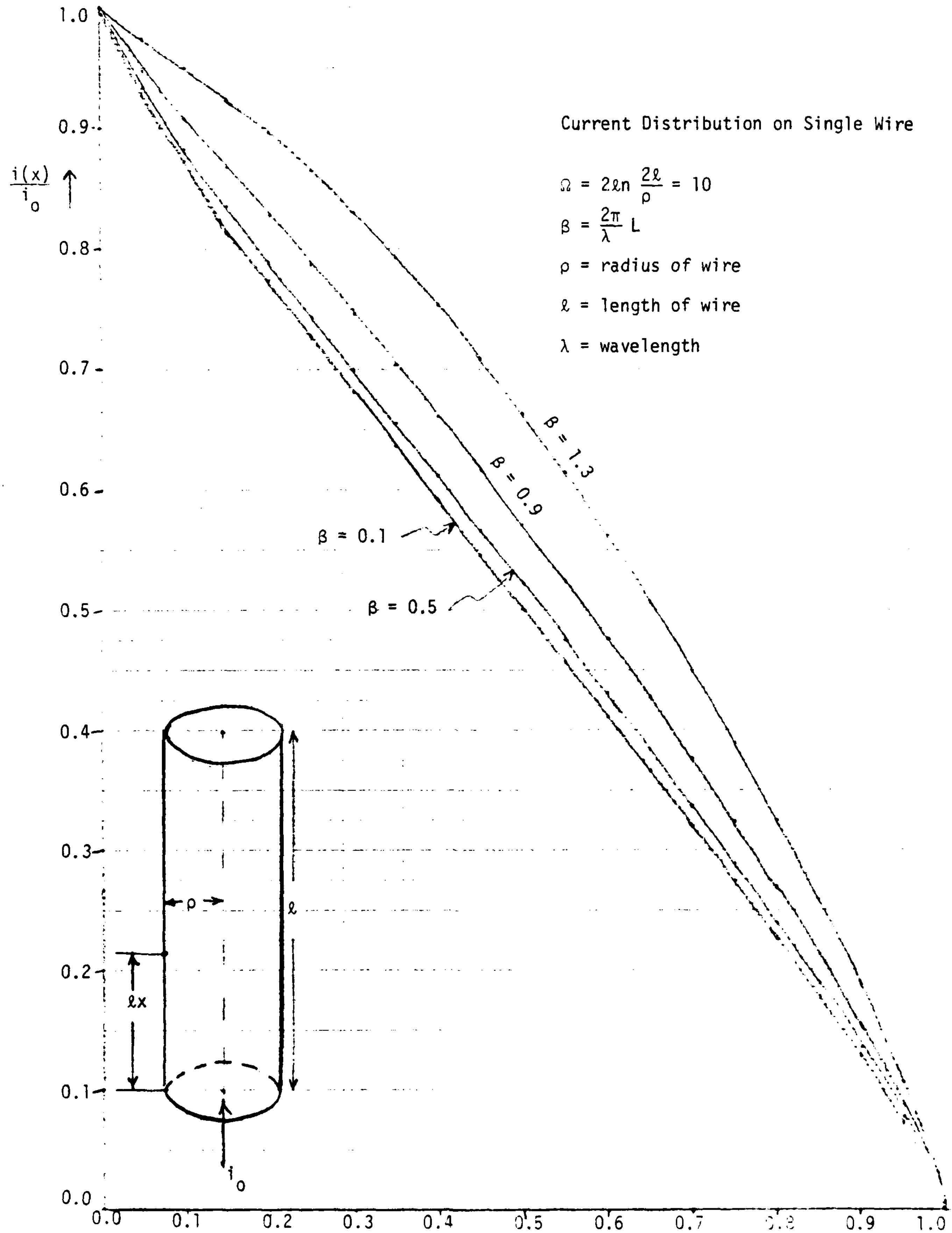


FIGURE 10 Current distribution of a cylindrical conductor with a current I_0 impressed at one end.

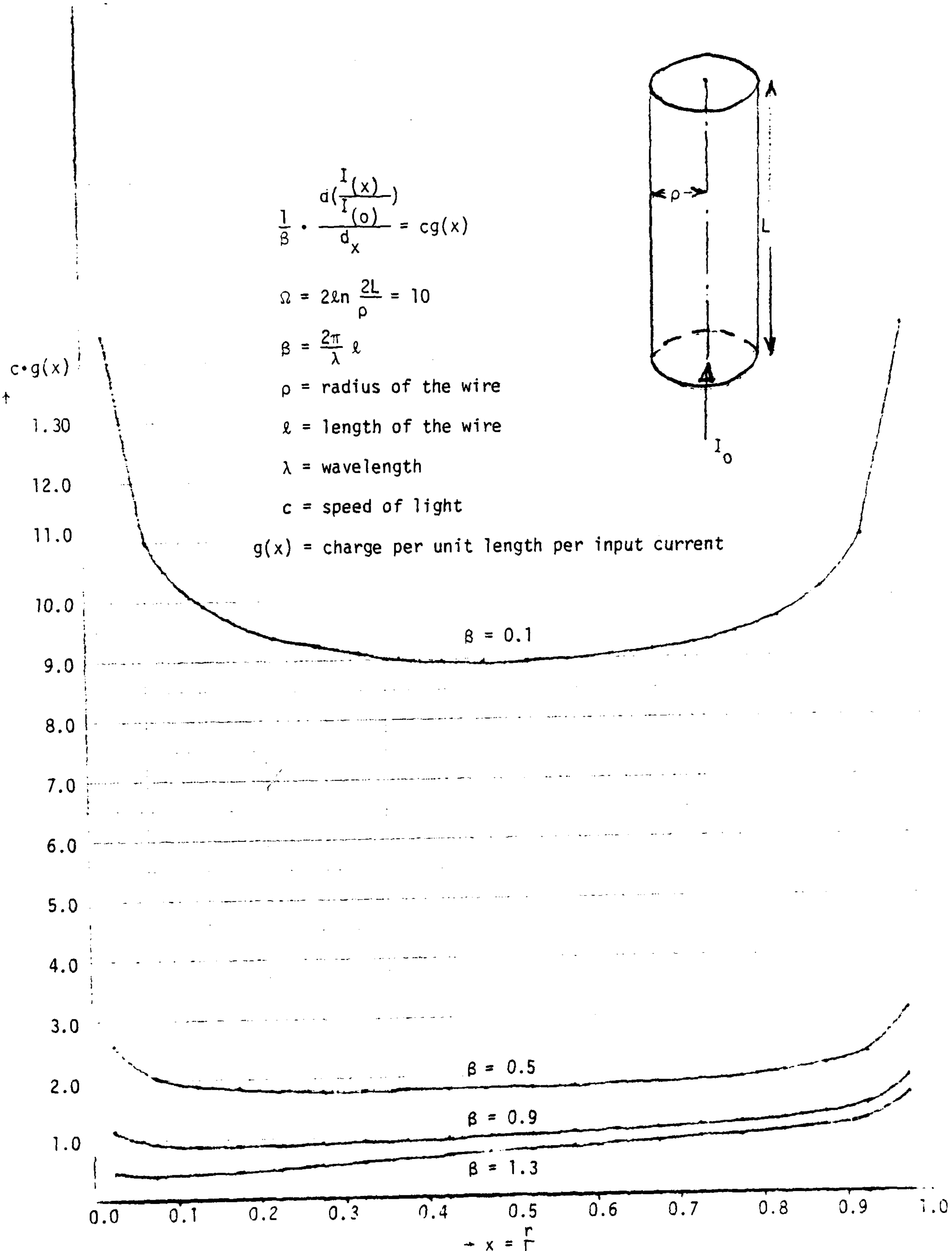


FIGURE 11 Charge distribution of a cylindrical conductor with a current I_o impressed at one end

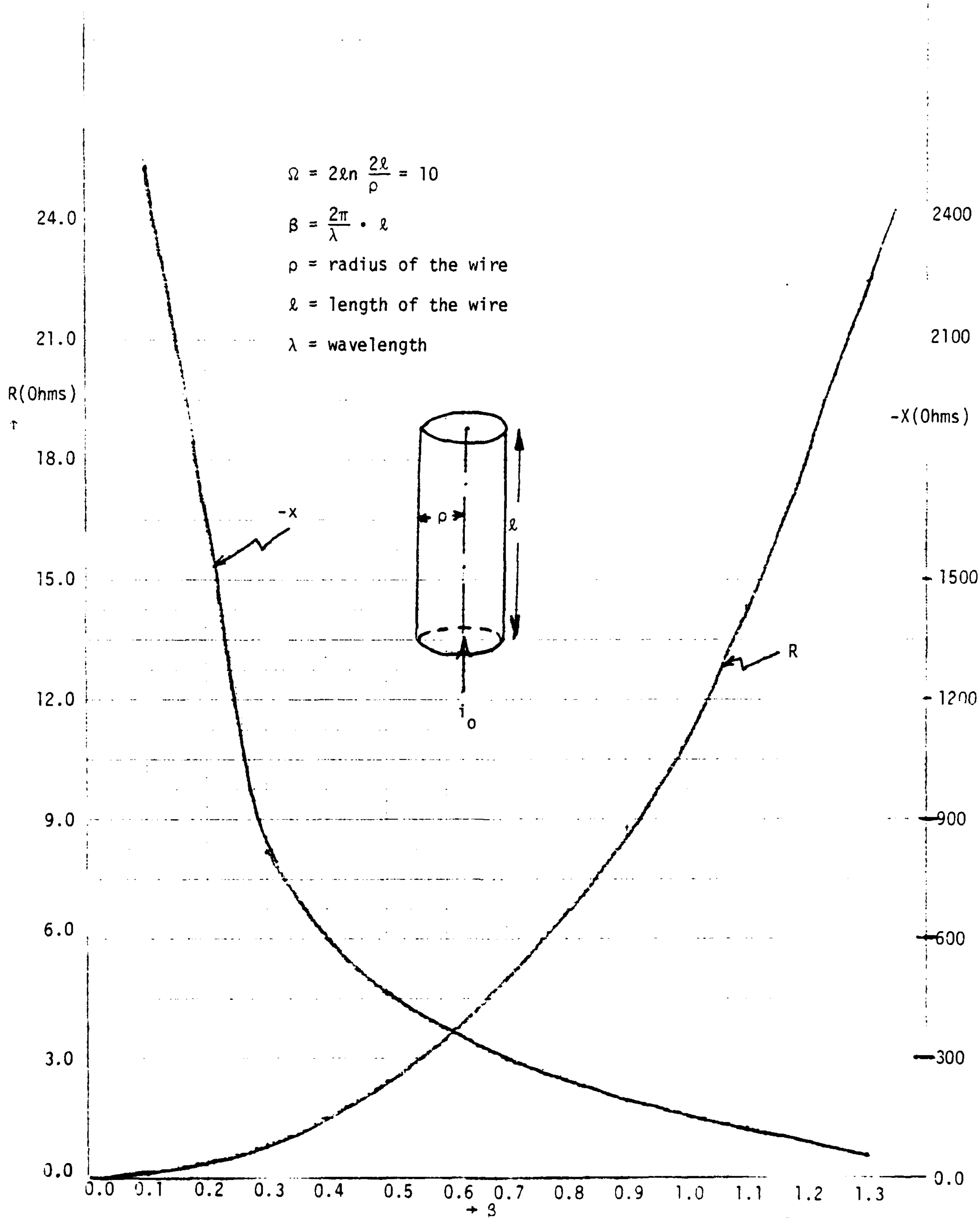
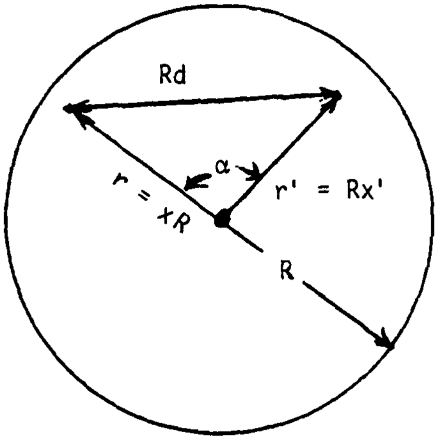


FIGURE 12 Input impedance of a cylindrical conductor with a current I_o impressed at one end

VIII. Dominant Current Distribution and Impedance of a Circular Disc Fed at the Center



$$2\pi x i(x) = I(x)$$

$$\beta = \frac{2\pi}{\lambda} R$$

$$A(x') = \frac{\mu}{4\pi^2} \int_0^1 \int_0^\pi I(x) \frac{e^{-j\beta d}}{d} \cos\alpha \, dx \, d\alpha \qquad \text{VIII.1}$$

$$\phi(x') = \frac{j}{4\pi^2 \varepsilon R\omega} \int_0^1 \int_0^\pi \frac{d}{dx} I(x) \frac{e^{-j\beta d}}{d} \, dx \, d\alpha \qquad \text{VIII.2}$$

Boundary condition

$$-j\omega A(x') - \frac{1}{R} \frac{d}{dx'} \phi(x') = 0 \qquad \text{VIII.3}$$

From the above three equations

$$\int_0^1 \int_0^\pi \left[\frac{d}{dx'} \left(\frac{e^{-j\beta d}}{d}\right) \frac{d}{dx} I(x) + \beta^2 \frac{e^{-j\beta d}}{d} \cos\alpha I(x)\right] dx \, d\alpha = 0 \qquad \text{VIII.4}$$

charge is zero at the center and takes the form of $(1-x^2)^{-1/2}$ at the edge. Therefore, we assume

$$\frac{d}{dx} I(x) = a_k \frac{x}{\sqrt{1-x^2}} \qquad x_{k-1} < x < x_k$$

Integrating

$$I(x_k) = \sum_{\lambda=k+1}^{N} a_\lambda \left(\sqrt{1-x_\lambda^2} - \sqrt{1-x_{\lambda-1}^2}\right) \qquad k = 0, \ldots, N-1$$

$$I(x) = I(x_k) + a_{k+1} \left(\sqrt{1-x_k^2} - \sqrt{1-x^2}\right) \qquad x_k < x \leq x_{k+1}$$

Equation VIII.4 can be reworked as:

$$\sum_{j=1}^{N} [a_j(g_j(x) + k_j(x)) + \sum_{\lambda=j}^{N} a_\lambda \ell_{\lambda j}(x)] = 0 \qquad 0 \leq x \leq 1 \qquad \text{VIII.5}$$

where

$$g_j(x) = \int_{x_{j-1}}^{x_j} \int_0^\pi \frac{d}{dx} \left(\frac{e^{-j\beta d}}{d}\right)\left(\frac{x'}{\sqrt{1-x'^2}}\right) d\alpha dx'$$

$$k_j(x) = \int_{x_{j-1}}^{x_j} \int_0^\pi \beta^2 \frac{e^{-j\beta d}}{d} \cos\alpha \left(\sqrt{1-x_{j-1}^2} - \sqrt{1-x^2}\right) d\alpha dx'$$

$$\ell_{\lambda j}(x) = \int_{x_{j-1}}^{x_j} \int_0^\pi \beta^2 \frac{e^{-j\beta d}}{d} \cos\alpha\left(\sqrt{1-x_\lambda^2} - \sqrt{1-x_{\lambda-1}^2}\right) d\alpha dx'$$

Rewriting VIII.5 and adding the terminal condition,

$$\sum_{j=1}^{N} a_j \left(g_j(x_i) + k_j(x_i) + \sum_{\lambda=1}^{j} \ell_{\lambda j}(x_i)\right) = 0 \qquad x_i = i-1/2$$

$$i = 1, \ldots, N$$

$$\sum_{j=1}^{N} a_j \left(\sqrt{1-x_j^2} - \sqrt{1-x_{j-1}^2}\right) = I_0$$

Solution of above equations yields the dominant current distribution.

Impedance is computed as

$$Z = \frac{\phi(x')}{I_0}\bigg|_{x'=0} = \frac{j}{4\pi\epsilon R\omega} \sum_{k=1}^{N} a_k \int_{x_{k-1}}^{x_k} \frac{e^{-j\beta x}}{x}\, dx$$

Program for Current and Impedance Calculation

of a Center Fed Circular Disk

[illegible]

```
      DATA WEIGHT /0.1?3421[illegible]
     [illegible]0.0787?1212/
      DATA WEIGHT /0.1?8421[illegible]000*
     [illegible]0.1[illegible],0.0[illegible]
     [illegible]0.0?30[illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]WEIGHT[illegible]BETA,RHO)*RE
     [illegible]BETA,RHO)
      [illegible]BETA,RHO)
      [illegible]
      [illegible]PRINT[illegible]
      [illegible]RESULT?
      RETURN
      END
C
C
      FUNCTION G(X1,X2,ALPHA,BETA,RHO)
      [illegible]
      [illegible]X1,X2,BETA,RHO,ALPHA,G
      [illegible]COS(ALPHA)*RHO**2)
     [illegible]DCOS(BETA*[illegible],-(0.[illegible]*DSIN(BETA*2,
     [illegible]*BETA*E/2)*(X1[illegible]DCOS(ALPHA)).1
      [illegible]
      [illegible]
C
C
      [illegible](X1,X2,ALPHA,BETA,RHO)
      [illegible]
      [illegible]X1,X2,BETA,RHO,ALPHA,G
      [illegible]COS(ALPHA)*RHO**2)
      [illegible]BETA*2,-(0.[illegible]
      [illegible]
      [illegible]
      [illegible]
```

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[illegible]
[illegible]
[illegible]
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[illegible]
[illegible]
[illegible]
[illegible]
1160 CALL INTEGR(0.0,3.1415,3,0,RESG,X1,X2,BETA,AHL)
1170 CALL INTEGR(0.0,3.1415,13,H,RESH,X1,X2,BETA,AHL)
1180 SUM=SUM+RESH
1190 A(I,J)=(DSQRT(1.0-(X1-STEP/2.0)**2)-DSQRT(1.0-(X2+STEP/2.0)**2))*RE
 [illegible]
 [illegible]
1200 X1=X1+STEP
1210 X2=X2+STEP
[illegible] X=STEP
1230 DO 30 J=1,N
[illegible] A=[illegible](DSQRT(1.0-X**2)-DSQRT(1.0-(X-STEP)**2)
[illegible] X=X+STEP
[illegible] DO 40 I=1,N
[illegible]
[illegible] DO 50 I=1,N
[illegible] DO 55 J=1,N
[illegible] REALPA=(A(I,J)+DCONJG(A(I,J)))/2.0D00
[illegible] IMAGPA=(A(I,J)-DCONJG(A(I,J)))/2.0D00/(0.0D00,1.0D00)
1320 REALBS=(AB(I)+DCONJG(AB(I)))/2.0D00
1330 IMAGBS=(AB(I)-DCONJG(AB(I)))/2.0D00/(0.0D00,1.0D00)
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
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[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
      WRITE (6,100)
      DO 110 I=1,N
  110 WRITE (6,*) RB(I),RB(I+N)
      WRITE (6,100)
      WRITE (6,2000)
[illegible]
      DO 120 I=1,N
      CHARGE=(RB(I)*(1.0D00,0.0D00)+RB(I+N)*(0.0D00,1.0D00))*X/DSQRT(1.
     *-X**2)
      MAGCHA=CDABS(CHARGE)
      MAGTIM=MAGCHA*BETA
      WRITE (6,3000) X,MAGTIM
  120 X=X+STEP
      WRITE (6,100)
      WRITE (6,2100)
      CURR=(0.0D00,0.0D00)
      X=1.0
      DO 130 I=1,N
      CURR=CURR+(DSQRT(1.0-X**2)-DSQRT(1.0-(X-STEP)**2))*(RB(N-I+1)*(1.0D
     *00,0.0D00)+RB(N-I+1+N)*(0.0D00,1.0D00))
      X=X-STEP
      MAGCUR=CDABS(CURR)
  130 WRITE (6,3000) X,MAGCUR
      WRITE (6,100)
      WRITE (6,2300)
      Z=(0.0D00,0.0D00)
      X2=0.0D0
      DO 170 I=1,N
      CALL INTEGR(X2,X2+STEP,...,FREQ,RESULT,0.0,0.0,BETA,RHO)
      Z=Z+RESULT*(RB(I)*(1.0D00,0.0D00)+RB(I+N)*(0.0D00,1.0D00))
  170 X2=X2+STEP
      Z=Z*[illegible]*(0.0D00,1.0D00)/4.0/3.1416/BETA
      WRITE (6,*) Z
  100 FORMAT(' ')
 2000 FORMAT(' DISTANCE          MAGN OF CHARGE TIMES VELOC OF LIGHT
 2100 FORMAT(' DISTANCE             MAGNITUDE OF CURRENT ')
 2200 FORMAT(' BETA,RHO ')
 2300 FORMAT(' IMPEDANCE ')
 3000 FORMAT(F8.4,...F20.8)
[illegible]
```

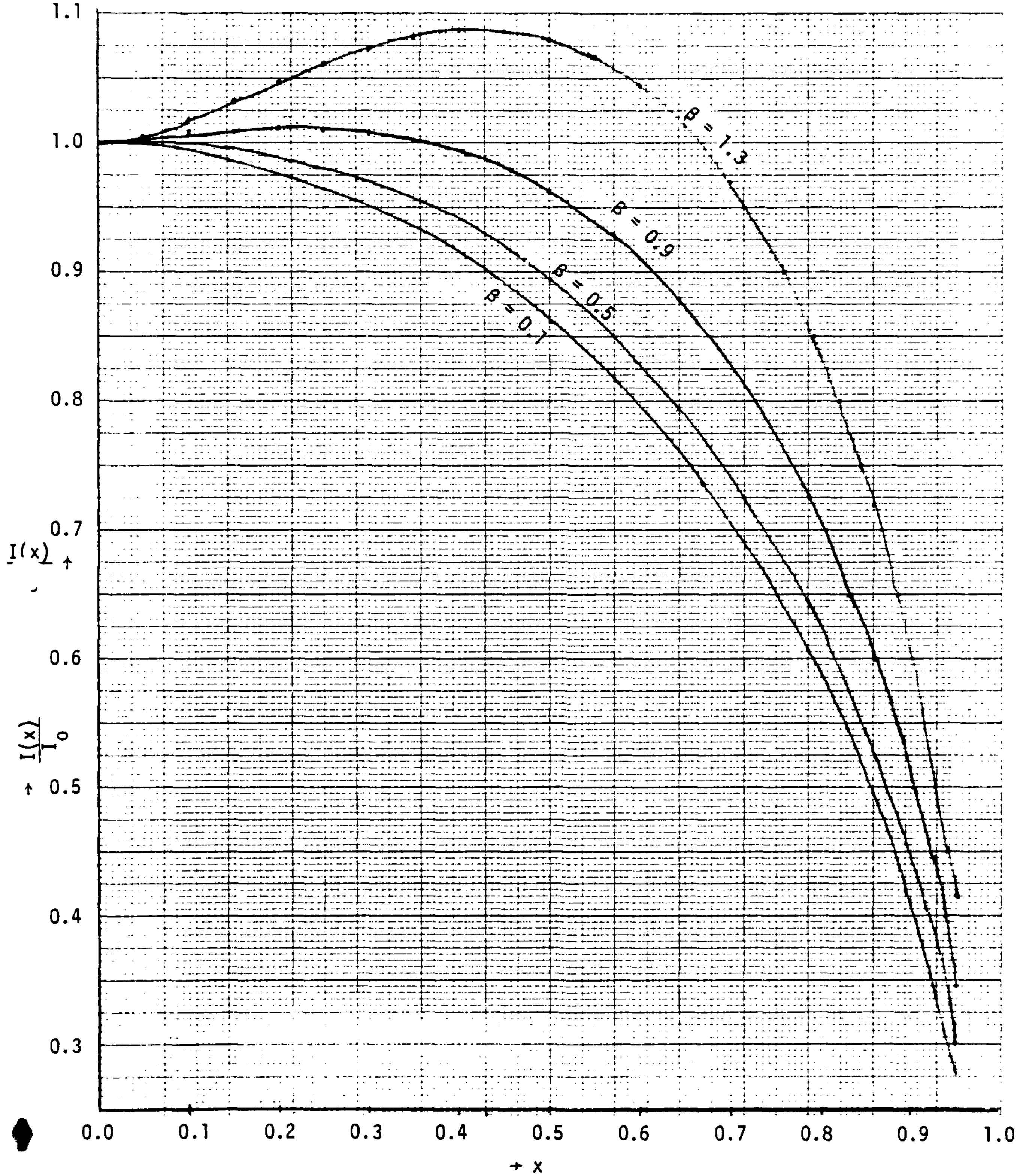


FIGURE 13 Current distribution on a Circular Disk fed at the center

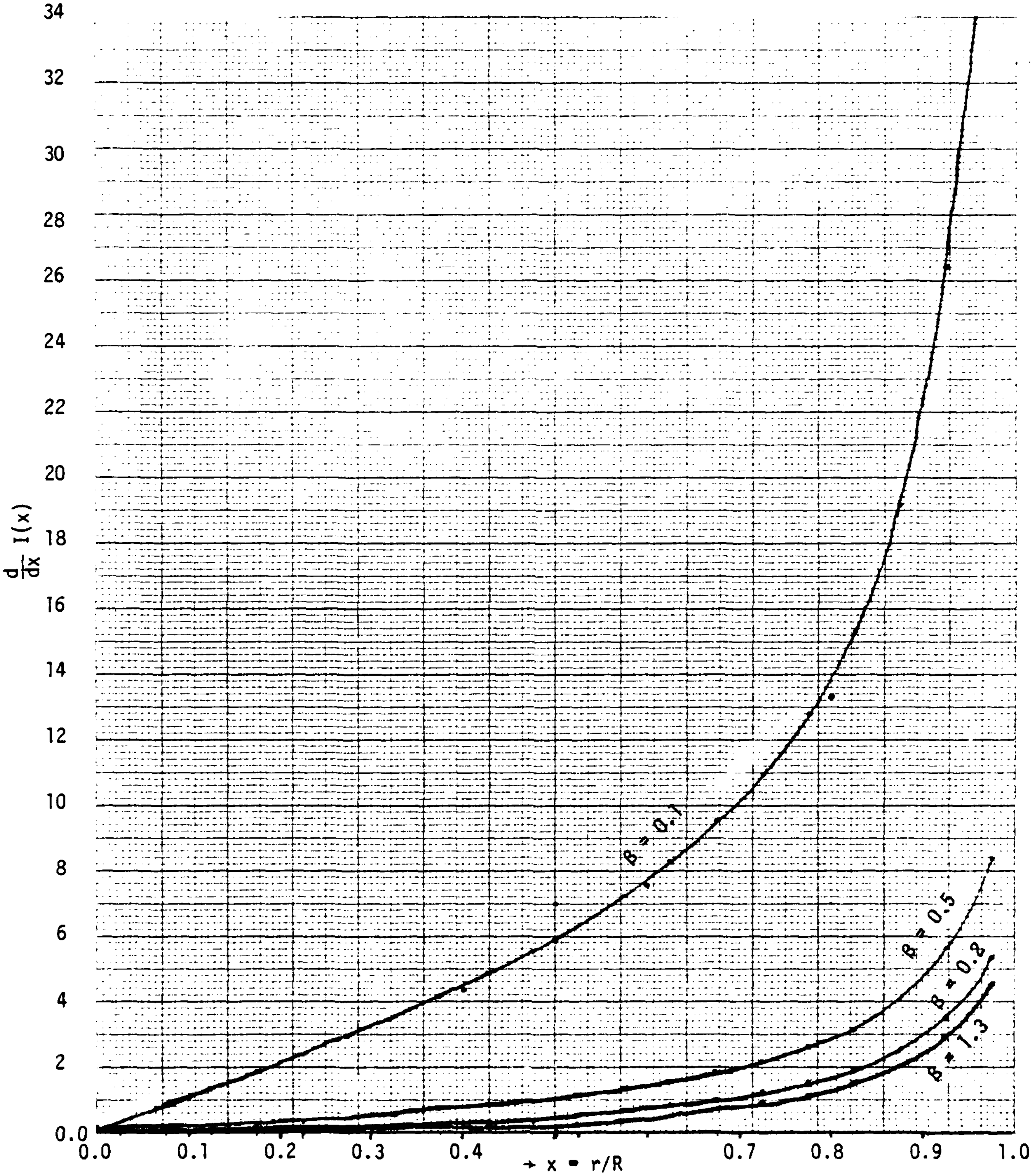


FIGURE 14 Charge Distribution on a Circular Disk

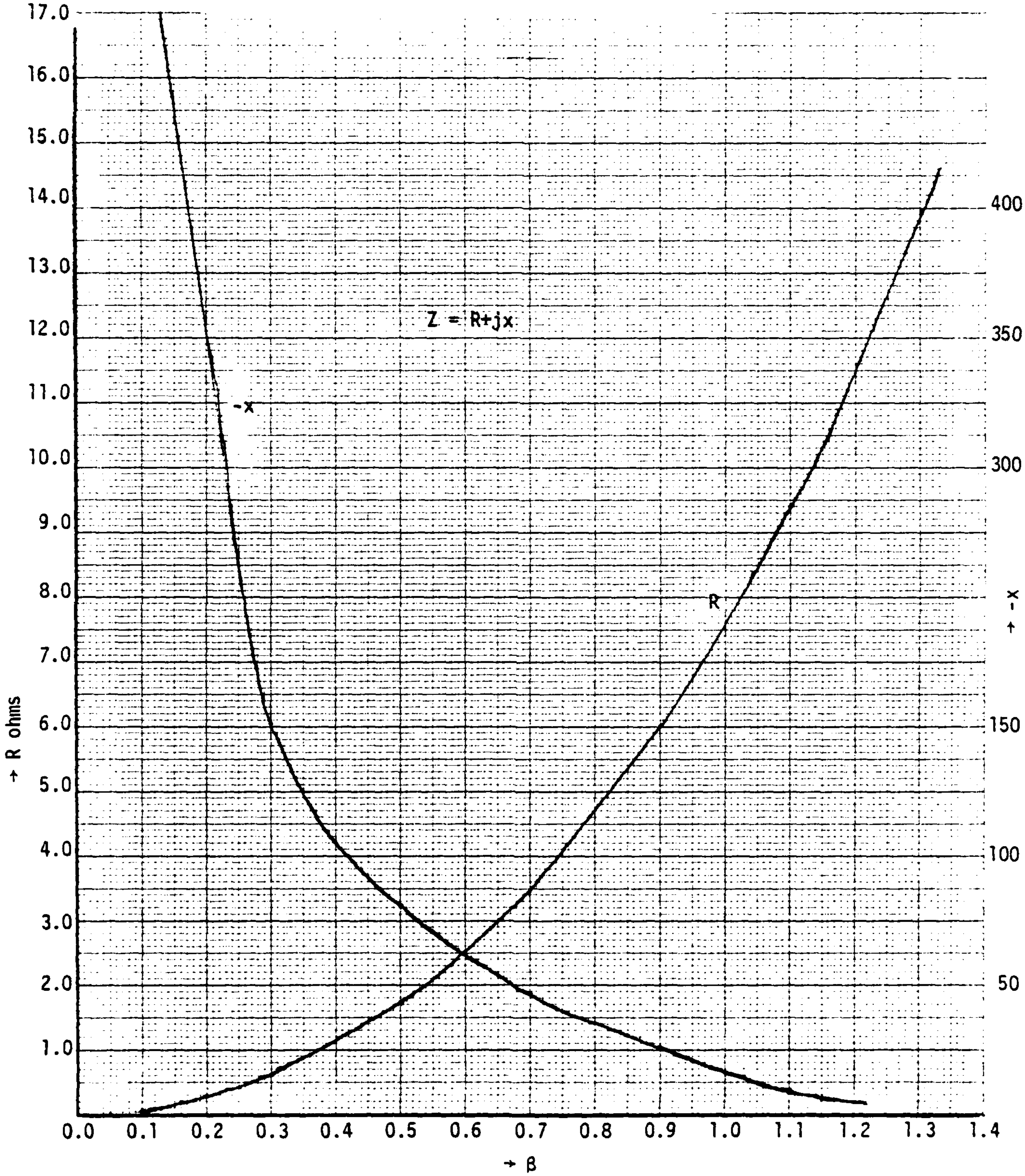
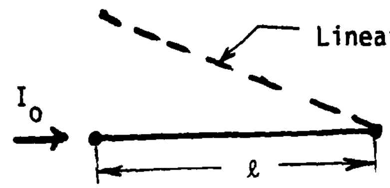


FIGURE 15 Impedance of a Center Fed Circular Disk

IX. Impedance Calculation of a Thin Wire with Linear Current Distribution



$$Z = \frac{jM}{\beta I_o^2} \int_0^1 \int_0^1 \int_0^\pi [\beta^2 i(x) i(x') \frac{e^{-j\beta d}}{d} - \frac{d}{dx} i(x) \frac{d}{dx} i(x') (\frac{e^{-j\beta d}}{d} + j\beta) \, dx dx' d\alpha$$

$$M = \frac{1}{4\pi^2} \sqrt{\frac{\mu}{\varepsilon}}$$

Assumption of linear current distribution and computation of the kernel at

$\alpha = \pi/3$

$$Z = \frac{jM}{\beta\pi} \int_0^1 \int_0^1 [\beta^2 (1-x)(1-x') \frac{e^{-j\beta d}}{d} - (\frac{e^{-j\beta d}}{d} + j\beta)] \, dx dx'$$

$$d = \sqrt{(x-x')^2 + \rho^2}$$

ρ = Radius of the wire

Program for Impedance of a Wire with Linear Current Distribution

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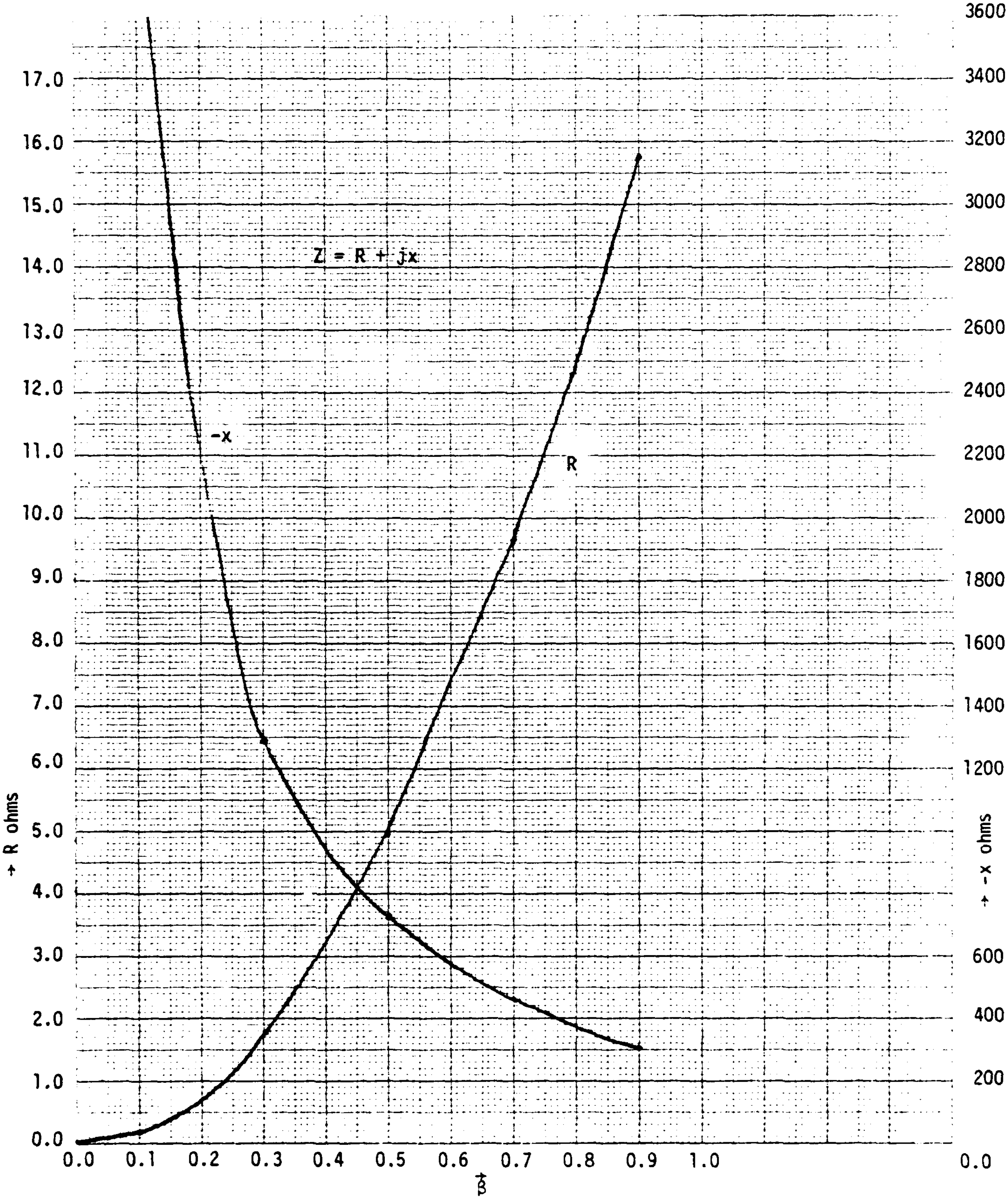
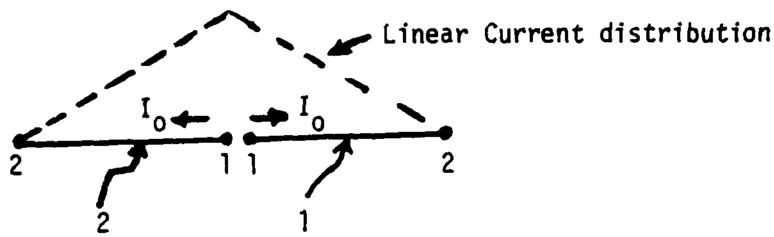


FIGURE 16 Dipole Impedance (Linear Current Distribution)

X. Impedance Calculation of a Dipole with Linear Current Distribution Via Diakoptic Theory



$$Z_{1,1}^{1,1} = \frac{j}{4\pi\beta}\sqrt{\frac{\mu}{\varepsilon}} \int_0^1 \int_0^1 \left[\beta^2(1-x)(1-x')\frac{e^{-j\beta d}}{d} - \left(\frac{e^{-j\beta d}}{d} + j\beta\right)\right] dx\,dx' = Z_o$$

$$Z_{1,1}^{2,1} = \frac{j}{4\pi\beta}\sqrt{\frac{\mu}{\varepsilon}} \int_0^1 \int_0^1 \left[\beta^2(1-x)(1-x')\frac{e^{-j\beta d_1}}{d_1} - \left(\frac{e^{-j\beta d_1}}{d_1} + j\beta\right)\right] dx\,dx' = Z_1$$

$$d = \sqrt{(x-x')^2 + \rho^2}\ , \qquad d_1 = \sqrt{(x-x')^2 + \rho^2}$$

$$Z = 2[Z_0 - Z_1]$$

Program to Compute Impedance of Dipole Assuming Linear Current Distribution

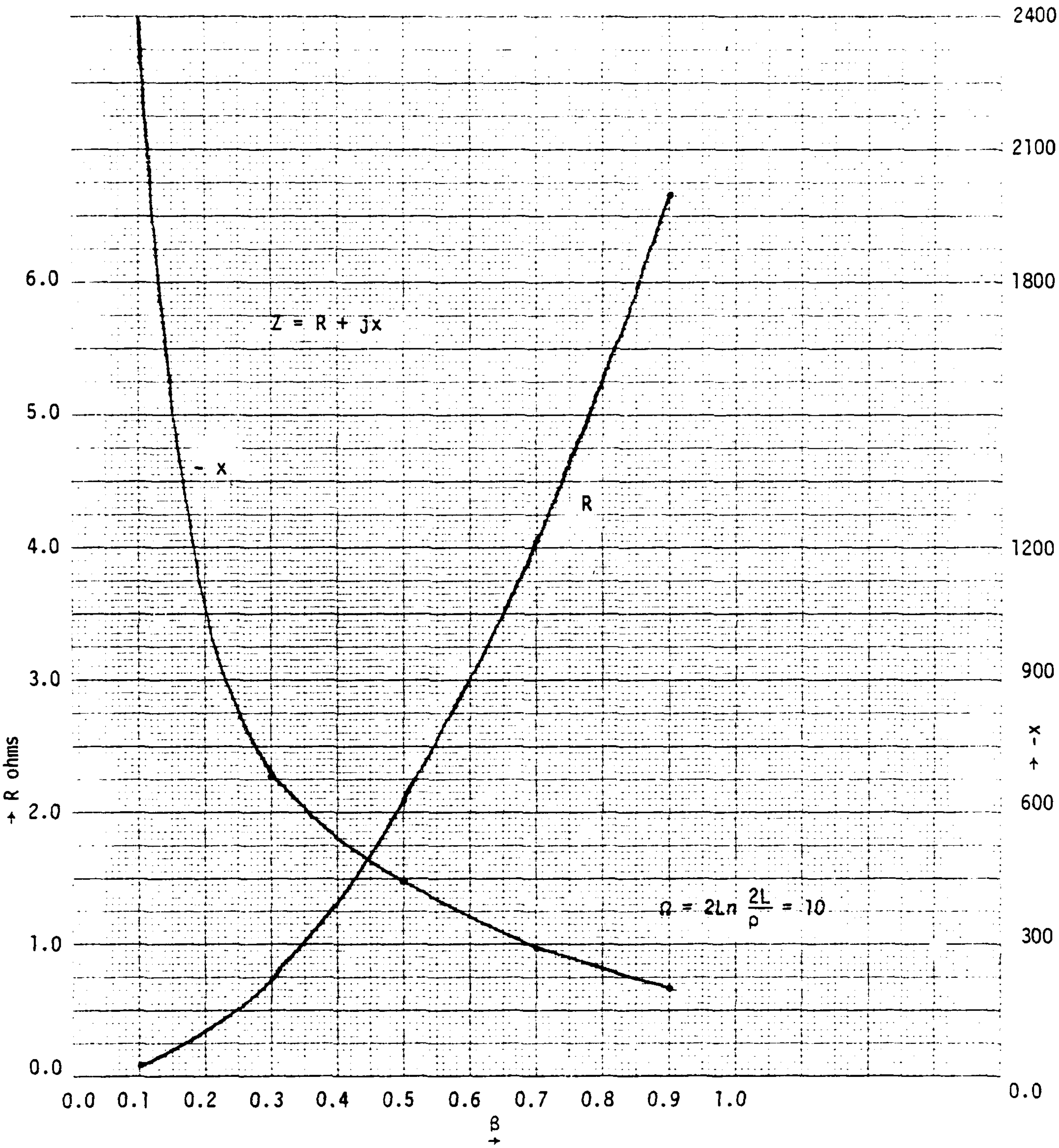


FIGURE 17 Impedance of Wire with Linear Current Distribution

XI. Computation of Dominant Current Distribution for all Frequencies Via Static Charge Distribution

Accurate computation of dominant current becomes one of the most important tasks in using Diakoptic Theory for complicated radiating structure analysis. For a general structure element it is a difficult task. Furthermore, if we can compute dominant current for all frequencies, the impedance spectrum becomes easy to compute. In many shapes, such as spheres, circular disks and cylindrical conductors, the static charge distribution is either known or easy to compute. In this section, we shall develop an algorithm to compute dominant current distribution from static charge distribution.

Let

$$\bar{i}(\bar{r}') = \sum_{0}^{n} (-jk)^n \, \bar{i}_n(r') \quad , \quad i_n(\bar{r}') = \bar{i}'_n \qquad \text{XI.1}$$

$$q(\bar{r}') = \sum_{0}^{u} (-jk)^u \, q_n(r') \quad , \quad q_n(\bar{r}') = q_n \qquad \text{XI.2}$$

$$q'_n = \bar{\nabla} \cdot \bar{i}'_n \quad , \quad k = \omega\sqrt{\mu\varepsilon}$$

Thus

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (-jk)^{n+m} \, \bar{A}_{n\lambda}(\bar{r})$$

$$= \frac{\mu}{4\pi} \sum_{\lambda=0}^{\infty} \sum_{n=0}^{\lambda} (-jk)^{\lambda} \, \bar{A}_{n\lambda} \qquad \text{XI.3}$$

$$\bar{A}_{n\lambda} = \int_{S'} \frac{\bar{i}_n(\bar{r}') D^{\lambda-n-1}}{(\lambda-n)!} \, dS' \quad , \quad D = |\bar{r} - \bar{r}'|$$

Similarly

$$\hat{\phi}(\bar{r}) = \frac{1}{4\pi\varepsilon} \sum_{\lambda=0}^{\infty} \sum_{n=0}^{\lambda} (-jk)^{\lambda-1} \Phi_{n\lambda}(\bar{r}) \qquad \text{XI.4}$$

where

$$\phi_{n\lambda}(\bar{r}) = \int_{S'} \frac{\nabla\cdot\bar{i}_n}{(\lambda-n)!} D^{\lambda-n-1} dS'$$

S' represents the total structure area.

Taking gradient of XI.4

$$-j\omega\mu\varepsilon\ \nabla\hat{\phi} = \frac{\mu}{4\pi} \sum_{\lambda=0}^{\infty} \sum_{n=0}^{\lambda} (-jk)^{\lambda}\ \bar{\ell}_{n\lambda}(\bar{r})$$

where

$$\bar{\ell}_{n\lambda}(\bar{r}) = \int_{S'} \frac{(\lambda-n-1)}{(\lambda-n)!} \bar{\nabla}\cdot\bar{i}_n'\ D^{\lambda-n-2} \frac{\partial d}{\partial r} \left(\frac{\bar{r}}{|\bar{r}|}'\right) dS'$$

Boundary Condition $E_{tan} = 0$, implies

$$(-k^2\bar{A}(\bar{r}) + j\omega\mu\varepsilon\nabla\hat{\phi})xd\bar{S} = 0, \quad d\bar{S} = \bar{n}dS \qquad \text{XI.5}$$

Substituting XI.3, XI.4 into XI.5 and equating powers of (-jk),

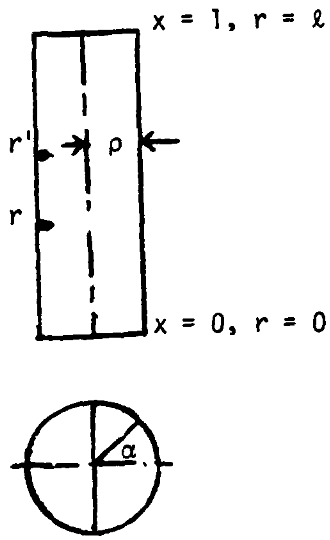
$$\ell_{00} = \int_{S'} (\bar{\nabla}\cdot\bar{i}_o')\frac{1}{D^2} \frac{\partial d}{\partial r} \frac{\bar{r}x\bar{n}}{|\bar{r}|} dS' = 0 \qquad \text{XI.6}$$

$$\ell_{11} = \int_{S'} (\bar{\nabla}\cdot\bar{i}_1') \frac{1}{D^2} \frac{\partial d}{\partial r} \frac{\bar{r}x\bar{n}}{|\bar{r}|} dS' = 0 \qquad \text{XI.7}$$

$$\int (\bar{\nabla}\cdot\bar{i}_{\lambda+1}') \frac{1}{D^2} \frac{\partial d}{\partial r} \frac{\bar{r}x\bar{n}}{|\bar{r}|} dS' = \sum_{n=0}^{\lambda-1} (-\ell_{n,\lambda+1} + \bar{A}_{n,\lambda-1}x\bar{n}) \qquad \lambda = 1,2,\ldots \quad n \leq \lambda \qquad \text{XI.8}$$

If the impressed current is assumed real at all frequencies, we obtain $i_1 \equiv 0$, $i_k(o) = 0$, $k = 2, \ldots, n$. Equations XI.6 and XI.8 compute the static current i_o and rest of the currents $i_2, i_3, \ldots, i_n$.

A. Cylindrical Conductor



Dominant current equations are simplified as:

$$\int_0^{\pi}\int_0^{1} (\bar{\nabla}\cdot\bar{i}_0(x'))\,\frac{(x-x')}{D^3}\,dx'\,d\alpha = 0$$

$$2\pi\rho\bar{i}_0(o) = I_0 \quad , \quad D = \sqrt{(x-x')^2 + (2\rho\sin\alpha/2)^2}$$

$$i_1(x) = 0$$

$$\int_0^{\pi}\int_0^{1} (\bar{\nabla}\cdot\bar{i}'_{\lambda+1})\,\frac{(x-x')}{D^3}\,dx'\,d\alpha = \sum_{n=0}^{\lambda-1}\int_0^{\pi}\int_0^{1}\left[\frac{(\bar{\nabla}\cdot\bar{i}'_n(\lambda-n)(x-x')-i'_n}{(\lambda-n+1)!}\right]D^{\lambda-n-2}\,dx'\,d\alpha$$

Tables 1 and 2 represent computed values of the current $I_n(x)$ and charge $q_n(x)$. Figures 19 and 20 show current $I_n(x)$ and charge $q_n(x)$ for different values of β.

Table 1, $\frac{I_n(x)}{I_o(0)}$ of a cylindrical conductor $\Omega = 2\ln \frac{2\ell}{\rho} = 10$

x	$\frac{I_o(x)}{I_o(0)}$	$\frac{I_2(x)}{I_o(0)}$	$\frac{I_3(x)}{I_o(0)}$	$\frac{I_4(x)}{I_o(0)}$
0.0	1.000	0.0	0.0	0.0
0.1	0.865	-0.0644	-0.0070	0.0095
0.2	0.768	-0.1000	-0.0110	0.0157
0.3	0.677	-0.1231	-0.0138	0.0206
0.4	0.588	-0.1357	-0.0155	0.0243
0.5	0.500	-0.1386	-0.0163	0.0264
0.6	0.412	-0.1323	-0.0160	0.0266
0.7	0.323	-0.1169	-0.0146	0.0247
0.8	0.232	-0.0923	-0.0120	0.0203
0.9	0.135	-0.0570	-0.0079	0.0141
1.0	0.0	0.0	0.0	0.0

Table 2. : $\frac{1}{I_0(o)} \cdot \frac{dI_n(x)}{dx}$ of a cylindrical conductor $\Omega = 2\ell n \frac{2\ell}{\rho} = 10$

x	$\frac{1}{I_0(o)} \cdot \frac{dI_0(x)}{dx}$	$\frac{1}{I_0(0)} \cdot \frac{dI_2(x)}{dx}$	$\frac{1}{I_0(0)} \cdot \frac{dI_3(x)}{dx}$	$\frac{1}{I_0(0)} \cdot \frac{dI_4(x)}{dx}$
0.1	-1.010	-0.427	-0.0466	0.0683
0.2	-0.929	-0.286	-0.0331	0.0557
0.3	-0.896	-0.176	-0.0225	0.0435
0.4	-0.881	-0.076	-0.0125	0.0292
0.5	-0.877	0.018	-0.0024	0.0124
0.6	-0.881	0.108	-0.0081	-0.0073
0.7	-0.896	0.199	-0.0195	-0.0304
0.8	-0.929	0.293	-0.0325	-0.0579
0.9	-1.010	0.414	-0.0500	-0.0957

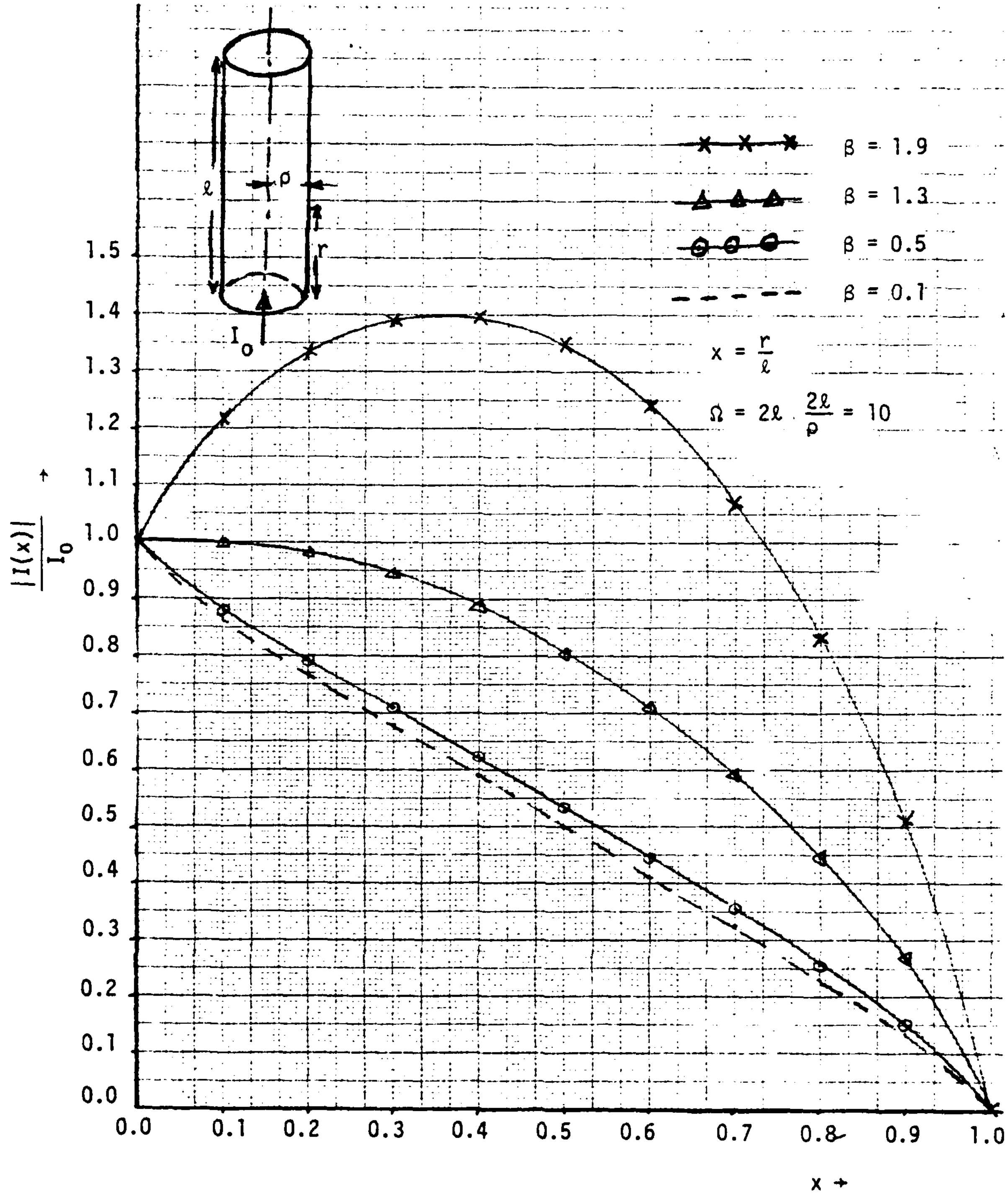


FIGURE 18 Current Distribution on Cylindrical Conductor

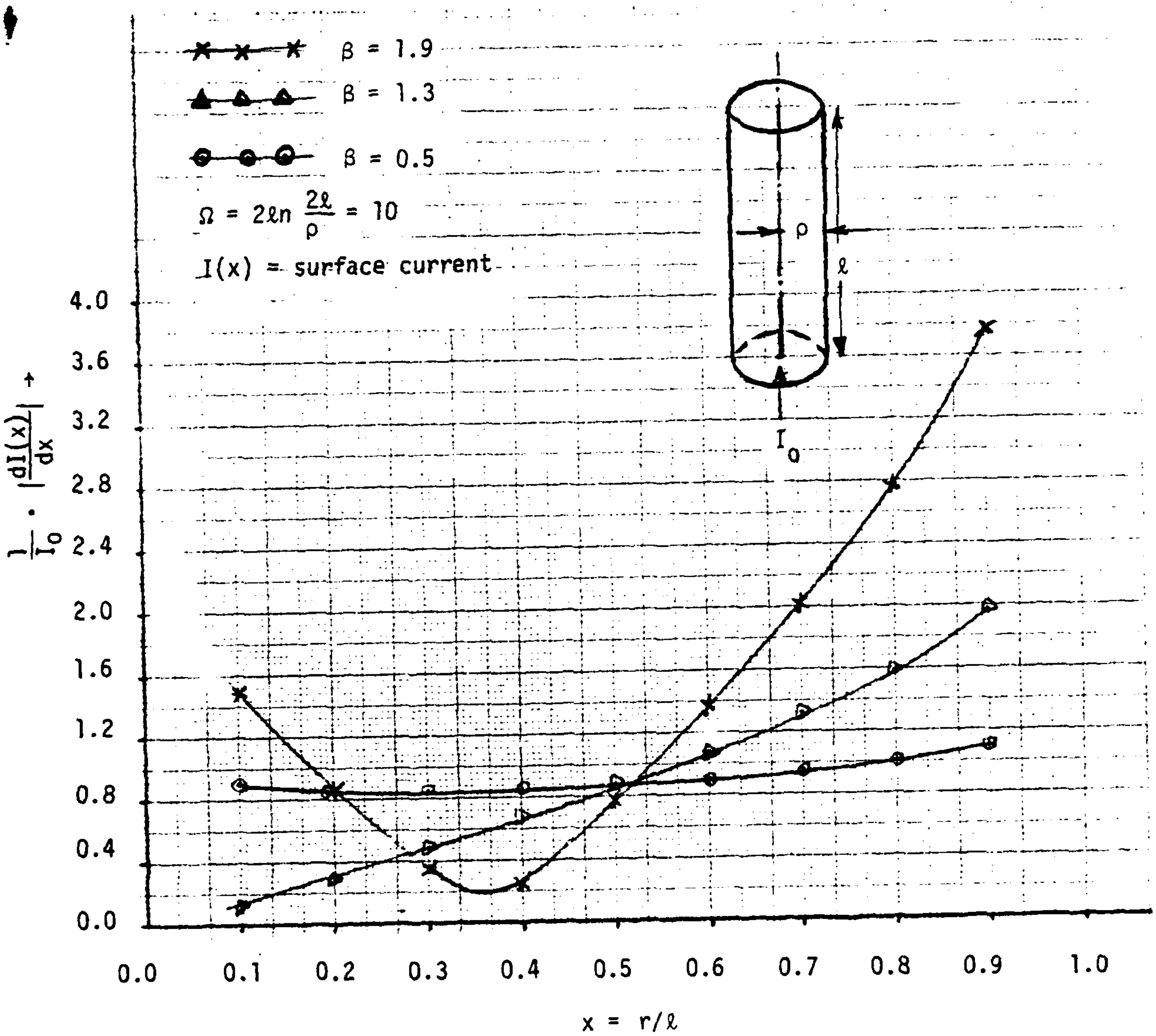


FIGURE 19 Charge Distribution on a Cylindrical Conductor

B. Circular Disc

$$2\pi x i(x) = I(x)$$

$$D^2 = (x^2 + x'^2 - 2xx'\cos\alpha)^{1/2}$$

$$x = r/R, \quad x' = r'/R$$

Static charge equation is

$$\int_0^{2\pi}\int_0^1 \frac{d}{dx'}\,(x' i_0')\,\frac{d}{dx}\,\left(\frac{1}{D}\right)\,dx'\,d\alpha = 0 \qquad\qquad 2\pi x i(x) = I_0$$

$$i_1(x) \equiv 0$$

Higher order current densities are obtained by

$$\int_0^{\pi}\int_0^1 \frac{d}{dx}\,(x' i'_{\lambda+1})\,\left(\frac{x-x'\cos\alpha}{D^3}\right)\,dx'\,d\alpha = \sum_{n=0}^{\lambda=1}\int_0^{\pi}\int_0^1 \Big[\left(\frac{\lambda-n}{\lambda-n+1!}\right)\frac{d}{dx'}\,(x' i_n')(x-x'\cos\alpha)$$

$$- \frac{i_n'\,x'}{\lambda-n+1!}\cos\alpha\Big]\,D^{\lambda-n-2}\,dx'\,d\alpha$$

$$\lim_{x\to 0}\; i_\lambda(x) = 0 \qquad\qquad \lambda = 1,2,\ldots$$

Tables 3 and 4 show computed values for various currents and charges. Figures 21 and 22 show current distribution and charges for different values of β.

X	Exact Static $\omega \to 0$ $\sqrt{1-x^2}$	$\frac{I_o(x)}{I_o(0)}$	$\frac{I_2(x)}{I_o(0)}$	$\frac{I_3(x)}{I_o(0)}$	$\frac{I_4(x)}{I_o(0)}$
0.0	1.000	1.000	0.0	0.0	0.0
0.1	0.995	0.995	-0.014	-0.001	0.00030
0.2	0.980	0.980	-0.039	-0.006	0.00105
0.3	0.954	0.954	-0.070	-0.012	0.00188
0.4	0.917	0.017	-0.101	-0.021	0.00241
0.5	0.866	0.866	-0.129	-0.031	0.00229
0.6	0.800	0.801	-0.150	-0.041	0.00125
0.7	0.714	0.715	-0.161	-0.049	-0.00074
0.8	0.600	0.600	-0.157	-0.054	-0.00333
0.9	0.436	0.433	-0.129	-0.049	-0.00526
1.0	0.0	0.0	0.0	0.0	0.0

Table 3 : $\frac{I_n(x)}{I_o(0)}$ of a circular plate fed at the center

X	Exact Static $\frac{-1}{2\pi\sqrt{1-x^2}}$	$\frac{\text{div } i(x)}{I_o(0)}$	$\frac{\text{div } i(x)}{I_o(0)}$	$\frac{\text{div } i(x)}{I_o(0)}$	$\frac{\text{div } i(x)}{I_o(0)}$
0.0	-0.159	-0.159	-0.647	-0.045	+0.011
0.1	-0.160	-0.160	-0.330	-0.045	00.009
0.2	-0.162	-0.162	-0.229	-0.043	0.007
0.3	-0.167	-0.166	-0.165	-0.041	0.004
0.4	-0.174	-0.173	-0.117	-0.037	0.001
0.5	-0.184	-0.182	-0.081	-0.033	-0.002
0.6	-0.199	-0.198	-0.043	-0.026	-0.004
0.7	-0.223	-0.222	-0.011	-0.017	-0.006
0.8	-0.265	-0.262	0.025	-0.004	-0.005
0.9	-0.365	-0.362	0.082	0.022	-0.00185

Table 4: $\frac{\text{div } i(x)}{I_o(0)}$ of a circular plate fed at the center

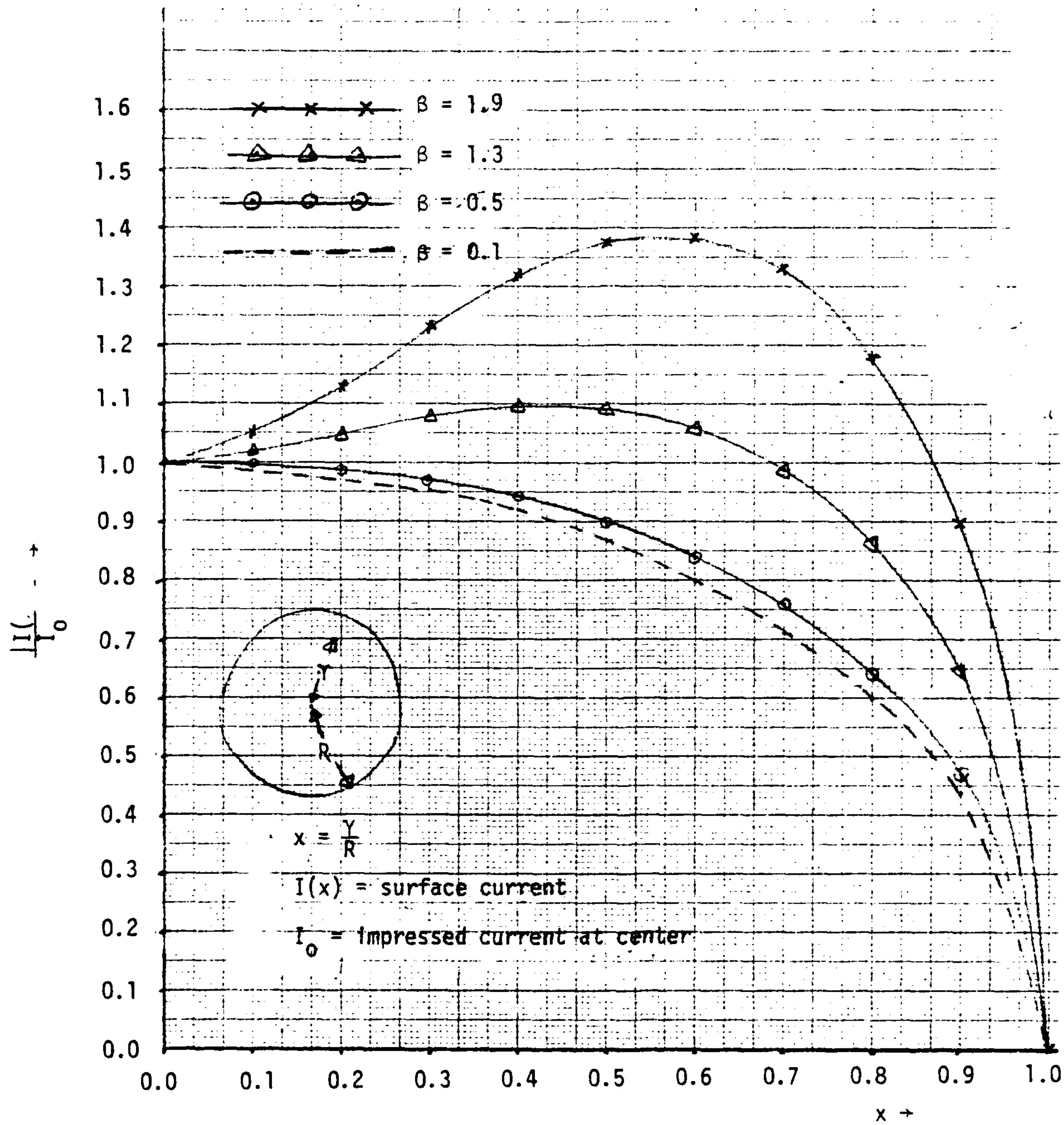


Figure 20 Current distribution on circular plate fed at the center

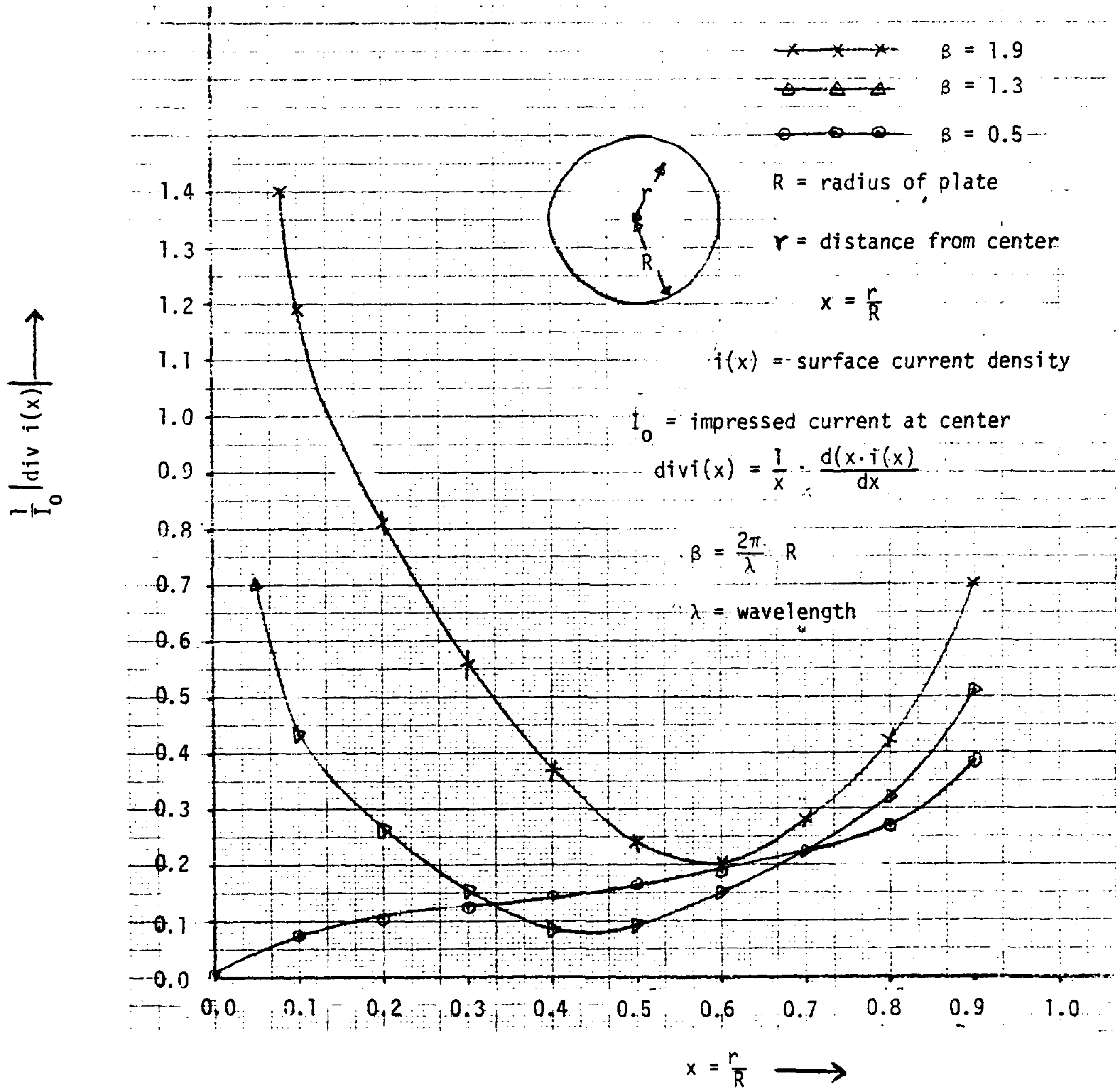


Figure 21 Charge distribution on a circular plate fed at the center

XII. Top Loaded Dipole Antenna

In this section we shall compute impedance characteristics of a top loaded dipole as shown in Figure XI-1

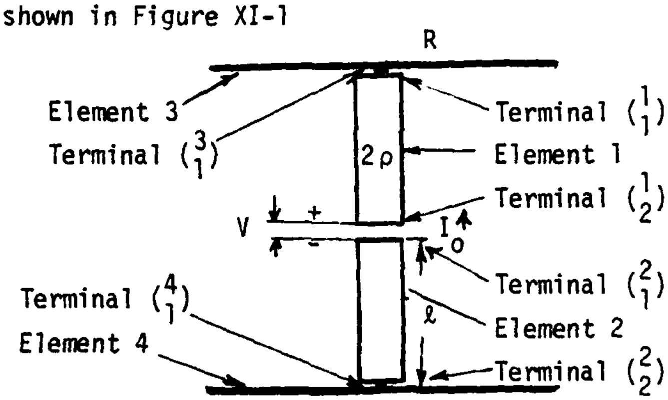


Fig. 22 Dipole with Circular Plates Capactive Loading

Section X is used to compute dominant current distribution on various radiating elements. Using the dominant current distributions, the various impedances are computed as

$$Z^{i,\ell}_{k,m} = \frac{j\omega}{I^i_k I^\ell_m} \int_{S^i} (\bar{A}^\ell_m \cdot \bar{I}^i_k + \hat{\phi}^\ell_m q^i_k)\, d\bar{S}^i \qquad . \qquad \text{XII-1}$$

where $\binom{i}{k}$ represents the impressed current terminal and $\binom{\ell}{m}$ represents the terminal where resulting potential is computed.

Matrix equation relating potentials to impressed currents is:

$$\begin{bmatrix} \Phi_1^3 \\ \Phi_2^1 \\ \Phi_1^1 \\ \Phi_1^2 \\ \Phi_2^2 \\ \Phi_1^4 \end{bmatrix} = \begin{bmatrix} Z_{11}^{33} & Z_{12}^{31} & Z_{11}^{31} & Z_{11}^{32} & Z_{12}^{32} & Z_{11}^{34} \\ Z_{21}^{13} & Z_{22}^{11} & Z_{21}^{11} & Z_{21}^{12} & Z_{22}^{12} & Z_{21}^{14} \\ Z_{11}^{13} & Z_{12}^{11} & Z_{11}^{11} & Z_{11}^{12} & Z_{12}^{12} & Z_{11}^{14} \\ Z_{11}^{23} & Z_{12}^{21} & Z_{11}^{21} & Z_{11}^{22} & Z_{12}^{22} & Z_{11}^{24} \\ Z_{21}^{23} & Z_{22}^{21} & Z_{21}^{21} & Z_{21}^{22} & Z_{22}^{22} & Z_{21}^{24} \\ Z_{11}^{43} & Z_{12}^{41} & Z_{11}^{41} & Z_{11}^{42} & Z_{12}^{42} & Z_{11}^{44} \end{bmatrix} \begin{bmatrix} I_1^3 \\ I_2^1 \\ I_1^1 \\ I_1^2 \\ I_2^2 \\ I_1^4 \end{bmatrix} \qquad \text{XII-2}$$

Let

$$Z_{11}^{11} = Z_{22}^{11} = Z_{11}^{22} = Z_{22}^{22} = Z_o$$

$$Z_{11}^{33} = Z_{11}^{44} = Z_o^1$$

$$Z_{21}^{24} = Z_{21}^{13} = Z_{12}^{42} = Z_{12}^{31} = Z_1^1$$

$$Z_{11}^{12} = Z_{11}^{21} = Z_1$$

$$Z_{11}^{24} = Z_{11}^{13} = Z_{11}^{42} = Z_{11}^{31} = Z_3^1$$

$$Z_{12}^{12} = Z_{21}^{21} = Z_{21}^{12} = Z_{12}^{21} = Z_3$$

AD-A096 685 RUTGERS - THE STATE UNIV PISCATAWAY NJ COLL OF ENGIN--ETC F/G 9/5
MATHEMATICAL MODELING OF MULTI-ELEMENT MONOPOLE ANTENNAS.(U)
MAR 81 N N PURI, G GOUBAU, A STAVRIDIS DAAG29-79-C-0201
UNCLASSIFIED ARO-15415.2-EL NL

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$$Z_{11}^{14} = Z_{11}^{32} = Z_{11}^{41} = Z_{11}^{23} = Z_4$$

$$Z_{12}^{11} = Z_{21}^{22} = Z_{21}^{11} = Z_{12}^{22} = Z_2$$

$$Z_{22}^{12} = Z_{22}^{21} = Z_5$$

$$Z_{12}^{32} = Z_{21}^{23} = Z_{21}^{14} = Z_{12}^{41} = Z_6$$

$$Z_{11}^{34} = Z_{11}^{43} = Z_7$$

Thus

$$\begin{bmatrix} \phi_1^3 \\ \phi_2^1 \\ \phi_3^1 \\ \phi_1^2 \\ \phi_2^2 \\ \phi_1^4 \end{bmatrix} = \begin{bmatrix} Z_o' & Z_1' & Z_3' & Z_4 & Z_6 & Z_7 \\ Z_1' & Z_o & Z_2 & Z_3 & Z_5 & Z_6 \\ Z_3' & Z_2 & Z_o & Z_1 & Z_3 & Z_4 \\ Z_4 & Z_3 & Z_1 & Z_o & Z_2 & Z_3' \\ Z_6 & Z_5 & Z_3 & Z_2 & Z_o & Z_1' \\ Z_7 & Z_6 & Z_4 & Z_3' & Z_1' & Z_o' \end{bmatrix} \begin{bmatrix} I_1^3 \\ I_2^1 \\ I_1^1 \\ I_1^2 \\ I_2^2 \\ I_1^4 \end{bmatrix}$$

Potential Condition: $\phi_1^3 = \phi_2^1$, $\phi_1^1 - \phi_1^2 = V$, $\phi_2^2 = \phi_1^4$

Continuity Condition: $I_1^3 = -I_2^1$, $I_1^1 = -I_1^2$, $I_2^2 = -I_1^4$

Thus

$$\begin{bmatrix} 0 \\ V \\ 0 \end{bmatrix} = \begin{bmatrix} Z_0'-Z_1'-Z_1'+Z_0 & Z_3'-Z_2-Z_4+Z_3 & Z_6-Z_5-Z_7+Z_6 \\ Z_3'-Z_4-Z_2+Z_3 & Z_0-Z_1-Z_1+Z_0 & Z_3-Z_2-Z_4+Z_3' \\ Z_6-Z_7-Z_5+Z_6 & Z_3-Z_4-Z_2+Z_3' & Z_5-Z_1'-Z_1'+Z_0' \end{bmatrix} \begin{bmatrix} I_1 \\ I_0 \\ I_1 \end{bmatrix}$$

Solving for I_0,

$$I_0 = \frac{(Z_0'-2Z_1'+Z_0+2Z_6-Z_5-Z_7)V}{2(Z_0-Z_1)(Z_0'-2Z_1'+Z_0+2Z_6-Z_5-Z_7)-(Z_3'-Z_2-Z_4+Z_3)(2Z_3'+2Z_3-2Z_2-2Z_4)}$$

Thus

$$Z_{in} = \frac{V_0}{I_0} = \frac{2(Z_0-Z_1)(Z_0'-2Z_1'+Z_0+2Z_6-Z_5-Z_7)-(Z_3'-Z_2-Z_4+Z_3')(2Z_3'+2Z_3-2Z_2-2Z_4)}{(Z_0'-2Z_1'+Z_0+2Z_6-Z_5-Z_7)}$$

```
?LF?
C     THIS PROGRAM COMPUTES THE SELF IMPEDANCES AS WELL AS THE
C     MUTUAL IMPEDANCES OF THE ELEMENTS OF AN ANTENNA CONSISTING OF
C     A DIPOLE WITH END CIRCULAR PLATES.
C     THE OUTPUT OF THIS PROGRAM ARE SELF AND MUTUAL IMPEDANCES
C     OF THE ELEMENTS, THE IMPEDANCE OF THE DIPOLE WITHOUT THE
C     PLATES,THE IMPEDANCE OF THE DIPOLE WITH THE PLATES AND
C     THE CURRENT AND CHARGE DISTRIBUTIONS ON THE CYLINDRICAL
C     CONDUCTORS AND THE PLATES.
C     THE DOMINANT CURRENT AND CHARGE DISTRIBUTIONS OF THE CIRCULAR
C     PLATE AND THE CYLINDRICAL CONDUCTOR ARE COMPUTED  IN TWO SEPARATE
C     PROGRAMS AND MUST BE SUPPLIED AS INPUT DATA TO THIS PROGRAM
      FUNCTION FW(X,J)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,M,K
      REAL*8 X
      COMPLEX*16 FW,CURVEC,CHAVEC,CURVED,CHAVED
      L = 10.0*X+1
      IF(L.LT.11)GO TO 50
      L = 10
   50 CONTINUE
      IF(L.EQ.1)GO TO 10
      IF(L.EQ.10)GO TO 20
      FW = CHAVEC(J,L)+(CHAVEC(J,L+1)-CHAVEC(J,L))/0.1D00*(X-(L-1)*0.1D0
     *0)
      RETURN
   10 FW = CHAVEC(J,2)+2.0D00*DSQRT(0.1D00)/0.1D00**2*(0.1D00-X)*CHAVEC(
     *J,1)
      RETURN
   20 FW= CHAVEC(J,10)+2.0D00*DSQRT(0.1D00)/0.1D00**2*(X-1.0D00+0.1D00)*
     *CH
     *AVEC(J,11)
      RETURN
      END
      FUNCTION FD(X,J)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,M,K
      REAL*8 X
      COMPLEX*16 FD,CURVEC,CHAVEC,CURVED,CHAVED
      L = 10.0*X+1
      IF(L.LT.11) GO TO 50
      L = 10
   50 CONTINUE
      IF(L.EQ.10)GO TO 20
      FD = CHAVED(J,L)+(CHAVED(J,L+1)-CHAVED(J,L))/0.1D00*(X-(L-1)*0.1D0
     *0)
      RETURN
   20 FD=CHAVED(J,10)+0.5D00*DSQRT(1.0D00-0.9D00**2)/((1.0D00-0.9D00)**3
     *(3.0D00-0.4[illegible]*(1.0D00-0.9D00**2)[illegible](X-0.9D00)*CHAVED(J,11)
```

```
      RETURN
      END
      SUBROUTINE ARCMAG(RES,MAGN,ARC)
      REAL*8 RRRES,RRES,ARC,MAGN
      COMPLEX*16 RES
      MAGN=CDABS(RES)
      IF(MAGN.EQ.0.0D00)GO TO 10
      RRRES=(RES+DCONJG(RES))/2.0D00/CDABS(RES)
      ARC=DARCOS(RRRES)
      RRES=(RES-DCONJG(RES))/2.0D00/(0.0D00,1.0D00)
      IF(RRES.GT.0.0D00) GO TO 56
      ARC=-ARC
   56 CONTINUE
      RETURN
   10 MAGN=0.0D00
      ARC=0.0D00
      RETURN
      END
      SUBROUTINE CURCHA(X,BETA,CHARGE,CURREN)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      REAL*8 X,BETA
      COMPLEX*16 RES,FW,CURVEC,CHAVEC,CURVED,CHAVED,CHARGE,CURREN
      RES=(0.0D00,0.0D00)
      DO 13 JJ=1,5
   13 RES = RES + FW(X,JJ)*((0.0D00,-1.0D00)*BETA)**(JJ-1)
      CHARGE=RES
      L=10.0*X+1
      RES=(0.0D00,0.0D00)
      DO 15 JJ=1,5
   15 RES=RES+(CURVEC(JJ,L)+(CURVEC(JJ,L+1)-CURVEC(JJ,L))/0.1D00
     **(X-(L-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(JJ-1)
      CURREN=RES
      RETURN
      END
      SUBROUTINE CUCHDI(X,BETA,CHARGE,CURREN)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      REAL*8 X,BETA
      COMPLEX*16 RES,FD,CURVEC,CHAVEC,CURVED,CHAVED,CHARGE,CURREN
      RES=(0.0D00,0.0D00)
      DO 13 JJ=1,5
   13 RES = RES + FD(X,JJ)*((0.0D00,-1.0D00)*BETA)**(JJ-1)
      CHARGE=RES
      L=10.0*X+1
      RES=(0.0D00,0.0D00)
      DO 15 JJ=1,5
   15 RES=RES+(CURVED(JJ,L)+(CURVED(JJ,L+1)-CURVED(JJ,L))/0.1D00
     **(X-(L-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(JJ-1)
      CURREN=RES
      RETURN
      END
```

```
      FUNCTION TGAUSS(A1X,B1X,A1Y,B1Y,A1Z,B1Z,MX,MY,MZ,...,
     *,FUNCTN)
      DIMENSION NPOINT(7),KEY(8),Z(24),WEIGHT(24)
      REAL*8 A1X,B1X,A1Y,B1Y,A1Z,B1Z,Z,WEIGHT,AX,BX,CX,AY,BY,CY
     *,AZ,BZ,CZ,DX,DY,DZ
      COMPLEX*16 TGAUSS,FUNCTN,SUM
      DATA NPOINT / 2,3,4,5,6,10,15 /
      DATA KEY / 1,2,4,6,9,12,17,25 /
      DATA  Z              / 0.577350269,0.0         ,0.774596669,
     1           0.339981044,0.861136312,0.0         ,0.538469310,
     2           0.906179846,0.238619186,0.661209387,0.932469514,
     3           0.148874339,0.433395394,0.679409568,0.865063367,
     4           0.973906529,0.0         ,0.201194094,0.394151347,
     5           0.570972173,0.724417731,0.848206583,0.937273392,
     6           0.987992518 /
      DATA  WEIGHT         / 1.0         ,0.888888889,0.555555556,
     1           0.652145155,0.347854845,0.568888889,0.478628671,
     2           0.236926885,0.467913935,0.360761573,0.171324492,
     3           0.295524225,0.269266719,0.219086363,0.149451349,
     4           0.066671344,0.101127121,0.198431485,0.186161000,
     5           0.166269206,0.139570678,0.107159221,0.070366047,
     6           0.030753242 /
      DO 1   I=1,7
      IF (MX.EQ.NPOINT(I)) GO TO 2
    1 CONTINUE
      TGAUSS=0.0
      RETURN
    2 JFIRSX = KEY(I)
      JLASTX = KEY(I+1)-1
      TGAUSS = 0.0
      DO 11 I =1,7
      IF (MY.EQ.NPOINT(I)) GO TO 12
   11 CONTINUE
      TGAUSS=0.0
      RETURN
   12 JFIRSY=KEY(I)
      JLASTY=KEY(I+1)-1
      TGAUSS=0.0
      DO 21 I=1,7
      IF (MZ.EQ.NPOINT(I)) GO TO 22
   21 CONTINUE
      TGAUSS=0.0
      RETURN
   22 JFIRSZ=KEY(I)
      JLASTZ=KEY(I+1)-1
      TGAUSS=0.0
      DO 7 IX = 1,KX
      DO 7 IY=1,KY
      DO 7 IZ=1,KZ
      AX=(IX-1)*(B1X-A1X)/KX+A1X
```

```
      BX=IX*(B1X-A1X)/KX+A1X
      CX=(BX-AX)/2.0
      DX=(BX+AX)/2.0
      AY=(IY-1)*(B1Y-A1Y)/KY+A1Y
      BY=IY*(B1Y-A1Y)/KY+A1Y
      CY=(BY-AY)/2.0
      DY=(BY+AY)/2.0
      AZ=(IZ-1)*(B1Z-A1Z)/KZ+A1Z
      BZ=IZ*(B1Z-A1Z)/KZ+A1Z
      CZ=(BZ-AZ)/2.0
      DZ=(BZ+AZ)/2.0
      SUM = 0.0
      DO 5 JX=JFIRSX,JLASTX
      DO 5 JY=JFIRSY,JLASTY
      DO 5 JZ=JFIRSZ,JLASTZ
    5 SUM = SUM+WEIGHT(JX)*WEIGHT(JY)*WEIGHT(JZ)*(FUNCTN(Z(JX)*CX
     *+DX,Z(JY)*CY+DY,Z(JZ)*CZ+DZ)+FUNCTN(Z(JX)*CX+DX,Z(JY)*CY+DY,
     *-Z(JZ)*CZ+DZ)+FUNCTN(Z(JX)*CX+DX,-Z(JY)*CY+DY,Z(JZ)*CZ+DZ)
     *+FUNCTN(Z(JX)*CX+DX,-Z(JY)*CY+DY,-Z(JZ)*CZ+DZ)
     *+FUNCTN(-Z(JX)*CX+DX,Z(JY)*CY+DY,Z(JZ)*CZ+DZ)
     *+FUNCTN(-Z(JX)*CX+DX,Z(JY)*CY+DY,-Z(JZ)*CZ+DZ)
     *+FUNCTN(-Z(JX)*CX+DX,-Z(JY)*CY+DY,Z(JZ)*CZ+DZ)
     *+FUNCTN(-Z(JX)*CX+DX,-Z(JY)*CY+DY,-Z(JZ)*CZ+DZ))
    7 TGAUSS = TGAUSS + CX*CY*CZ*SUM
      RETURN
      END
      FUNCTION F1111(X,XPR,ALPHA)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,M,K
      COMPLEX*16 F1111,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
     *,CURR1,CURR2,CHAR1,CHAR2
      REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
      L = 10.0*X+1
      LPR=10.0*XPR+1
      CHAR1=(0.0D00,0.0D00)
      CHAR2=(0.0D00,0.0D00)
      CURR1=(0.0D00,0.0D00)
      CURR2=(0.0D00,0.0D00)
      DO 10 II=1,5
      CHAR1=CHAR1+FW(X,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
      CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
      CURR1=CURR1+(CURVEC(II,L)+(CURVEC(II,L+1)-CURVEC(II,L))/0.1D00
     **(X-(L-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
   10 CURR2=CURR2+(CURVEC(II,LPR)+(CURVEC(II,LPR+1)-CURVEC(II,LPR))/
     *0.1D00*(XPR-(LPR-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
      D=DSQRT((X-XPR)**2+RHO**2)
      F1111=CURR1*CURR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))
     **BETA/D
     *-CHAR1*CHAR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))/D
     **(0.0D00,1.0D00)*BETA)/BETA
```

```
      RETURN
      END
      FUNCTION F1122(X,XPR,ALPHA)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,N,K
      COMPLEX*16 F1122,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
     *,CURR1,CURR2,CHAR1,CHAR2
      REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
      L = 10.0*X+1
      CURR1=(0.0D00,0.0D00)
      CURR2=(0.0D00,0.0D00)
      CHAR1=(0.0D00,0.0D00)
      CHAR2=(0.0D00,0.0D00)
      LPR=10.0*XPR+1
      DO 10 II=1,5
      CHAR1=CHAR1+FW(X,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
      CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
      CURR1=CURR1+(CURVEC(II,L)+(CURVEC(II,L+1)-CURVEC(II,L))/0.1D00
     **(X-(L-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
   10 CURR2=CURR2+(CURVEC(II,LPR)+(CURVEC(II,LPR+1)-CURVEC(II,LPR))
     */0.1D00*(XPR-(LPR-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
      D=DSQRT((X+XPR)**2+RHO**2)
      F1122=-CURR1*CURR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
     *)*BETA/D
     *-CHAR1*CHAR2*((DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))/D
     **(0.0D00,1.0D00)*BETA)/BETA
      RETURN
      END
      FUNCTION F1211(X,XPR,ALPHA)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,N,K
      COMPLEX*16 F1211,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
     *,CURR1,CURR2,CHAR1,CHAR2
      REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
      L = 10.0*X+1
      LPR=10.0*XPR+1
      CHAR1=(0.0D00,0.0D00)
      CHAR2=(0.0D00,0.0D00)
      CURR1=(0.0D00,0.0D00)
      CURR2=(0.0D00,0.0D00)
      DO 10 II=1,5
      CHAR1=CHAR1+FW(X,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
      CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
      CURR1=CURR1+(CURVEC(II,L)+(CURVEC(II,L+1)-CURVEC(II,L))/0.1D00
     **(X-(L-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
   10 CURR2=CURR2+(CURVEC(II,LPR)+(CURVEC(II,LPR+1)-CURVEC(II,LPR))
     */0.1D00*(XPR-(LPR-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
      D=DSQRT((1.0D00-X-XPR)**2+RHO**2)
      F1211=-CURR1*CURR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
```

```
[illegible]
[illegible] CHAR2*((DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))*D
*-[illegible](1.0D00)*BETA)/BETA
 RETURN
 END
 FUNCTION F1121(X,XPR,ALPHA)
 COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
 COMMON /DEF/ RHO,R,BETA
 COMMON /GHI/ I,M,K
 COMPLEX*16 F1121,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
*,CURR1,CURR2,CHAR1,CHAR2
 REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
 L = 10.0*X+1
 LPR=10.0*XPR+1
 CHAR1=(0.0D00,0.0D00)
 CHAR2=(0.0D00,0.0D00)
 CURR1=(0.0D00,0.0D00)
 CURR2=(0.0D00,0.0D00)
 DO 10 II=1,5
 CHAR1=CHAR1+FW(X,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
 CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
 CURR1=CURR1+(CURVEC(II,L)+(CURVEC(II,L+1)-CURVEC(II,L))/0.1D00
**(X-(L-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
10 CURR2=CURR2+(CURVEC(II,LPR)+(CURVEC(II,LPR+1)-CURVEC(II,LPR))
*/0.1D00*(XPR-(LPR-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
 D=DSQRT((1.0D00+X-XPR)**2+RHO**2)
 F1121=CURR1*CURR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
*)*BETA/D
*-CHAR1*CHAR2*((DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))/D
**(0.0D00,1.0D00)*BETA)/BETA
 RETURN
 END
 FUNCTION F1221(X,XPR,ALPHA)
 COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
 COMMON /DEF/ RHO,R,BETA
 COMMON /GHI/ I,M,K
 COMPLEX*16 F1221,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
*,CURR1,CURR2,CHAR1,CHAR2
 REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
 L = 10.0*X+1
 LPR=10.0*XPR+1
 CHAR1=(0.0D00,0.0D00)
 CHAR2=(0.0D00,0.0D00)
 CURR1=(0.0D00,0.0D00)
 CURR2=(0.0D00,0.0D00)
 DO 10 II=1,5
 CHAR1=CHAR1+FW(X,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
 CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
 CURR1=CURR1+(CURVEC(II,L)+(CURVEC(II,L+1)-CURVEC(II,L))/0.1D00
**(X-(L-1)*0.1D00))*((0.0D00,-1.0D00)*BETA)**(II-1)
10 CURR2=CURR2+(CURVEC(II,LPR)+(CURVEC(II,LPR+1)-CURVEC(II,LPR))
```

```
*(0.1D00*(XPR-(LPR-1)*0.1D00))*((0.0D00,-1.0D00)*BETA*R)**(II-1)
 D=DSQRT(X**2+XPR**2-2.0D00*X*XPR*DCOS(ALPHA))*R
 FZ121=CURR1*CURR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))*(1.
*D00+(0.0D00,1.0D00)*BETA*D)
*-CHAR1*CHAR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))*(
*(0.0D00+1.0D00)*BETA)/BETA
 RETURN
 END
 FUNCTION FZ131(X,XPR,ALPHA)
 COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
 COMMON /DEF/ RHO,R,BETA
 COMMON /GHI/ I,M,K
 COMPLEX*16 FZ131,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
*,CURR1,CURR2,CHAR1,CHAR2
 REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
 PI = 3.141592653589793D00
 L = 10.0*X+1
 LPR=10.0*XPR+1
 CHAR1=(0.0D00,0.0D00)
 CHAR2=(0.0D00,0.0D00)
 CURR1=(0.0D00,0.0D00)
 CURR2=(0.0D00,0.0D00)
 DO 10 II=1,5
 CHAR1=CHAR1+FD(X,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
 CHAR2=CHAR2+FD(XPR,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
 CURR1=CURR1+(CURVED(II,L)+(CURVED(II,L+1)-CURVED(II,L))/0.1D00
**(X-(L-1)*0.1D00))*((0.0D00,-1.0D00)*BETA*R)**(II-1)
10 CURR2=CURR2+(CURVED(II,LPR)+(CURVED(II,LPR+1)-CURVED(II,LPR))/
*0.1D00*(XPR-(LPR-1)*0.1D00))*((0.0D00,-1.0D00)*BETA*R)**(II-1)
 D=DSQRT(X**2+XPR**2-2.0D00*X*XPR*DCOS(ALPHA))*R
 FZ131=CURR1*CURR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))
*/BETA/D*R**2*4.0D00/PI**2
*-CHAR1*CHAR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))*(
*(0.0D00+1.0D00)*BETA)/BETA*X*XPR
 RETURN
 END
 FUNCTION FZ141(X,XPR,ALPHA)
 COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
 COMMON /DEF/ RHO,R,BETA
 COMMON /GHI/ I,M,K
 COMPLEX*16 FZ141,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
*,CURR1,CURR2,CHAR1,CHAR2
 REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
 PI = 3.141592653589793D00
 L = 10.0*X+1
 LPR=10.0*XPR+1
 CHAR1=(0.0D00,0.0D00)
 CHAR2=(0.0D00,0.0D00)
 CURR1=(0.0D00,0.0D00)
 CURR2=(0.0D00,0.0D00)
 DO 10 II=1,5
```

```
      CHAR1=CHAR1+FD(X,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
      CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
      CURR1=CURR1+(CURVED(II,L)+(CURVED(II,L+1)-CURVED(II,L))*(X-0.1D0
     *(L-1))/0.1D00)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
   10 CURR2=CURR2+(CURVED(II,LPR)+(CURVED(II,LPR+1)-CURVED(II,LPR))
     */0.1D00*(XPR-(LPR-1)*0.1D00))*((0.0D00,-1.0D00)*BETA*R)**(II-1)
      D=DSQRT(X**2+XPR**2-2.0D00*X*XPR*DCOS(ALPHA)+4.0D00*RHO**2)*R
      F3141=CURR1*CURR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D)
     *)*BETA/D/8.0D00/PI**2
     *-CHAR1*CHAR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))/D
     **(0.0D00,1.0D00)*BETA/BETA*X*XPR
      RETURN
      END
      FUNCTION F3111(X,XPR,ALPHA)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,M,K
      COMPLEX*16 F3111,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
     *,CURR1,CURR2,CHAR1,CHAR2
      REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
      L = 10.0*X+1
      LPR=10.0*XPR+1
      CHAR1=(0.0D00,0.0D00)
      CHAR2=(0.0D00,0.0D00)
      CURR1=(0.0D00,0.0D00)
      CURR2=(0.0D00,0.0D00)
      DO 10 II=1,5
      CHAR1=CHAR1+FD(X,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
   10 CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
      D=DSQRT((R*X)**2+(1.0D00-XPR)**2)
      F3111=-CHAR1*CHAR2*(DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))/
     *D*(0.0D00,1.0D00)*BETA/BETA
      RETURN
      END
      FUNCTION F3112(X,XPR,ALPHA)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,M,K
      COMPLEX*16 F3112,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
     *,CURR1,CURR2,CHAR1,CHAR2
      REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
      L = 10.0*X+1
      LPR=10.0*XPR+1
      CHAR1=(0.0D00,0.0D00)
      CHAR2=(0.0D00,0.0D00)
      CURR1=(0.0D00,0.0D00)
      CURR2=(0.0D00,0.0D00)
      DO 10 II=1,5
      CHAR1=CHAR1+FD(X,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
   10 CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
      D=DSQRT((R*X)**2+(1.0D00+XPR)**2)
```

```
      F3112=-CHAR1*CHAR2*X*((DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))/
     *D+(0.0D00,1.0D00)*BETA)/BETA
      RETURN
      END
      FUNCTION F3121(X,XPR,ALPHA)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,M,K
      COMPLEX*16 F3121,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
     *,CURR1,CURR2,CHAR1,CHAR2
      REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
      L = 10.0*X+1
      LPR=10.0*XPR+1
      CHAR1=(0.0D00,0.0D00)
      CHAR2=(0.0D00,0.0D00)
      CURR1=(0.0D00,0.0D00)
      CURR2=(0.0D00,0.0D00)
      DO 10 II=1,5
      CHAR1=CHAR1+FD(X,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
   10 CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
      D=DSQRT((R*X)**2+(2.0D00-XPR)**2)
      F3121=-CHAR1*CHAR2*X*((DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))/
     *D+(0.0D00,1.0D00)*BETA)/BETA
      RETURN
      END
      FUNCTION F3122(X,XPR,ALPHA)
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,M,K
      COMPLEX*16 F3122,CURVEC,CHAVEC,CURVED,CHAVED,FW,FD
     *,CURR1,CURR2,CHAR1,CHAR2
      REAL*8 X,XPR,ALPHA,RHO,R,BETA,D
      L = 10.0*X+1
      LPR=10.0*XPR+1
      CHAR1=(0.0D00,0.0D00)
      CHAR2=(0.0D00,0.0D00)
      CURR1=(0.0D00,0.0D00)
      CURR2=(0.0D00,0.0D00)
      DO 10 II=1,5
      CHAR1=CHAR1+FD(X,II)*((0.0D00,-1.0D00)*BETA*R)**(II-1)
   10 CHAR2=CHAR2+FW(XPR,II)*((0.0D00,-1.0D00)*BETA)**(II-1)
      D=DSQRT((R*X)**2+(1.0D00+XPR)**2)
      F3122=-CHAR1*CHAR2*X*((DCOS(BETA*D)-(0.0D00,1.0D00)*DSIN(BETA*D))/
     *D+(0.0D00,1.0D00)*BETA)/BETA
      RETURN
      END
      COMMON /ABC/ CURVEC(5,11),CHAVEC(5,11),CURVED(5,11),CHAVED(5,11)
      COMMON /DEF/ RHO,R,BETA
      COMMON /GHI/ I,M,K
      EXTERNAL FW,FD,F1111,F1122,F1211,F1121,F1221,F3131,F3141,F3111
     *,F3112,F3121,F3122
```

```
      COMPLEX*16 TGAUSS,FW,RES,FB,CURVEC,CHAVEC,CURVED,CHAVED,RES1
     1,RES2,RES3,RES4,RES5,RES6,RES7,RES8,RES9,RES10
     *,RES11,RES12,RES13,RES14,RES15,RES16,RES17,RES18,RES19,RES20
     *,CII,ZIN,ZBW,F1111,F1122,F1211,F1121,F1221,F3131,F3141,F3111
     K,F3112,F3121,F3122
      REAL*8 RHO,R,BETA,PI,AALN,ARG,RRES,RRRES,X
      PI=3.14159265358979D00
      WRITE(6,5602)
 5602 FORMAT(' RHO,R,BETA ')
      READ(5,*) RHO,R,BETA,(CURVED(I,J),CHAVED(I,J),CURVEC(I,J),CHAVEC(I,J),I=1,5,J=1,11)
      WRITE(6,5601) RHO,R,BETA,
 5601 FORMAT(' RHO = ',F12.7,' R = ',F12.5,' BETA = ',F10.4)
      RES = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,4,4,1,F1111)
      RES=RES*(0.0D00,1.0D00)/4.0D00/PI**2*377.0D00
      WRITE(6,5111) RES
 5111 FORMAT(' Z0 = Z1111 = ',2F14.6)
      RES1 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,3,3,1,F1122)
      RES1=RES1*(0.0D00,1.0D00)/4.0D00/PI**2*377.0D00
      WRITE(6,5110) RES1
 5110 FORMAT(' Z1 = Z1122 = ',2F14.6)
      RES2 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,3,3,1,F1211)
      RES2 = RES2*(0.0D00,1.0D00)/4.0D00/PI**2*377.0D00
      WRITE (6,5100) RES2
 5100 FORMAT(' Z2 = Z1211 = ',2F14.6)
      RES3 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,3,3,1,F1121)
      RES3 = RES3*(0.0D00,1.0D00)/4.0D00/PI**2*377.0D00
      WRITE(6,5101) RES3
 5101 FORMAT(' Z3 = Z1121 = ',2F14.6)
      RES4 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,3,3,1,F1221)
      RES4 = RES4*(0.0D00,1.0D00)/4.0D00/PI**2*377.0D00
      WRITE(6,5102) RES4
 5102 FORMAT(' Z5 = Z1221 = ',2F14.6)
      RES5 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,15,2,2,1,F3131)
      RES5 = RES5*(0.0D00,1.0D00)*377.0D00
      WRITE(6,5103) RES5
 5103 FORMAT(' Z0PR = Z3131 = ',2F14.6)
      RES6 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,15,1,1,1,F3141)
      RES6 = RES6*(0.0D00,1.0D00)*377.0D00
      WRITE(6,5104)RES6
 5104 FORMAT(' Z7 = Z3141 = ',2F14.6)
      RES7 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,1,1,1,F3111)
      RES7 =  RES7*(0.0D00,1.0D00)/2.0D00/PI*377.0D00
      WRITE(6,5105) RES7
 5105 FORMAT(' Z3PR = Z3111 = ',2F14.6)
```

```
      RES8 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,1,1,1,F3112)
      RES8 = RES8*(0.0D00,1.0D00)/2.0D00/PI*377.0D00
      WRITE (6,5106) RES8
 5106 FORMAT(' Z1PR = Z3112 = ',2F14.6)
      RES9 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,1,1,1,F3121)
      RES9 = RES9*(0.0D00,1.0D00)/2.0D00/PI*377.0D00
      WRITE (6,5107) RES9
 5107 FORMAT(' Z5 = Z3121 = ',2F14.6)
      RES10 = TGAUSS(0.0,1.0,0.0,1.0,0.0,PI,15,15,2,1,1,1,F3122)
      RES10 = RES10*(0.0D00,1.0D00)/2.0D00/PI*377.0D00
      WRITE(6,5108) RES10
 5108 FORMAT(' Z4 = Z3122 = ',2F14.6)
      ZDW=2.0D00*(RES-RES1)
      WRITE(6,5110) ZDW
 5110 FORMAT(' IMPEDANCE OF DOUBLE WIRE = ',2F14.6)
      CI1 = (RES2+RES10-RES7-RES3)/(RES5-2.0D00*RES8+RES+2.0D00*RES9
     *-RES4-RES6)
      ZIN = (2.0D00*(RES-RES1)*(RES5-2.0D00*RES8+RES+2.0D00*RES9
     *-RES4-RES6)
     *-(RES7-RES2-RES10+RES3)*(2.0D00*RES7+2.0D00*RES3-2.0D00*RES2
     *-2.0D00*RES10))
     */(RES5-2.0D00*RES8+RES+2.0D00*RES9-RES4-RES6)
      WRITE (6,5120) ZIN
 5120 FORMAT(' IMPEDANCE OF DOUBLE  WIRE WITH TOP CAPACITORS = ',
     *2F14.6)
      DO 11 J=1,11
      X=(J-1)*0.1D00
      CALL CURCHA(X,BETA,RES1,RES2)
      CALL CURCHA(1.0D00-X,BETA,RES3,RES4)
      RES5=RES1-RES3*CI1
      RES6=RES2+RES4*CI1
      CALL ARCMAG(RES5,MAGN,ARC)
      CALL ARCMAG(RES6,MAGN1,ARC1)
   11 WRITE (6,2005) X,RES5,MAGN,ARC,RES6,MAGN1,ARC1
 2005 FORMAT(' X=',F4.2,'  DI/DX=',2F9.6,'  MAGDI/DX=',F7.4,'  ARCDI/DX=
     *',
     *F9.6,'  I=',2F9.6,'  MAGI=',F7.4,'  ARCI=',F9.6)
      DO 13 J=1,11
      X=(J-1)*0.1D00+1/J*RHO
      CALL CUCHDI(X,BETA*R,RES1,RES2)
      RES1=RES1*CI1*2.0D00*PI*X
      RES2=RES2*CI1
      CALL ARCMAG(RES1,MAGN,ARC)
      CALL ARCMAG(RES2,MAGN1,ARC1)
   13 WRITE (6,2006) X,RES1,MAGN,ARC,RES2,MAGN1,ARC1
 2006 FORMAT(' X=',F4.2,'  DI/DX=',2F9.6,'  MAGDI/DX=',F7.4,'  ARCDI/DX=
     *',F9.6
     *,'  I=',2F9.6,'  MAGI=',F7.4,'  ARCI=',F9.6)
  123 CONTINUE
      STOP
      END
```

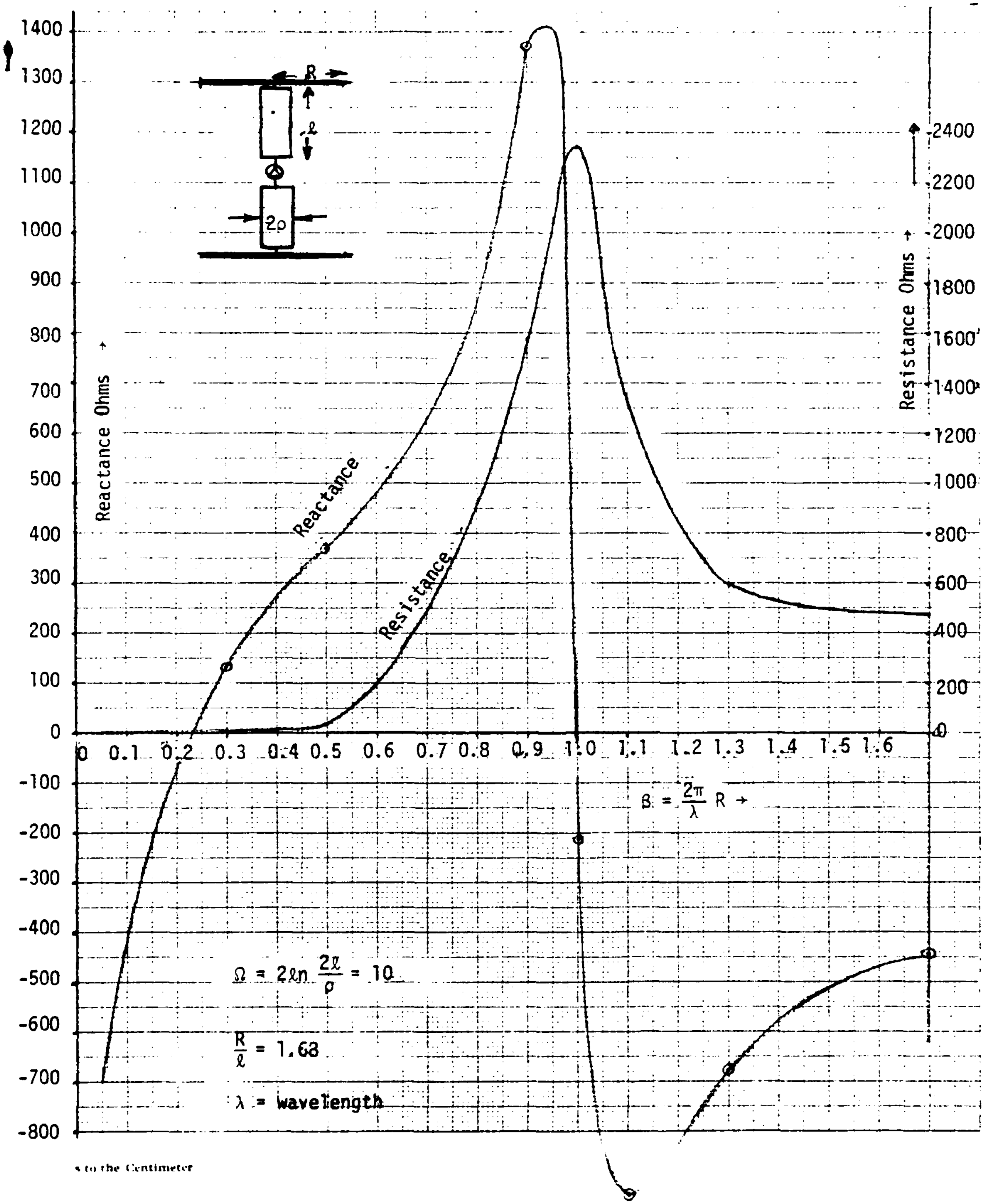

Figure 23 : Impedance of a dipole with top circular plates

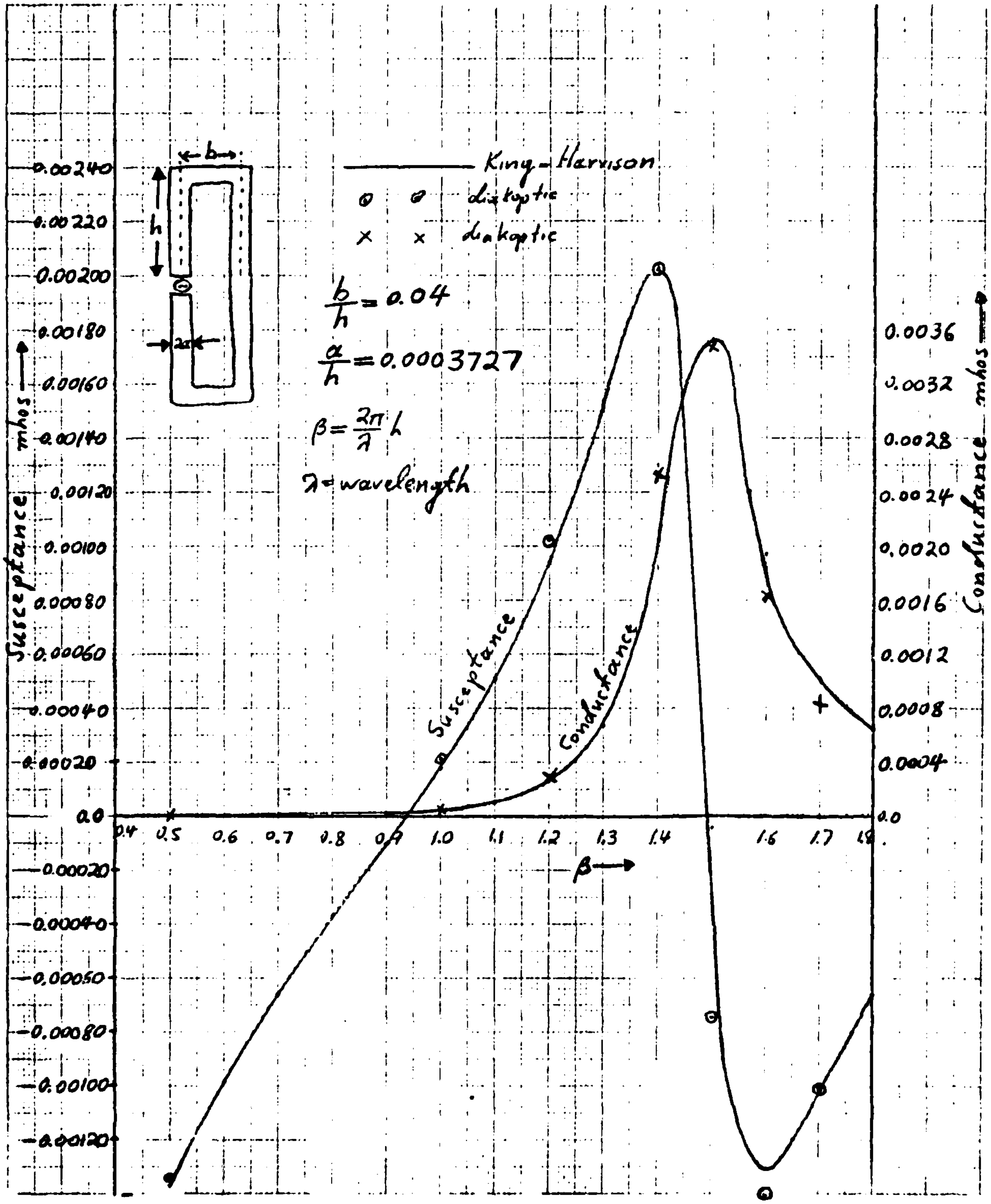

FIGURE 24 Comparison of Folded Dipole Admittance Calculated with Diakoptic Theory vs. King, Harrison.

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Conclusion

The major advantage of the diakoptic theory for multielement antennas, is that the problem of determining the current distribution on the antenna need not be solved for the structure as a whole, but only for the individual structure elements. Excitation of each structure element is ascribed to the currents at its junction with adjacent elements, and to the fields of the surface currents on all the other elements. The current distributions produced by the junction currents have been termed dominant current distributions, because they constitute the major portion of the currents on the composite antenna structure. The remainder of the currents are made up by scatter currents which are produced by field coupling. Field coupling, as a first approximation, is determined by the dominant current distributions, while coupling by the scatter currents in general is negligible. Introduction of impedances for the characterization of structure elements and their interaction permits utilization of network theory concepts for the determination of the junction currents and the input impedance of the antenna. Formulation of all impedances by stationary expressions renders the results insensitive to computational errors in the current distributions. As demonstrated by the example given in the paper, even rather crude approximations to the dominant current distributions can yield good results.

Appendix I

Equivalence between Current and Charge Excitation

Consider a structure element excited by an oscillating charge Q which is placed at a distance $d \to 0$ above the (plane) contact area σ(Fig. 4). The charge Q produces an electric potential field $-\bar{\nabla}\hat{\phi}_p$ which acts as the primary field for the excitation of the structure element. The induced current and charge distribution $\bar{i}, q$ radiates a Maxwell field which is characterized by the retarded potentials $\bar{A}$ and $\hat{\phi}$. The total field satisfies the boundary condition $\bar{E}_{tan} = 0$:

$$[j\omega\bar{A} + (\bar{\nabla}\hat{\phi} + \bar{\nabla}\hat{\phi}_p)] \times d\bar{S} = \bar{0} \text{ on } S \text{ and } \sigma \tag{A1.1}$$

Current and charge distribution satisfy the continuity condition

$$\bar{\nabla}\cdot\bar{i} + j\omega q = \bar{0} \qquad \text{on } S \text{ and } \sigma \tag{A1.2}$$

Let

$$i = i_S \text{ on } S,\ \bar{i} = \bar{i}_\sigma \text{ on } \sigma,\ q = q_S \text{ on } S,\ q = q_\sigma \text{ on } \sigma \tag{A1.3}$$

$$\bar{A} = \bar{A}_S + \bar{A}_\sigma \qquad \hat{\phi} = \hat{\phi}_S + \hat{\phi}_\sigma \tag{A1.4}$$

where $\bar{A}_S$ and $\hat{\phi}_S$ refer to the current and charge distribution on S, and $\bar{A}_\sigma$ and $\hat{\phi}_\sigma$ to the current and charge distribution on σ. Since $\sigma \ll S$, the contribution of $\bar{A}_\sigma$ to the total vector potential $\bar{A}$ can be neglected. The charge distribution q_σ consists essentially of the counter-charge to Q:

$$\int_\sigma q_\sigma \, d\sigma = -Q \tag{A1.5}$$

There is a small additional induced charge on σ which is a continuation of the charge distribution on S into the contact area. This charge can be neglected since $\sigma \ll S$.

When d approaches zero, the potential field of the oscillating charge Q is compensated by that of the counter-charge:

$$\text{Thus } \hat{\phi}_p + \hat{\phi}_\sigma = 0 \text{ , } \bar{\nabla}\hat{\phi}_p \approx \bar{\nabla}\hat{\phi}_S \tag{A1.6}$$

This means, the entire field is practically only determined by the current and charge distribution on S which satisfies the boundary condition

$$(j\omega\bar{A}_S + \bar{\nabla}\phi_S) \times d\bar{S} = \bar{0} \text{ on } S \tag{A1.7}$$

and the continuity condition

$$\bar{\nabla}\cdot\bar{I}_S + j\omega q_S = 0 \tag{A1.8}$$

Moreover, since the net charge on the structure element is zero, the charge on S is Q, and the current flux through the boundary Γ of the contact area is $j\omega Q$.

Thus, the current and charge distribution on S, produced by the external charge Q, are identical with the dominant current and charge distribution produced by an impressed current $I = j\omega Q$.

Since $j\omega Q$ is the displacement current which enters the structure element at the contact area it is obvious that excitation by an impressed displacement current is equivalent to excitation by an impressed conduction current.

Appendix 2

Derivation of Equation III.19

Consider a structure element with several terminals and let k and j be any two terminals where currents I_k and I_j are impressed. The corresponding dominant current and charge distributions $\bar{I}_k$, q_k and $\bar{I}_j$, q_j produce the fields $\bar{E}_k$ and $\bar{E}_j$ which satisfy the boundary conditions

$$\bar{E}_k \times d\bar{S} = -(j\omega\bar{A}_k + \bar{\nabla}\hat{\phi}_k) \times d\bar{S} = \bar{0} \qquad (A2.1)$$

$$\bar{E}_j \times d\bar{S} = -(j\omega\bar{A}_j + \bar{\nabla}\hat{\phi}_j) \times d\bar{S} = \bar{0} \qquad (A2.2)$$

Since the currents $\bar{I}_k$ and $\bar{I}_j$ are tangential to the surfaces, it follows from the boundary conditions

$$\int_S (j\omega\bar{A}_k + \bar{\nabla}\hat{\phi}_k) \cdot \bar{I}_k \, dS = 0 \qquad (A2.3)$$

$$\int_S (j\omega\bar{A}_k + \bar{\nabla}\hat{\phi}_k) \cdot \bar{I}_j \, dS = 0 \qquad (A2.4)$$

Using the vector identity

$$\bar{\nabla} \cdot (\hat{\phi}\bar{I}) = \bar{\nabla}\hat{\phi} \cdot \bar{I} + \hat{\phi}(\bar{\nabla}\cdot\bar{I}) \quad \text{with } \bar{\nabla} \cdot \bar{I} = -j\omega q \qquad (A2.5)$$

and applying Gauss' theorem as in (8) of Section II, equations (A2.3) and (A2.4) can be written in the form

$$-\int_S [\bar{\nabla} \cdot (\hat{\phi}_k\bar{I}_k)] \, dS = \hat{\phi}_{kk} I_k = j\omega\int_S (\bar{A}_k \cdot \bar{I}_k + \hat{\phi}_k q_k) dS \qquad (A2.6)$$

$$- \int_S [\bar{\nabla} \cdot (\hat{\phi}_k \bar{I}_j)] \, dS = \hat{\Phi}_{jk} I_j = j\omega \int_S (\bar{A}_k \cdot \bar{I}_j + \hat{\phi}_k q_j) dS \qquad \text{(A2.7)}$$

where $\hat{\Phi}_{kk}$ is the potential at the terminal k due to I_k, and $\hat{\Phi}_{jk}$ that at the terminal j due to I_k. For j = k, (A2.7) transforms into (A2.6).

With

$$\hat{\Phi}_{jk} = Z_{jk} I_k \qquad \text{(A2.8)}$$

one obtains from (A2.7) the expression for Z_{jk} given in the first line of III.19.

The formulation in the second line of III.19 is obtained if the potentials $\bar{A}_k$ and $\hat{\phi}_k$ are expressed by III.1 and III.12 respectively.

Appendix 3

Derivation of Equation III.23

Let $\bar{i}_k^i$ be a dominant current distribution on the surface S^i and δ_k^{in} be the scatter current distribution on S^n produced by $\bar{i}_k^i$ $(n = 1, 2, \ldots, N)$.

$$\delta\bar{A}_k^i(\bar{r}) = \frac{\mu}{4\pi} \sum_{n=1}^{N} \int_{S^n} \delta\bar{i}_k^{in}(\bar{r}')G(\bar{r},\bar{r}')dS^n(\bar{r}') \tag{A3.1}$$

Multiplying (A3.1) with $\bar{i}_k^i(\bar{r})$ and integrating over S^i

$$\int_{S^i} \delta A_k^i(\bar{r}) \cdot \bar{i}_k^i(\bar{r})dS^i = \frac{\mu}{4\pi} \int_{S^i} \bar{i}_k^i(\bar{r}) \cdot \sum_{n=1}^{N} \int_{S^n} \delta\bar{i}_k^{in}(\bar{r}')G(\bar{r},\bar{r}')dS^n(\bar{r}')dS^i(\bar{r})$$

$$= \frac{\mu}{4\pi} \sum_{n=1}^{N} \int_{S^n} \delta\bar{i}_k^{in}(\bar{r}') \cdot \int_{S^i} \bar{i}_k^i(\bar{r})G(\bar{r},\bar{r}')dS^i(\bar{r})dS^n(\bar{r}')$$

$$= \sum_{n=1}^{N} \int_{S^n} \delta\bar{i}_k^{in}(\bar{r}) \cdot \bar{A}_k^i(\bar{r})dS^n, \quad (G(\bar{r},\bar{r}') = G(\bar{r}',\bar{r})) \tag{A3.2}$$

Similarily it can be shown that

$$\int_{S^i} \delta\hat{\phi}_k^i \, q_k^i \, dS^i = \sum_{n=1}^{N} \int_{S^n} \delta q_k^{in} \, \hat{\phi}_k^i \, dS^n \tag{A3.3}$$

The proof for III.31 in the body of the paper follows the same outline given above.

Appendix 4

Proof for the Stationary Formulation of the Impedances

a) To prove that

$$Z = \frac{j\omega}{I^2}\int_S (\bar{A}\cdot\bar{I} + \hat{\phi}q)dS \qquad \text{(A4.1)}$$

represents a stationary formulation of the intrinsic impedance, we assume that the dominant current distribution $\bar{I}$ has an error of $\Delta\bar{I}$. The corresponding errors of q, $\bar{A}$ and $\hat{\phi}$ shall be denoted Δq, $\Delta\bar{A}$ and $\Delta\hat{\phi}$. Then

$$Z + \Delta Z = \frac{j\omega}{I^2}\int_S [(\bar{A} + \Delta\bar{A})\cdot(\bar{I} + \Delta\bar{I}) + (\hat{\phi} + \Delta\hat{\phi})(q + \Delta q)]dS \qquad \text{(A4.2)}$$

The boundary condition for the correct dominant current distribution yields

$$\int_S (j\omega\bar{A} + \bar{\nabla}\hat{\phi}) \cdot \Delta\bar{I}\, dS = 0 \qquad \text{(A4.3)}$$

Since the dominant current distribution is the continuation of the impressed current which is assumed to be unchanged, $\Delta\bar{I}$ is zero at the terminal, and (A4.3) can be written in the form

$$j\omega\int_S (\bar{A} \cdot \Delta\bar{I} + \hat{\phi}\Delta q)dS = 0 \qquad \text{(A4.4)}$$

Using the relations

$$\int_S \bar{A} \cdot \Delta\bar{I}\, dS = \int_S \Delta\bar{A} \cdot \bar{I}\, dS; \int_S \hat{\phi}\Delta q\, dS = \int_S \Delta\hat{\phi} q\, dS \qquad \text{(A4.5)}$$

along with (A4.4), one obtains

$$\int_S (\bar{A} \cdot \Delta\bar{I} + \hat{\phi}\Delta q)dS = \int_S (\Delta\bar{A} \cdot \bar{I} + \Delta\hat{\phi}q)dS \tag{A4.6}$$

Thus, from (A4.1), (A4.3), and (A4.6)

$$\Delta Z = \frac{j\omega}{I^2} \int_S (\Delta\bar{A} \cdot \Delta\bar{I} + \Delta\hat{\phi}\Delta q)dS \tag{A4.7}$$

This means, ΔZ is of second order.

b) In the case of a mutual intrinsic impedance

$$Z_{jk} = \frac{j\omega}{I_k I_j} \int_S (\bar{A}_k \cdot \bar{I}_j + \hat{\phi}_k q_j)dS \tag{A4.8}$$

both the dominant current distributions $\bar{I}_k$ and $\bar{I}_j$ may have errors $\Delta\bar{I}_k$ and $\Delta\bar{I}_j$. Thus

$$\Delta Z_{jk} = \frac{j\omega}{I_k I_j} \int_S [(\Delta\bar{A}_k \cdot \bar{I}_j + \Delta\hat{\phi}_k q_j) + (\bar{A}_k \cdot \Delta\bar{I}_j + \hat{\phi}_k q_j) + (\Delta\bar{A}_k \cdot \Delta\bar{I}_j + \Delta\hat{\phi}_k \Delta q_j)]dS \tag{A4.9}$$

Because the correct dominant current distributions satisfy the boundary condition $\bar{E} \times d\bar{S} = \bar{0}$,

$$\int_S (j\omega\bar{A}_k + \bar{\nabla}\hat{\phi}_k) \cdot \Delta\bar{I}_j dS = 0$$

or

$$j\omega\int_S (\bar{A}_k \cdot \Delta\bar{I}_j + \hat{\phi}_k \Delta q_j)dS = 0 \tag{A4.10}$$

Furthermore

$$\int_S (\Delta\bar{A}_k \cdot \bar{I}_j + \Delta\hat{\phi}_k q_j)dS = \int_S (\bar{A}_j \cdot \Delta\bar{I}_k + \hat{\phi}_j \Delta q_k)dS = 0 \tag{A4.11}$$

From (A4.10) and (A4.11), (A4.9) reduces to

$$\Delta Z_{jk} = \frac{j}{I_k I_j} \int_S (\Delta\bar{A}_k \cdot \Delta\bar{I}_j + \Delta\hat{\phi}_k \Delta q_j)dS \tag{A4.12}$$

Thus ΔZ_{jk} is of second order.

c) To prove that III.33 is a stationary expression for the field coupling impedances we treat the assembly of disconnected structure elements like a single body. This means, when a current is impressed on terminal $\binom{i}{k}$ we consider the dominant current distribution $\bar{I}_k^i$ together with the associated scatter currents $\delta\bar{I}_k^i$ which are distributed over all the elements as a dominant current distribution of the system. The coupling impedances between any two terminals can then be formulated like mutual intrinsic impedances (A4.8):

$$Z(F)_{k,m}^{i,\ell} = \frac{j\omega}{I_k^i I_m^\ell} \int_{\sum S^n} [(\bar{A}_m^\ell + \delta\bar{A}_m^\ell)\cdot(\bar{I}_k^i + \delta\bar{I}_k^i) + (\hat{\phi}_m^\ell + \delta\hat{\phi}_m^\ell)(q_m^\ell + \delta q_m^\ell)]dS \tag{A4.13}$$

The error $\Delta Z_{km}^{i\ell}$ produced by errors in the current distributions $\bar{I}_k^i$, $\delta\bar{I}_k^i$ and $\bar{I}_m^\ell$, $\delta\bar{I}_m^\ell$ is obtained from (A4.12):

$$\Delta Z(F)_{k,m}^{i,\ell} = \frac{j\omega}{I_k^i I_m^\ell} \int_{\sum S^n} [(\Delta\bar{A}_k^i + \Delta\delta\bar{A}_k^i)\cdot(\Delta I_m^\ell + \Delta\delta\bar{I}_m^\ell) + (\Delta\hat{\phi}_k^i + \delta\hat{\phi}_k^i)(\Delta q_m^\ell + \Delta\delta q_m^\ell)]dS \tag{A4.14}$$

and is of second order. This relation can also be derived from III.33 but only in a rather cumbersome manner.

Appendix 5

If a current is impressed on any terminal of a diakopted structure there will be capacitive currents between the contact areas of the disconnected elements, which have not been considered in the derivation of the field coupling impedances. One might therefore conclude that the formulas are approximations which require the gaps between adjacent contact areas to be so large that capacitive currents are negligible. The purpose of this appendix is to show that the expressions for $Z(F)^{i,i}_{k,k}$ and $Z(F)^{i,\ell}_{k,m}$ are correct even if the gaps are infinitely small.

Figure 25 shows two structure elements, a clylindrical rod 1 and a disc 2 with the opposing contact areas σ^1_2 and σ^2_1. If a current is impressed on the terminal $(^1_1)$ of the rod, there will be a potential difference between σ^1_2 and σ^2_1 which, in turn, produces a displacement current between these terminals. The potential difference which is the line integral of the electric potential field between σ^1_2 and σ^2_1 is essentially determined by the charges on the contact areas. If the gap is made smaller and smaller, the potential difference approaches zero, and the total current distribution becomes the dominant current distribution of the interconnected elements. As shown in Appendix 1 displacement currents at contact areas are equivalent to impressed currents. Thus, the situation discussed above is the excitation of a diakopted structure not by one, but by three impressed currents. To produce excitation by one impressed current in accordance with our theory the displacement currents must be compensated so that there is no current flux from the contact area onto the surface S of the element (S, by definition does not contain the contact areas of the element). The magnitude of these compensating currents does not enter into

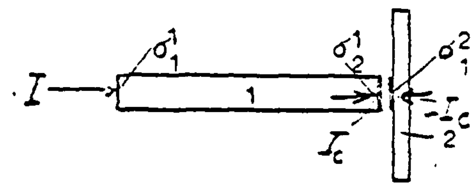

FIGURE 25

Compensation of Capacitive Currents at Contact Areas the analysis because, if the impressed currents of the diakopted structure are identical with the junction currents of the interconnected structure there are no displacement currents between adjacent contact areas and the sum of all the compensating currents is zero.

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